Ellipsoidal Collapse

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Ringberg, July 2021

The spherical top hat

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2} = -\frac{4\pi G}{3}\overline{\rho}(1+\overline{\delta})R$$

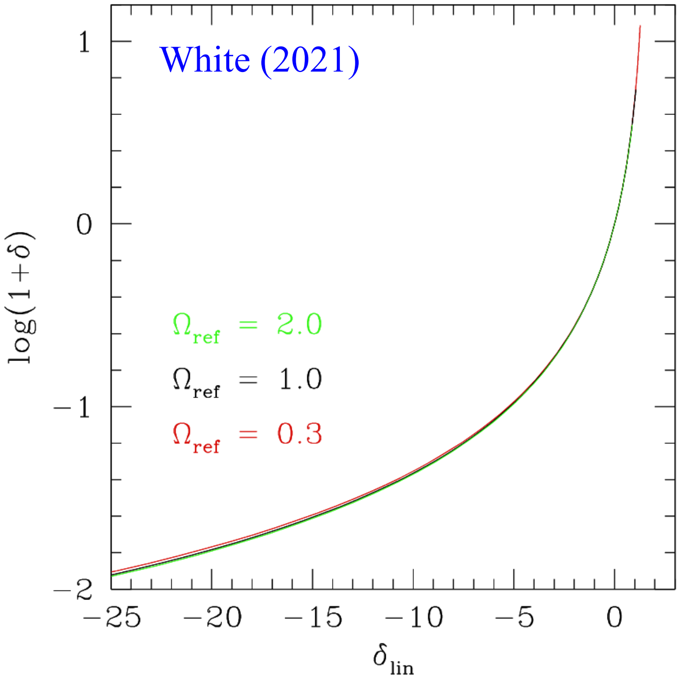
A spherically symmetric perturbation evolves like a separate universe

$$R/R_m = \frac{1}{2}(1-\cos\eta); \quad t/t_m = (\eta - \sin\eta)/\pi$$

Overdense regions collapse in finite time..

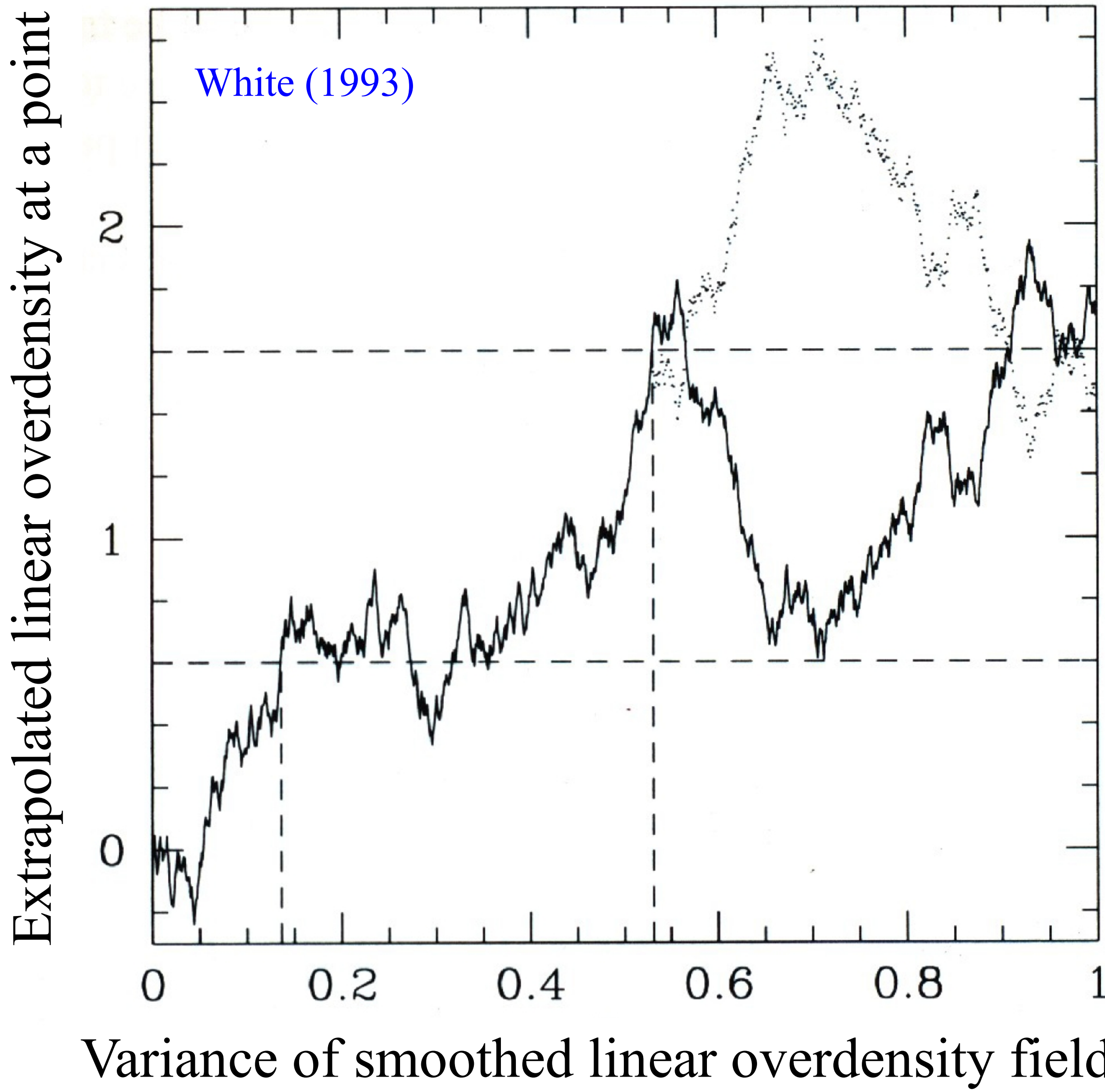
$$\delta_{lin,\,coll} = \overline{\delta}(2t_m) = \frac{3}{20}(12\pi)^{2/3} = 1.686$$

..when their extrapolated <u>linear</u> overdensity is 1.69 (almost) independent of Ω



Until the moment of collapse, the nonlinear density, ρ / ρ, of a spherical over- or underdensity is approximately a **cosmology-independent** function of its extrapolated linear overdensity

1-D excursion set model for structure formation



rms fluctuation →

← mass, smoothing scale

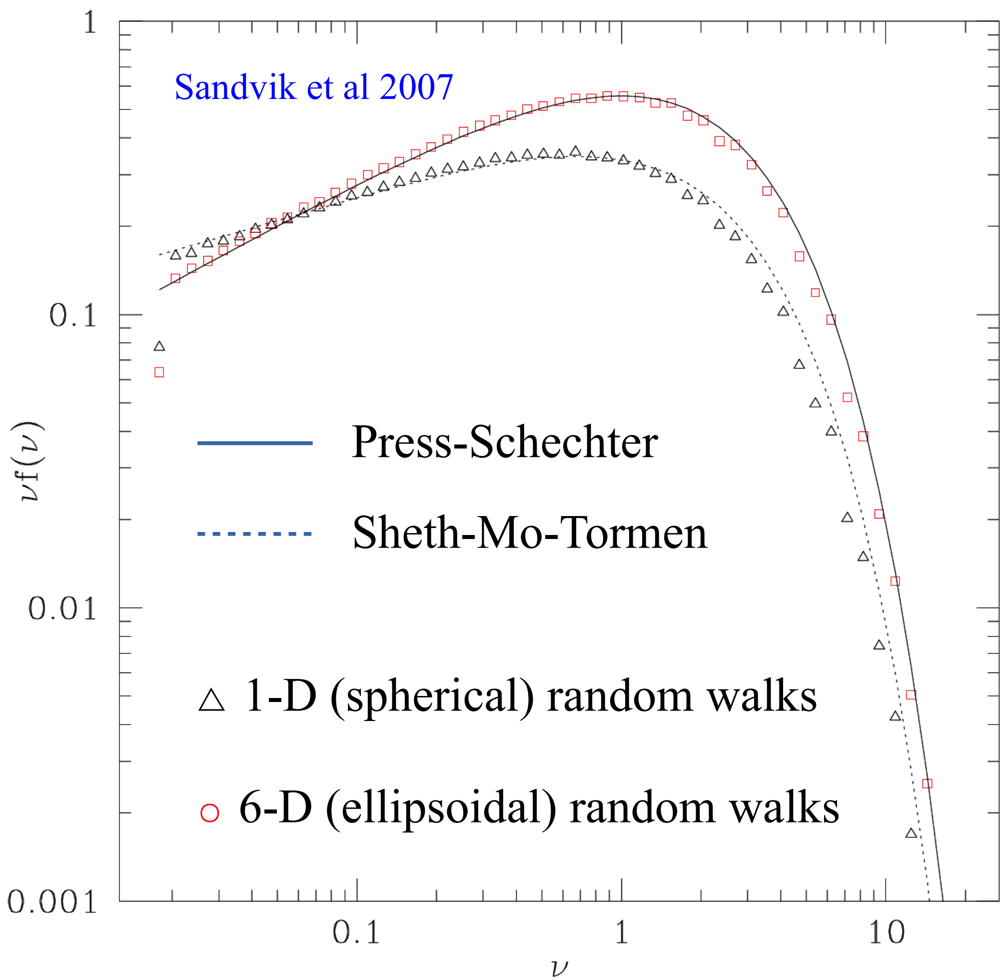
Markov random walk

Threshold crossing sets the halo mass of the element

Smoothed overdensity on larger scale sets environment density on that scale

This model determines the statistics of halo properties directly from the (gaussian) initial density field.

Excursion set mass functions



1-D excursion set theory leads to the Press-Schechter (1974) mass function

SMT (2001) show (approximate) ellipsoidal collapse fits simulations better

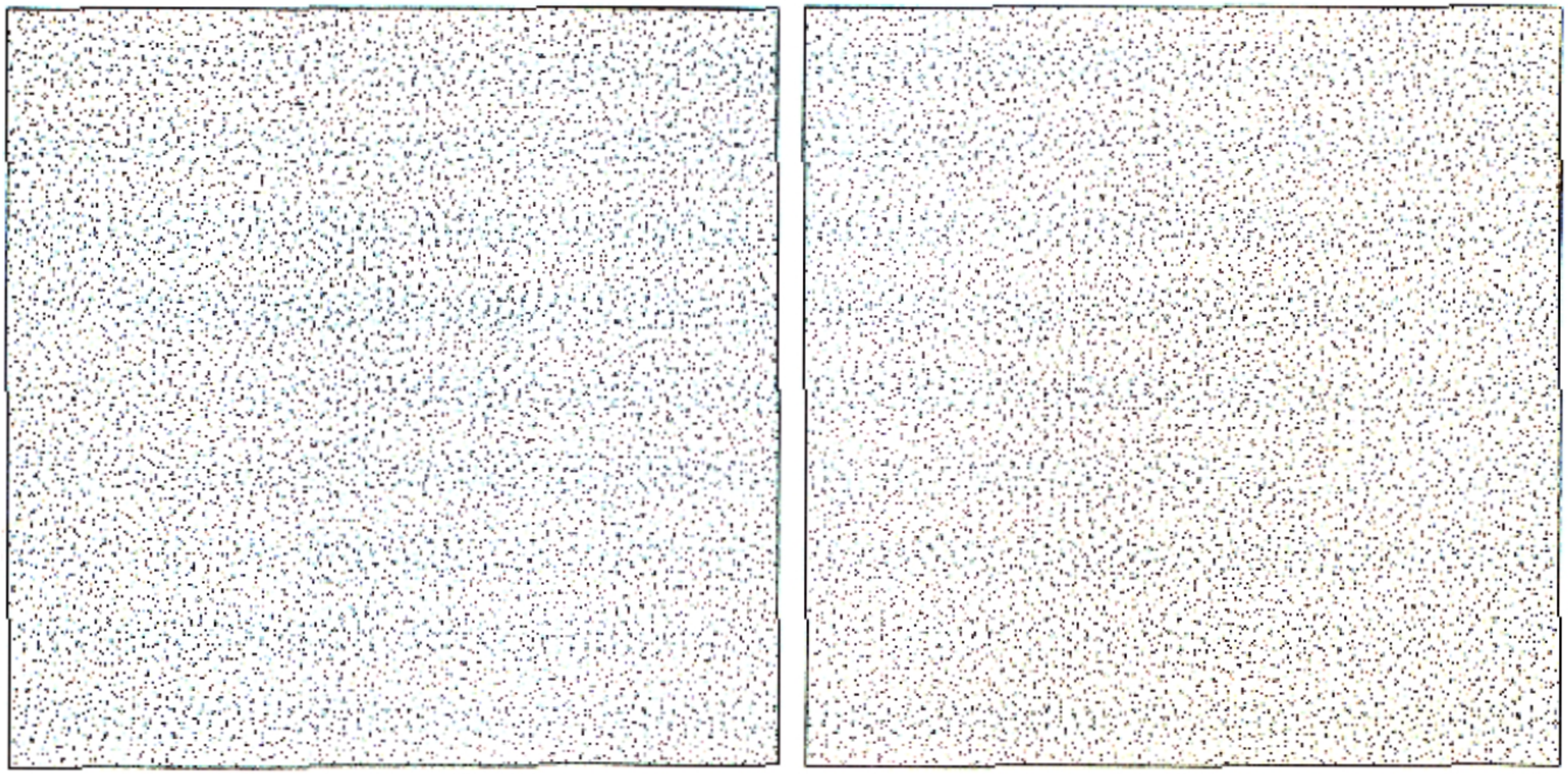
SMLW (2007) reproduce this with 6-D random walks + ellipsoidal collapse

The ellipsoidal top hat

$$a_1 : a_2 : a_3 = 1 : 1.25 : 1.5$$

$$\delta_{lin} = 0.1$$

White (1993)

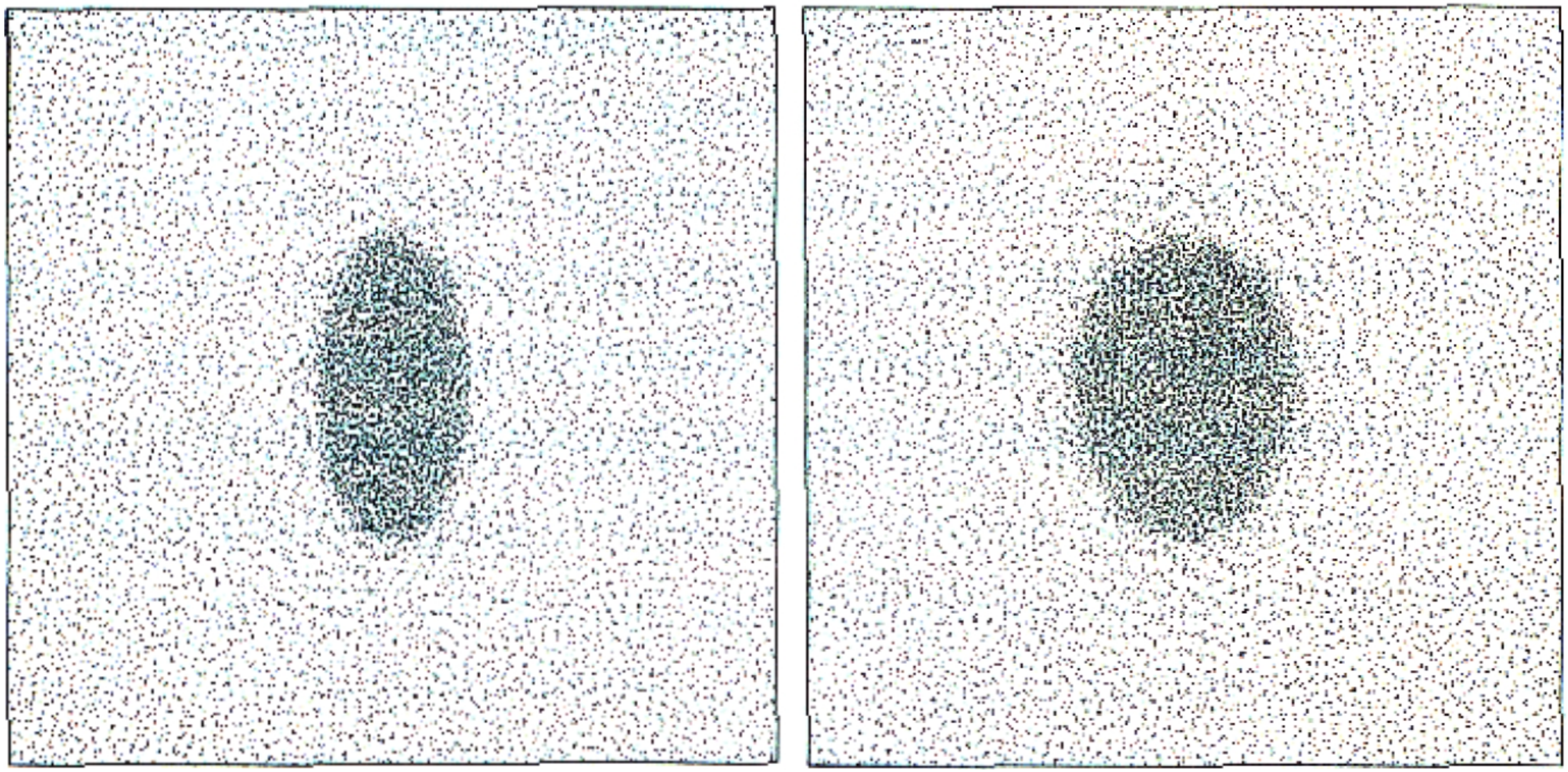


The ellipsoidal top hat

$$a_1 : a_2 : a_3 = 1 : 1.25 : 1.5$$

$$\delta_{lin} = 1.0$$

White (1993)

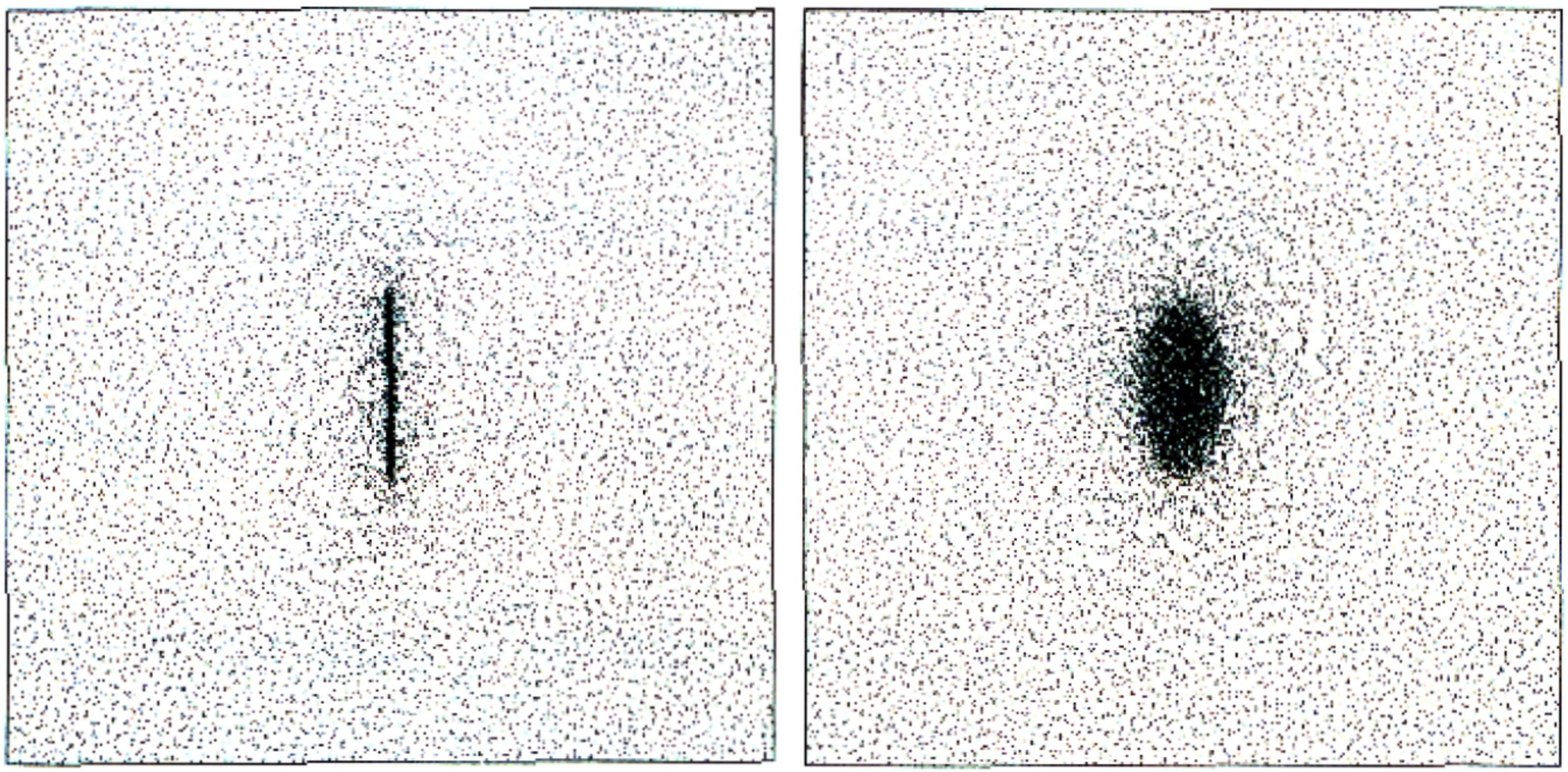


The ellipsoidal top hat

$$a_1 : a_2 : a_3 = 1 : 1.25 : 1.5$$

$$\delta_{lin} = 1.6$$

White (1993)



The ellipsoidal top hat

$a_1 : a_2 : a_3 = 1 : 1.25 : 1.5$

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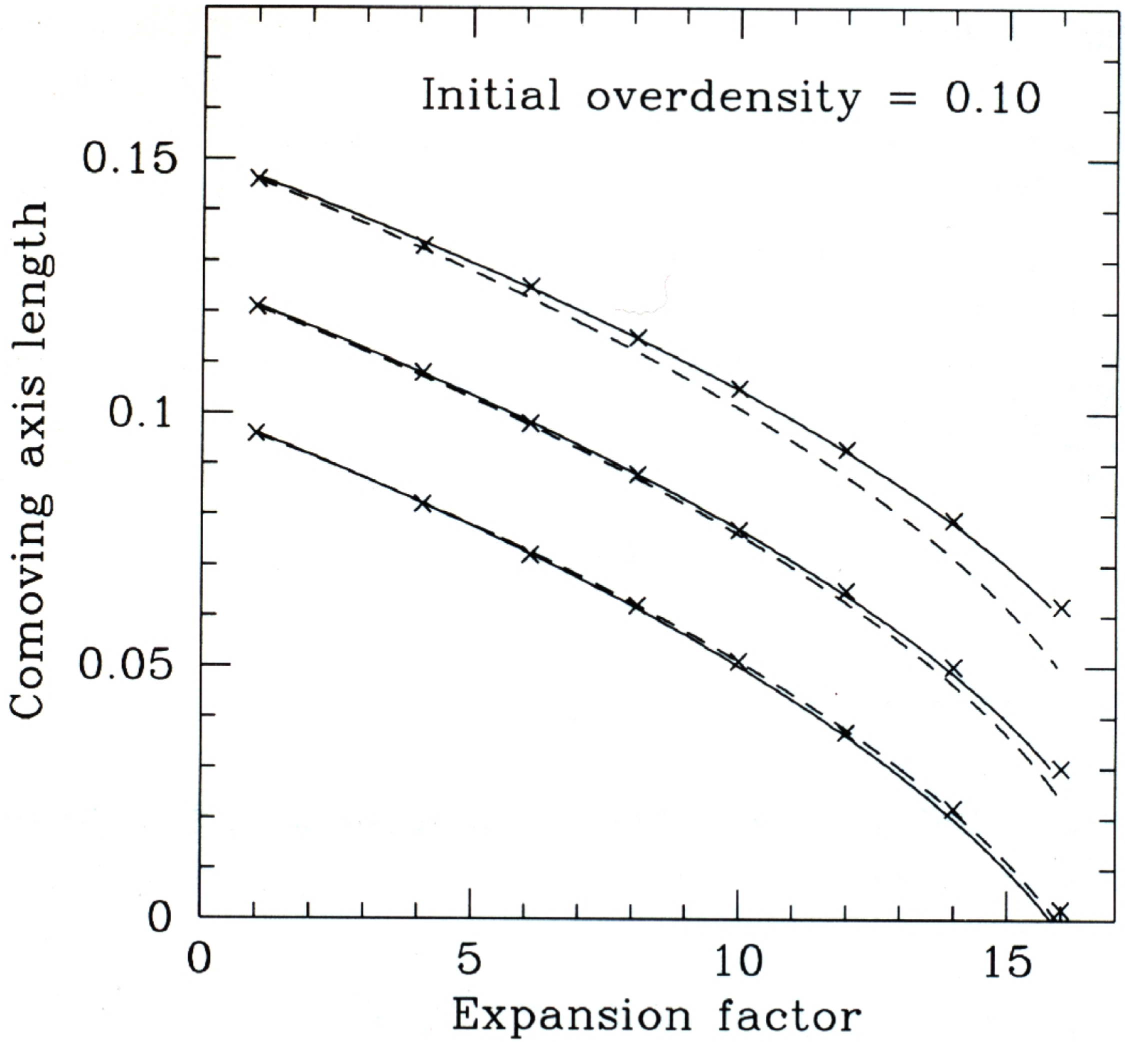


White & Silk (1979)

$$\frac{d^2 a_i}{dt^2} = -2\pi G[\rho_e \alpha_i + (\tfrac{2}{3} - \alpha_i)\rho_b] a_i \, ,$$

$$\frac{d^2 R_b}{dt^2} = -\frac{4\pi}{3} G \rho_b R_b \, ,$$

$$\rho_e a_1 a_2 a_3 = \text{const.} \, ,$$

$$\rho_b R_b^3 = \text{const.} \, ,$$

$$\alpha_i = a_1 a_2 a_3 \int_0^\infty (a_i^2 + \lambda)^{-1} \prod_{j=1}^{3} (a_j^2 + \lambda)^{-1/2} d\lambda \, ,$$

An ellipsoidal top hat stays uniform and ellipsoidal as it evolves. The axis ratios become more extreme as it collapses

Ellipsoidal dynamics of peaks

Bond & Myers (1996) approximate the dynamics of a peak in a gaussian random field by an homogeneous ellipsoid in an external tidal field by using equations,

$$\frac{d^2}{dt^2} X_i = -4\pi G \bar{\rho}_{\rm nr} X_i \left[\frac{1}{3} + \frac{\delta_{\rm nr}}{3} + \frac{b_i'}{2} \delta_{\rm nr} + \lambda_{vi}'(t) \right]$$

Converting to a consistent notation

$$\frac{d^2 R}{dt^2} = -\frac{4\pi}{3} G \overline{\rho} R$$

Homogeneous and isotropic universe (Friedmann 1922)

$$\frac{d^2 R_i}{dt^2} = -\frac{4\pi}{3} G \overline{\rho} R_i [1 + \frac{3}{2} \alpha_i \delta]$$

Ellipsoidal top hat (White & Silk 1979)

$$\frac{d^2 R_i}{dt^2} = -\frac{4\pi}{3} G \overline{\rho} R_i [1 + \frac{3}{2} \alpha_i \delta + T_{{\rm ext},i}]$$

Peak collapse (Bond & Myers 1996)
$T_{{\rm ext},i}$ are e-values of the external tidal field and align with the ellipsoid (??)

Geodesic Deviation Equation – ellipsoidal collapse

Stücker et al (2017)

$$\dot{\mathbf{x}} = a^{-2}\mathbf{p}, \quad \dot{\mathbf{p}} = -\nabla\phi, \quad \nabla^2\phi = 4\pi G a^2\bar{\rho}\delta = 4\pi G a^{-1}\rho_0\delta,$$

Equations of motion or a particle trajectory

$$\dot{D}_{ij} = a^{-2}P_{ij}, \quad \dot{P}_{ij} = -T_{ik}D_{kj}\ , \quad \bar{\rho}(t)/|\det(\underline{\underline{D}})| = \bar{\rho}(1+\delta),$$

Geodesic deviation equation for neighboring trajectories

where

$$D_{ij} = \frac{\partial x_i}{\partial q_j}, \quad P_{ij} = \frac{\partial p_i}{\partial q_j}, \quad S_{ij} = \frac{\partial s_i}{\partial q_j} \text{ and } T_{ij} = \frac{\partial^2\phi}{\partial x_i \partial x_j}.$$

In linear theory D_{ij}, P_{ij} and T_{ij} all share the same unchanging principal axes.
As long as this remains true, their e-values satisfy the "ellipsoidal" equations

$$\dot{X}_i = a^{-2}P_i, \quad \dot{P}_i = -T_i X_i, \quad 1+\delta = 1/X_1X_2X_3$$

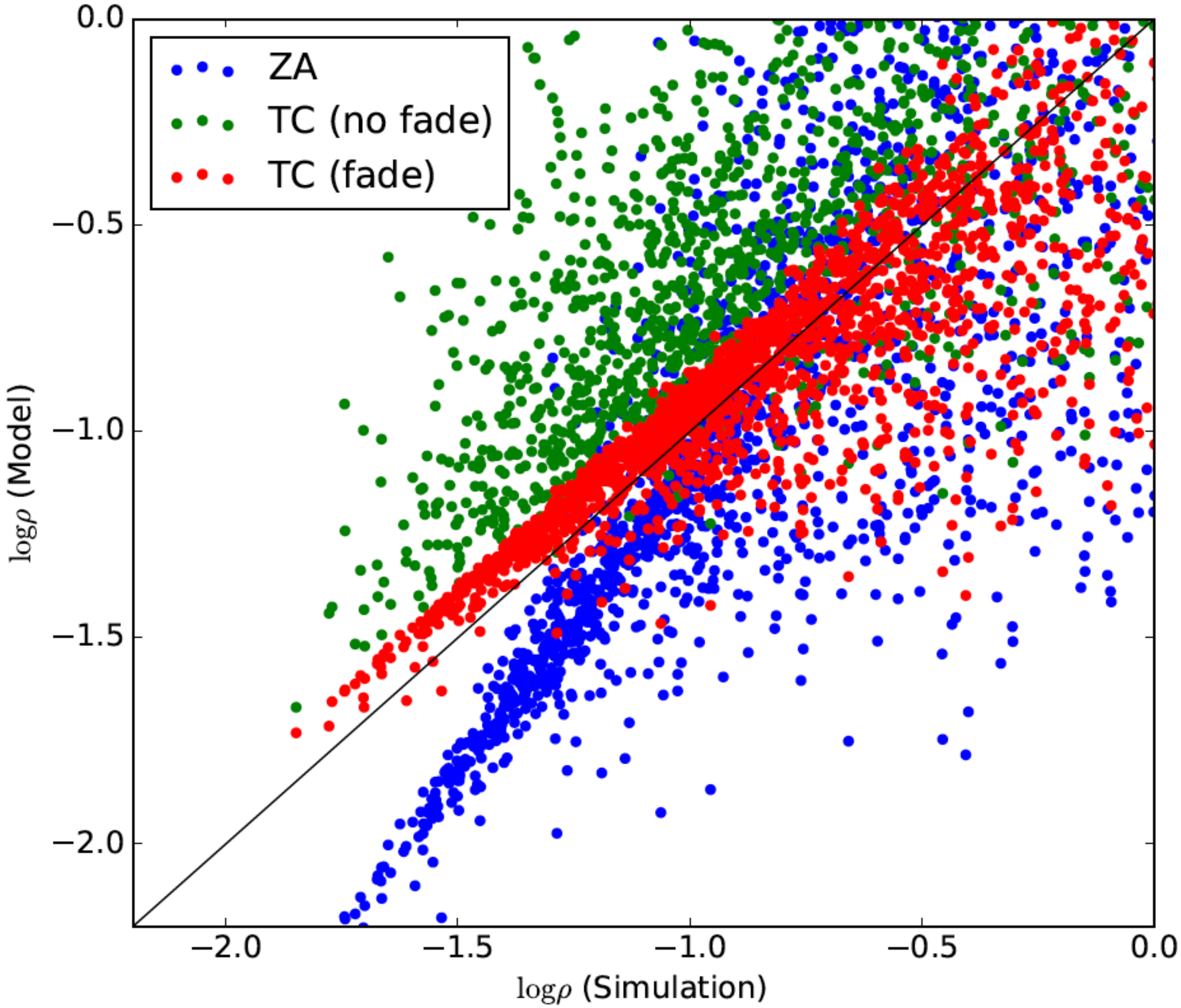
$$\dot{X}_i = a^{-2}P_i, \quad \dot{P}_i = -\left(\frac{4\pi G\rho_0}{3a}\delta + T_{\mathrm{ext},i}\right)X_i.$$

Anisotropy evolution is driven purely by the *external* tide – there is **no** local ellipsoidal contribution.

We need a <u>model</u> for the <u>external</u> tide – linear theory *à la* Bond+Myers96?

Simulation tests of GDE ellipsoidal collapse

Stücker et al (2017)



Model density vs GDE simulation results for 1000 random "uncollapsed" particles in a high-resolution WDM simulation

Blue points assume linear theory (Zeldovich approx.) dynamics

Green points assume GDE model with linear T_{ext} evolution.

Red points assume tidal evolution weakens for nonlinear densities

$$T_{ext} = T_{ext,lin} / (1 + \sigma)$$

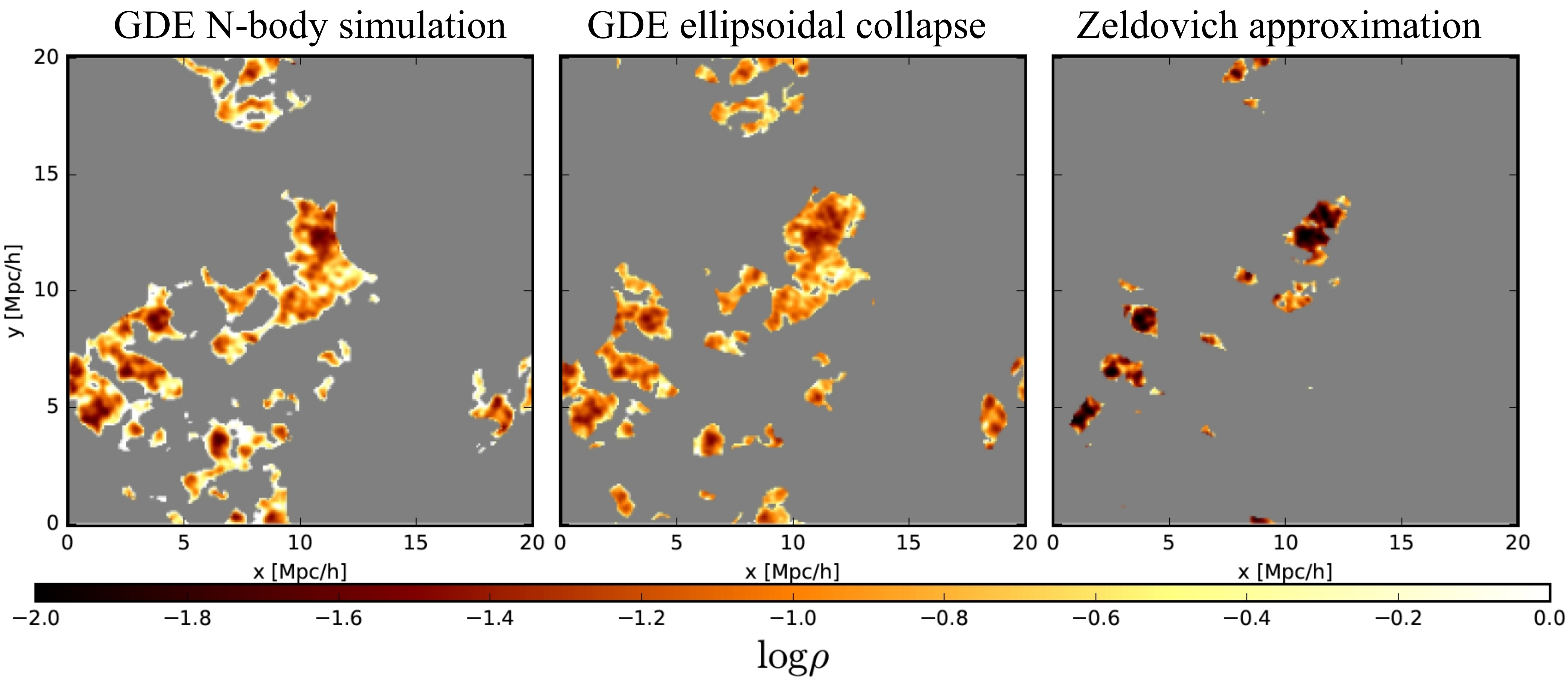
rms (extrapolated) linear overdensity

Simulation tests of GDE ellipsoidal collapse

Stücker et al (2017)

Densities of uncollapsed particles plotted in Lagrangian (initial position) space

Grey regions are collapsed in 1D so are part of sheets, filaments or halos



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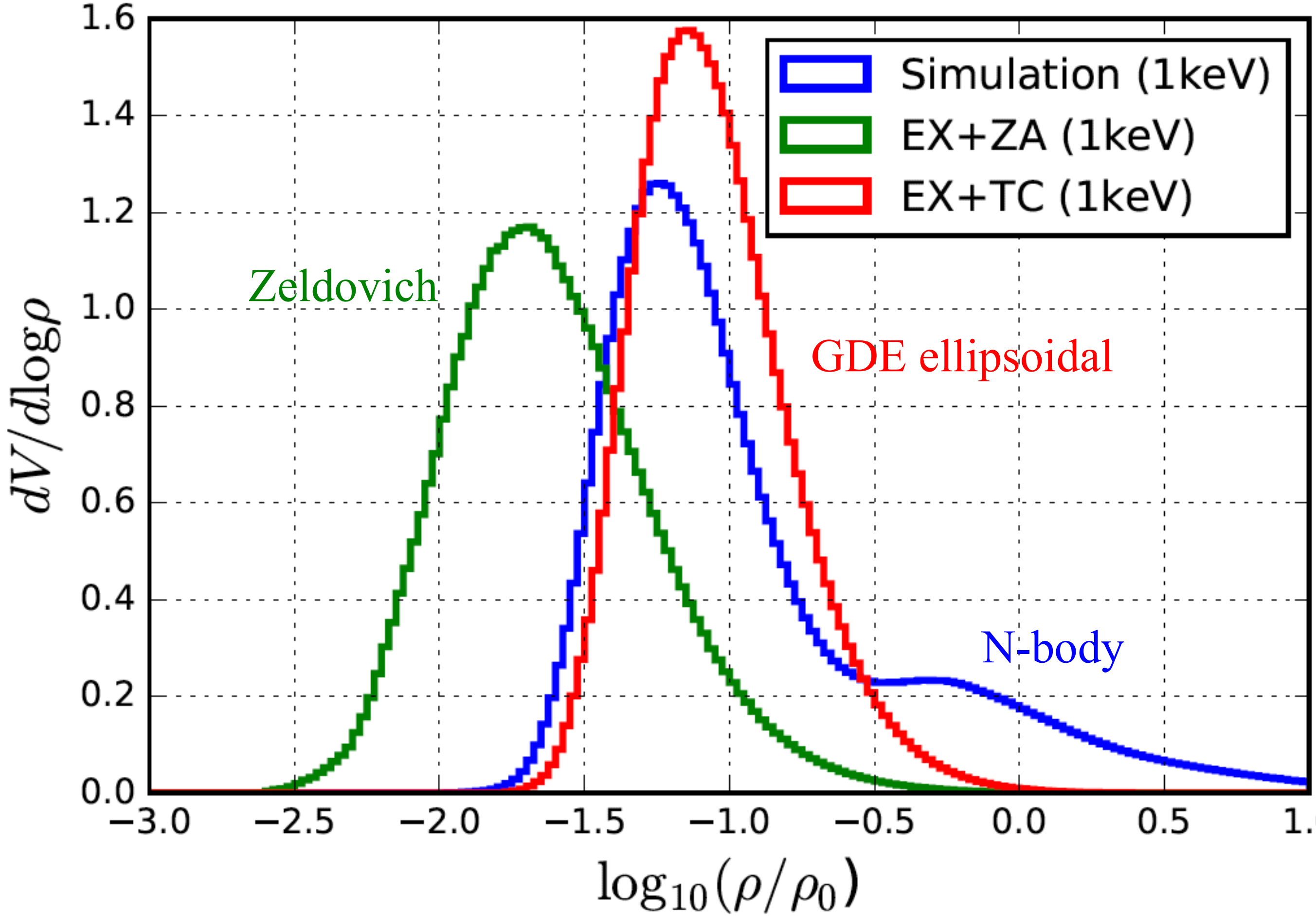


The cosmic volume fraction occupied by mass elements as a function of density

Collapsed elements have infinite density/zero volume in Zeldovich and GDE cases

Collapse thresholds for the GDE ellipsoidal model

$$\frac{dX_i}{da} = \frac{3}{2}a^{-3/2}(t_0 P_i),$$
$$\frac{d(t_0 P_i)}{da} = -a^{-1/2}\Big(\delta/3 + \alpha(\lambda_i - \delta_0/3)\Big)X_i, \quad (34)$$
$$\delta = (X_1 X_2 X_3)^{-1} - 1, \quad \alpha = a/(1+\sigma_s a), \quad \delta_0 = \sum_i \lambda_i.$$

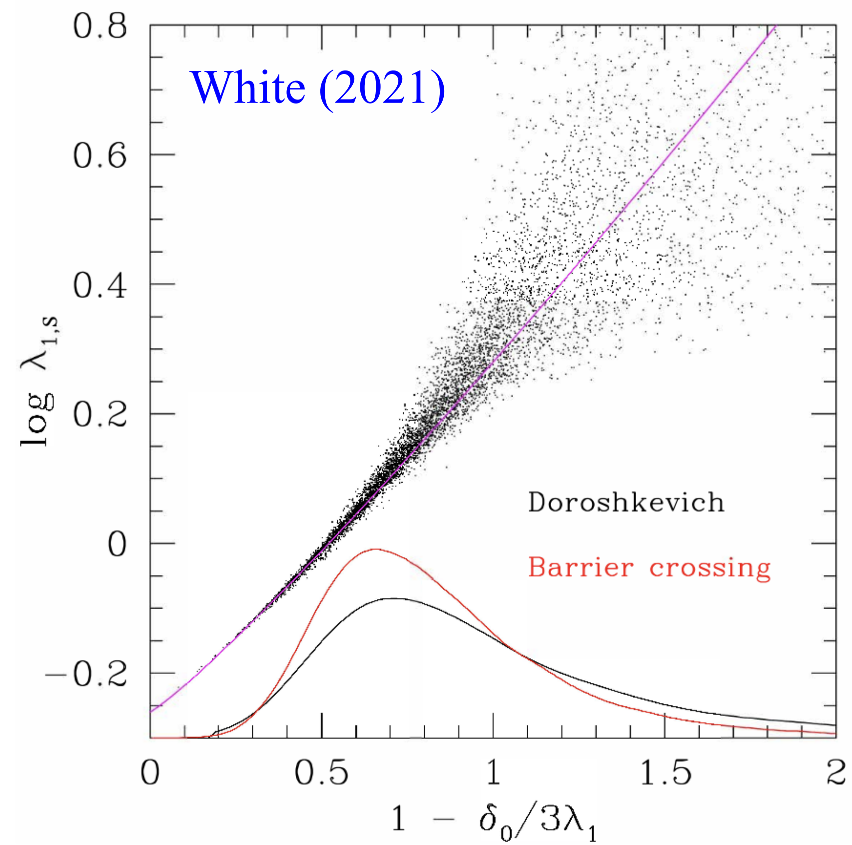
For any chosen set of λ_i, this closed set of equations can be integrated forwards starting from initial conditions given by

$$X_i = 1 - a\lambda_i, \quad t_0 P_i = -\frac{2}{3}a^{3/2}\lambda_i, \quad a \ll 1, \quad (35)$$

and stopping either at the present day ($a = 1$) or when collapse occurs (for which $\lambda_1 > 0$ is a necessary but not sufficient condition).

The equations with fading tidal field require λ_1, the largest e-value of the tidal tensor to be positive for collapse, but the linear overdensity can be negative!

A collapse threshold **cannot** refer to linearly extrapolated overdensity.



Eventual collapse in 1-D is predicted for 85% of elements selected from a gaussian random field, hence for 70% of *underdense* regions.

Nearly spherical evolution is the exception

Current state of play

- The GDE casts doubt on the model normally used to describe ellipsoidal collapse in excursion set theories of structure formation

- An ellipsoidal model derived from the GDE predicts that many initially underdense regions will undergo tidally driven collapse

- As a result collapse barriers cannot be expressed as a thresholds for the linearly extrapolated overdensity, only for the linearly extrapolated value of λ_1

- This may effect the mass function predictions based on ellipsoidal collapse (e.g. Sheth-Mo-Tormen) but this depends on the validity of models for collapse of sheets to filaments to halos

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This project is not finished so these results may change!