Cosmology

TUM WS 2019/2020

Lecture 8

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(http://www.eso.org/~bleibund/Cosmology)

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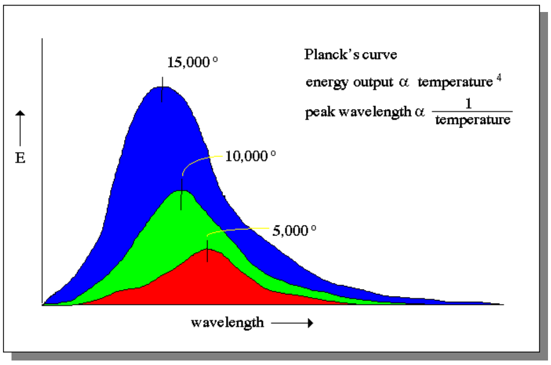
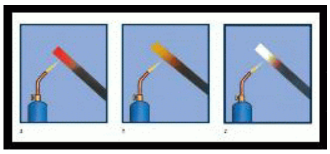
Black body radiation





- A hot body is brighter than a cool one ($L \propto T^4$, Stefan-Boltzmann's law)
- A hot body's spectrum is bluer than that of a cool one ($\lambda_{max} \propto 1/T$, Wien's law)

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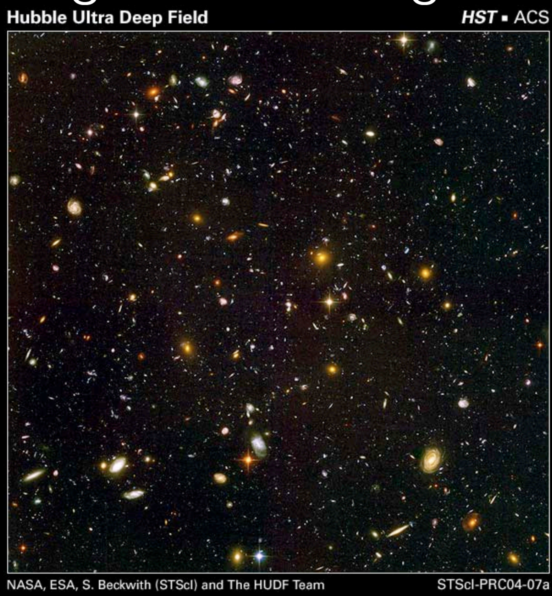
Extragalactic Backgrounds



Extragalactic backgrounds

- Pointing at an ‘empty’ place on the sky one observes “background” light
  - origins:
    - radio sources
    - CMB
    - Cosmic IR background
    - Optical
    - UV
    - X-ray
    - γ-ray

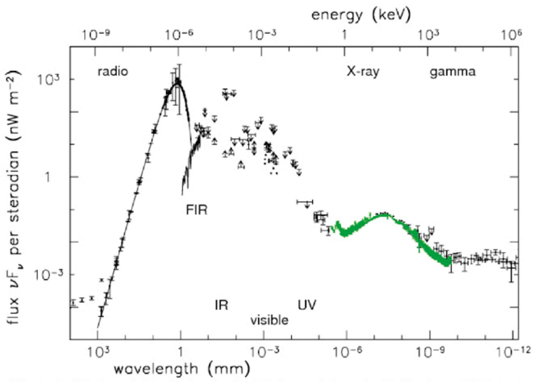
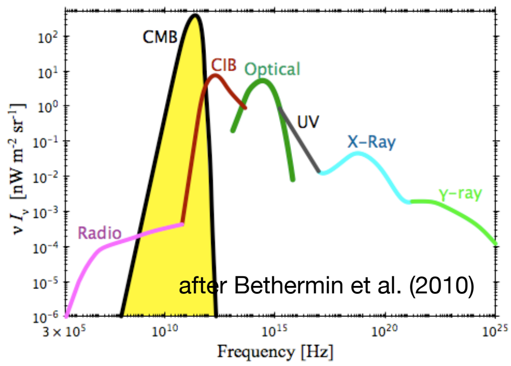


Fig 1.19 (D. Scott) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Extragalactic backgrounds

- CMB is the highest intensity
- X-rays and γ-rays are unresolved sources → AGN
- CIB is coming from dust
- Optical/UV: unresolved galaxies and stars



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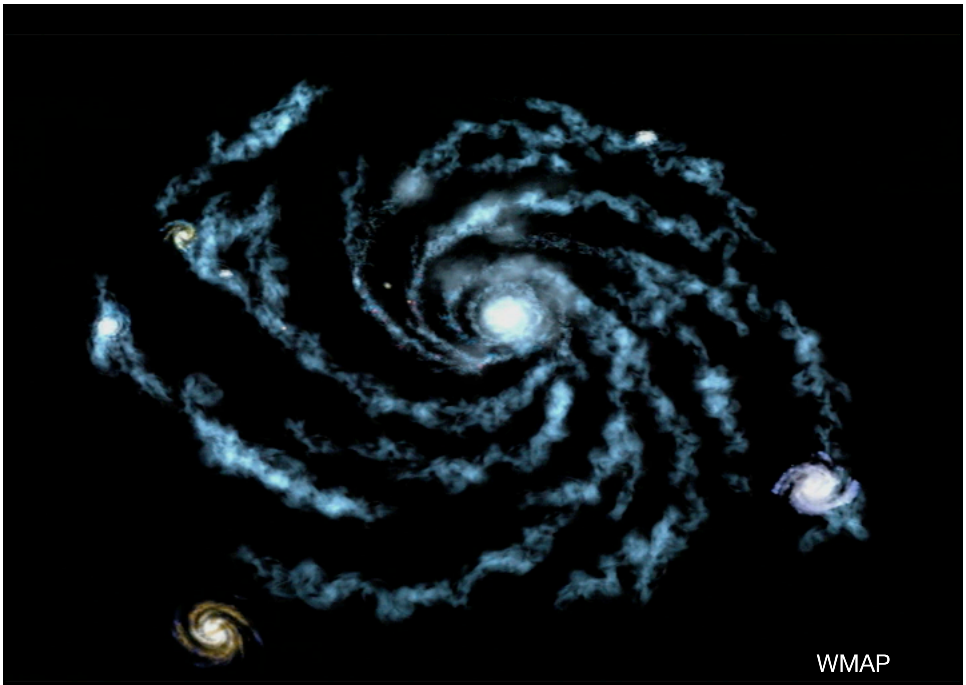

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Cosmic Microwave Background

- Relic radiation from the Big Bang
- Direct observation of the early universe
- Fantastic source of cosmological information
- The number density of photons in the black body radiation

$$n_T(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{e^{\frac{h\nu}{k_b T}} - 1}$$

→ Several 100 photons/cm^3 today!

Invariance of black body

What happens to the black body radiation when the photons decouple?

- Consider the change in frequency

$$\nu(t) = \nu \frac{a(t)}{a(t_L)}$$

the new number density then becomes

$$n(\nu,t)d\nu = \left(\frac{a(t_L)}{a(t)}\right)^3 n_{T(t_L)}\left(\nu \frac{a(t)}{a(t_L)}\right) d\left(\nu \frac{a(t)}{a(t_L)}\right)$$

(the cubic factor is the increase in volume)

Invariance of black body

This leads to

$$n(\nu,t)d\nu = \frac{8\pi\nu^2 d\nu}{e^{\frac{h\nu}{k_B T}} - 1} = n_{T(t)}(\nu)d\nu$$

with

$$T(t) = T(t_L)\frac{a(t_L)}{a(t)}$$

The photon number density maintains the black body distribution and the temperature simply decrease with $(1+z)$

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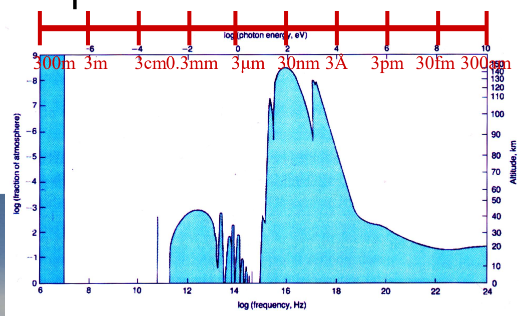
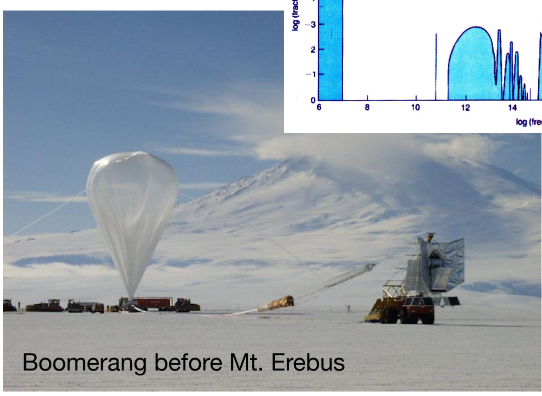
Balloon experiments

Move above the atmosphere




Boomerang before Mt. Erebus

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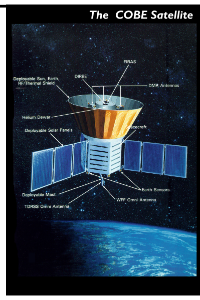
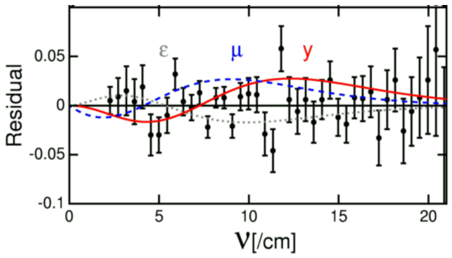
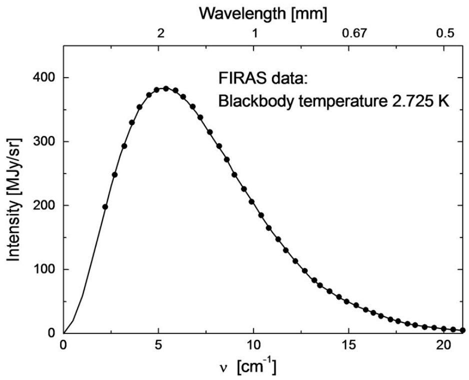
Black body



Blackbody radiation measured to exquisite accuracy with COBE





Cobe (1996)

Energy density

- Energy density is the integral of the photon energy times the number density

$$\int_0^\infty h\nu\, n(\nu) d\nu = \sigma_B T^4$$

with $\sigma_B = \frac{8\pi^5 k_B^4}{15 h^3 c^3} = 7.56577 \cdot 10^{-15}$ erg cm^3 deg^{-4}

- The energy density today is then

$$\sigma_{\gamma 0} = \sigma_B T_{\gamma 0}^4 = 4.64 \cdot 10^{-34} \text{ g cm}^{-3}$$

- compared to the critical density this is

$$\Omega_\gamma \equiv \frac{\rho_{\gamma 0}}{\rho_{crit,0}} = 2.47 \cdot 10^{-5}\, h^{-2}$$

$(\rho_{crit,0} \approx 10^{-29}\, g\, cm^{-3}; H_0 = 70\, km\, s^{-1}\, Mpc^{-1})$

Radiation density

- If we want to account for all relativistic particles, we need to include the neutrinos and the radiation density becomes
$$\rho_{R0} = \left[1 + 3\left(\frac{7}{8}\right)\left(\frac{4}{11}\right)^{\frac{4}{3}}\right]\rho_{\gamma 0} = 7.80 \cdot 10^{-34}\ g\ cm^{-3}$$
and the total radiation density
$$\Omega_R \equiv \frac{\rho_{R0}}{\rho_{crit,0}} 4.15 \cdot 10^{-5}\ h^2$$
- This is the reason that Ω_R is often neglected for the distance calculations today

Photon number

The photon number is the integral over the density
$$n_{\gamma 0} = \int_0^\infty \frac{8\pi\nu^2 d\nu}{e^{\frac{h\nu}{k_B T}} - 1} = \frac{30\zeta(3)}{\pi^4}\frac{\sigma_B T^3}{k_B} = 0.3702\frac{\sigma_B T^3}{k_B} = 20.28 \cdot T^3\ \ cm^{-3}$$
with the ζ function from the integral (used: $\zeta(3) = 1.202057$).

- With the CMB temperature today ($T = 2.725K$) this becomes
$$n_{\gamma 0} \approx 410\, photons\ cm^{-3}$$

Photon number

Compare this to the number of nucleons

$$n_{B0} = \frac{3\Omega_B H_0^2}{8\pi G m_N} = 1.123 \cdot 10^{-5} \Omega_B h^2 \; nucleons \; cm^{-3}$$

Both number densities vary with time as $a^{-3}(t)$ and hence this ratio has not changed since the photons decoupled.

Temperature of Equality

The photon energy density is proportional to

$$\rho_R \propto a^{-4},$$

while the matter density scales with the volume ($\rho_M \propto a^{-3}$).

- Hence the ratio $\frac{\rho_R}{\rho_M} \propto a^{-1} \propto T$
- The temperature when the two energy densities were equal is then

$$T_{EQ} = T_{\gamma 0} \frac{\Omega_M}{\Omega_R} = 6.56 \cdot 10^4 \; K \times \Omega_M h^2$$

Equilibrium between photons and electrons

- Interaction on free electrons is through Thomson scattering $\Lambda_\gamma = \sigma_T n_e c$ with $\sigma_T = 6.66525 \cdot 10^{-25}$ cm^{-2} the Thomson scattering cross section
- Temperature still high enough that all atoms are fully ionised, i.e. the number of electrons is equal to the number of protons
  - count the protons (remember in H and He)

Equilibrium between γ and e

- Ratio of number densities
  - (76% H and 24% He)

$$\frac{n_e}{n_B} = 0.76 + \frac{0.24}{2} = 0.88$$

- Density scales with T^3 thus

$$n_e = 0.88\, n_B = 0.88\, n_{B0} \left(\frac{T}{T_{\gamma 0}}\right)^3$$

- Hence the scattering rate is

$$\Lambda_\gamma = 0.88\, n_{B0} \left(\frac{T}{T_{\gamma 0}}\right)^3 \sigma_T c = 1.97 \cdot 10^{-19}\, s^{-1} \times \Omega_B h^2 \left(\frac{T}{T_{\gamma 0}}\right)^3$$

- Important is the momentum exchange between γ and e which is $\propto \frac{k_B T}{m_e}$

Equilibrium between γ and e

- The rate of the momentum exchange is then

$$\Gamma_\gamma \cong \left(\frac{k_B T}{m_e}\right) \Lambda_\gamma \approx 9.0 \cdot 10^{-20}\ s^{-1} \left(\frac{T}{T_{\gamma 0}}\right)^2$$

- As long as this is faster than the expansion rate (radiation-dominated)

$$H = \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_R \frac{T^4}{T_{\gamma 0}^4}} = 2.1 \cdot 10^{-20}\ s^{-1} \left(\frac{T}{T_{\gamma 0}}\right)^2$$

- Equality between rate and expansion is

$$T_{freeze} = \frac{1.5 \cdot 10^4\ K}{\sqrt{\Omega_B h^2}}\ (\approx 10^5 K)$$

Equilibrium between γ and e

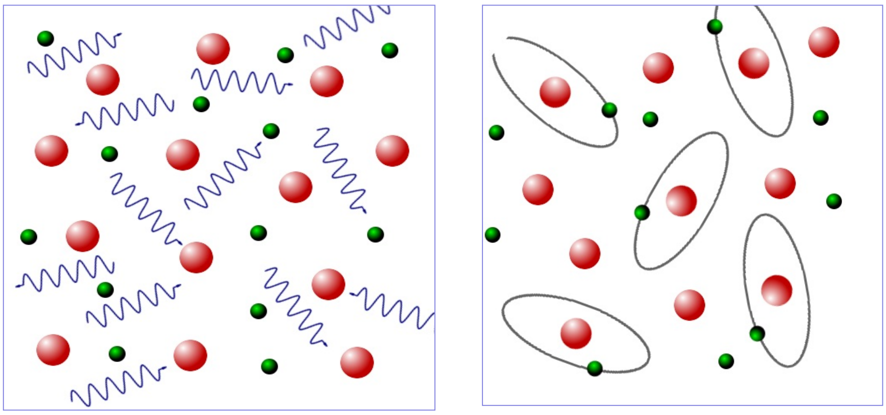
- Although no momentum is exchanged any longer the photons still scatter on the free electrons
- At lower temperatures the expansion is matter-dominated and hence

$$H = H_0 \sqrt{\Omega_M \frac{T^3}{T_{\gamma 0}^3}} = 3.3 \cdot 10^{-18}\ s^{-1} \sqrt{\Omega_M h^2} \left(\frac{T}{T_{\gamma 0}}\right)^{3/2}$$

- Equality here would be at

$$T = 18\ K \frac{\Omega_M^{1/3}}{\Omega_B^{2/3}} \approx 130\ K$$

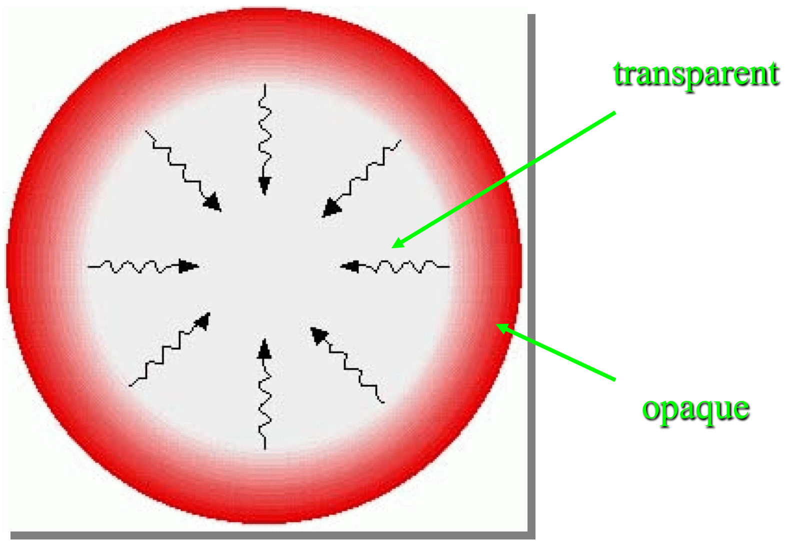
- However there is more physics to be considered → see following



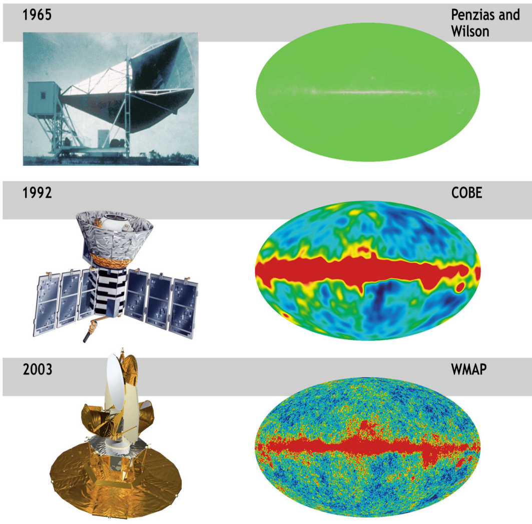
Before recombination: *The Universe is opaque*
After recombination: *The Universe is transparent*

Transition ~ 300 000 years after the Big Bang

Last scattering surface



Uncovering the CMB



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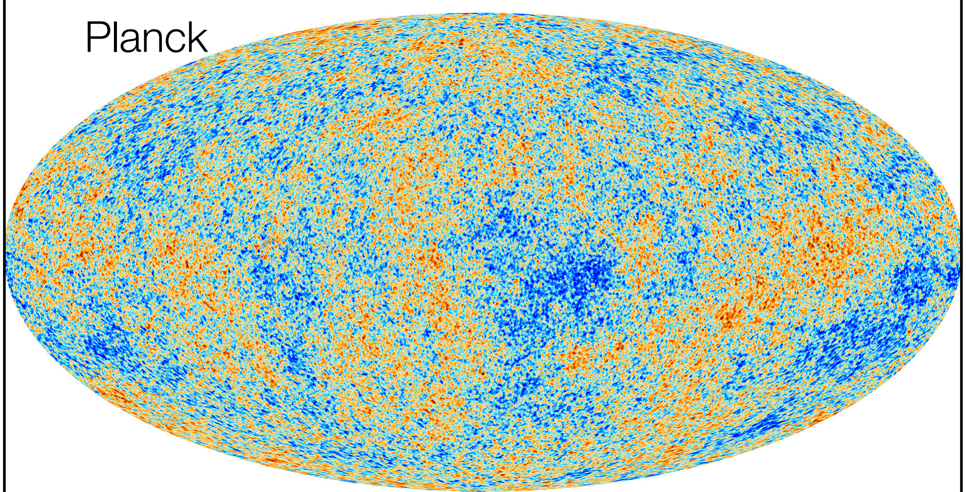
Uncovering the CMB



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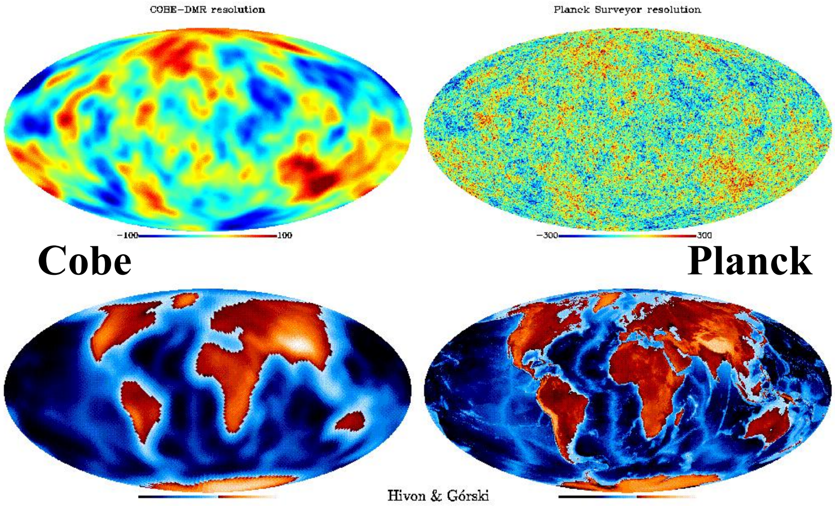
Comparison with a familiar surface



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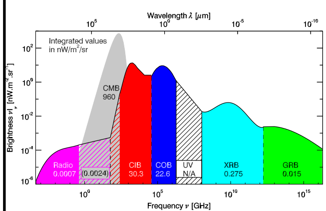
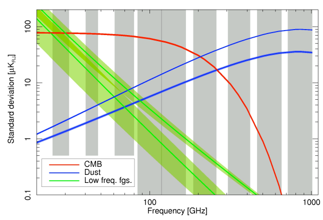
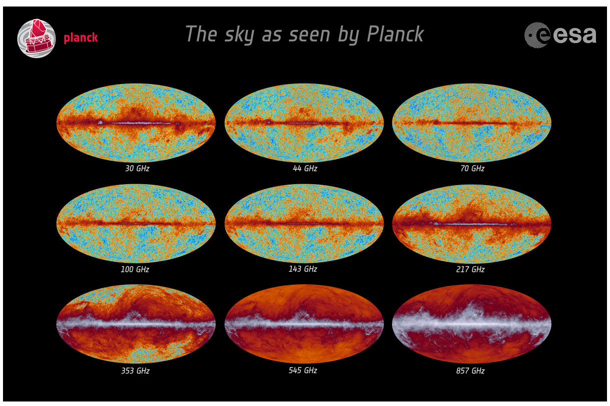
Foregrounds

CMB signal is hidden behind Galactic foregrounds







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Recombination and last scattering

- The free electrons are removed when they form atoms with the nuclei formed in the Big Bang
- Consider the Maxwell-Boltzmann density distribution

$$n_i = \frac{g_i}{(2\pi\hbar)^3} \mathrm{e}^{\frac{\mu_i}{k_B T}} \int e^{-\frac{\left(m_i + \frac{p^2}{2m_i}\right)}{k_B T}} d^3p$$

- Particles of interest are photons, electrons, baryons,hydrogen atoms at different excitation states (1s, 2d, 3p, etc.)

Recombination

- Relevant factors
  - fermions → half spins: $g_p = g_e = 2$
- Hydrogen 1s ground state has two hyperfine states (spin 0 and 1): $g_{1s} = 4$
- In ionization equlibrium the chemical potential is the sum of the individual

$$\mu_{1s} = \mu_p + \mu_e$$

- The integrals work out to be

$$\frac{1}{(2\pi\hbar)^3} \int d^3p \, e^{-\left(\frac{p^2}{2mk_B T}\right)} = \left(\frac{mk_B T}{2\pi\hbar^2}\right)^{3/2}$$

Recombination

Because of $\mu_{1s} = \mu_p + \mu_e$ the ratio of the number densities becomes

$$\frac{n_{1s}}{n_e n_p} = \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-\frac{3}{2}} e^{\frac{B_1}{k_B T}}$$

with $B_1 = m_p + m_e - m_H = 13.6\ eV$ the energy of the ground state of Hydrogen.

Since the universe is charge neutral, we have the same amount of electrons and protons $n_e = n_p$.

Recombination

- It is also okay to assume that the hydrogen is in its ground state
  - the number density of excited states scales with $e^{-\frac{\Delta E}{k_B T}}$ and the excitation from $n = 1$ to $n = 2$ is $10.6\ eV$. Hence for $T < 4200\ K$ the exponential is $< 6 \cdot 10^{-13}$
- Assume 24% in Helium we find

$$n_p + n_{1s} = 0.76 n_B$$

with n_B the baryon density

Recombination

The hydrogen ionisation fraction is $X \equiv \frac{n_p}{n_p+n_{1s}}$ then described by the Saha equation

$$X\left(1+\frac{(n_p+n_{1s})n_{1s}}{n_p^2}X\right) = X\left(1+0.76n_B X\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-\frac{3}{2}} e^{\frac{B_1}{k_B T}}\right) = 1$$

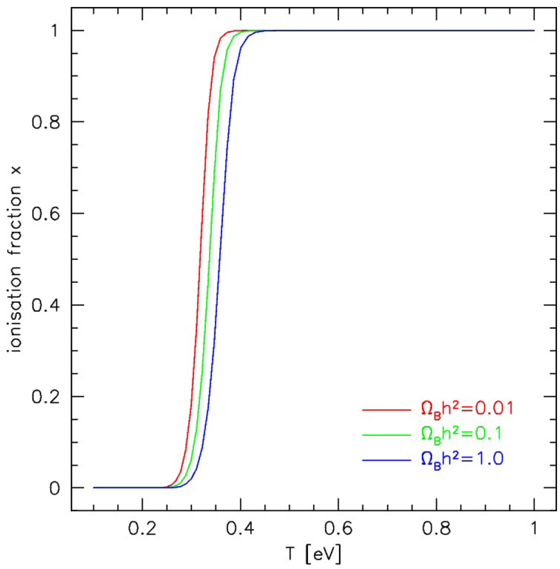
The second term describes the ionisation state as a function of temperature and with $n_B = n_{B0}\left(\frac{T}{T_{\gamma 0}}\right)^3$ the transition can be written as a function of baryon density.
(See next table)

Recombination

Change of H ionisation as a function of temperature (and baryon density)

T(K)	$\Omega_B h^2 = 0.01$	$\Omega_B h^2 = 0.02$	$\Omega_B h^2 = 0.03$
4500	0.999	0.998	0.997
4200	0.990	0.981	0.971
4000	0.945	0.900	0.863
3800	0.747	0.634	0.565
3600	0.383	0.290	0.244
3400	0.131	0.094	0.078
3200	0.0337	0.0240	0.0196
3000	0.00693	0.00491	0.00401
2800	0.00112	0.00079	0.00065
2.725	$2.8\ 10^{-12571}$	$2.0\ 10^{-12571}$	$1.6\ 10^{-12571}$

$T_{recombination}$



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Recombination – a complication

Recombination rate in hydrogen depends on the density. The transition from 2p to 1s creates a photon (Lyman α for astronomers), which can immediately excite another hydrogen atom to the 2p state. This means that hydrogen gets trapped in an ionised state and these need to be taken into account.

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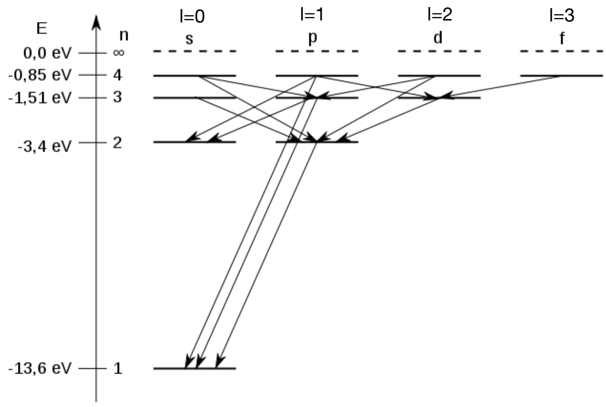
Ionisation structure of H

- Grotrian diagram of Hydrogen



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Recombination – a complication

Conditions to be considered

- rapid transitions, which means thermal equilibrium, except for the 1s ground state, which is slow. Number density is

$$n_{nl} = (2l+1)n_{2s}e^{\frac{B_2-B_1}{k_BT}}$$

- net rate of population of 1s state is given by the rate of radiative decays from 2s and 2p states minus the rate of excitation from the 1s state
- radiative processes much faster than the reduction in density due to the expansion

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Recombination

- These effects change the ionisation rates and hence modify the Saha equation
- Exact calculation tedious involving the details of the radiative decays and excitations
  - e.g. 2-photon decay from 2s to 1s state, which is strongly suppressed
- Following table is the result of a more exact calculation

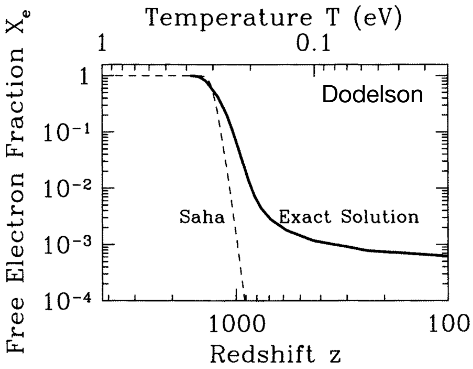
Recombination

Due to the atomic transitions in hydrogen the suppression of the Lyman α photons leads to modification of the ionisation fraction

z	T(K)	T(yrs)	$X_{\Omega_B h^2=0.01}$	$X_{\Omega_B h^2=0.02}$	$X_{\Omega_B h^2=0.03}$
1550	4226	202600	0.992	0.984	0.982
1500	4090	213200	0.976	0.958	0.954
1450	3954	225900	0.935	0.902	0.878
1400	3818	239800	0.861	0.815	0.780
1350	3681	255200	0.759	0.703	0.659
1300	3545	272000	0.645	0.580	0.529
1250	3409	290600	0.526	0.456	0.402
1200	3273	311300	0.409	0.339	0.289
1150	3136	334600	0.299	0.236	0.194
1100	3000	360400	0.205	0.154	0.122
1050	2864	389600	0.129	0.0928	0.0721
1000	2728	422600	0.0752	0.0520	0.0396
950	2591	460500	0.0405	0.0270	0.0203
900	2455	503600	0.0210	0.0136	0.0101
800	2183	611400	0.00662	0.00387	0.00276
700	1910	761300	0.00319	0.00174	0.00120
600	1638	977700	0.00203	0.00107	0.000731
500	1365	$1.312\ 10^6$	0.00147	0.000762	0.000517
250	684	$3.922\ 10^6$	0.000829	0.000423	0.000285
100	275	$1.604\ 10^7$	0.000632	0.000321	0.000216
50	139	$4.535\ 10^7$	0.000579	0.000294	0.000197
10	30	$4.568\ 10^7$	0.000537	0.000272	0.000183

Recombination

The effect is that at low redshifts the fraction of free electrons is approaching a constant level



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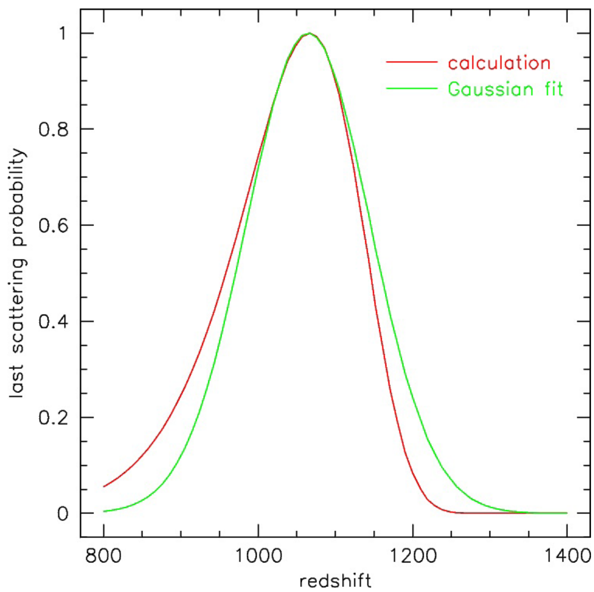
Recombination 'shell'



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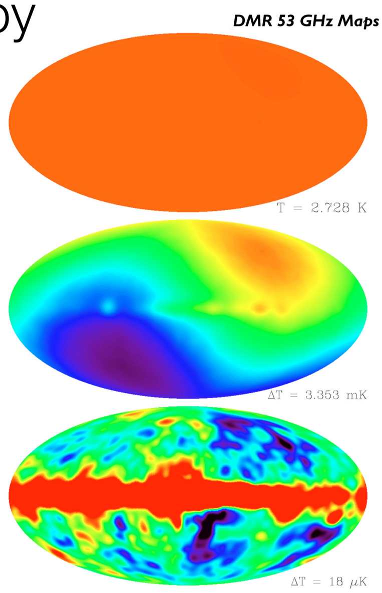
Anisotropy

- CMB extremely uniform
- only at 3.35mK is there a dipole visible



Dipole anisotropy

- Due to Earth's motion relative to the CMB a dipole is induced
  - blueshift in direction of motion
  - redshift in opposite direction
- Measured dipole is $\Delta T = 3.346 \pm 0.017\ mK$ as measured by the Wilkinson Microwave Anisotropy Probe (WMAP)
- Velocity is then

$$v = \frac{\Delta T}{T} c = \frac{0.00335}{2.725} \cdot 3 \cdot 10^5 km\ s^{-1} \approx 370\ km\ s^{-1}$$