Observational Cosmology

TUM WS 2019/2020

Lecture 1

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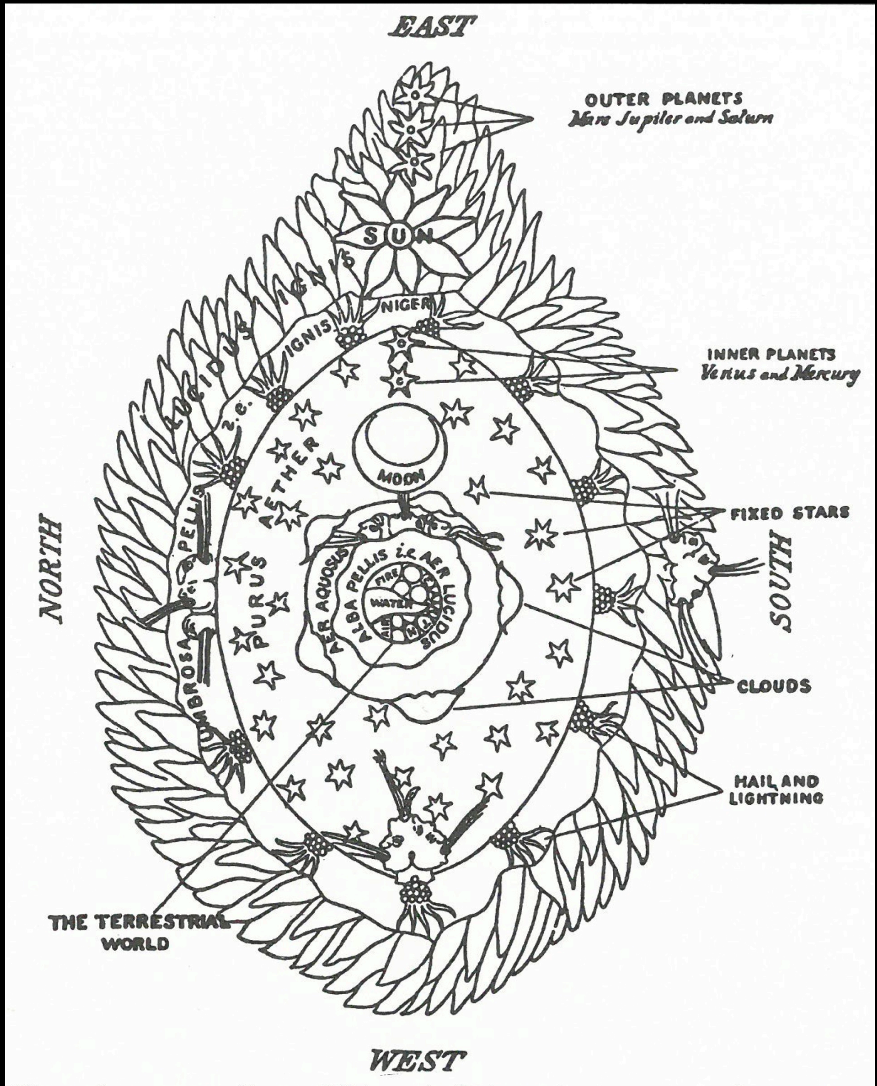
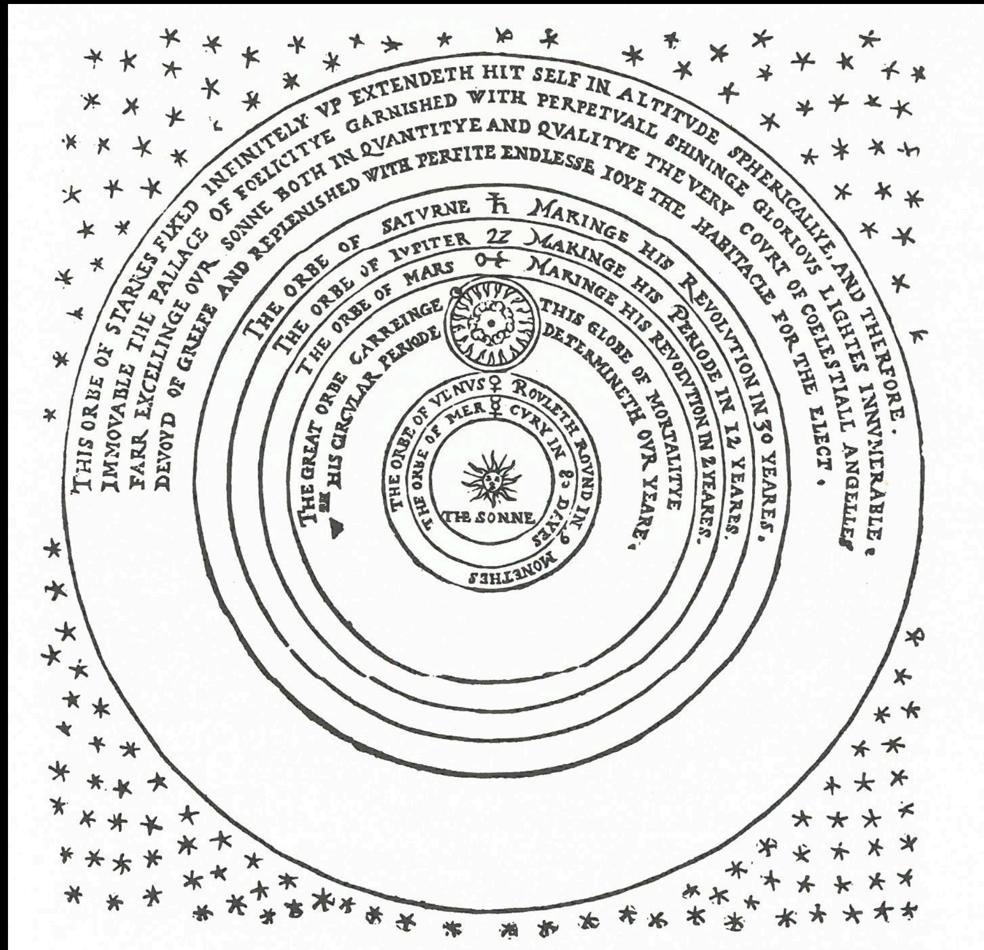
Slides available at

http://www.eso.org/~bleibund/Cosmology

World Views

Past World Views

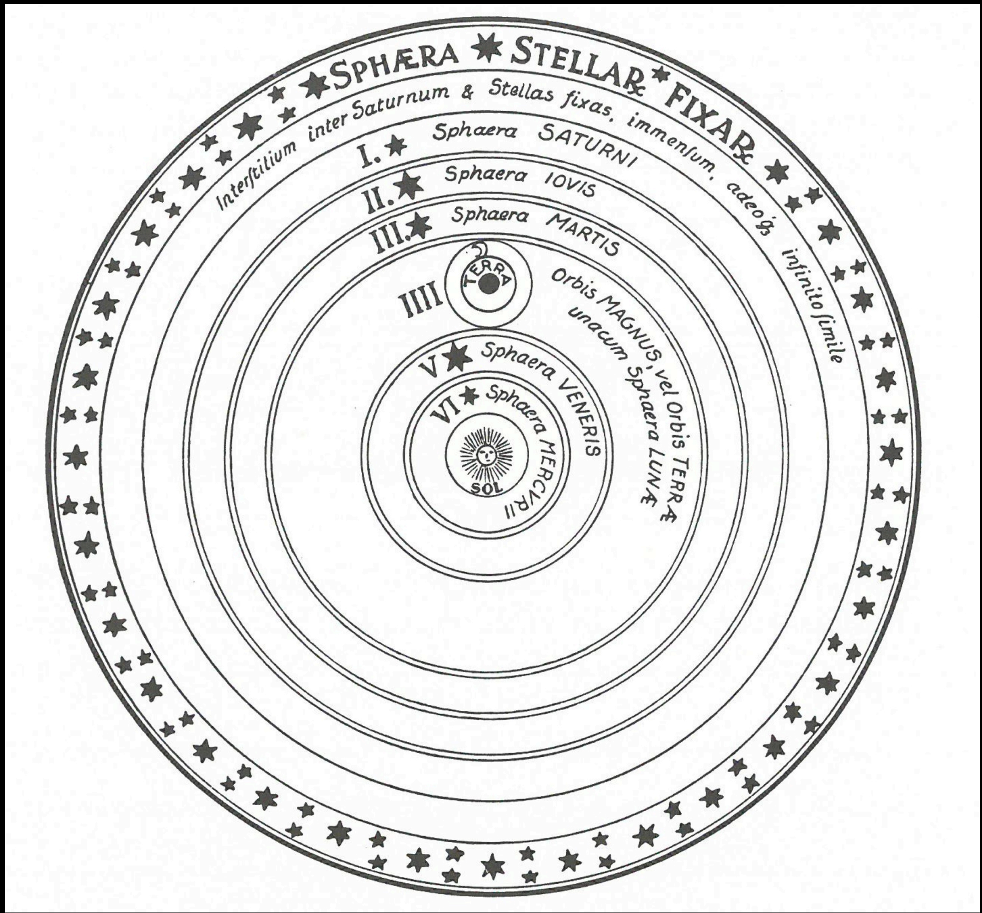




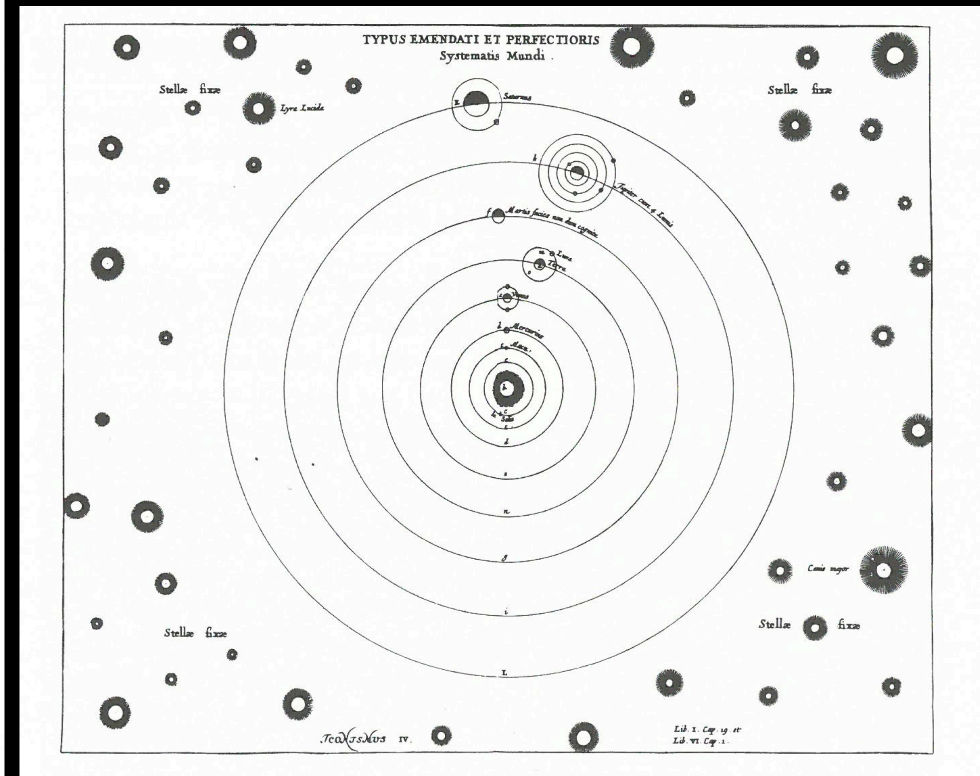
Hildegard von Bingen (1150)

Thomas Digges (1576)

Past World Views

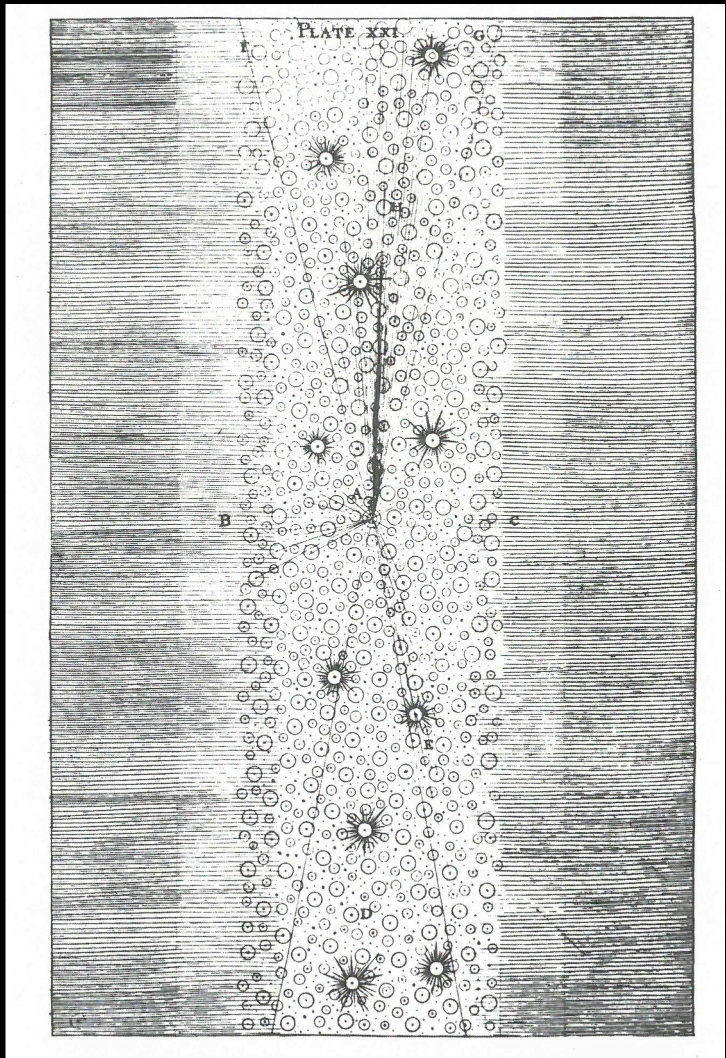


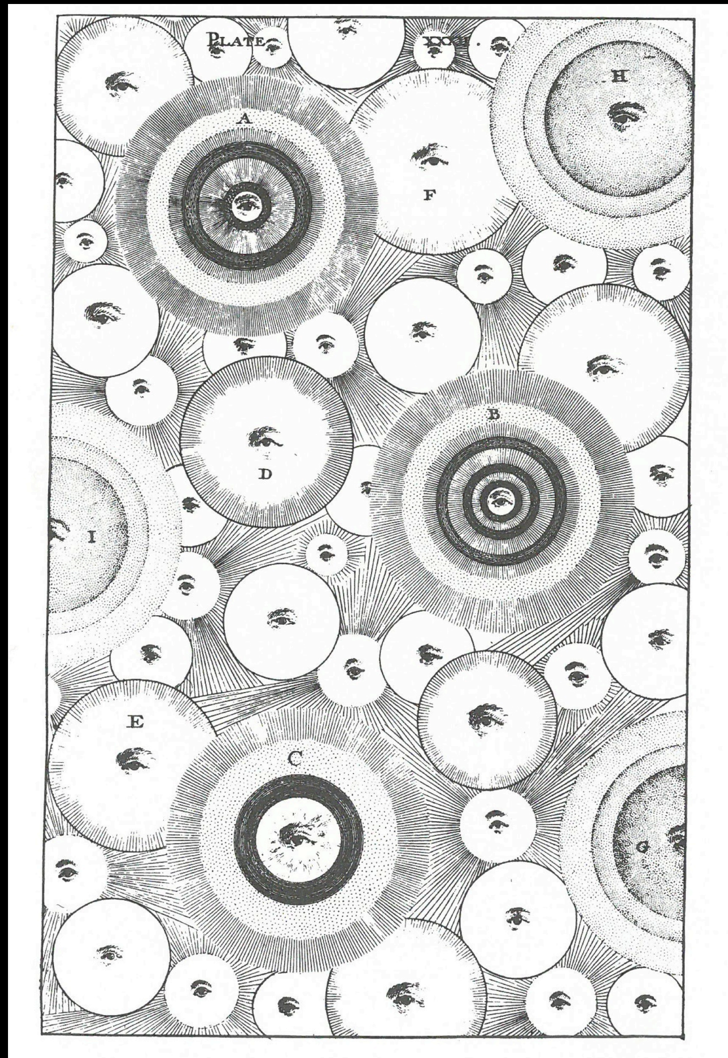
Johannes Kepler (1596)



Otto von Guericke (1672)

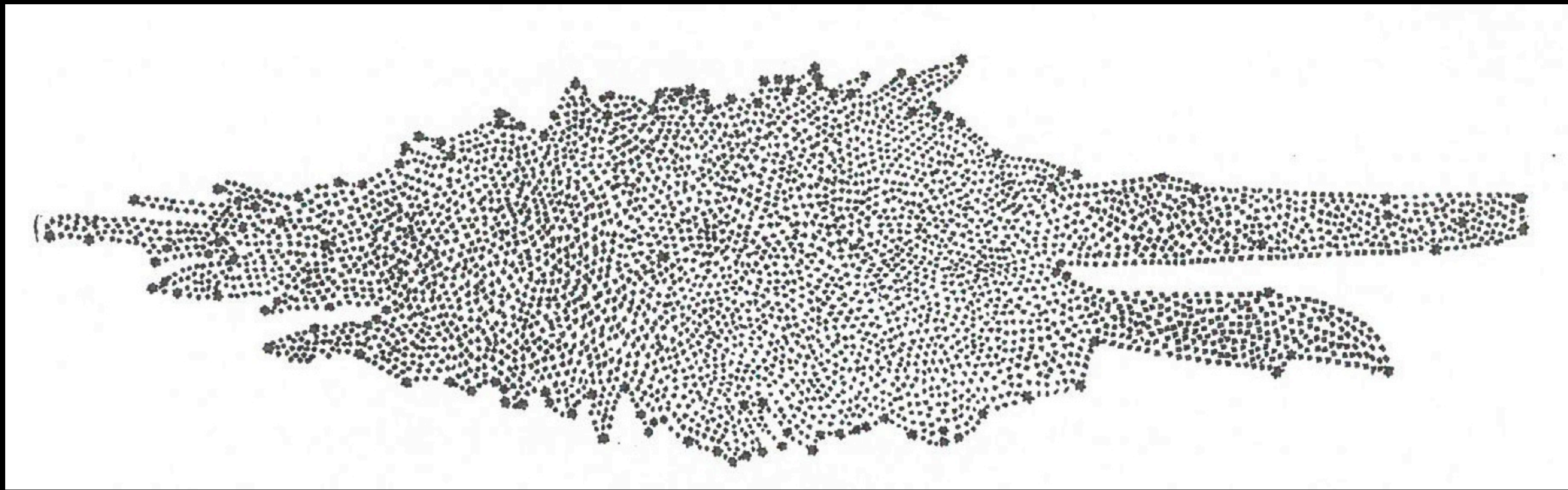
Past World Views

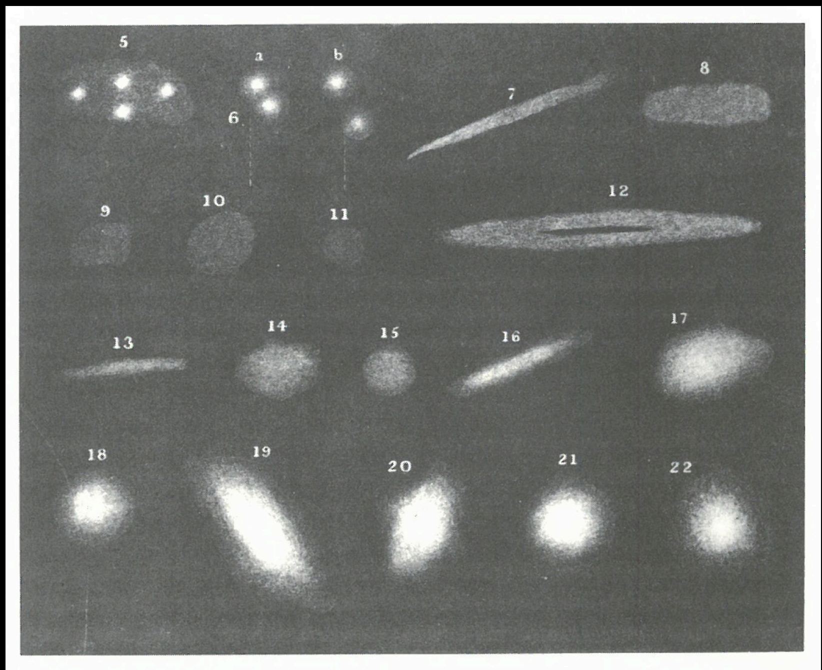




Thomas Wright of Durham (1750)

Past World Views

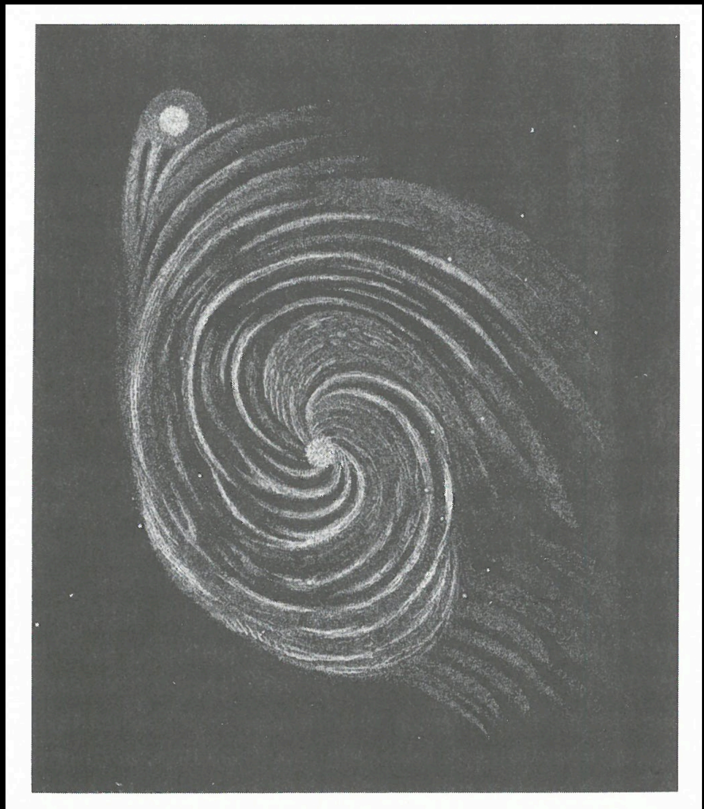




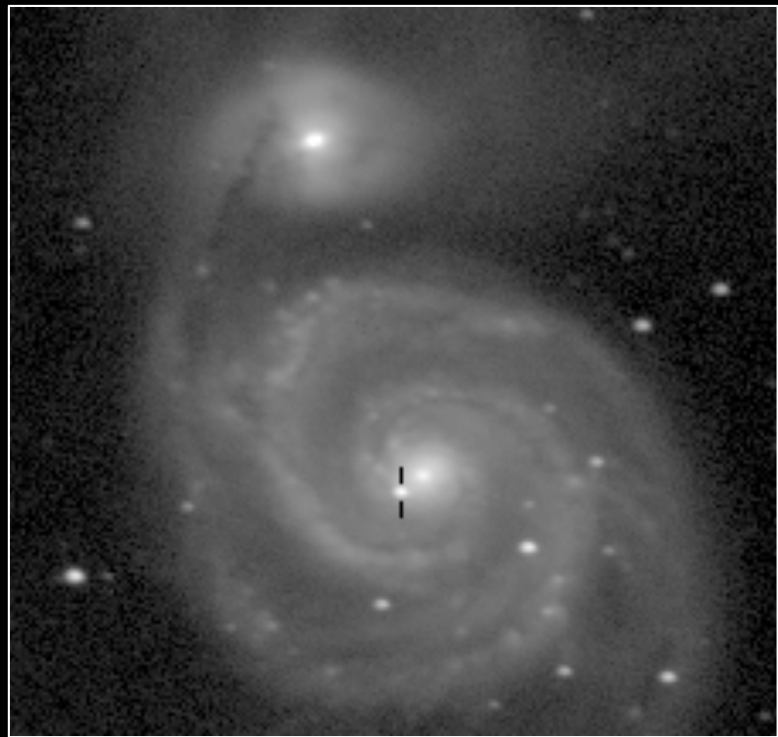
William und Caroline Herschel
(late 18th century)

Past World Views

Messier 51



William Parson (1845)



Modern Image (1991)

Hubble Ultra Deep Field

HST ▪ ACS

NASA, ESA, S. Beckwith (STScI) and The HUDF Team

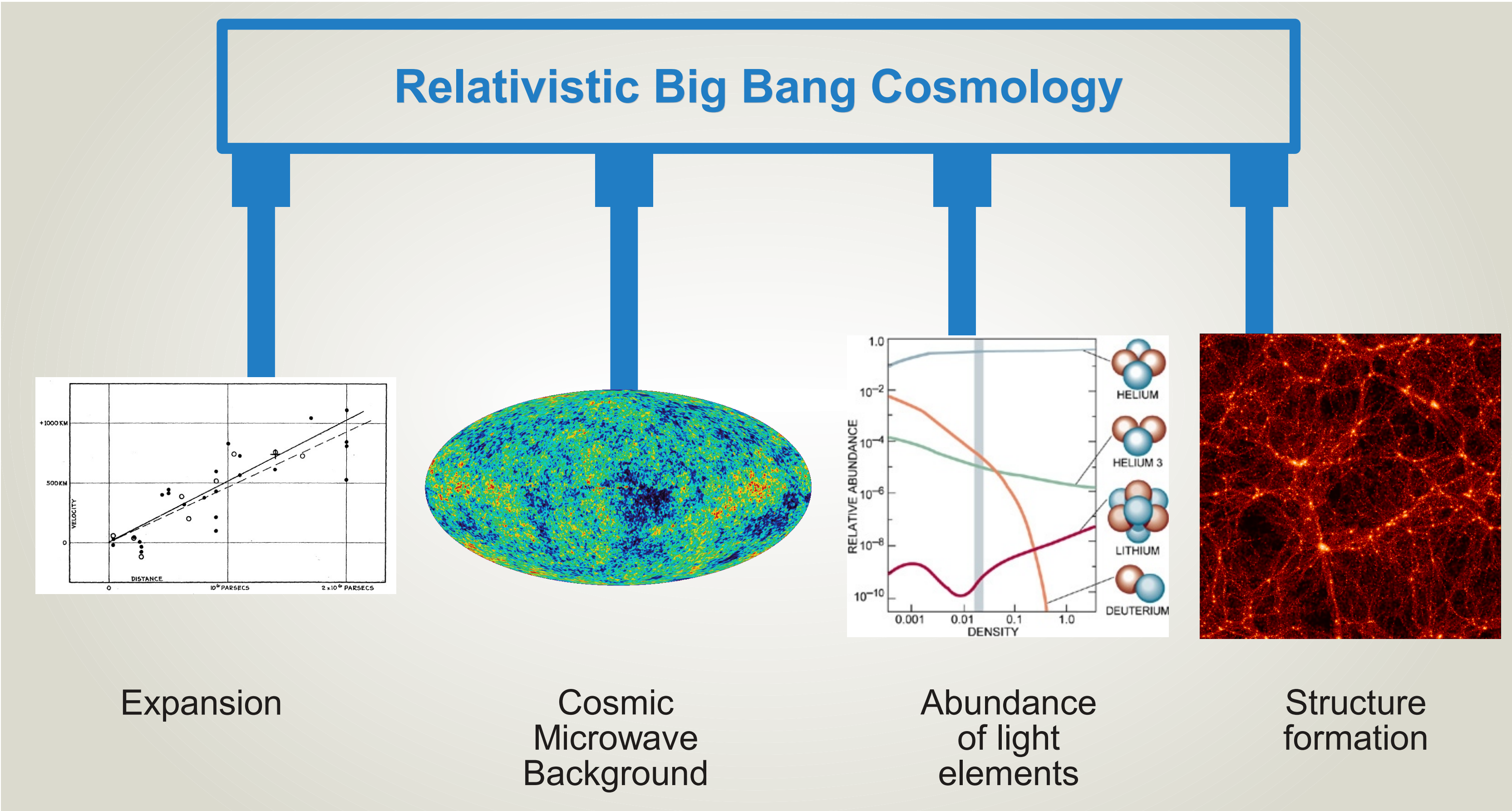
STScI-PRC04-07a

Astrophysics Nobel Prizes

Year	Winner	Titel
1967	Hans Bethe	Energy production in stars, nuclear reactions
1974	Martin Ryle, Antony Hewish	Radio astronomy, pulsars
1978	Arno Penzias, Robert Wilson	Cosmic Microwave Background
1983	Subramanyan Chandrasekhar, William Fowler	Structure of stars and chemical enrichment
2002	Riccardo Giacconi Raymond Davies, Masatoshi Koshiba	X-ray astrophysics cosmic neutrino
2006	John Mather, George Smoot	Cosmic Microwave Background (COBE)
2011	Saul Perlmutter, Brian Schmidt, Adam Riess	Accelerating Universe
2017	Rainer Weiss, Barry Barish, Kip Thorne	Gravitational Waves
2019	James Peebles Michel Mayor, Didier Queloz	“theoretical discoveries in physical cosmology“ “for the discovery of an exoplanet orbiting a solar-type star“

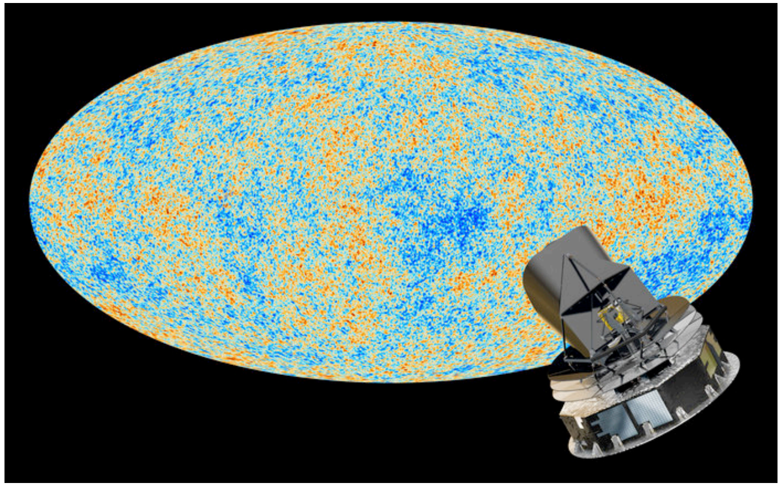
Pillars of Cosmology

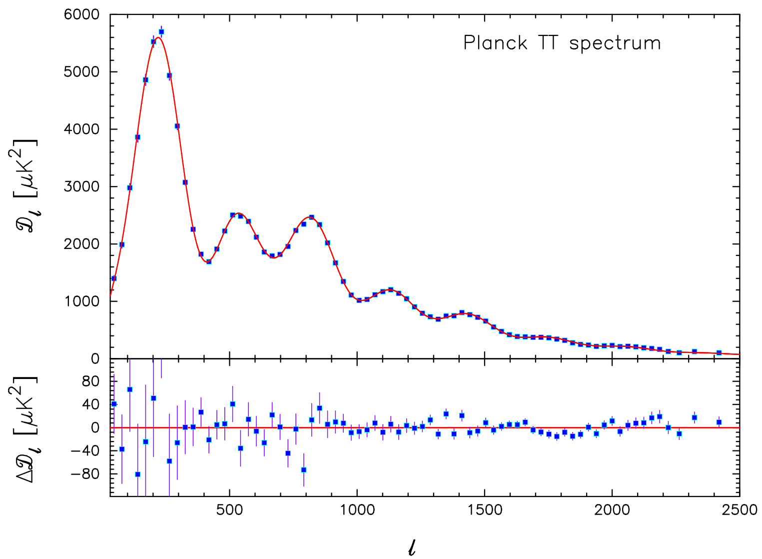
Relativistic Big Bang Cosmology



J. Liske

Cosmological Parameters



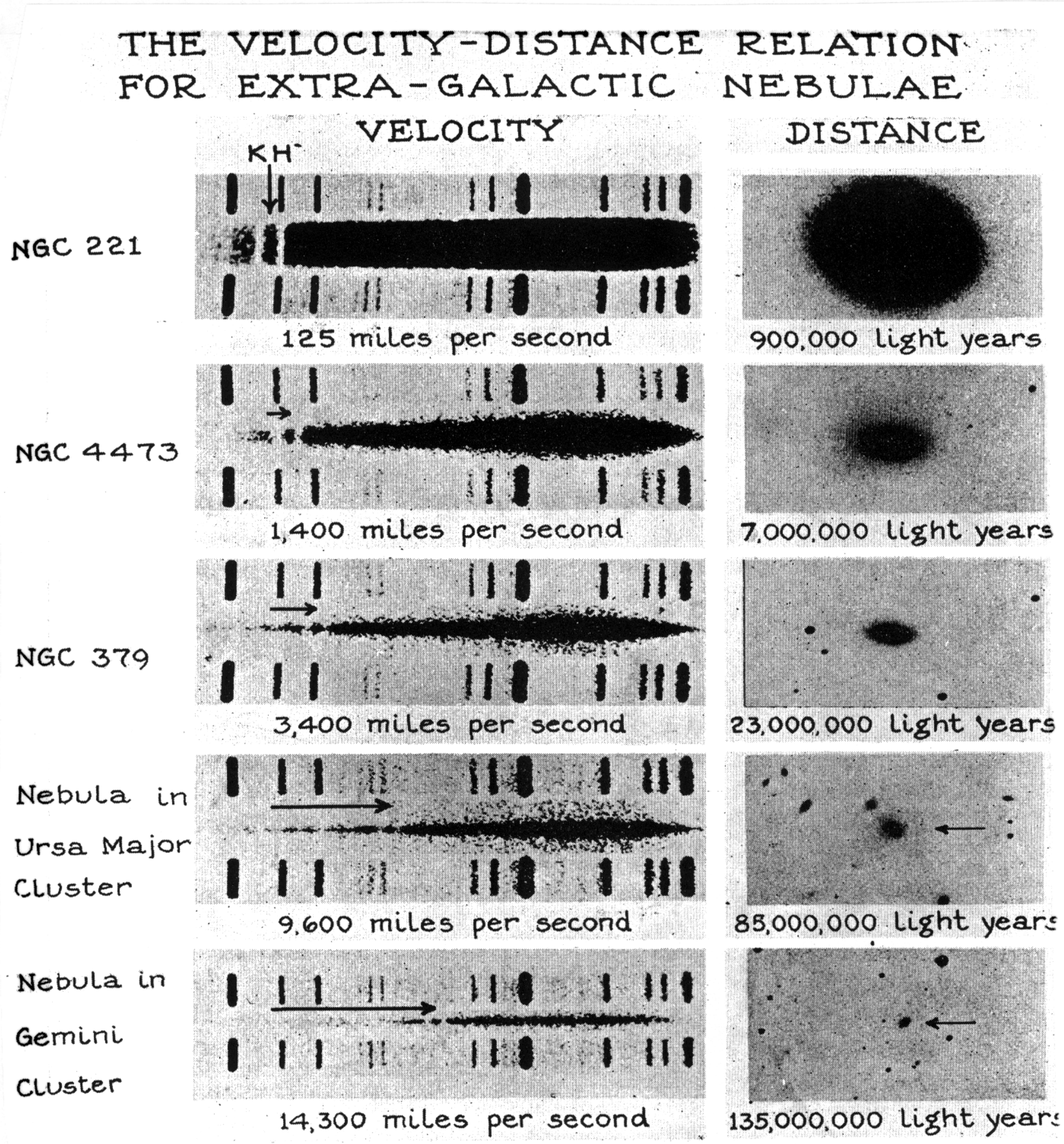


Parameter	*Planck*	
	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068
τ	0.0925	0.097 ± 0.038
n_s	0.9624	0.9616 ± 0.0094
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072
Ω_Λ	0.6825	0.686 ± 0.020
Ω_m	0.3175	0.314 ± 0.020
σ_8	0.8344	0.834 ± 0.027
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$
H_0	67.11	67.4 ± 1.4
$10^9 A_s$	2.215	2.23 ± 0.16
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059
Y_P	0.247710	0.24771 ± 0.00014
Age/Gyr	13.819	13.813 ± 0.058
z_*	1090.43	1090.37 ± 0.65
r_*	144.58	144.75 ± 0.66
$100\theta_*$	1.04139	1.04148 ± 0.00066
z_{drag}	1059.32	1059.29 ± 0.65
r_{drag}	147.34	147.53 ± 0.64
k_D	0.14026	0.14007 ± 0.00064
$100\theta_D$	0.161332	0.16137 ± 0.00037
z_{eq}	3402	3386 ± 69
$100\theta_{eq}$	0.8128	0.816 ± 0.013
$r_{drag}/D_V(0.57)$	0.07130	0.0716 ± 0.0011

Cosmological Topics

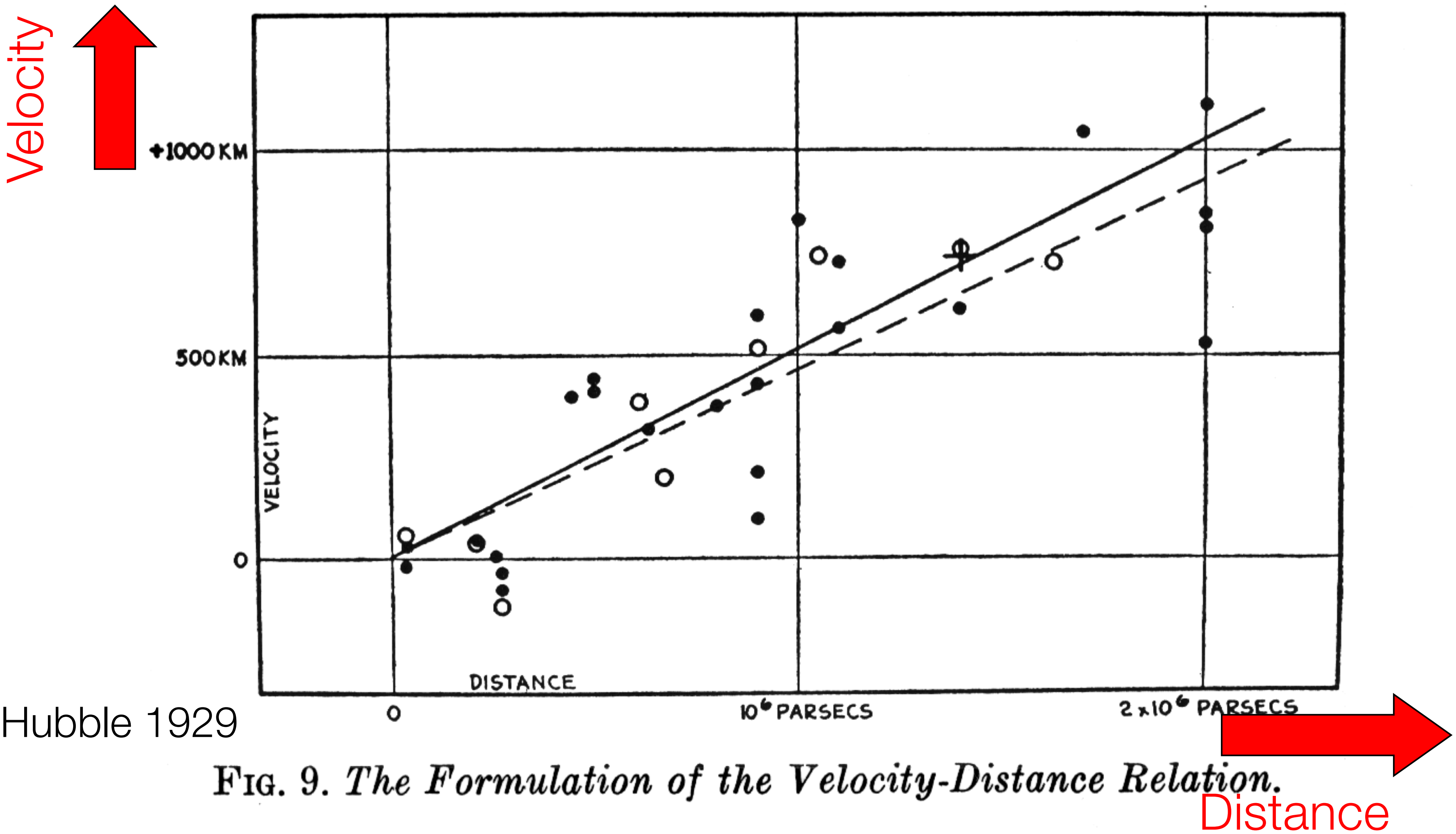
- Cosmic expansion
  - H_0, q_0
- Expansion history of the universe
  - age t_0
- (Energy) contents of the universe
  - baryons, (cold) dark matter, neutrinos, dark energy, ???
- Space geometry
  - metric
- Formation of elements
  - Big Bang nucleosynthesis → First Three Minutes
- Formation of structure
  - r (T/S)
- Growth of structure
  - spectral index

Expansion of the Universe



Hubble 1936

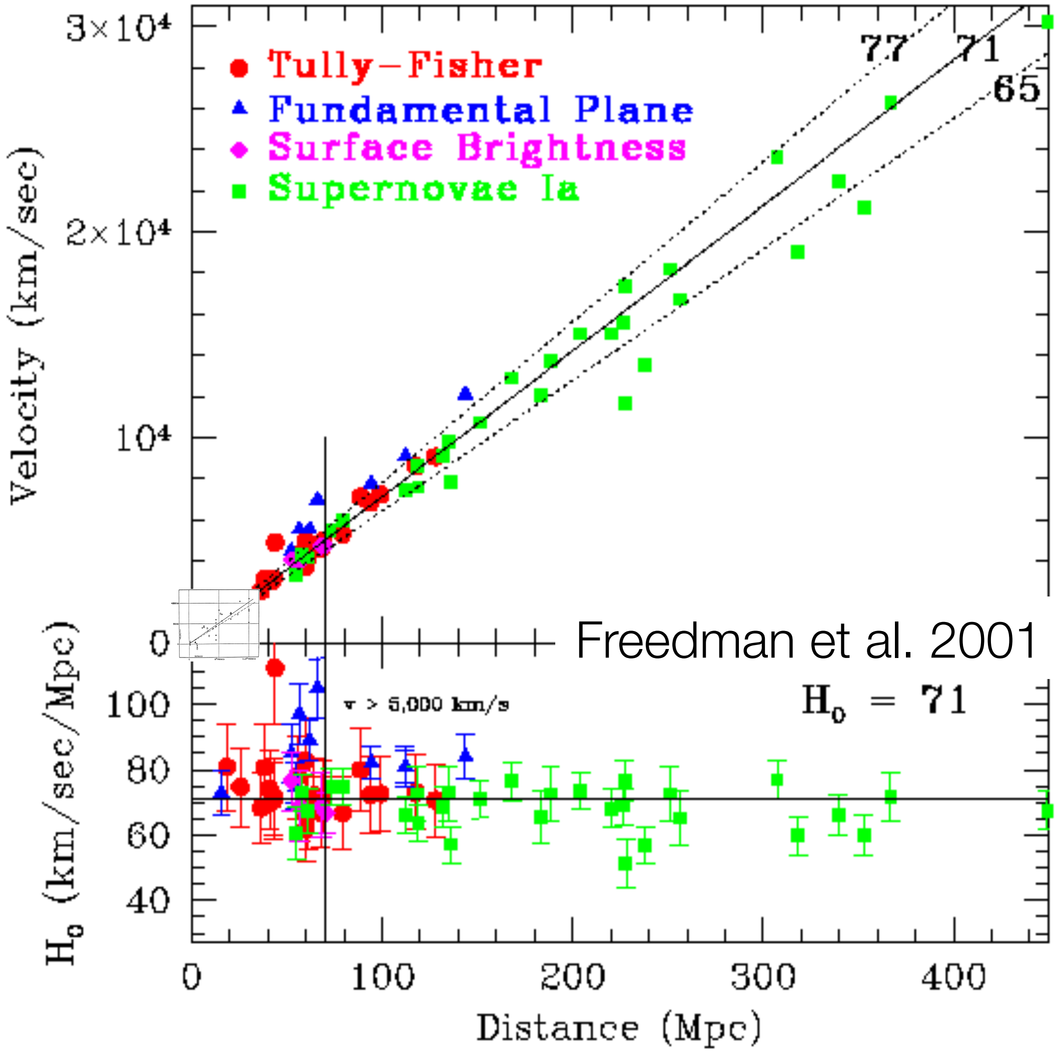
The Original Hubble Diagram



Hubble 1929

Fig. 9. *The Formulation of the Velocity-Distance Relation.*

A modern Hubble Diagram

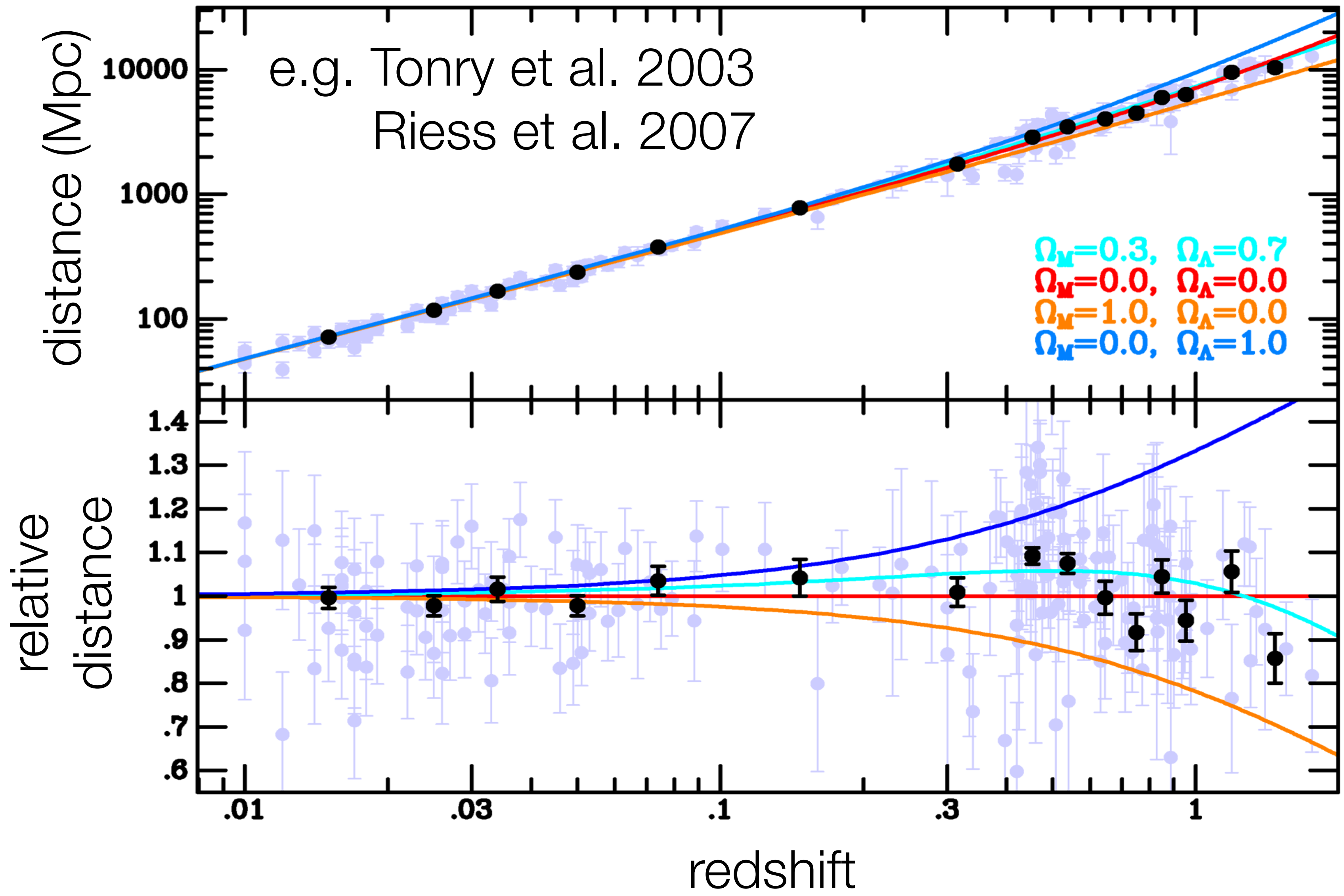


Freedman et al. 2001

Expansion history of the universe

- Changes in the expansion rate
  - influenced by gravity → geometry
  - determined by the different constituents in the universe
    - neutrinos, radiation, baryons, dark matter, dark energy, ???
- Sets the age of the universe

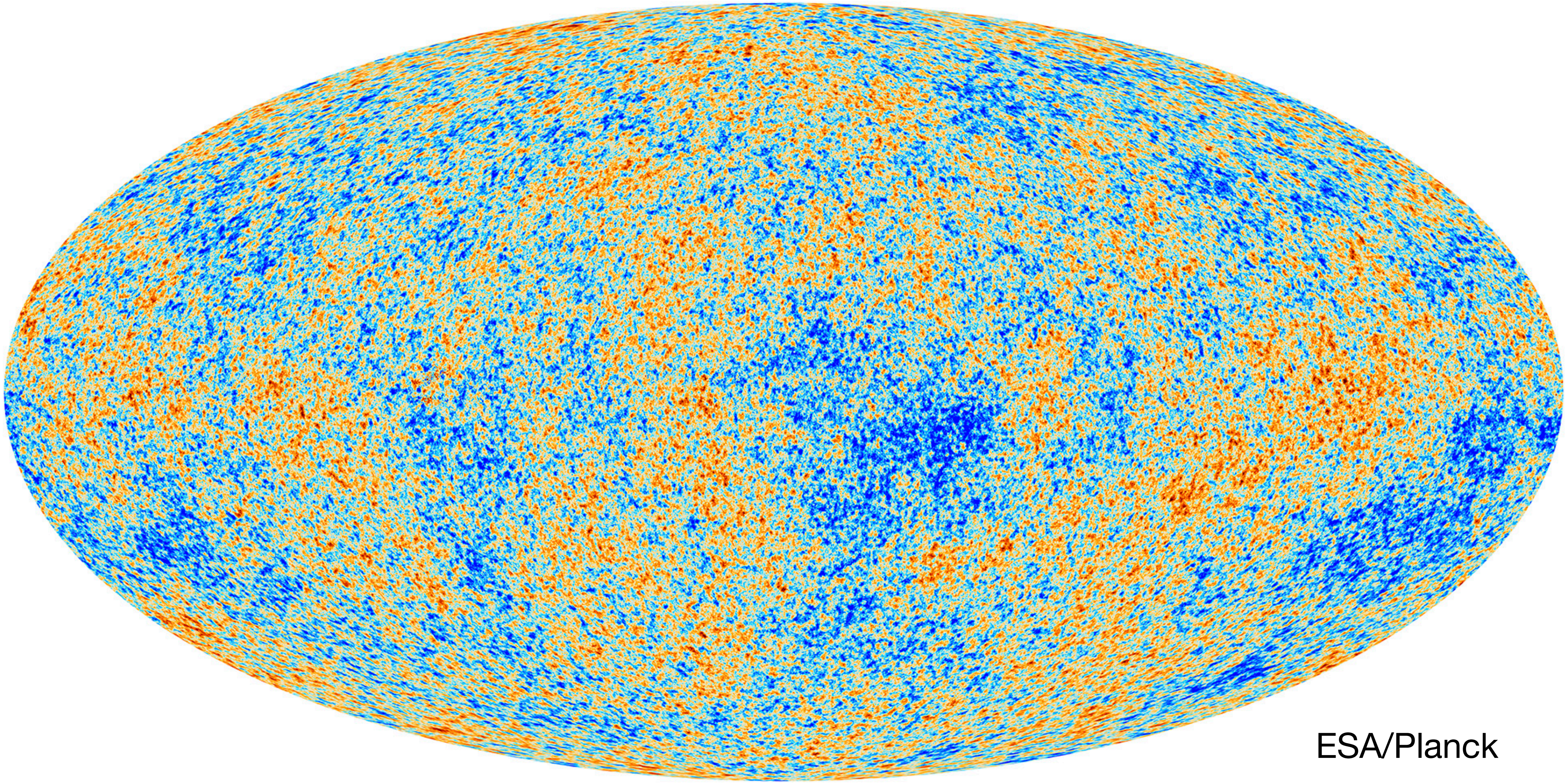
Expansion History of the Universe



Expansion History

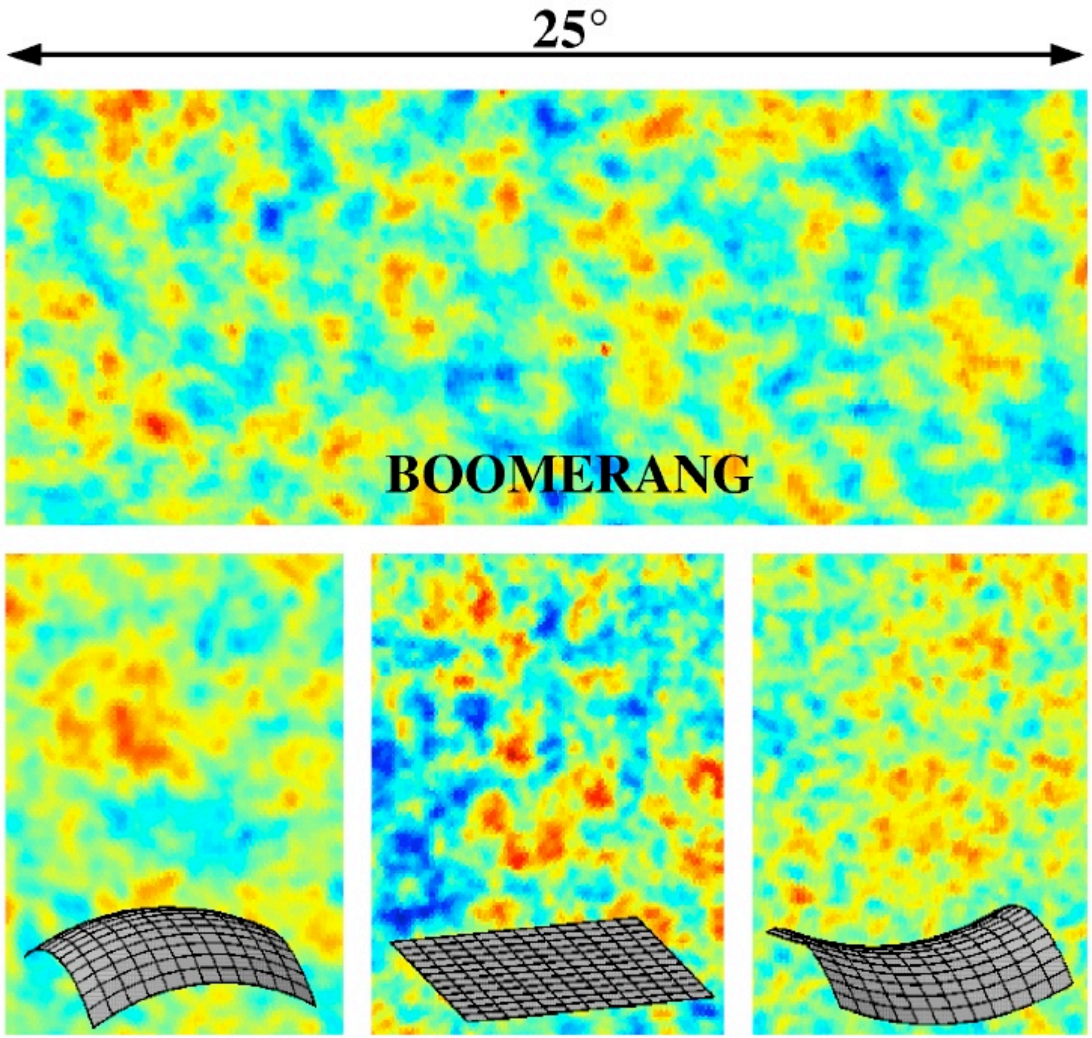
- Note move from velocities to redshifts
- introduction of the density parameters
  - Ω_M (all) matter
  - Ω_Λ cosmological constant/dark energy
- tiny signal → big implications
- uncertainty in the data

Cosmic Microwave Background

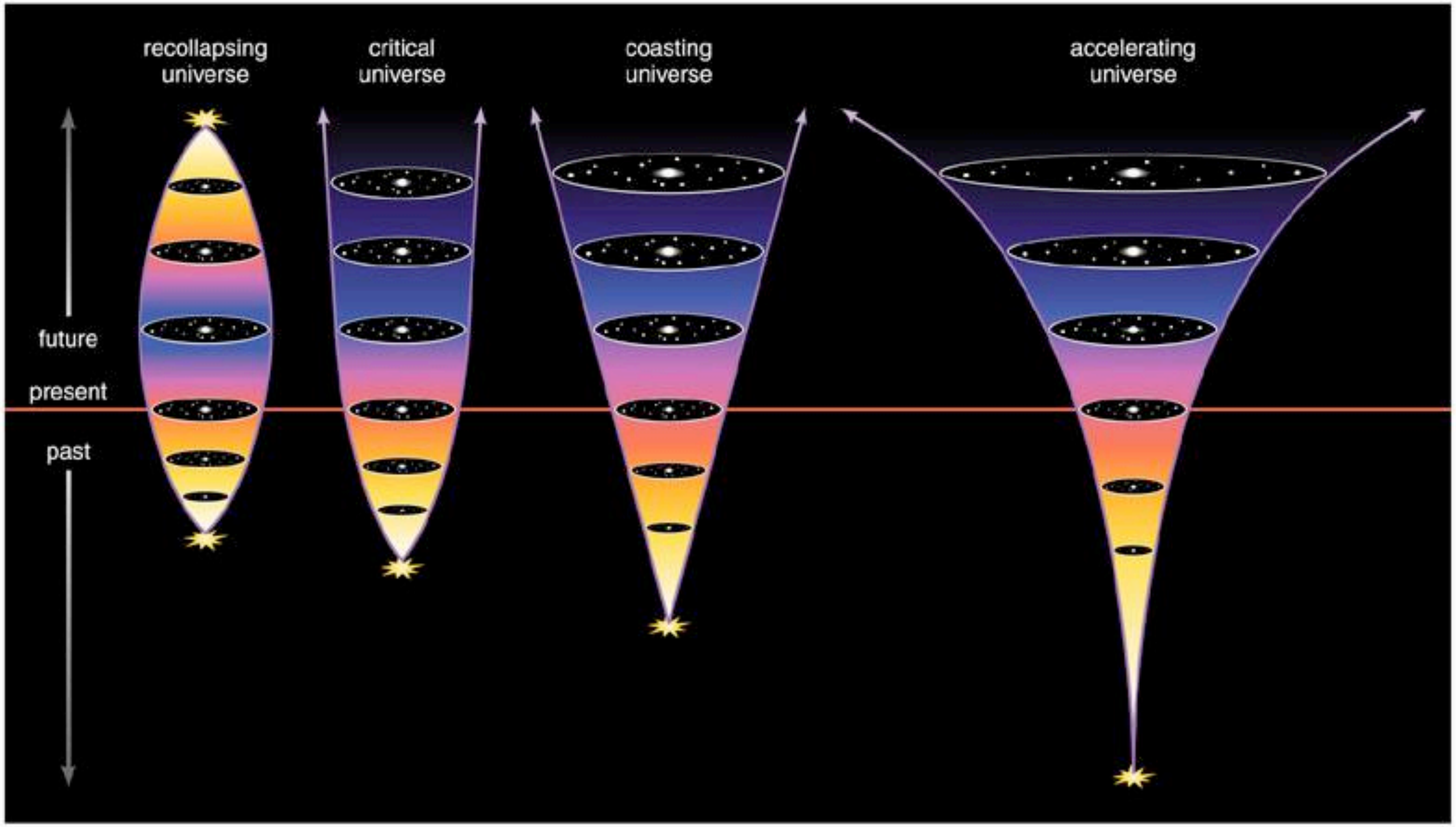


ESA/Planck

CMB and Geometry



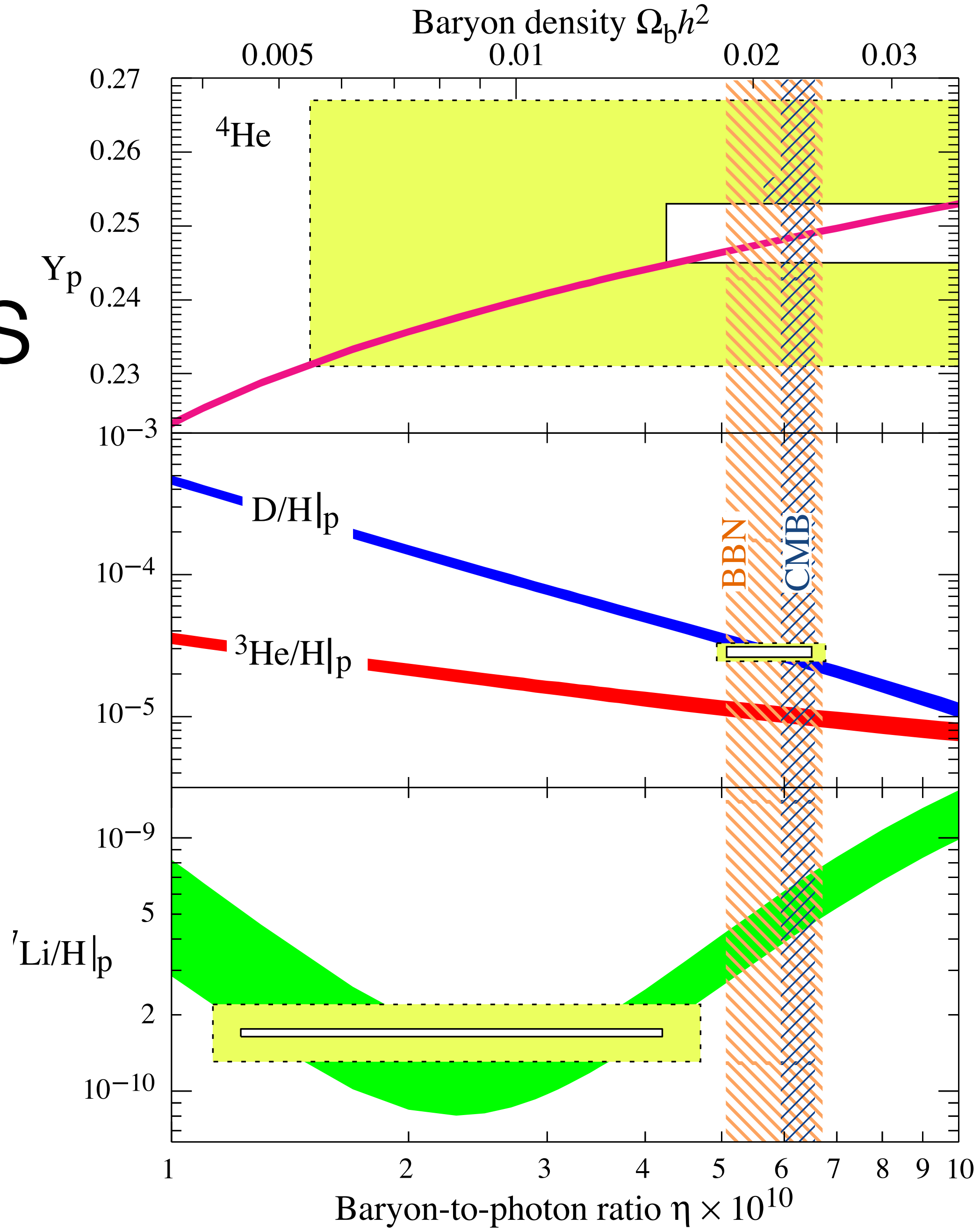
Age and Geometry



Cosmic microwave background

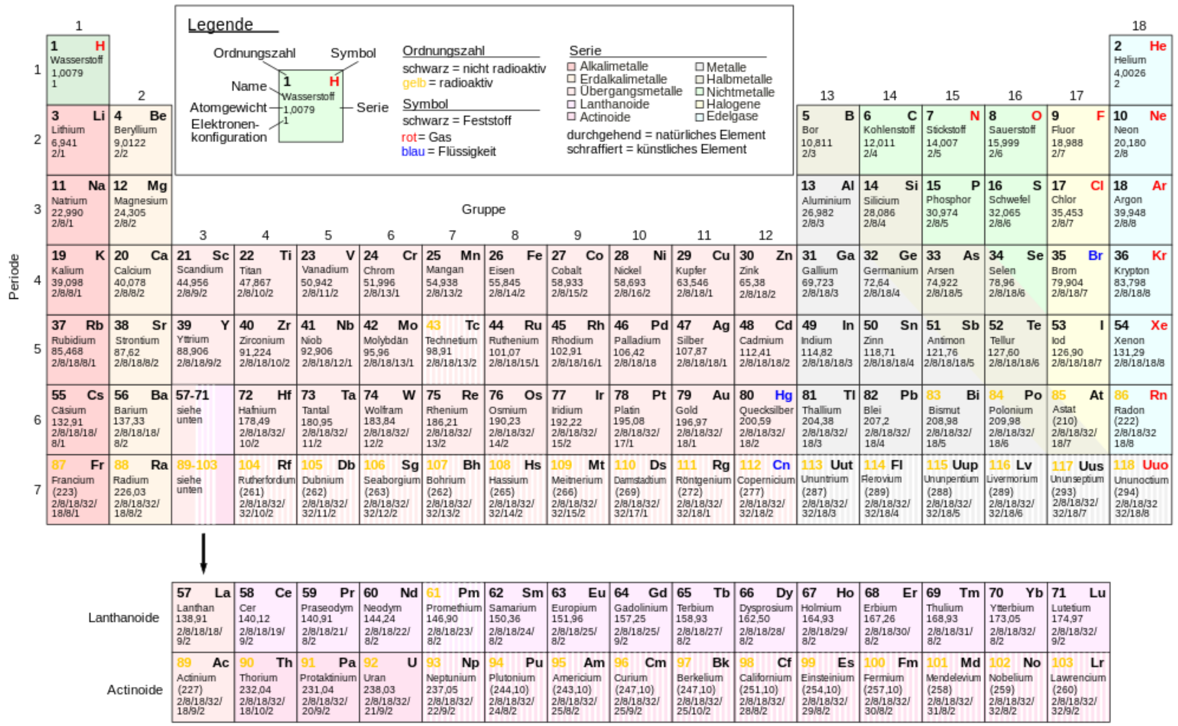
- nearly perfect blackbody radiation
- tiny (10^{-5} K) temperature variations
  - isotropy!
- signature of the hot phase of the (early) universe
- fluctuations seeded by quantum fluctuations during the inflationary phase
- depends on structure and geometry of the universe

Big Bang Nucleosynthesis

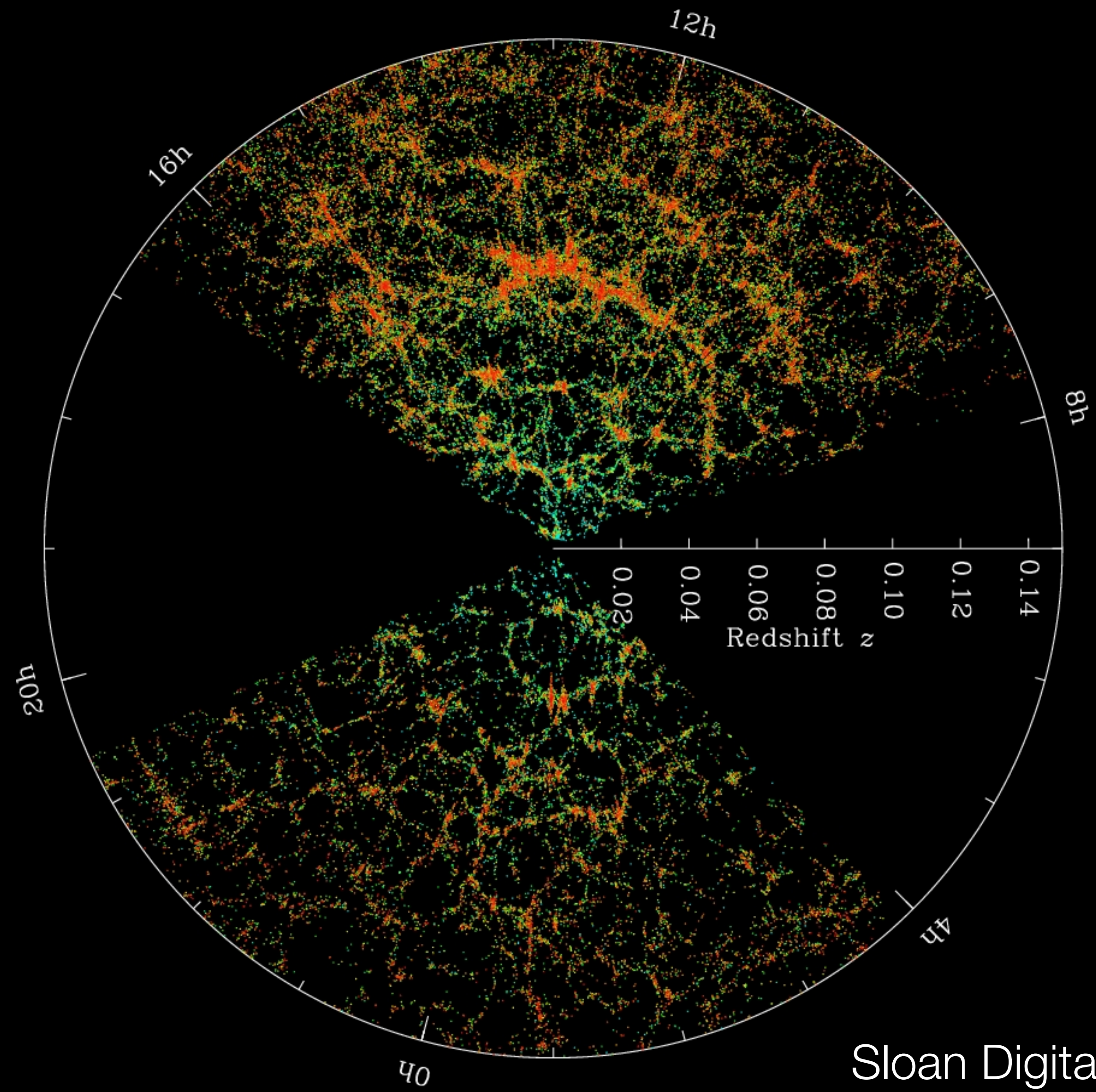


Big Bang Nucleosynthesis

- Creation of the first elements
  - H, D, He, Li, (Be)
- All in the First Three Minutes



Large Scale Structure



Sloan Digital Sky Survey

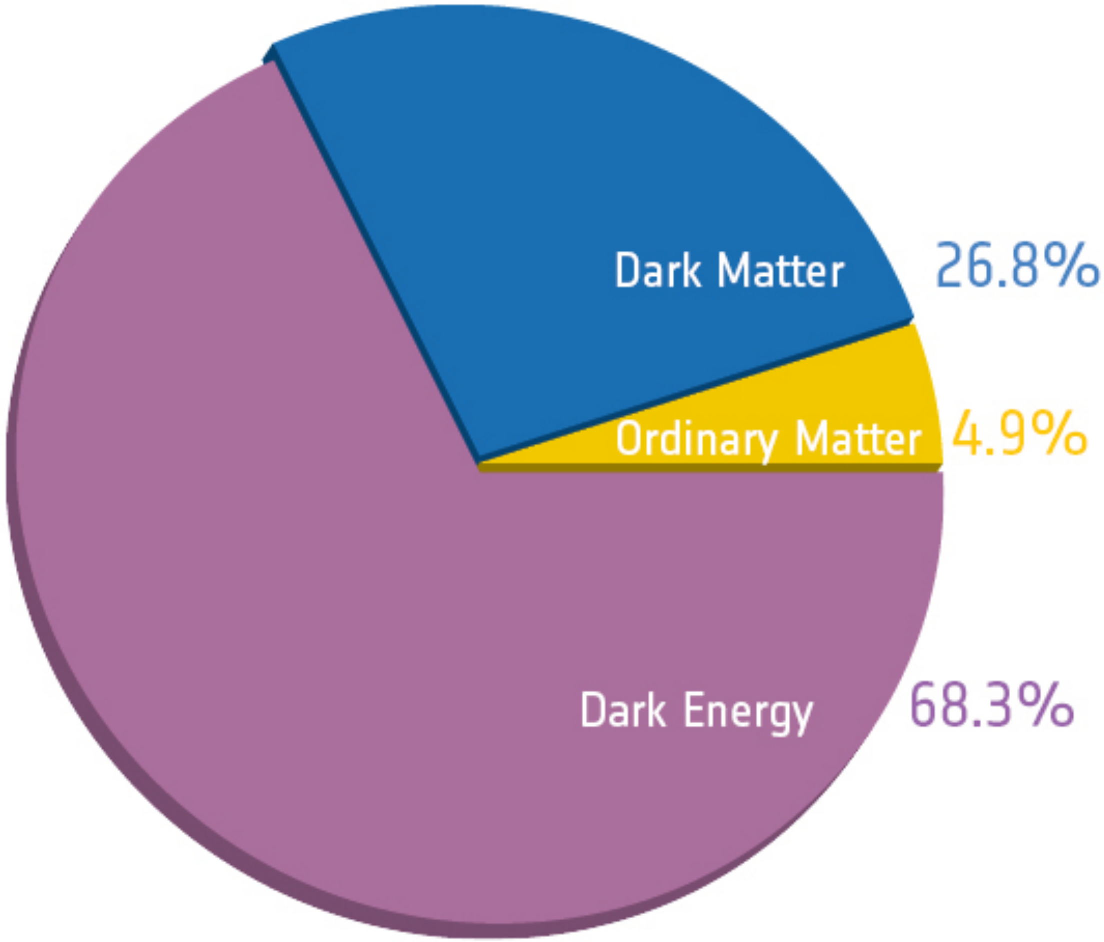
Simulating the universe

Simulating the Universe

Structure in the universe

- Forms out of the smooth background
- Growth of the seeds seen in the CMB
- Need of Dark Matter to form the structure of today
- Is the universe homogeneous?
- Interaction between baryons and dark matter?

Composition of the universe

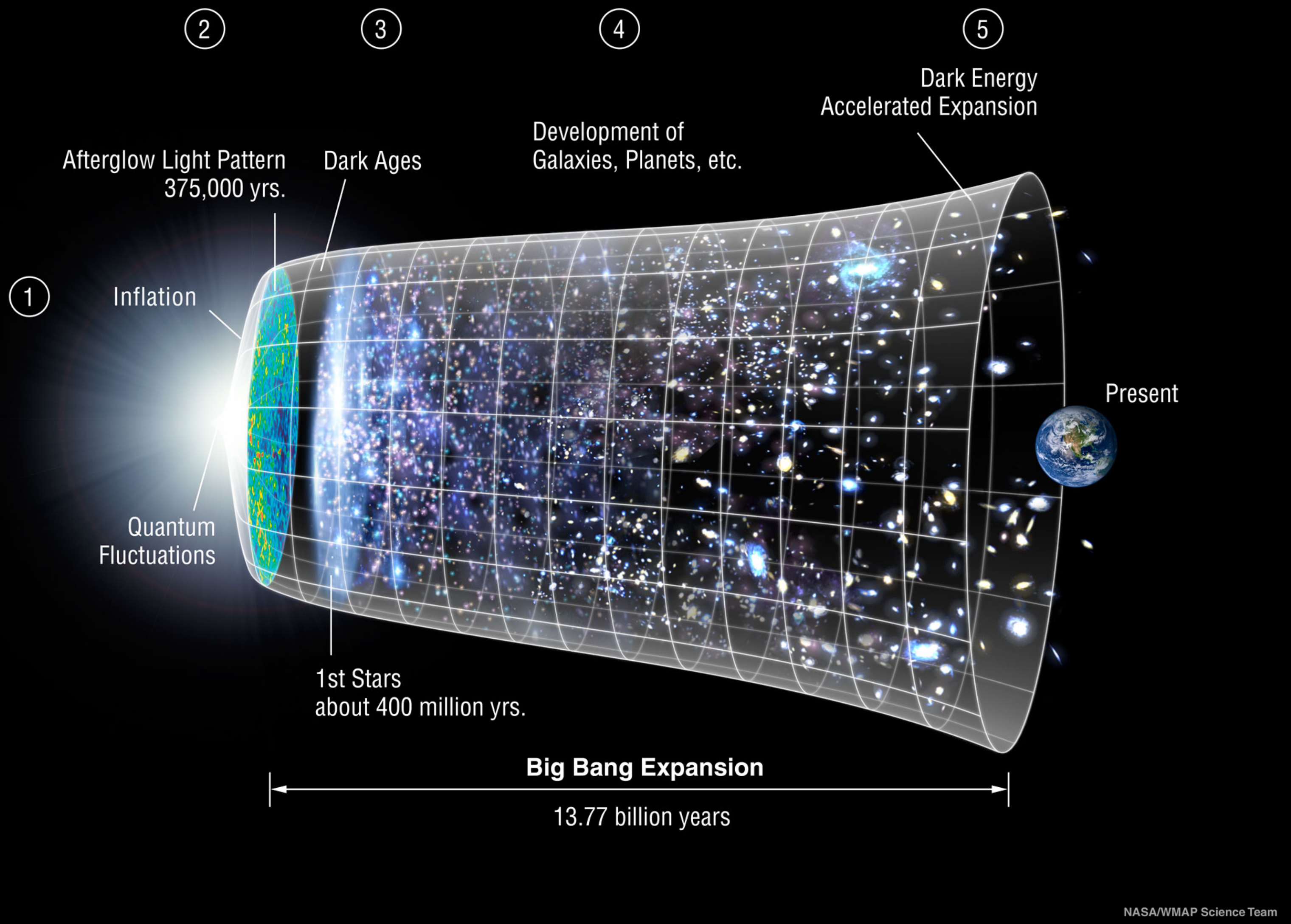


ESA/Planck

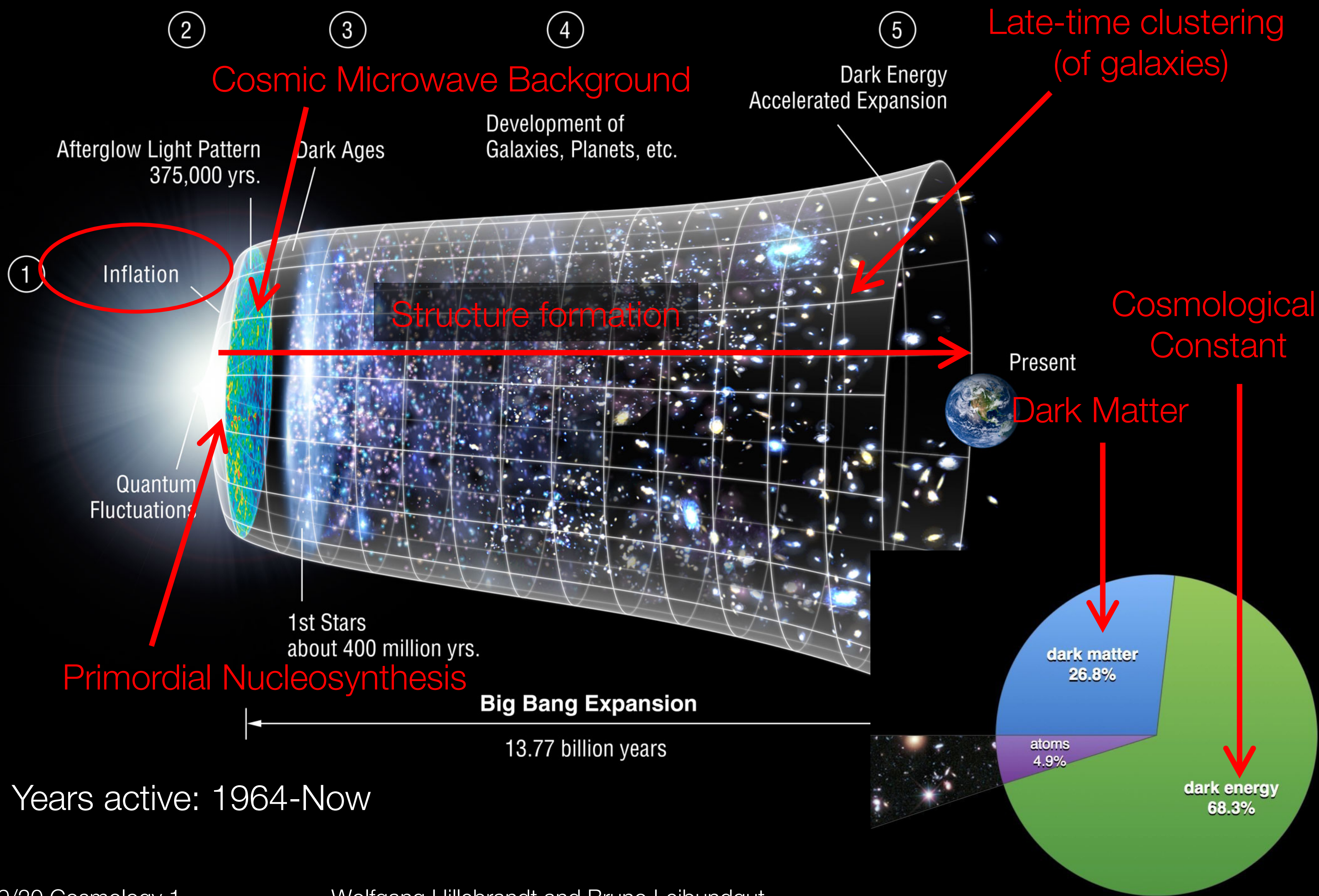
Composition of the universe

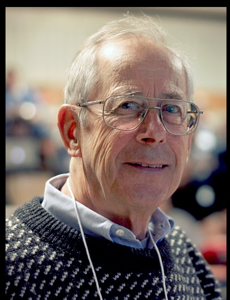
- What is Dark Energy?
- What is Dark Matter?
- Neutrinos?
- Radiation?
- Baryons are ‘special’

Our current picture of (the history of) the Universe



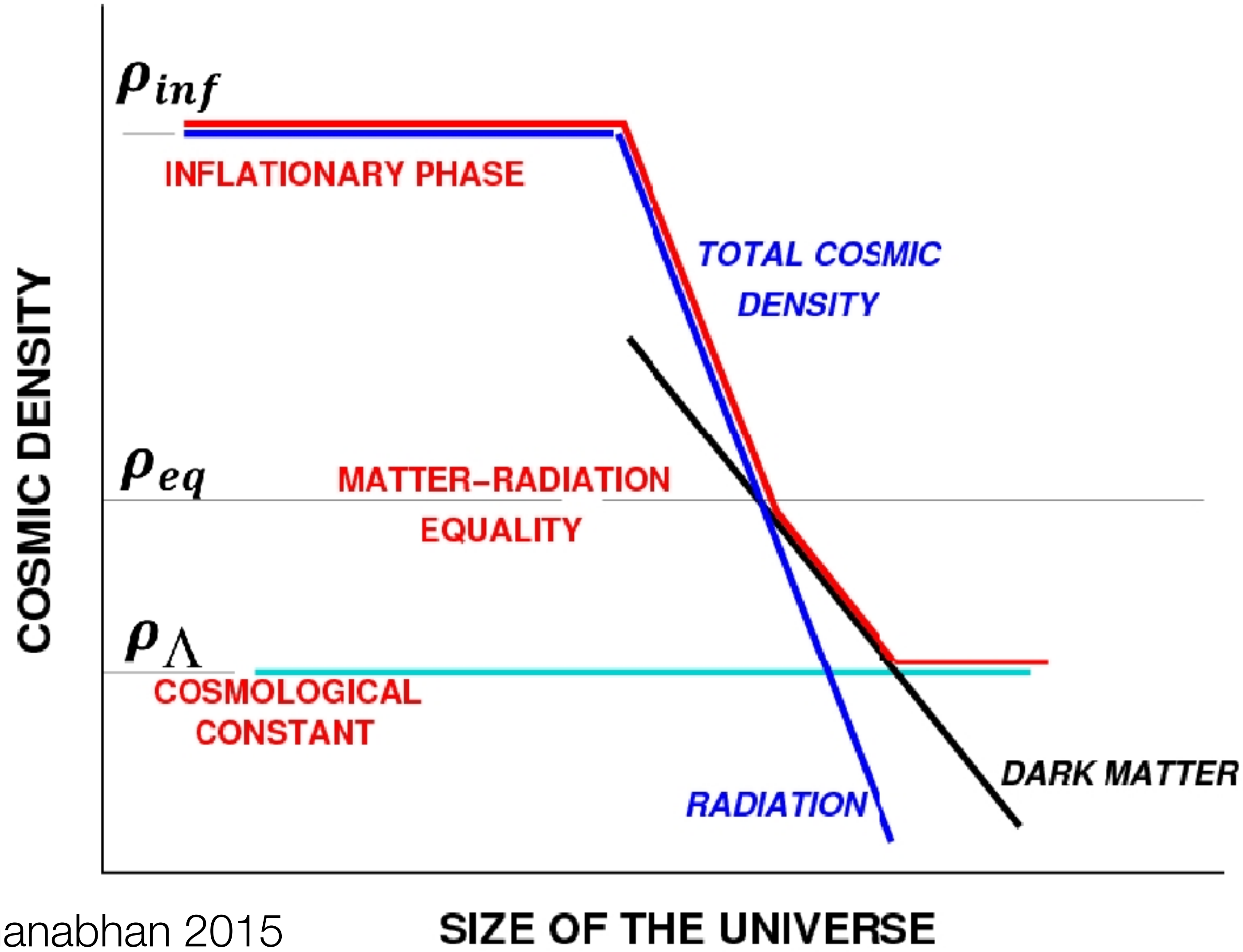
Our current picture of (the history of) the Universe





Years active: 1964-Now

Understand the evolution of the Universe



Padmanabhan 2015

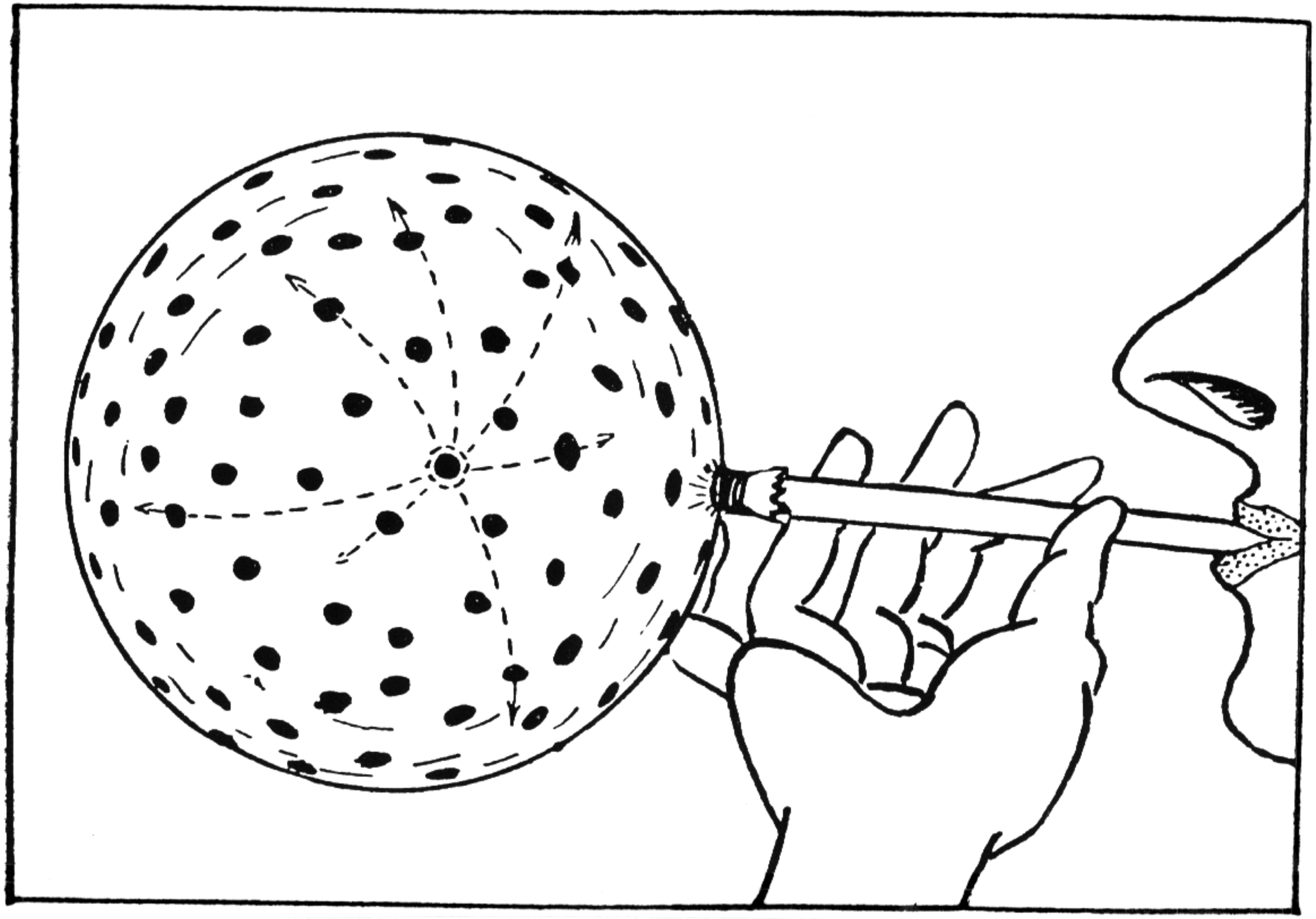
Forces in the Universe

- Gravity is the dominant force
- Nuclear forces only short range
  - only importance in the very early universe
- Electromagnetic force is based on charges
  - in a neutral universe not important
  - magnetic fields!
- Need a theory for gravity → relativity

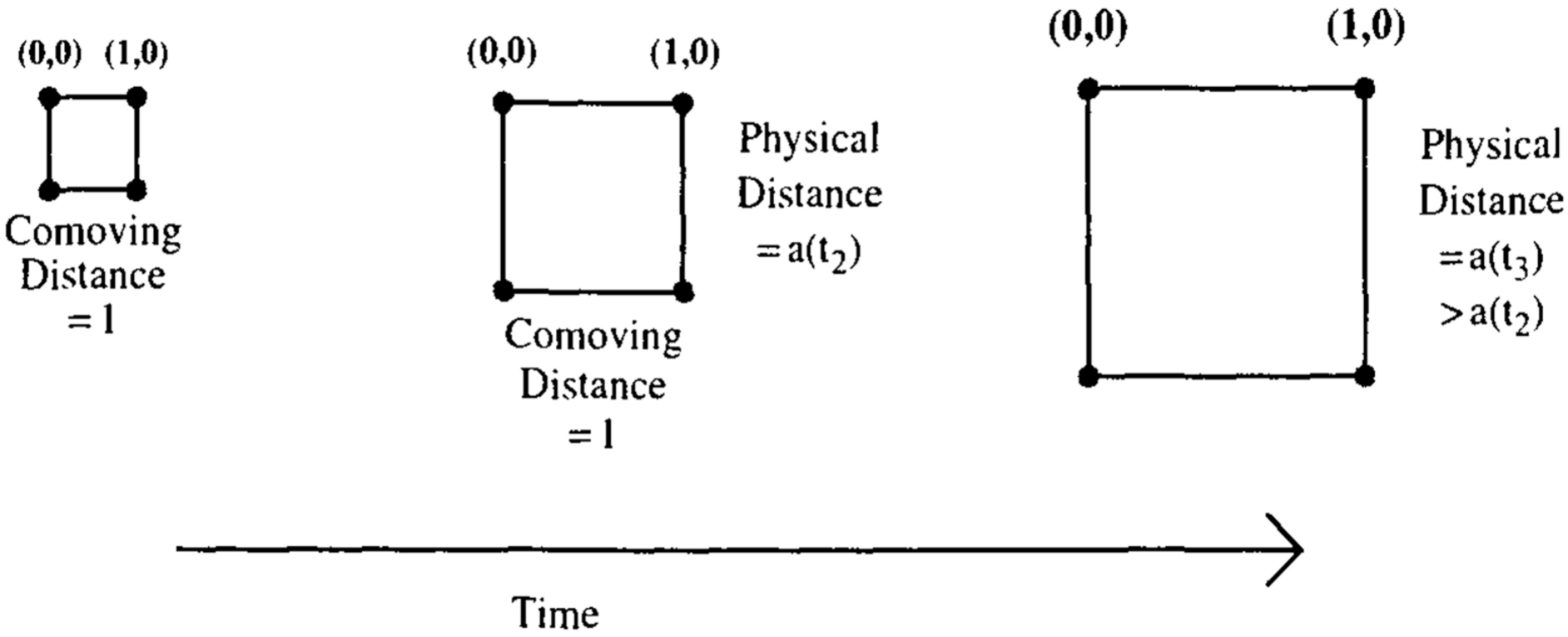
Metric

- Why do we have to bother with the metric?
  - Euclidian (flat space) is not good enough
- Reason: expansion of the universe
- Due to the expansion the coordinates are moving
  - need a translation from the coordinates to the physical distances
- Example: coordinates on an inflating balloon

Example of a moving coordinate system

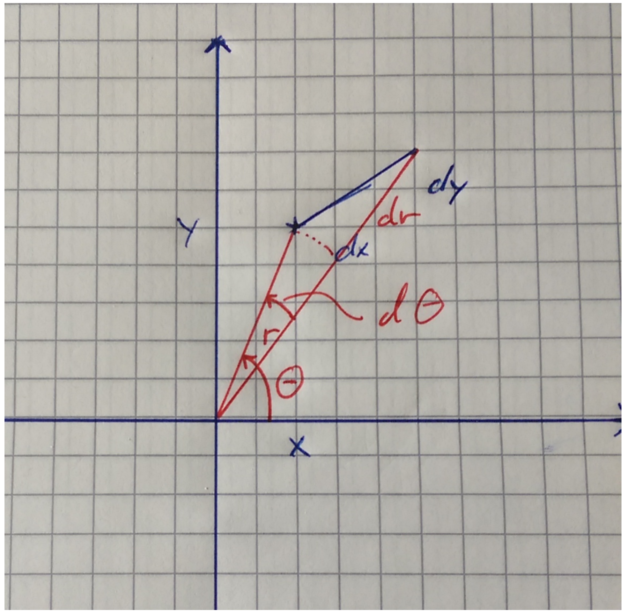


More formally



Calculating distances in general

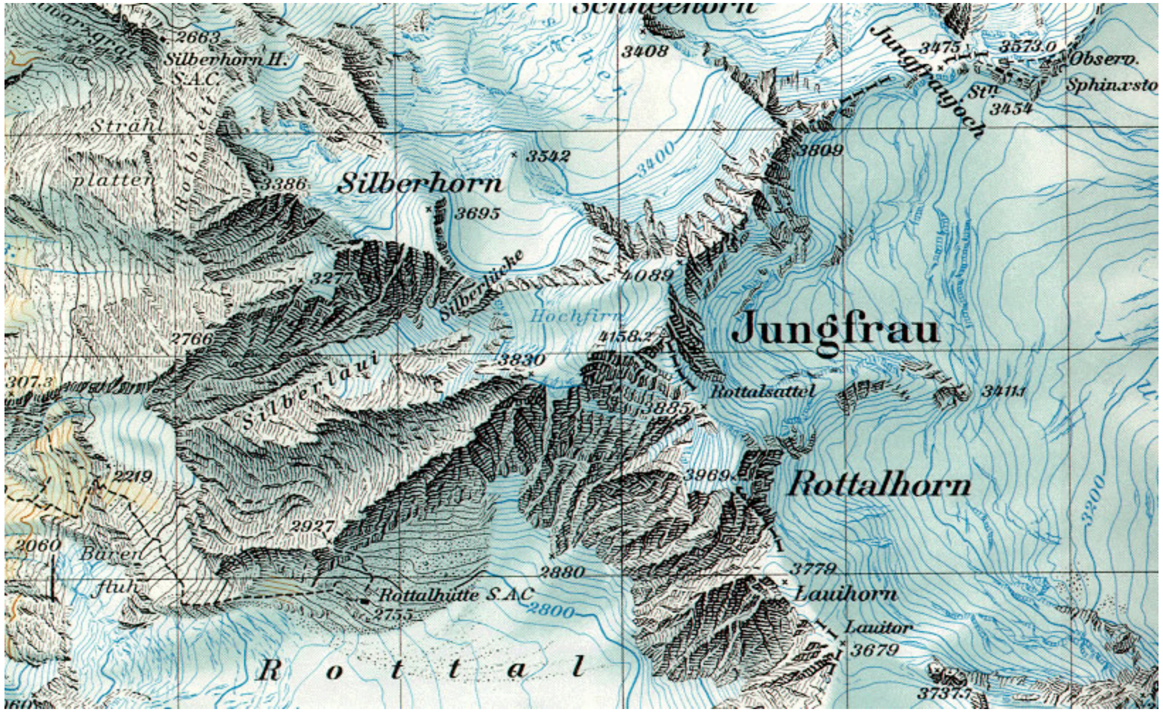
- Simple example of distances in flat space:
- Coordinates x and y
- Distance: $dl^2 = dx^2 + dy^2$ *(Euclid)*
- Coordinates r and Θ
- Distance: $dl^2 = dr^2 + r^2\, d\Theta^2$
- (think of crane) or Earth and Sky
- In general:
  $(dl)^2 = \Sigma_{i,j=1,2}\, g_{ij} dx^i\, dx^j$



An everyday r-Θ system

Calculating distances

- explicitly:
  - $$(dl)^2 = \begin{pmatrix} dx & dy \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} dr & d\Theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix} \begin{pmatrix} dr \\ d\Theta \end{pmatrix}$$
- in 3 dimensions think of contour lines on a map



Calculating distances

- In 4 dimensions (with time as the 0th coordinate) this becomes

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu \equiv g_{\mu\nu} dx^\mu dx^\nu$$

  using the (Einstein summation) convention where repeated indices are summed

- or explicitly:
  – Minkowski (flat) space

$$ds^2 = \begin{pmatrix} cdt & dx & dy & dz \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

Calculating distances

- Expanding universe with scale parameter a(t)

$$ds^2 = \begin{pmatrix} cdt & dx & dy & dz \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

  – called Friedmann-Robertson-Walker (FRW) metric for an isotropic and homogeneous universe

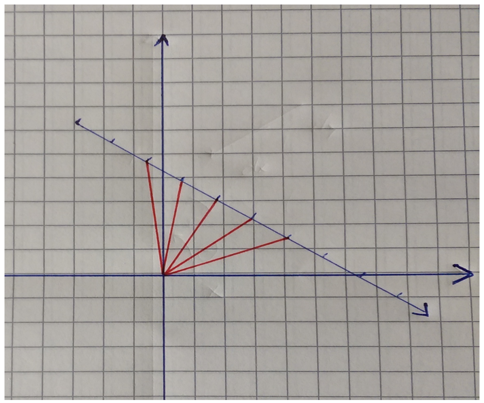
- ds^2 is proper space and the metric $g_{\mu\nu}$ is the conversion from the coordinates dx^μ

Calculating a geodesic in an expanding universe

- Simple case: Minkowski space (flat):
- movement of a force-free particle (geodesic):

$$\frac{d^2x^i}{dt^2} = 0$$ with $x^i = (x, y)$, i.e. Cartesian coordinates

- How would this look like in polar coordinates?
  – $x'^i = (r, \Theta)$
  – now $\frac{d^2x^i}{dt^2} \neq 0$!!!



Coordinate transformation

- need to transform to the polar coordinates

$$\frac{dx^i}{dt} = \frac{\partial x^i}{\partial x'^j}\frac{dx'^j}{dt}$$

  – with $\frac{\partial x^i}{\partial x'^j}$ as transformation matrix from one coordinate system to the other

  – in this special case:

    - $x^1 = x = r\cos(\Theta)$; $x^2 = y = r\sin(\Theta)$

Coordinate transformation

- Hence

$$\frac{\partial x^1}{\partial x'^1} = \frac{\partial x}{\partial r} = \cos\Theta \qquad \frac{\partial x^1}{\partial x'^2} = \frac{\partial x}{\partial \Theta} = -r\sin\Theta$$

$$\frac{\partial x^2}{\partial x'^1} = \frac{\partial y}{\partial r} = \sin\Theta \qquad \frac{\partial x^2}{\partial x'^2} = \frac{\partial y}{\partial \Theta} = r\cos\Theta$$

- Putting it all together

$$\frac{\partial x^i}{\partial x'^j} = \begin{pmatrix} \cos x'^2 & -x'^1 \sin x'^2 \\ \sin x'^2 & x'^1 \cos x'^2 \end{pmatrix} = \begin{pmatrix} \cos\Theta & -r\sin\Theta \\ \sin\Theta & r\cos\Theta \end{pmatrix}$$

Back to geodesics

- Back to the geodesic equation (no force):

$$\frac{d}{dt}\left[\frac{dx^i}{dt}\right] = \frac{d}{dt}\left[\frac{\partial x^i}{\partial x'^j}\frac{dx'^j}{dt}\right] = 0$$

- now use

$$\frac{d}{dt}\left(\frac{\partial x^i}{\partial x'^j}\right) = \frac{\partial}{\partial x'^j}\left(\frac{dx^i}{dt}\right) = \frac{\partial^2 x^i}{\partial x'^j \partial x'^k}\frac{dx'^k}{dt}$$

- and substitute in the equation above

$$\frac{d}{dt}\left[\frac{\partial x^i}{\partial x'^j}\frac{dx'^j}{dt}\right] = \frac{\partial x^i}{\partial x'^j}\frac{d^2 x'^j}{dt^2} + \frac{\partial^2 x^i}{\partial x'^j \partial x'^k}\frac{dx'^k}{dt}\frac{dx'^j}{dt} = 0$$

… some algebra …

- Goal: isolate the time derivatives
  - multiplying with the inverse of the transformation matrix $\left(\frac{\partial x^i}{\partial x'^j}\right)^{-1} = \left(\left\{\frac{\partial x}{\partial x'}\right\}^{-1}\right)^l_j$
- leads to $$\frac{d^2 x'^l}{dt^2} + \left[\left(\left\{\frac{\partial x}{\partial x'}\right\}^{-1}\right)^l_i \frac{\partial^2 x^i}{\partial x'^j \partial x'^k}\right] \frac{dx'^k}{dt}\frac{dx'^j}{dt} = 0$$
  - introduce the Christoffel symbol

$$\Gamma^l_{jk} = \left[\left(\left\{\frac{\partial x}{\partial x'}\right\}^{-1}\right)^l_i \frac{\partial^2 x^i}{\partial x'^j \partial x'^k}\right]$$

- Finally $$\frac{d^2 x'^l}{dt^2} + \Gamma^l_{jk}\frac{dx'^k}{dt}\frac{dx'^j}{dt} = 0$$

Geodesic

- General equation of a freely moving particle $$\frac{d^2x'^l}{dt^2} + \Gamma^l_{jk}\frac{dx'^k}{dt}\frac{dx'^j}{dt} = 0$$
- Note $\Gamma^l_{jk} = 0$ for Cartesian coordinates
- We need 4 dimension (time and space) and a new definition of time derivative

Geodesic

- Use progress along path (here denoted λ)

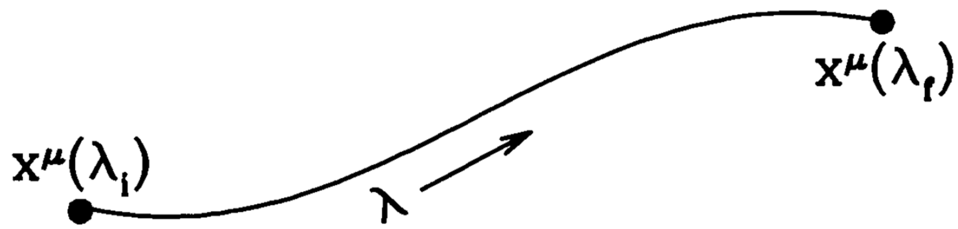


Figure 2.2. A particle's path is parametrized by λ, which monotonically increases from its initial value λ_i to its final value λ_f.

- General form

$$\frac{d^2 x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

  – moved to greek letters (to indicate the 4 dimensions)

Christoffel symbols

- Can be derived from the metric

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left[\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right]$$

- Note $g^{\mu\nu} = \left(g_{\mu\nu}\right)^{-1}$
- Examples:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = g^{\mu\nu} \qquad g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^{-2} & 0 & 0 \\ 0 & 0 & a^{-2} & 0 \\ 0 & 0 & 0 & a^{-2} \end{pmatrix}$$

Flat space (Minkowski) (smooth) expanding universe

Why all this?

We now have a general form for the force-free movement of a particle for **any** metric. Since the universe is expanding, we can now translate the metric (coordinates) into our physical universe. Einstein realised that the metric relates to matter and energy and hence we are half way to understand the dominant force in the universe.

Einstein's Field Equations

- Gravity described by the geometry (metric) and the energy-momentum

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

  – with $G_{\mu\nu}$ as the Einstein tensor, $R_{\mu\nu}$ the Ricci tensor, R the Ricci scalar ($R=g^{\mu\nu}R_{\mu\nu}$), G Newton constant, c speed of light and $T_{\mu\nu}$ the energy-momentum tensor.

The Energy-Momentum Tensor

- Use the form for the 'perfect fluid'

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$
A. EINSTEIN
13-7
2003
FOR WORLD
CUP
VOLENDAM

Recap Einstein Equations

- Gravity is the dominant force in the universe → General Relativity
- Need the most general form of the metric → transformations between coordinate systems
  - find 'invariant' parameters
- Equation of motion for a force-free particle ($\ddot{x} = 0$) in GR leads to affine connections → Christoffel symbols
- Putting this together with the geometry and the energy content → Einstein Equations

Einstein's Field Equation

- The (time) evolution of the scale factor depends only on the time-time component of the Einstein equation:

$$R_{00} - \frac{1}{2} g_{00} R = \frac{8\pi G}{c^4} T_{00}$$

  - $T_{00}=\rho c^2$ *(energy density)*
  - *time part* $R_{00} - \frac{1}{2} g_{00} R = \frac{3}{c^2}\left(\frac{\dot{a}}{a}\right)$

Friedmann equation

- The equation governing the expansion of the (flat) universe is

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2(t) = \frac{8\pi G}{3}\rho(t)$$

- and dividing by the Hubble constant H_0

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{crit}} \equiv \Omega$$

  - with $\rho_{crit} = \frac{3H_0^2}{8\pi G} \approx 10^{-26}\,kg/m^3$

- ρ*(t)* includes all energy forms in the universe

Curved geometry

- Consider the spatial part

$$dl^2 = d\mathbf{x}^2$$

- This is invariant under translations and rotations of the coordinate system

$$dl^2 = d\mathbf{x}^2 + dz^2; \mathbf{x}^2 + z^2 = a^2$$

- This is also true for the hyperbolic case

$$dl^2 = d\mathbf{x}^2 - dz^2; \mathbf{x}^2 - z^2 = a^2$$

- rescale with ***x'*=*ax*** and *z'*=*az*

Curved space

- gives

$$dl^2 = a^2[d\mathbf{x}^2 \pm dz^2]; z^2 \pm \mathbf{x}^2 = 1$$

- Differentiating $z^2 = 1 \mp \mathbf{x}^2$ gives $zdz = \mp \mathbf{x} d\mathbf{x}$
- $$dl^2 = a^2\left[d\mathbf{x}^2 \pm \frac{(\mathbf{x}d\mathbf{x})^2}{1 \mp \mathbf{x}^2}\right]$$
- and in general

$$dl^2 = a^2\left[d\mathbf{x}^2 + k\frac{(\mathbf{x}d\mathbf{x})^2}{1 - k\mathbf{x}^2}\right]$$

- with

$$k = \begin{cases} +1 & \text{spherical} \\ -1 & \text{hyperspherical} \\ 0 & \text{flat (Euclidian)} \end{cases}$$

Curved space

- The line element becomes

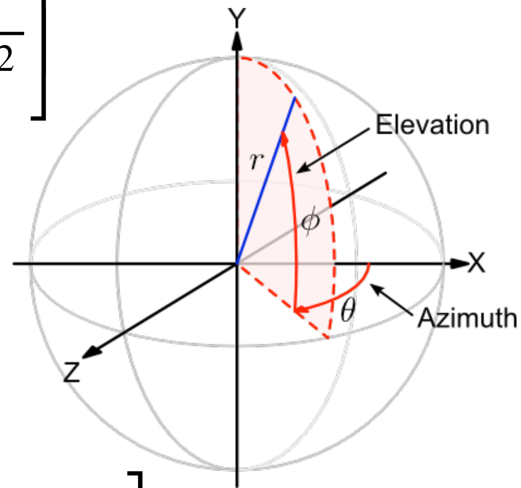
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2(t)\left[d\mathbf{x}^2 + k\frac{(\mathbf{x}d\mathbf{x})^2}{1-k\mathbf{x}^2}\right]$$

- The elements of the metric are then

$$g_{00} = -1; \quad g_{i0} = 0; \quad g_{ij} = a^2(t)\left[\delta_{ij} + k\frac{x^i x^j}{1-k\mathbf{x}^2}\right]$$

- Consider polar coordinates

$$d\mathbf{x}^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi\right)$$



- leads to

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Curved space

- With the (diagonal) metric

$$g_{00} = -1; \quad g_{rr} = \frac{a^2(t)}{1-kr^2}; \quad g_{\theta\theta} = a^2(t)r^2; \quad g_{\phi\phi} = a^2(t)r^2\sin^2\theta$$

- Going through the same steps again to calculate the contributions in the Einstein equations and then determine the Friedmann equation for curves space

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t)$$

Gravity in Einstein's Equations

- Consider an enclosed mass in a sphere

$$M(x) = \frac{4\pi}{3}\rho_0 x^3 = \frac{4\pi}{3}\rho(t) r^3(t) = \frac{4\pi}{3}\rho(t) a^3(t) x^3$$

  – here we converted the fixed density in comoving coordinates first into the density in the observed coordinate and then replaced it with the expansion factor

  – in principle this resulted in $\rho_0 = \rho(t) a^3(t)$

Gravity in Einstein's Equations

- Acceleration of a particle on the surface of the sphere is

$$\ddot{r}(t) = \frac{d^2 r}{dt^2} = -\frac{GM(x)}{r^2} = -\frac{4\pi G}{3}\frac{\rho_0 x^3}{r^2}$$

  – now use $r(t) = a(t)x$ to change to the expansion factor

$$\ddot{a}(t) = \frac{\ddot{r}(t)}{x} = -\frac{4\pi G}{3}\frac{\rho_0}{a^2(t)} = -\frac{4\pi G}{3}\rho(t)a(t)$$

- This is the gravitational part of the field equations – GR modifies this part

Friedmann equation

- Time evolution of the scale factor is described through the time part of the Einstein equations
- Assume a metric for a homogeneous and isotropic universe (metric is diagonal in polar coordinates) and a perfect fluid

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t)$$

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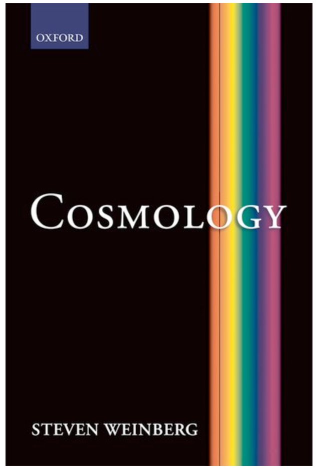

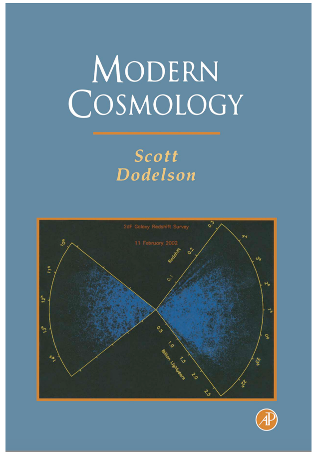

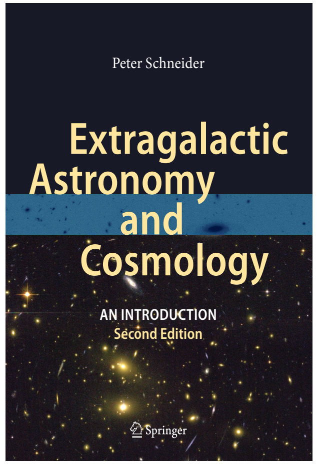

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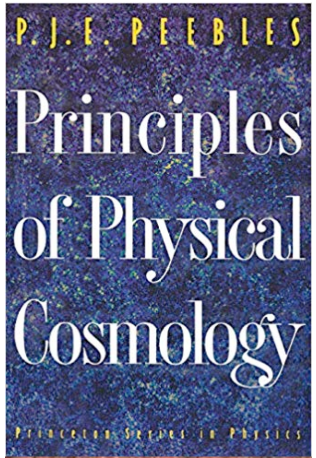

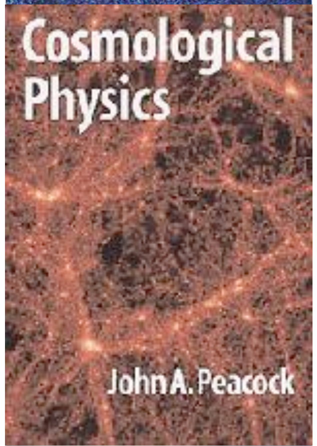

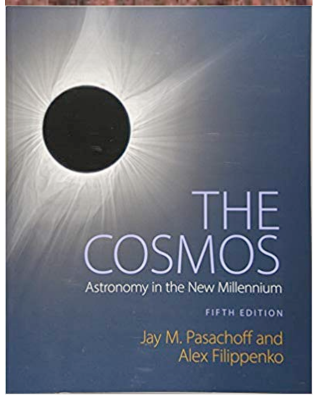

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