

DESY workshop “Dark Matter”
June 2011

The structure of the dark matter distribution on laboratory scales

Simon White
Max-Planck-Institute for Astrophysics

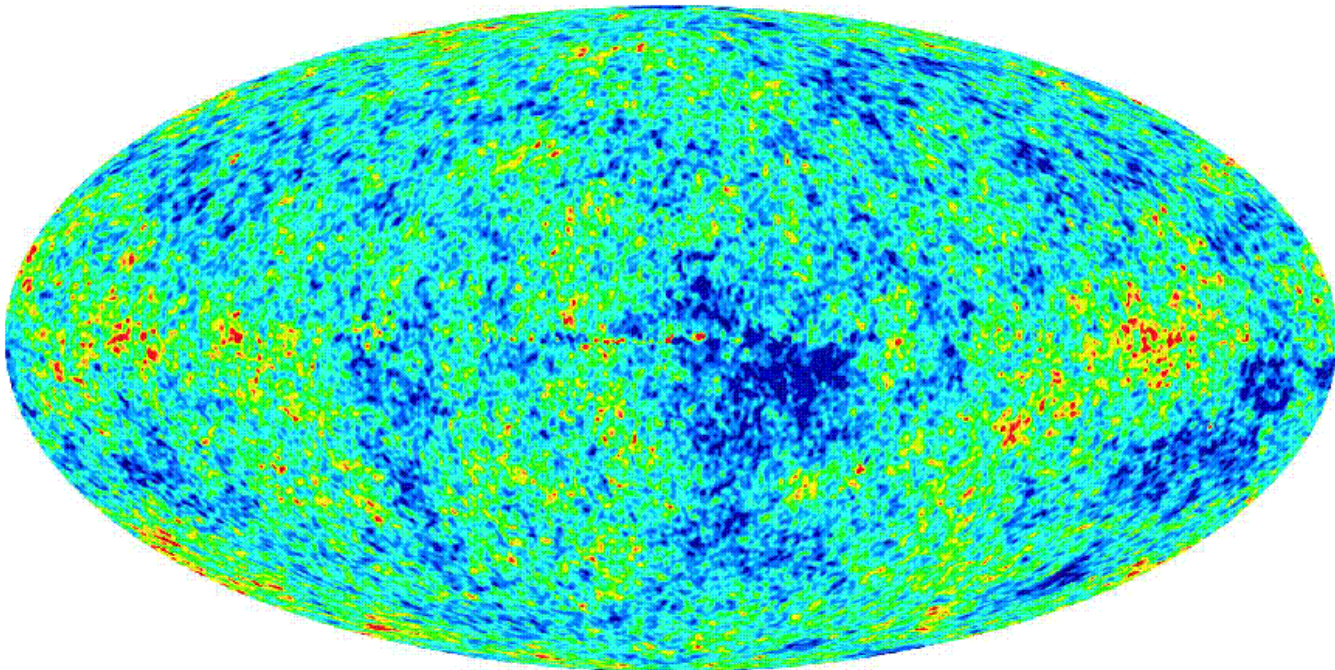
STScI Symposium, “Dark Matter”
May 2011



Mark Vogelsberger

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WMAP7+BAO+SN

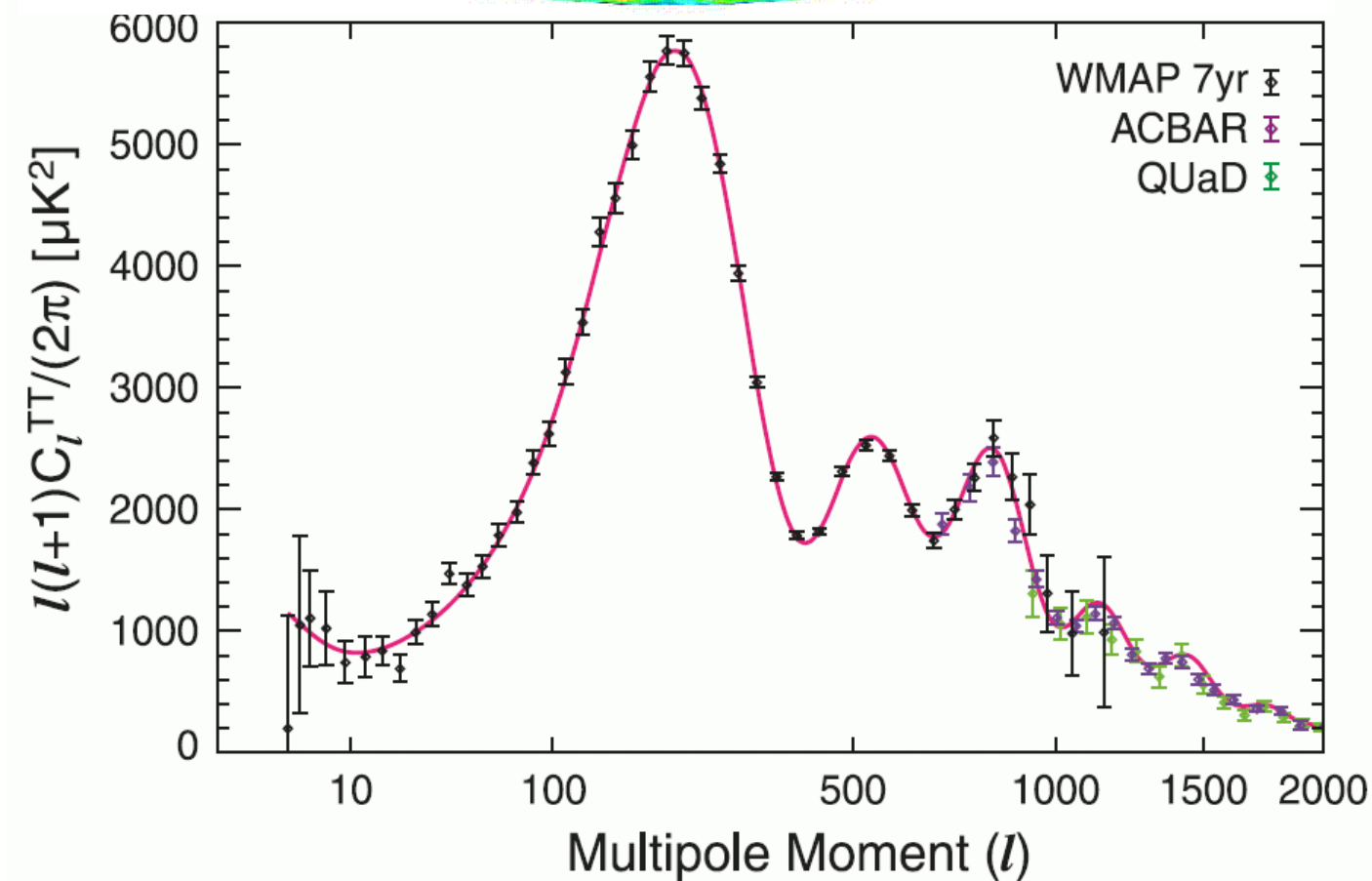
$$\Omega_{\text{bar}} = 0.046 \pm 0.002$$

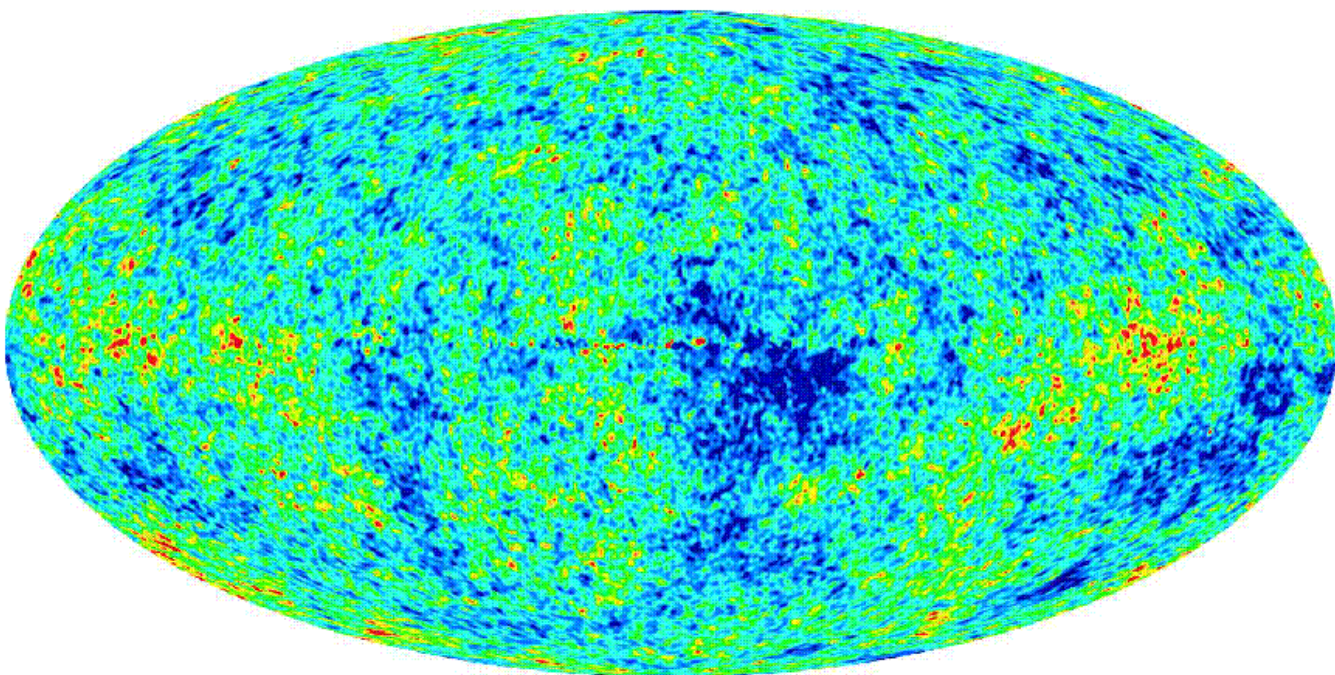
$$\Omega_{\text{DM}} = 0.229 \pm 0.015$$

$$\Omega_{\text{tot}} = 0.994 \pm 0.007$$

$$\sigma_8 = 0.82 \pm 0.02$$

$$n_s = 0.968 \pm 0.012$$





WMAP7+BAO+SN

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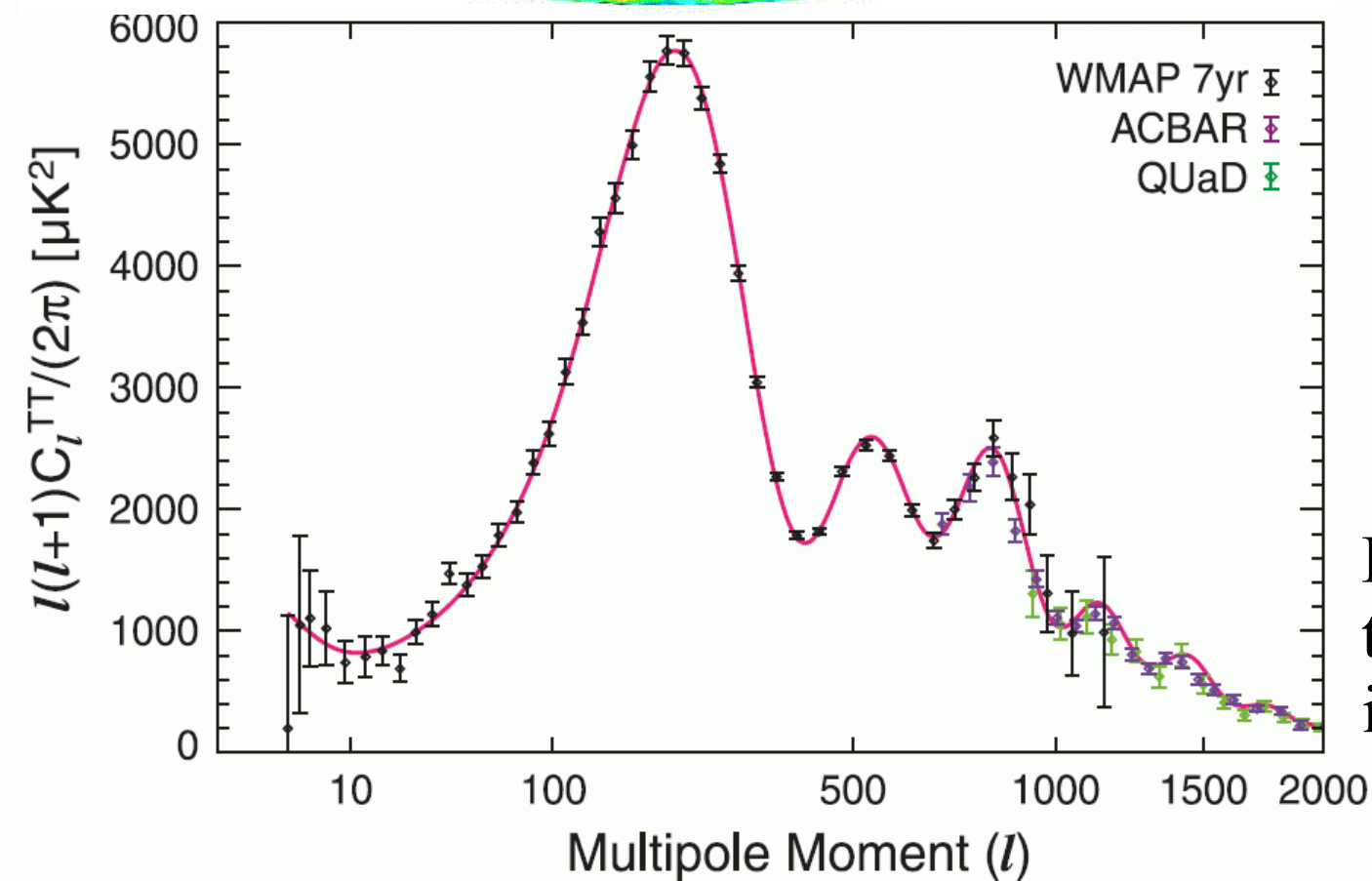
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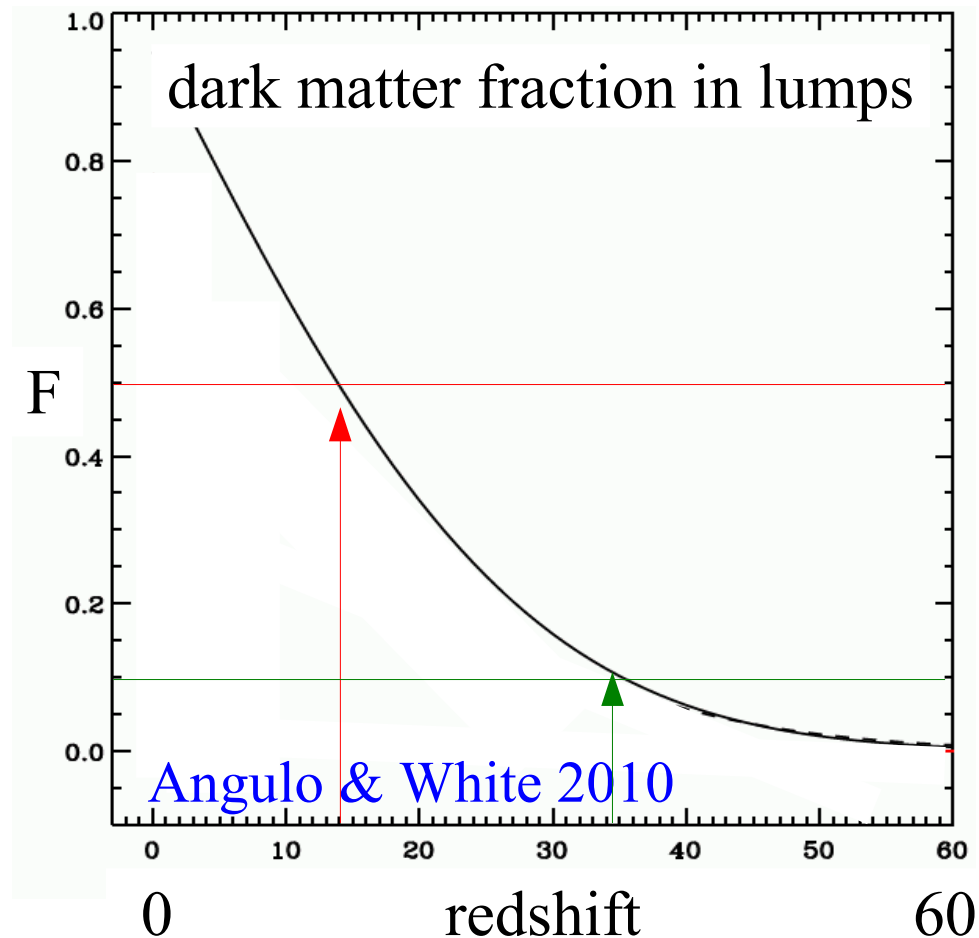


Best current evidence for
the existence of weakly
interacting, nonbaryonic
dark matter



The growth of nonlinear dark matter structures

- Structure grows through gravitational amplification of the seed fluctuations visible in the CMB
- Nonlinear dark matter objects (“halos”) like that in which the Milky Way lives grow by the infall of diffuse material and smaller halos



All dark matter is diffuse at $z > 60$

90% is diffuse at $z > 35$

50% is diffuse at $z > 13$

All nonlinear structure forms late,
even halos of Earth mass or smaller



The four elements of Λ CDM halos

I Smooth background halo

- NFW-like cusped density profile
- near-ellipsoidal equidensity contours

II Bound subhalos

- most massive typically 1% of main halo mass
- total mass of all subhalos $\lesssim 10\%$
- less centrally concentrated than the smooth component

III Tidal streams

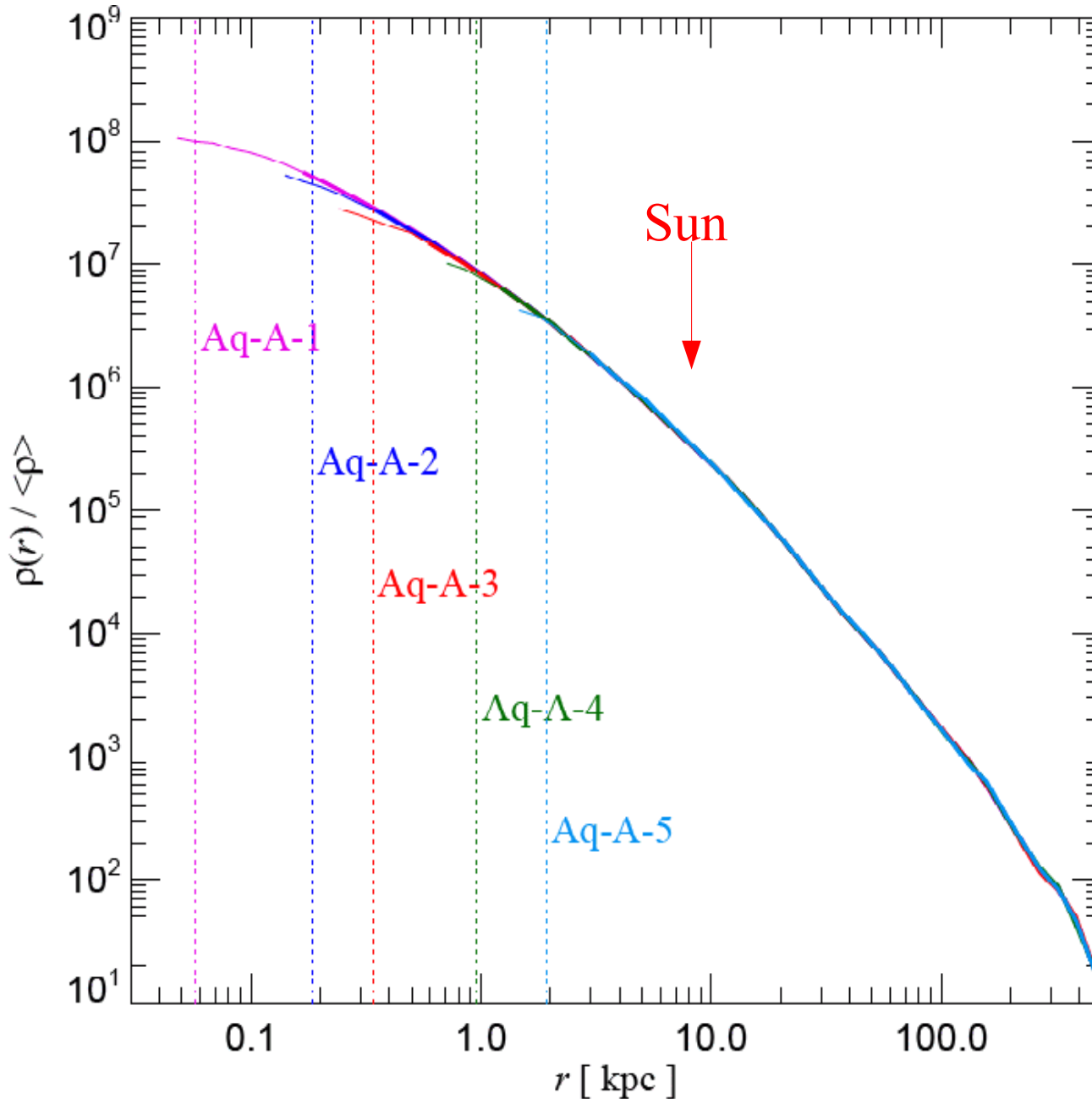
- remnants of tidally disrupted subhalos

IV Fundamental streams

- consequence of smooth and cold initial conditions
- very low internal velocity dispersions
- produce density caustics at projective catastrophes

I. Smooth background halo

Aquarius Project: Springel et al 2008



- Density profiles of simulated DM-only Λ CDM halos are now very well determined -- to radii well inside the Sun's position

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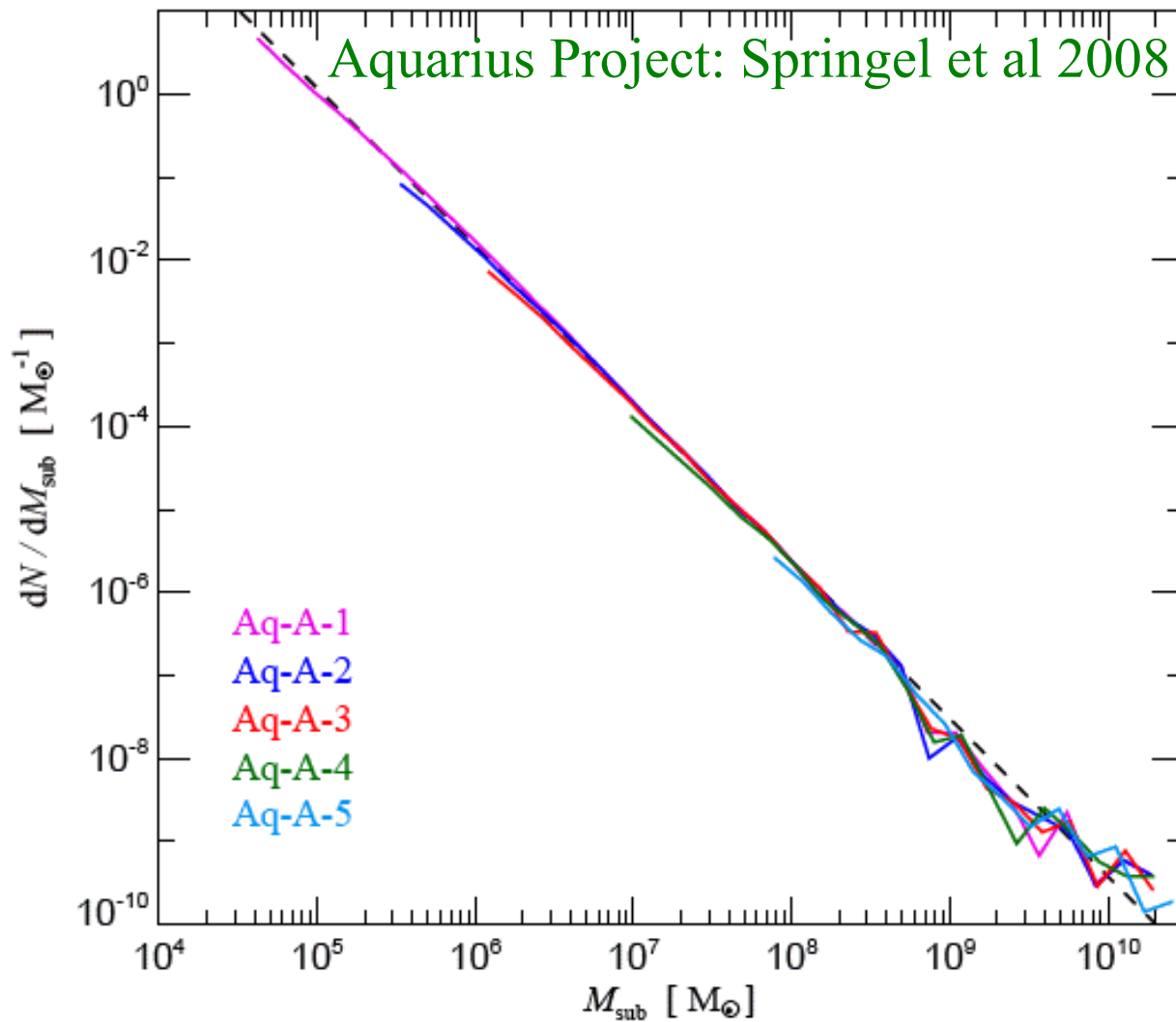
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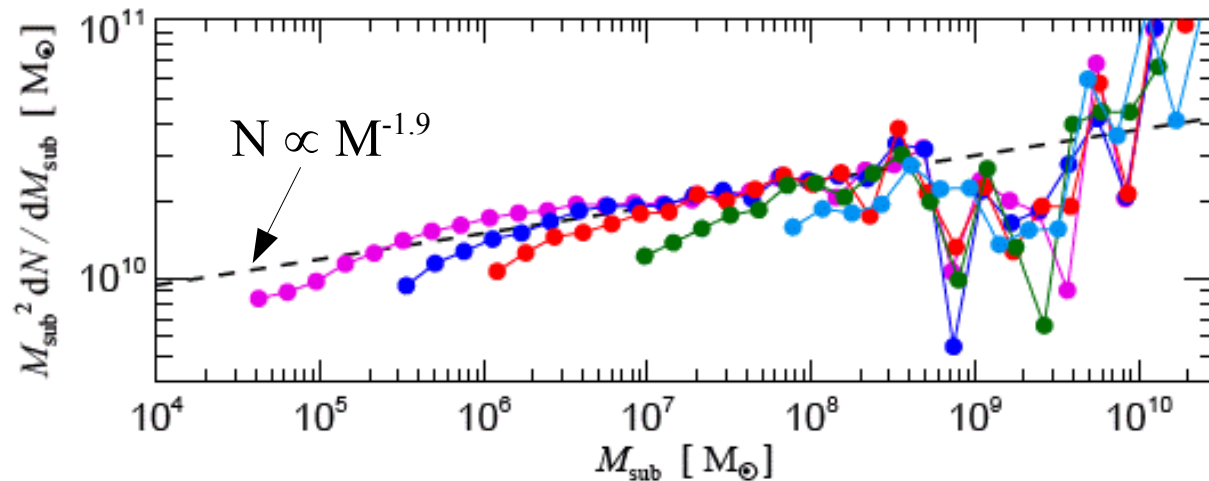
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Aquarius Project: Springel et al 2008



II. Bound subhalos

- Abundance of self-bound subhalos is measured to below $10^{-7} M_{\text{halo}}$
- Most subhalo mass is in the biggest objects (just)



Bound subhalos: conclusions

- Substructure is primarily in the outermost parts of halos
- The radial distribution of subhalos is almost mass-independent
- The total mass in subhalos converges (weakly) at small m
- Subhalos contain a very small mass fraction in the inner halo ($\sim 0.1\%$ near the Sun) and so will *not* be relevant for direct detection experiments

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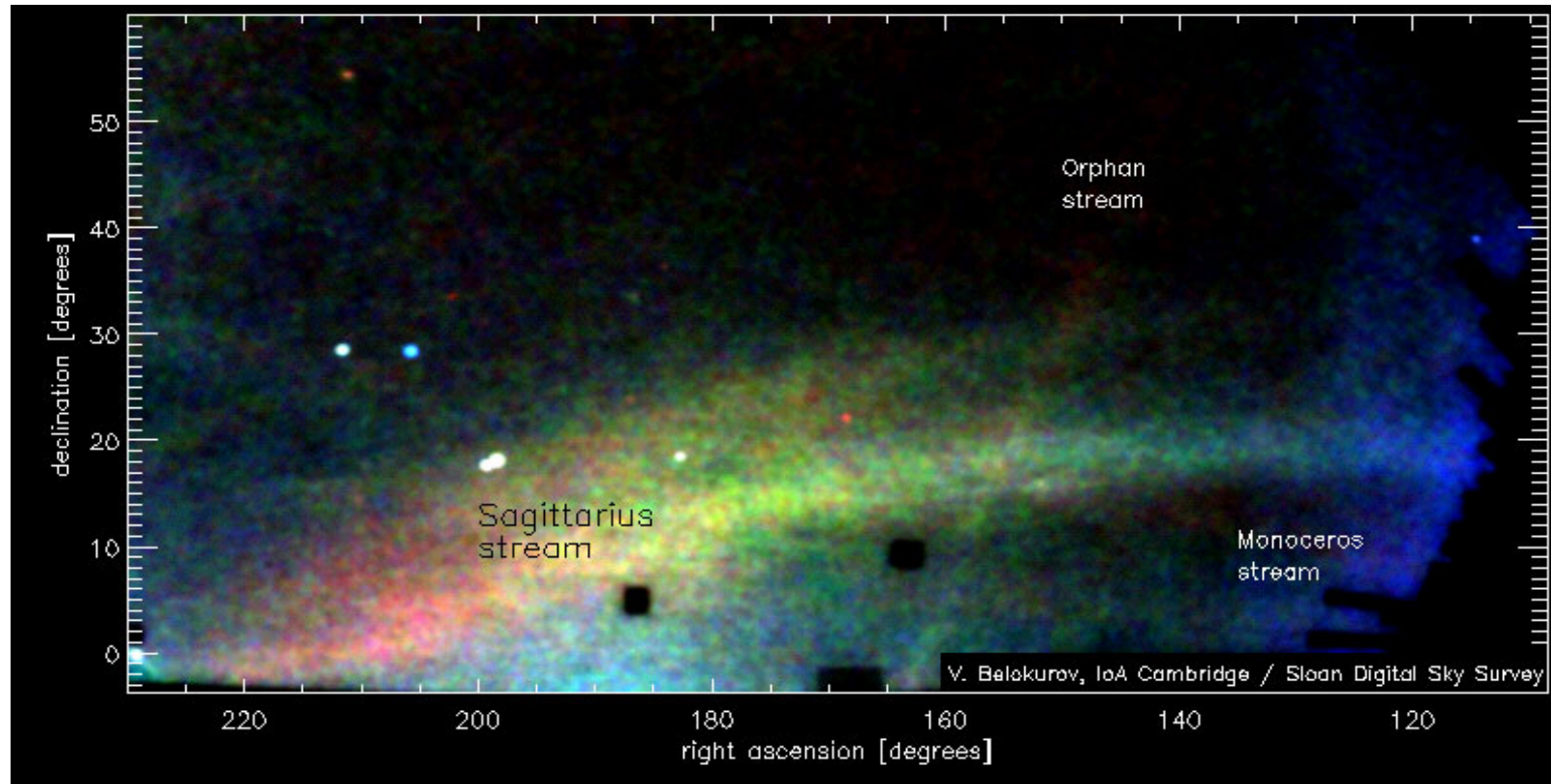
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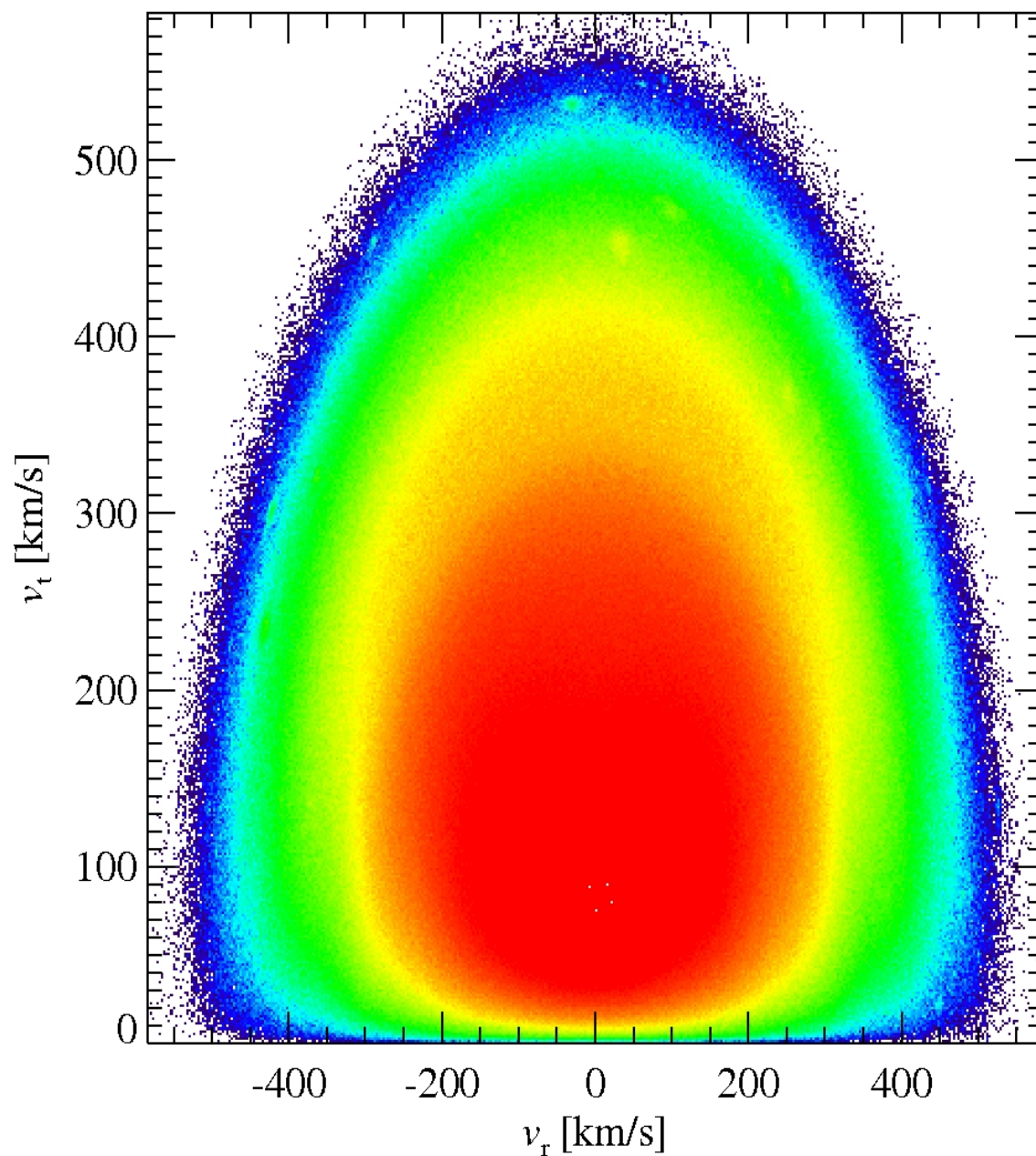
III. Tidal Streams



- Produced by partial or total tidal disruption of subhalos
- Analogous to observed stellar streams in the Galactic halo
- Distributed along/around orbit of subhalo (c.f. meteor streams)
- Localised in almost 1-D region of 6-D phase-space (\underline{x} , \underline{v})

Dark matter phase-space structure in the inner MW

M. Maciejewski



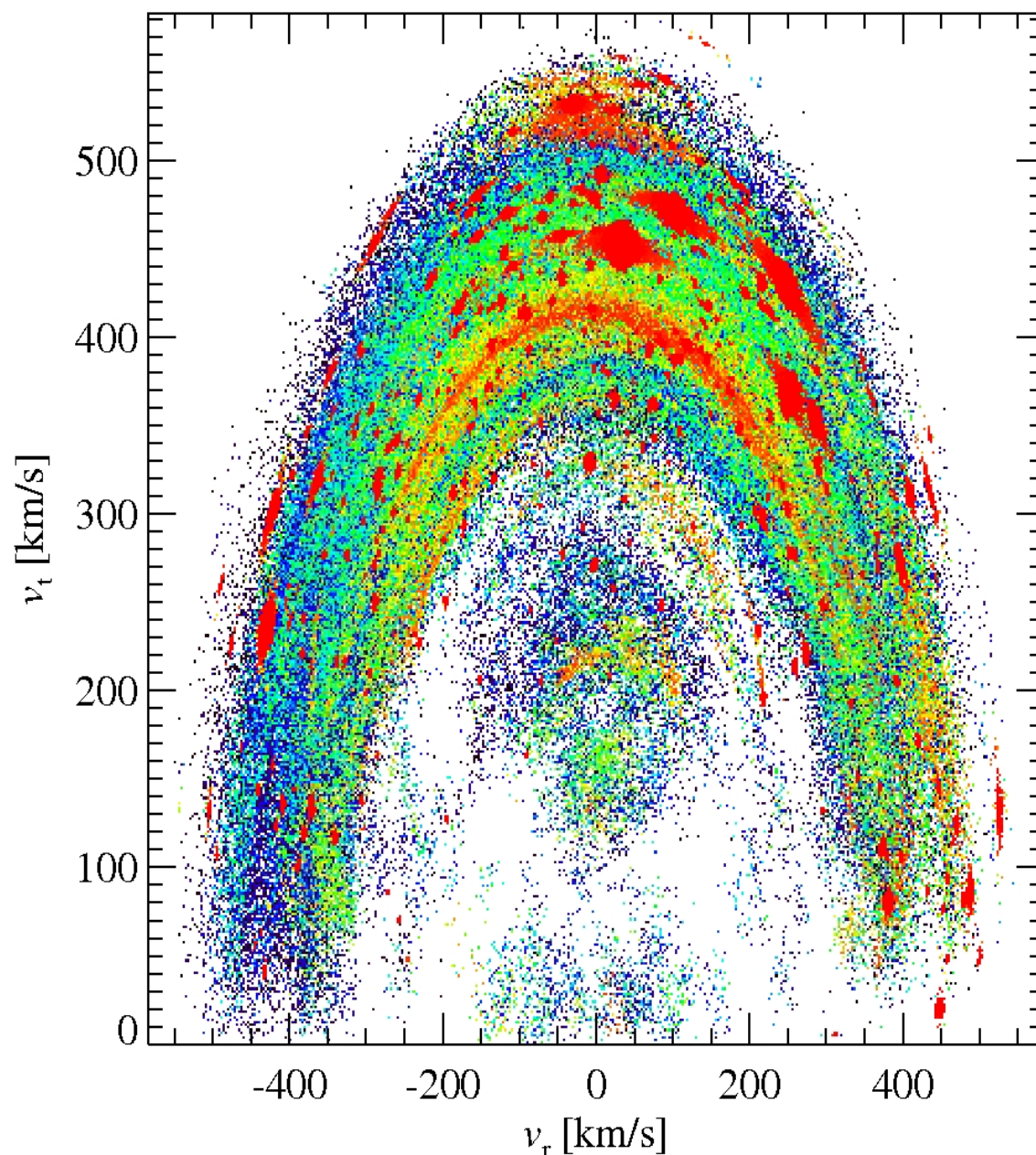
$6 \text{ kpc} < r < 12 \text{ kpc}$

All particles

$N = 3.8 \times 10^7$

Dark matter phase-space structure in the inner MW

M. Maciejewski



$6 \text{ kpc} < r < 12 \text{ kpc}$

Particles in detected
phase-space structure

$N = 2.6 \times 10^5$
in tidal streams

$N = 3.9 \times 10^4$
in subhalos

→ only $\sim 1\%$ of the
DM signal is in strong
tidal streams

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IV. Fundamental streams

After CDM particles become nonrelativistic, but *before* nonlinear objects form (e.g. $z > 100$) their distribution function is

$$f(\mathbf{x}, \mathbf{v}, t) = \rho(t) [1 + \delta(\mathbf{x}, t)] N[\{\mathbf{v} - \mathbf{V}(\mathbf{x}, t)\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\mathbf{x}, t)$ is a Gaussian random field with finite variance $\ll 1$,

$\mathbf{V}(\mathbf{x}, t) = \nabla \psi(\mathbf{x}, t)$ where $\nabla^2 \psi \propto \delta$,

and N is normal with $\sigma^2 \ll \langle |\mathbf{V}|^2 \rangle$ (today $\sigma \sim 0.1$ cm/s)

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto \mathbf{x} -space is near-uniform.

$Df/Dt = 0 \longrightarrow$ only a 3-D subspace is occupied at *all* times.

Nonlinear evolution leads to multi-stream structure and caustics

IV. Fundamental streams

Consequences of $Df/Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least one sheet must pass through every point \mathbf{x}
- In nonlinear objects there are typically many sheets at each \mathbf{x}
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f = \text{const.}$ $\longrightarrow \sigma \sim \rho^{-1/3}$
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{\mathbf{X}} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{\mathbf{X}} = \begin{bmatrix} \delta\mathbf{v} \\ \mathbf{T} \cdot \delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot \delta\mathbf{X}$; $\mathbf{T} = -\nabla(\nabla\phi)$

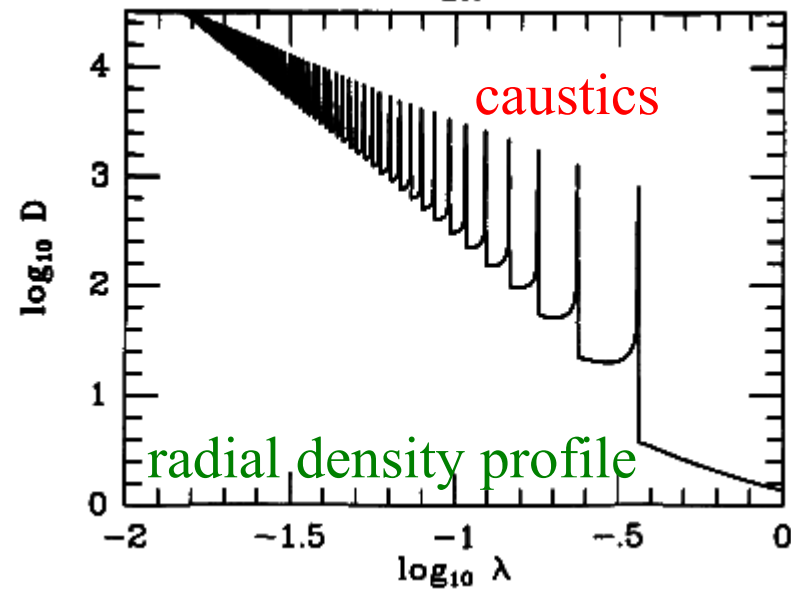
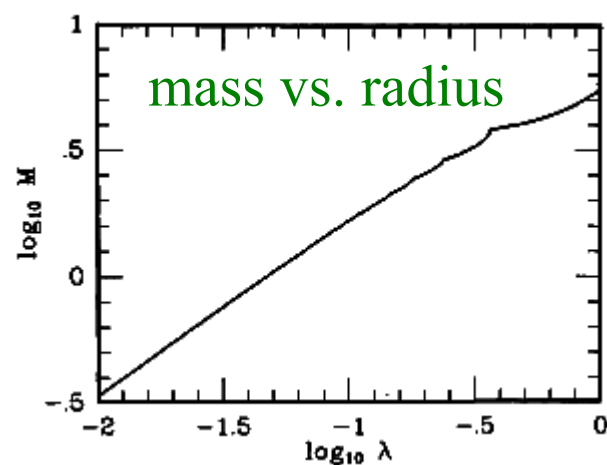
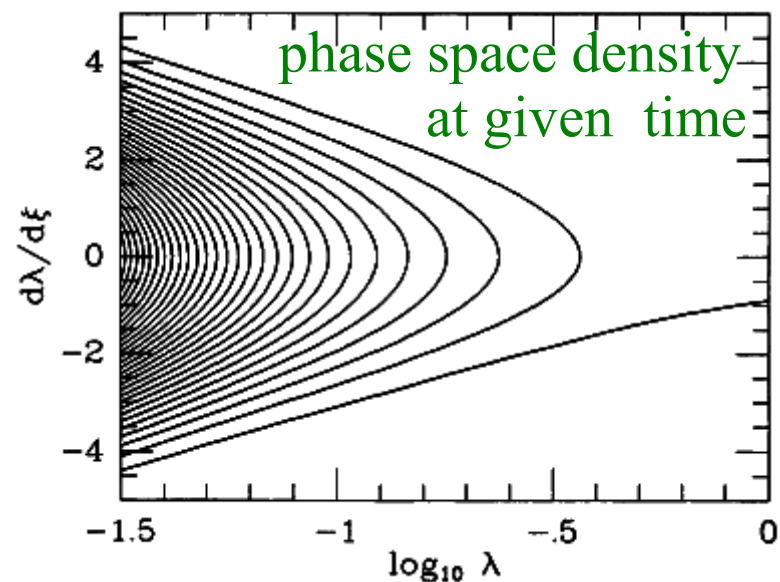
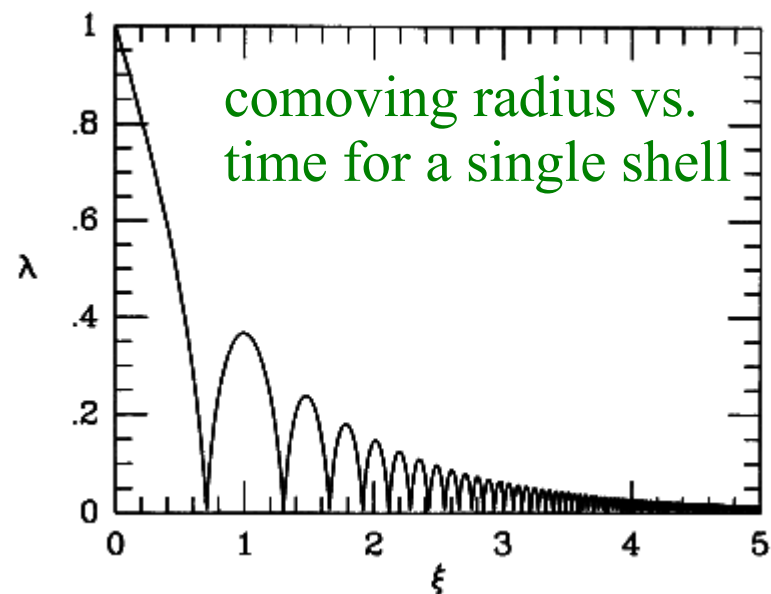
Write $\delta\mathbf{X}(t) = \mathbf{D}(\mathbf{X}_0, t) \cdot \delta\mathbf{X}_0$, then differentiating w.r.t. time gives,

$$\dot{\mathbf{D}} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot \mathbf{D} \quad \text{with } \mathbf{D}_0 = \mathbf{I}$$

- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- $\det(\mathbf{D}) = 1$ at all times by Liouville's theorem
- For CDM, $1/|\det(\mathbf{D}_{\mathbf{xx}})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

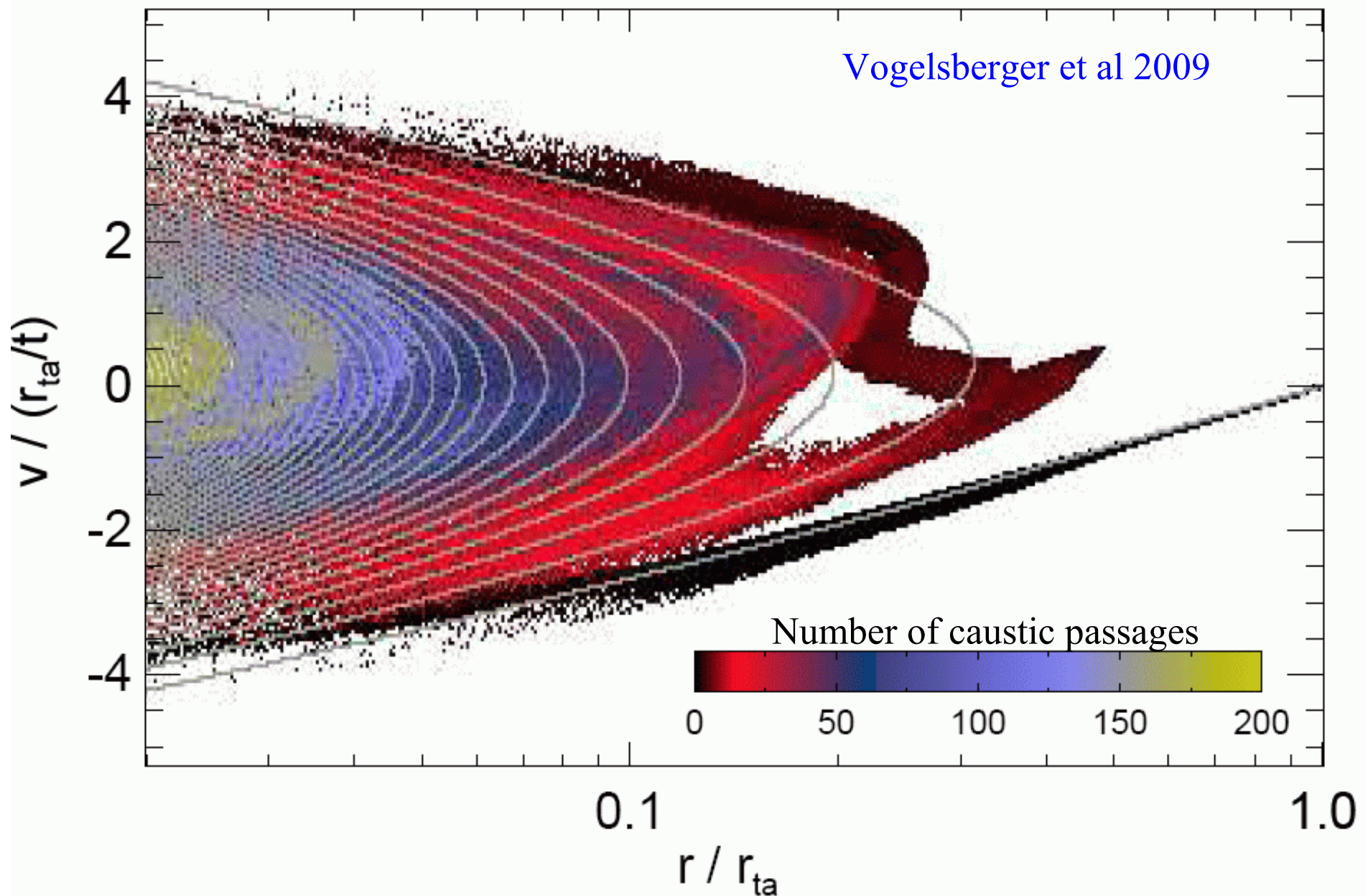
Similarity solution for spherical collapse in CDM

Bertschinger 1985

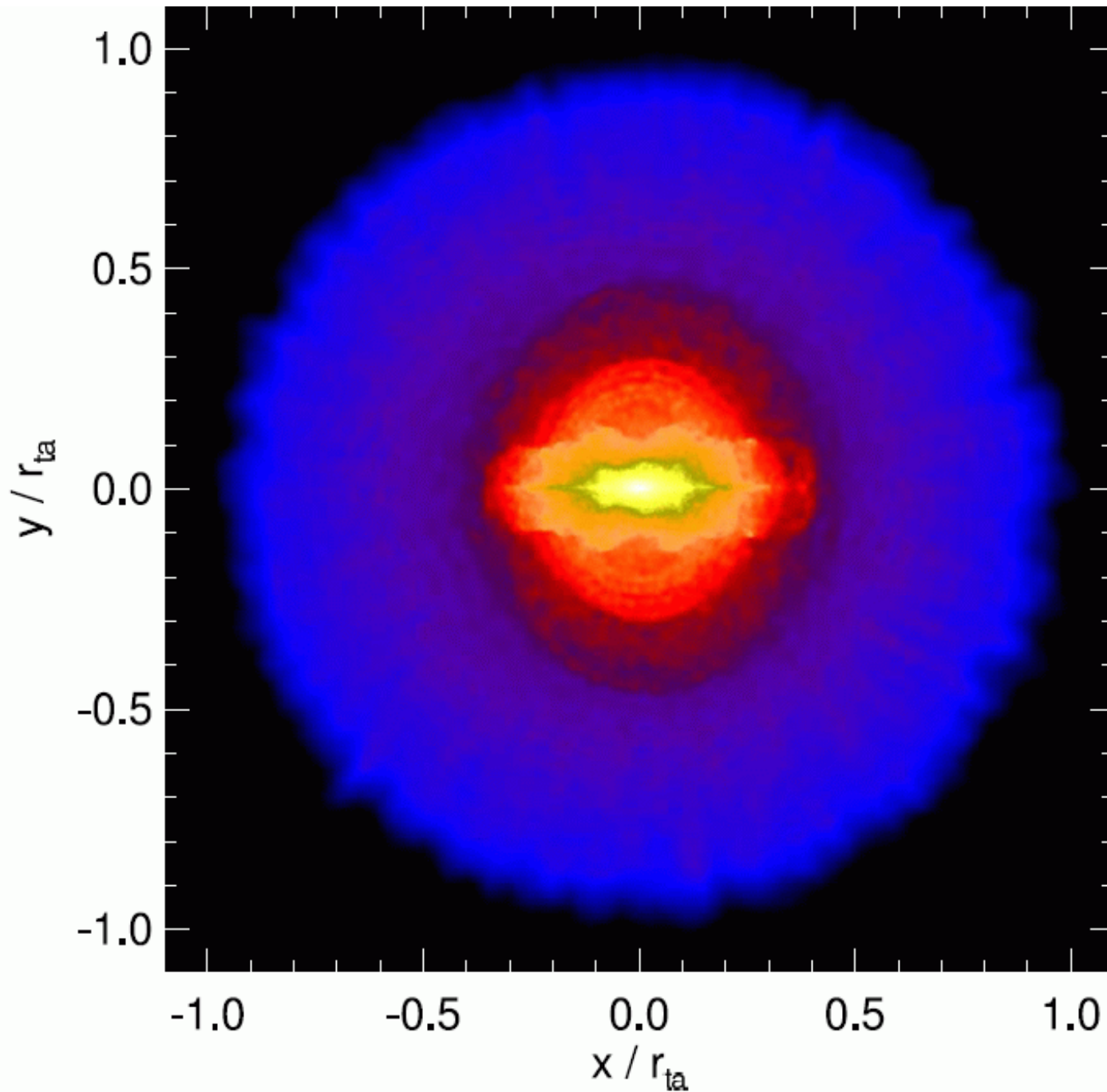


Simulation from self-similar spherical initial conditions

Geodesic deviation equation \longrightarrow phase-space structure local to each particle



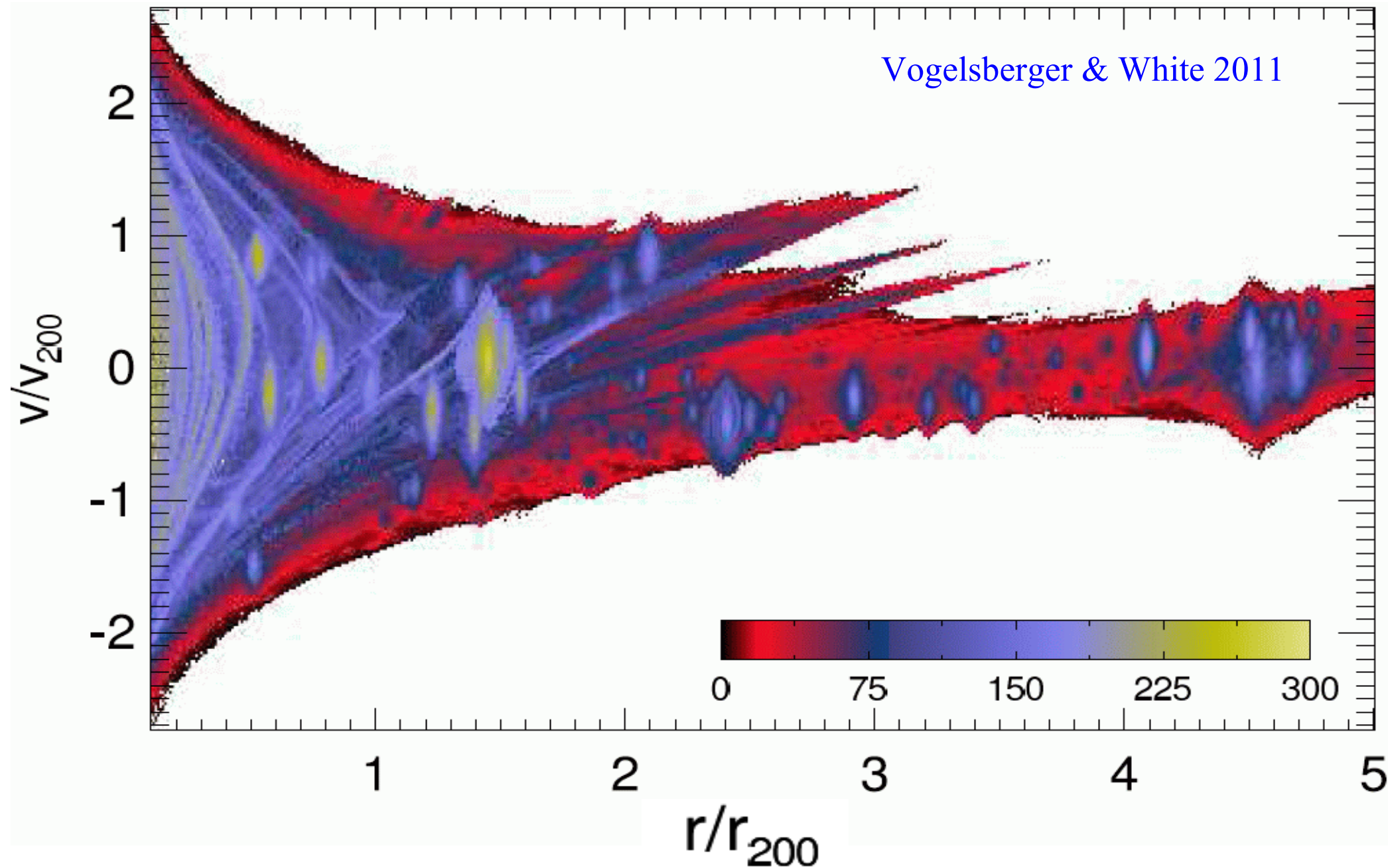
Simulation from self-similar spherical initial conditions



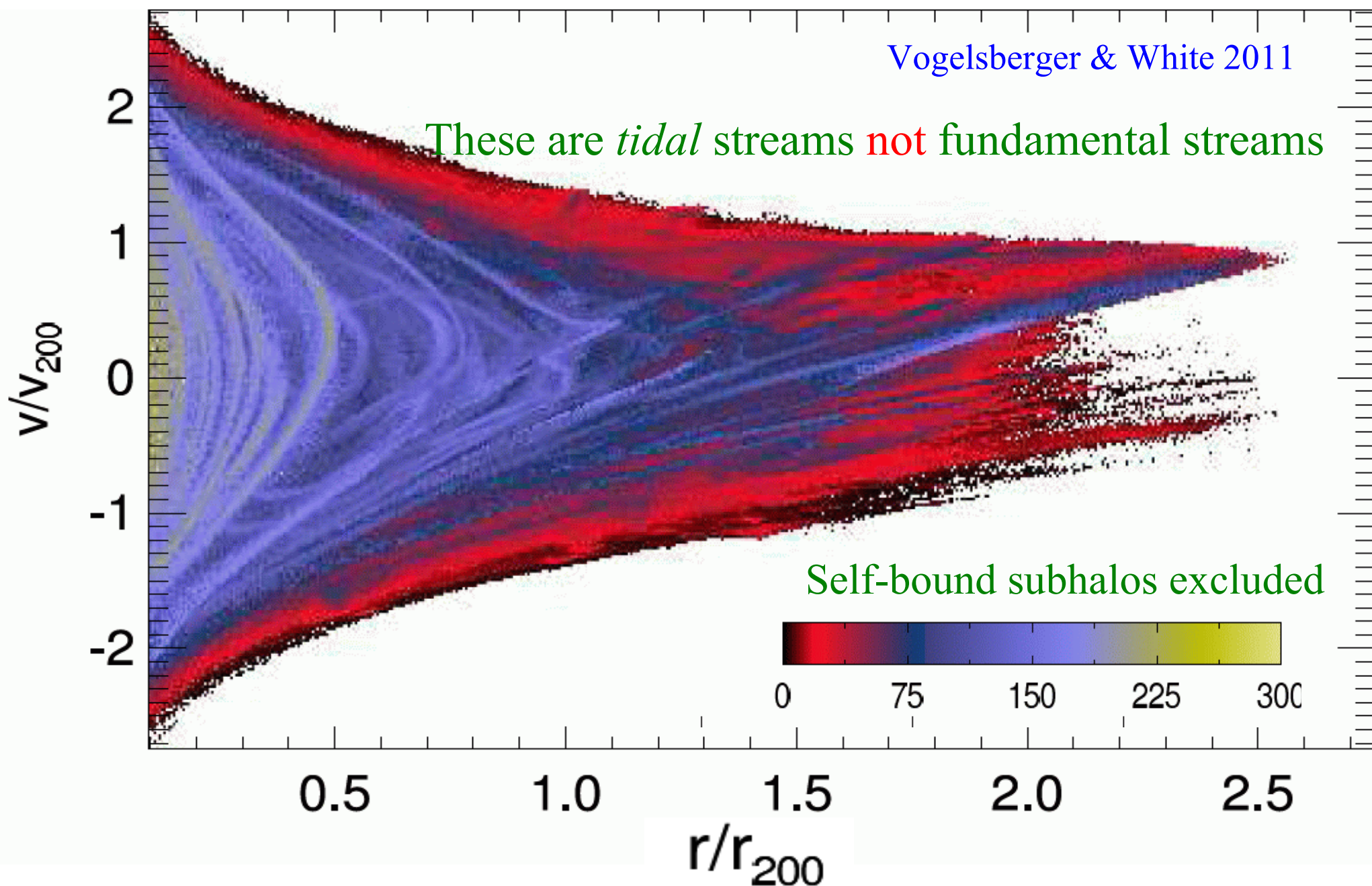
Vogelsberger et al 2009

The radial orbit instability leads to a system which is strongly prolate in the inner nonlinear regions

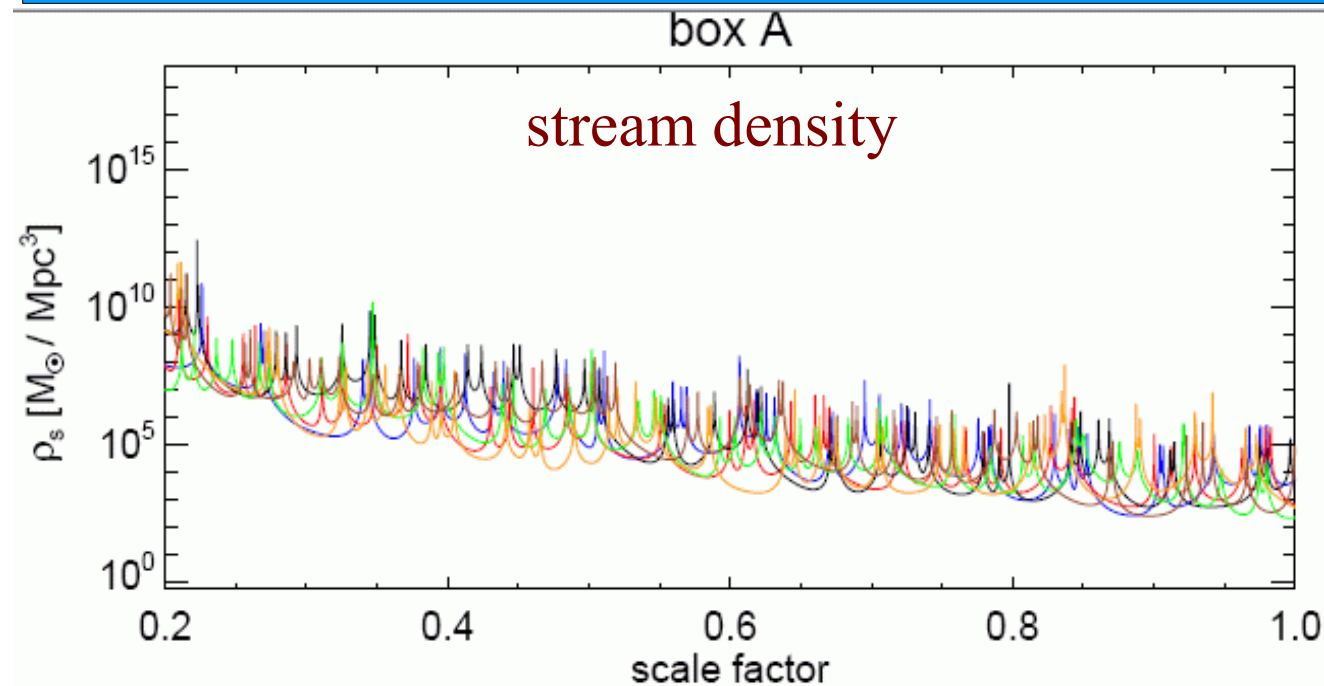
Caustic crossing counts in a Λ CDM Milky Way halo



Caustic crossing counts in a Λ CDM Milky Way halo



Stream density variations along orbits in a Λ CDM halo

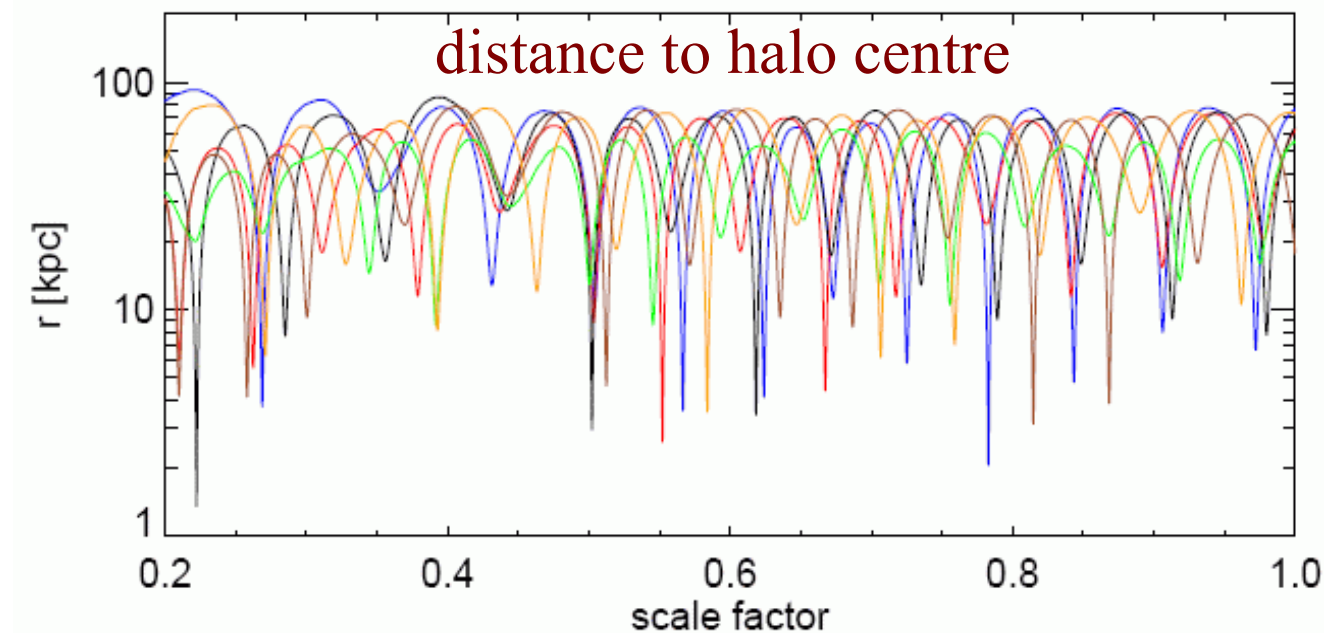


Orbital properties for six particles chosen so:

-- in main halo at $z = 4$

-- 40 caustics in $4 > z > 0$

-- typical drop in ρ_{stream}

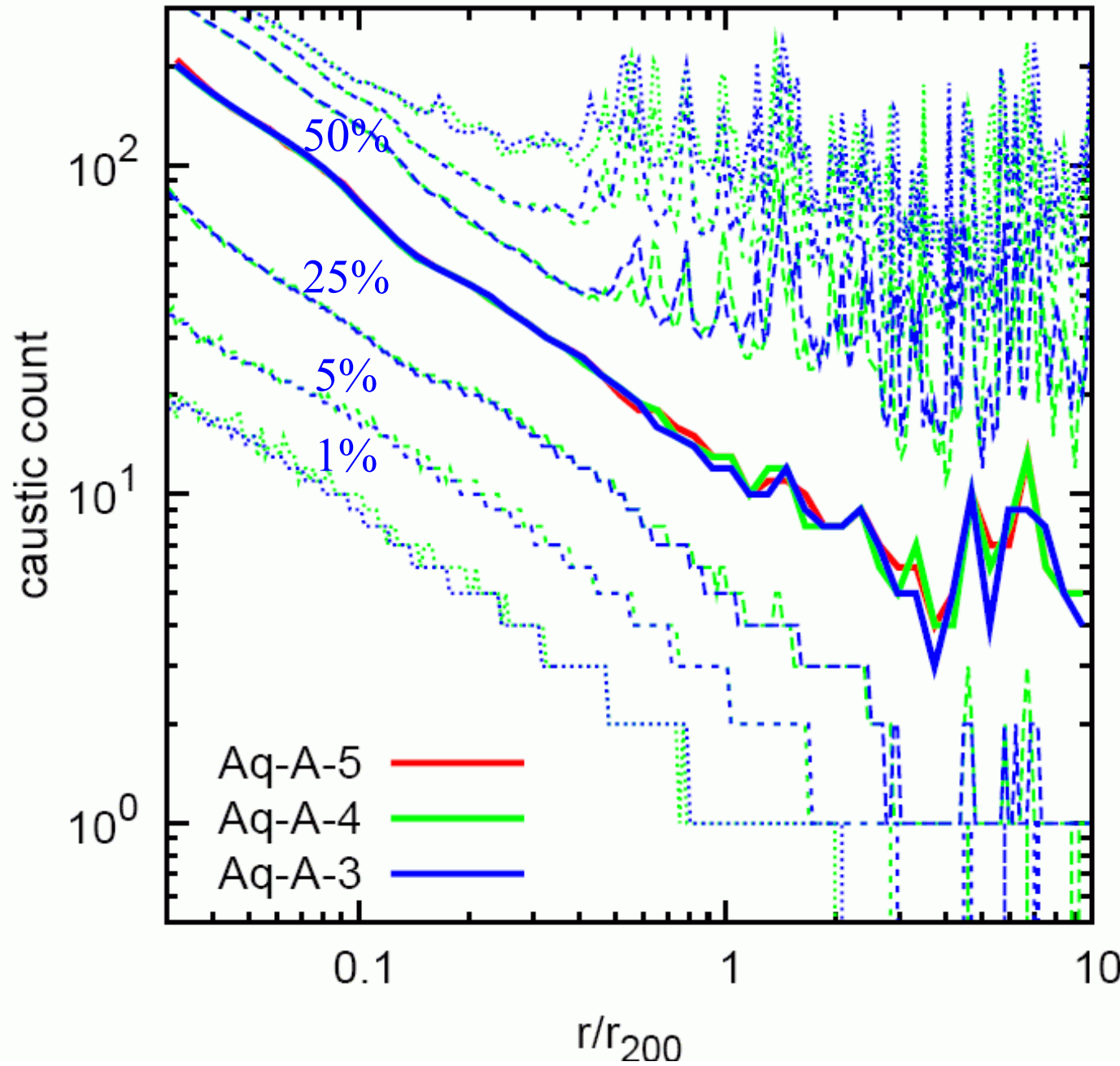


An average of 3 caustic crossings per orbit

Large drops in minimum ρ_{stream} often follow close pericentre passages

Caustic count profiles for Aquarius halos

Vogelsberger & White 2011



Note agreement of simulations of the same object with

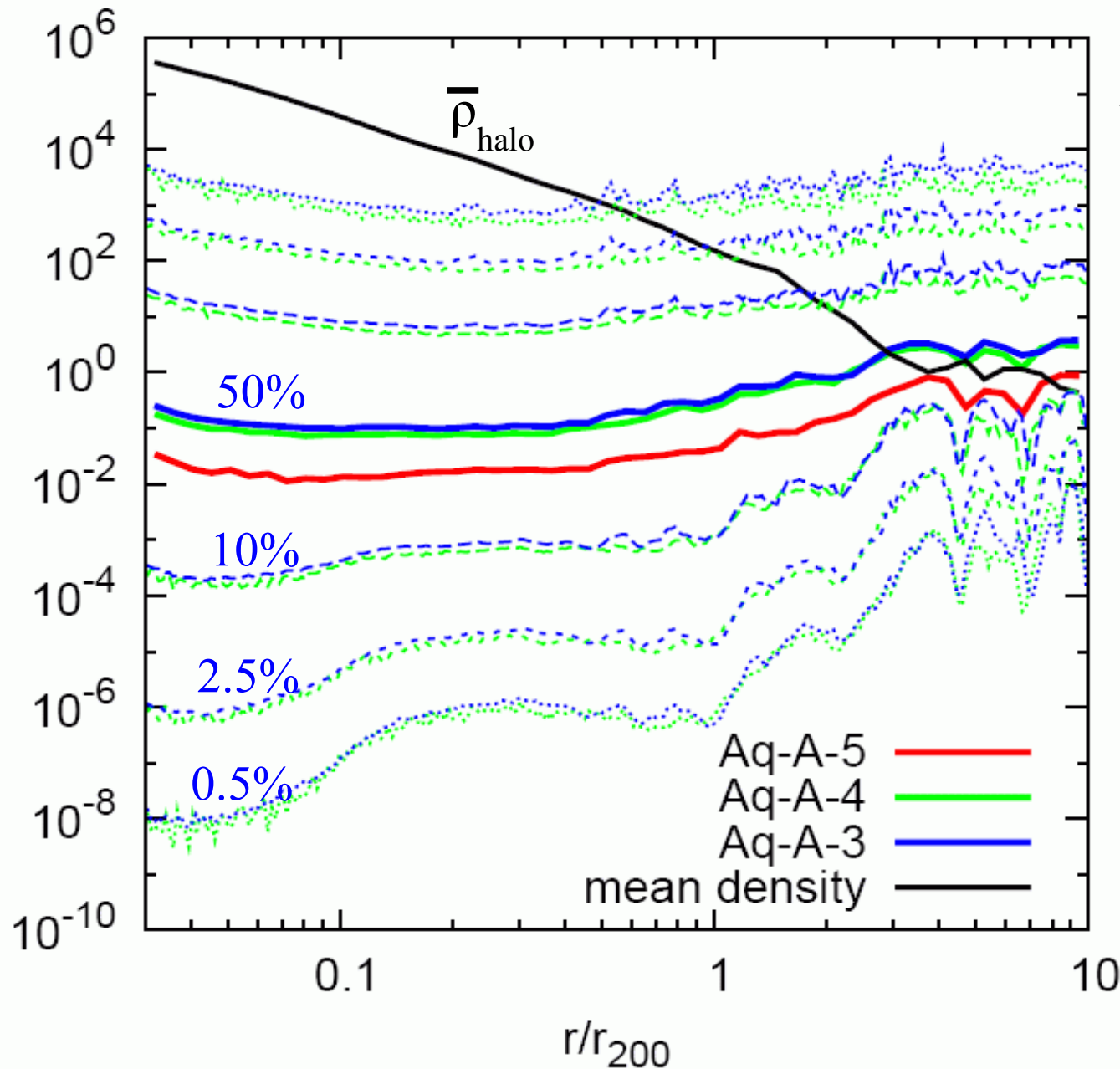
$$N = 8.1 \times 10^5$$

$$N = 6.4 \times 10^6$$

$$N = 5.1 \times 10^7$$

Stream density distribution in Aquarius halos

Vogelsberger & White 2011



Note the convergence with varying N .

With conventional methods detecting a stream with

$$\rho_{\text{stream}} = 10^{-8} \rho_b$$

requires particle mass

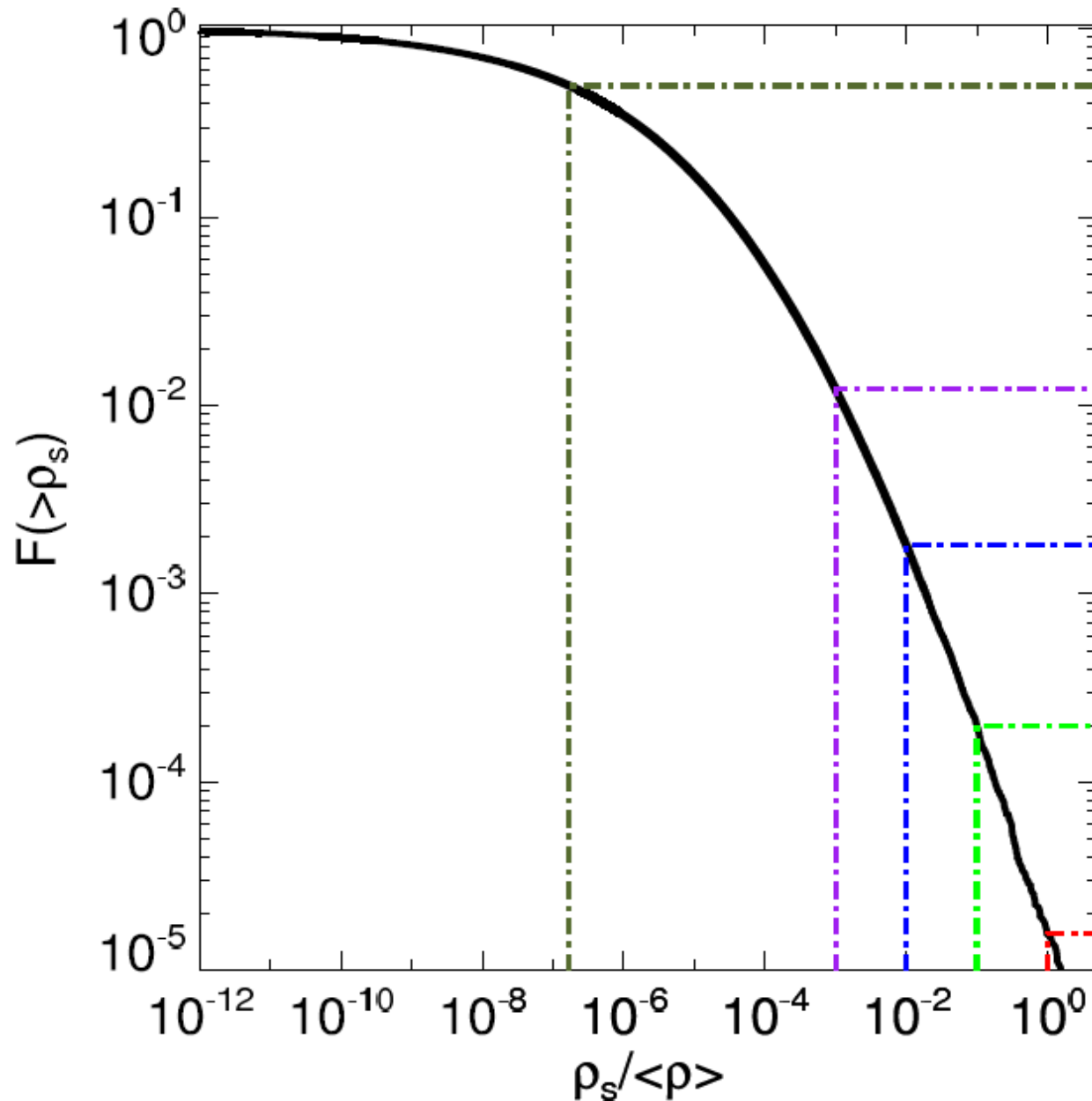
$$m_p \sim 10^{-7} M_{\odot},$$

thus a simulation with

$$N \sim 10^{20}$$

Stream density distribution at the Sun

Vogelsberger & White 2011



Cumulative stream density distribution for particles with $7 \text{ kpc} < r < 13 \text{ kpc}$

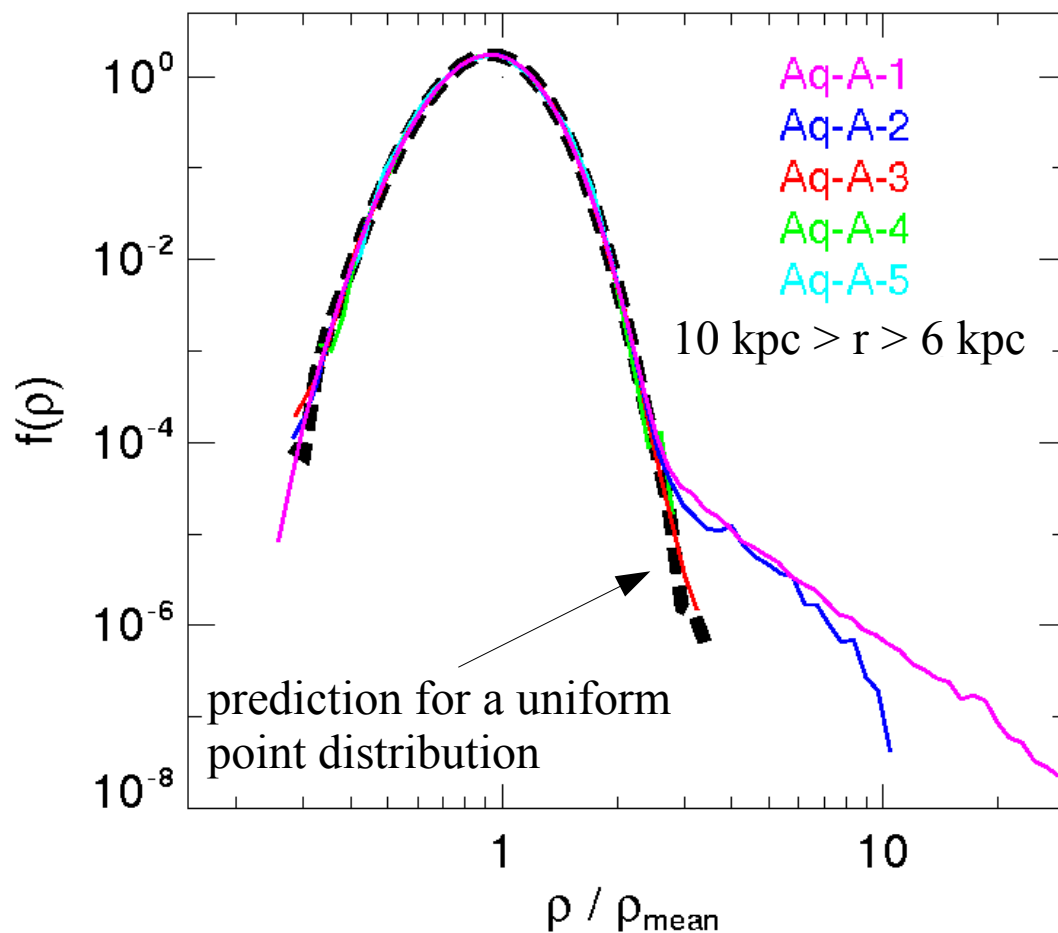
Probability that the Sun is in a stream with density $> X \langle\rho\rangle$ is P

X	P
1.0	0.00001
0.1	0.002
0.01	0.2
0.001	~1

A typical particle has $\rho_{\text{stream}} \sim 10^{-7} \langle\rho\rangle$

Local density in the inner halo compared to a smooth ellipsoidal model

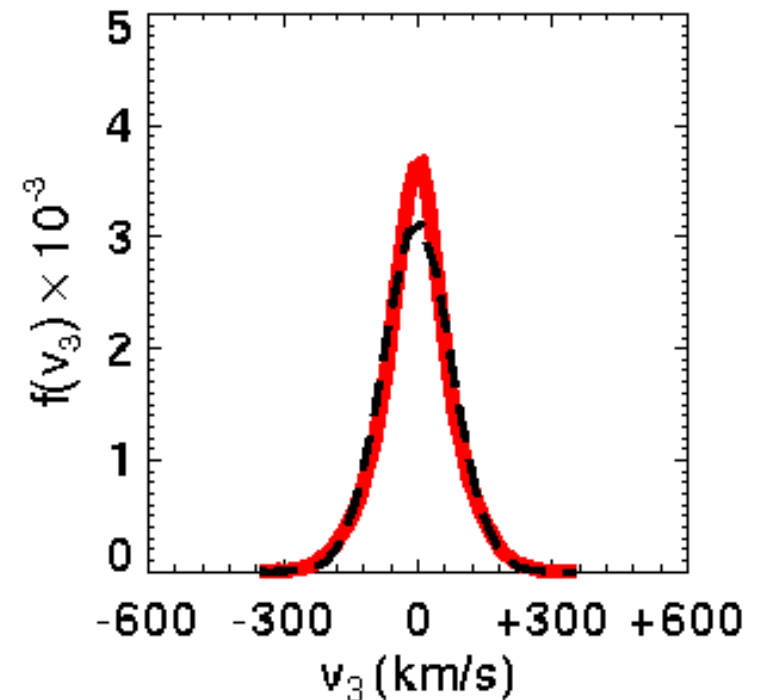
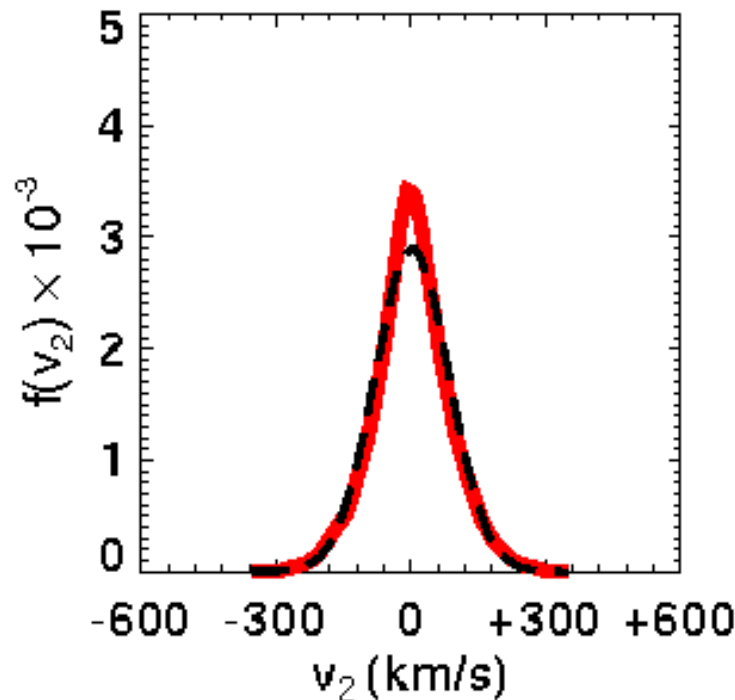
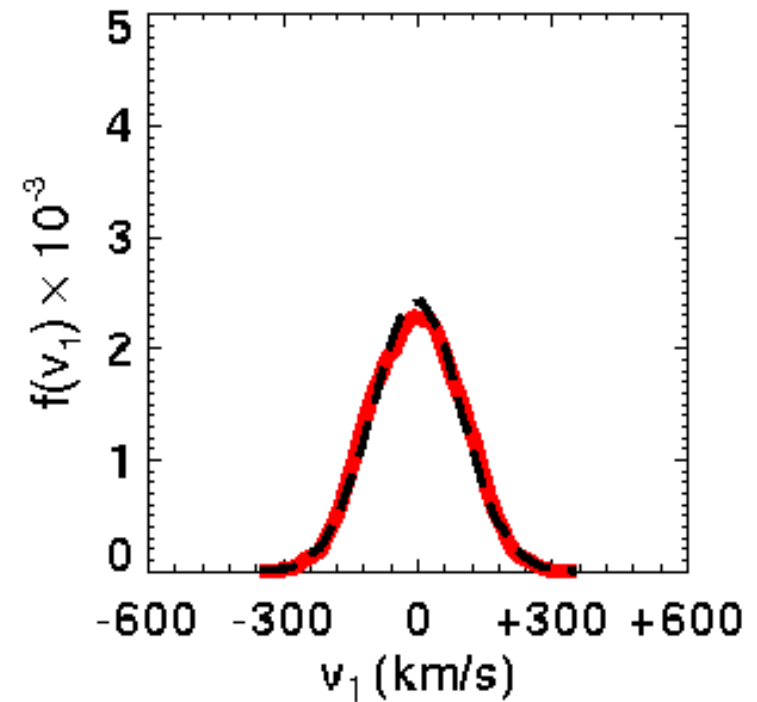
Vogelsberger et al 2008



- Estimate a density ρ at each point by adaptively smoothing using the 64 nearest particles
- Fit to a smooth density profile stratified on similar ellipsoids
- The chance of a random point lying in a substructure is $< 10^{-4}$
- The *rms* scatter about the smooth model for the remaining points is only about 4%

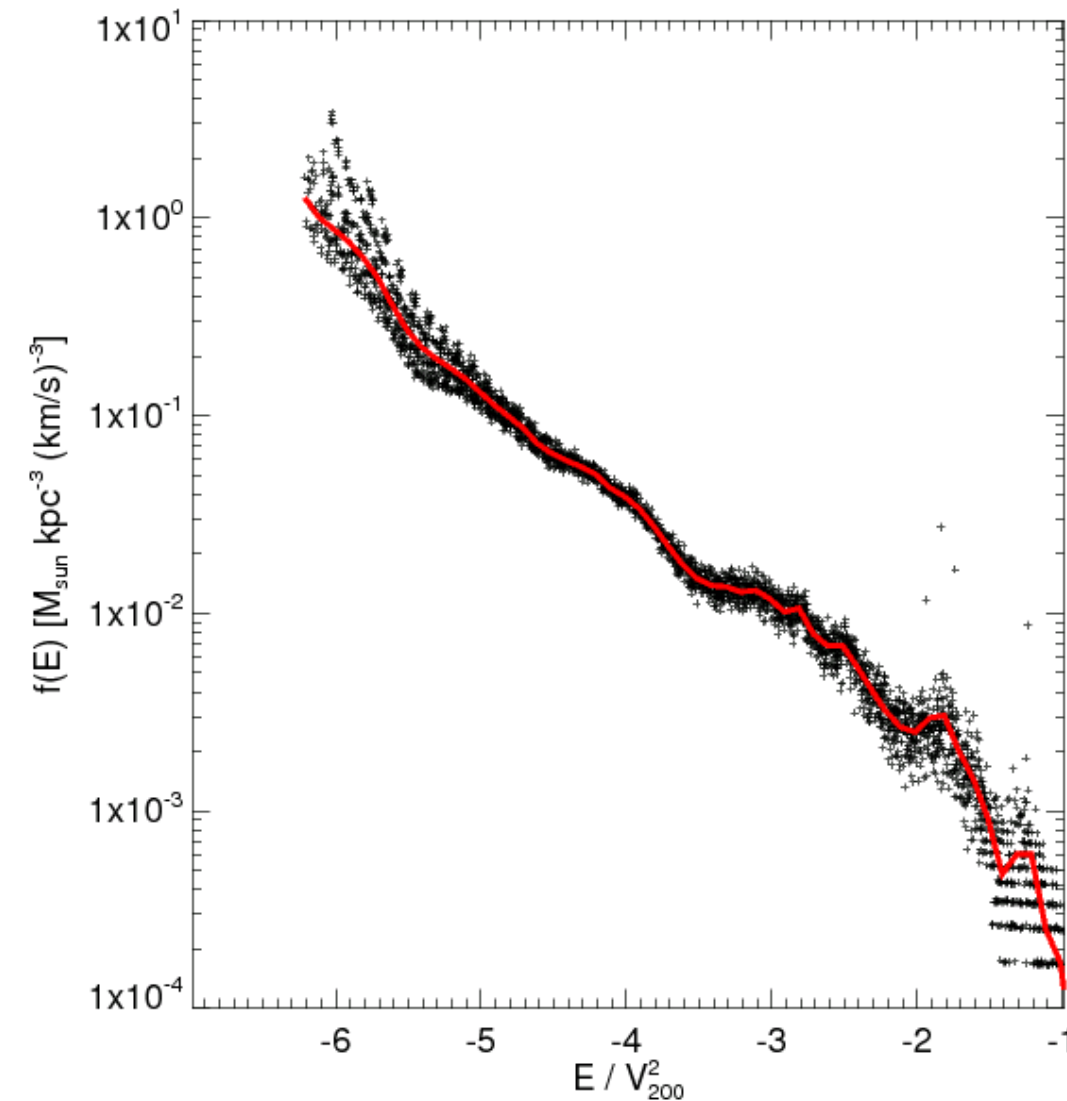
Local velocity distribution

- Velocity histograms for particles in a typical $(2\text{kpc})^3$ box at $R = 8\text{ kpc}$
- Distributions are smooth, near-Gaussian and different in different directions
- No individual streams are visible



Energy space features – fossils of formation

Vogelsberger et al 2008



The energy distribution within $(2 \text{ kpc})^3$ boxes shows bumps which

- repeat from box to box
- are stable over Gyr timescales
- repeat in simulations of the same object at varying resolution
- are different in simulations of different objects

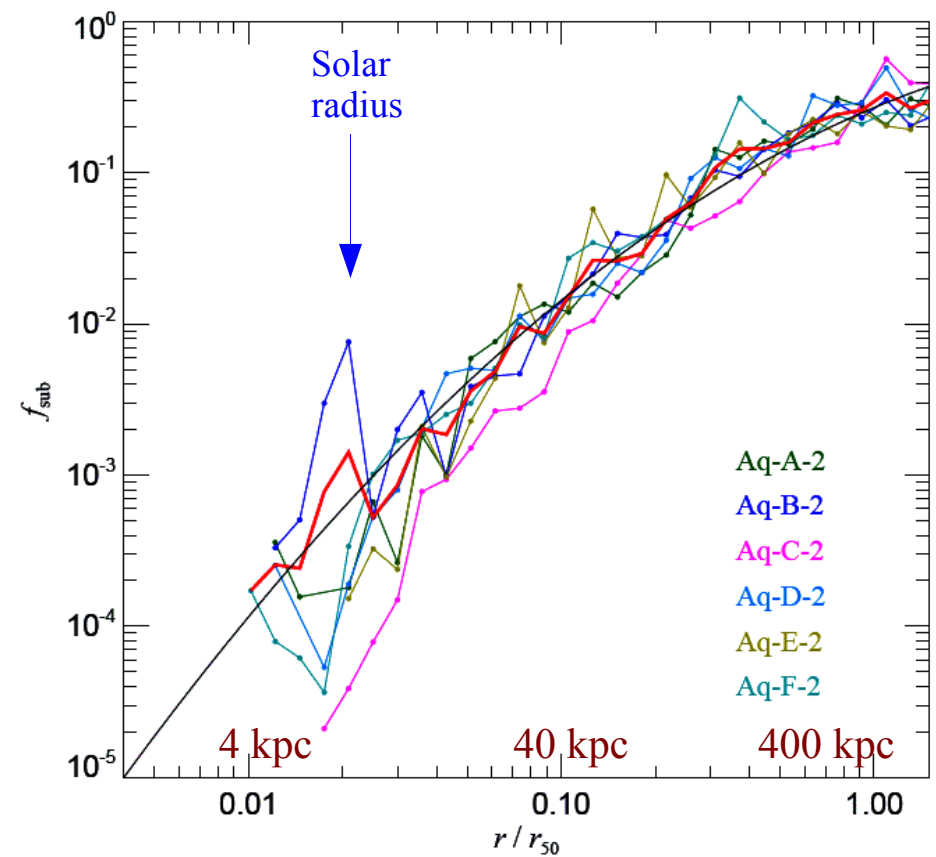
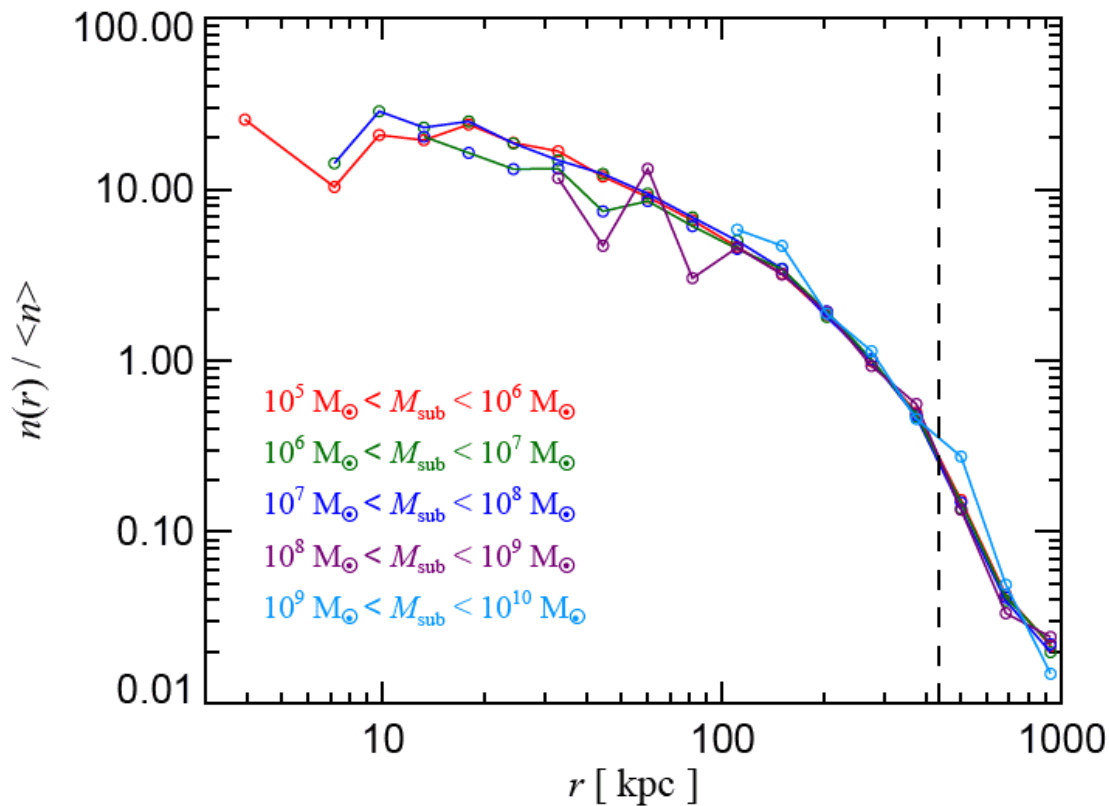
These are potentially observable fossils of the formation process

Conclusions for direct detection experiments

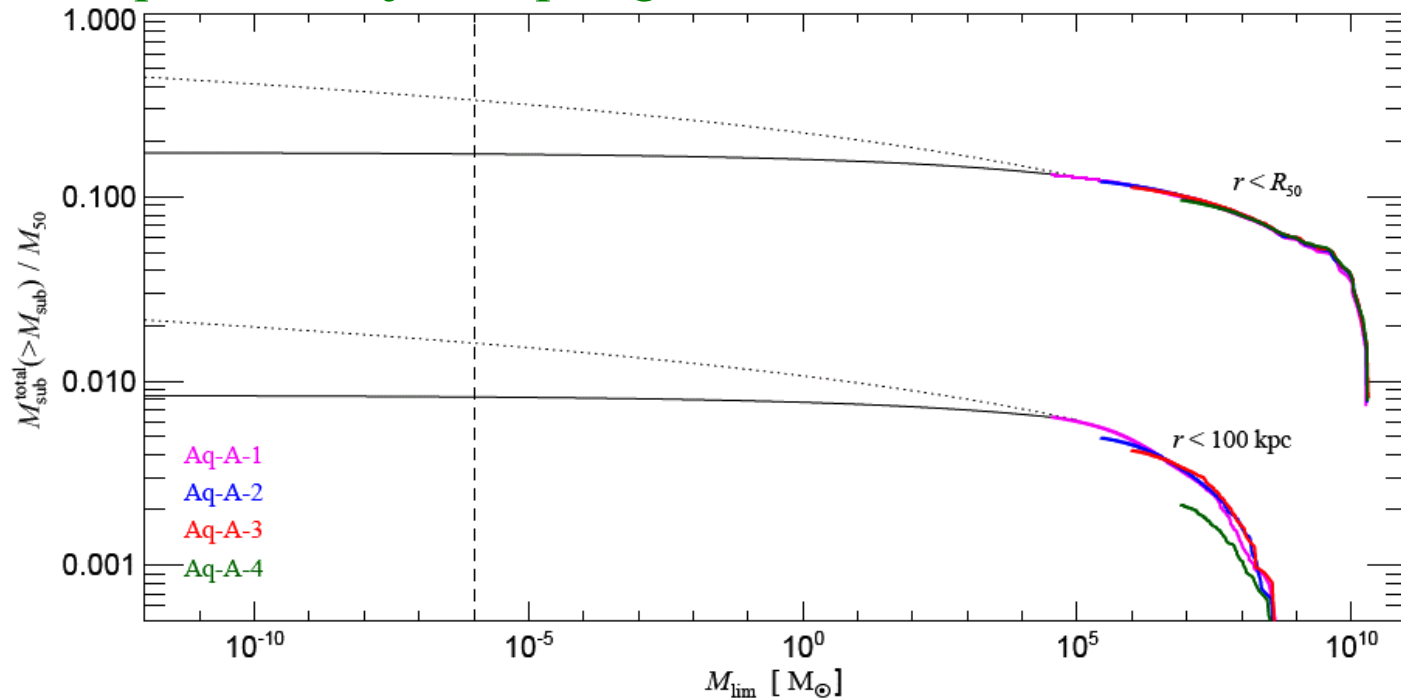
- With more than 99.9% confidence the Sun lies in a region where the DM density differs from the smooth mean value by $< 20\%$
- The local velocity distribution of DM particles is similar to a trivariate Gaussian with no measurable “lumpiness” due to individual DM streams
- The strongest stream at the Sun should contain about 10^{-3} of the local DM density. Its energy width is $\Delta E/E < 10^{-10}$ so it would be detectable as a “spectral line” in an axion experiment.
- The energy distribution of DM particles should contain broad features with $\sim 20\%$ amplitude which are the fossils of the detailed assembly history of the Milky Way's dark halo



Dark matter astronomy



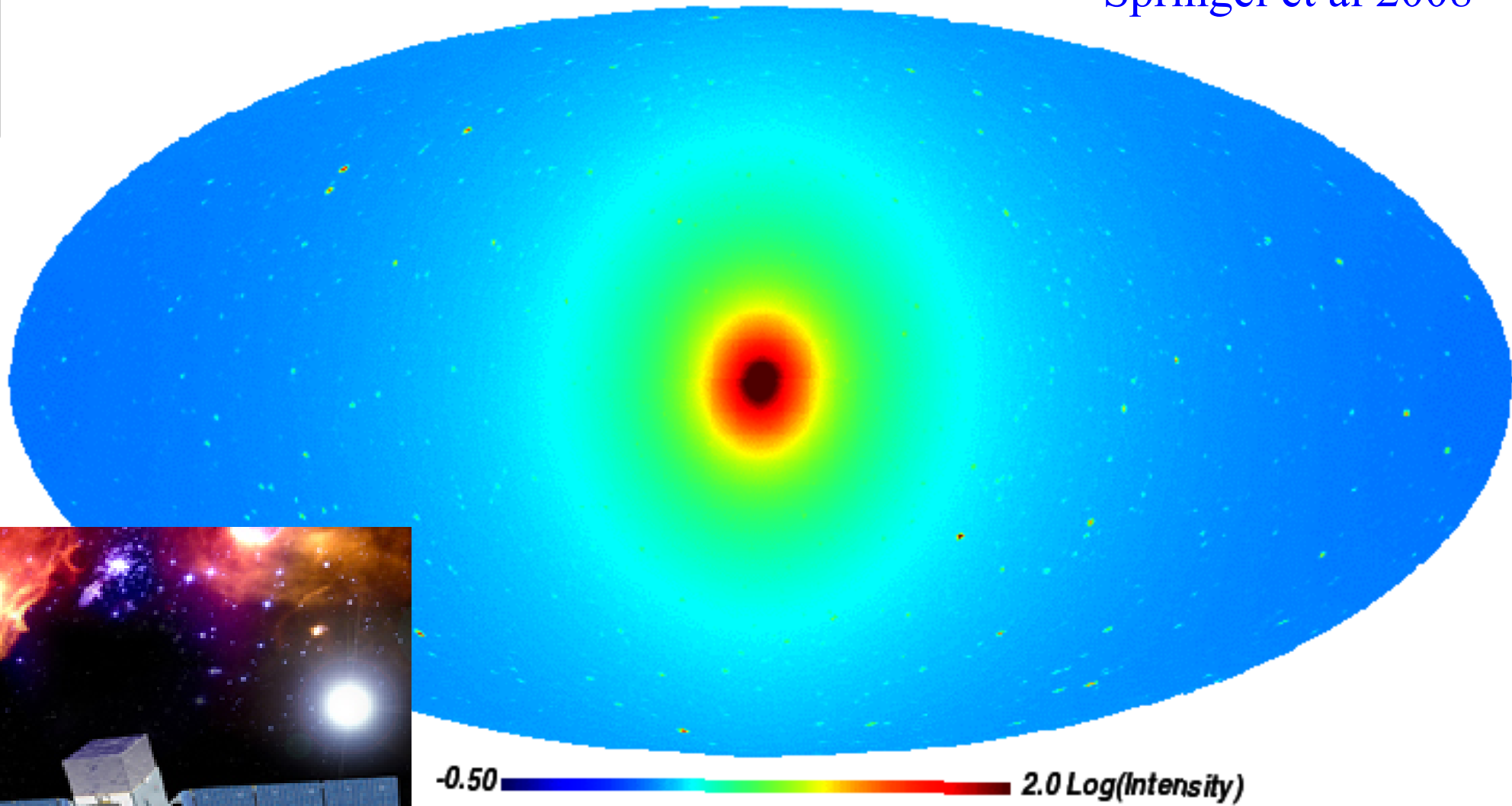
Aquarius Project: Springel et al 2008



- All mass subhalos are similarly distributed
- A small fraction of the inner mass in subhalos
- $\ll 1\%$ of the mass near the Sun is in subhalos

total emission

Springel et al 2008

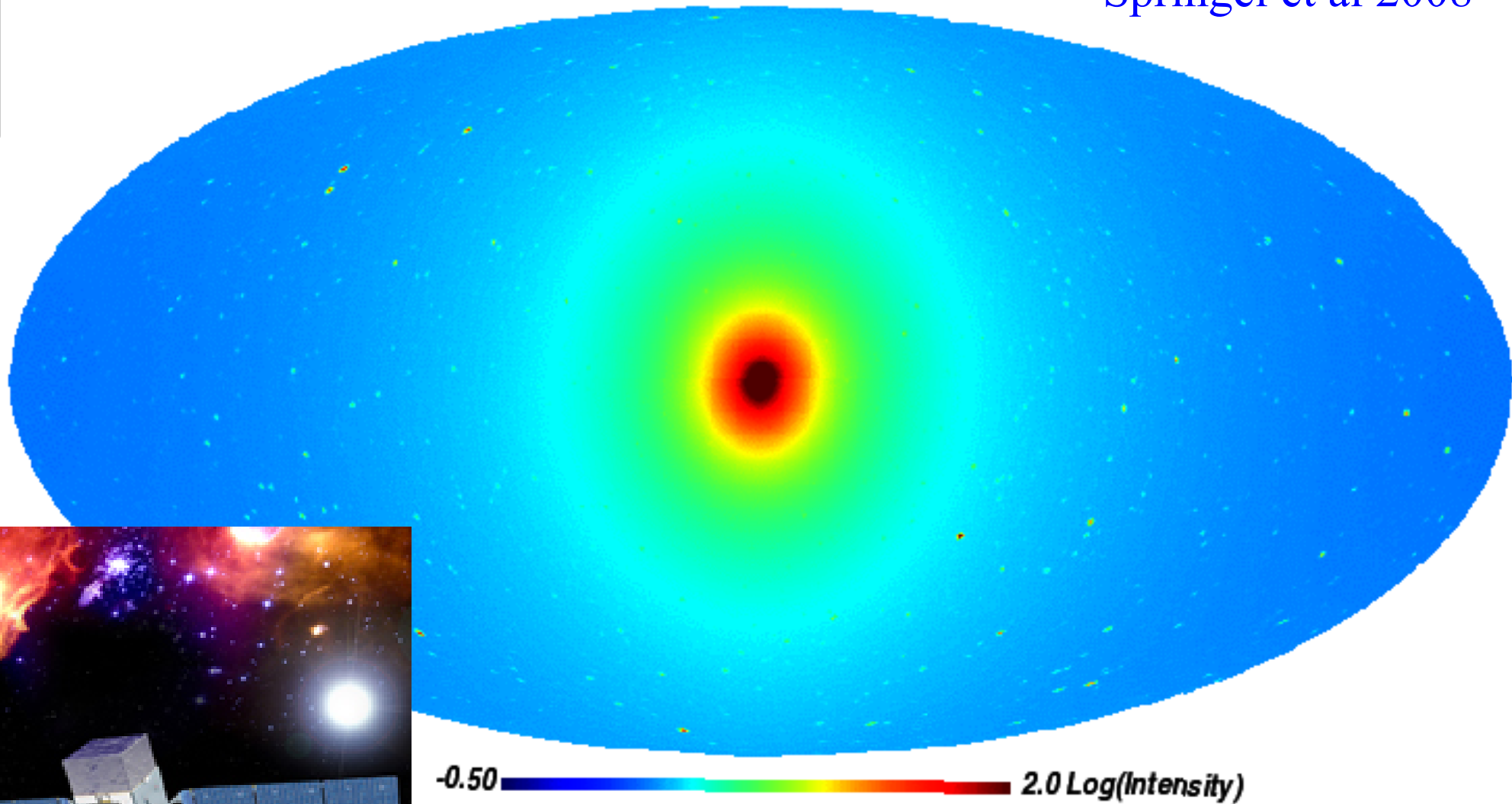


Maybe the annihilation of Dark Matter will be seen by Fermi?

Fermi γ -ray observatory

total emission

Springel et al 2008

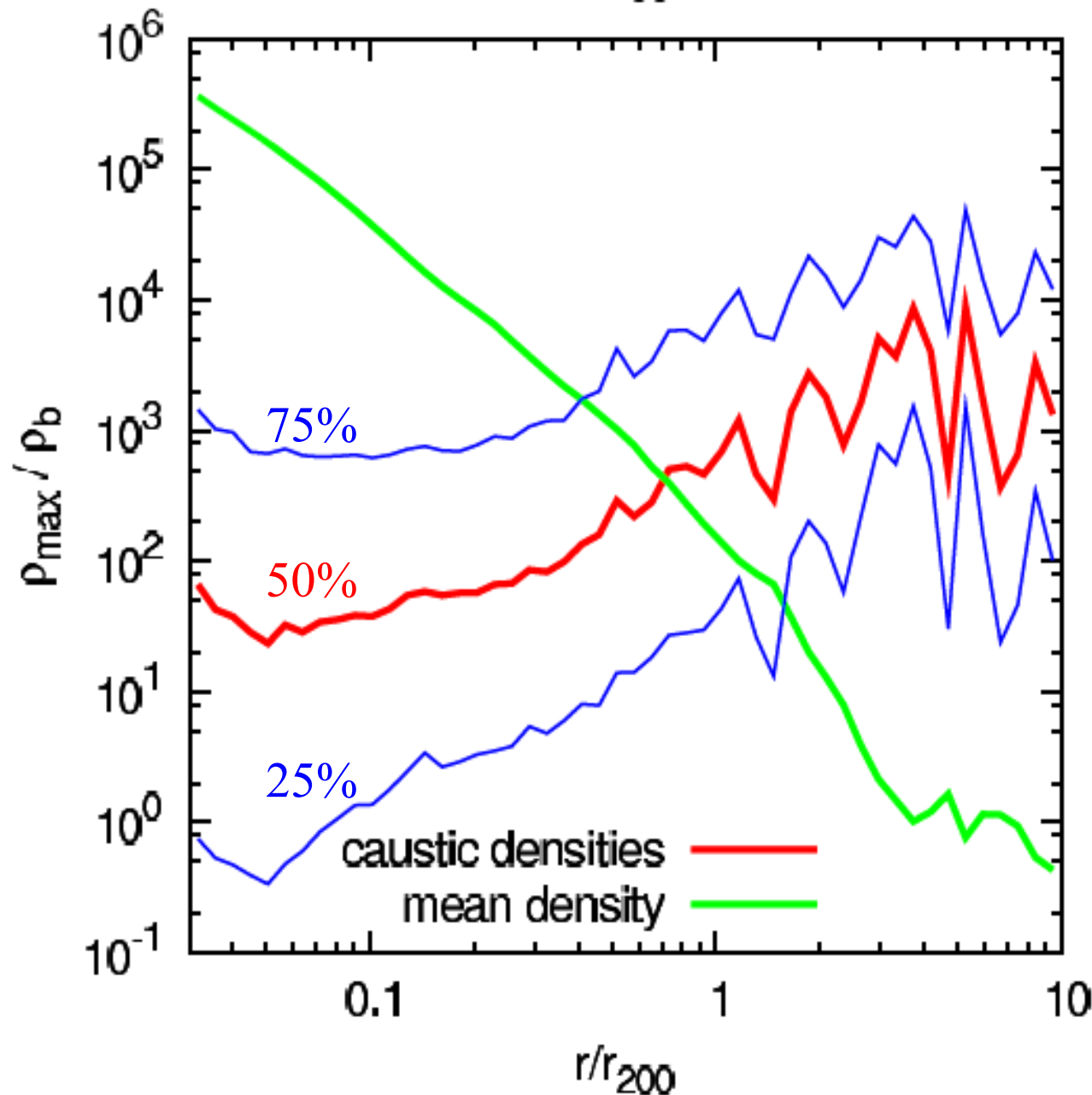


Maybe the annihilation of Dark Matter will be seen by Fermi?
Might caustics be visible?

Fermi γ -ray observatory

Radial distribution of peak density at caustics

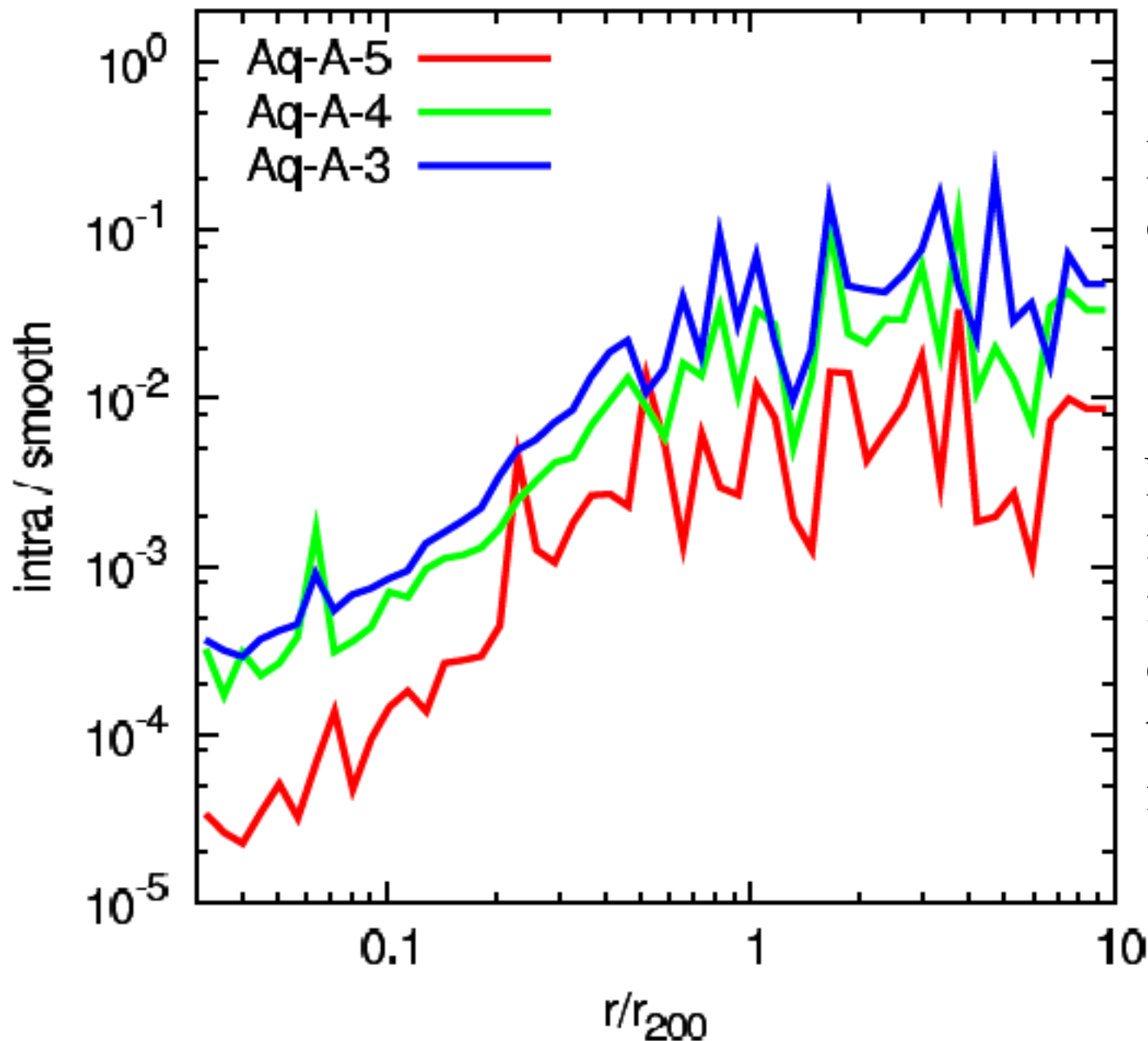
Vogelsberger & White 2011



Initial velocity
dispersion assumes a
standard WIMP with
 $m = 100 \text{ GeV}/c^2$

Fraction of annihilation luminosity from caustics

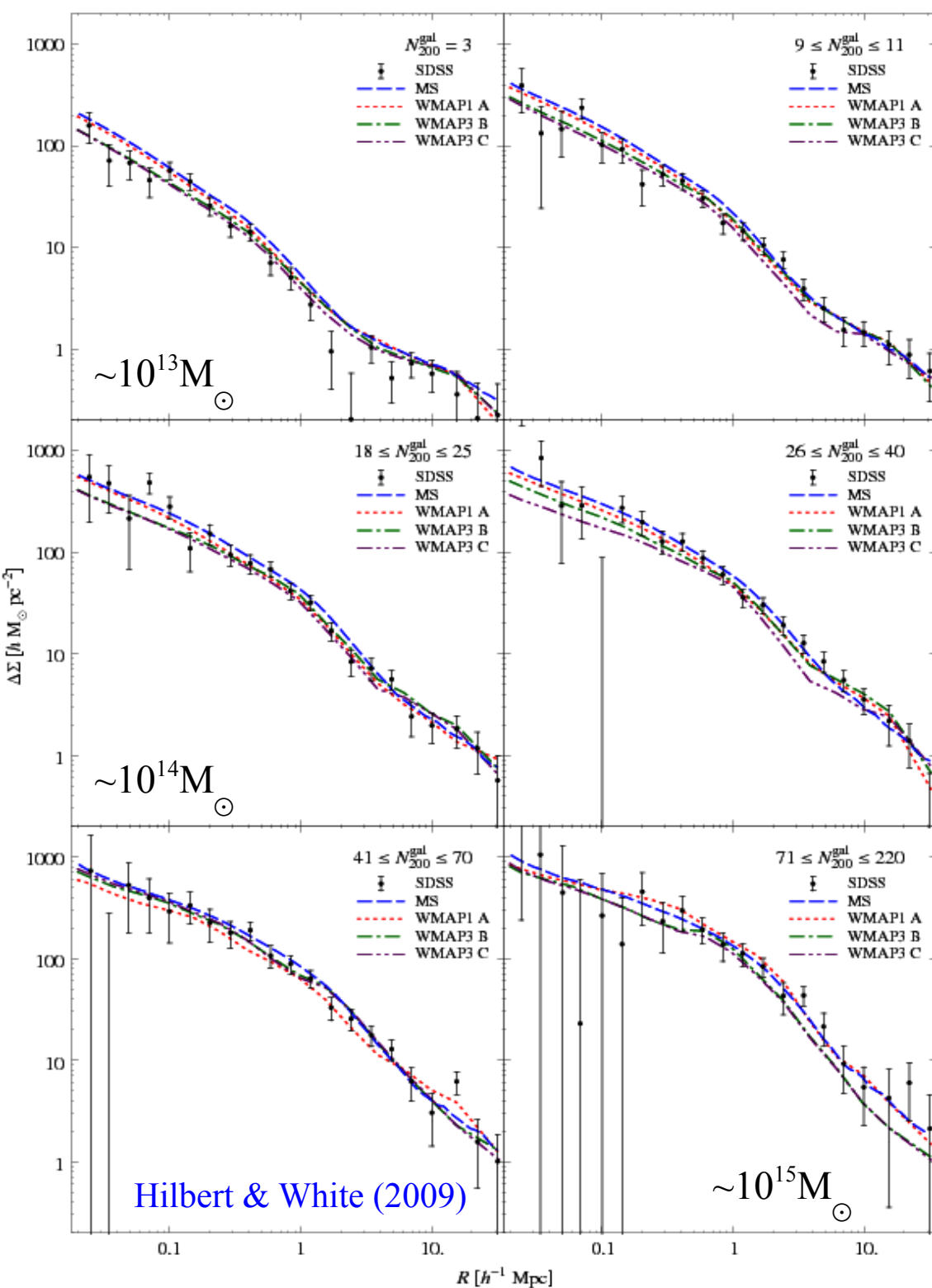
Vogelsberger & White 2011



Initial velocity dispersion assumes a standard WIMP with $m = 100 \text{ GeV}/c^2$

Note: caustic emission is compared to that from the smooth DM component here, but the dominant emission is from small subhalos

Galaxy formation simulations fit low-z groups and clusters



The simulated cluster population fits the *detailed* shape of the mean mass profile of groups and clusters as a function of richness

This holds for total masses
 $10^{13} M_\odot \leq M_{200} \leq 10^{15} M_\odot$

Lensing data from SDSS/maxBCG
 (Sheldon et al 2007)