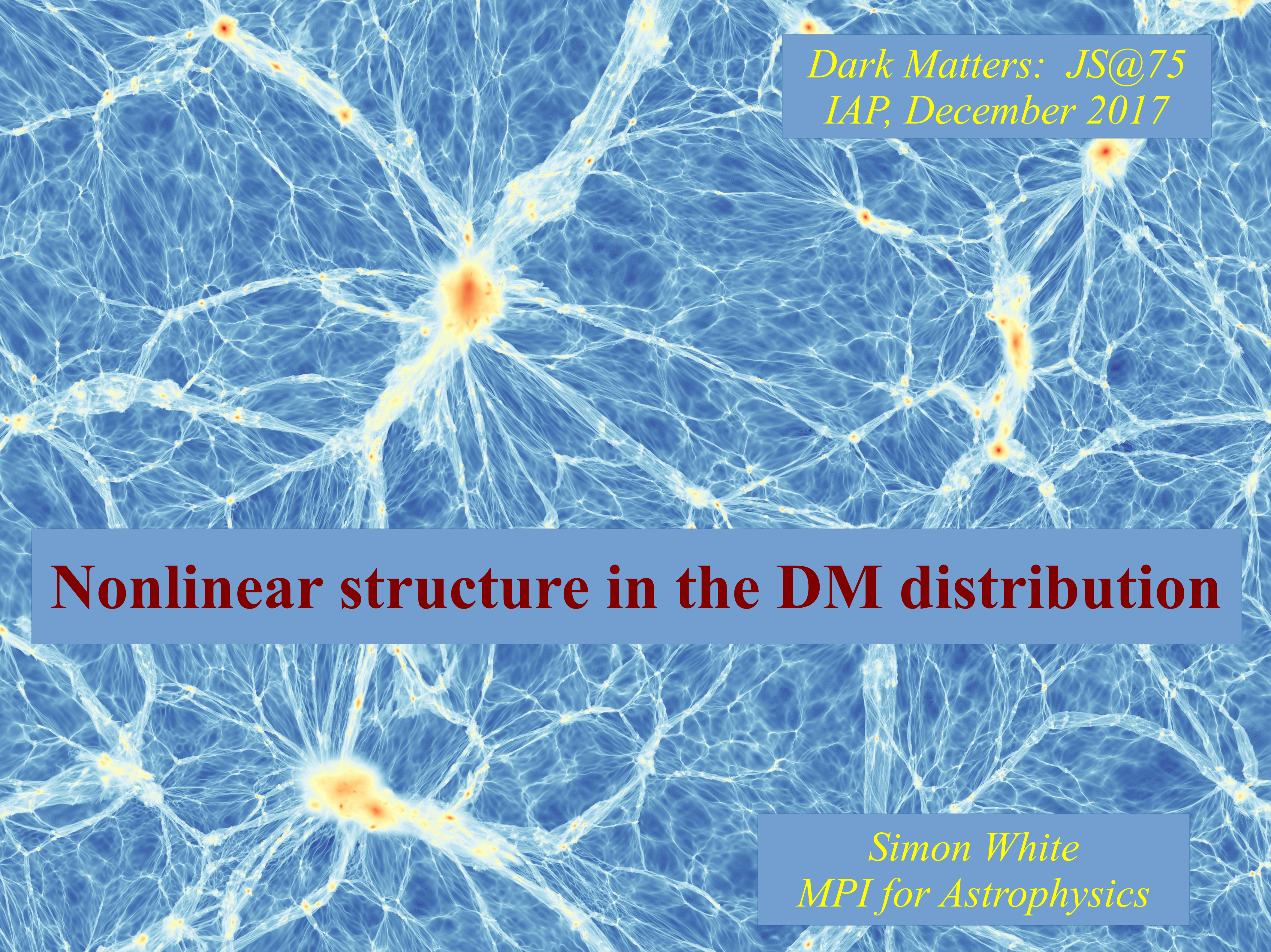


Dark Matters: JS@75
IAP, December 2017

Nonlinear structure in the DM distribution

Simon White
MPI for Astrophysics

1 [1979ApJ...231....1W]	132.000	07/1979	A F G R C U	
White, S. D. M.; Silk, J.	The growth of aspherical structure in the universe - Is the local supercluster an unusual system			
2 [1978ApJ...226L.103S]	122.000	12/1978	F G R C U	
Silk, J.; White, S. D. M.	The determination of Q_0 using X-ray and microwave observations of galaxy clusters			
3 [1978ApJ...223L..59S]	84.000	07/1978	A F G R C	
Silk, J.; White, S. D.	The development of structure in the expanding universe			
4 [1980ApJ...241..864W]	57.000	11/1980	A F G R C	
White, S. D. M.; Silk, J.	The X-ray structure of two rich galaxy clusters and its implications for the detectability of microwave diminutions			
5 [1981ApJ...251L..65W]	54.000	12/1981	A F G R C	
White, S. D. M.; Silk, J.; Henry, J. P.	The X-ray structure of a galaxy cluster at Z = 0.54 - Implications for cluster evolution and cosmology			

The total potential within a homogeneous ellipsoidal overdensity in an otherwise unperturbed universe is

$$V = -\pi G \sum_i [(\rho_e - \rho_b)\alpha_i + \tfrac{2}{3}\rho_b]x_i^2$$

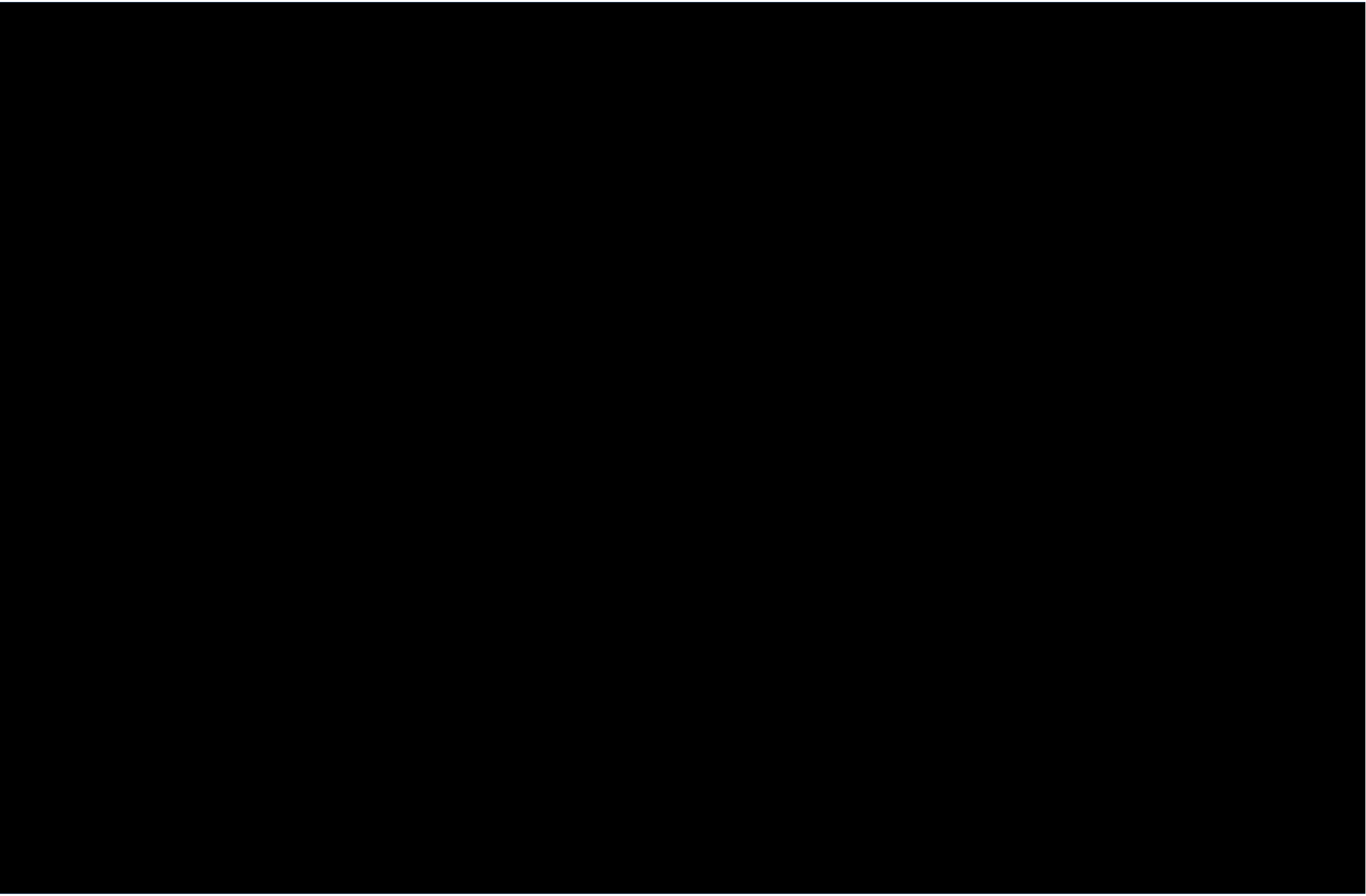
$$\frac{d^2 a_i}{dt^2} = -2\pi G[\rho_b \alpha_i + (\tfrac{2}{3} - \alpha_i)\rho_b]a_i ,$$

c.f. $$\frac{d^2 R_b}{dt^2} = -\frac{4\pi}{3} G\rho_b R_b ,$$

approx. $$a_i(t)/a_i(t_0) = \tfrac{3}{2}\alpha_i(t_0)R_e(t) + [1 - \tfrac{3}{2}\alpha_i(t_0)]R_b(t)$$

JS + DEFW circa 1982

Springel et al 2008



Newtonian “experiment” with 100 million bodies – forming a dark matter halo

The four elements of ΛCDM halos

I Smooth background halo

-- NFW-like cusped density profile

-- near-ellipsoidal equidensity contours

II Bound subhalos

-- most massive typically 1% of main halo mass

-- total mass of all subhalos $\lesssim 10\%$

-- less centrally concentrated than the smooth component

III Tidal streams

-- remnants of tidally disrupted subhalos

IV Fundamental streams

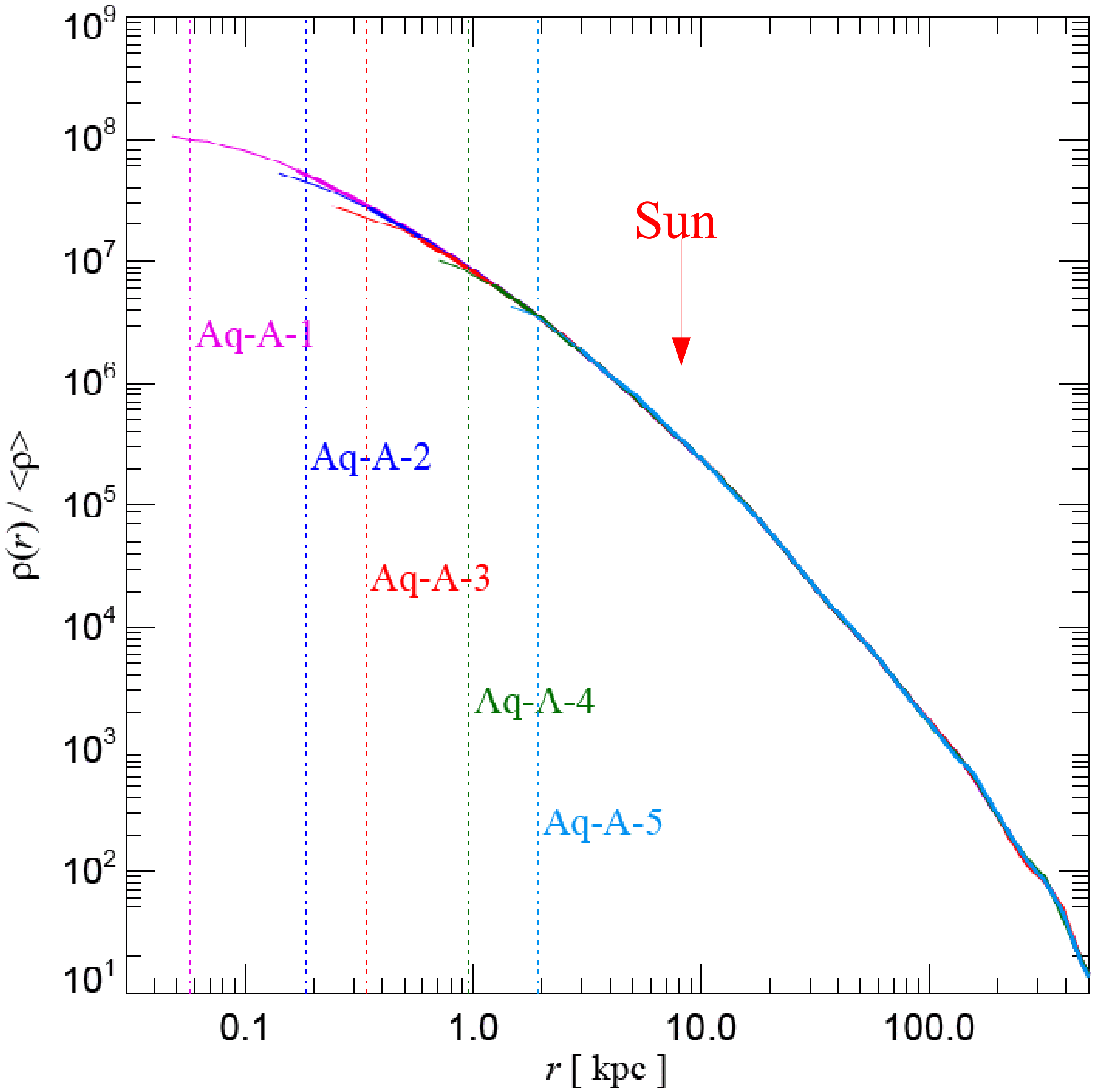
-- consequence of smooth and cold initial conditions

-- very low internal velocity dispersions

-- produce density caustics at projective catastrophes

I. Smooth background halo

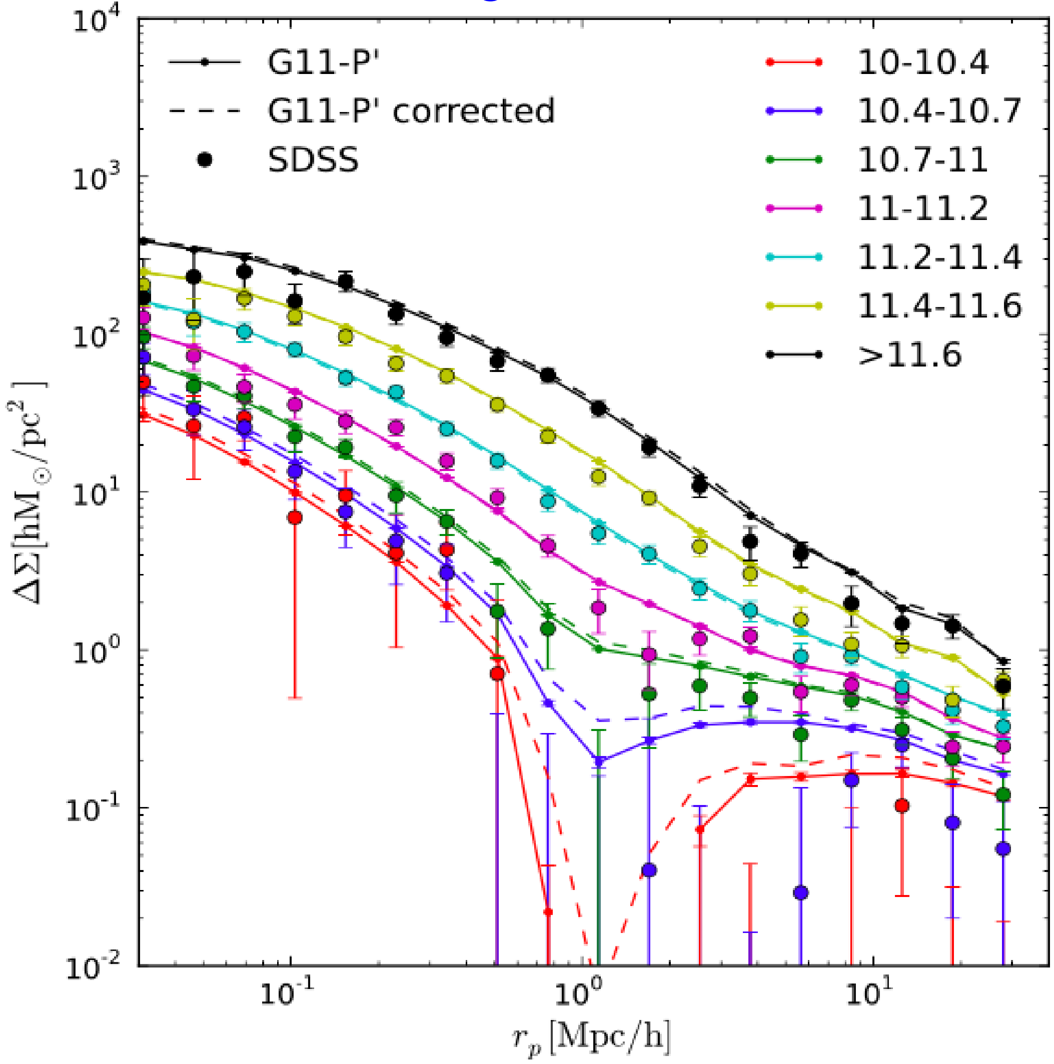
Aquarius Project: Springel et al 2008



Density profiles of simulated DM-only ΛCDM halos are now very well determined -- to radii well inside the Sun's position

ΛCDM halo profiles vs lensing observations

Wang et al 2016



Weak lensing profiles around stacks of isolated SDSS galaxies as a function of their stellar mass.

Predictions from a SDSS “mock” catalogue made from a SAM in the Planck cosmology with parameters adjusted to fit galaxy abundances.

No further parameter adjustment to fit lensing/clustering observations.

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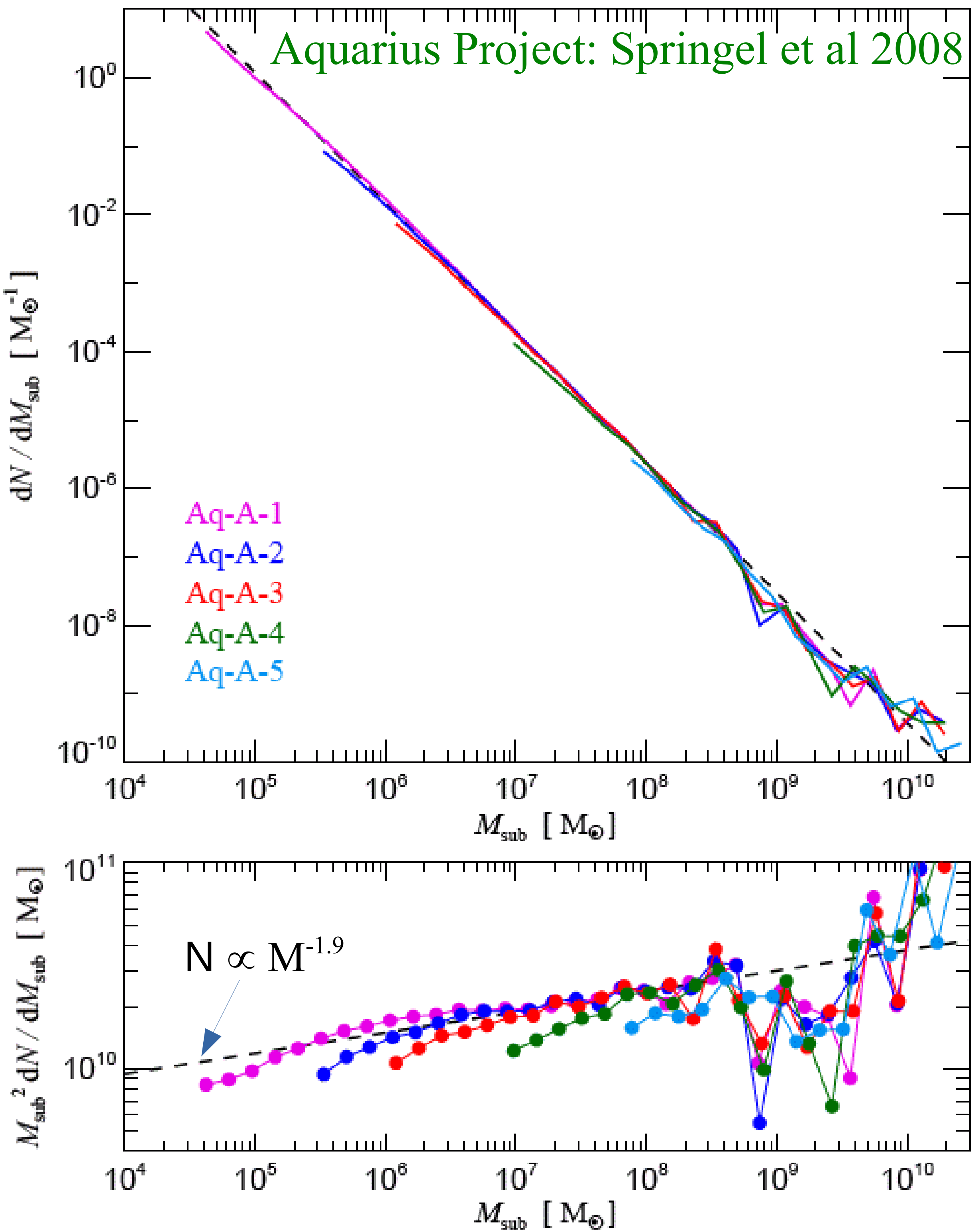
II. Bound subhalos



Abundance of self-bound subhalos is measured to below 10^{-7} M_{halo}

Most subhalo mass is in the biggest objects (just)

Bound subhalos: conclusions

Substructure is primarily in the outermost parts of halos

The radial distribution of subhalos is almost mass-independent

The total mass in subhalos converges (weakly) at small m

Subhalos contain a very small mass fraction in the inner halo (~0.1% near the Sun) and so will *not* be relevant for direct detection experiments

(Small) subhalos *dominate* the total annihilation luminosity at large radius

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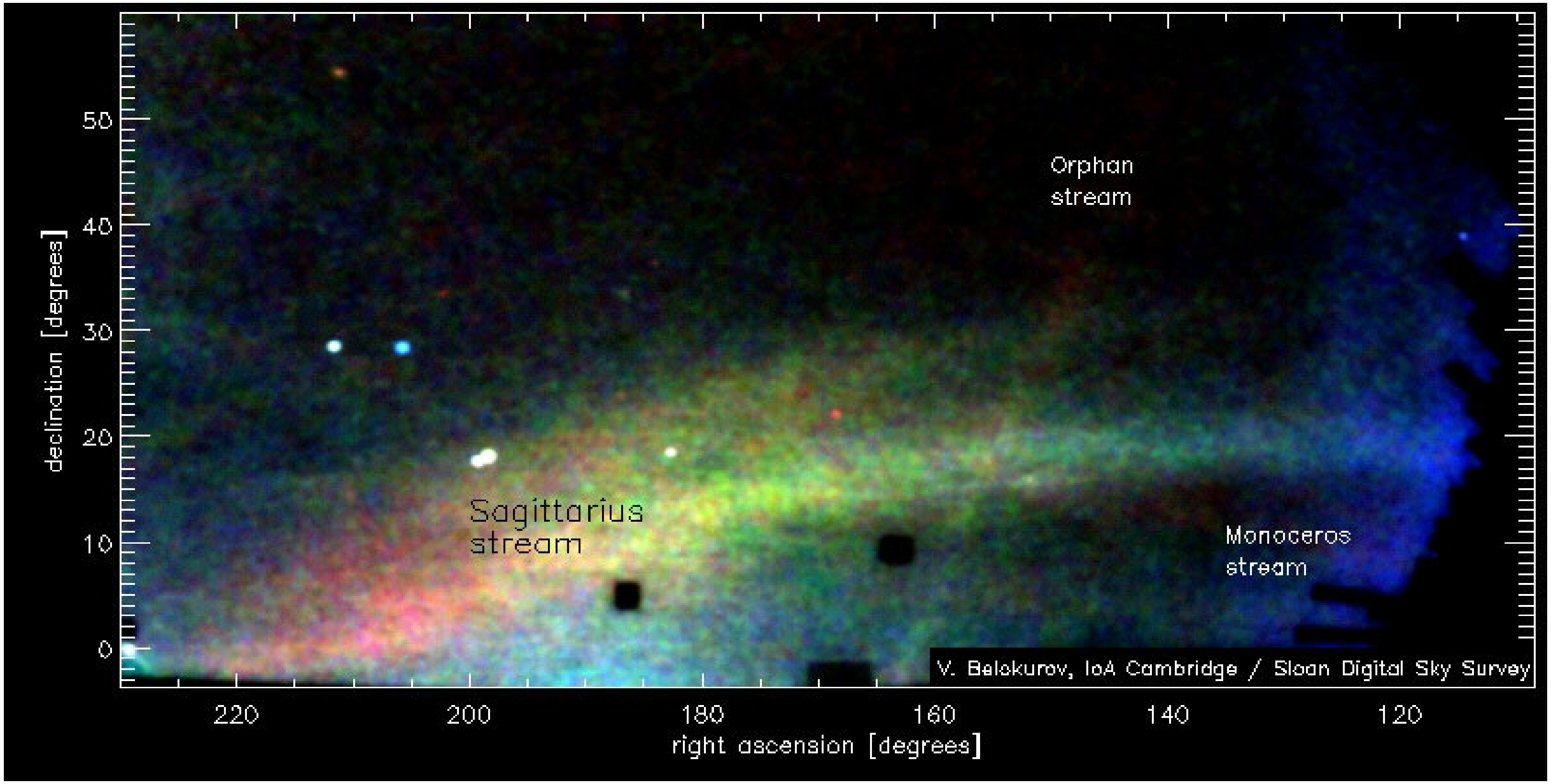
III. Tidal Streams



Produced by partial or total tidal disruption of subhalos

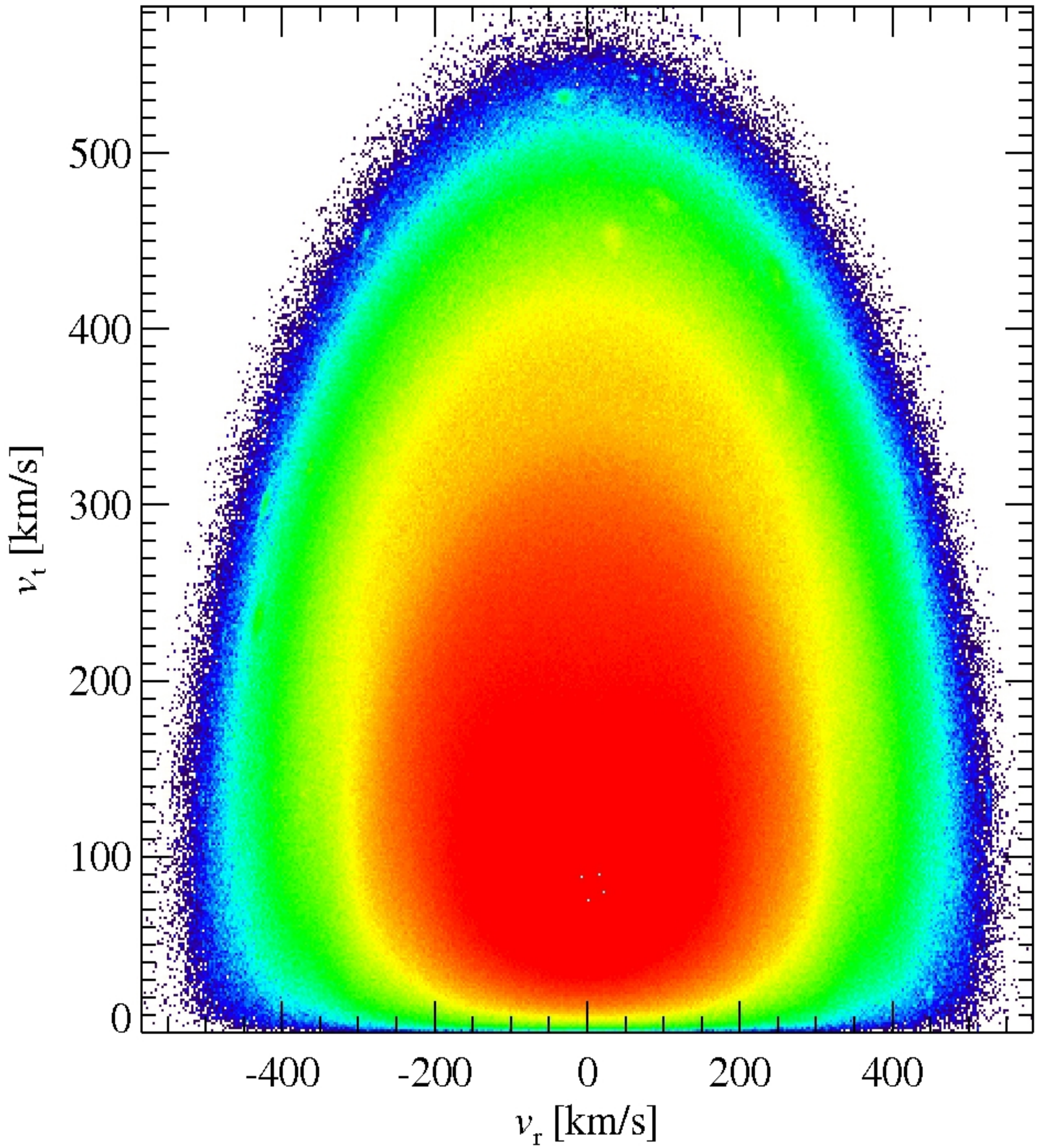
Analogous to observed stellar streams in the Galactic halo

Distributed along/around orbit of subhalo (c.f. meteor streams)

Localised in almost 1-D region of 6-D phase-space (**x**, **v**)

Dark matter phase-space structure in the inner MW

M. Maciejewski



6 kpc < r < 12 kpc

All particles

$N = 3.8 \times 10^7$

Dark matter phase-space structure in the inner MW

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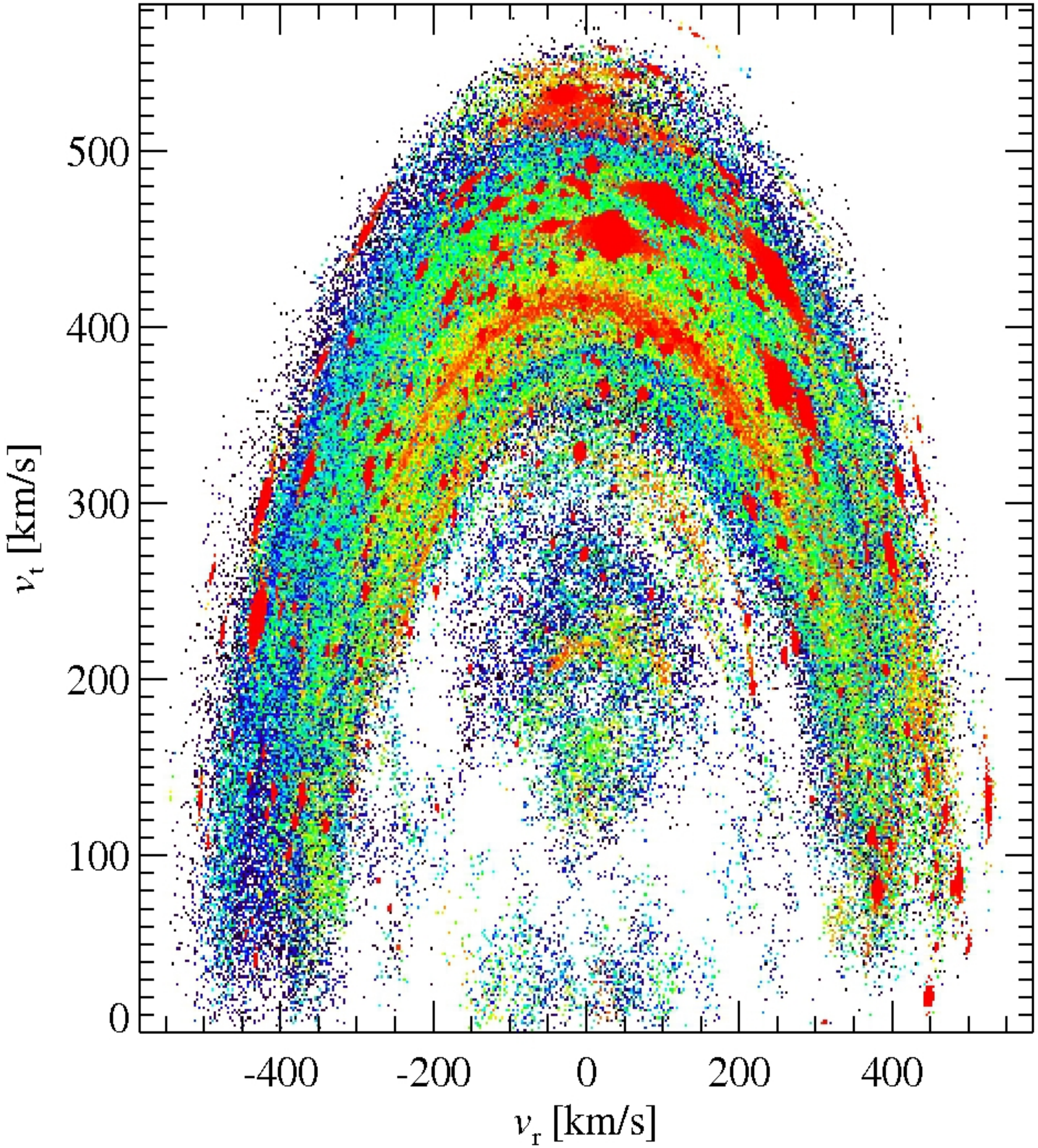


6 kpc < r < 12 kpc

Particles in detected phase-space structure

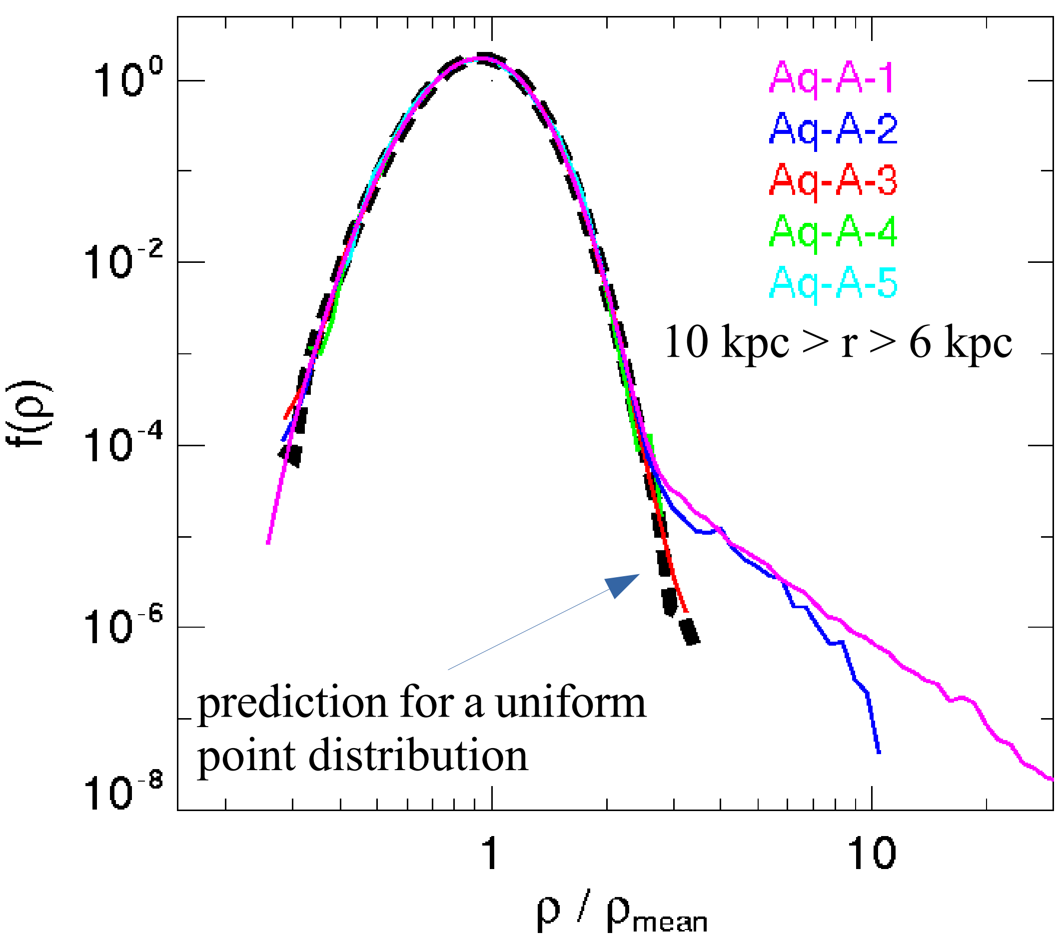
$N = 2.6 \times 10^5$ in tidal streams

$N = 3.9 \times 10^4$ in subhalos

→ only ~1% of the DM signal is in strong tidal streams

Local density in the inner halo compared to a smooth ellipsoidal model

Vogelsberger et al 2008



Estimate a density ρ at each point by adaptively smoothing using the 64 nearest particles

Fit to a smooth density profile stratified on similar ellipsoids

The chance of a random point lying in a substructure is $< 10^{-4}$

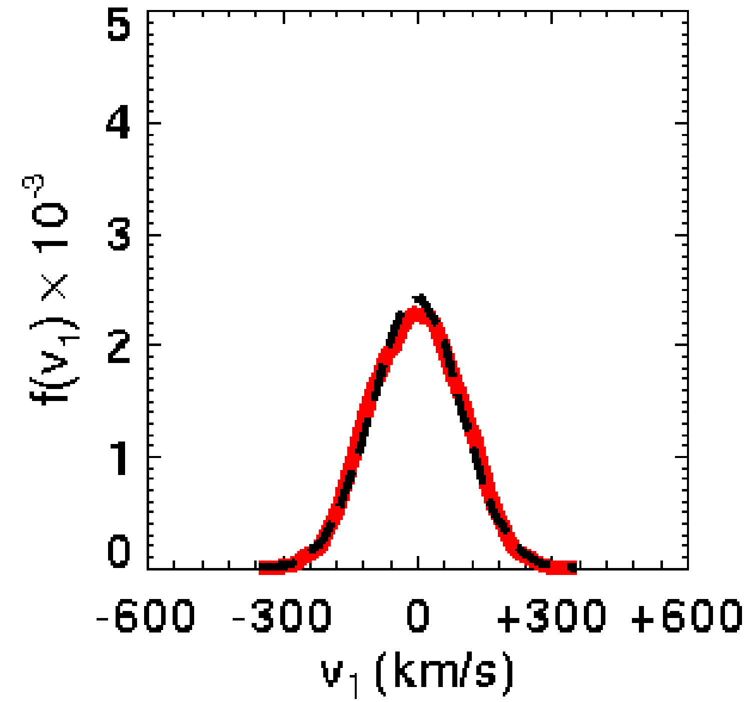
The *rms* scatter about the smooth model for the remaining points is only about 4%

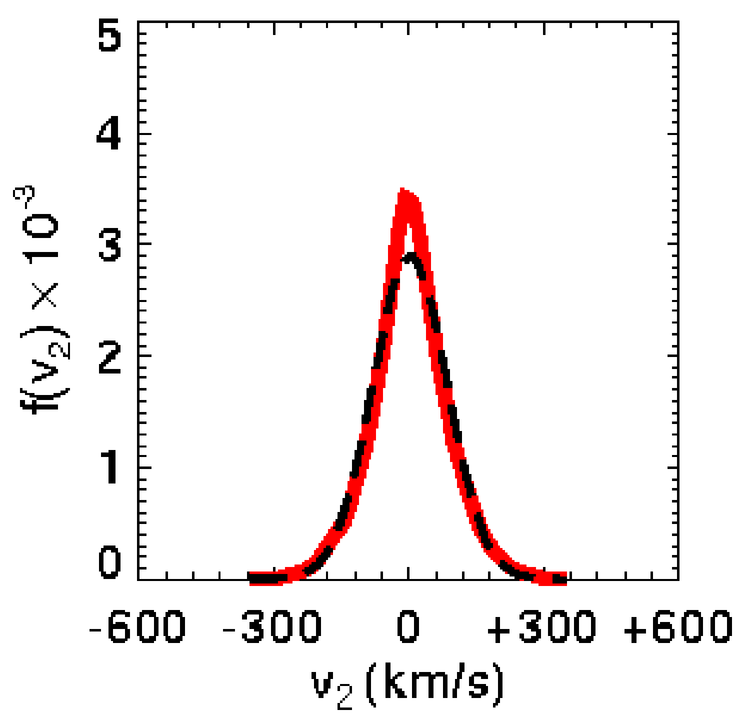
Local velocity distribution

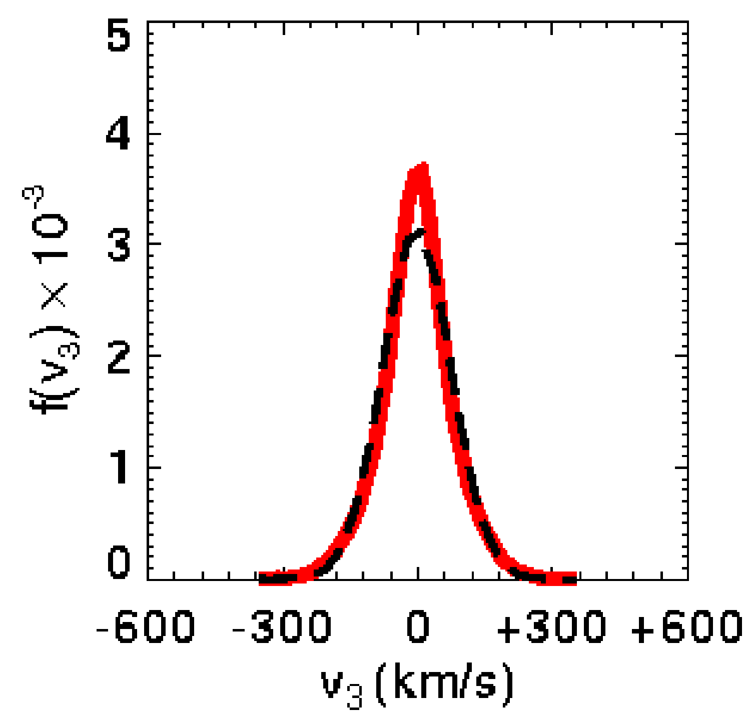
Velocity histograms for particles in a typical $(2\text{kpc})^3$ box at R = 8 kpc

Distributions are smooth, near-Gaussian and different in different directions

No individual streams are visible







Vogelsberger et al 2008

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IV. Fundamental streams

After CDM particles become nonrelativistic, but *before* nonlinear objects form (e.g. z > 100) their distribution function is

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x},t)]\, N\,[\{\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x},t)\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\boldsymbol{x},t)$ is a Gaussian random field with finite variance $\ll 1$,

$\boldsymbol{V}(\boldsymbol{x},t) = \nabla\psi(\boldsymbol{x},t)$ where $\nabla^2\psi \propto \delta$,

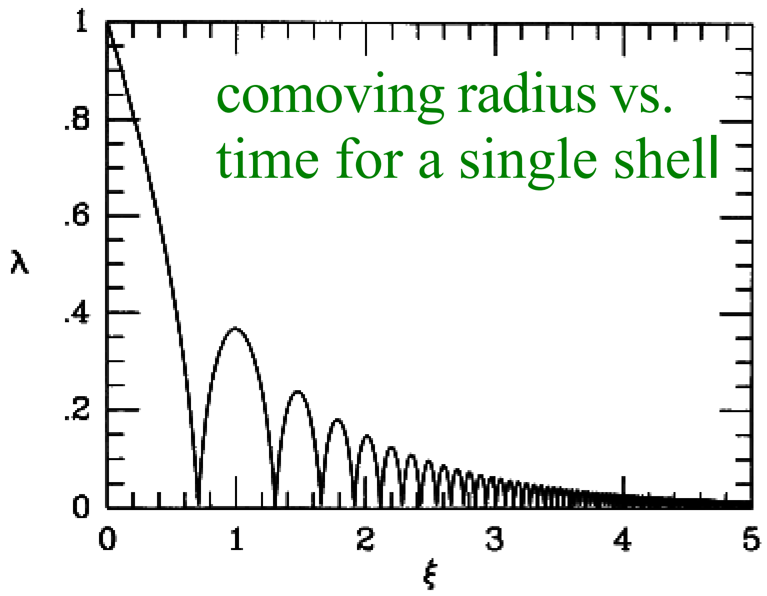
and N is normal with $\sigma^2 << \langle|\mathbf{V}|^2\rangle$ (today $\sigma \sim 0.1$ cm/s)

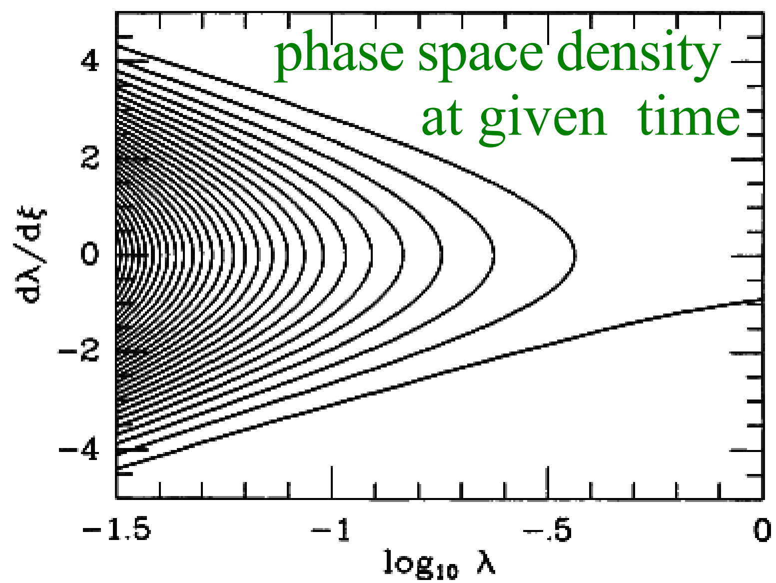
CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

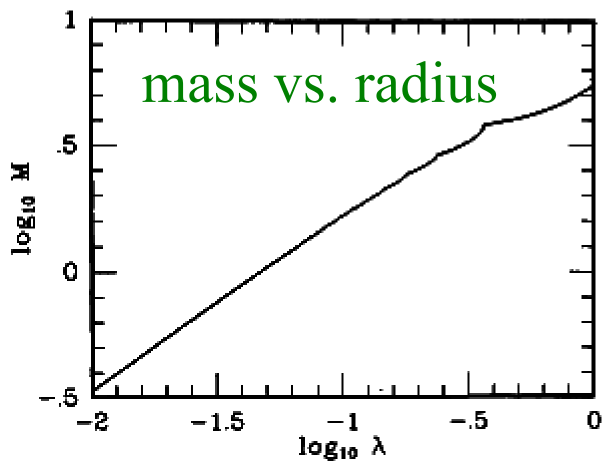
$Df / Dt = 0$ → only a 3-D subspace is occupied at *all* times.
Nonlinear evolution leads to <u>multi-stream</u> structure and <u>caustics</u>

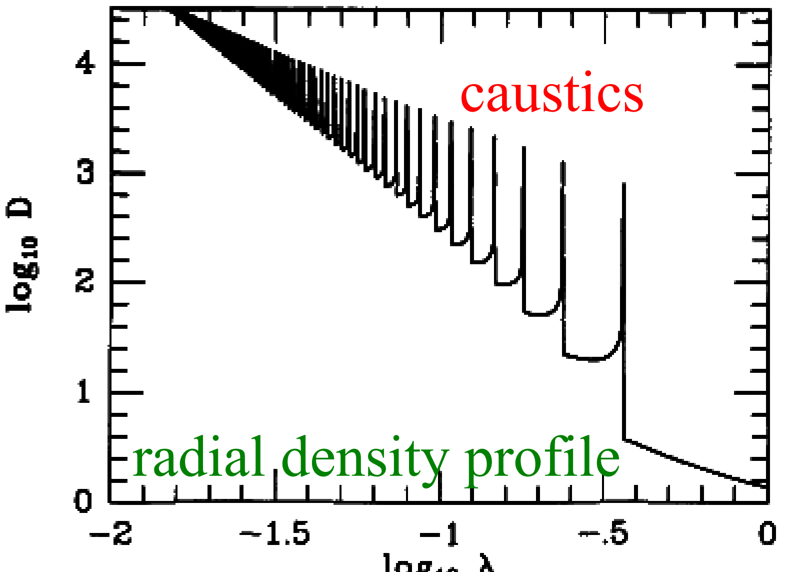
Similarity solution for spherical collapse in CDM

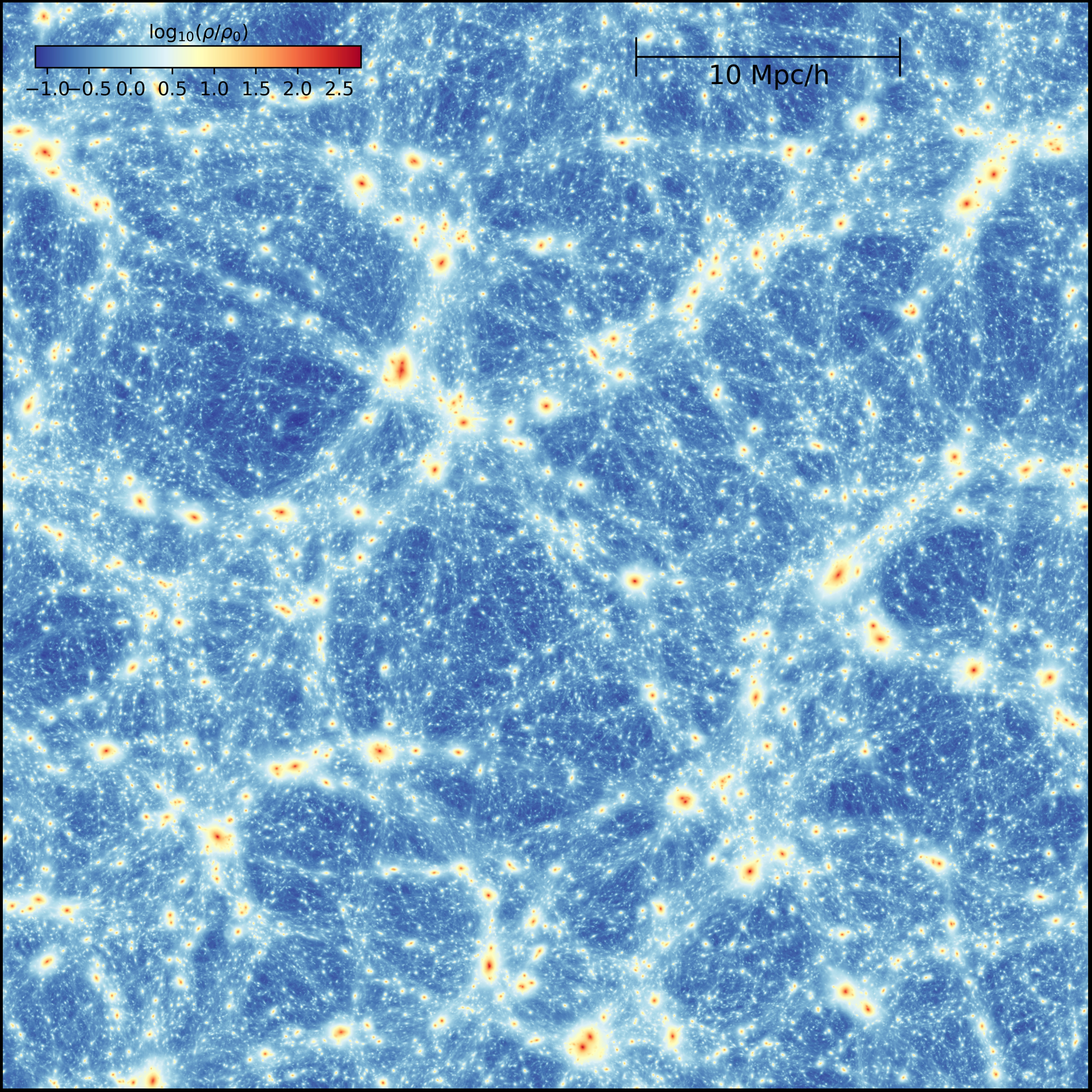
Bertschinger 1985



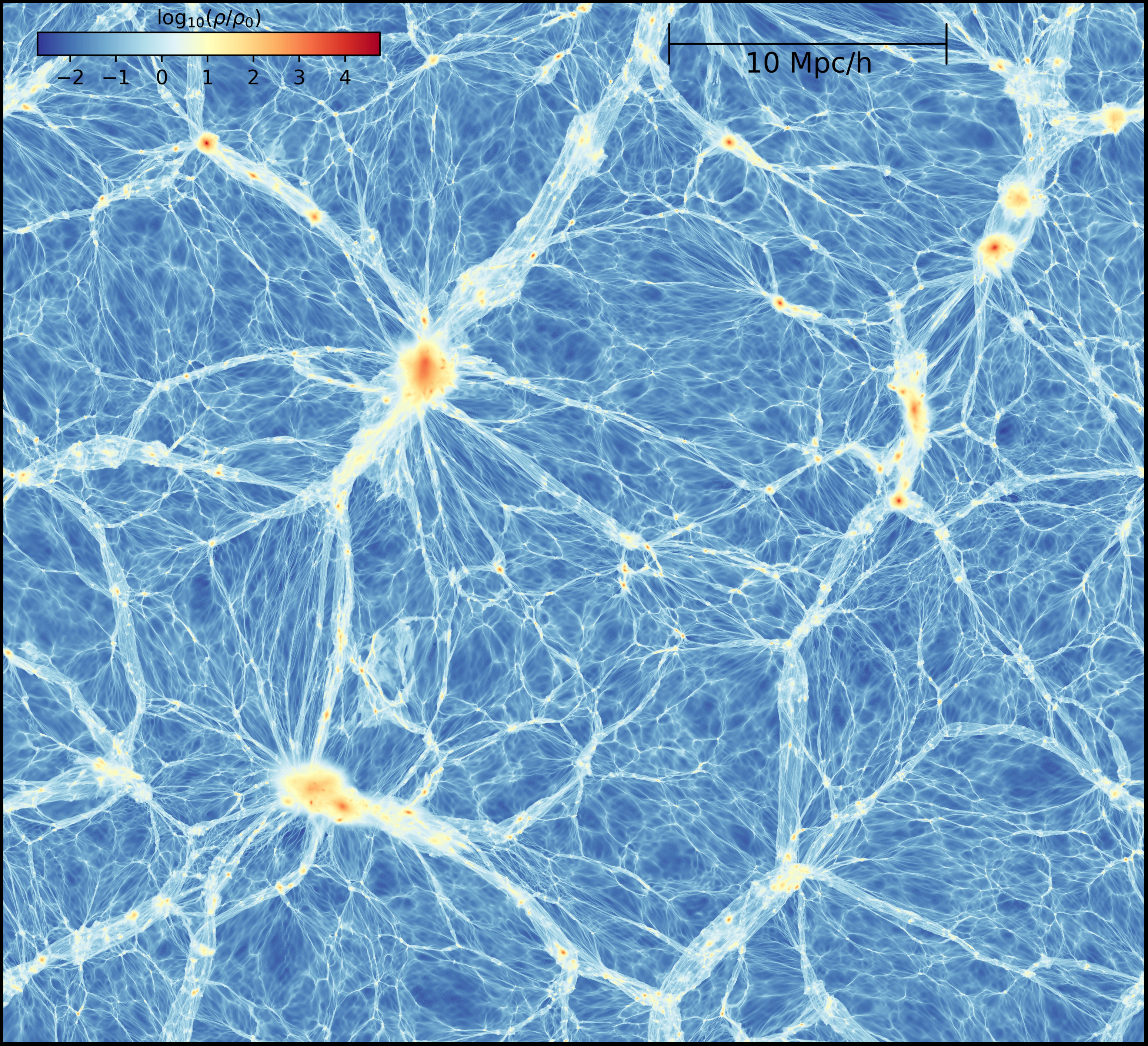








log10(ρ/ρ0)
−1.0 −0.5 0.0 0.5 1.0 1.5 2.0 2.5
10 Mpc/h



$\log_{10}(\rho/\rho_0)$
−2
−1
0
1
2
3
4
10 Mpc/h

IV. Fundamental streams

Consequences of $Df / Dt = 0$

The 3-D phase sheet can be stretched and folded but not torn

At least one sheet must pass through every point **x**

In nonlinear objects there are typically many sheets at each **x**

Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain f = const. ⟶ $\sigma \sim \rho^{1/3}$

At a caustic, at least one velocity dispersion must ⟶ ∞

All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot\delta X\,;\; T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = I$$

Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution

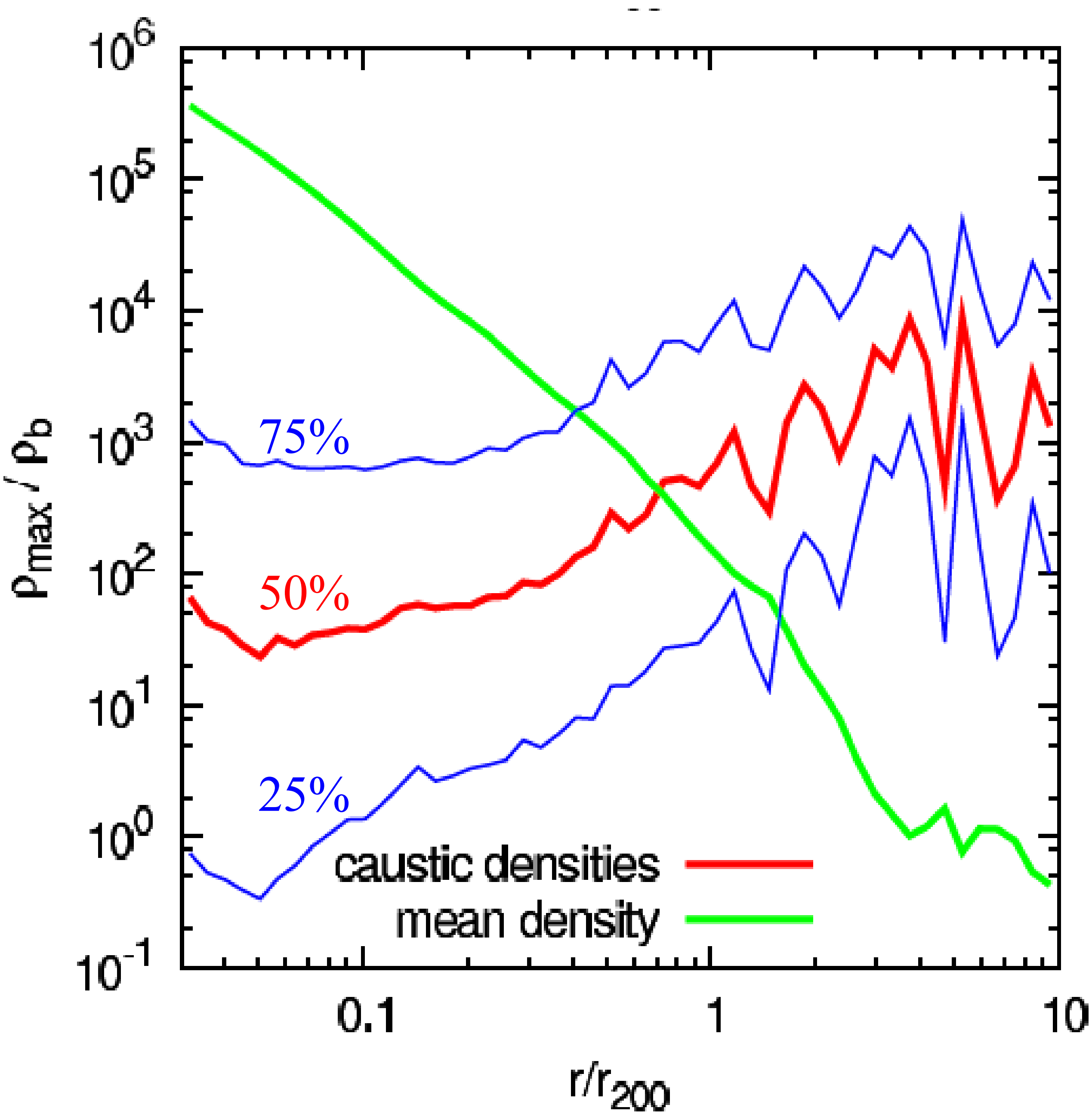
No symmetry or stationarity assumptions are required

det(D) = 1 at all times by Liouville's theorem

For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

Radial distribution of peak density at caustics

Vogelsberger & White 2011



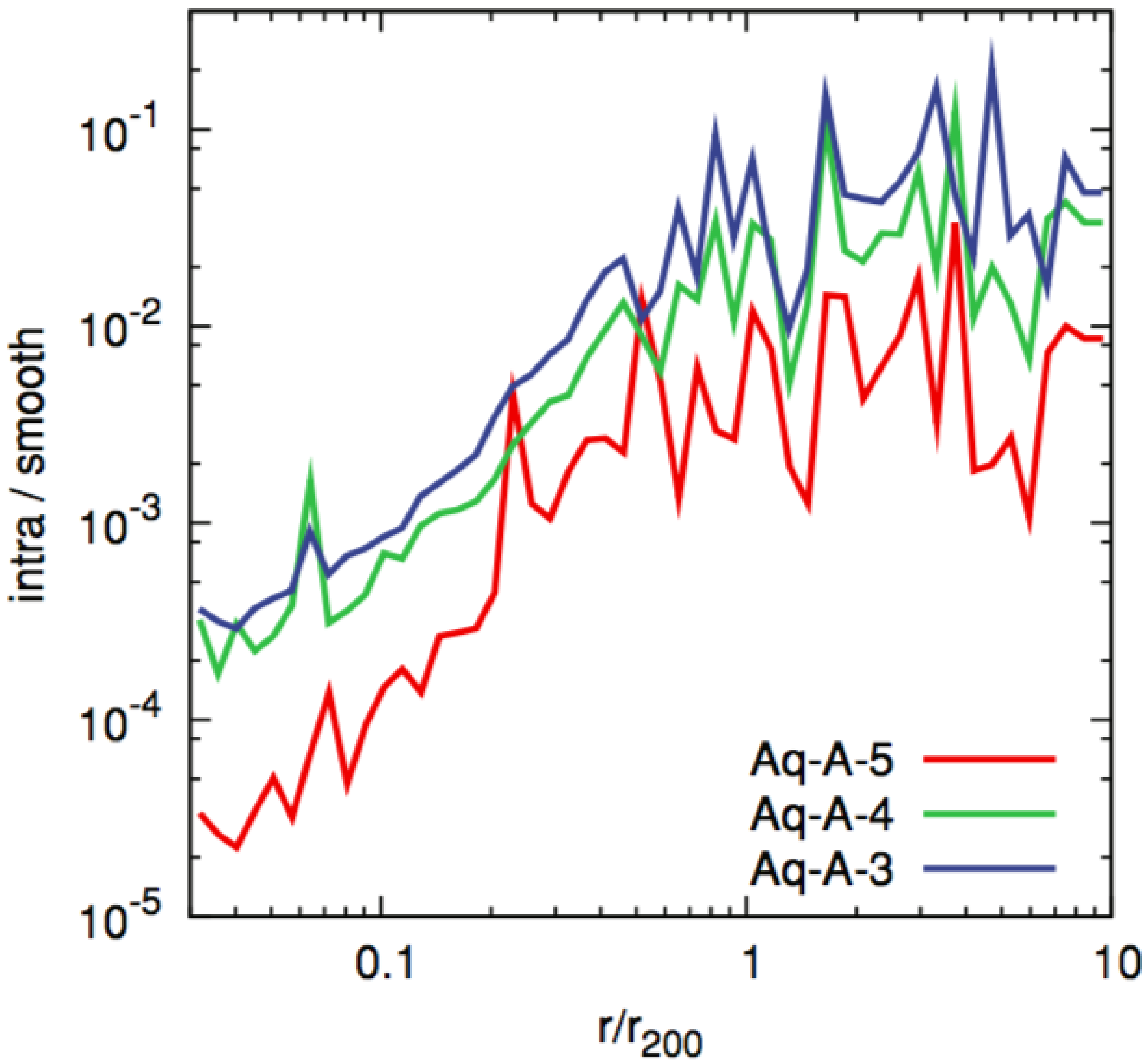
Milky Way mass halo

Initial velocity dispersion assumes a standard WIMP with

$m = 100$ GeV/c^2

Fraction of annihilation luminosity from caustics

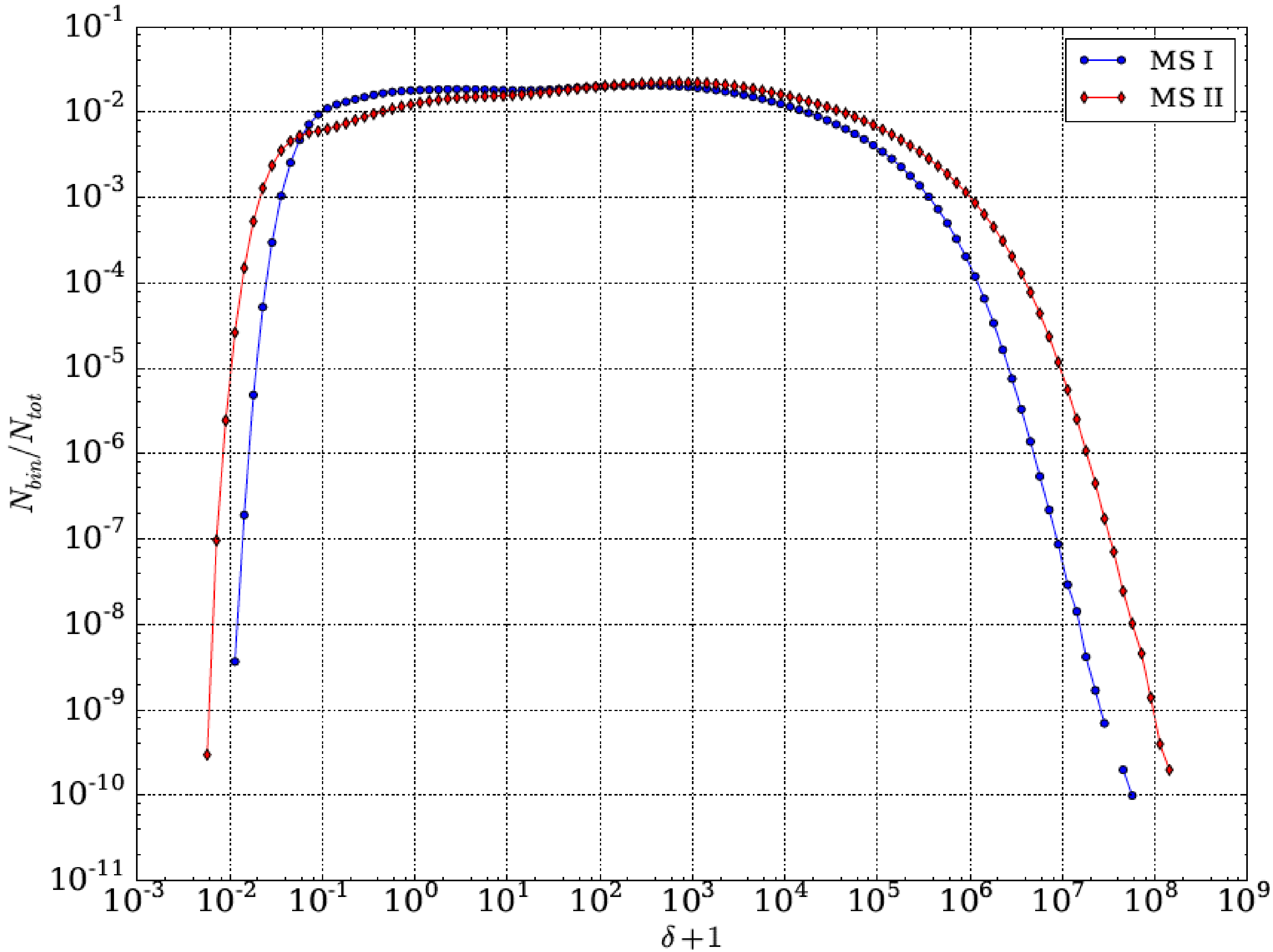
Vogelsberger & White 2011



Initial velocity dispersion assumes a standard WIMP with
$m = 100$ GeV/c^2

Note: caustic emission is compared to that from the smooth DM component here, but the dominant emission at large radius is from small subhalos

Voronoi-estimated DM densities at the particle positions in the two Millennium Simulations, estimated as: $\rho_i \propto 1 / V_{Vor,i}$



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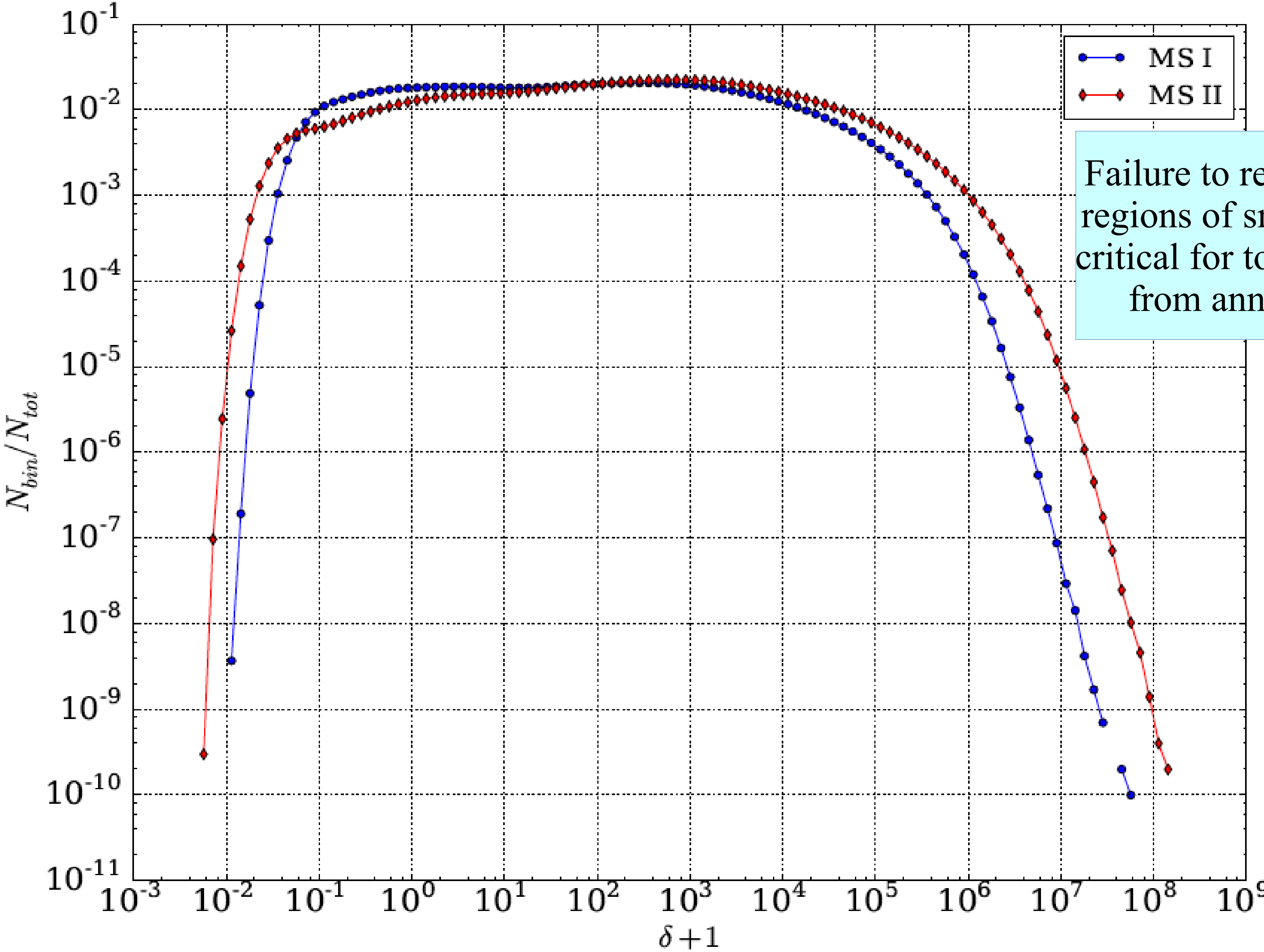
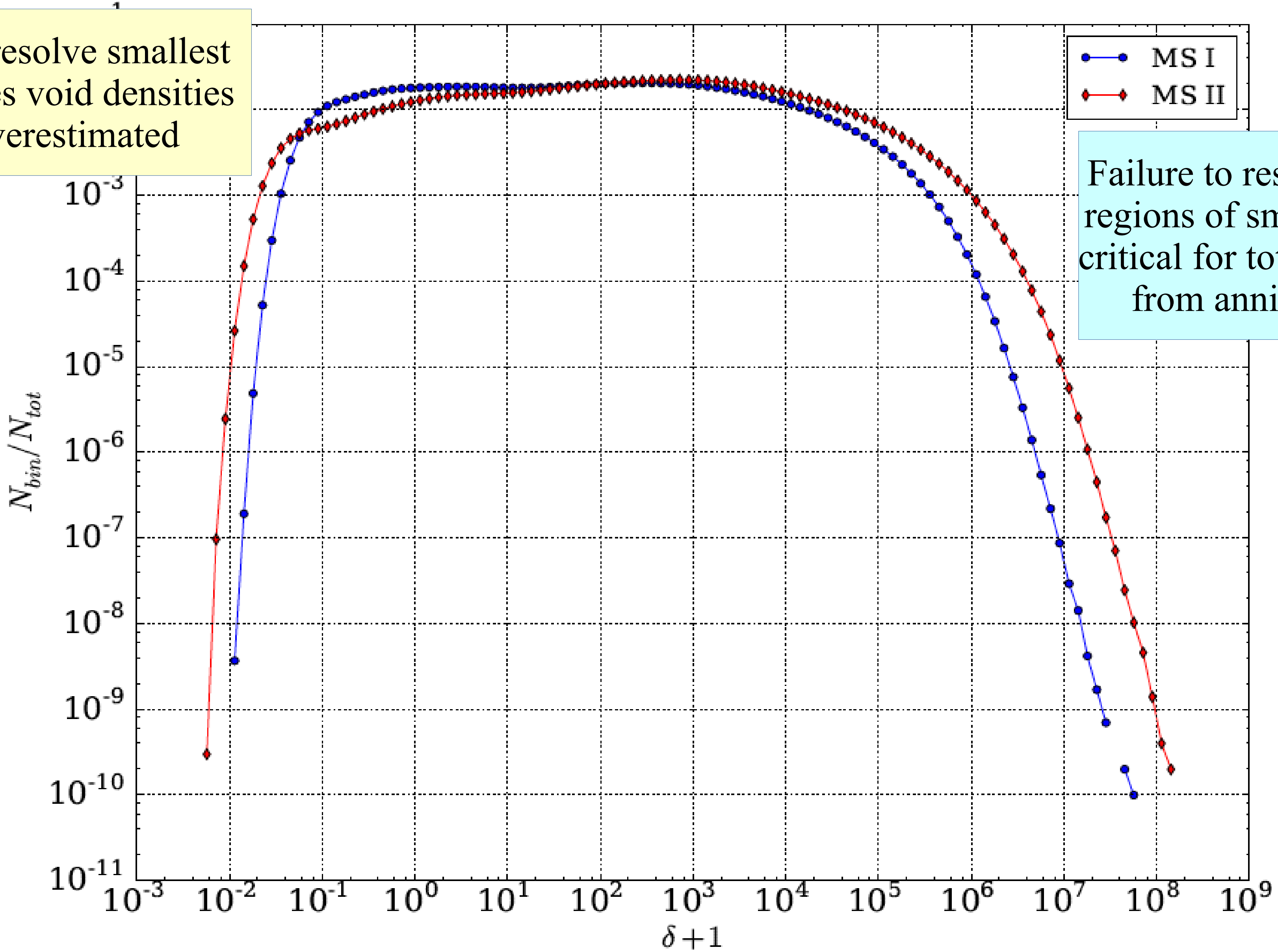


Failure to resolve inner regions of small halos is critical for total emission from annihilation

Voronoi-estimated DM densities at the particle positions in the two Millennium Simulations, estimated as: $\rho_i \propto 1 / V_{Vor,i}$

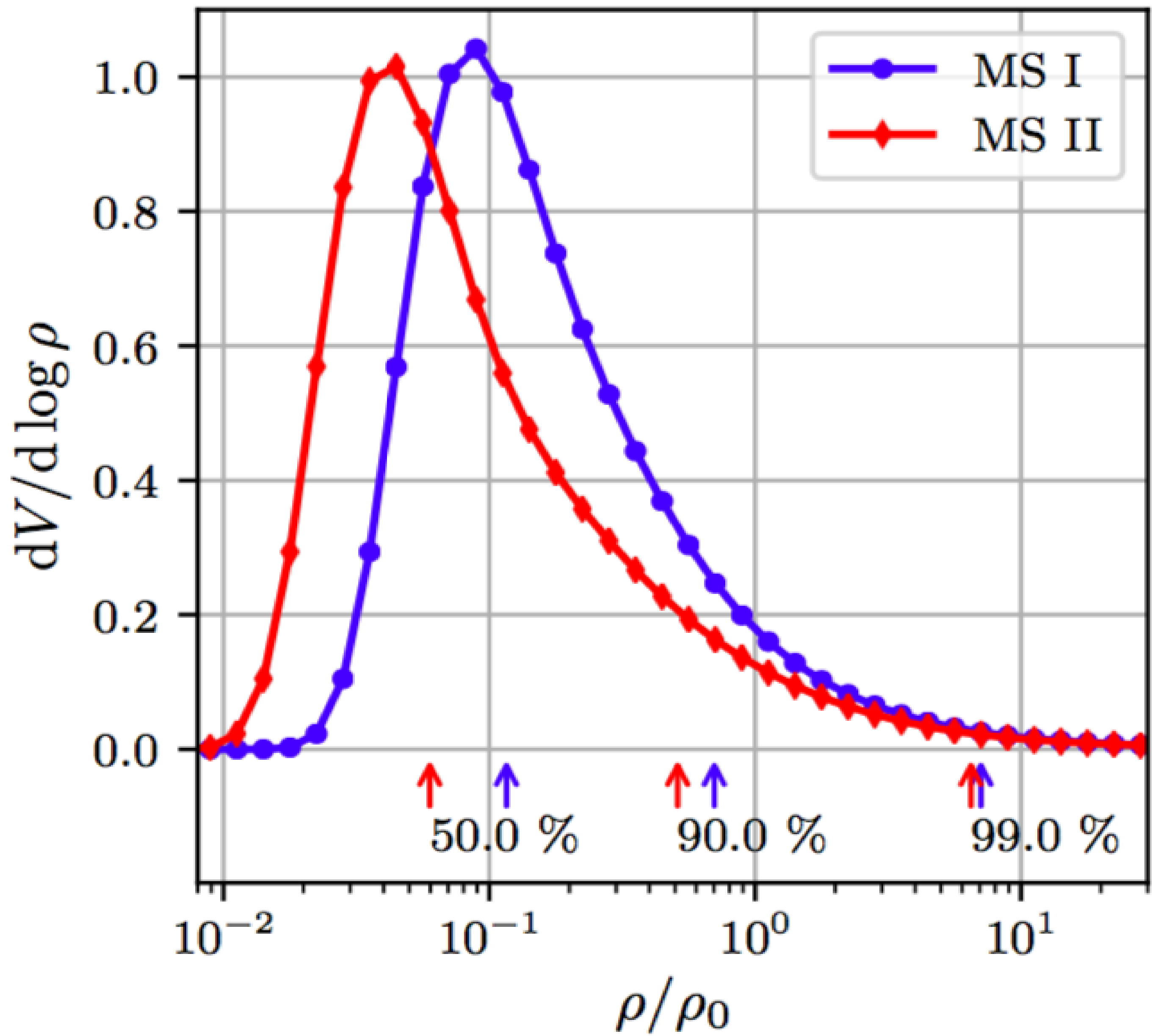
Failure to resolve smallest halos causes void densities to be overestimated



Failure to resolve inner regions of small halos is critical for total emission from annihilation

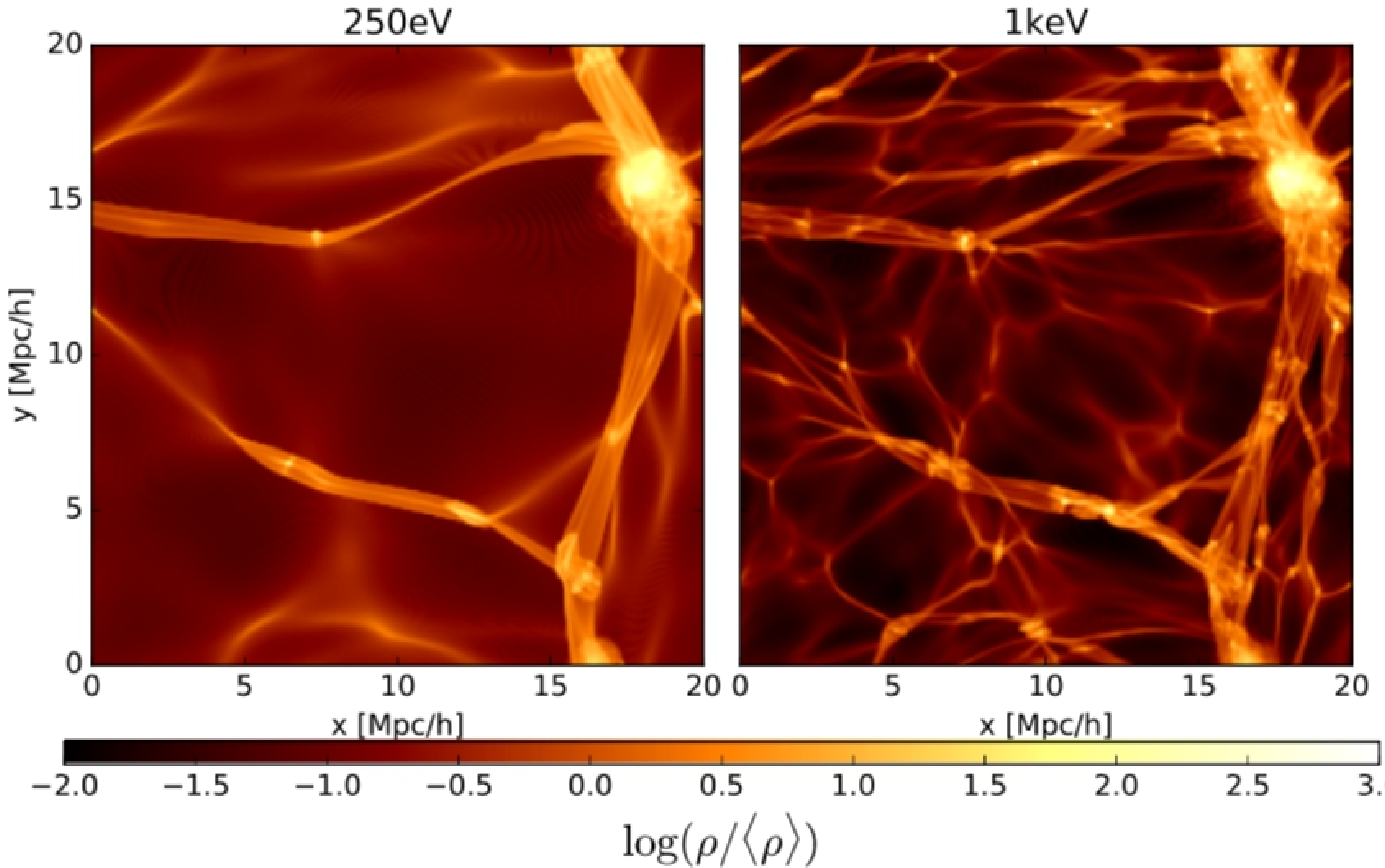
Volume-weighted density distributions in the two MS.

Stuecker et al 2017



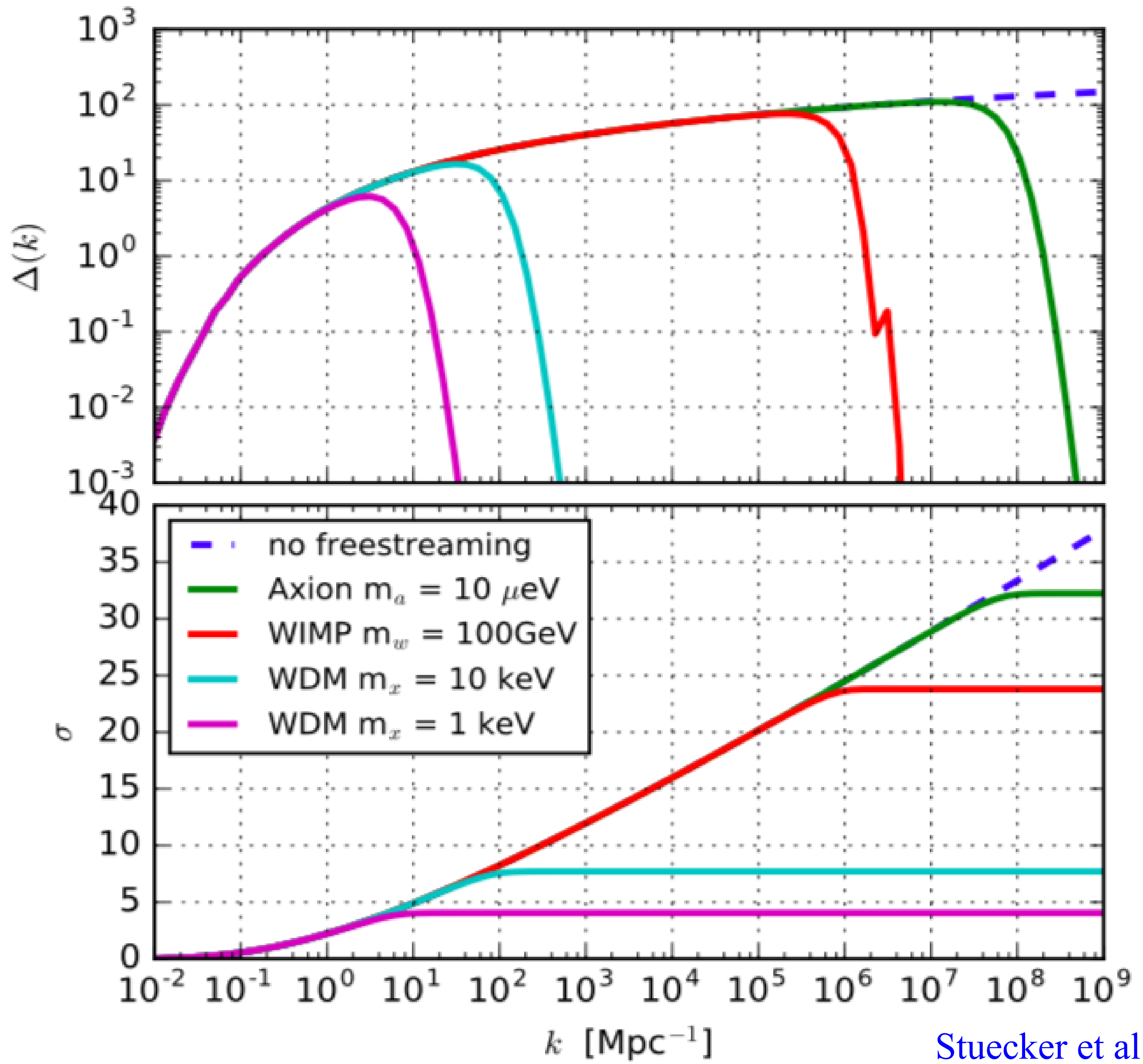
What is the <u>median</u> density of the Universe?

The median density is sensitive to the amount of small-scale structure: voids are emptier with more small-scale structure.



Stuecker et al 2017

The amount of small-scale structure depends on the nature of the dark matter.



Stuecker et al 2017

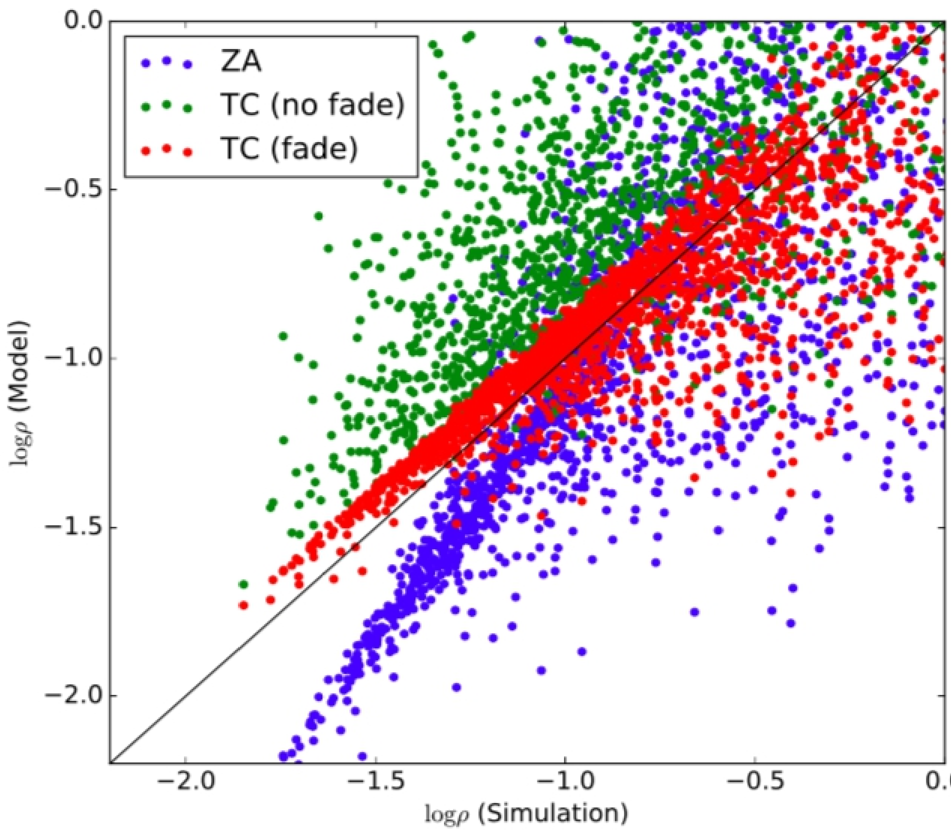
An excursion set model for single-stream regions

Most cosmic volume is in single-stream regions where the matter has never passed through a caustic. Their Lagrangian to Eulerian mapping involves stretching but no folding of the phase sheet. The GDE can then be approximated by

$$\dot{x}_i = a^{-2} p_i$$

$$\dot{p}_i = a^{-1} x_i \left(-\frac{4\pi G}{3} \rho_{bg} \delta + T_{ext,i} \right)$$

$$\delta = \frac{1}{|\det D_{xq}|} - 1 = \frac{1}{x_1 x_2 x_3} - 1$$



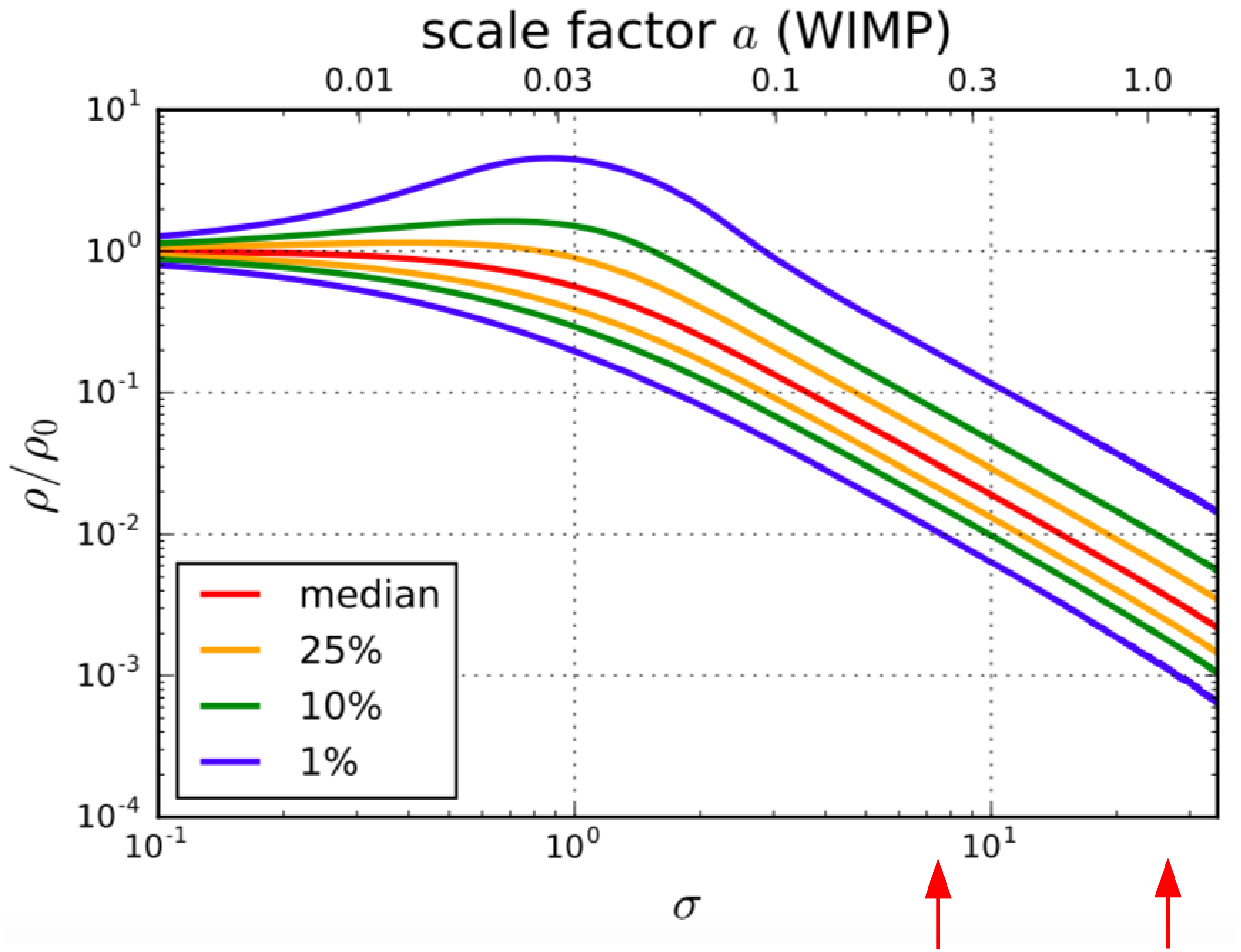
Stuecker et al 2017

together with a model for the evolution of the external tide T_{ext}

In an excursion set model, no caustic-crossing corresponds to a threshold in $T_{ext,0}$ as σ grows (smoothing shrinks)

A suitable tidal model agrees well with simulation for $\sigma = 4.1$

In an excursion set model, the density distribution in single stream regions depends only on σ, hence on the nature of DM



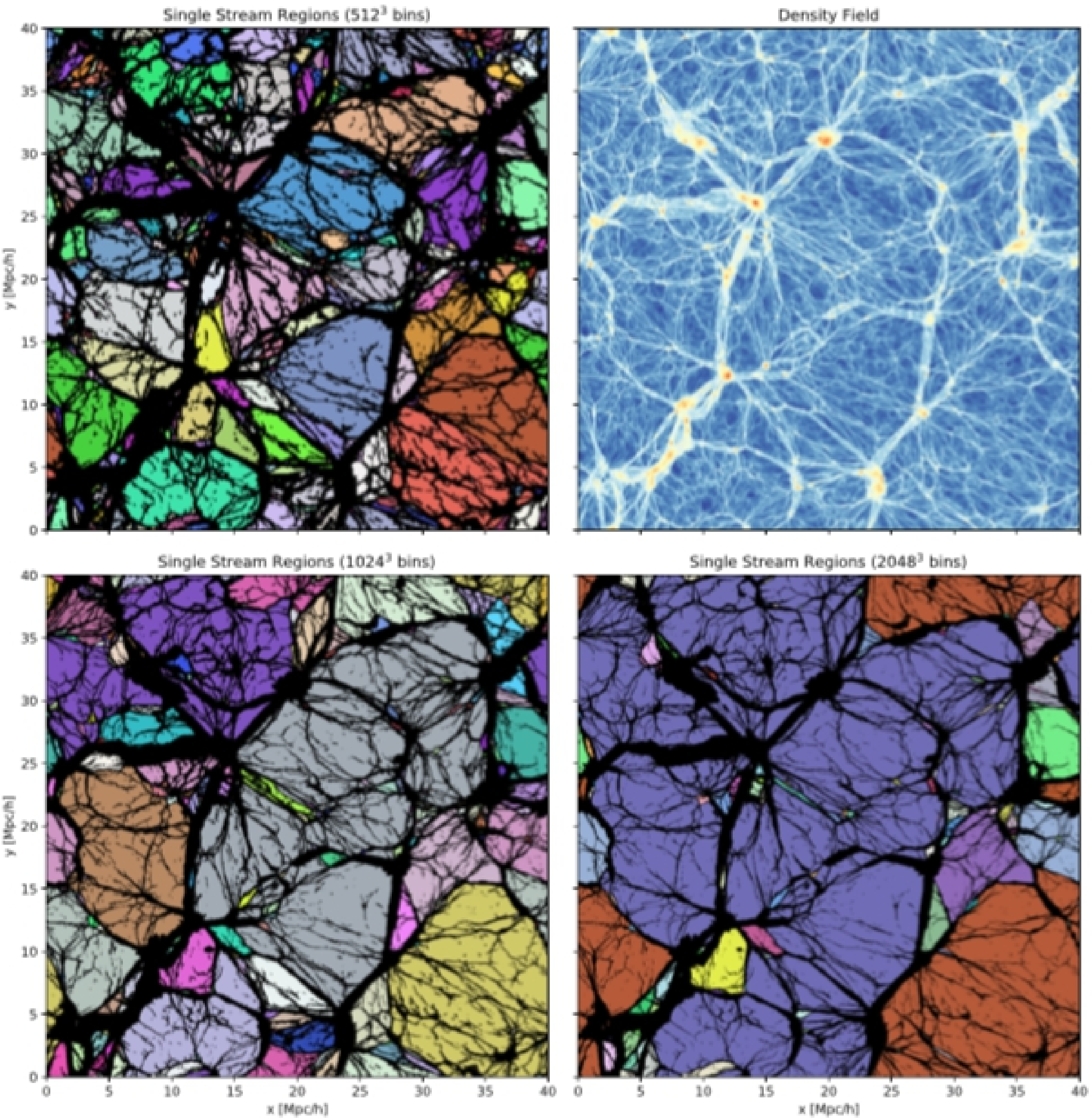
10 keV thermal relic

100 GeV WIMP

Stuecker et al 2017

Do single-stream regions percolate?

Stuecker et al 2017



In Eulerian space the answer is strongly resolution-dependent

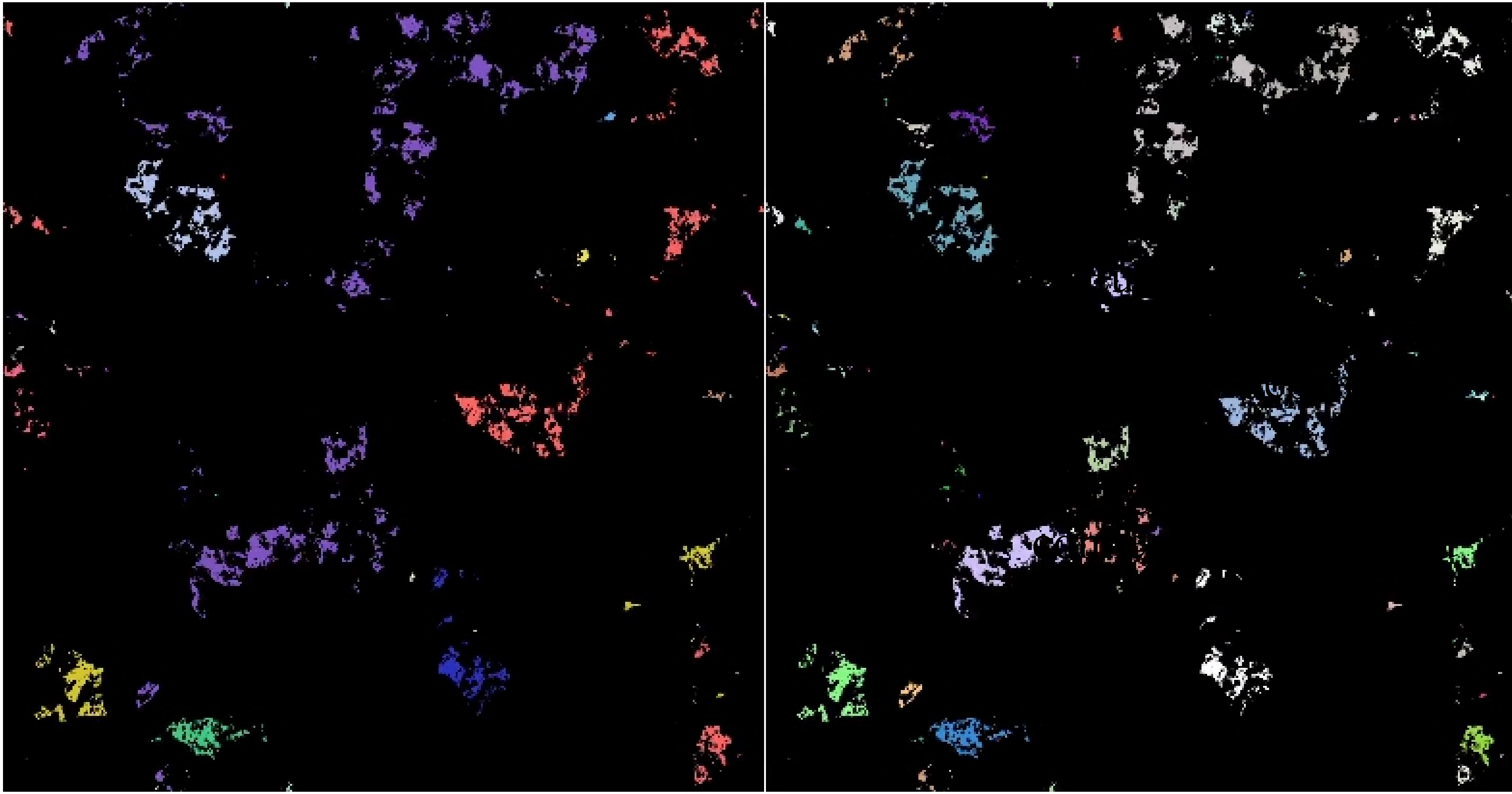
At higher resolution more connections are found

$\sigma = 6.4$

Do single-stream regions percolate?

Stuecker et al 2017

Colours from Eulerian connectivity. Colours from Lagrangian connectivity.



In Lagrangian space, however, they do not percolate for $\sigma = 6.4$ and seem less likely to percolate for larger σ

Conclusion?

- There are still many aspects of the nonlinear DM distribution that we do not understand well, even for vanilla CDM

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→ **Still lots of work to do, Joe! Many happy returns!**