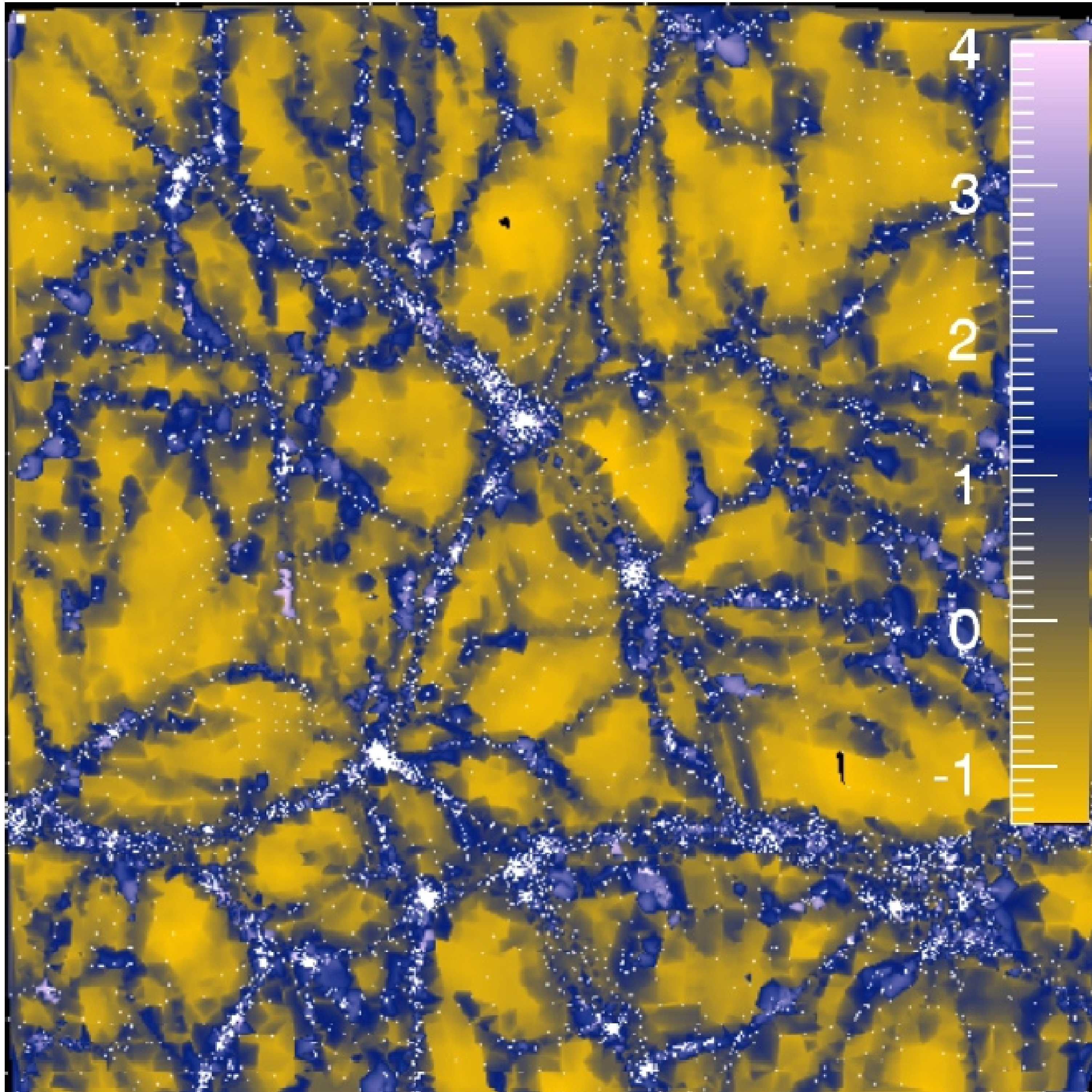
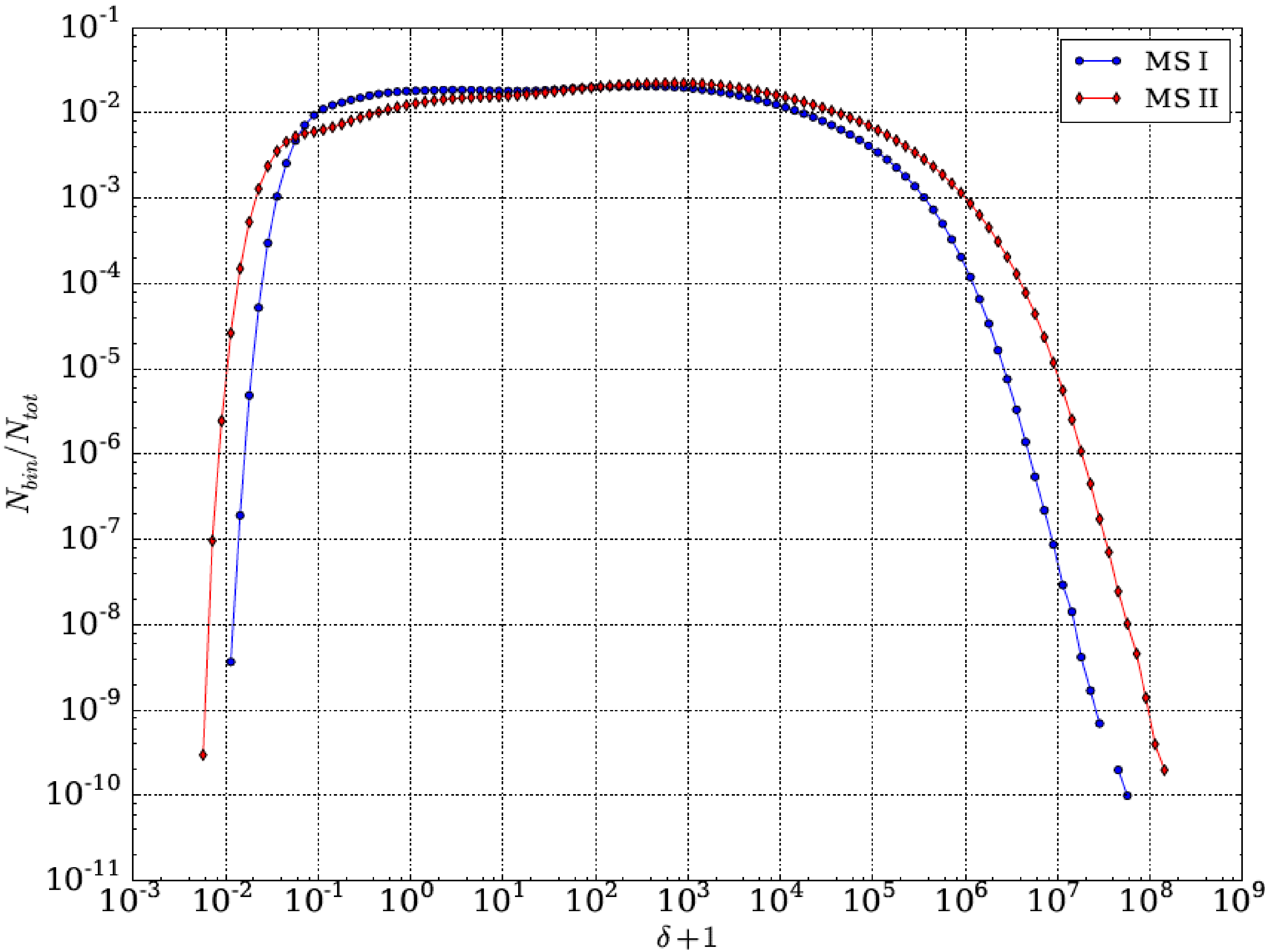
Consider a very large number of one meter cubes placed at random throughout the low-redshift universe. Consider the distribution of the average density within individual cubes according to the standard LCDM/LWDM paradigms.

What are the median, the mean, the standard deviation, and the 90% range of this distribution?
What are the shapes of its high and low density tails?
How is the distribution affected by caustics?
What is the local velocity distribution of DM particles in a cube and how does it depend on the density?
Which aspects differ between LCDM and LWDM?
How can one estimate/calculate these quantities?
What changes if one considers parsec cubes? kiloparsec cubes?

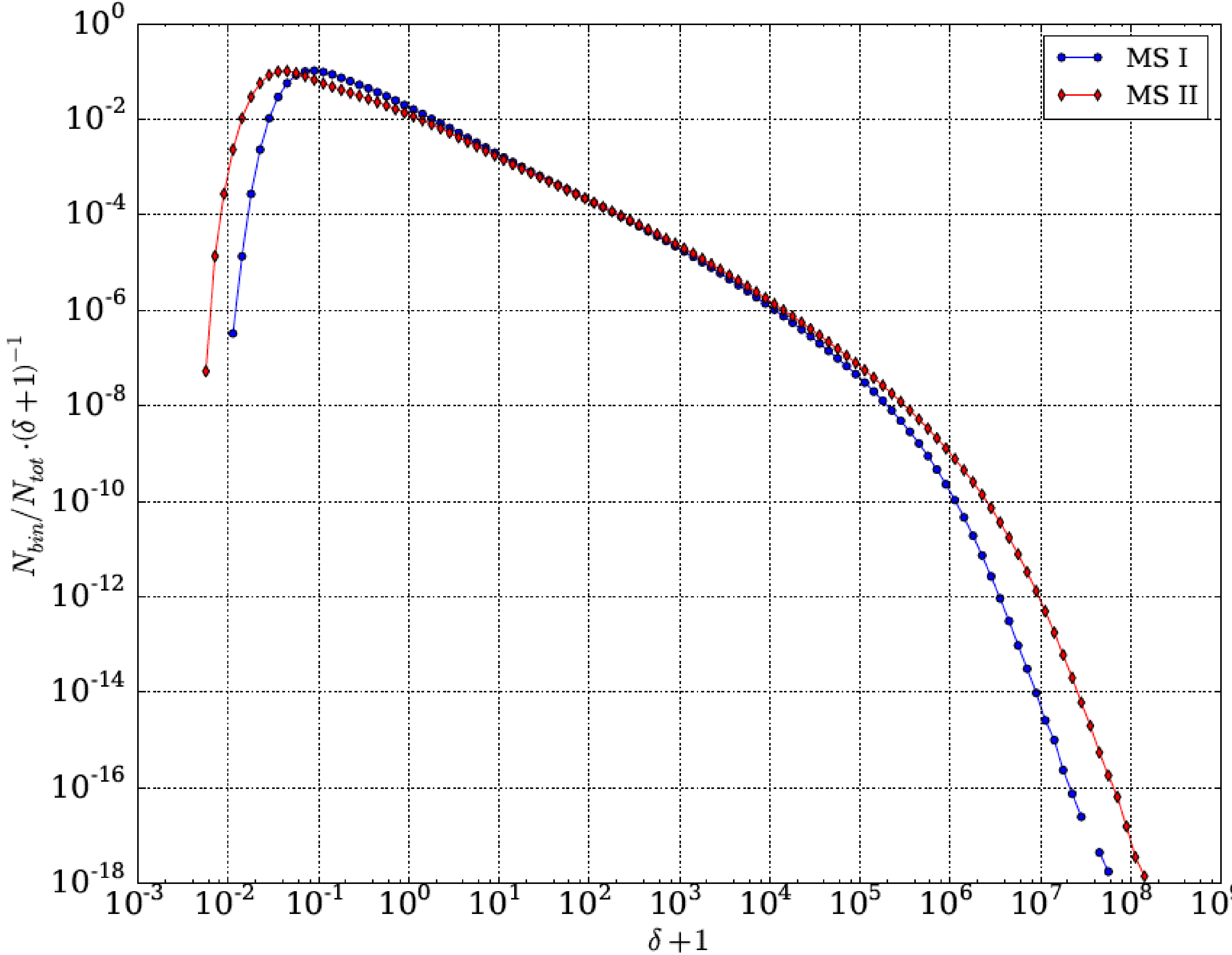
If you understand this, then answer all the questions again for the case where the one meter cubes are distributed over the region within 100 parsecs of the Sun.



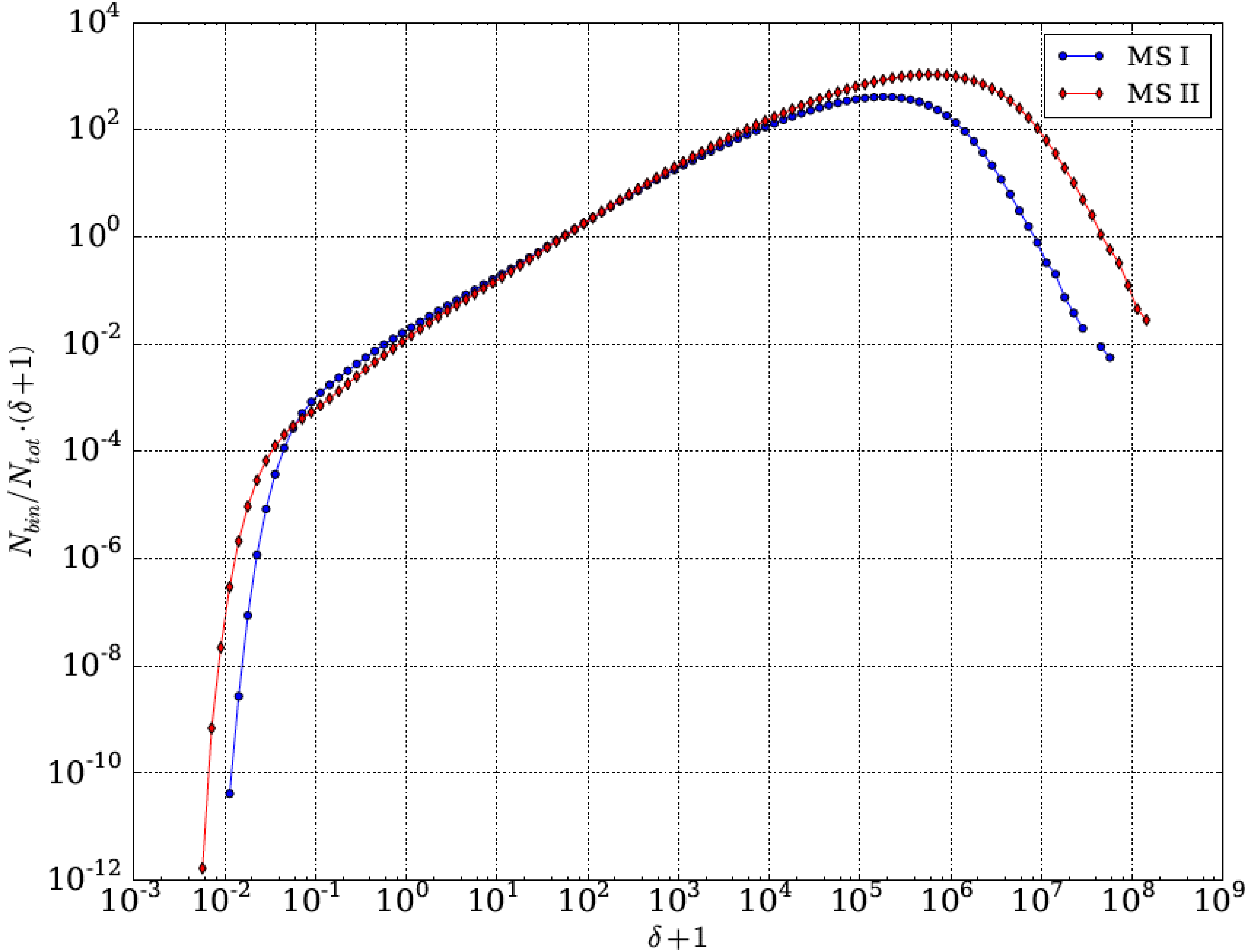
Abel et al 2012



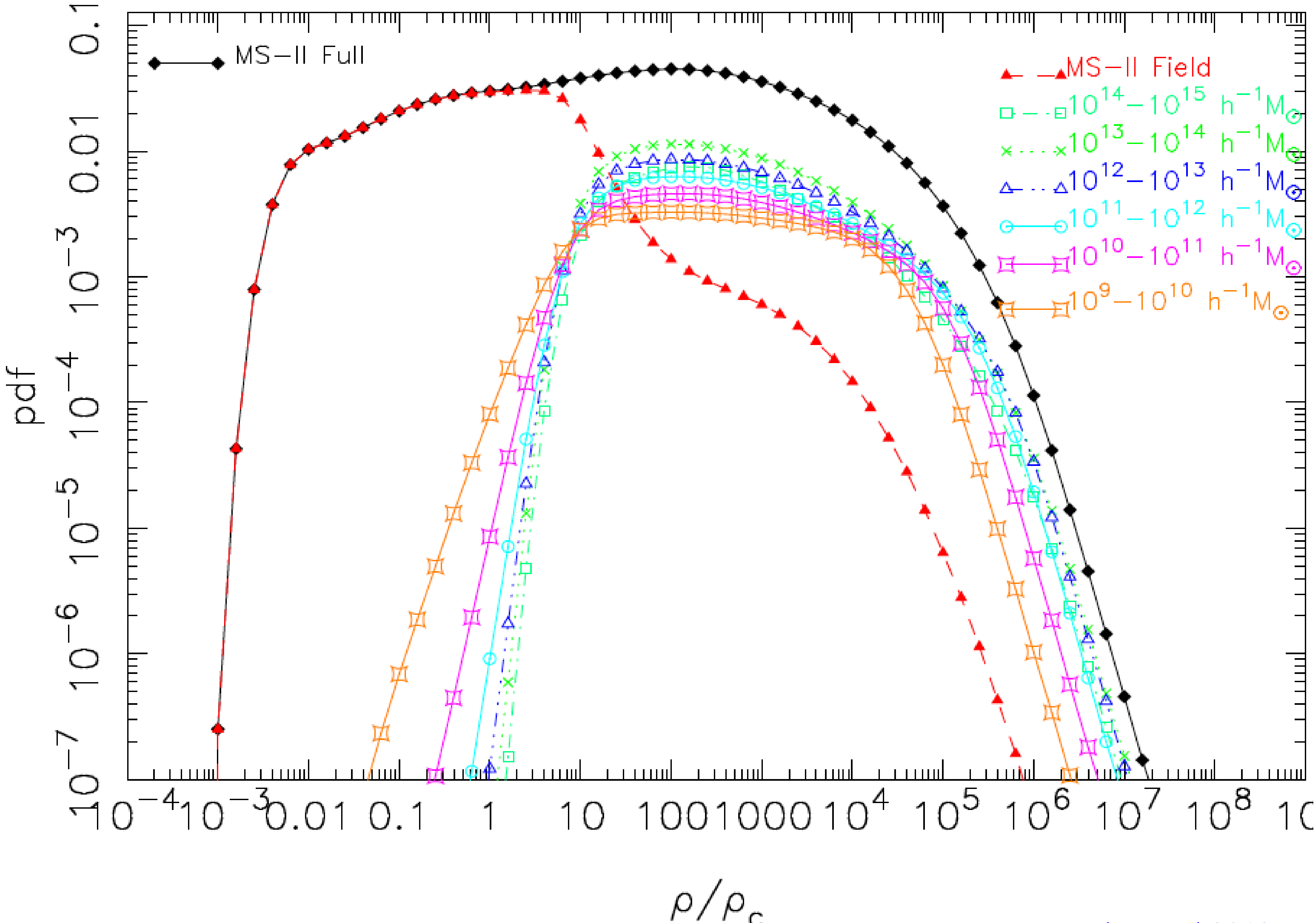
MS I
MS II
N_{bin}/N_{tot}
$\delta+1$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9}



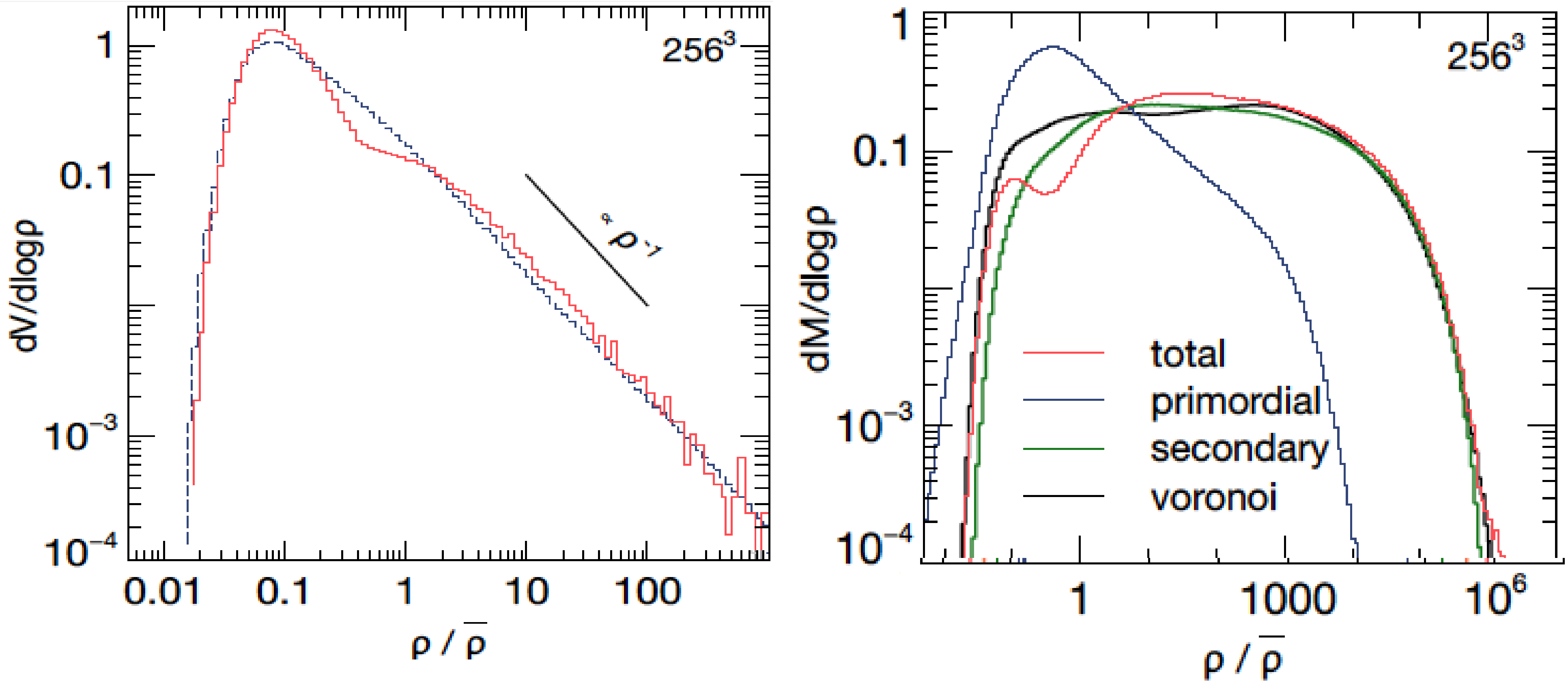
MS I
MS II
$N_{bin}/N_{tot} \cdot (\delta+1)^{-1}$
$\delta+1$
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}



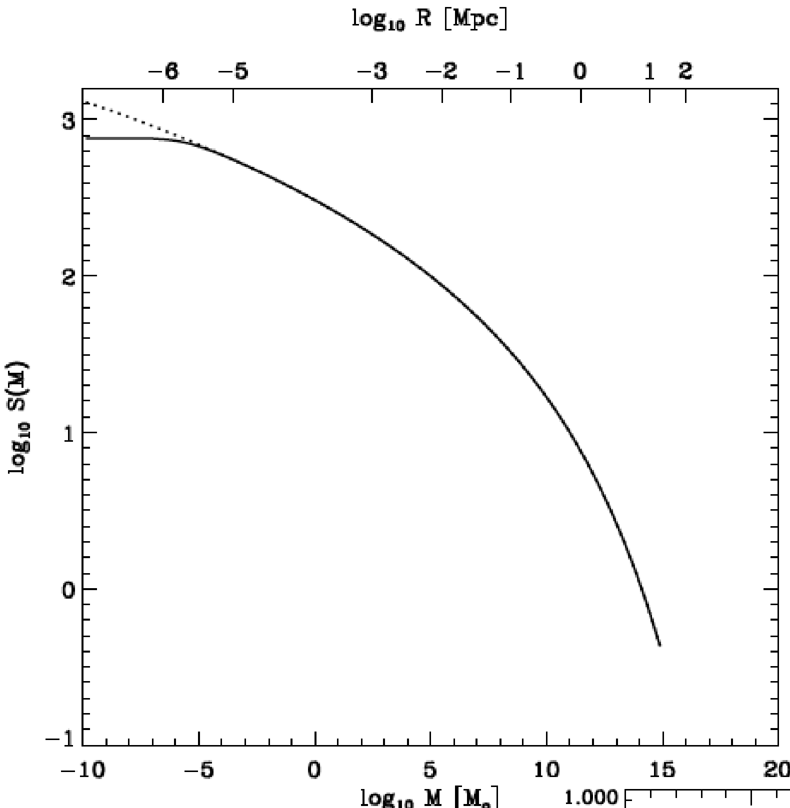
MS I
MS II
$N_{bin}/N_{tot} \cdot (\delta+1)$
$\delta+1$
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}

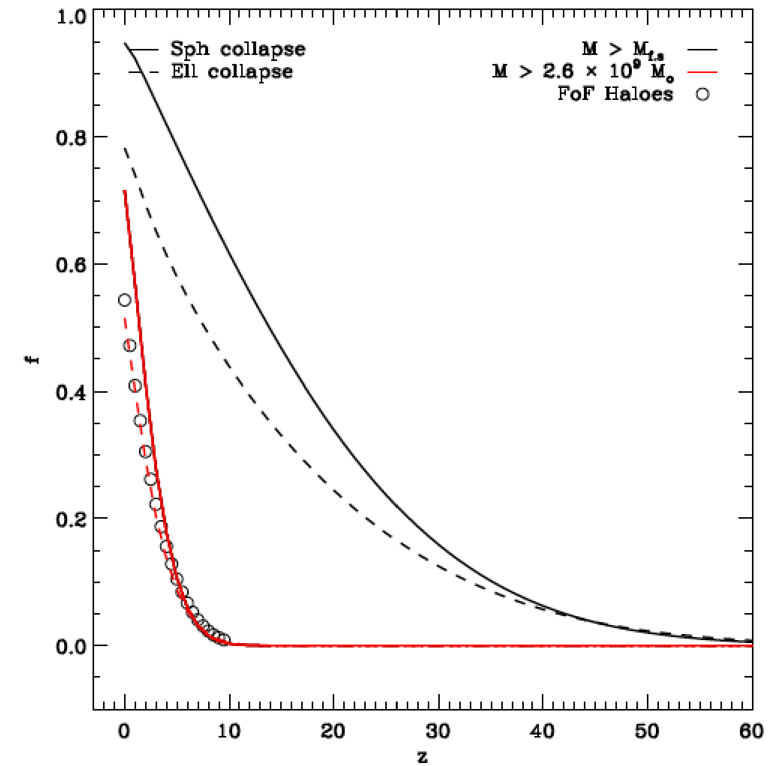


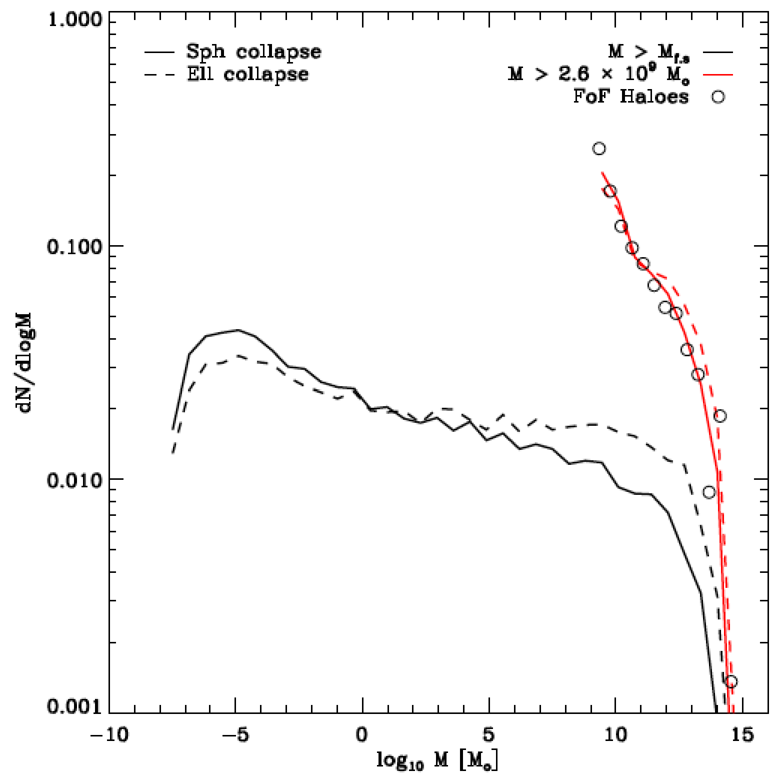
Pandey et al 2013



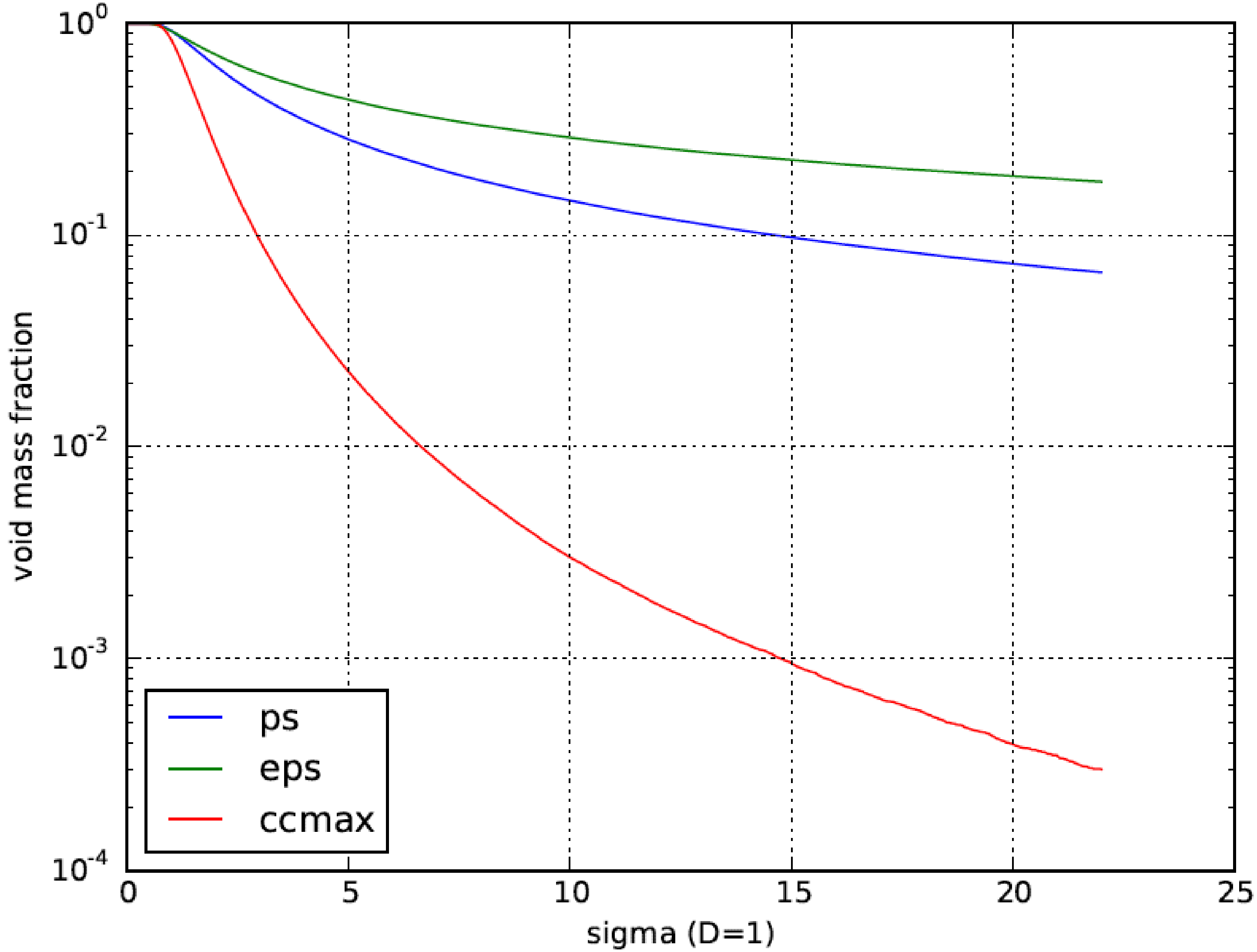
Abel et al 2012





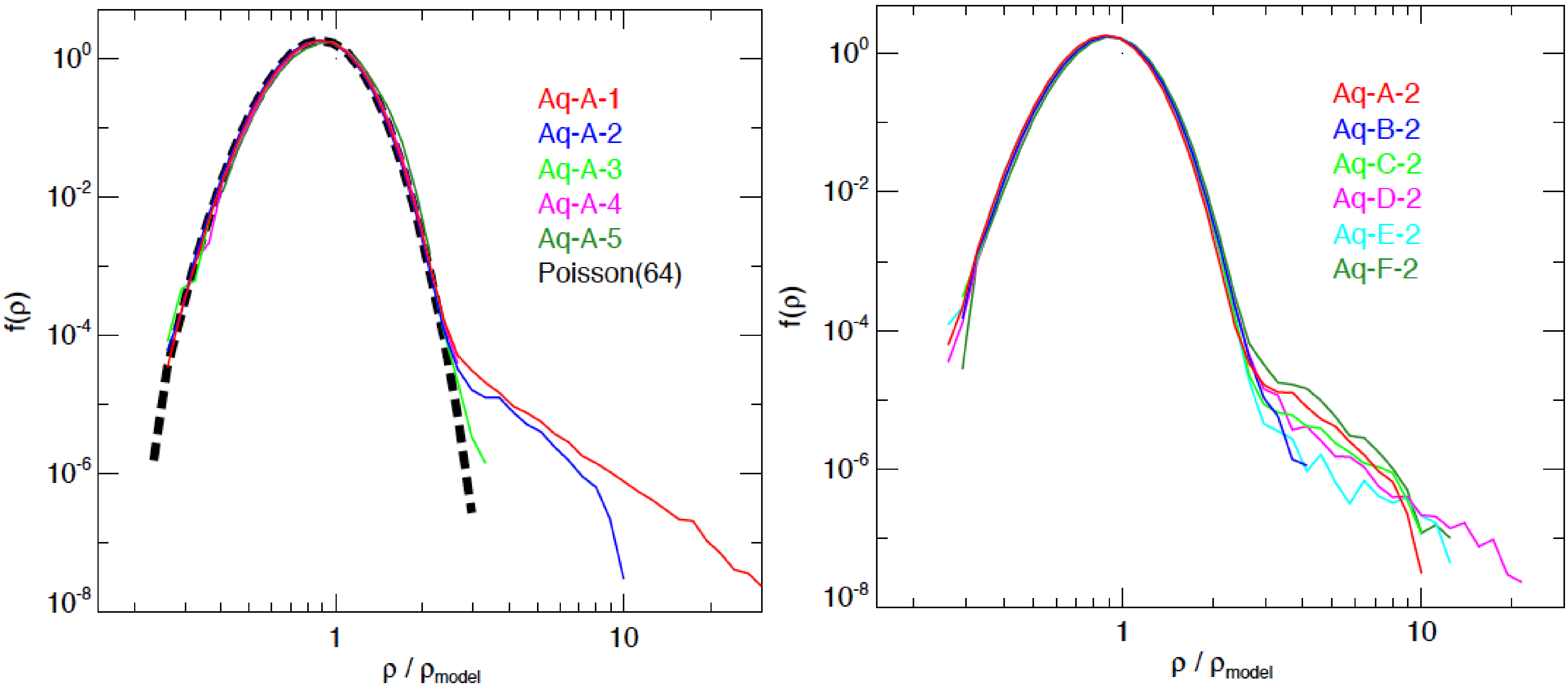


Angulo & White 2010



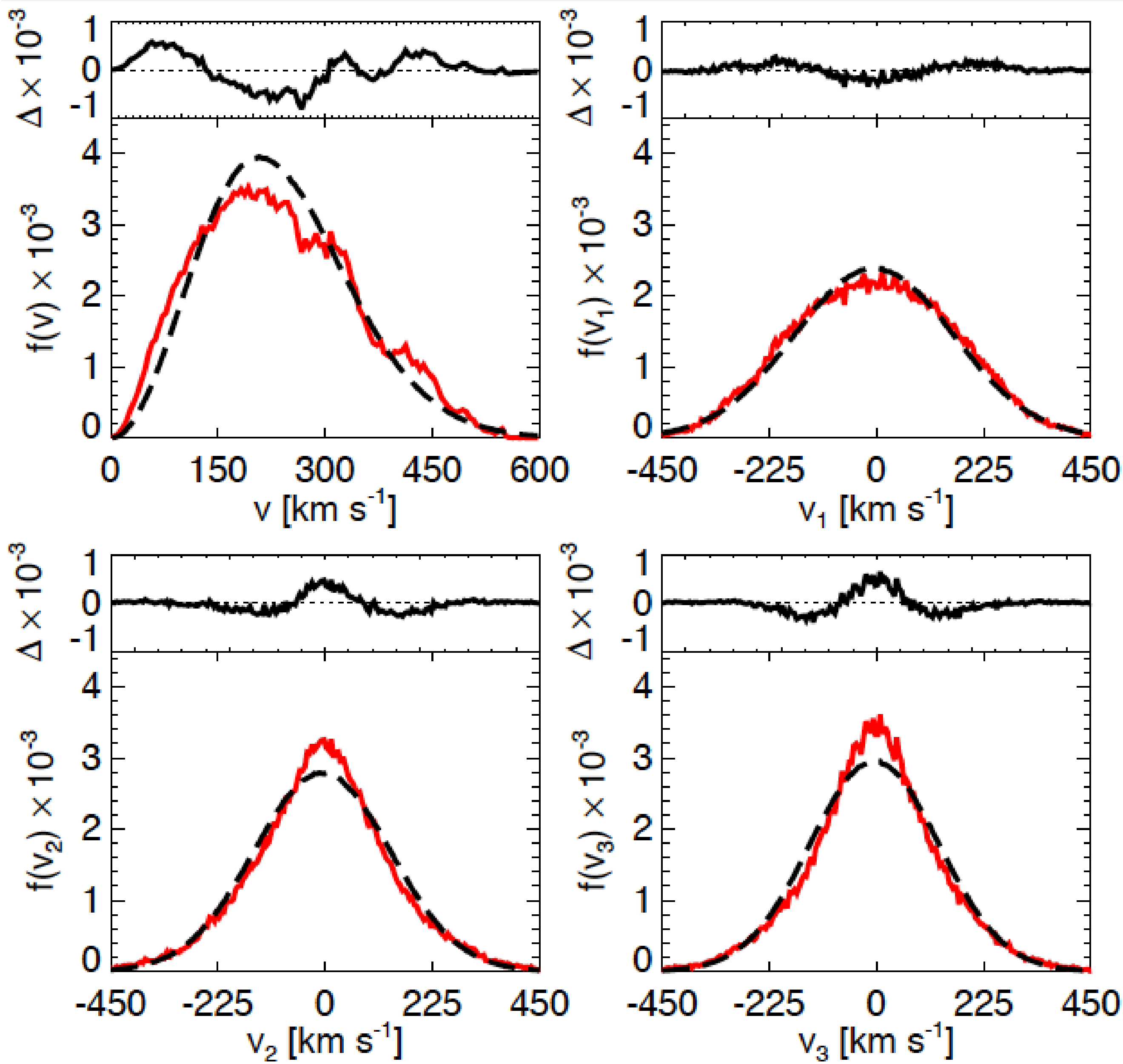
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
void mass fraction
0
5
10
15
20
25
sigma (D=1)
ps
eps
ccmax

Vogelsberger et al 2009

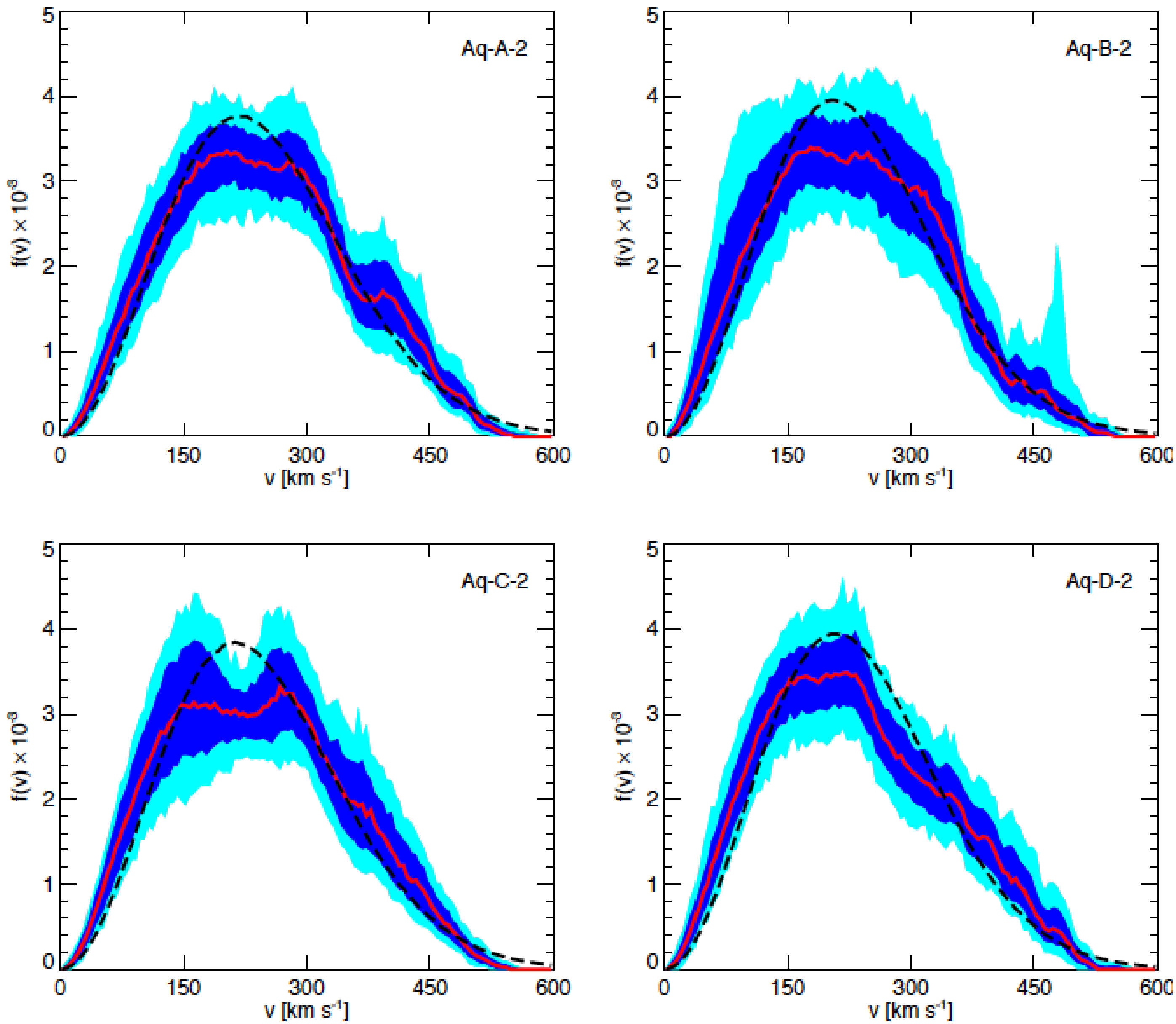


Scatter in local density around a power-law, ellipsoidal model is 2.2% over the radial range 6 to 12 kpc

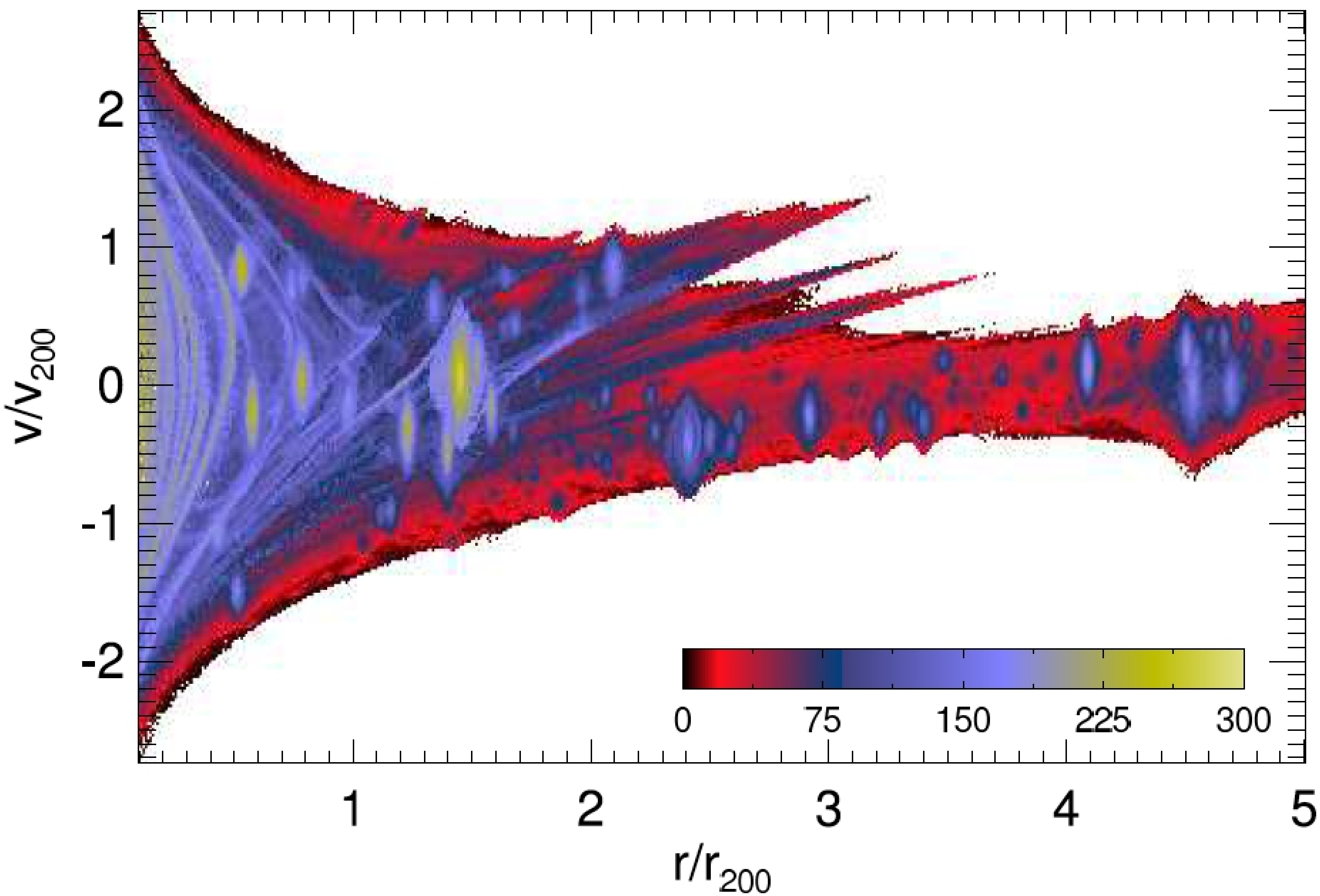
Velocity distributions in a 2kpc cube at R=8kpc



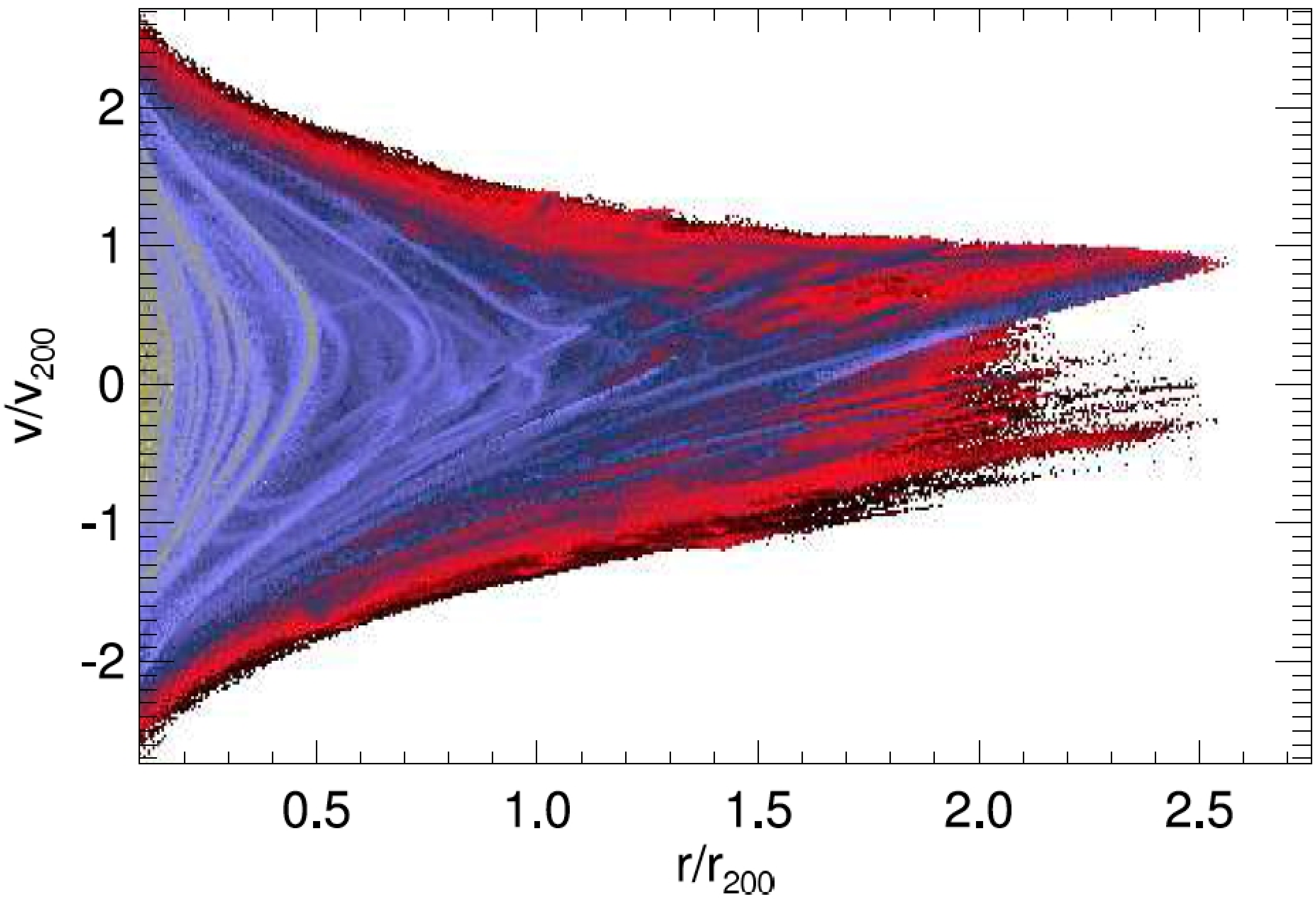
Vogelsberger et al 2009



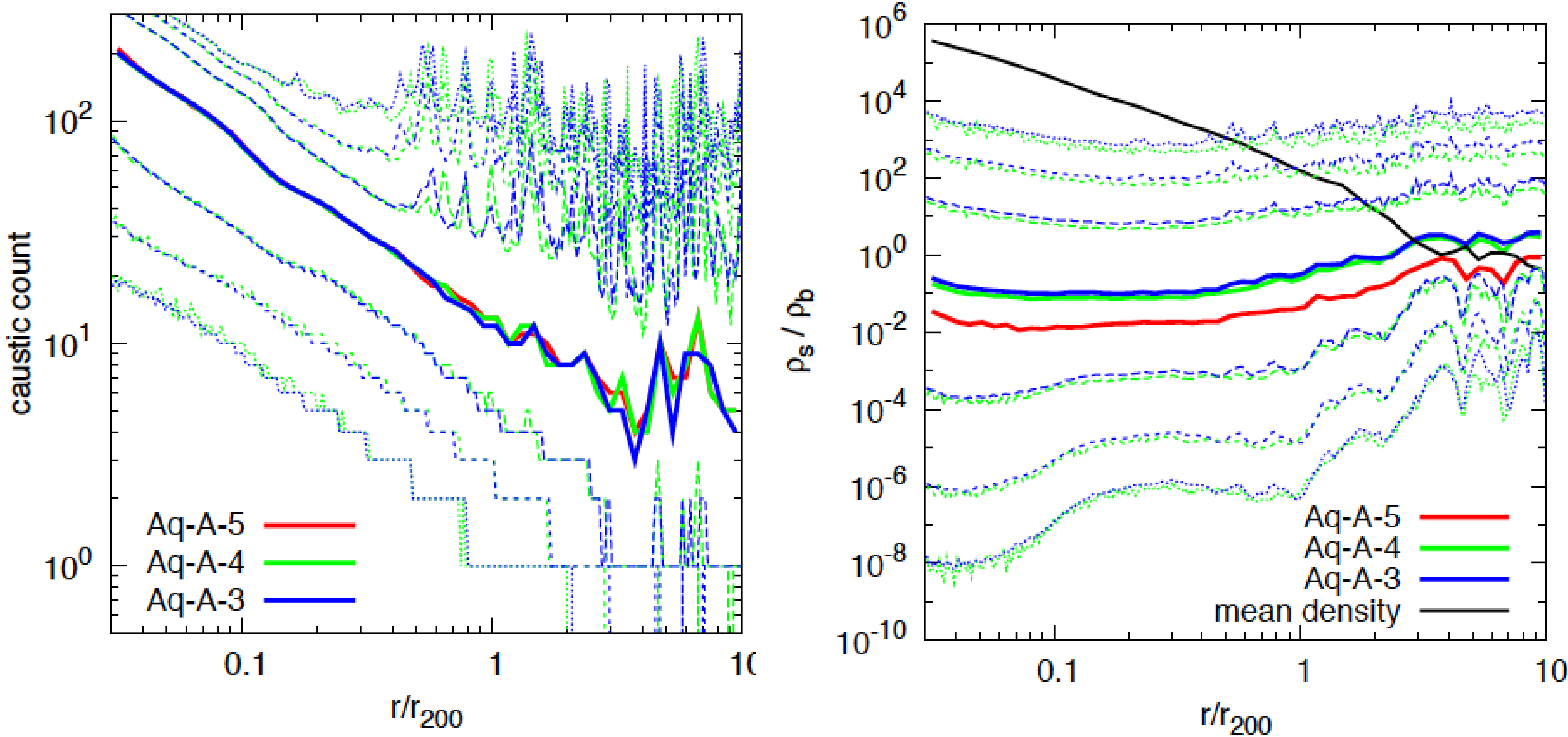
Vogelsberger et al 2009



2
1
0
-1
-2
v/v_{200}
1
2
3
4
5
r/r_{200}
0
75
150
225
300



2
1
0
-1
-2
v/v_{200}
0.5
1.0
1.5
2.0
2.5
r/r_{200}


caustic count
10^2
10^1
10^0
0.1
1
10
r/r200
Aq-A-5
Aq-A-4
Aq-A-3
ρs / ρb
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
0.1
1
10
r/r200
Aq-A-5
Aq-A-4
Aq-A-3
mean density

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot\delta X\,;\ T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = I$$

Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution

No symmetry or stationarity assumptions are required

det(D) = 1 at all times by Liouville's theorem

For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.