PiTP 2009: "Computational Astrophysics" Lectures on Collisionless Dynamics and SPH

Exercise Set 3

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1 SPH in two dimensions: Sampling noise and the clumping instability

This exercise carries out a couple of simple experiments with 2D dimensional SPH in a squared region. We start with an innocent (but slightly naive) attempt to construct a homogenous particle load. This leads to a surprise, and prompts us to think a bit more about the kernel.

1.1 Making an equilibrium particle configuration in SPH

A Cartesian grid is an obvious possibility to realize a constant density gas, but it has preferred directions and is therefore not natural for the SPH formalism. Instead, it is sometimes desirable to use an irregular distribution where the particles are spread out evenly and the repulsive pressure forces all cancel, yet preferred directions are avoided. Such a thing we could call an 'SPH-glass' in analogy to the gravity glasses that are sometimes used for CDM initial conditions.

Let's try to create such a thing in a simple way. Make a random distribution of $N = 1000$ particles in a periodic square box of unit length on a side (i.e. a Poisson sample in 2D). Assign unit density and pressure to the particles. (An example of how this could be done in IDL is provided on the web-site.) The density fluctuations in these initial conditions will trigger some gas motions that we can damp away with a moderate setting of the artificial viscosity.

Run this set-up with the Gadget2 code in 2D SPH mode (TWODIMS switch). Make sure that you switch of gravity in the makefile (NOGRAVITY switch). In the parameterfile, set the system of units factors all to 1, and ask the code to evolve the system from

0 to 10 time units, with relatively finely spaced outputs in time (e.g. with TimeBetSnapshot=0.05) such that you can take a look at the particle motion when you display the dumps as particle dot plots.

It is normally expected that the kernel interpolation approach of SPH should become more accurate when a larger number of neighbors is used for the smoothing. With this in mind, let's set DesNumNgb = 96 in the parameterfile, motivated by the desire to have low noise in the density field. Pick a relatively small viscosity, say ArtBulkViscConst=0.3.

Look carefully at the motion of the points, and the final particle configuration. What happened here? Also, create a scatter plot of the density values at the final time, and look at the *rms* density fluctuations as a function of time.

1.2 The role of the number of neighbors

Repeat the experiment above with a smaller number of neighbors, say DesNumNgb = 18. Again, look at the final particle configuration and the density noise. What changed? What's determining the neighbor number that one should pick? What should be taken in 3D?

1.3 Kernel steepening

One idea to suppress the clumping instability that you witnessed in the first simulation lies in modifying the kernel shape, preventing that the repulsive force smoothly drops to zero towards the origin, see e.g. Steinmetz (1996, MN, 278, 1005), or the recent paper by Read et al. (2009, arXiv:0906.0774).

The kernel normally used by Gadget2 for SPH is the standard cubic spline,

$$W(u) = \frac{8}{\pi} \begin{cases} 1 - 6u^2 + 6u^3, & 0 \le u \le \frac{1}{2}, \\ 2(1-u)^3, & \frac{1}{2} < u \le 1, \\ 0, & u > 1. \end{cases} \tag{1}$$

which is here normalized for 3D use.

Construct a new kernel $\tilde{W}$ which has the same first derivate as W, except for all radii smaller than the radius u_0 where the minimum of $W'(u)$ is attained. For radii $u < u_0$ simply keep this value of the derivative, i.e. set $\tilde{W}'(u) = \min(W')$ for $u < u_0$. The new kernel will then still have a continuous second derivative, except at the origin, where it will be peaky. Renormalize the new kernel properly for 2D use.

Now, let's locate the kernel evaluation routines in Gadget2, which you can find in the files *density.c* and *hydro.c*, approximately at lines 543 and 463/483, respectively. Create a patched copy of Gadget2 where you implement your new kernel shape with the 'peaked kernel'. Then repeat the run with 96 neighbors from above. What has changed?

1.4 Interpolation accuracy

Are such ad-hoc modifications of the kernel shape 'allowed'? They are certainly allowed in principle, but what ultimately counts is the accuracy that is achieved.

Look at the density estimates at the final time produced by the run with 96 neighbors and the peaky kernel. How well is the mean density (which should be unity) reproduced? Now repeat the run with just 18 neighbors and the peaked kernel. Look at the mean density that is estimated in this case, and compare with the result obtained with the ordinary kernel. Explain the difference. What does the result imply for the utility of the peaked kernel shape?

1.5 Making a good SPH glass

Go back to your simulation with the ordinary SPH kernel and 18 neighbors. For the final output, look at a scatter plot of the density versus the x-coordinate and similarly for the pressure. Why is the scatter in the density apparently larger than in the pressure?

Now go back to the initial conditions code and activate *flag_entropy* in the header of the initial conditions file. Also, in the *u*-block of the ICs, store P/ρ^γ instead of $u = P/[(\gamma-1)\rho]$. This means you initialize the entropic function of each particle instead of the temperature. Furthermore, disable the production of entropy in the code – to this end, locate line 320 in *hydra.c* and set the *DtEntropy* variable to zero there. Run again, perhaps also with a higher setting of the artificial viscosity. Look at the density scatter at the final time. Why is this approach for making a 'glass' better than the ones considered above?