Cosmology

TUM WS 2019/2020

Lecture 9

Wolfgang Hillebrandt and Bruno Leibundgut

(http://www.eso.org/~bleibund/Cosmology)

1

Dates of Oral Exams

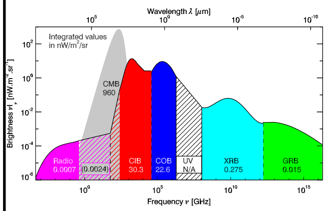
- After February 28

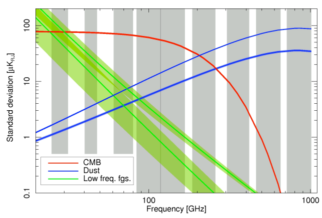
Contact: wfh@mpa-garching.mpg.de
or
bleibundgut@eso.org

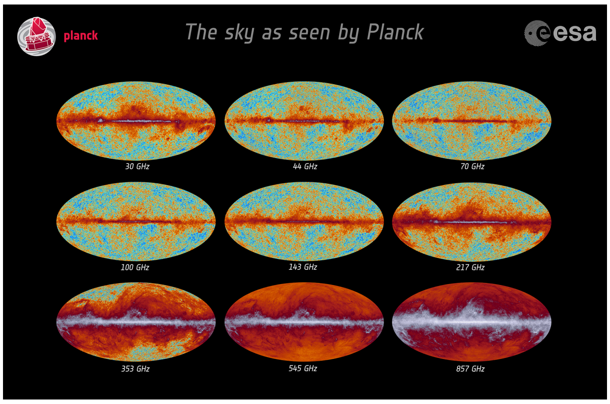
2

Foregrounds

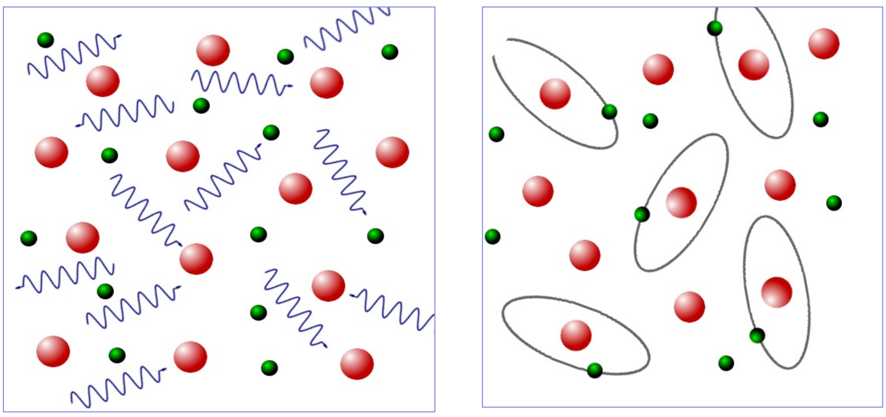
CMB signal is hidden behind Galactic foregrounds







3



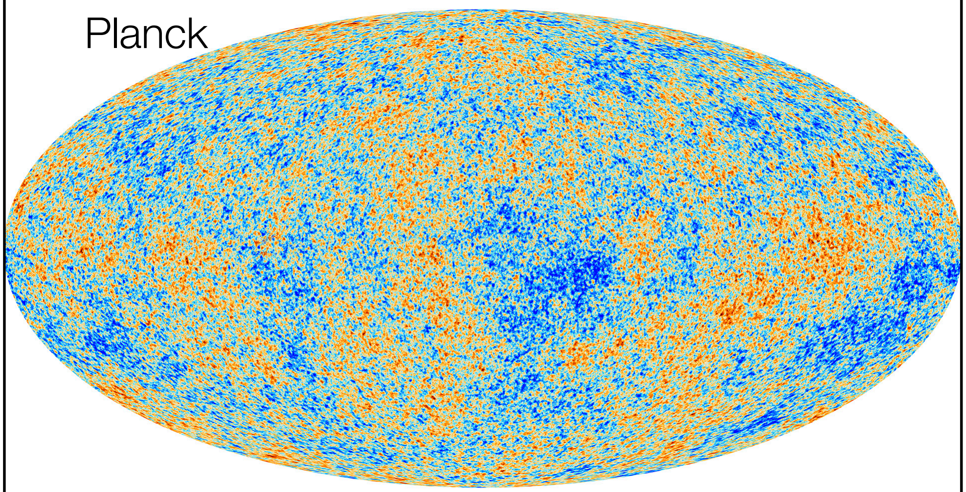
Before recombination: *The Universe is opaque*
After recombination: *The Universe is transparent*

Transition ~ 300 000 years after the Big Bang

4

Uncovering the CMB

Planck



Recombination and last scattering

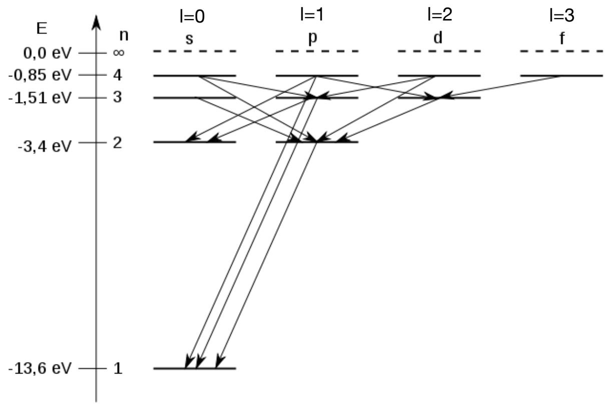
- The free electrons are removed when they form atoms with the nuclei formed in the Big Bang
- Consider the Maxwell-Boltzmann density distribution

$$n_i = \frac{g_i}{(2\pi\hbar)^3} \mathrm{e}^{\frac{\mu_i}{k_B T}} \int e^{-\frac{\left(m_i + \frac{p^2}{2m_i}\right)}{k_B T}} d^3 p$$

- Particles of interest are photons, electrons, baryons,hydrogen atoms at different excitation states (1s, 2d, 3p, etc.)

Ionisation structure of H

- Grotrian diagram of Hydrogen



Recombination

- Relevant factors
  - fermions → half spins: $g_p = g_e = 2$
- Hydrogen 1s ground state has four hyperfine states (↑↑,↓↓,↓↑,↑↓): $g_{1s} = 4$
- In ionization equlibrium the chemical potential is the sum of the individual

$$\mu_{1s} = \mu_p + \mu_e$$

- The integrals work out to be

$$\frac{1}{(2\pi\hbar)^3}\int d^3p\, e^{-\left(\frac{p^2}{2mk_BT}\right)} = \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}$$

Recombination

Because of $\mu_{1s} = \mu_p + \mu_e$ the ratio of the number densities becomes

$$\frac{n_{1s}}{n_e n_p} = \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-\frac{3}{2}} e^{\frac{B_1}{k_B T}}$$

with $B_1 = m_p + m_e - m_H = 13.6\ eV$ the energy of the ground state of Hydrogen.

Since the universe is charge neutral, we have the same amount of electrons and protons $n_e = n_p$.

Recombination

- It is also okay to assume that the hydrogen is in its ground state
  - the number density of excited states scales with $e^{-\frac{\Delta E}{k_B T}}$ and the excitation from $n = 1$ to $n = 2$ is $10.6\ eV$. Hence for $T < 4200\ K$ the exponential is $< 6 \cdot 10^{-13}$
- Assume 24% in Helium we find

$$n_p + n_{1s} = 0.76 n_B$$

with n_B the baryon density

Recombination

The hydrogen ionisation fraction is $X \equiv \frac{n_p}{n_p + n_{1s}}$ then described by the Saha equation

$$X\left(1 + \frac{(n_p + n_{1s})n_{1s}}{n_p^2} X\right) = X\left(1 + 0.76 n_B X \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-\frac{3}{2}} e^{\frac{B_1}{k_B T}}\right) = 1$$

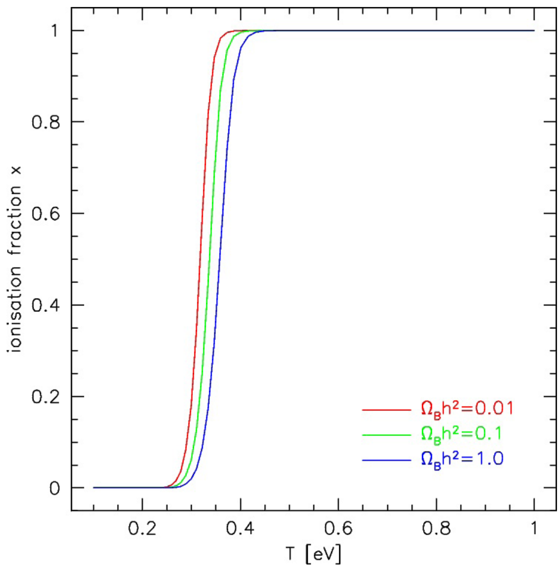
The second term describes the ionisation state as a function of temperature and with $n_B = n_{B0}\left(\frac{T}{T_{\gamma 0}}\right)^3$ the transition can be written as a function of baryon density.
(See next table)

Recombination

Change of H ionisation as a function of temperature (and baryon density)

T(K)	$\Omega_B h^2 = 0.01$	$\Omega_B h^2 = 0.02$	$\Omega_B h^2 = 0.03$
4500	0.999	0.998	0.997
4200	0.990	0.981	0.971
4000	0.945	0.900	0.863
3800	0.747	0.634	0.565
3600	0.383	0.290	0.244
3400	0.131	0.094	0.078
3200	0.0337	0.0240	0.0196
3000	0.00693	0.00491	0.00401
2800	0.00112	0.00079	0.00065
2.725	$2.8\ 10^{-12571}$	$2.0\ 10^{-12571}$	$1.6\ 10^{-12571}$

T_recombination



TUM WS19/20 Cosmology 9 Wolfgang Hillebrandt and Bruno Leibundgut 13

13

Recombination – a complication

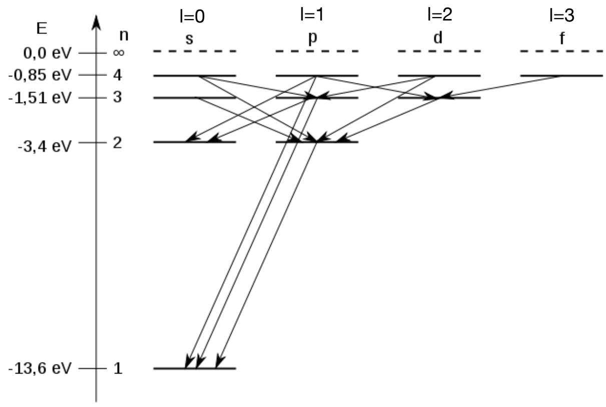
Recombination rate in hydrogen depends on the density. The transition from 2p to 1s creates a photon (Lyman α for astronomers), which can immediately excite another hydrogen atom to the 2p state. This means that hydrogen gets trapped in an ionised state and these need to be taken into account.

TUM WS19/20 Cosmology 9 Wolfgang Hillebrandt and Bruno Leibundgut 14

14

Ionisation structure of H

- Grotrian diagram of Hydrogen



Recombination – a complication

Conditions to be considered

- rapid transitions, which means thermal equilibrium, except for the 1s ground state, which is slow. Number density is

$$n_{nl} = (2l+1)n_{2s}e^{\frac{B_2-B_1}{k_BT}}$$

- net rate of population of 1s state is given by the rate of radiative decays from 2s and 2p states minus the rate of excitation from the 1s state
- radiative processes much faster than the reduction in density due to the expansion

Recombination

- These effects change the ionisation rates and hence modify the Saha equation
- Exact calculation tedious involving the details of the radiative decays and excitations
  - e.g. 2-photon decay from 2s to 1s state, which is strongly suppressed
- Following table is the result of a more exact calculation

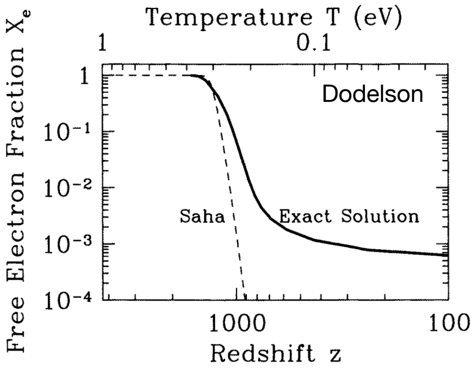
Recombination

Due to the atomic transitions in hydrogen the suppression of the Lyman α photons leads to modification of the ionisation fraction

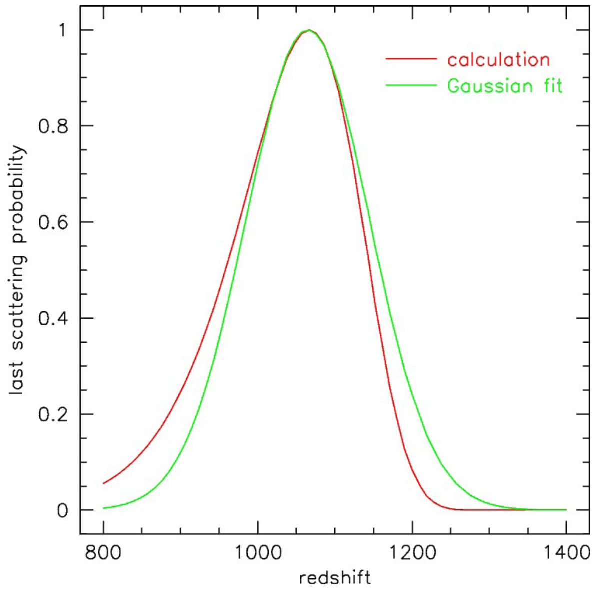
z	T(K)	T(yrs)	$X_{\Omega_B h^2=0.01}$	$X_{\Omega_B h^2=0.02}$	$X_{\Omega_B h^2=0.03}$
1550	4226	202600	0.992	0.984	0.982
1500	4090	213200	0.976	0.958	0.954
1450	3954	225900	0.935	0.902	0.878
1400	3818	239800	0.861	0.815	0.780
1350	3681	255200	0.759	0.703	0.659
1300	3545	272000	0.645	0.580	0.529
1250	3409	290600	0.526	0.456	0.402
1200	3273	311300	0.409	0.339	0.289
1150	3136	334600	0.299	0.236	0.194
1100	3000	360400	0.205	0.154	0.122
1050	2864	389600	0.129	0.0928	0.0721
1000	2728	422600	0.0752	0.0520	0.0396
950	2591	460500	0.0405	0.0270	0.0203
900	2455	503600	0.0210	0.0136	0.0101
800	2183	611400	0.00662	0.00387	0.00276
700	1910	761300	0.00319	0.00174	0.00120
600	1638	977700	0.00203	0.00107	0.000731
500	1365	$1.312\ 10^6$	0.00147	0.000762	0.000517
250	684	$3.922\ 10^6$	0.000829	0.000423	0.000285
100	275	$1.604\ 10^7$	0.000632	0.000321	0.000216
50	139	$4.535\ 10^7$	0.000579	0.000294	0.000197
10	30	$4.568\ 10^7$	0.000537	0.000272	0.000183

Recombination

The effect is that at low redshifts the fraction of free electrons is approaching a constant level

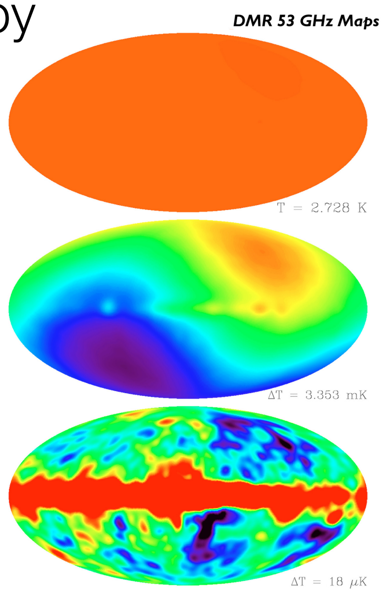


Recombination 'shell'



Anisotropy

- CMB extremely uniform
- only at 3.35mK is there a dipole visible



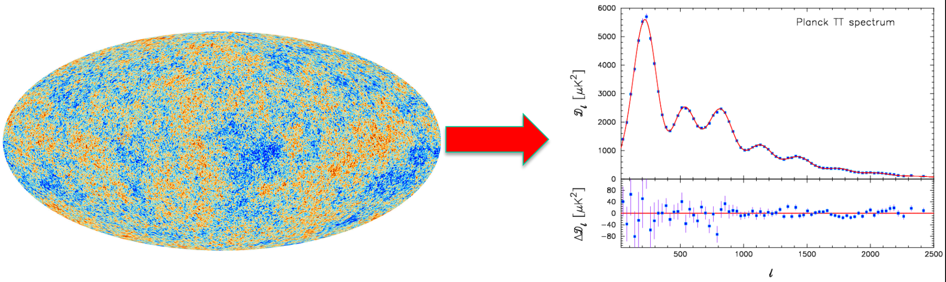
Dipole anisotropy

- Due to Earth's motion relative to the CMB a dipole is induced
  - blueshift in direction of motion
  - redshift in opposite direction
- Measured dipole is $\Delta T = 3.346 \pm 0.017\ mK$ as measured by the Wilkinson Microwave Anisotropy Probe (WMAP)
- Velocity is then

$$v = \frac{\Delta T}{T} c = \frac{0.00335}{2.725} \cdot 3 \cdot 10^5 km\ s^{-1} \approx 370\ km\ s^{-1}$$

Challenge

- How to we get from the temperature maps to the power spectrum?



Primary fluctuations in the microwave background

- Projection of small perturbations onto a sphere
  - ➔use spherical harmonics
  - ➔Small perturbations in the gravitational potential
  - ➔deviations from the smooth background
  - ➔local mass densities
- Use power spectra to look at the relevant scales
  - ➔work in Fourier space

Spherical Harmonics

- Describe the temperature distribution as an expansion in spherical harmonics

$$\Delta T(\hat{n}) \equiv T(\hat{n}) - T_0 = \sum_{lm} a_{lm} Y^m{}_l(\hat{n})$$

with $$T_0 \equiv \frac{1}{4\pi}\int d^2\hat{n}\, T(\hat{n})$$

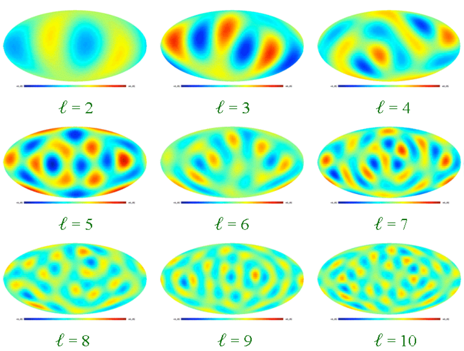
the average temperature over the whole sky
($\hat{n}$ is the unit vector in a given direction)

- The sums run over integer $l>0$ and m from $-l<m<l$

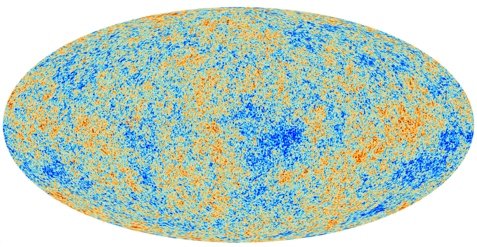
Spherical Harmonics

- Map the sky by looking at the modes in spherical harmonics

First 10 modes in the WMAP data



Planck observations



Averages

- The important parameters are the averages (and not the individual values).
- Assuming rotational invariance yields

$$\left\langle a_{lm} a^{*}_{l'm'} \right\rangle = \delta_{ll'} \delta_{mm'} C_l$$

- The product of temperature differences is

$$\left\langle \Delta T(\hat{n}) \Delta T(\hat{n}') \right\rangle = \sum_{lm} C_l Y_l^m(\hat{n}) Y_l^{-m}(\hat{n}') = \sum_l C_l \left(\frac{2l+1}{4\pi} \right) P_l(\hat{n} \cdot \hat{n}')$$

- P_l are the Legendre polynomials
- Remarkably the second part depends only on l and no longer on m

Averages

- The C_l are found by inverting the Legendre transformation

$$C_l = \frac{1}{4\pi} \int d^2\hat{n} d^2\hat{n}' P_l(\hat{n} \cdot \hat{n}') \left\langle \Delta T(\hat{n}) \Delta T(\hat{n}') \right\rangle$$

- The observations yield

$$C_l^{obs} = \frac{1}{2l+1} \sum_m a_{lm} a_{l-m} = \frac{1}{4\pi} \int d^2\hat{n} d^2\hat{n}' P_l(\hat{n} \cdot \hat{n}) \Delta T(\hat{n}) \Delta T(\hat{n}')$$

- Subtle difference → no averaging in the observed C_l^{obs}
- Difference is known as *cosmic variance*

Cosmic Variance

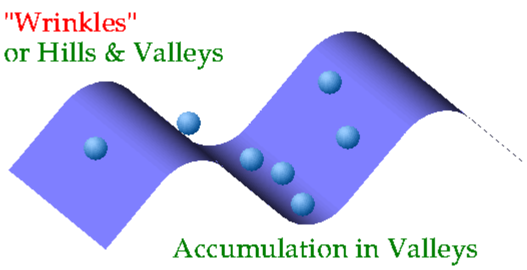
- The cosmic variance is the difference between the true (averaged) fluctuations and the observed ones

$$\left\langle \left(\frac{C_l - C_l^{obs}}{C_l} \right)^2 \right\rangle = \frac{2}{2l+1}$$

- For small l there remains an unresolved discrepancy ("only one universe observable") while the average decreases rapidly for $l > 5$

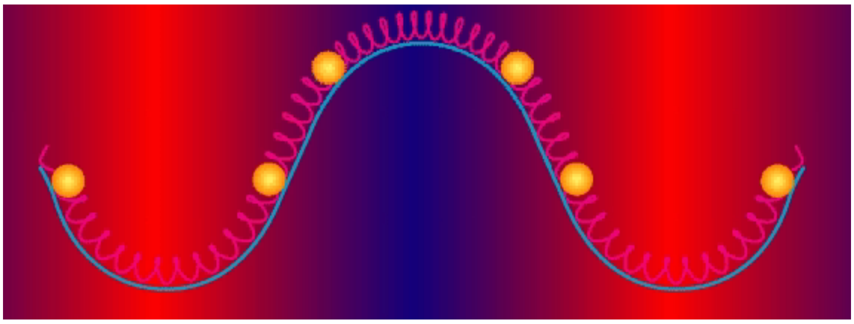
Where do the CMB fluctuations come from ?

- Wrinkles: some regions have a slightly higher gravity, some a slightly lower ("potential wells")
- Matter falls into potential wells



Can we "see" the "sound" of the universe ?

- Compressed gas heats up



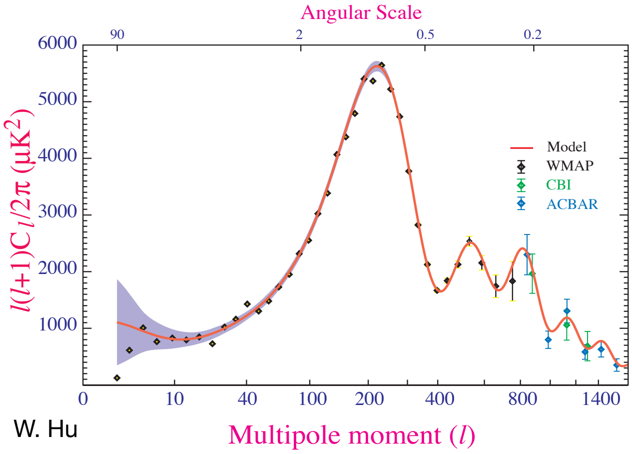
⇒ temperature fluctuations

Physical reasons for the temperature fluctuations

1. Intrinsic temperature fluctuations the electron-baryon-photon plasma at last scattering ($z \approx 1090$)
2. Doppler effect due to velocity fluctuations
3. Gravitational red- or blueshift due to fluctuations in the gravitational potential at last scattering: *Sachs-Wolfe effect*
4. Gravitational red- or blueshift due to time-dependent fluctuations in the gravitational potential between last scattering and now → *Integrated Sachs-Wolfe effect*

CMB Power Spectrum

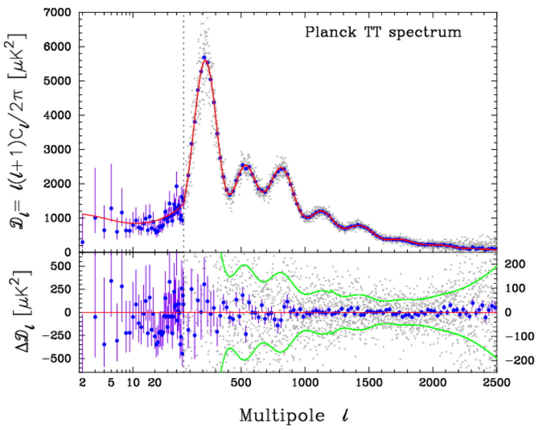
- Observed CMB power spectrum shows various features



W. Hu

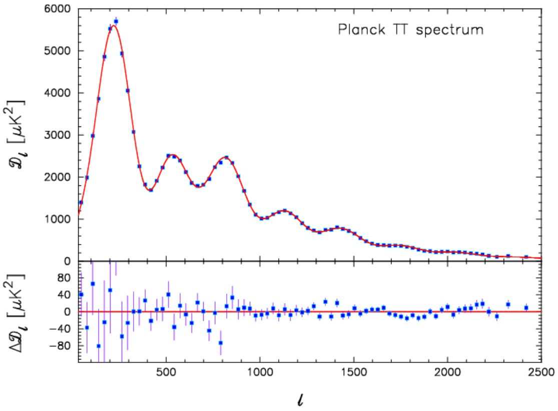
The latest CMB temperature power spectrum

- Planck collaboration (2013)



CMB cleaned from foregrounds

- After removing the foreground radiation and adding the small scales from other experiments (ACT, SPT)



A historical note

- All-sky surveys were extremely important to achieve sufficient signal on many l's for the angular power spectrum to be measured.
- First achieved by WMAP