Ensenada
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Galaxy halos at (very) high resolution

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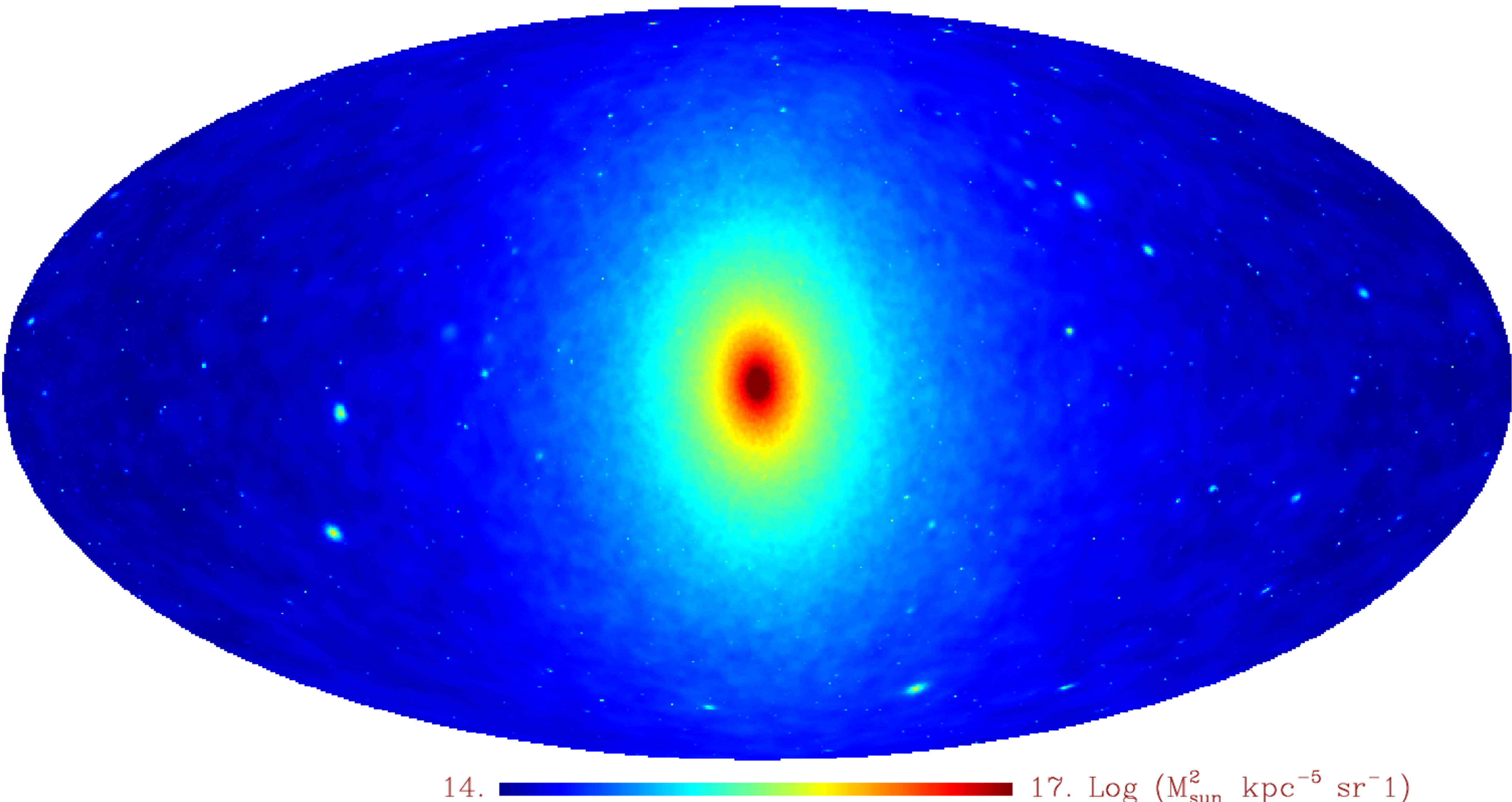
Milky Way halo seen in DM annihilation radiation

Aquarius simulation: $N_{200} = 190{,}000{,}000$



Small-scale structure of the CDM distribution

- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
  - -- what is the density structure between m and kpc scales?
  - -- how many streams intersect the detector at any time?
- Intensity of annihilation radiation depends on

$$\int \rho^2(\boldsymbol{x}) \langle \sigma v \rangle \, dV$$

  - -- what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales <u>far</u> below those accessible to simulation

→ We require a good theoretical understanding of mixing

Cold Dark Matter at high redshift (e.g. $z \sim 10^5$)

Well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, their distribution function is

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x})]\, N\,[\{\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x})\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,

$\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$

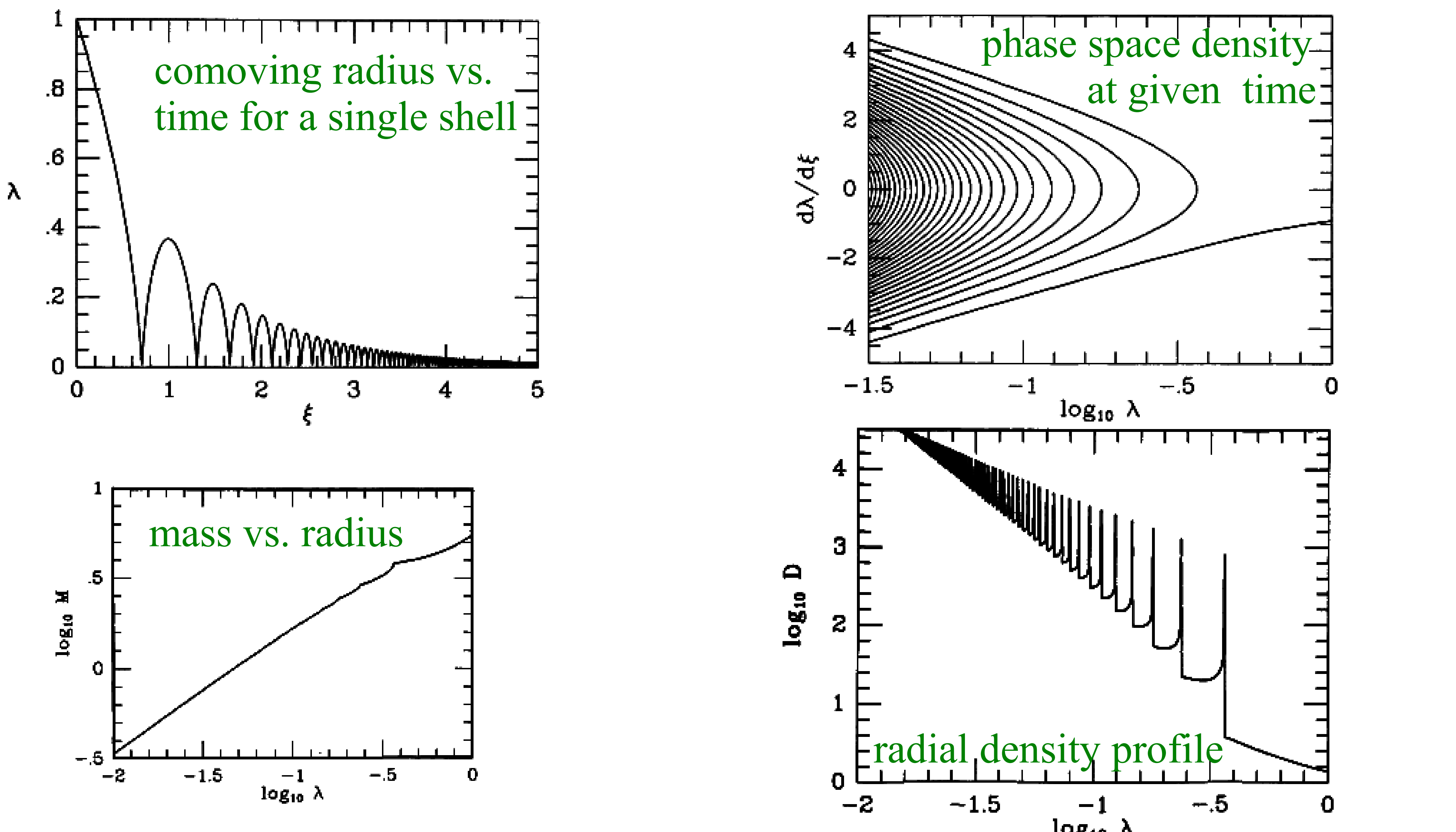
and N is standard normal with $\sigma^2 << \langle|\mathbf{V}|^2\rangle$

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

$Df\,/\,Dt = 0$ → only a 3-D subspace is occupied at later times. Nonlinear evolution leads to a complex, multi-stream structure.

Similarity solution for spherical collapse in CDM

Bertschinger 1985



Evolution of CDM structure

Consequences of $Df / Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least 1 sheet must pass through every point $\mathbf{x}$
- In nonlinear objects there are typically many sheets at each $\mathbf{x}$
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f =$ const.
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot\delta X\,;\ \ T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = I$$

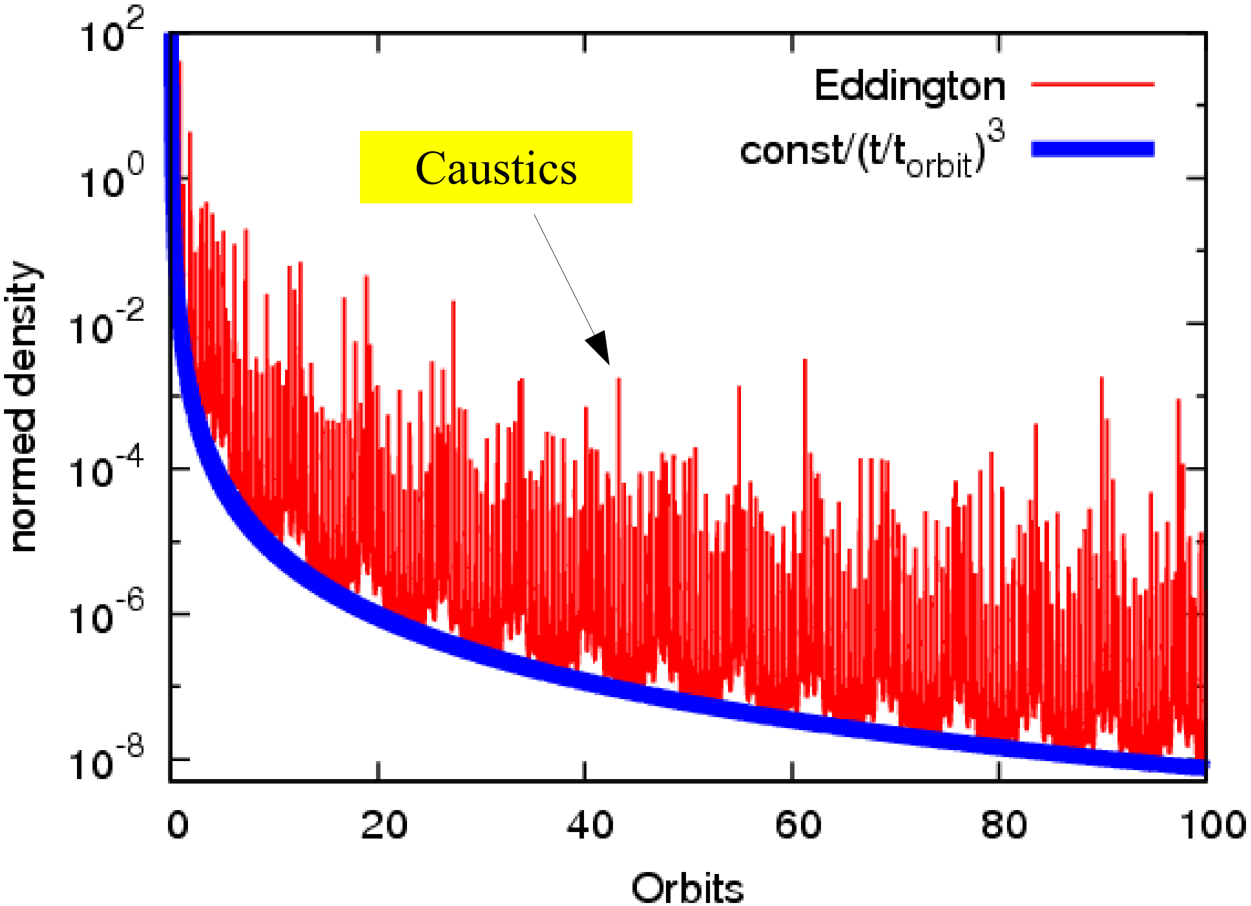
- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- det(D) = 1 at all times by Liouville's theorem
- For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

Static highly symmetric potentials

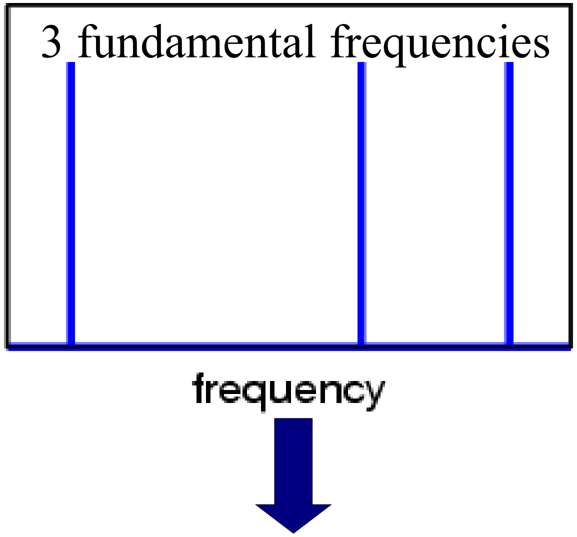
Mark Vogelsberger, Amina Helmi, Volker Springel

Axisymmetric Eddington potential

$$\Phi(r,\theta) = v_{\rm h}^2 \log\left(r^2 + d^2\right) + \frac{\beta^2 \cos^2\theta}{r^2}$$



Spectral analysis of orbit:

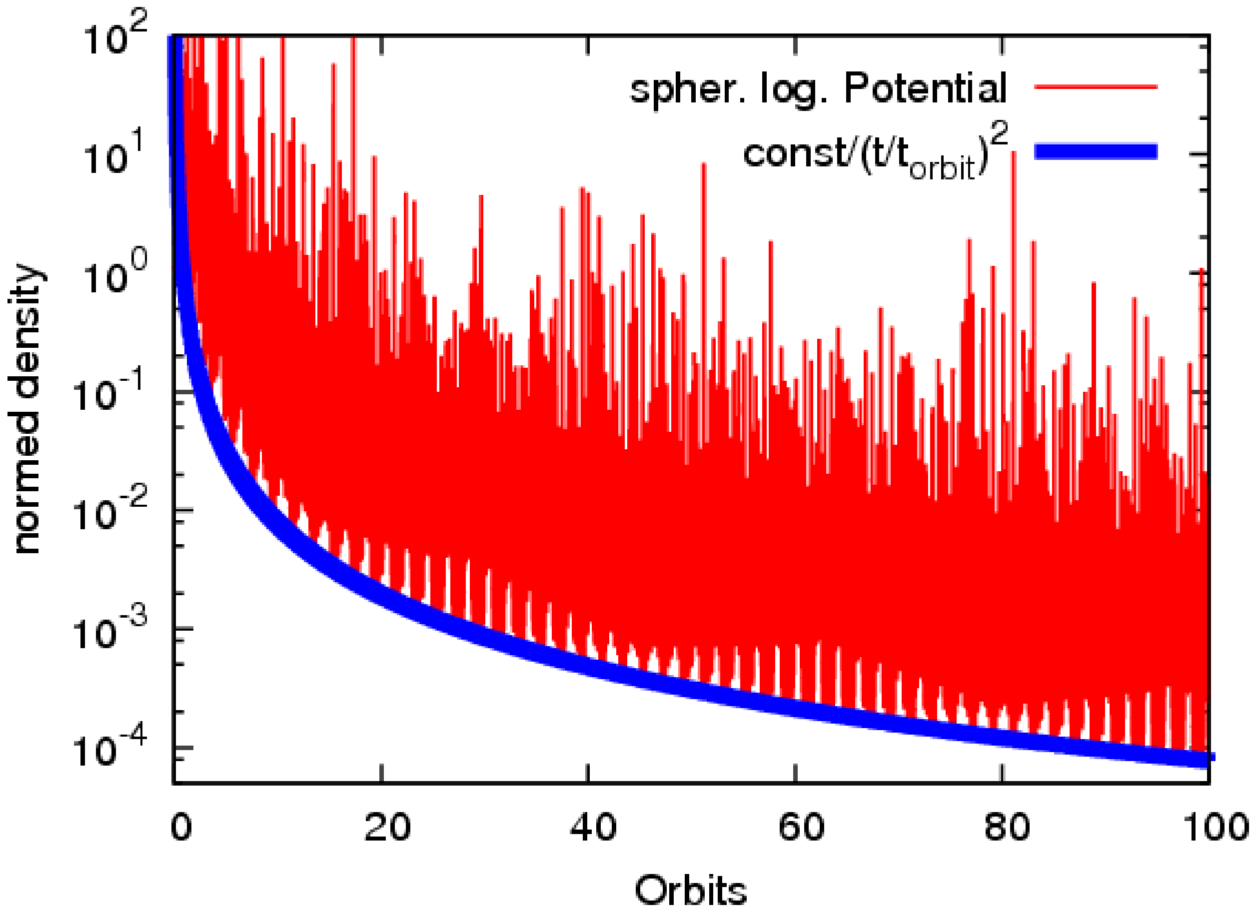


density decreases like $1/t^3$

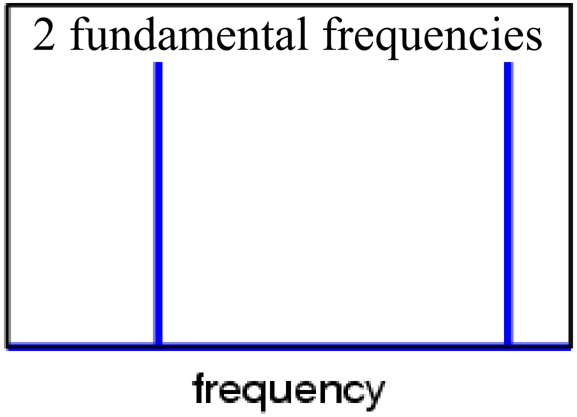
Changing the number of frequencies

Spherical logarithmic potential

$$\Phi(r,\theta) = v_{\rm h}^2 \log\left(r^2 + d^2\right)$$



Spectral analysis of orbit:



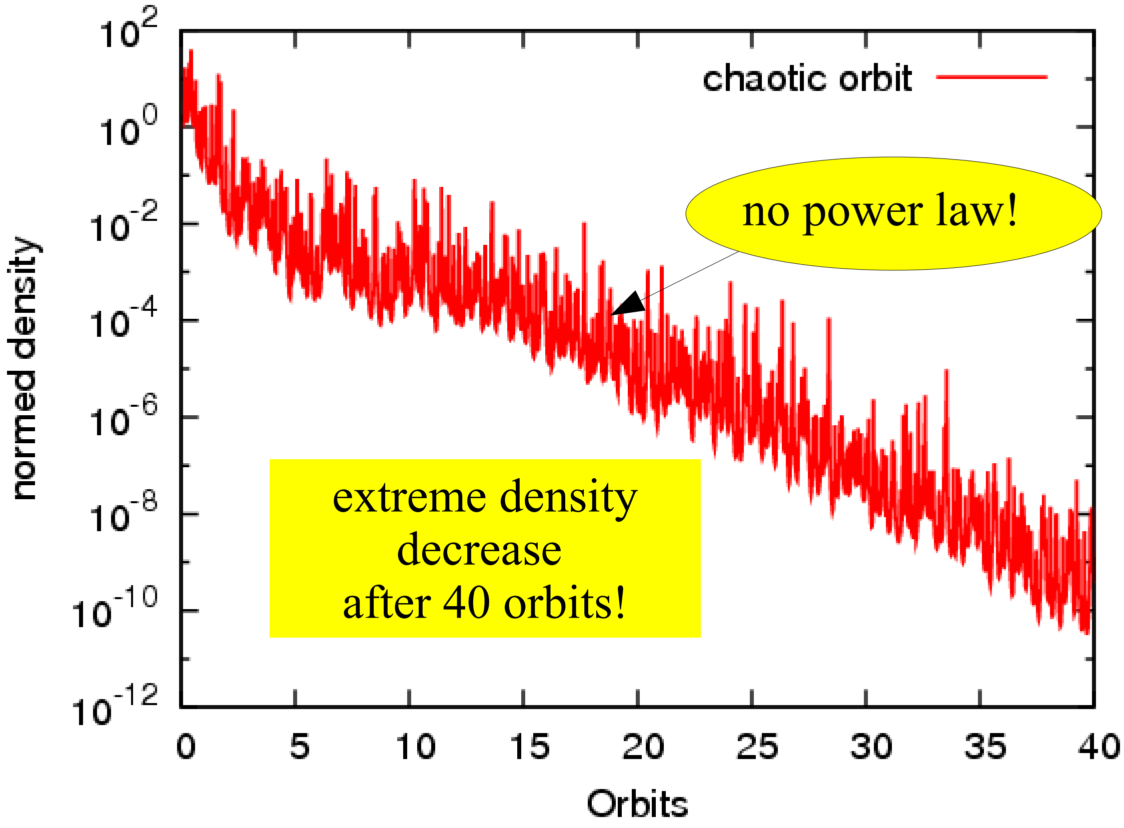
density decreases like $1/t^2$

Number of fundamental frequencies dictates the density decrease of the stream

Chaotic mixing

chaotic motion implies a **rapid stream density decrease** → **rapid mixing**

↳ density decrease is **not** like a power law anymore

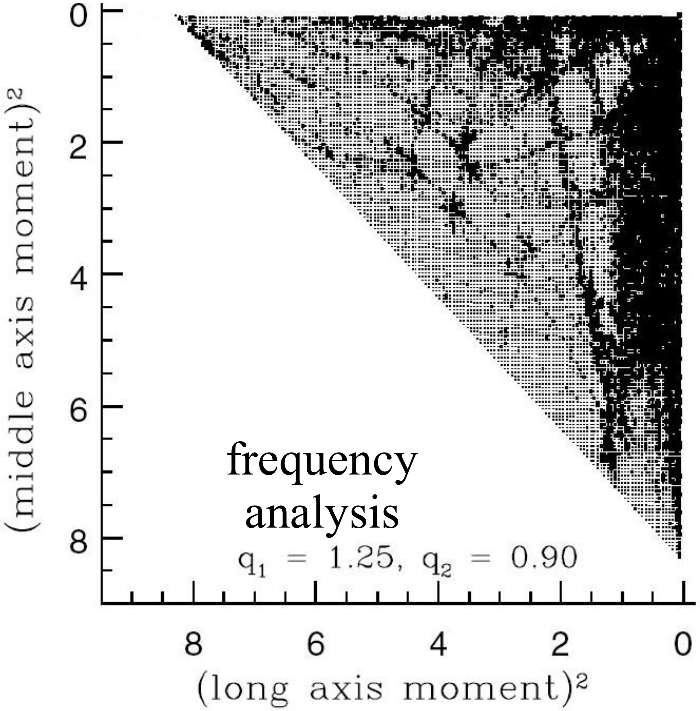


how to find chaotic regions in phase space?

Common method:

- Lyapunov exponents
- frequency analysis (NAFF)
- ...

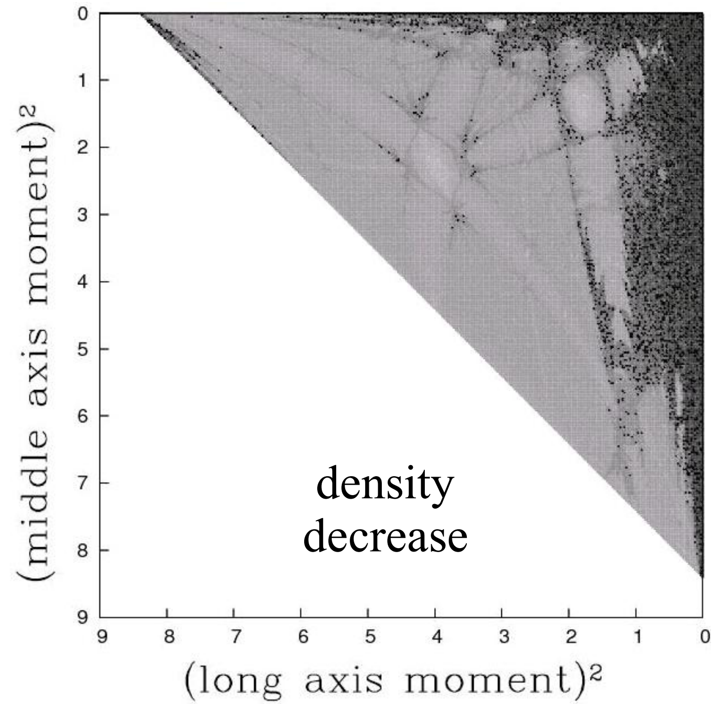
Compare frequency analysis results with geodesic deviation equation results



moderate triaxiality

small fraction of chaotic orbits

stream density mostly decaying like a power law



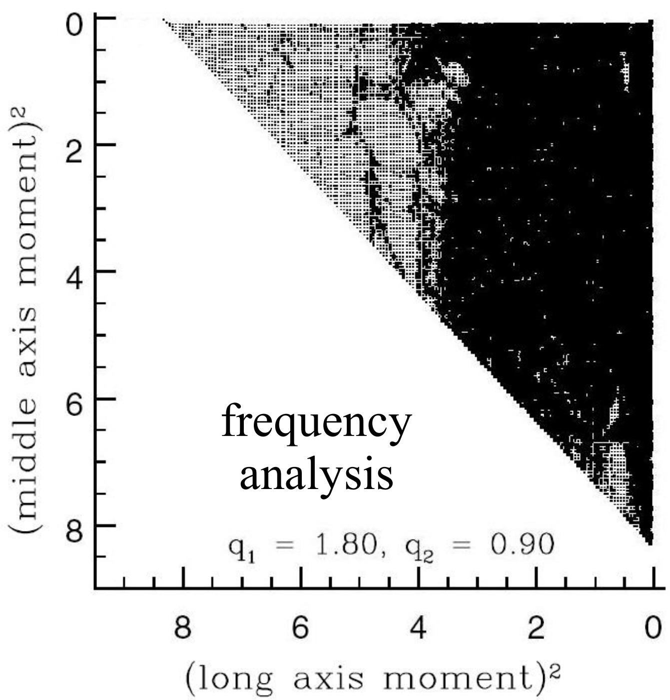
Papaphilippou & Laskar 1998

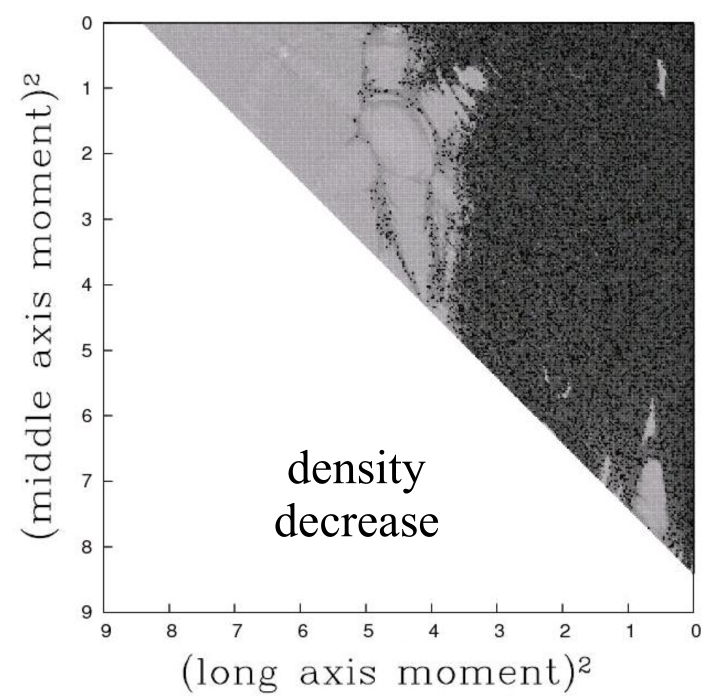
high triaxiality

large fraction of chaotic orbits

stream density mostly decaying much faster than a power law

integrate 10^5 different orbits

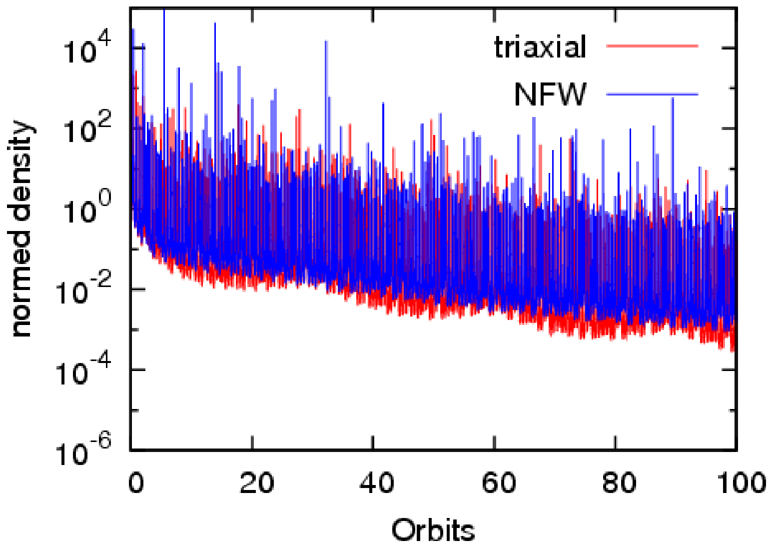




Dark matter streams in a triaxial NFW

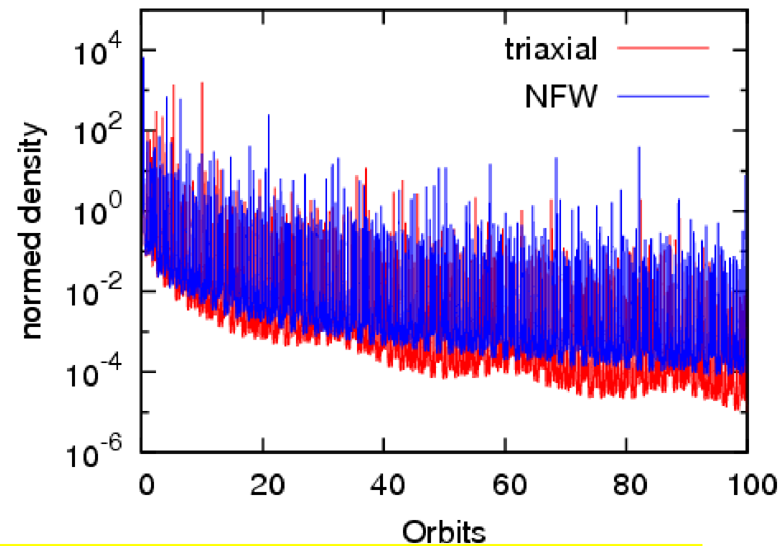
Inner potential shape a : b : c ~ 1.0 : 0.9 : 0.8

outer orbit: similar stream behaviour in spherical and triaxial cases



Apo: $8.9r_s$

Peri: $6.1r_s$

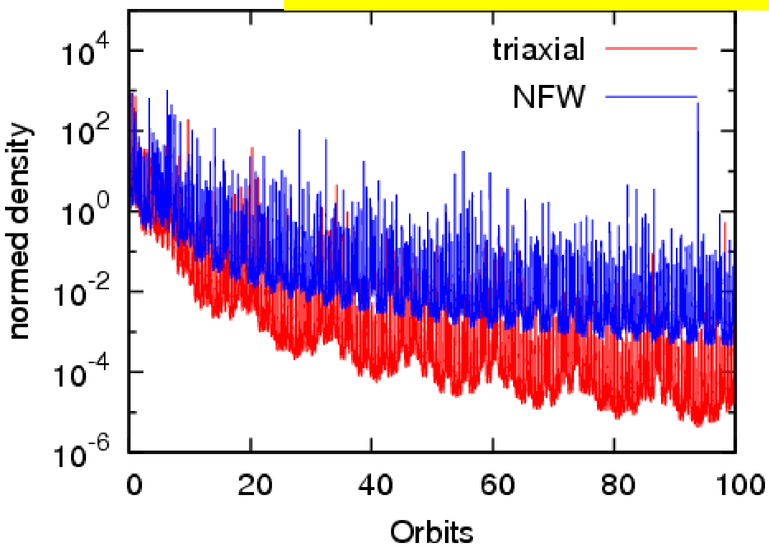


Apo: $8.7r_s$

Peri: $4.8r_s$

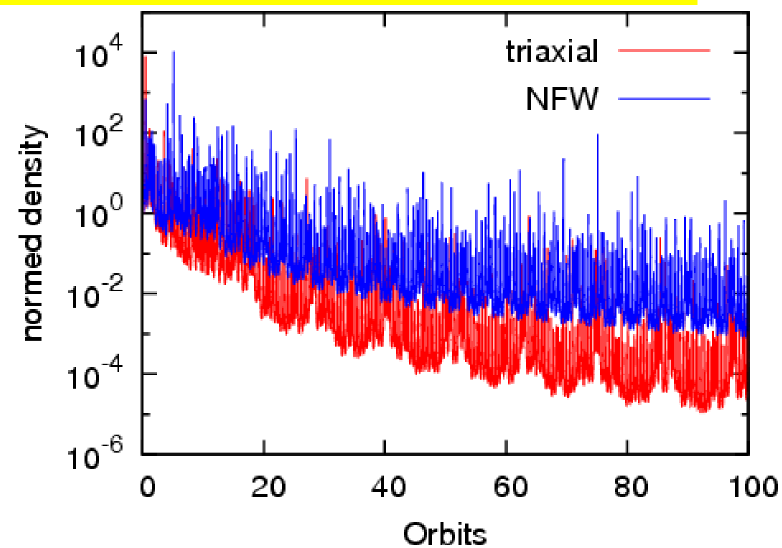
inner orbits: ~100 times lower stream density after 100 orbits

➡ 100 times more sheets at each point



Apo: $2.2r_s$

Peri: $0.5r_s$

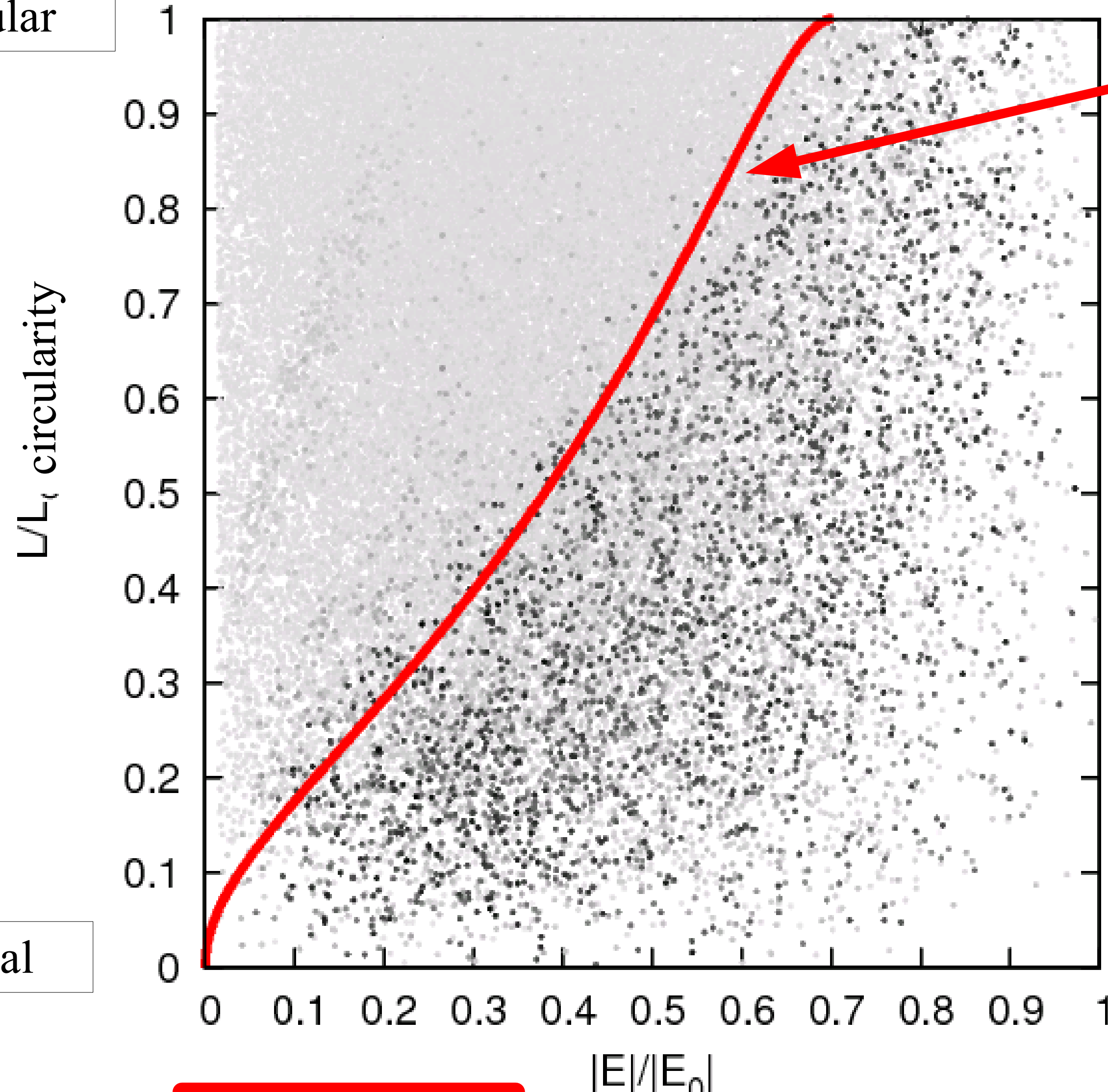


Apo: $1.2r_s$

Peri: $0.4r_s$

Chaotic mixing in a triaxial NFW?

circular



radial

constant pericentre:
~1/3 NFW scale radius

~40.000 orbits

Orbits reaching inner part of the halo are affected by the triaxiality

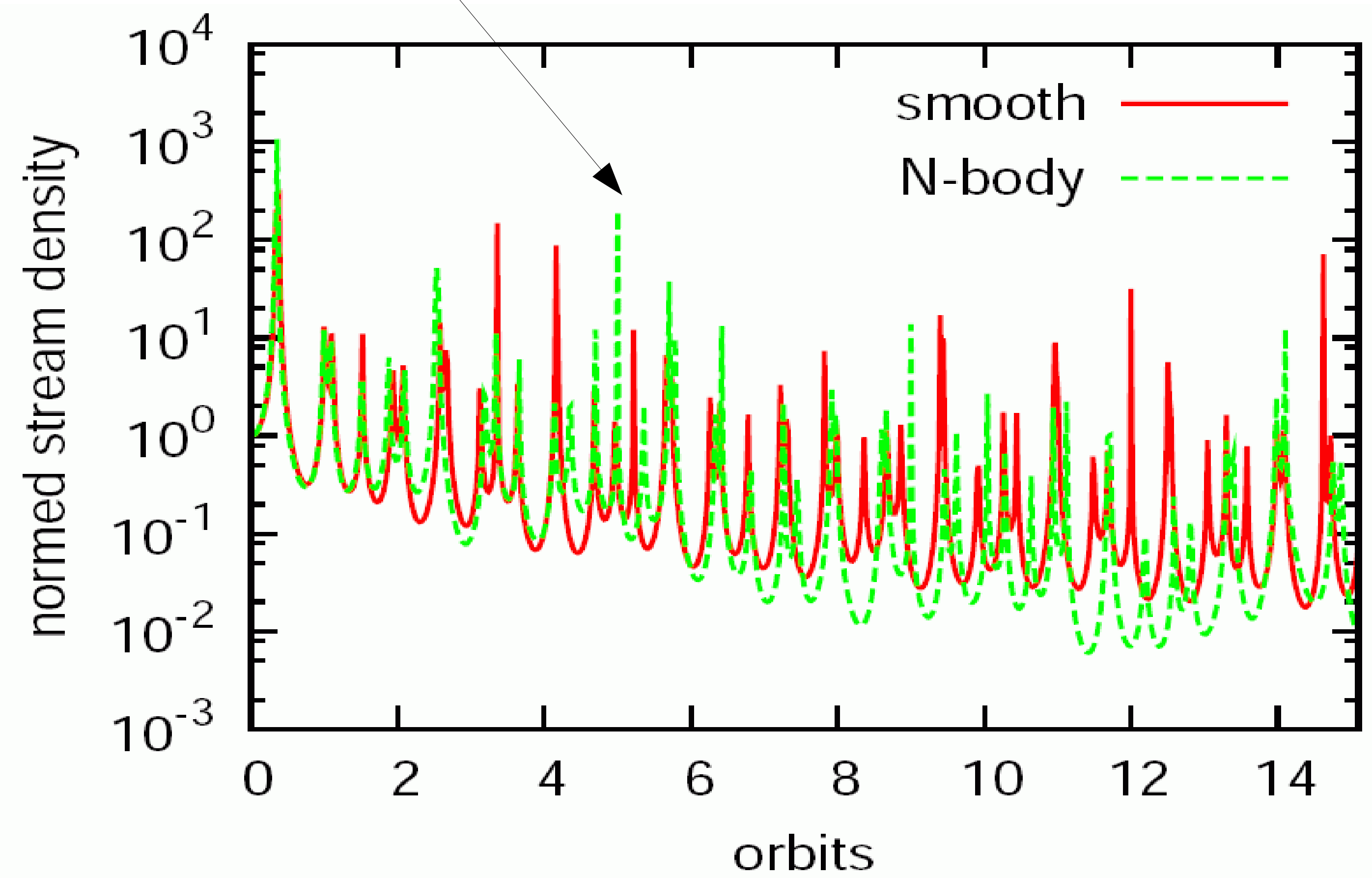
Chaotic mixing

A particle orbit in a live Halo

spherical Hernquist density profile

$$\rho(r) = \frac{M}{2\pi}\,\frac{a}{r}\,\frac{1}{(r+a)^3}$$

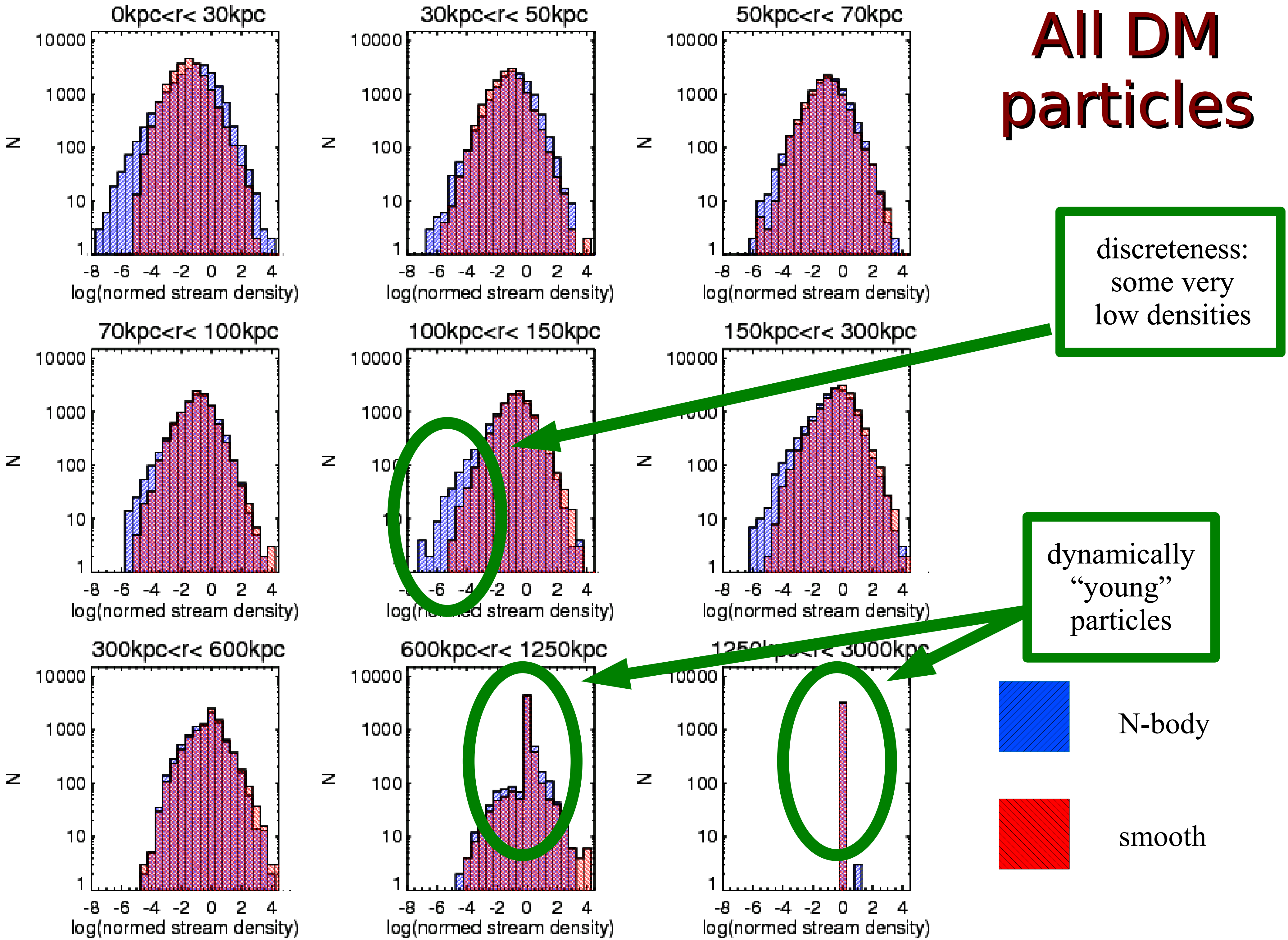
caustics resolved in N-body live halo!



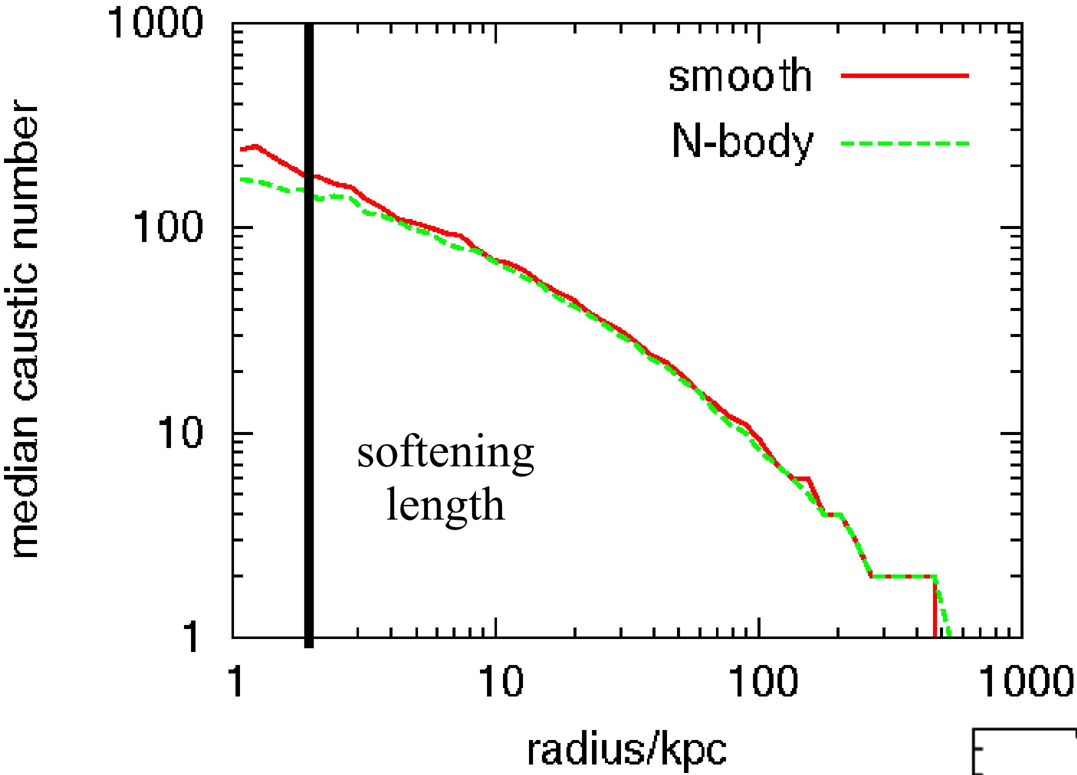
general shape and caustic spacing/number very similiar!

phase-space density conservation:10^{-8}

All DM particles



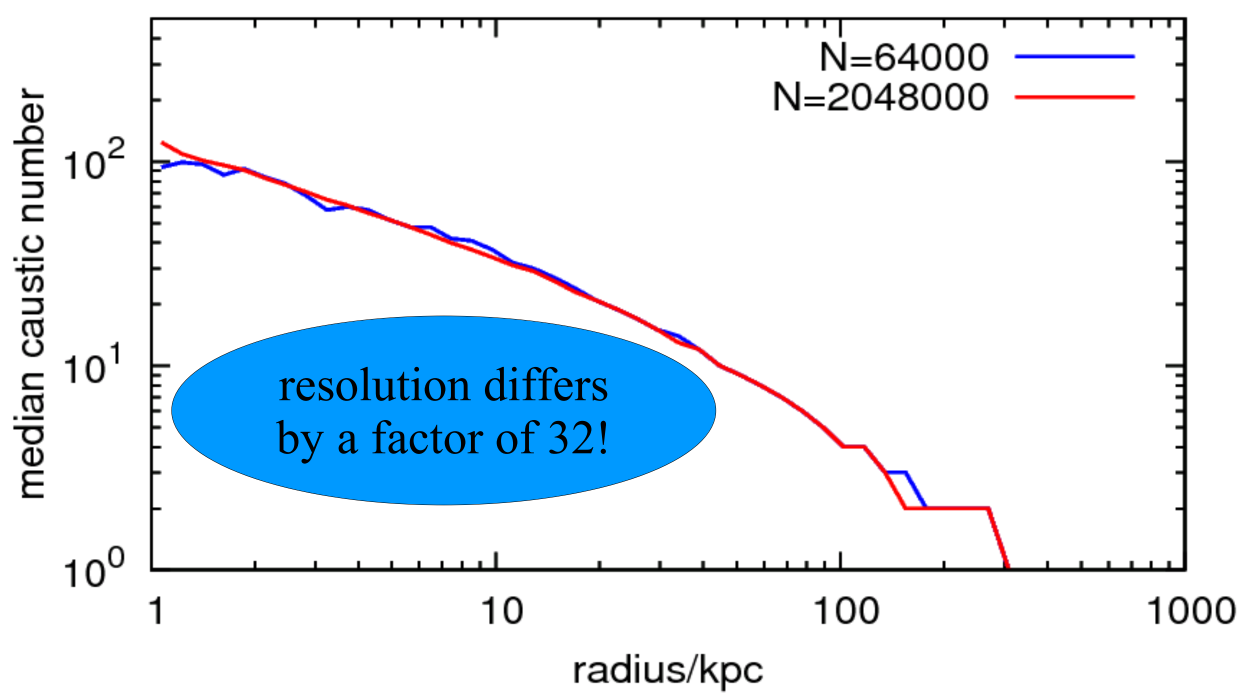
Number of Caustic Passages

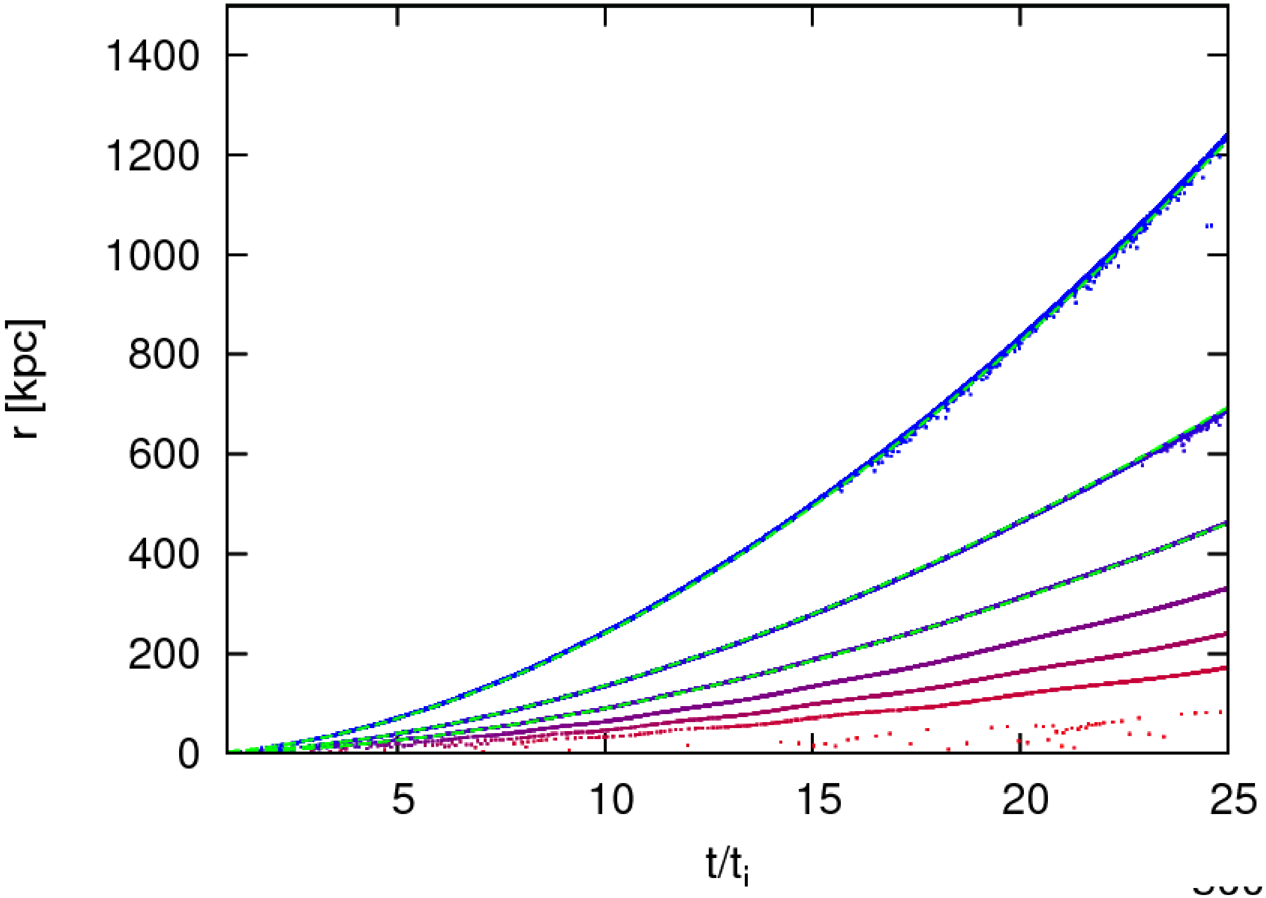


analytic and N-body results nearly the same!

Annihilation boost factor estimates due to caustics should be very robust!

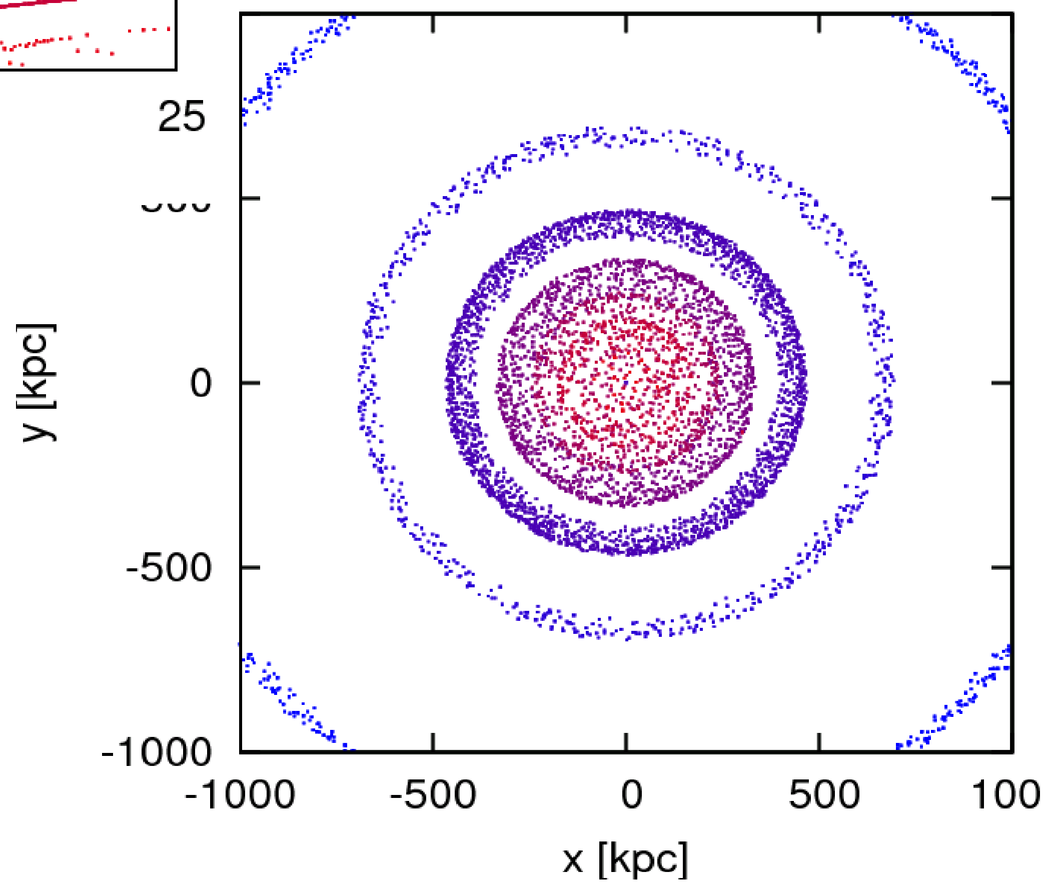
Very stable against particle number and softening length!

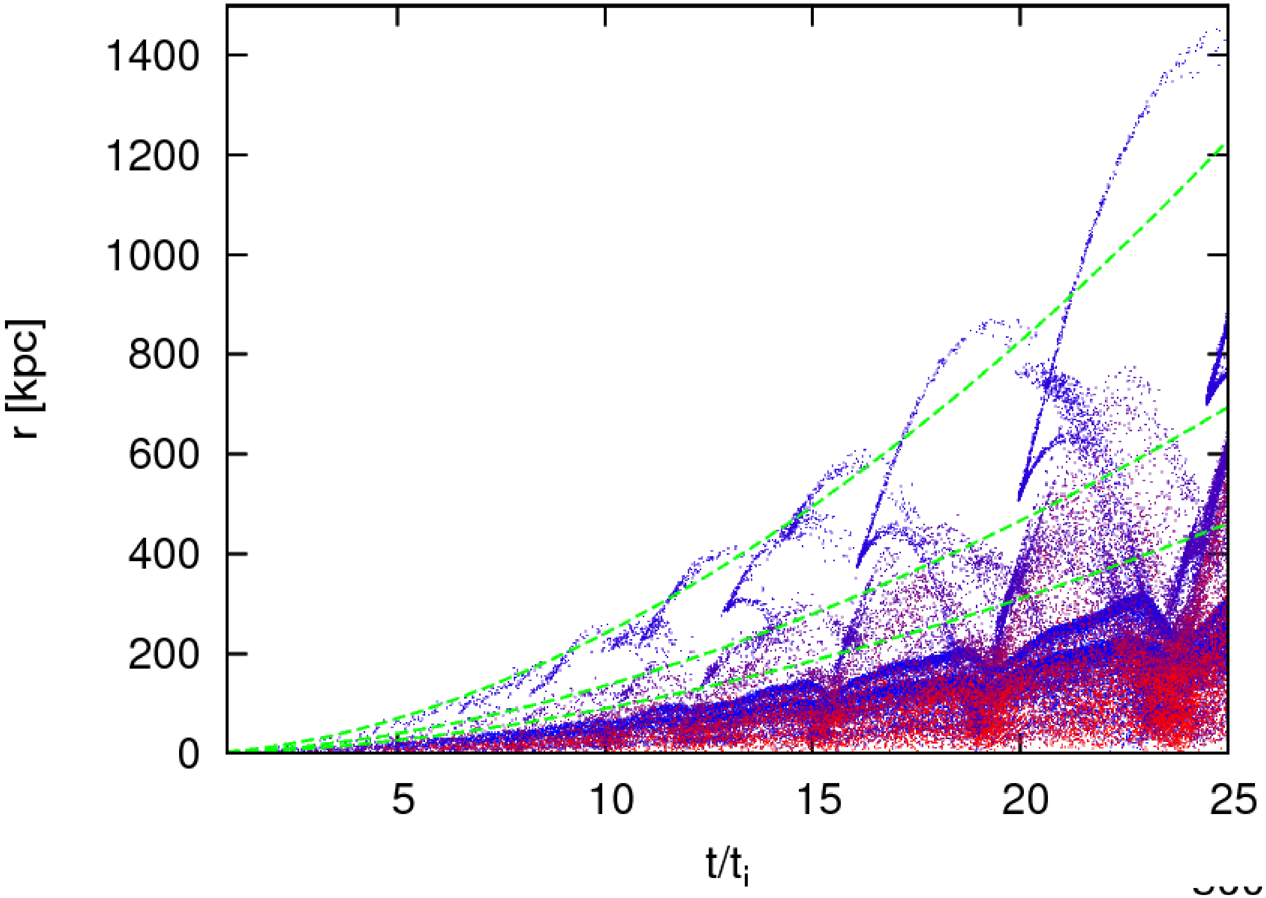




Caustic structure and evolution in growth from spherical self-similar IC's

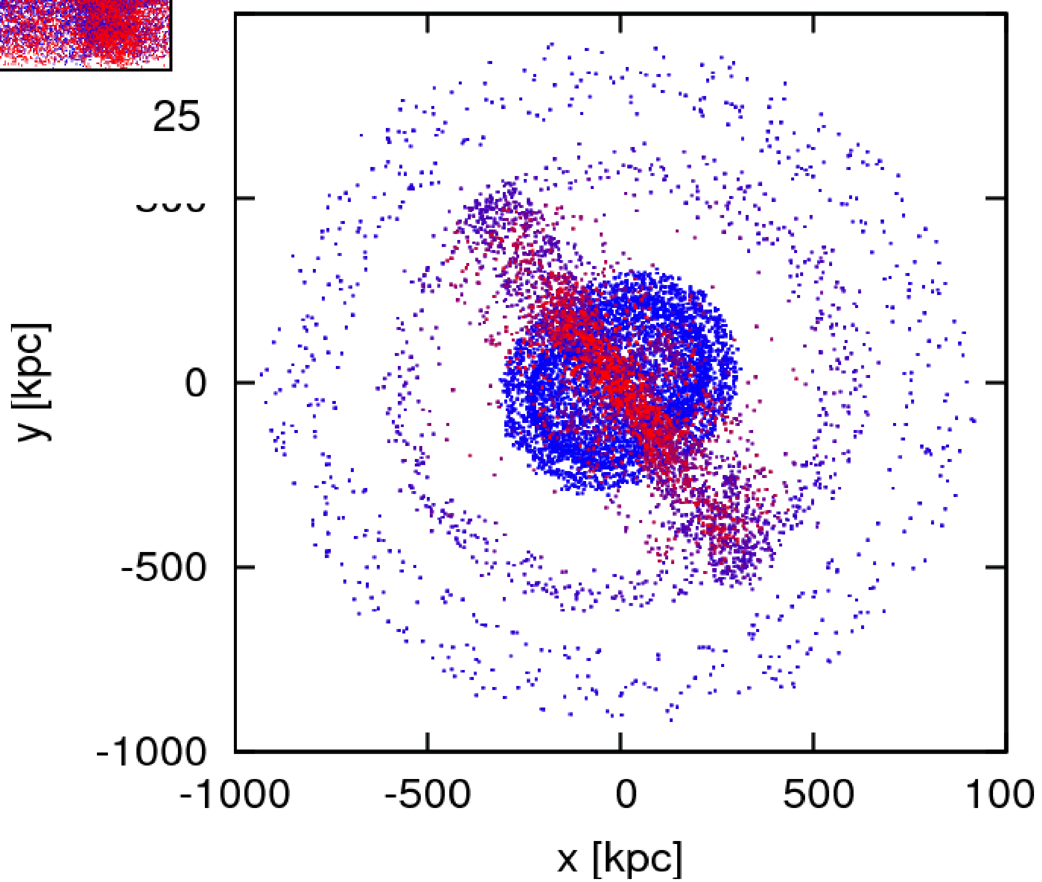
1-D (spherical) gravity





Caustic structure and evolution in growth from spherical self-similar IC's

Fully 3-D gravity



Conclusions (so far)

- GDE robustly identifies caustic passages and gives fair stream density estimates for particles in fully 3-D CDM simulations

- Many streams are present at each point well inside a CDM halo (at least 100,000 at the Sun's position)

  → quasi-Gaussian signal in direct detection experiments

- Caustic structure is more complex in realistic 3-D situations than in matched 1-D models but the caustics are weaker

  → negligible boosting of annihilation signal due to caustics

- Boost due to small substructures still uncertain but appears modest