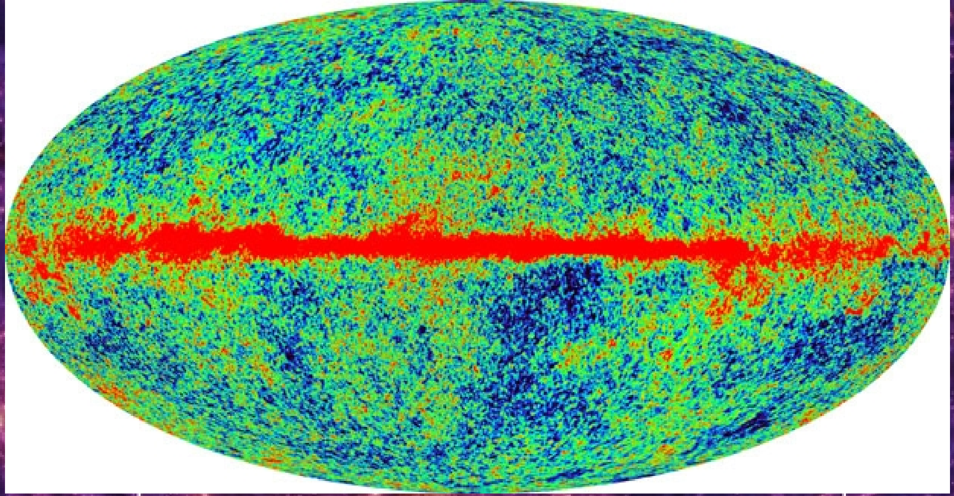
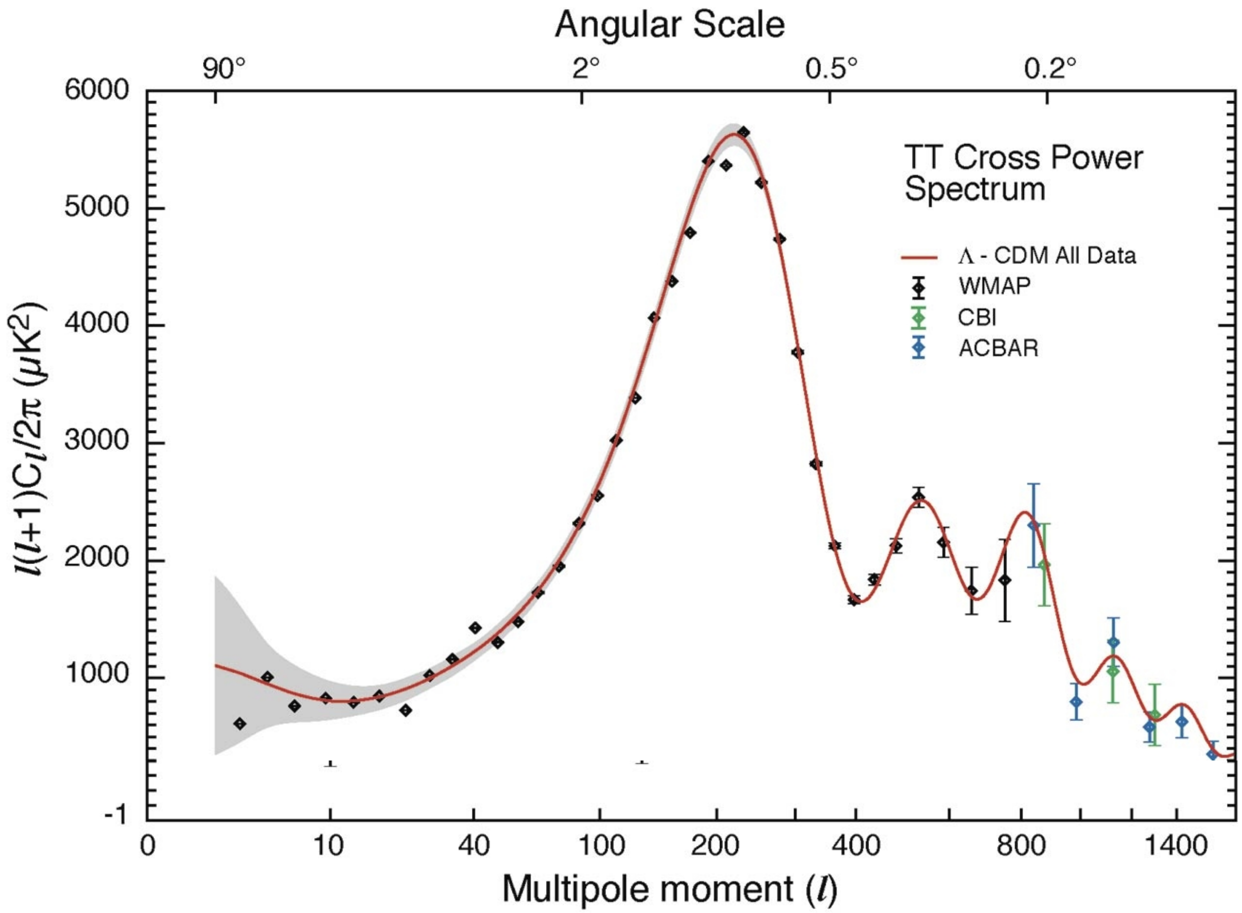


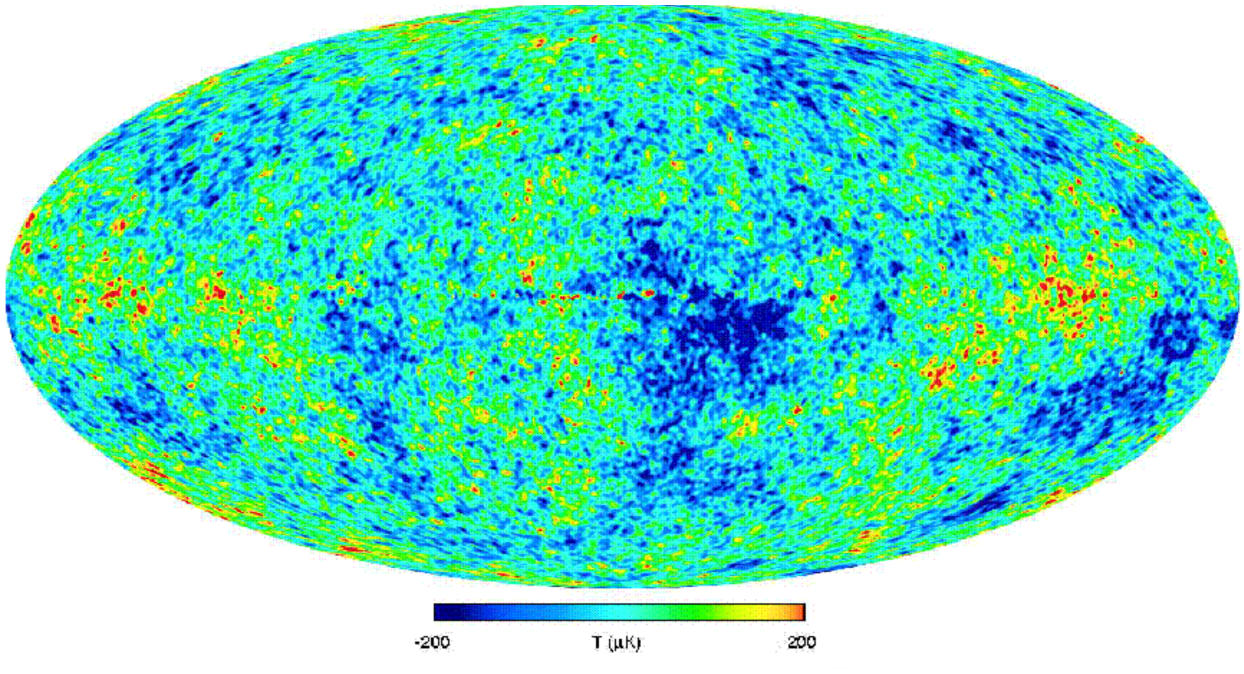
The Dynamics of Galaxies
St Petersburg
August 2007

The Dynamics of Cold Dark Matter

Simon White & Mark Vogelsberger
Max Planck Institute for Astrophysics

The Emergence of the Cosmic Initial Conditions





- Temperature-temperature and temperature-polarisation power spectra from *WMAP1* and interferometers
- Best ΛCDM model

t_o= 13.7±0.2 Gyr

h=0.71±0.03 σ_8=0.84±0.04

Ω_t=1.02±0.02 Ω_m=0.27±0.04

Ω_b=0.044±0.004

τ_e=0.17±0.07

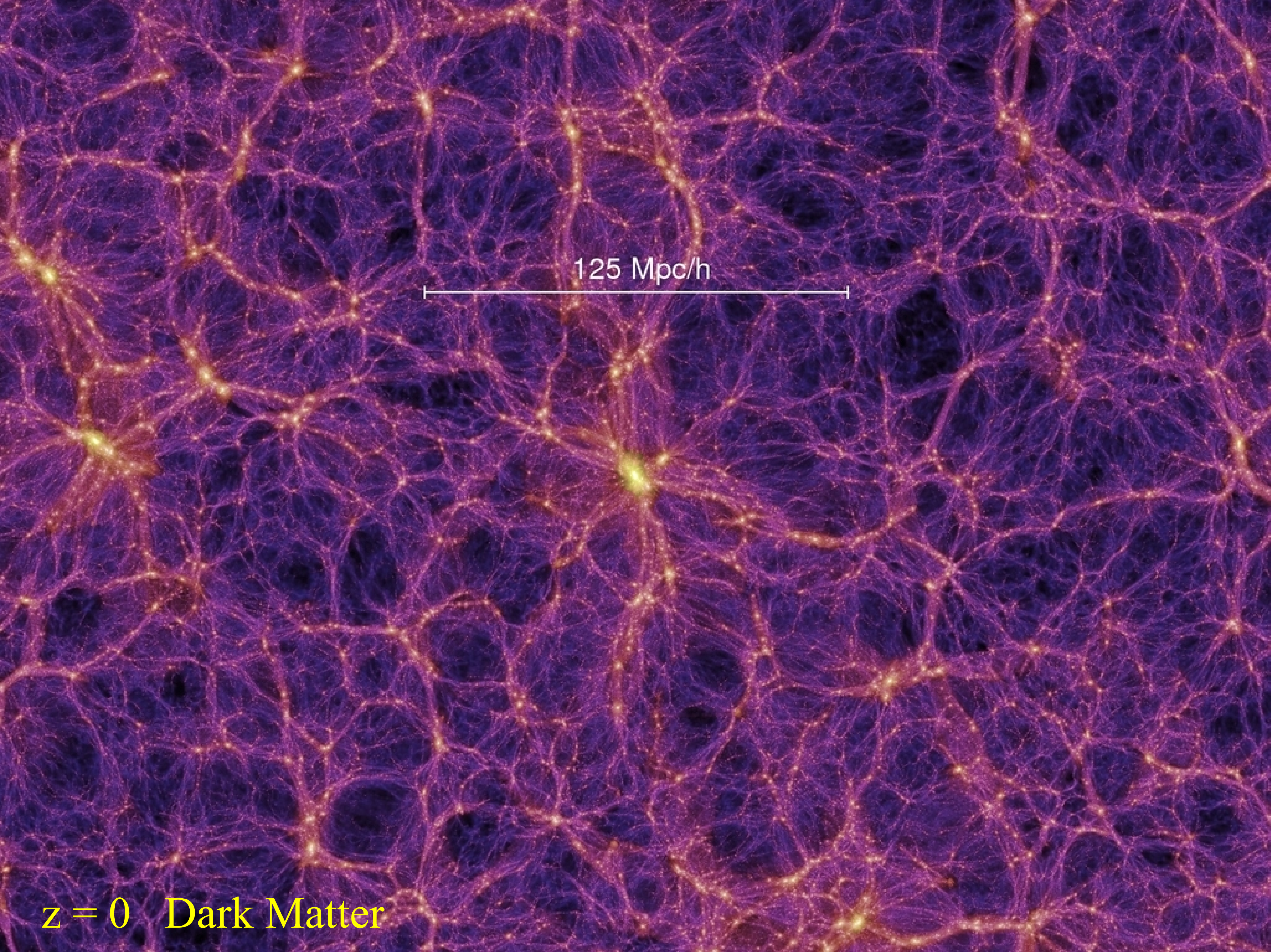
- Parameters in excellent agreement with other astronomical data

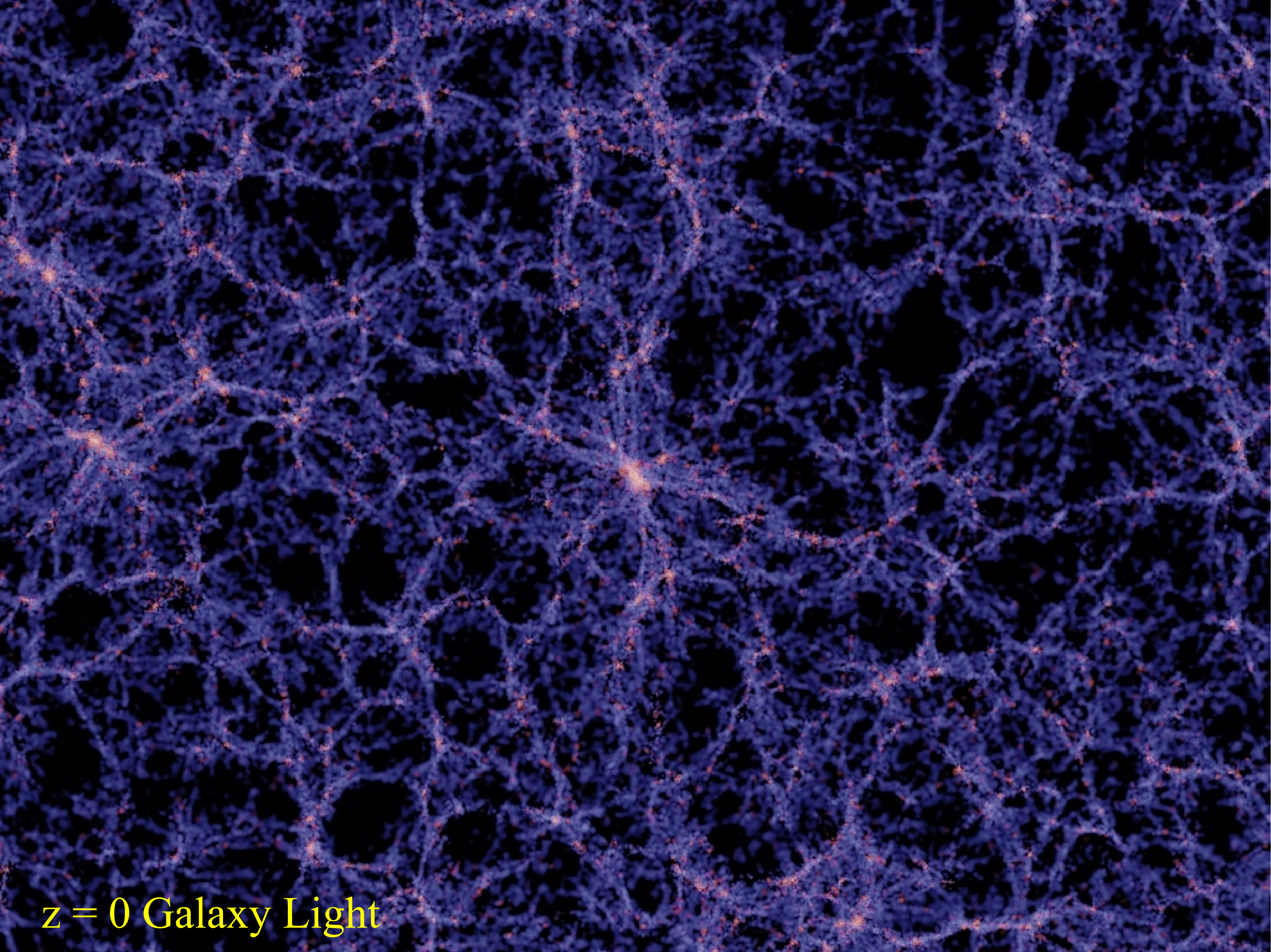
WMAP3 team

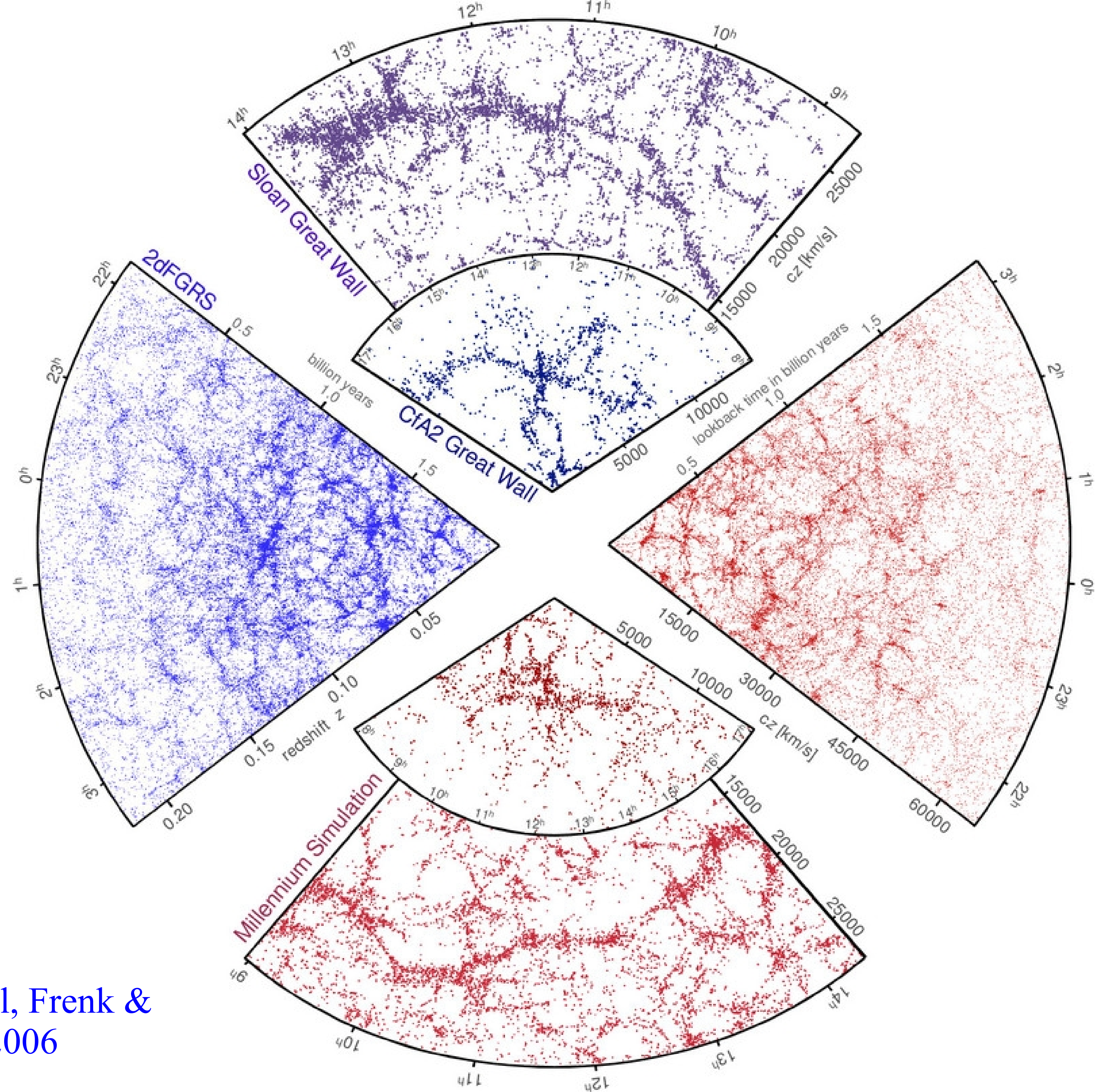
Parameter	First Year Mean	WMAPext Mean	Three Year Mean
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05

In just 2 years the Universe:

- Got ionized later (z ~ 10 rather than z ~ 15)
- Lost weight
- Got smoother, particularly on small scales


125 Mpc/h
z = 0 Dark Matter


z = 0 Galaxy Light

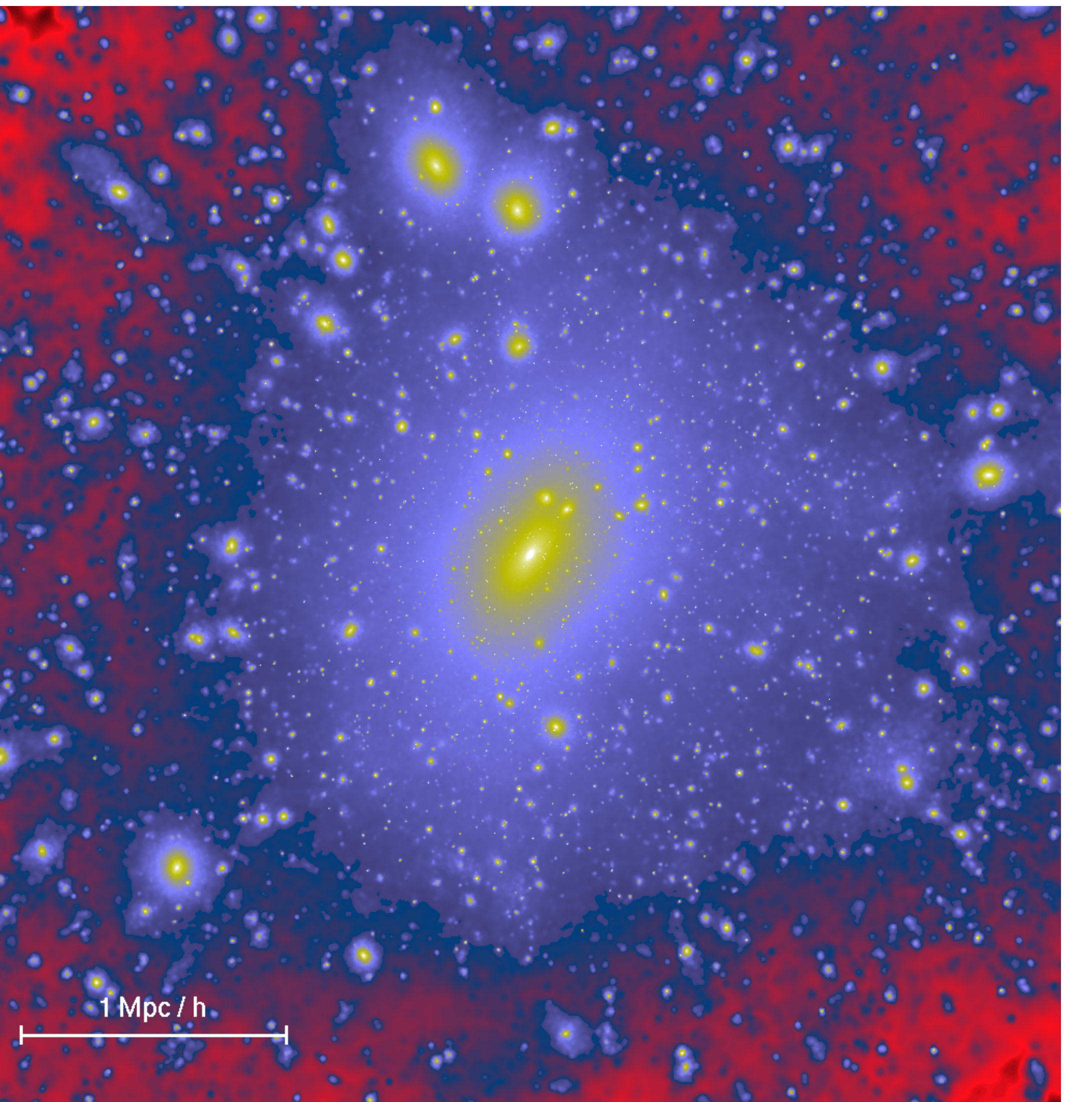


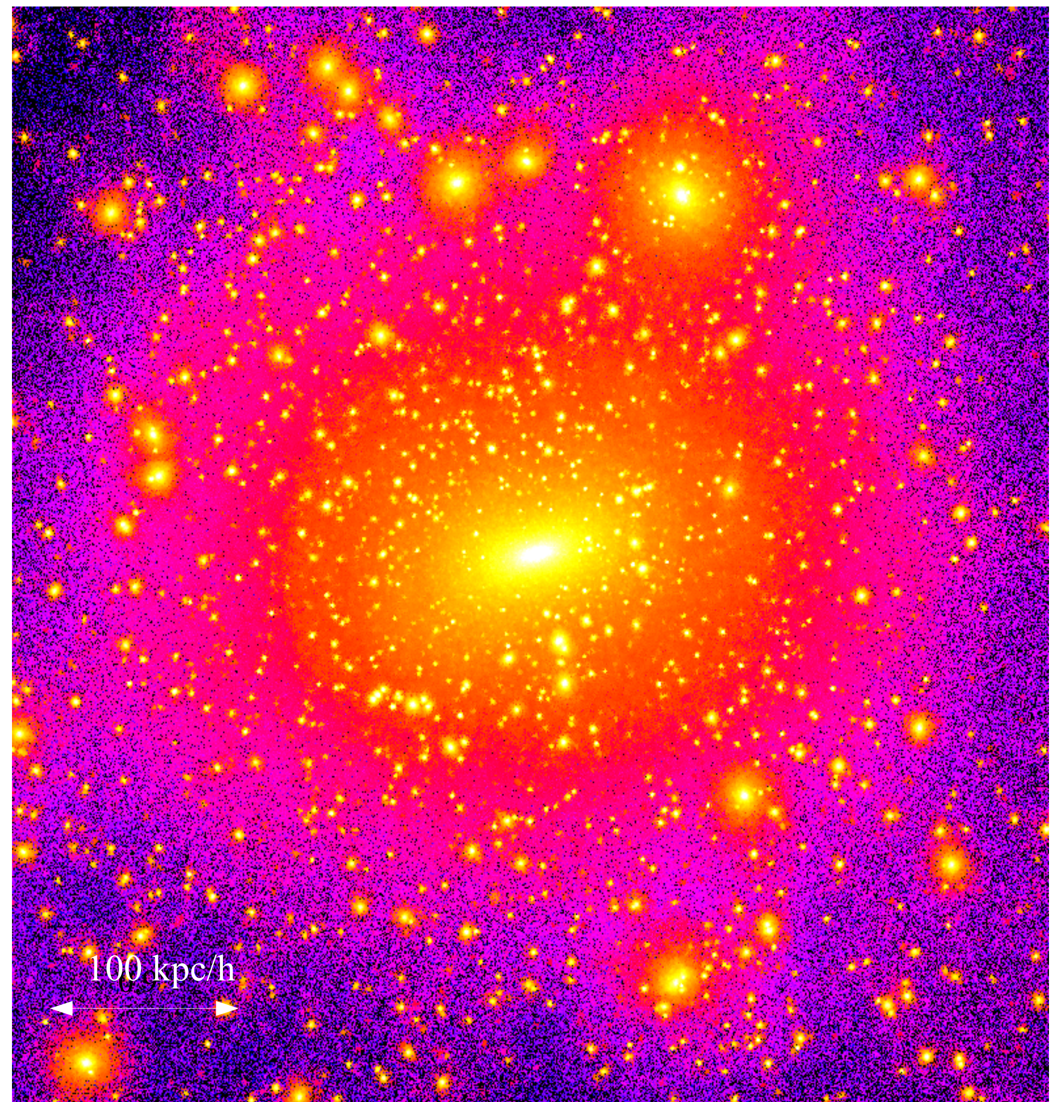
Springel, Frenk & White 2006

Small-scale structure in ΛCDM halos

A rich galaxy cluster halo
Springel et al 2001

A 'Milky Way' halo
Power et al 2002





ΛCDM galaxy halos (without galaxies!)

- Halos extend to ~10 times the 'visible' radius of galaxies and contain ~10 times the mass in the visible regions

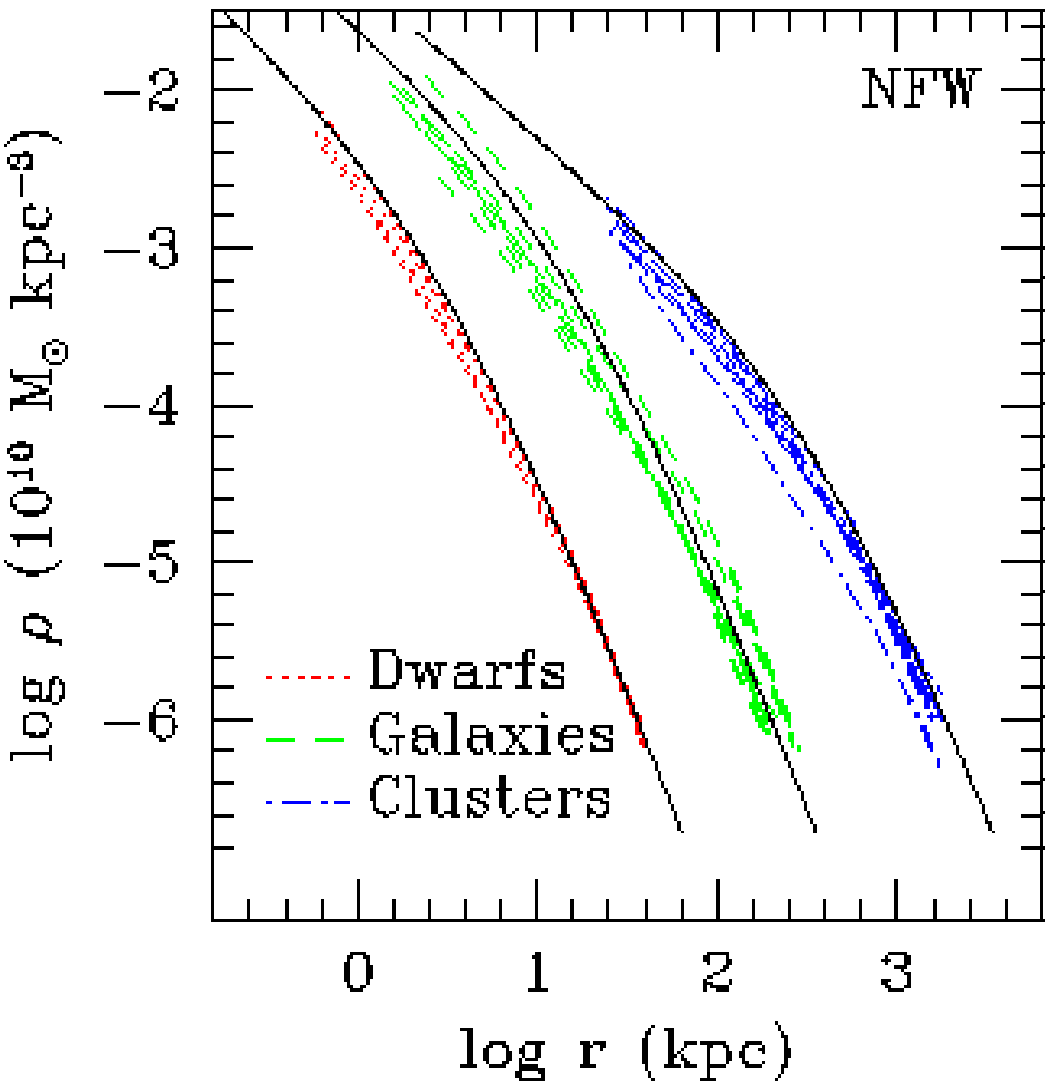
- Halos are not spherical but approximate triaxial ellipsoids
  -- more prolate than oblate
  -- axial ratios greater than two are common

- "Cuspy" density profiles with outwardly increasing slopes
  -- $d \ln \varrho / d \ln r = \gamma$ with $\gamma < -2.5$ at large r
  $\gamma > -1.2$ at small r

- Substantial numbers of self-bound subhalos contain ~10% of the halo's mass and have $dN/dM \sim M^{-1.8}$

→ Most substructure mass is in most massive subhalos

Density profiles of dark matter halos



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times

-- a universal profile shape --

$$\rho(r)/\rho_{crit} \approx \delta \; r_s \,/\, r(1 + r/r_s)^2$$

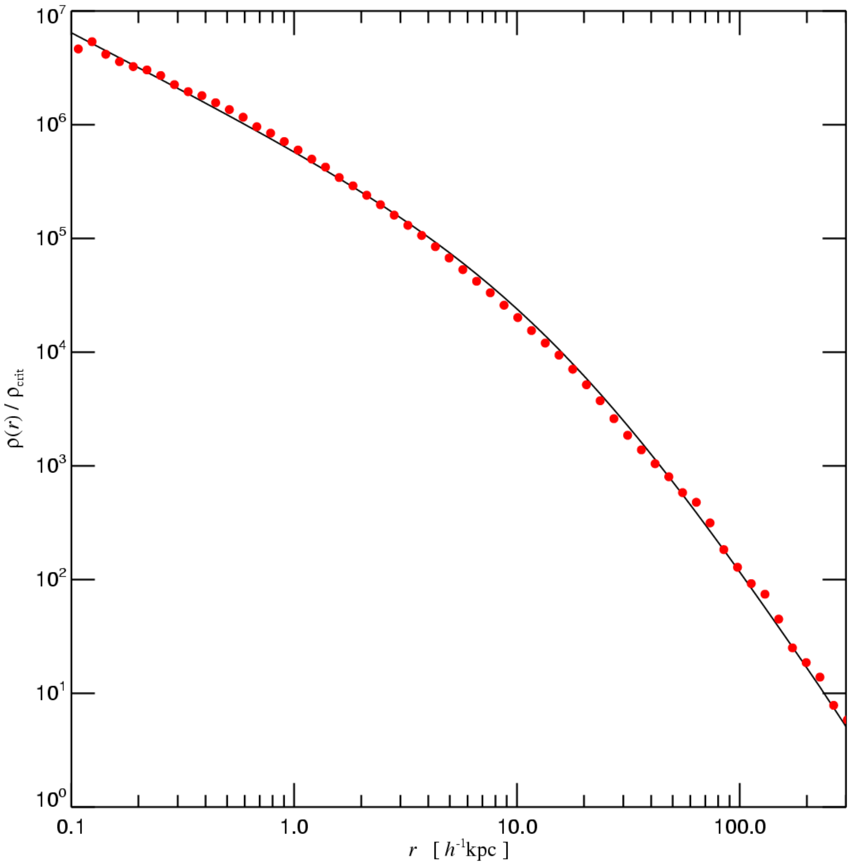
More massive halos and halos that form earlier have higher densities (bigger δ)

A high-resolution Milky Way halo

Navarro et al 2006

$N_{200} \sim 3 \times 10^7$

600 kpc

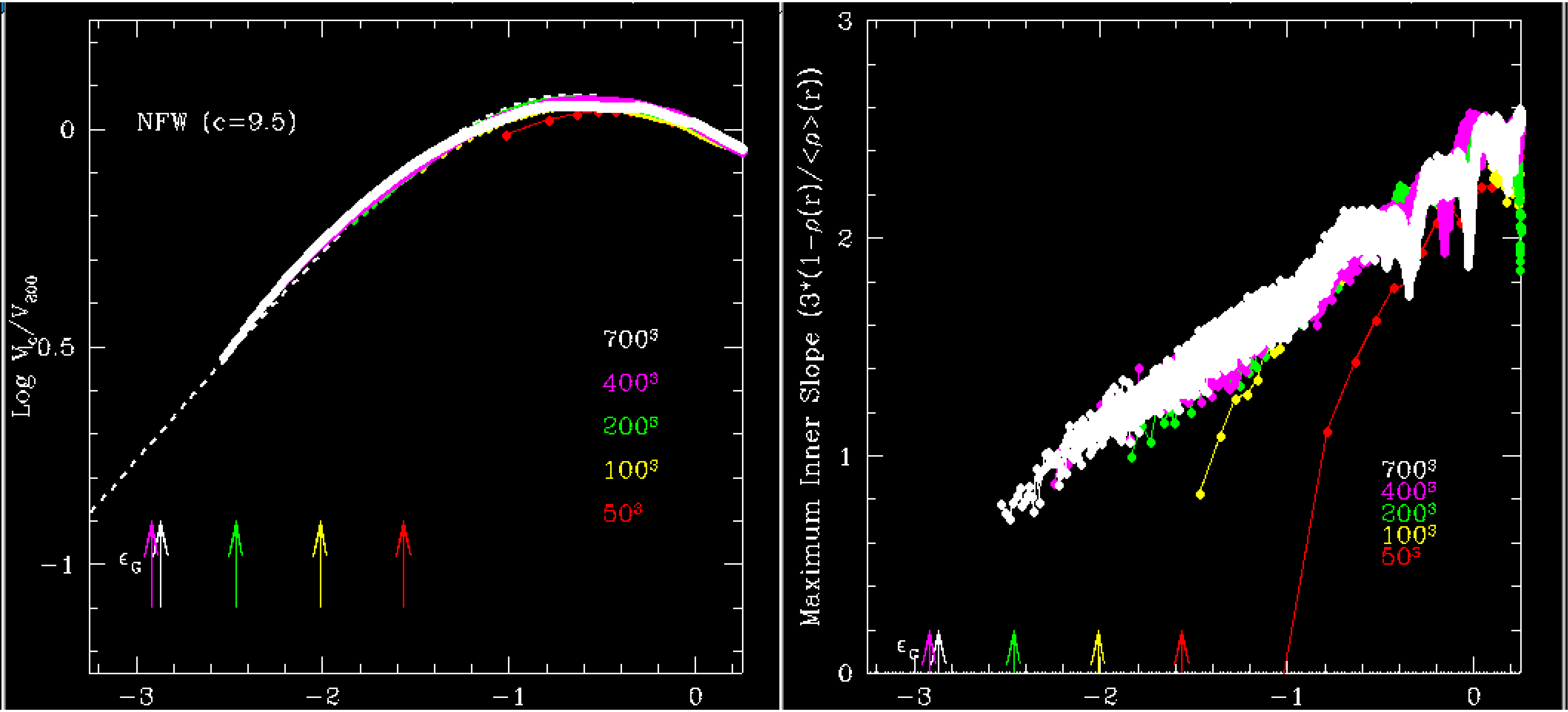


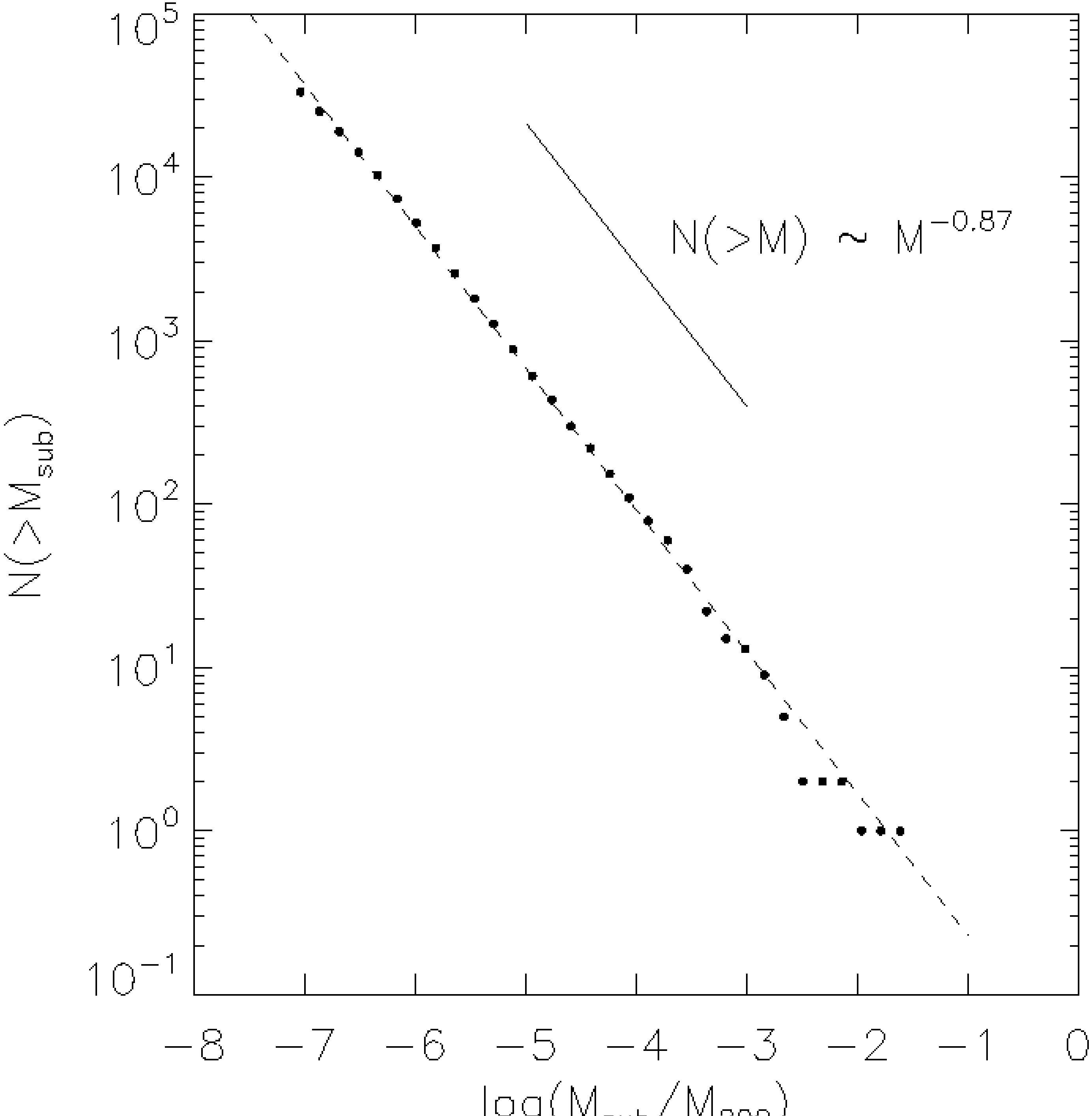
Convergence tests on density profile shape

Navarro et al 2006

DM profiles are converged to a few hundred parsecs

The inner asymptotic slope must be shallower than – 0.9

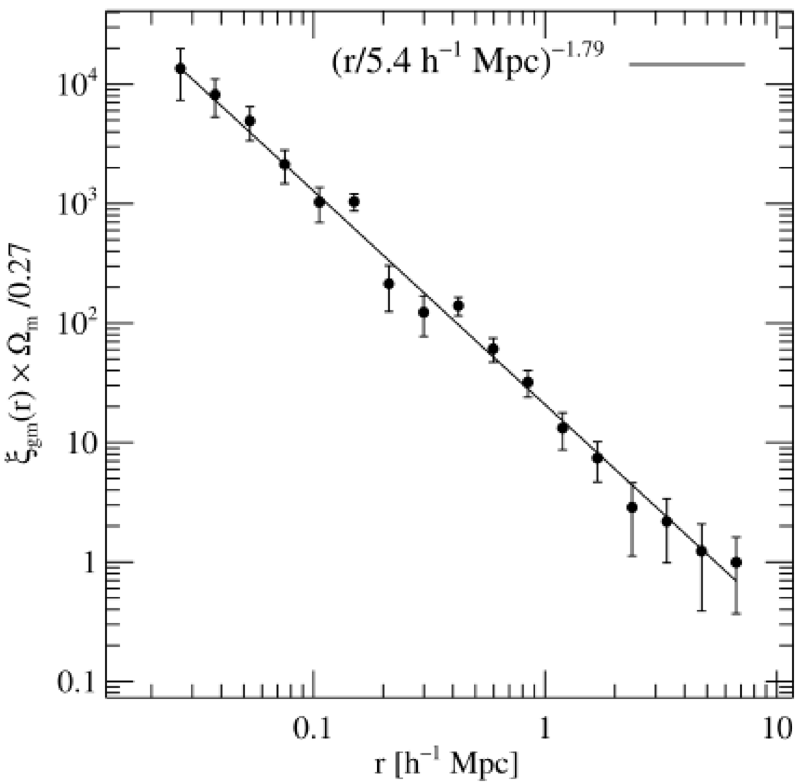




- $N_{200} = 1.60 \times 10^8$
- >30,000 subhalos
- 8% of mass within R_{200} in subhalos
- Total subhalo mass (weakly) convergent as $m_{sub} \to 0$

Observed mean mass profile of galaxy halos

Sheldon et al 2004



Stacked SDSS data

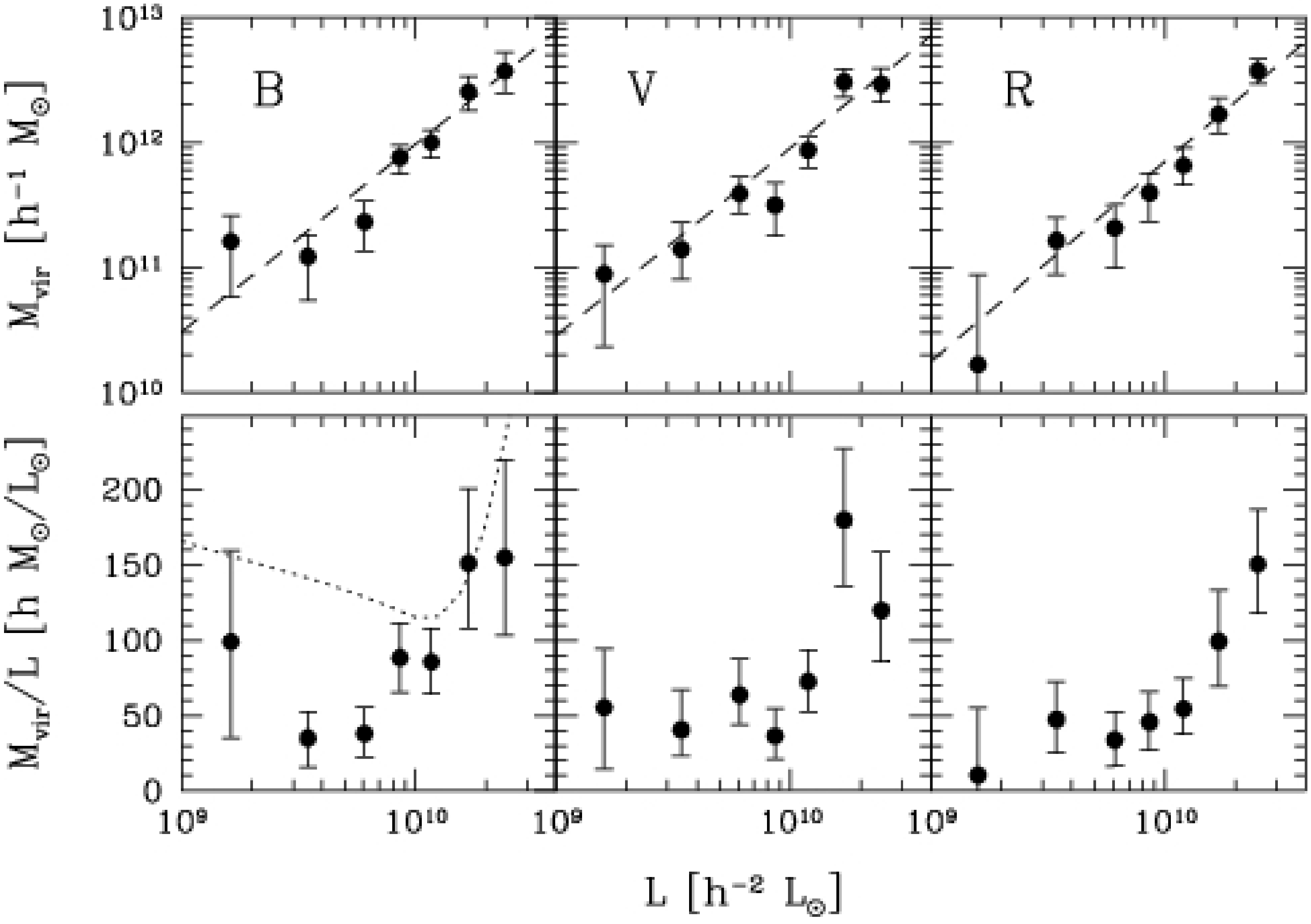
120,000 lenses with spectroscopic redshifts!

9 million sources with photometric redshifts!

Mean profile measured from 25 kpc to 5 Mpc

Observed mean halo mass as a function of L

Sheldon et al 2004



$M \sim L^{1.5}$

Likely Identification of Cold Dark Matter?

- A neutralino?
  - -- lightest supersymmetric partner of the known particles
  - -- interacts very weakly with photons/baryons/leptons
  - -- likely stable or metastable
  - -- expected mass ~ 100 GeV
  - -- thermal velocity in unclustered regions $\sim 10^{-4}(1+z)$ cm/s
  - -- detectable through energy deposition in a bolometer
  - -- annihilation into γ-rays potentially observable

- An axion?
  - -- proposed to solve strong CP problem
  - -- weak interactions
  - -- mass strongly constrained by astrophysics ~ 10 μeV
  - -- may form as a Bose condensate → zero thermal velocity
  - -- detectable through resonant interaction with microwaves

Cold Dark Matter at high redshift (e.g. $z \sim 10^6$)

At epochs well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, the inflationary model for the origin of structure predicts the distribution function:

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x})]\,\delta_D(\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x}))$$

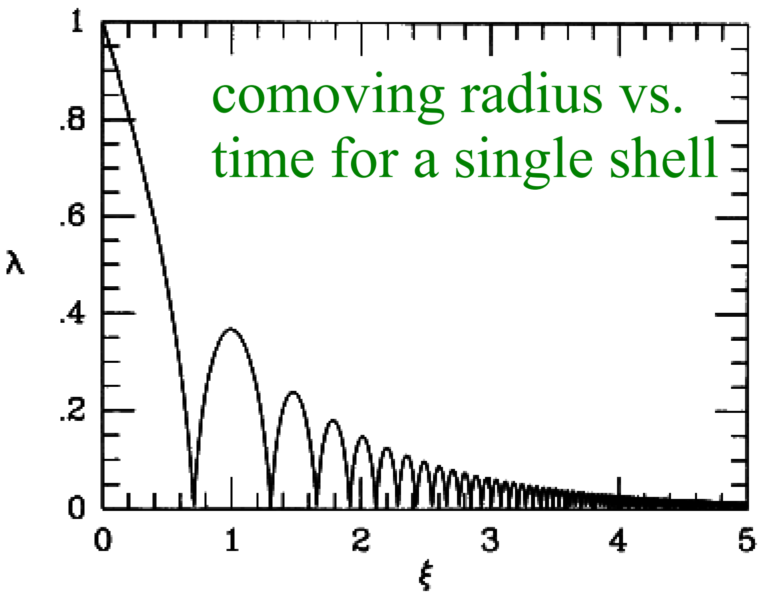
where $\rho(t)$ is the mean mass density of CDM,
$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,
and $\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$

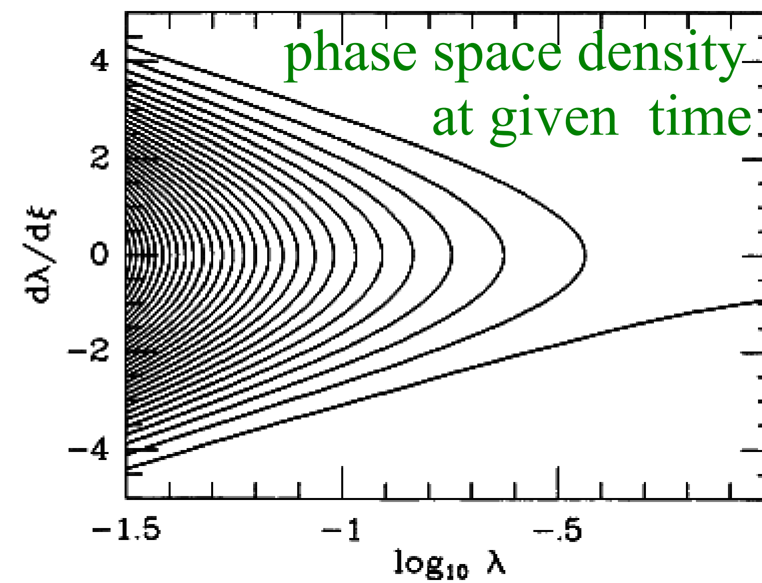
The phase density of CDM occupies a 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

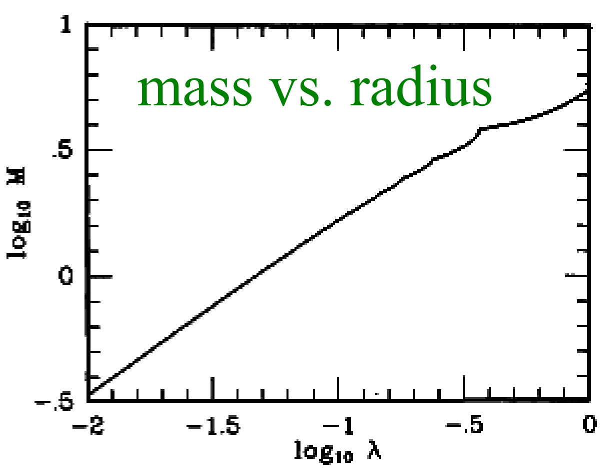
$Df / Dt = 0$ → only a 3-D subspace is occupied at later times.
Nonlinear evolution leads to a complex, multi-stream structure.

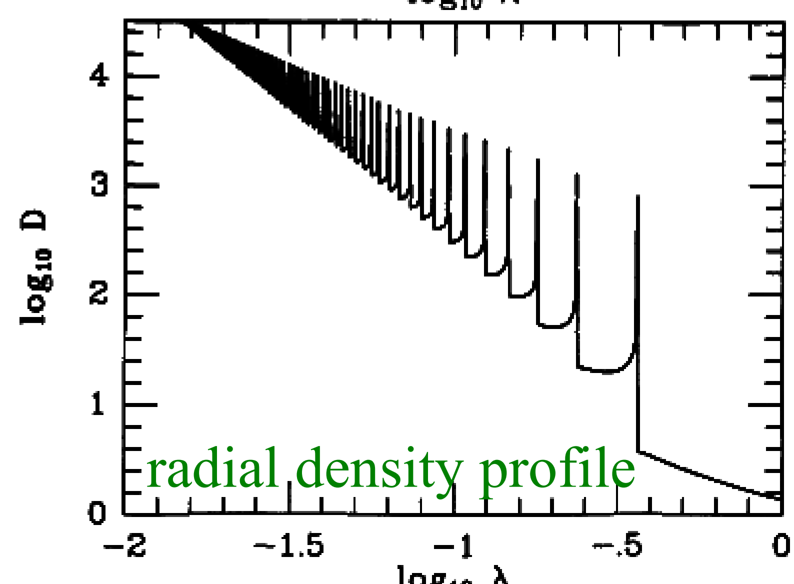
Similarity solution for a 1-D collapse in CDM

Bertschinger 1985









Small-scale structure of the CDM distribution

- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
  -- what is the density structure between m and kpc scales?
  -- how many streams intersect the detector at any time?

- Intensity of annihilation radiation depends on

$$\int \rho^2(\boldsymbol{x}) \langle \sigma v \rangle dV$$

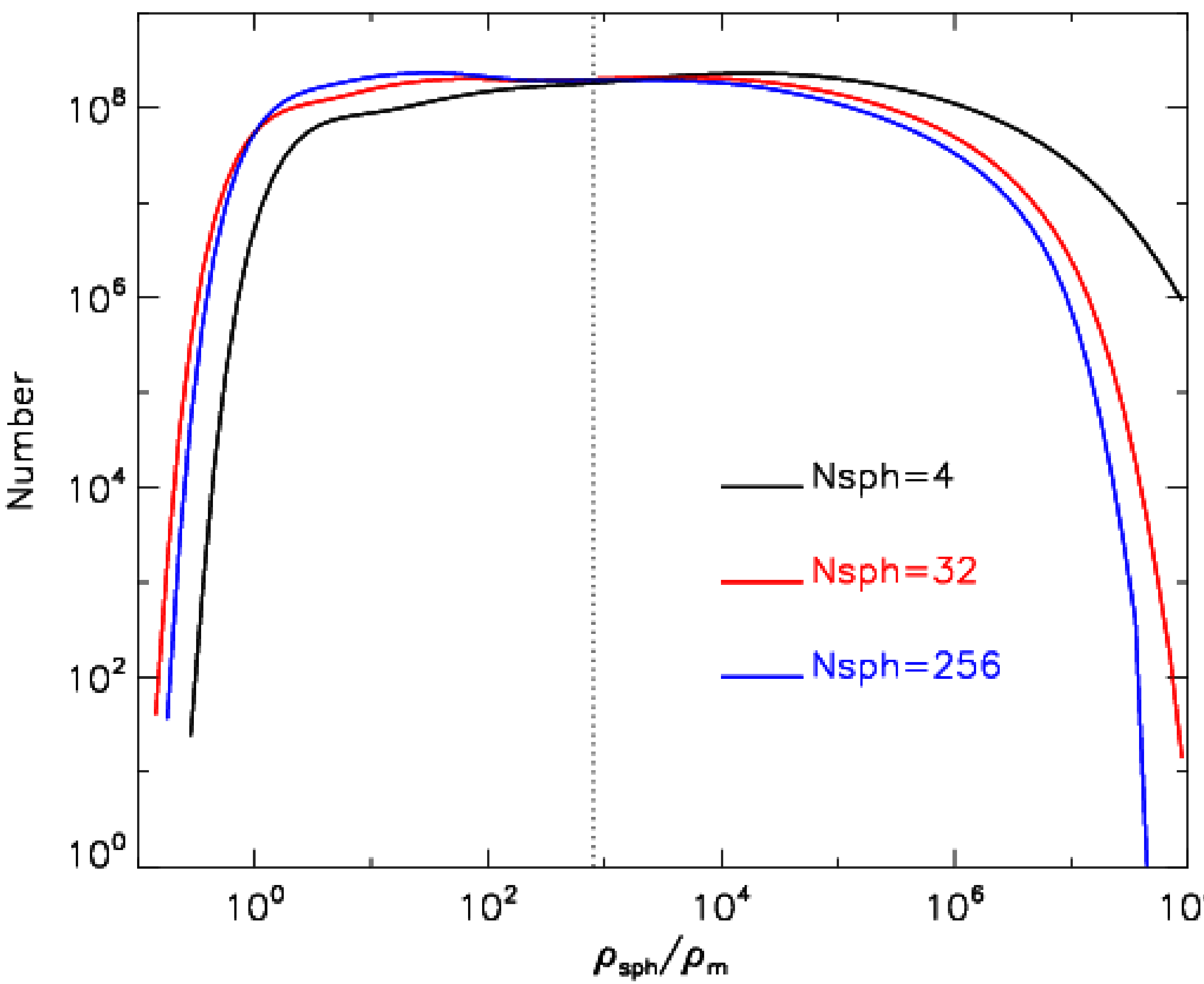
  -- what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales <u>far</u> below those accessible to simulation

→ We require a good theoretical understanding of mixing

Lagrangian DM density at the present day

Gao et al 2007



- Lagrangian smoothing gives density today on given *mass* scale
- Distribution function is flat over at least 6 orders of magnitude
- It is very far from lognormal
- No clear linear-nonlinear transition
- No clear convergence as resolution improves

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ \mathrm{T}\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{I} \\ \mathrm{T} & 0 \end{bmatrix}\cdot\delta X\,;\ \ \mathrm{T} = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & \mathrm{I} \\ \mathrm{T} & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = \mathrm{I}$$

- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- det(D) = 1 at all times by Liouville's theorem
- For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

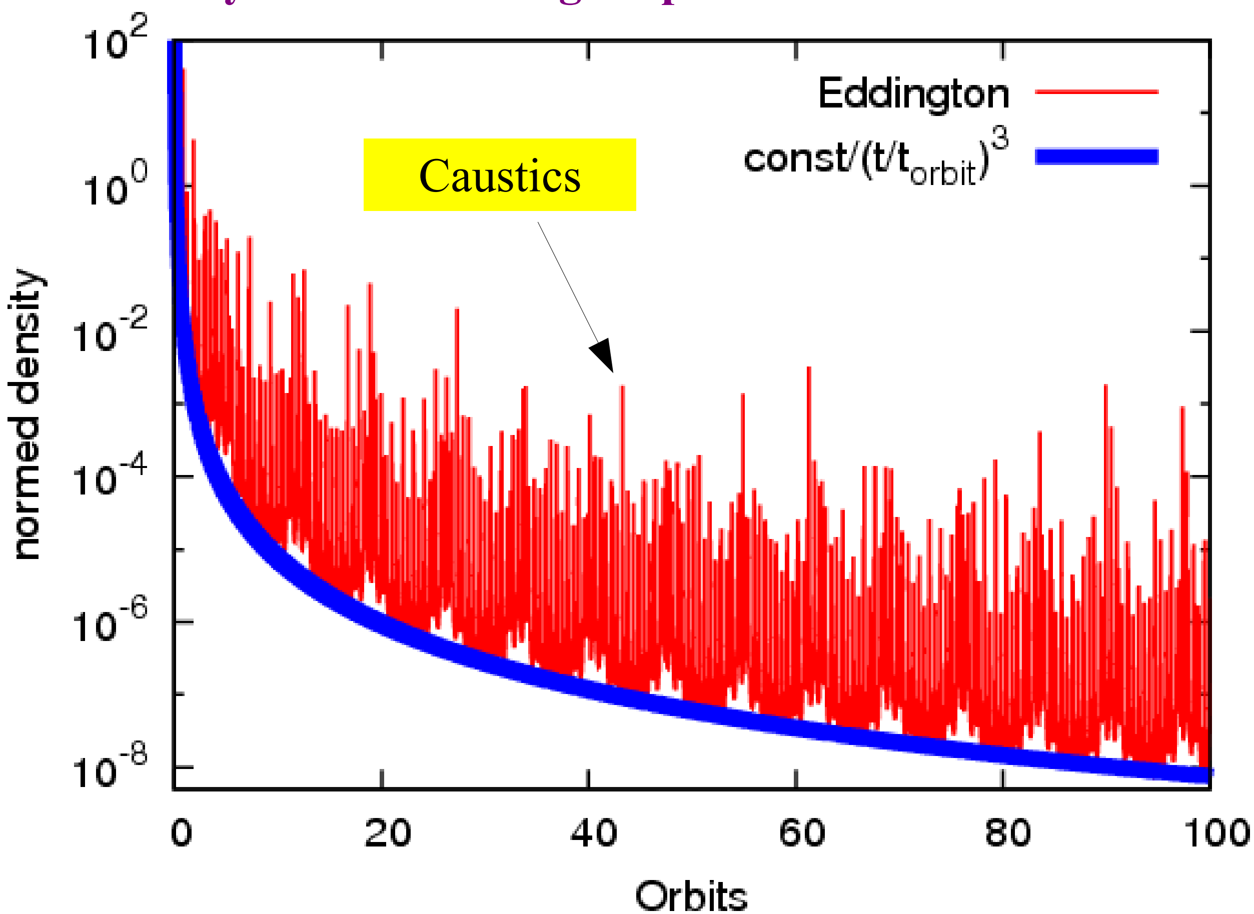
Static highly symmetric potentials

Code *DaMaFlow* developed for static potentials:

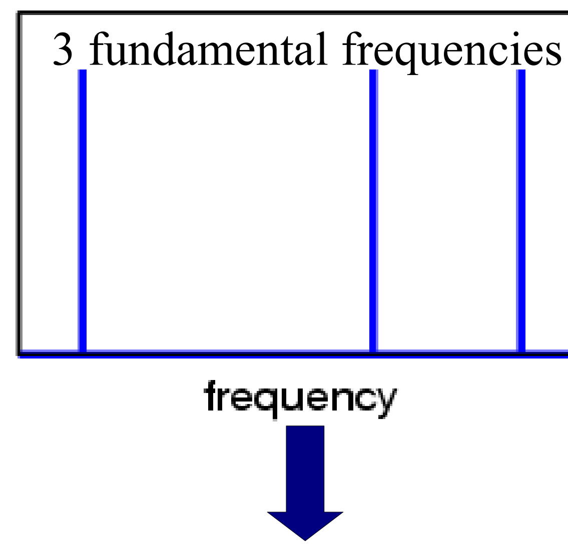
- orbit + geodesic deviation integrator (symplectic DKD/KDK Leapfrog + DOPRI853)
- modular design allows large variety of potentials to be analyzed
- precise spectral analysis on the fly (NAFF algorithm, $1/T^4$ accuracy) with integer programming to get the fundamental frequencies of motion
- automated stream density fitting

$$\Phi(r,\theta) = v_{\rm h}^2 \log\left(r^2 + d^2\right) + \frac{\beta^2 \cos^2\theta}{r^2}$$

Axisymmetric Eddington potential



Spectral analysis of orbit:

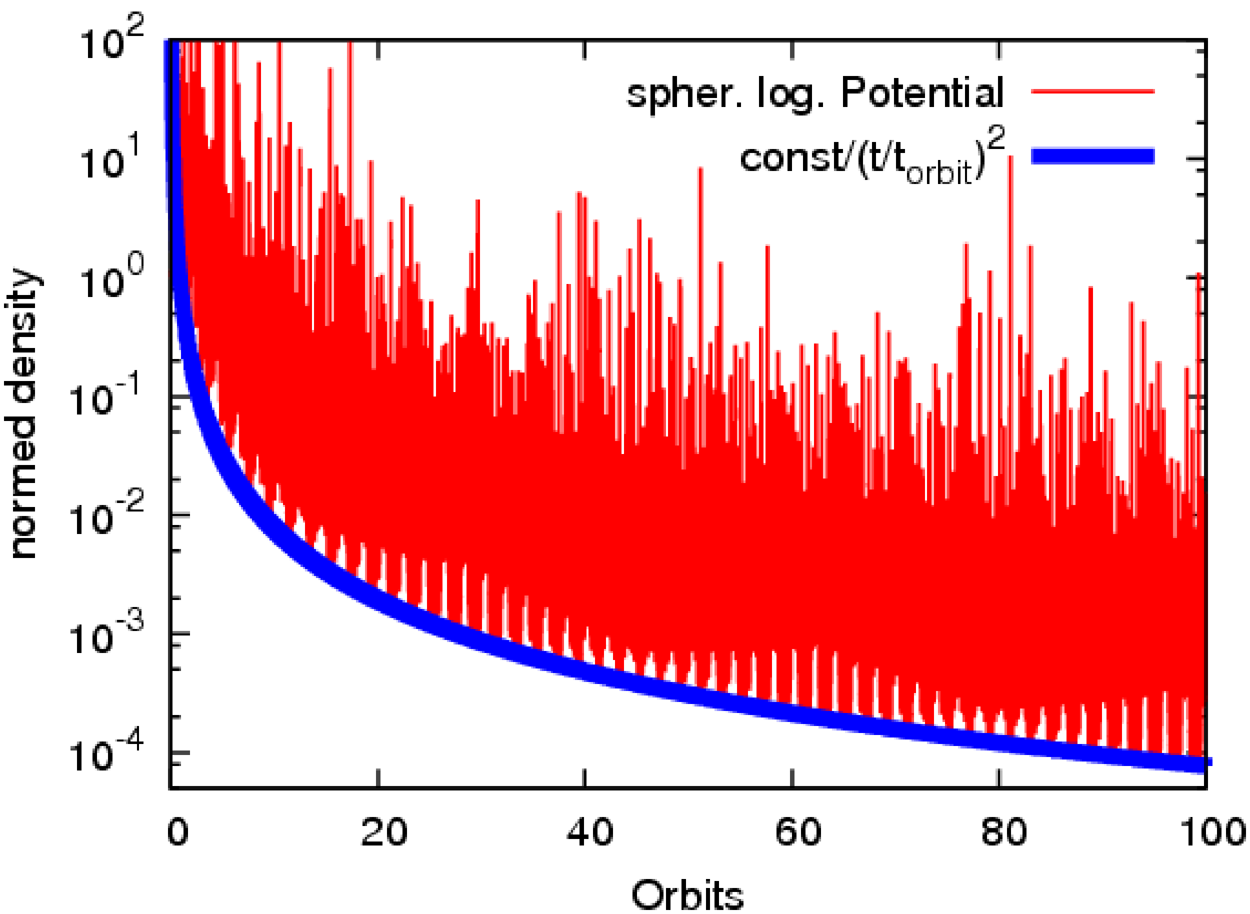


density decreases like $1/t^3$

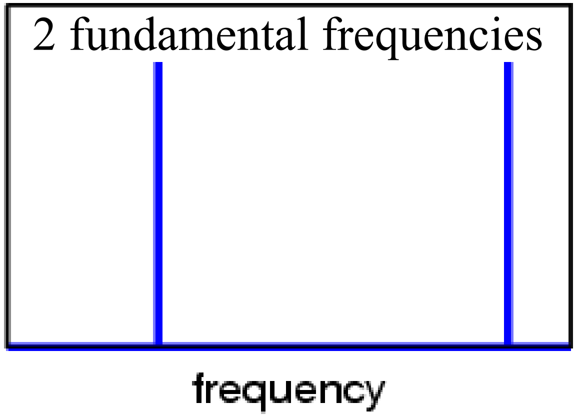
Changing the number of frequencies

Spherical logarithmic potential

$$\Phi(r,\theta) = v_{\rm h}^2 \log\left(r^2 + d^2\right)$$



Spectral analysis of orbit:



density decreases like $1/t^2$

Number of fundamental frequencies dictates the density decrease of the stream

What about non-trivial potentials?

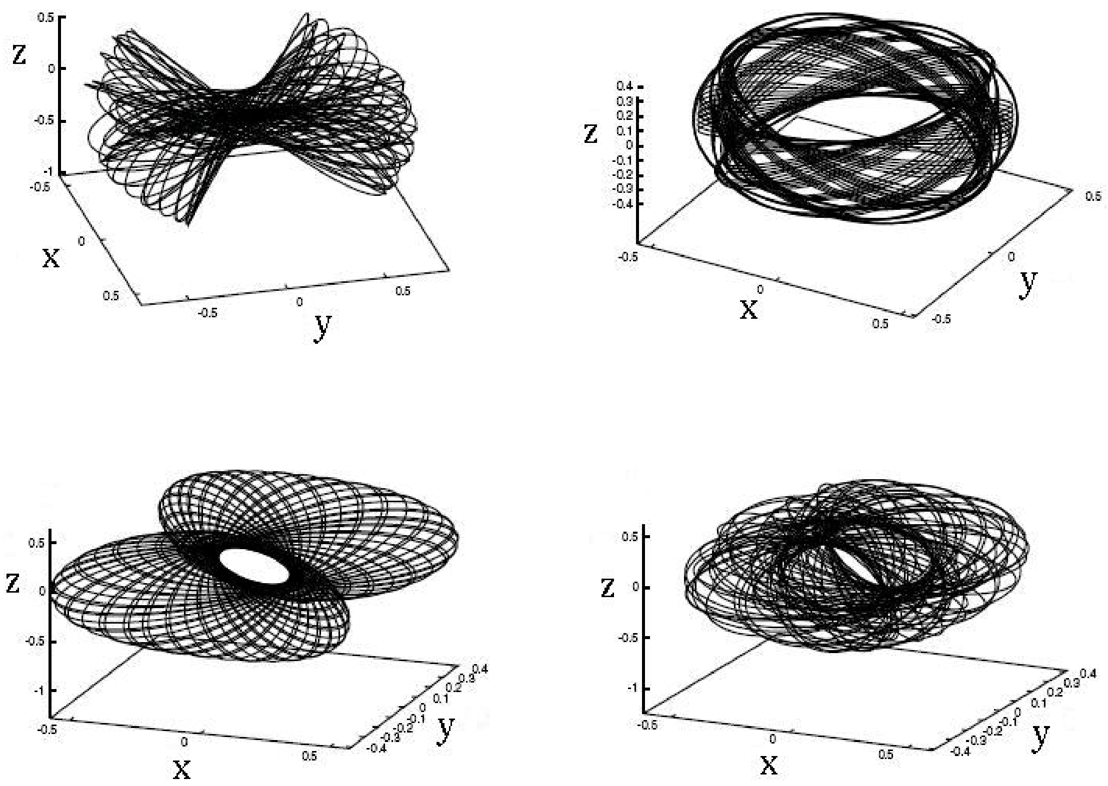
integrable systems give only rise to **regular motion**

non integrable (more realistic) systems:

have more complicated phase space structure, possibly with **chaotic regions**

this has an impact on dark matter stream density

Try to get more insights with our new approach!



Example:

triaxial logarithmic potential with core

$$H = \frac{1}{2}\left(X^2 + Y^2 + Z^2\right) + \ln\left(x^2 + \frac{y^2}{q_1^2} + \frac{z^2}{q_2^2} + R_c^2\right)$$

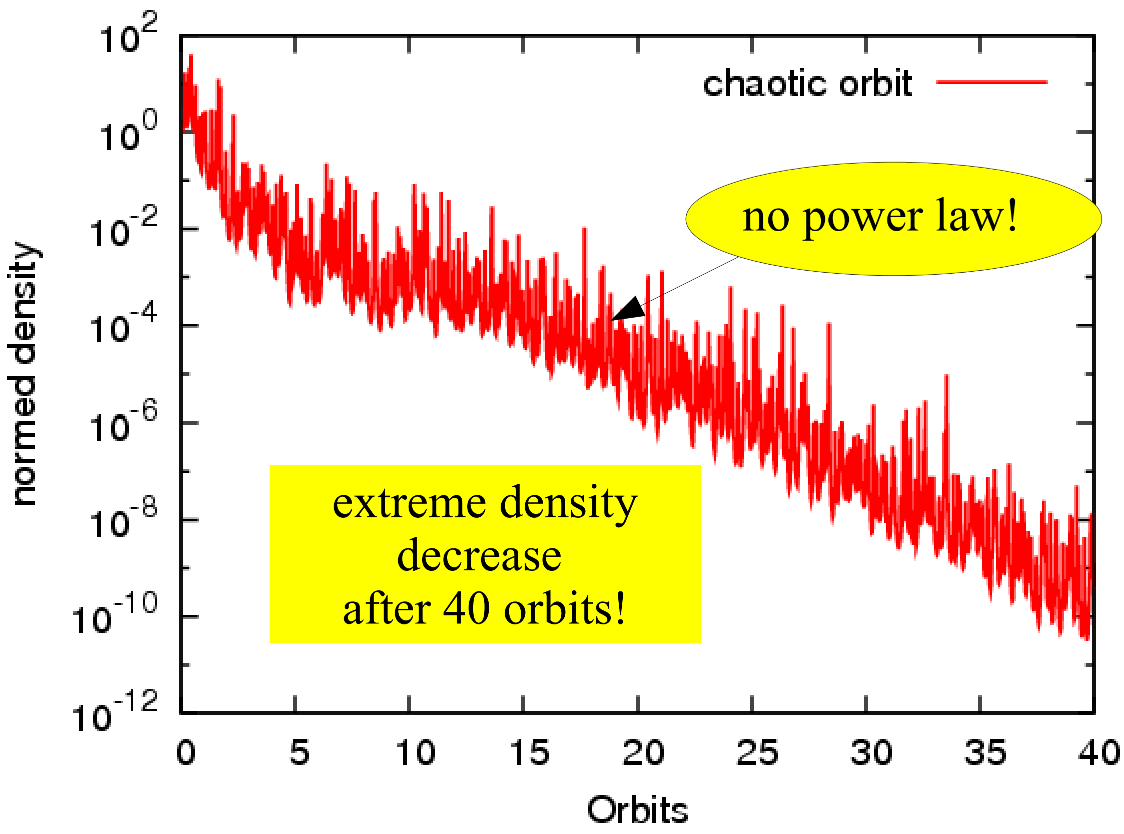
regular motion

box and tube orbits; density decreasing like a power law in time for regular motion

Chaotic mixing

chaotic motion implies a **rapid stream density decrease** → **rapid mixing**

density decrease is **not** like a power law anymore

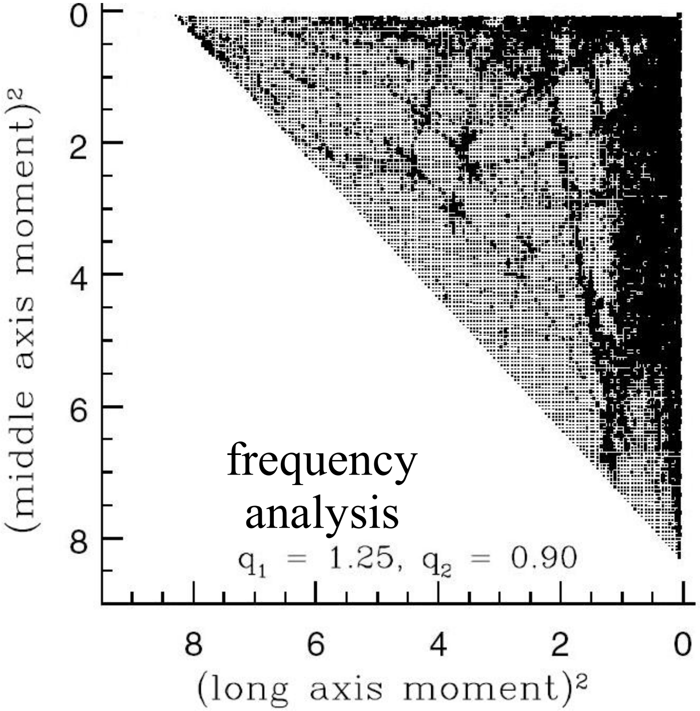


how to find chaotic regions in phase space?

Common method:

- Lyapunov exponents
- frequency analysis (NAFF)
- ...

Compare frequency analysis results with geodesic deviation equation results

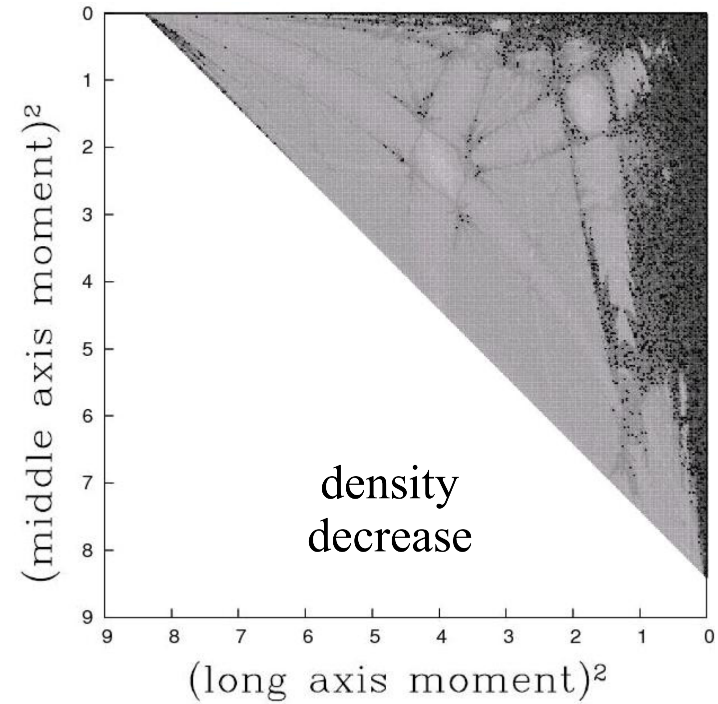


moderate triaxiality

↓

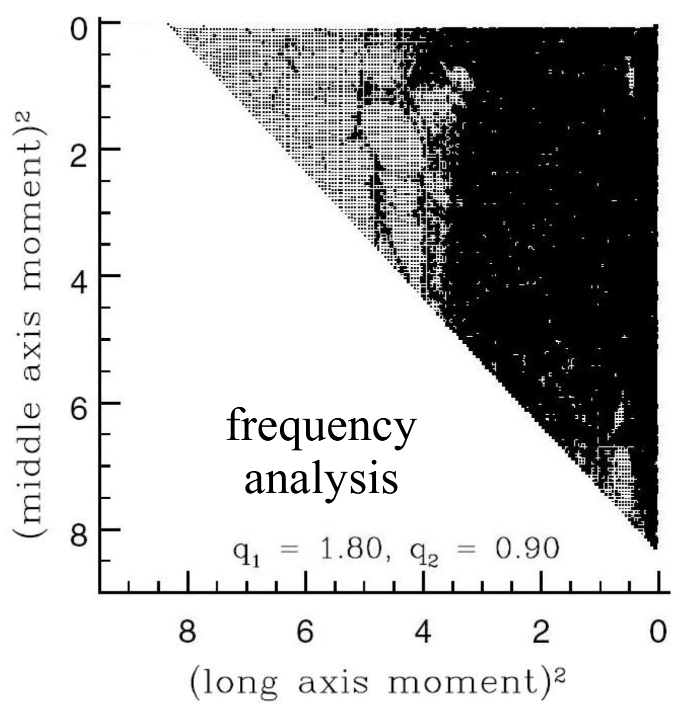
small fraction of chaotic orbits

stream density mostly decaying like a power law



Papaphilippou & Laskar 1998

integrate 10^5 different orbits

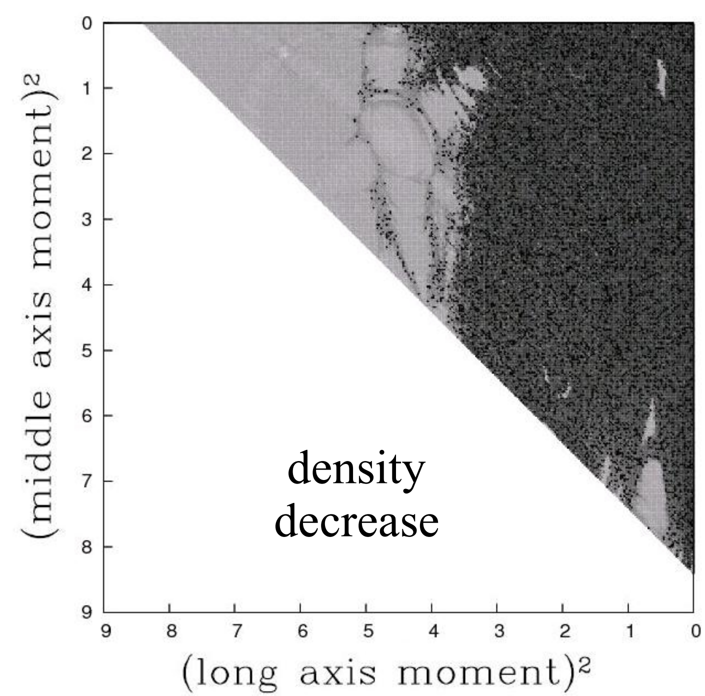


high triaxiality

↓

large fraction of chaotic orbits

stream density mostly decaying much faster than a power law



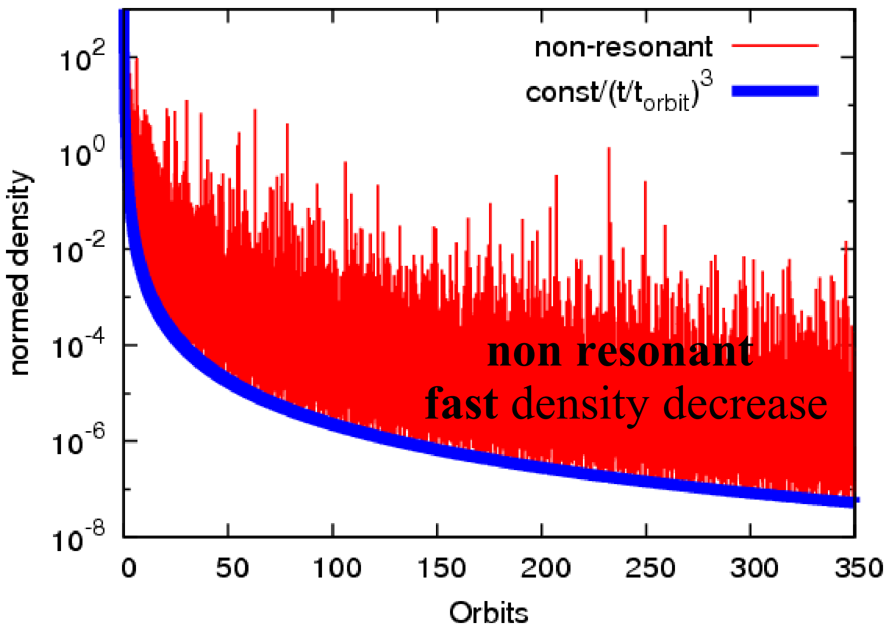
Resonances in phase space

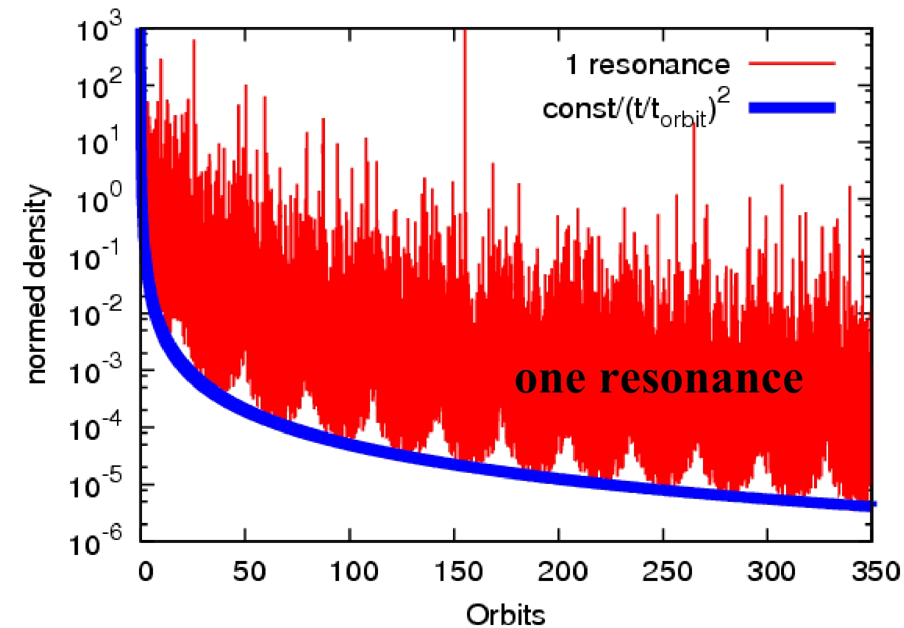
motion might be resonant in certain portions of phase space

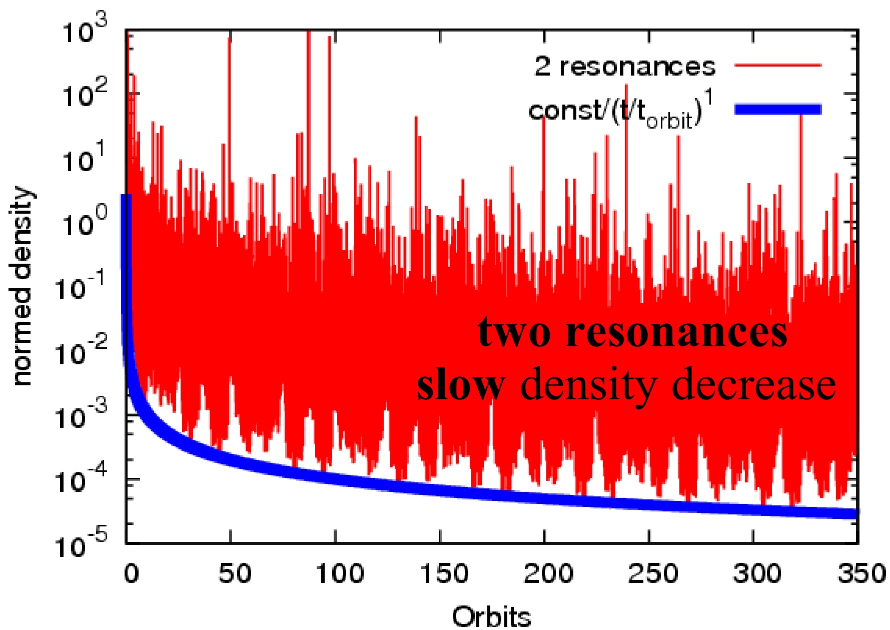
$$m_1\omega_1+m_2\omega_2+m_3\omega_3=0$$

KAM torus not completely covered

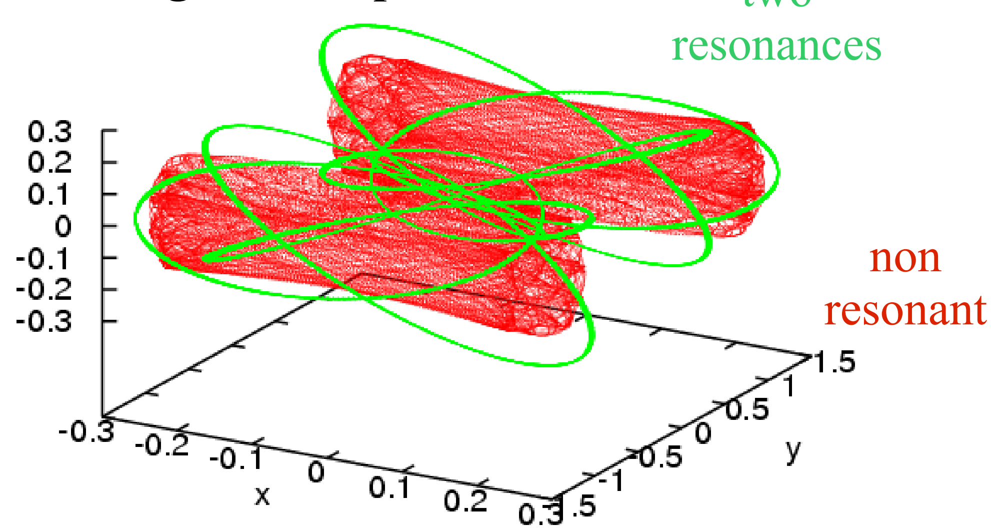
stream density decreases slower



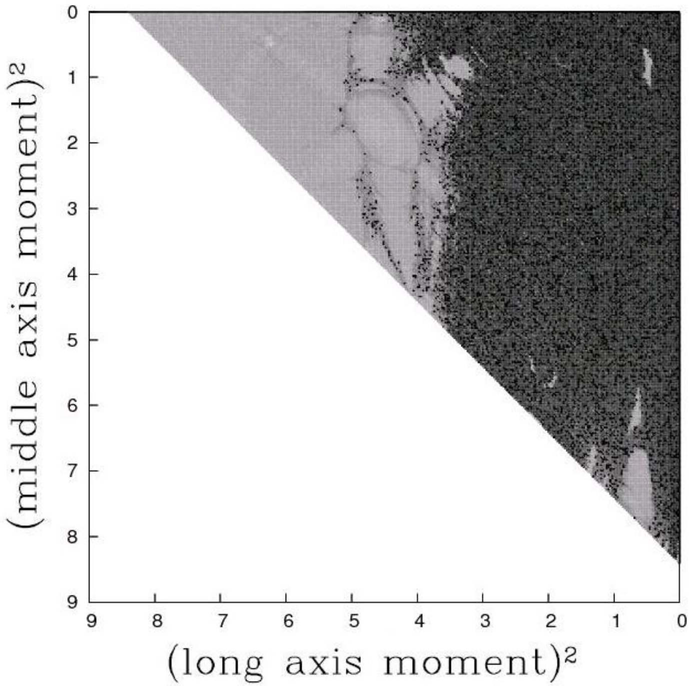




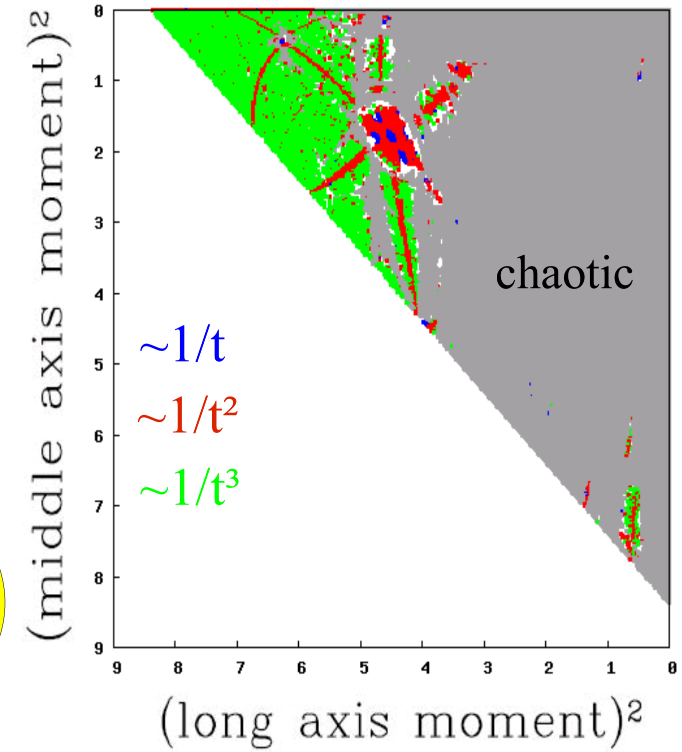
Configuration space



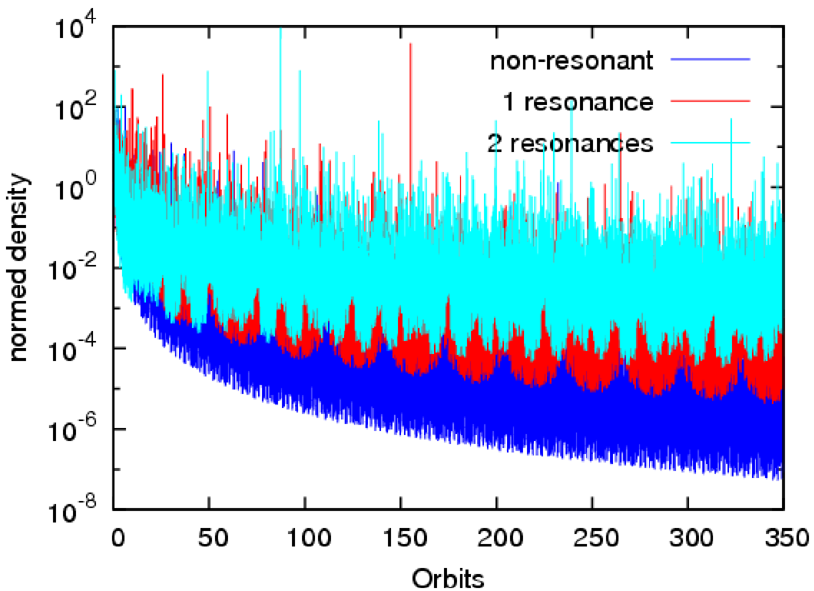
Resonances: scanning phase space



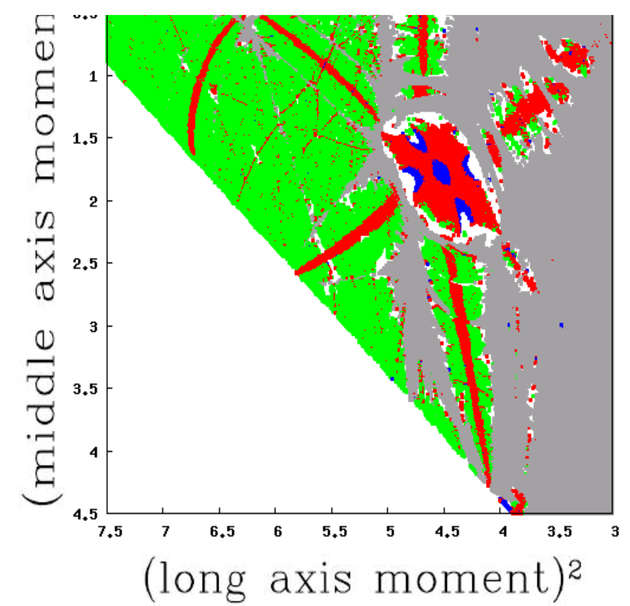
fitting / binning
density decrease



phase space
of non-integrable
systems <u>highly</u>
complex



stream density has
very different
evolution

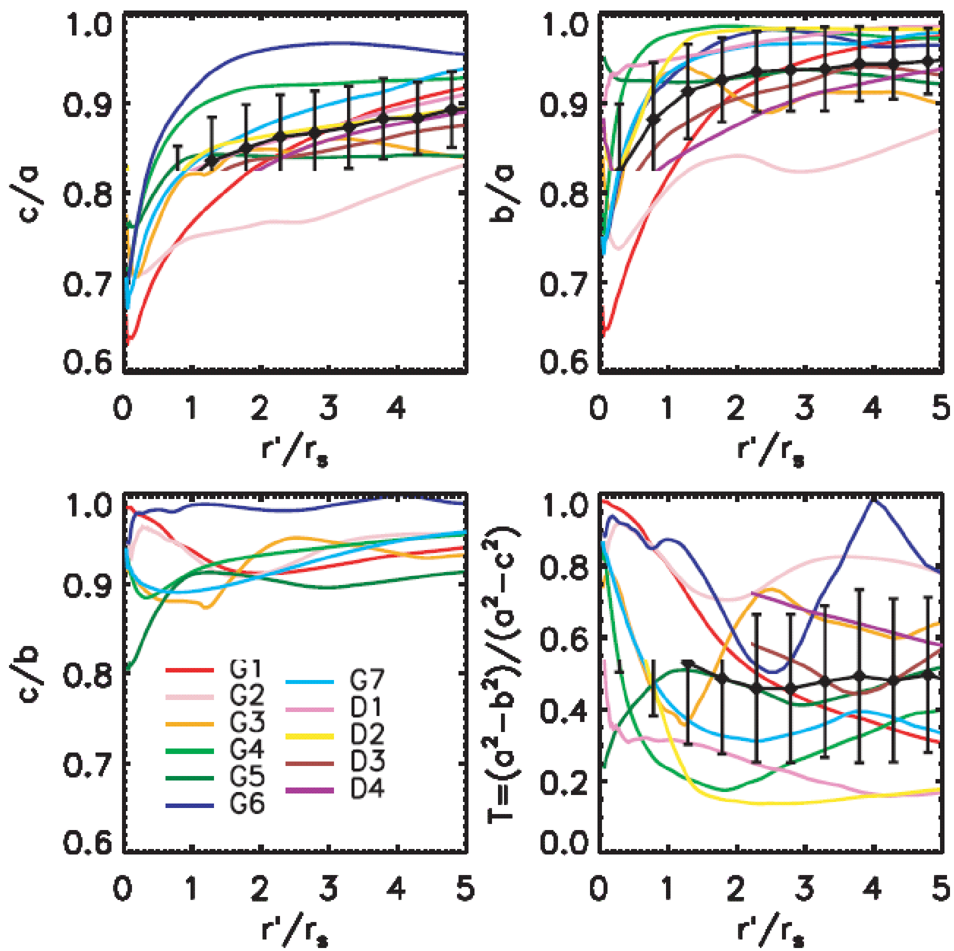


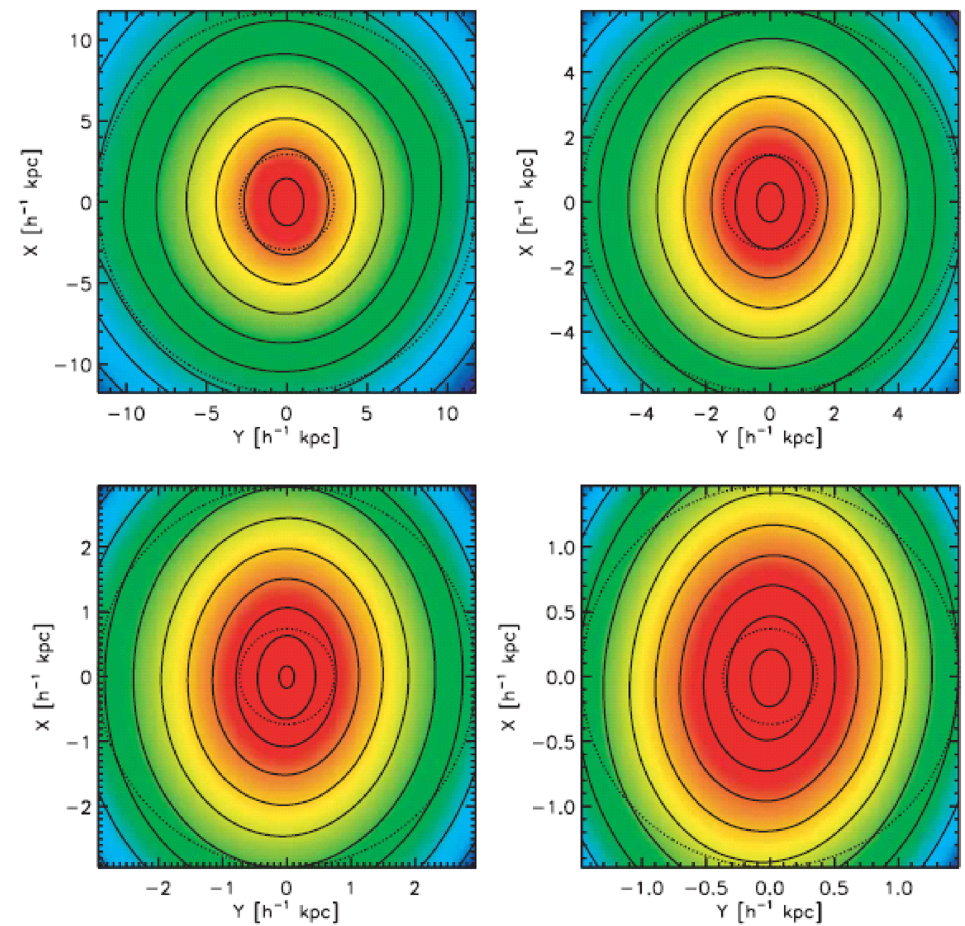
Realistic dark matter halo potentials

Cosmological simulations:

Hayashi et al, 2007

- outer regions *spherical*
- inner regions *aspherical*
- principal axes well aligned over radius
- halos tend to become more prolate near center





shape of potential very relevant for **stream dynamics**

build a model for the potential with similar qualitative behaviour

Simple model for triaxial halo potential

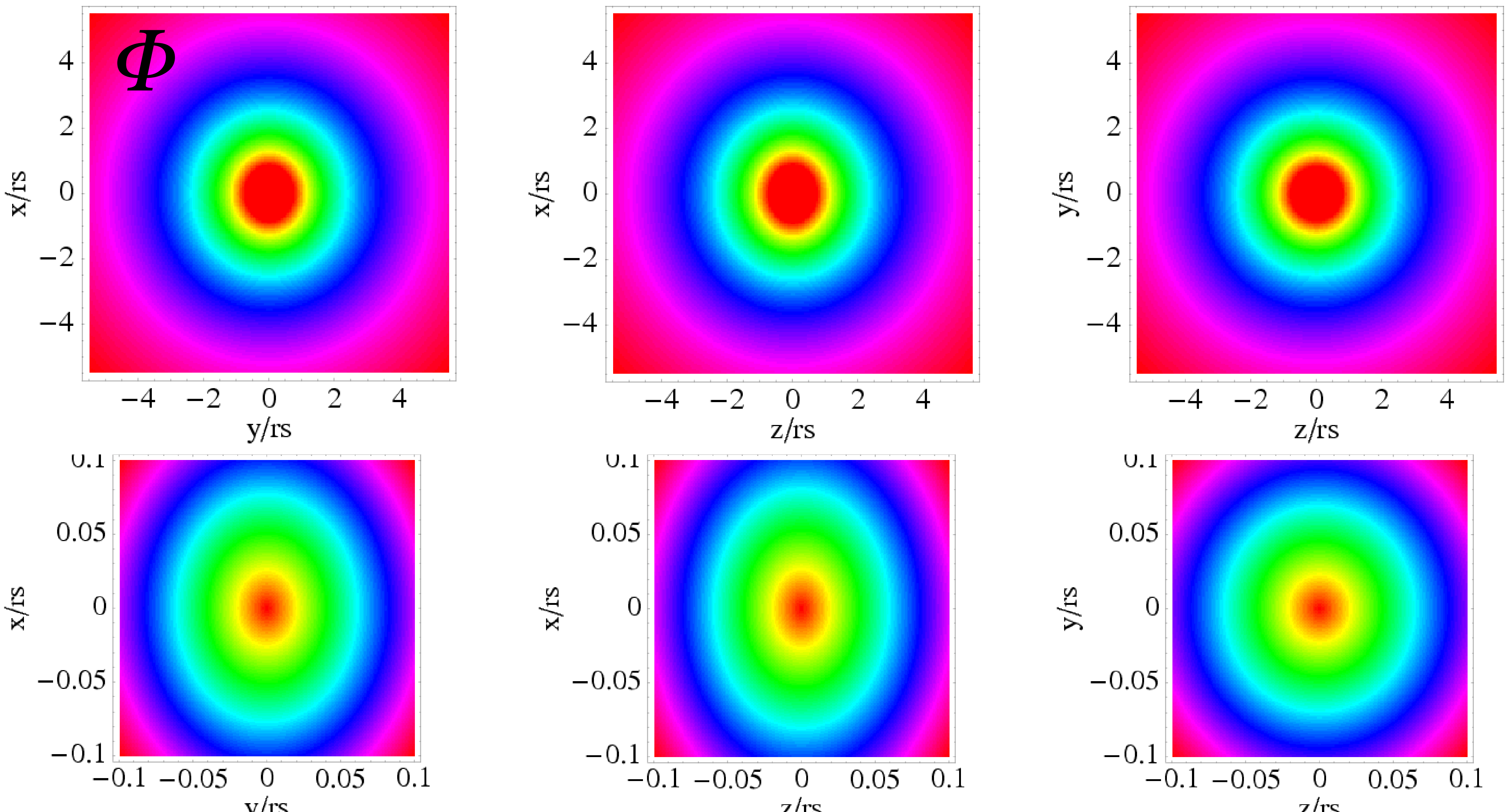
Potential:

$$\Phi(x,y,z)=\Phi_{NFW}(\tilde{r}(x,y,z))$$

$$\tilde{r}=\frac{(r_a+r)r^E}{(r_a+r^E)}$$

transition radius

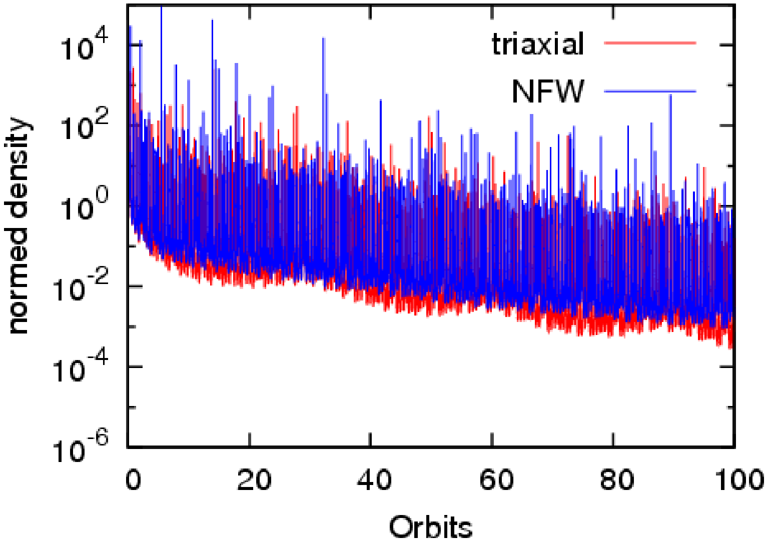
$$r^E=\sqrt{\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2}$$



Dark matter streams in a triaxial NFW

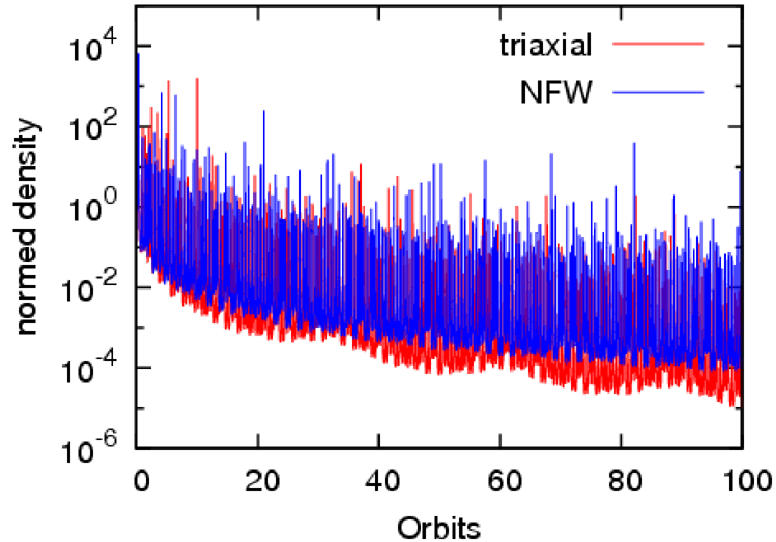
How does the radial shape variation influence the stream density evolution?

outer orbit: similar stream behaviour



Apo:8.9r_s

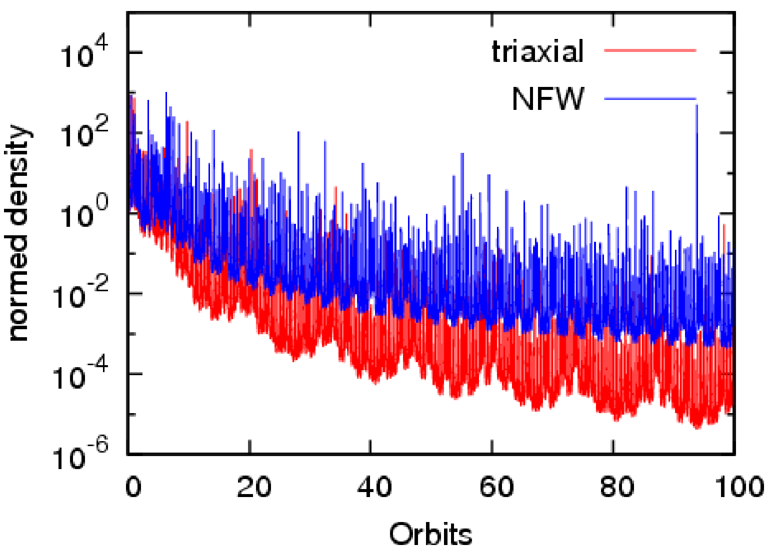
Peri:6.1r_s



Apo:8.7r_s

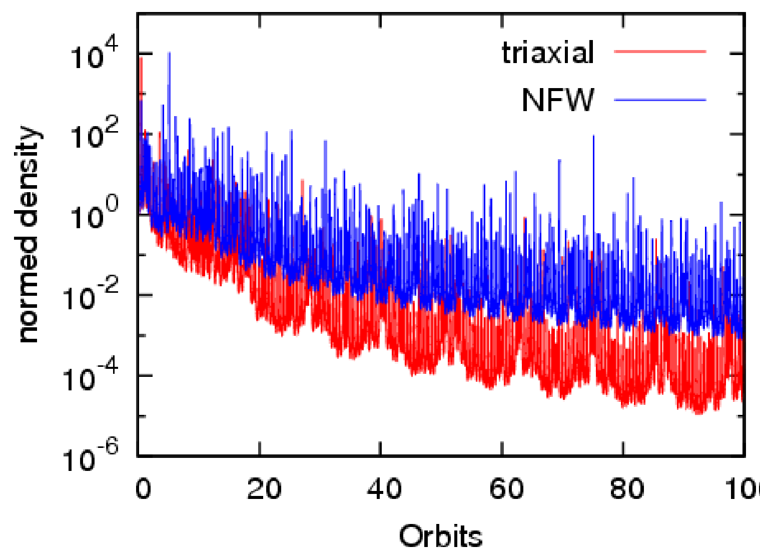
Peri:4.8r_s

inner orbits: ~100 times lower stream density after 100 orbits



Apo:2.2r_s

Peri:0.5r_s



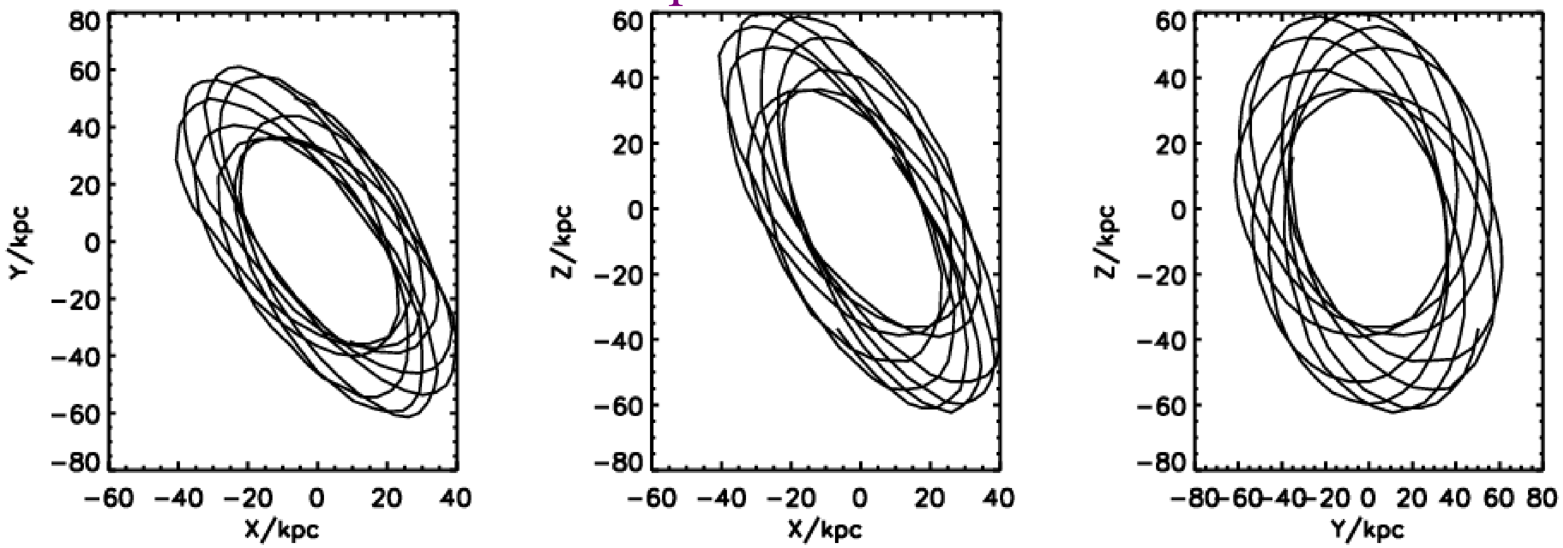
Apo:1.2r_s

Peri:0.4r_s

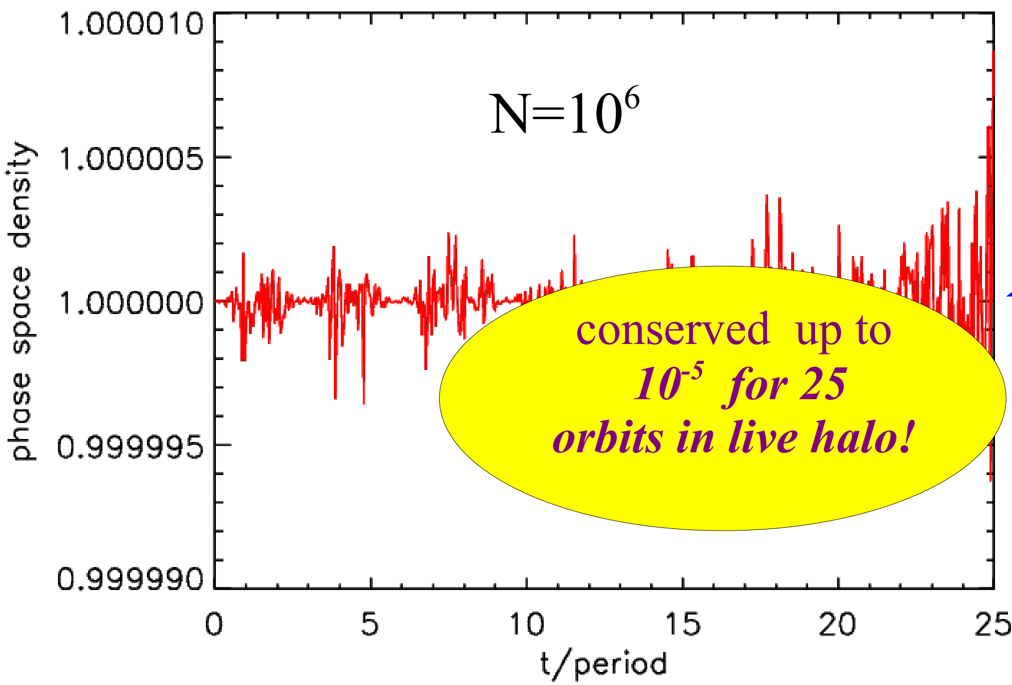
Results for live Halo:

A spherical Hernquist model

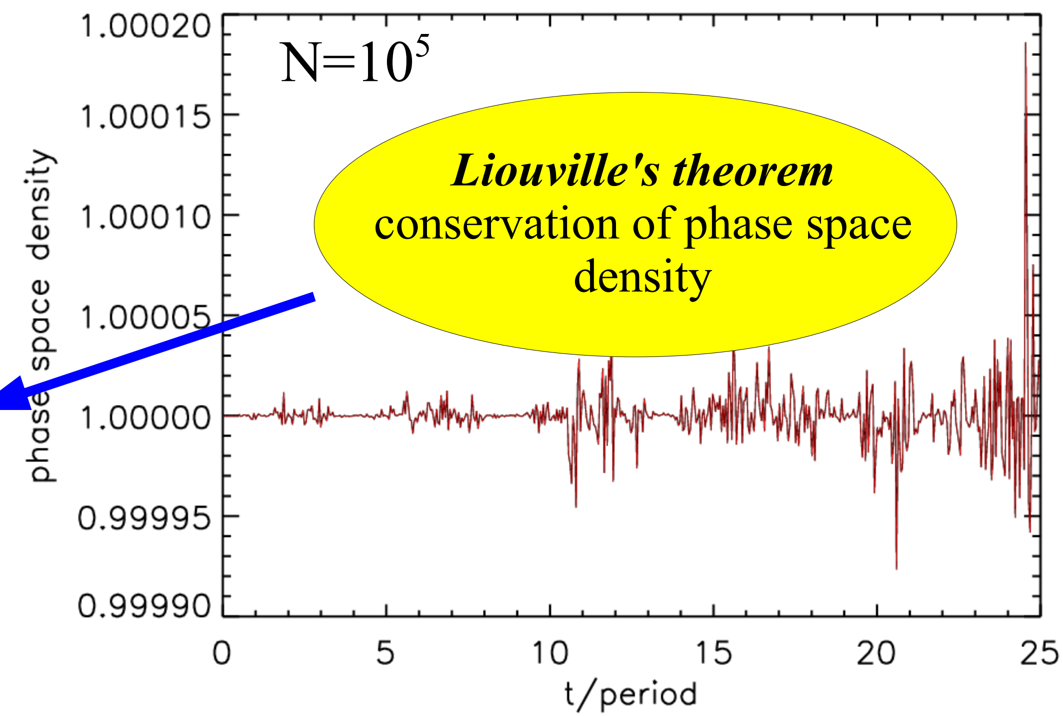
Shape of orbit:



Phase space density:



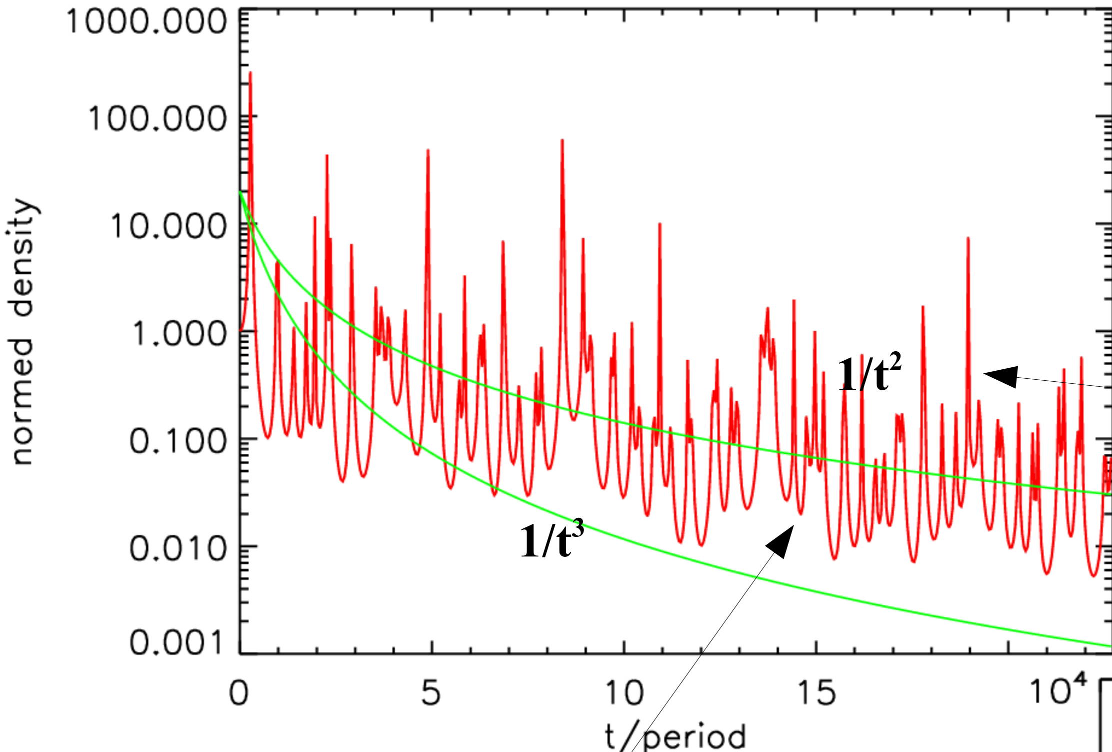
conserved up to ***10^{-5} for 25 orbits in live halo!***



Liouville's theorem conservation of phase space density

Stream density in live halo

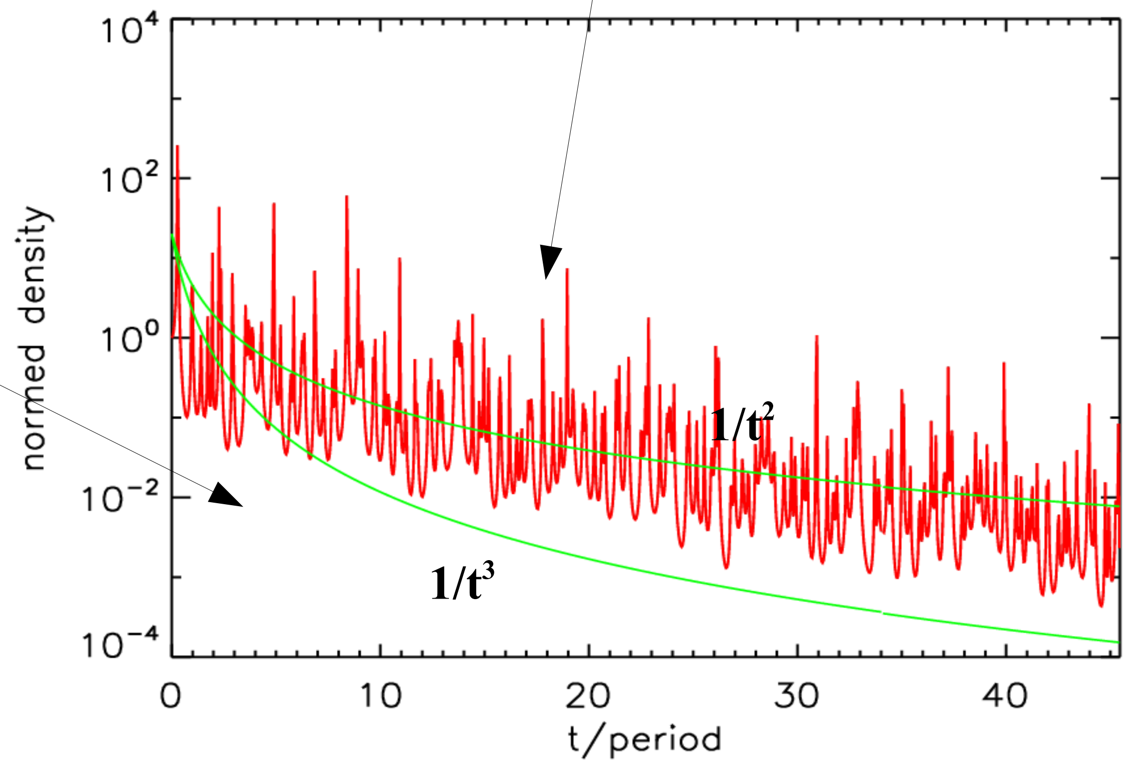
~20 orbits



A spherical Hernquist model

Caustics can be resolved!

time dependence as expected over 40 orbits!

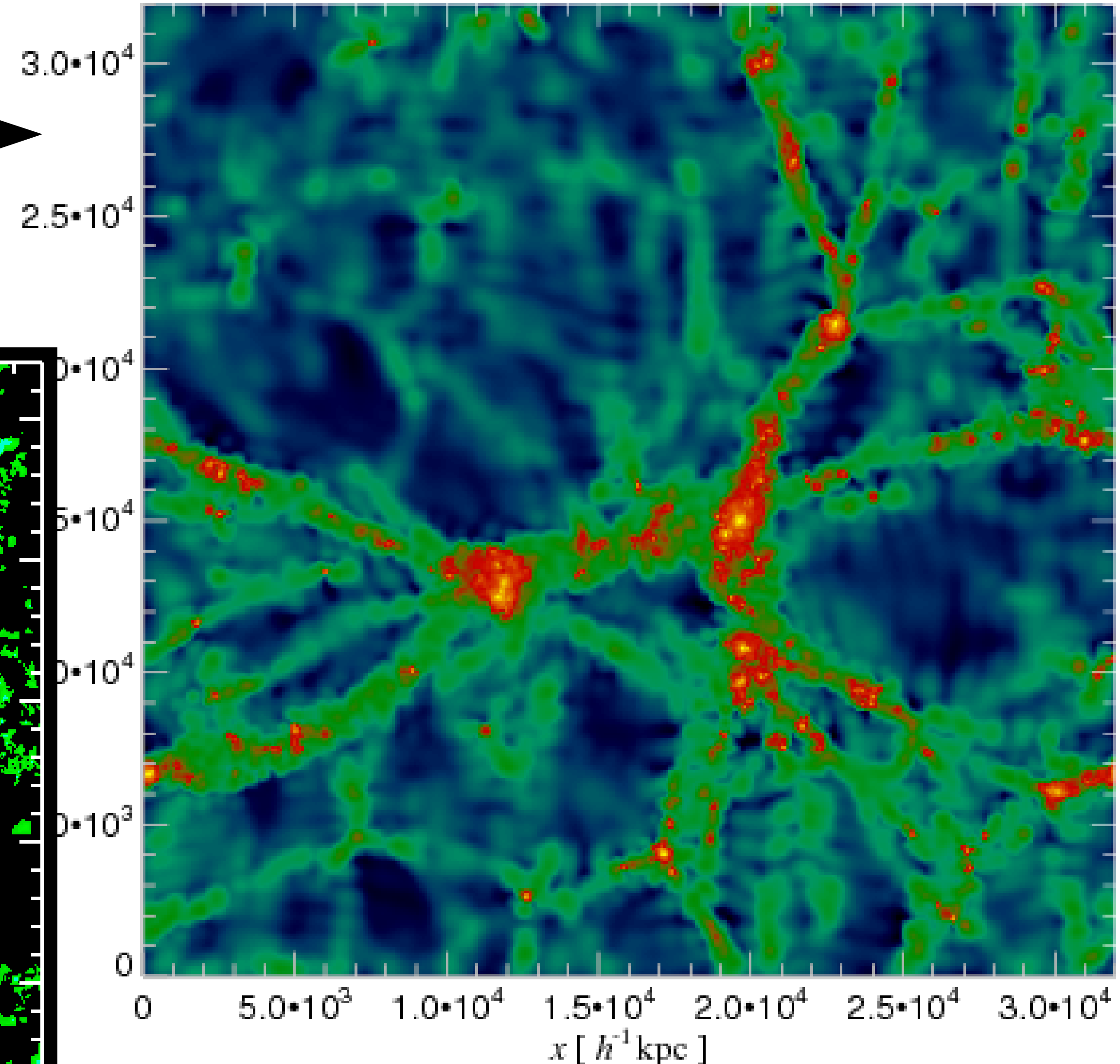


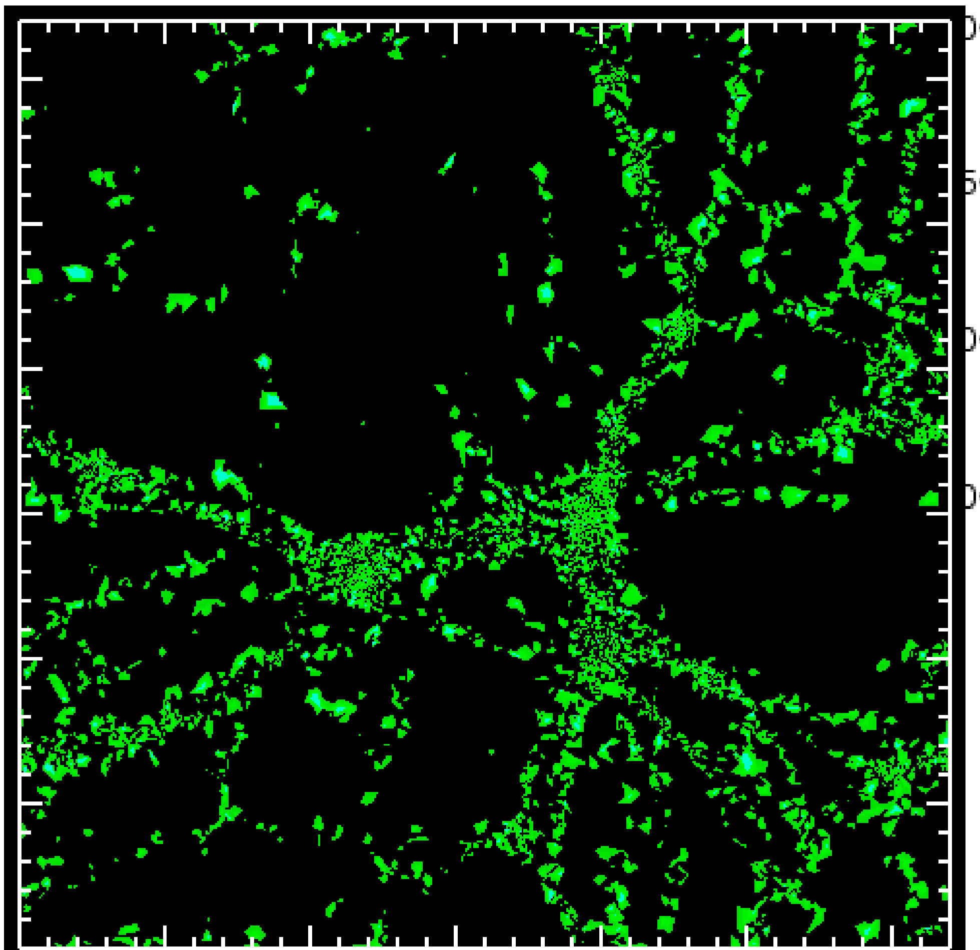
~40 orbits

A first view on stream densities in a tiny (32 Mpc/h)^3 cosmological box

DM distribution →

PRELIMINARY RESULT!





← multistream regions

Conclusions and uncertainties

- Near the Sun most CDM particles should belong to streams with very low space density
- Since $t_{orb} \sim 10^8$yrs at this radius, the Solar System should intersect $\sim 10^5$ different streams → smooth anisotropic *f(v)*
- Individual caustics are generally very weak but there are very many of them → insignificant gravitational effects
- What are the effects of the early phases of hierarchical clustering on the phase-space structure?
- What fraction of particles remain in small bound structures rather than large-scale streams?
- What is the distribution of local DM density in the immediate neighborhood of DM particles? Annihilation in caustics?