

IAU Symposium #225
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Numerical Simulations: the Nonlinear Mass and Galaxy Distributions

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Simulation Input to Lensing Science

- Precision Large-Scale Structure: Cosmic Shear
 - $P(k, z)$, $N_{\text{halo}}(M, z)$, $S_3(\kappa, z)$, $S_4(\kappa, z)$, $w(z)$...
- Halo Core Structure and Ellipticity: Arc Abundances
 - Cross-sections for tangential/radial arcs
 - Implications for nature of DM, assembly history of galaxies
- Substructure Abundances: Flux Ratios in multiply imaged QSO's
 - Detection of 'invisible' subhaloes
 - Test of CDM power spectrum and nature of DM
- Relation of Halo to Galaxy Properties: Galaxy-Galaxy Lensing
 - Shapes of galaxy halos
 - Luminosity/stellar mass/halo mass relations
 - Halo truncation in clusters
 - Evolution of bias -- tomography to obtain $w(z)$

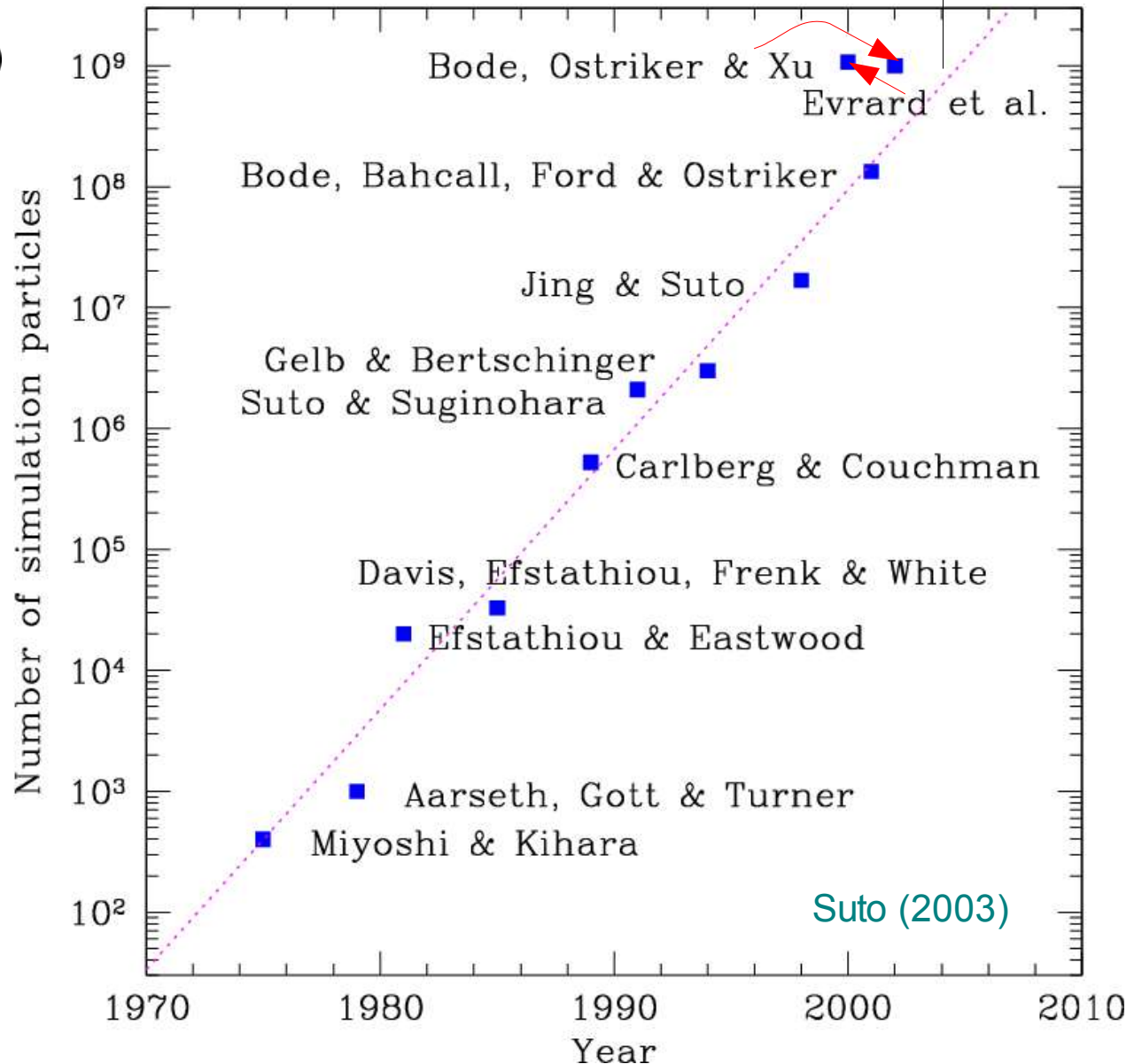
Requirements for a Precision Simulation

- Large volume to reduce cosmic variance
- Small particle mass to suppress shot-noise/2-body effects
- Proper representation of Λ CDM initial conditions
- Proper representation of growing mode velocity field
- Accurate forces in near uniform and highly non-uniform regimes
- Accurate time integration, even at high density

Moore's Law for Cosmological N-body Simulations

$$N = 400 \times 10^{0.215(\text{Year}-1975)}$$

- Computers double their speed every 18 months
- A naive N-body force calculation needs N^2 op's
- Simulations double their size every 17 months
- Thus $N = 10^{10}$ should be reached in 2010
- But it has already been completed...



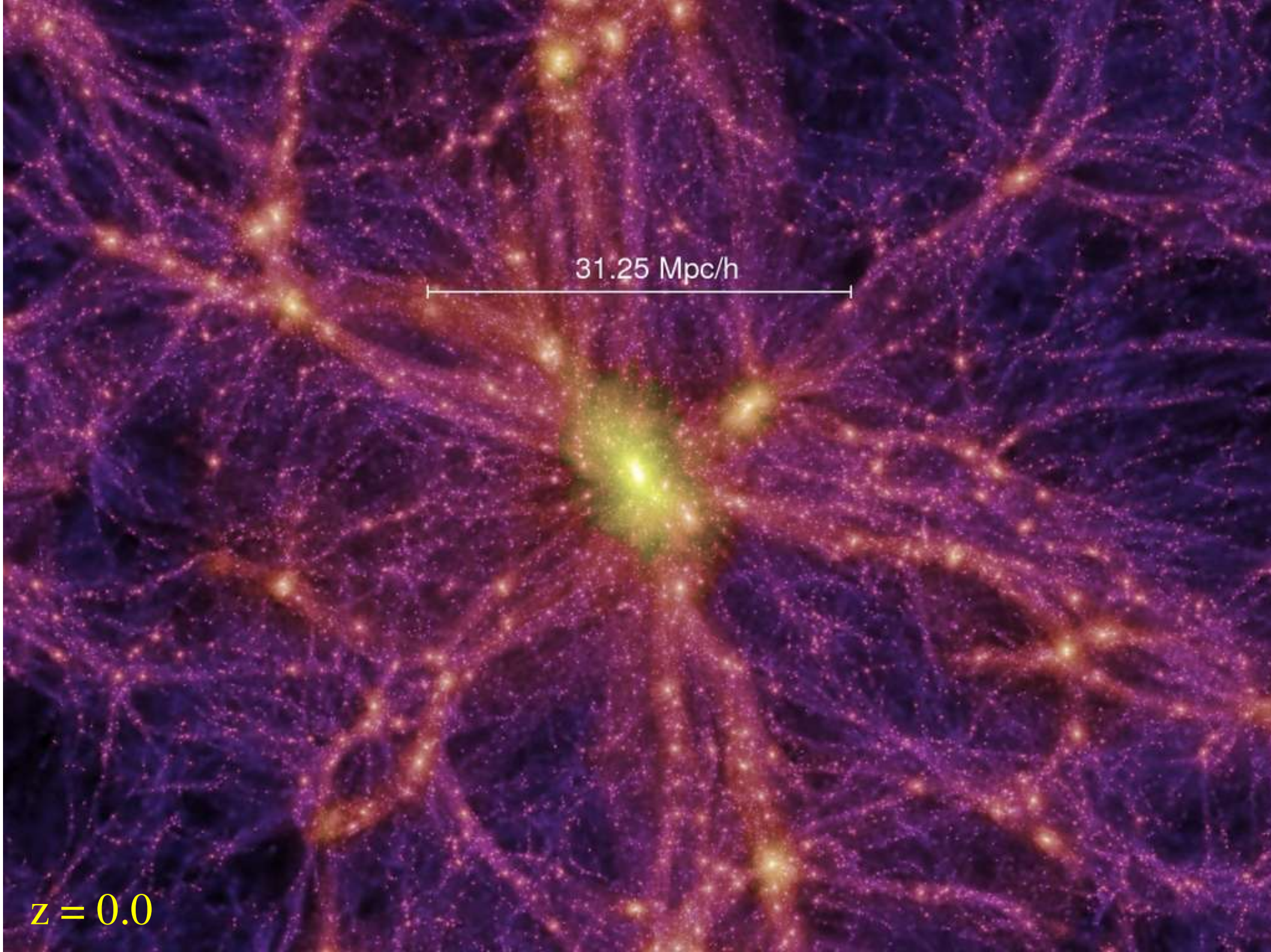
Millennium Run Statistics

Volker Springel and the Virgo Consortium

- Particle number: $N = 2160^3 = 10,077,696,000 \approx 10^{10}$
- Box size: $L = 500 \text{ Mpc}/h$, Softening: $\epsilon = 5 \text{ kpc}/h \longrightarrow L/\epsilon = 10^5$
- Initial redshift: $z_{\text{init}} = 127$
- Cosmology: $\Omega_{\text{tot}}=1$, $\Omega_{\text{m}}=0.25$, $\Omega_{\text{b}}=0.045$, $h=0.73$, $n=1$, $\sigma_8=0.9$
- 343,000 processor-hrs on an IBM Regatta (~ 1 machine-month)
- Full raw and reduced data stored at 64 redshifts

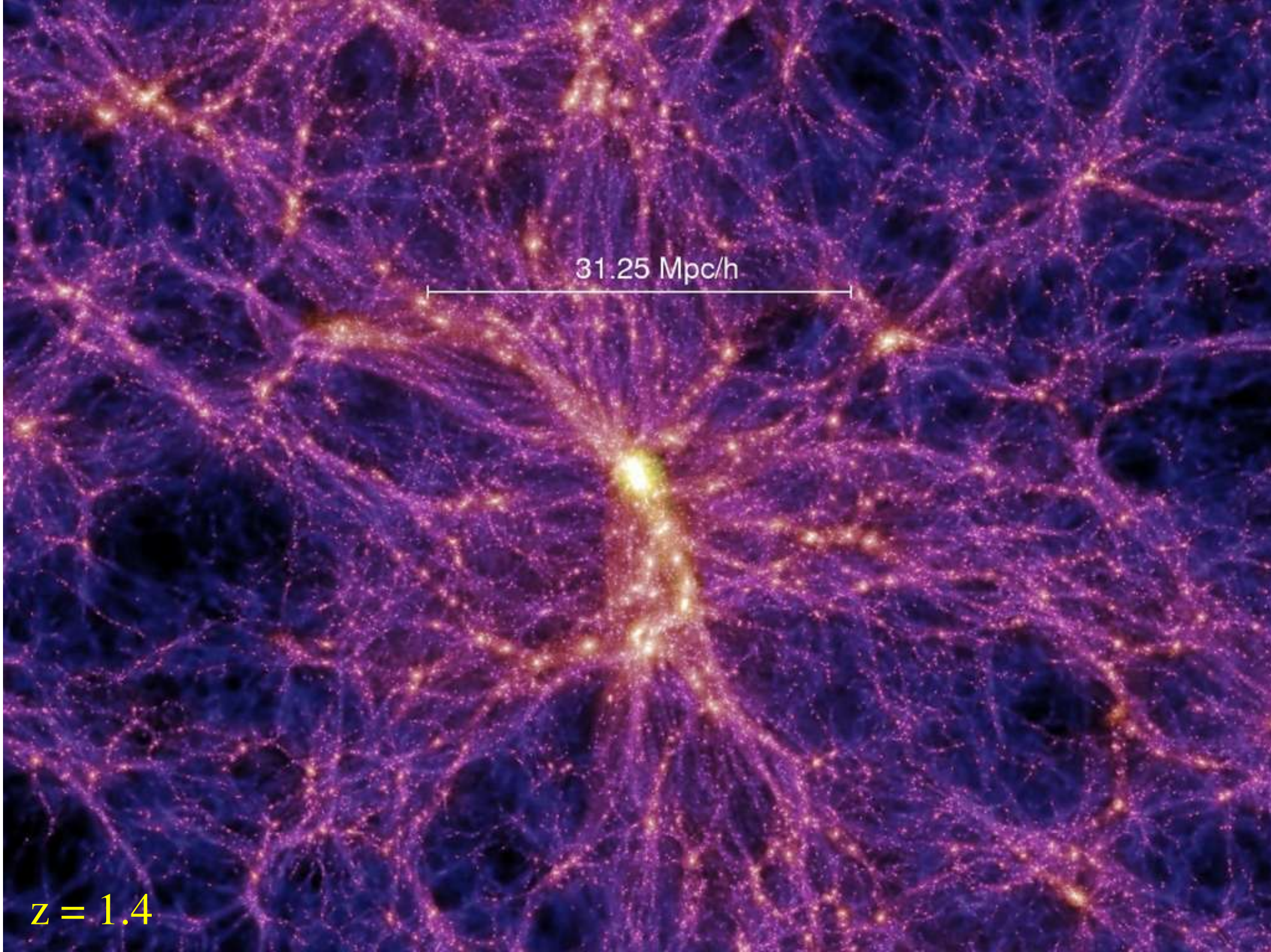
 **27 Tbytes of stored data**

Archive for a Theoretical Virtual Observatory



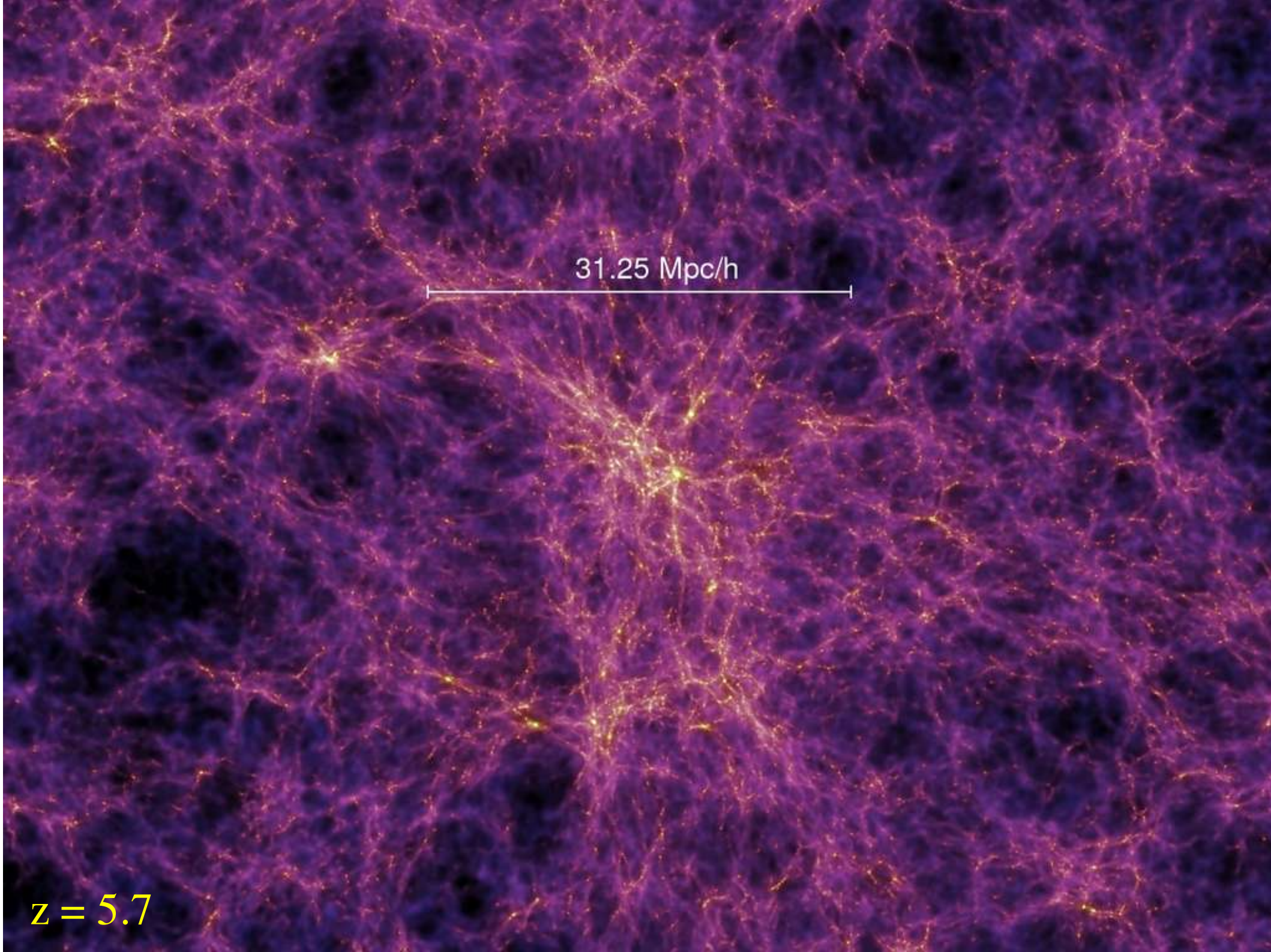
31.25 Mpc/h

$z = 0.0$



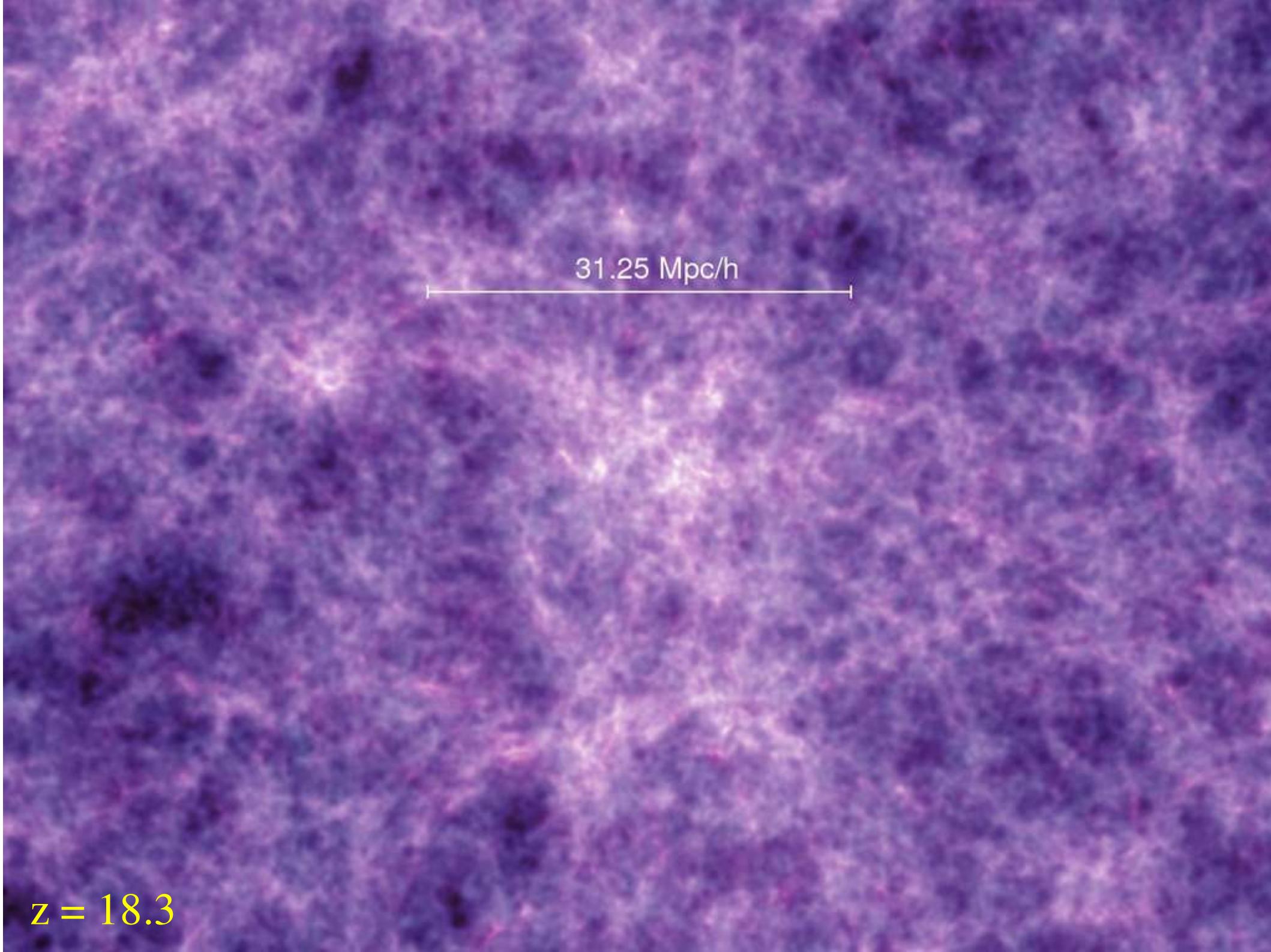
31.25 Mpc/h

$z = 1.4$



31.25 Mpc/h

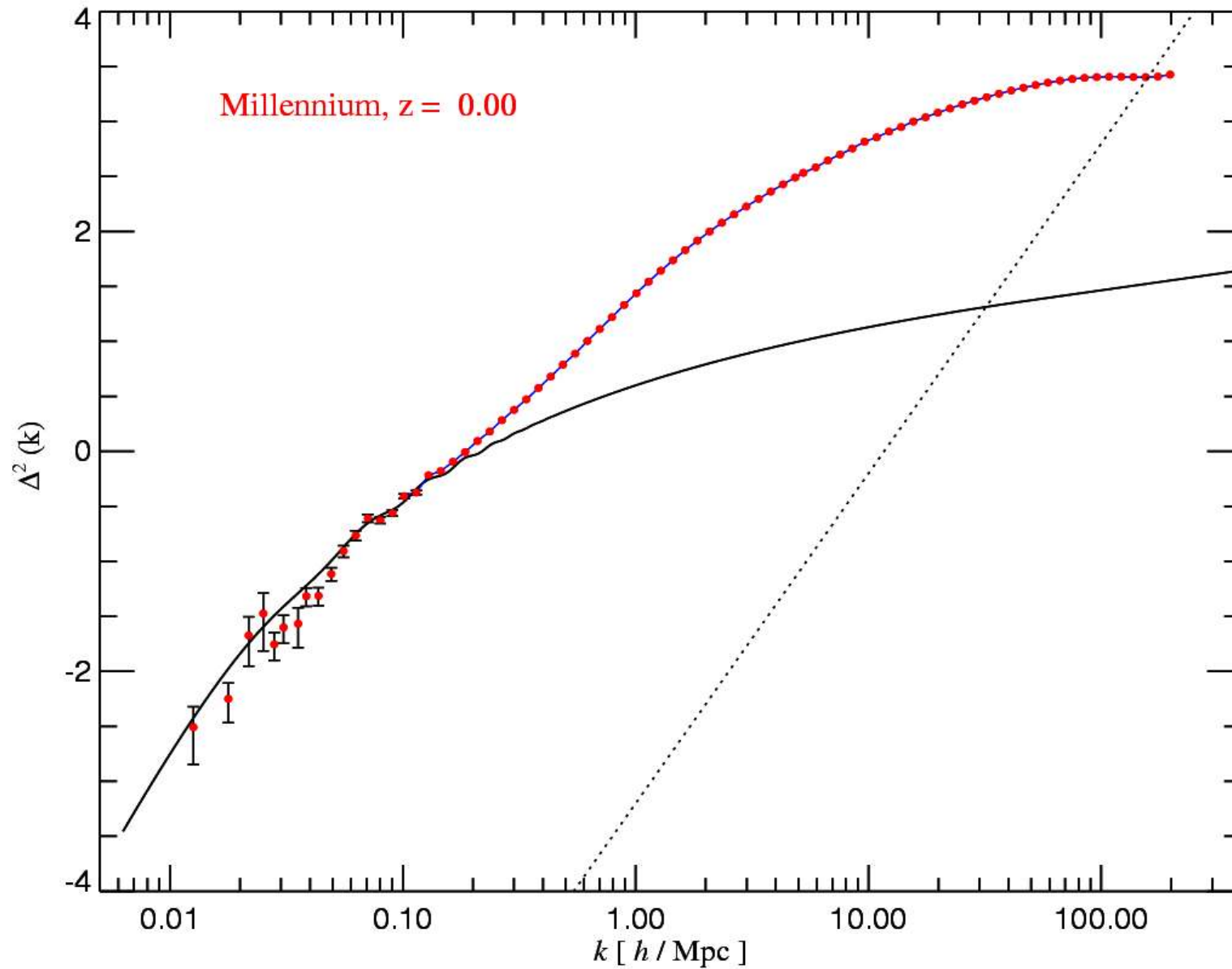
$z = 5.7$



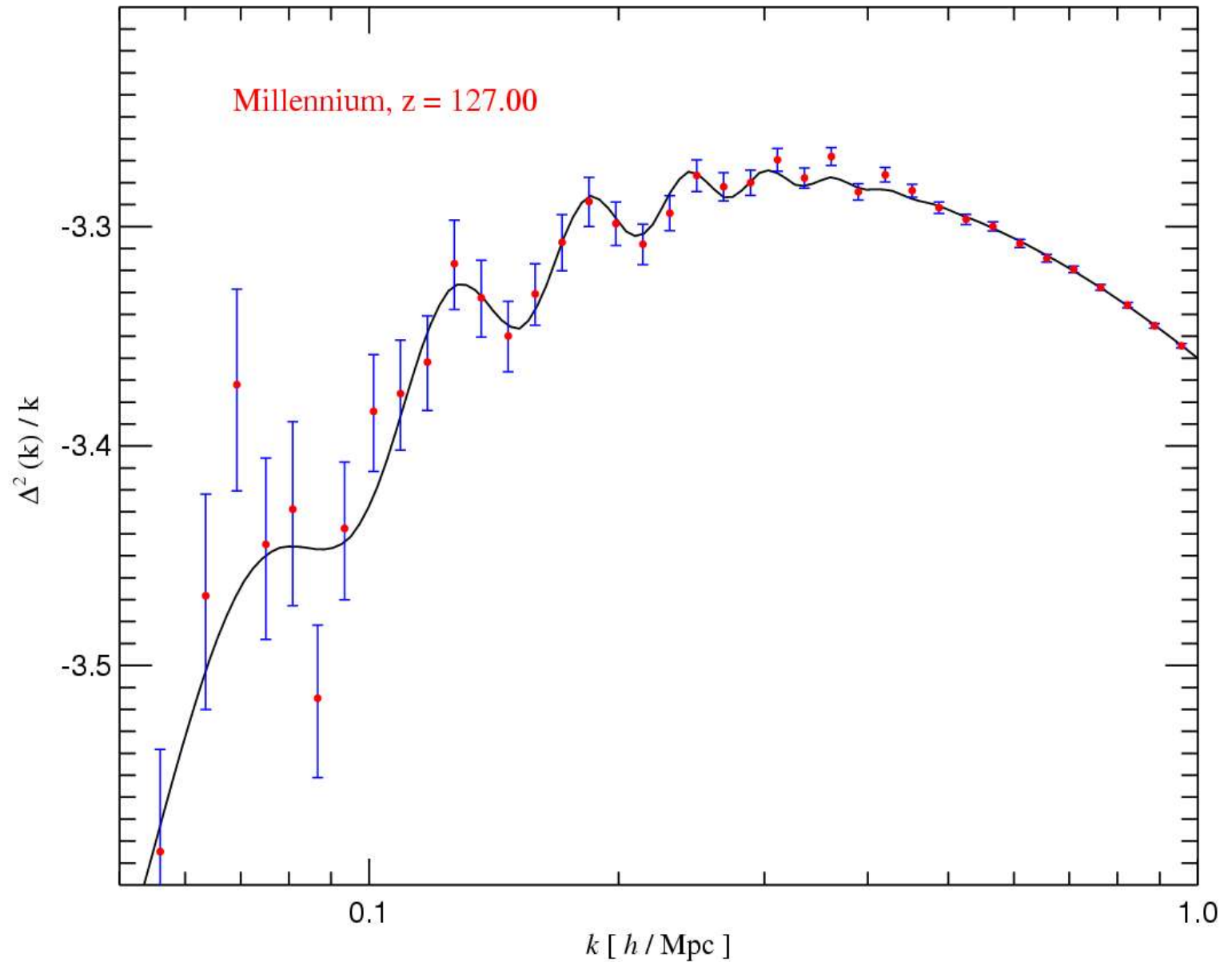
31.25 Mpc/h

$z = 18.3$

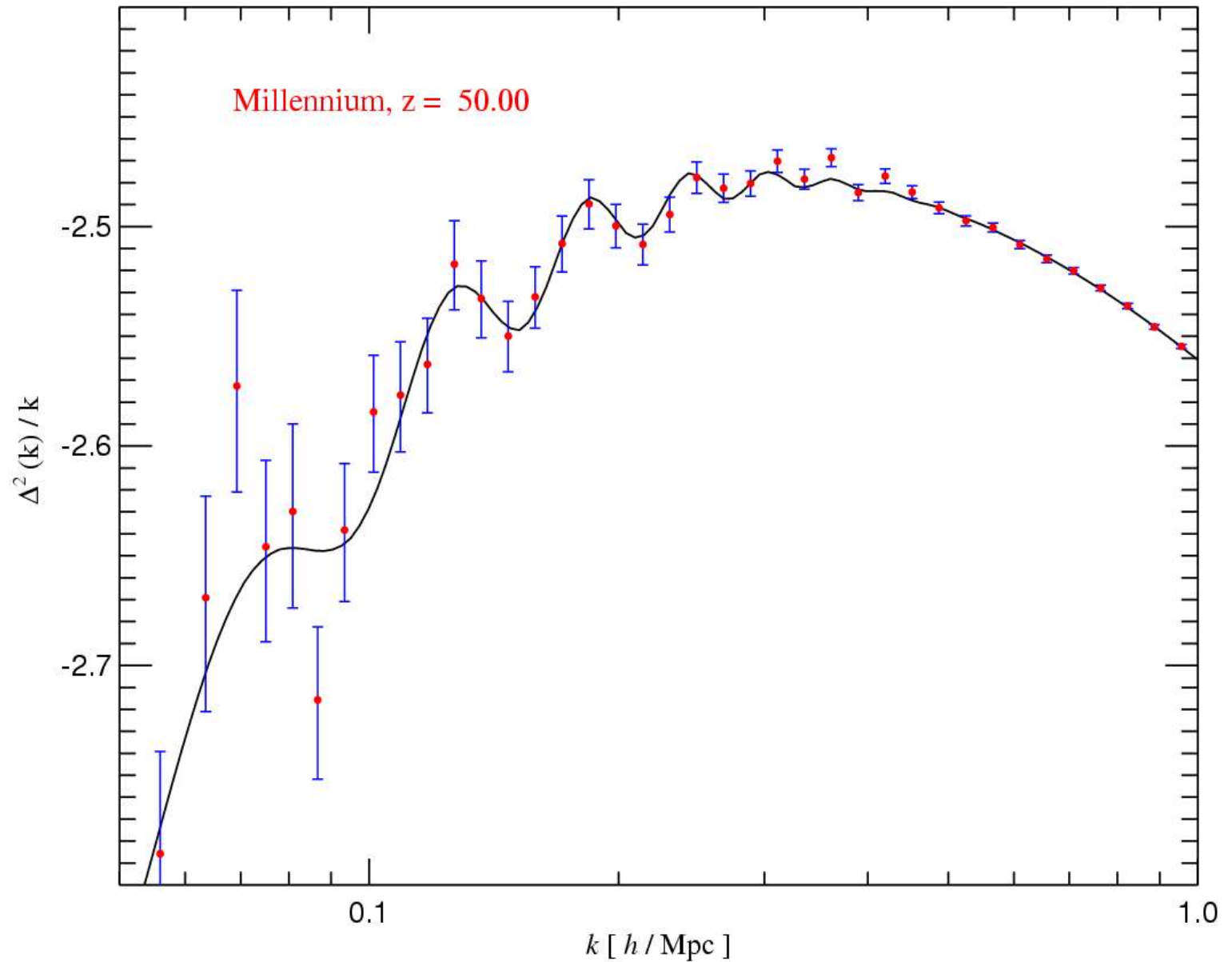
Nonlinear Mass Power spectrum



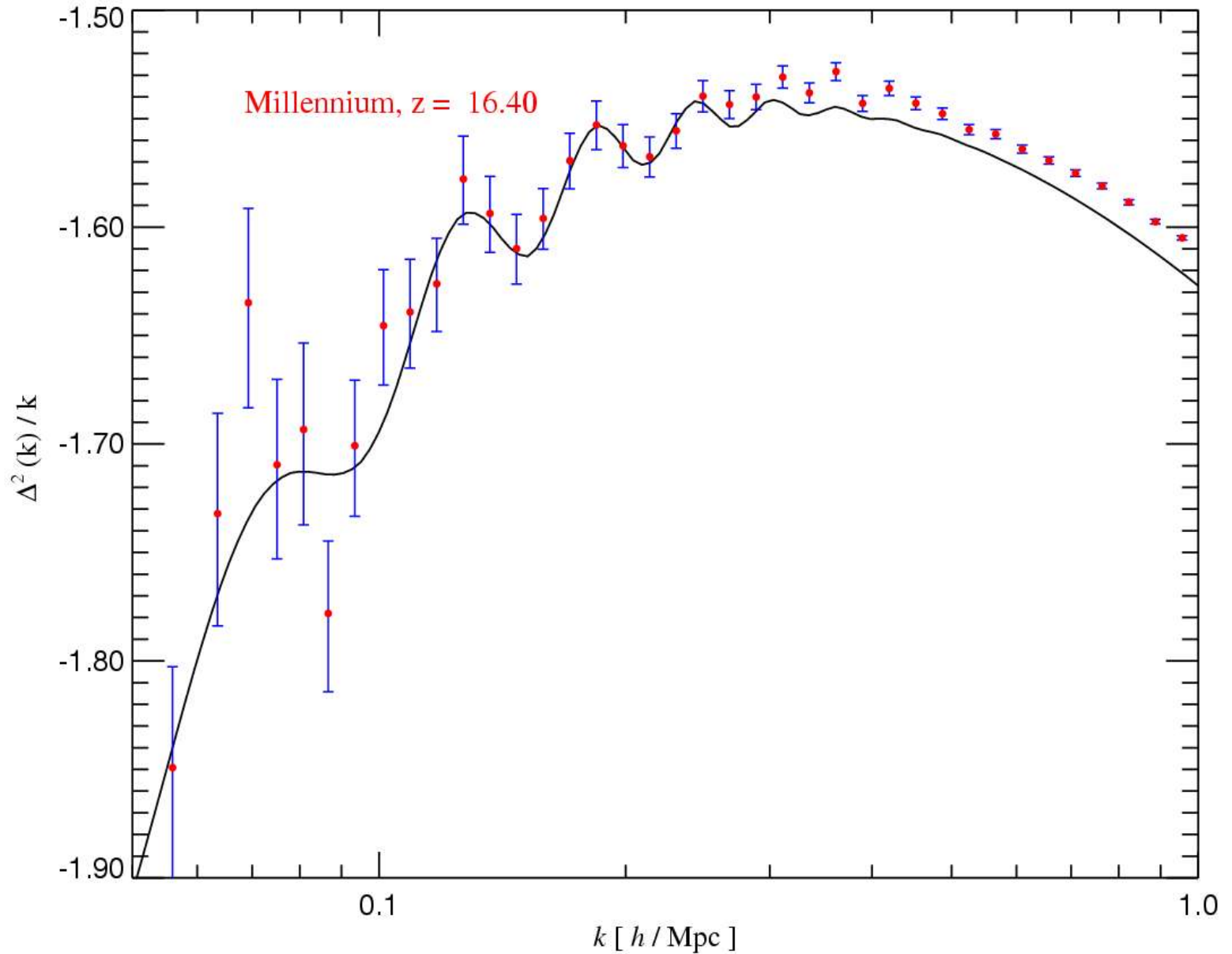
The Evolution of the Baryonic 'Wiggles'



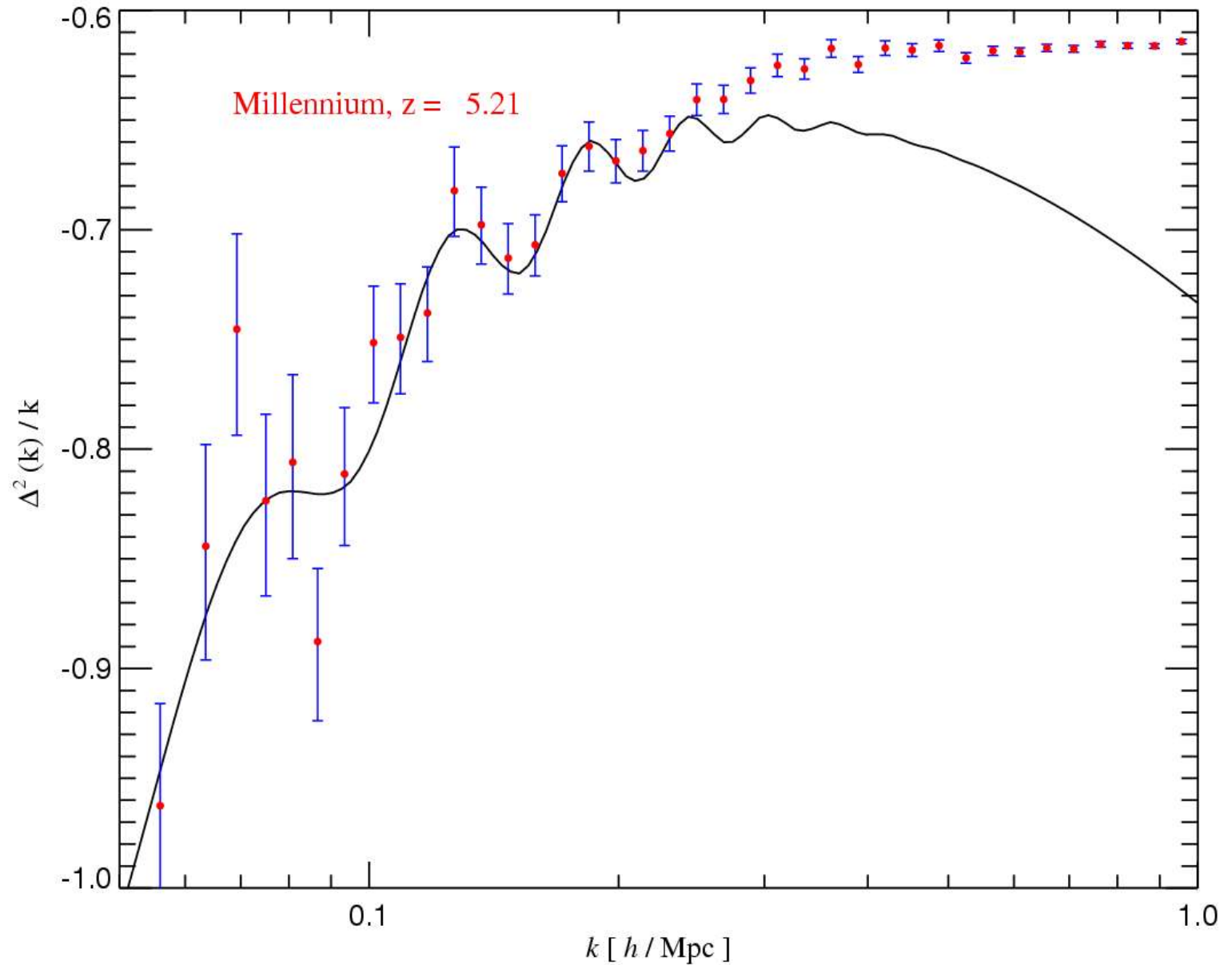
The Evolution of the Baryonic 'Wiggles'



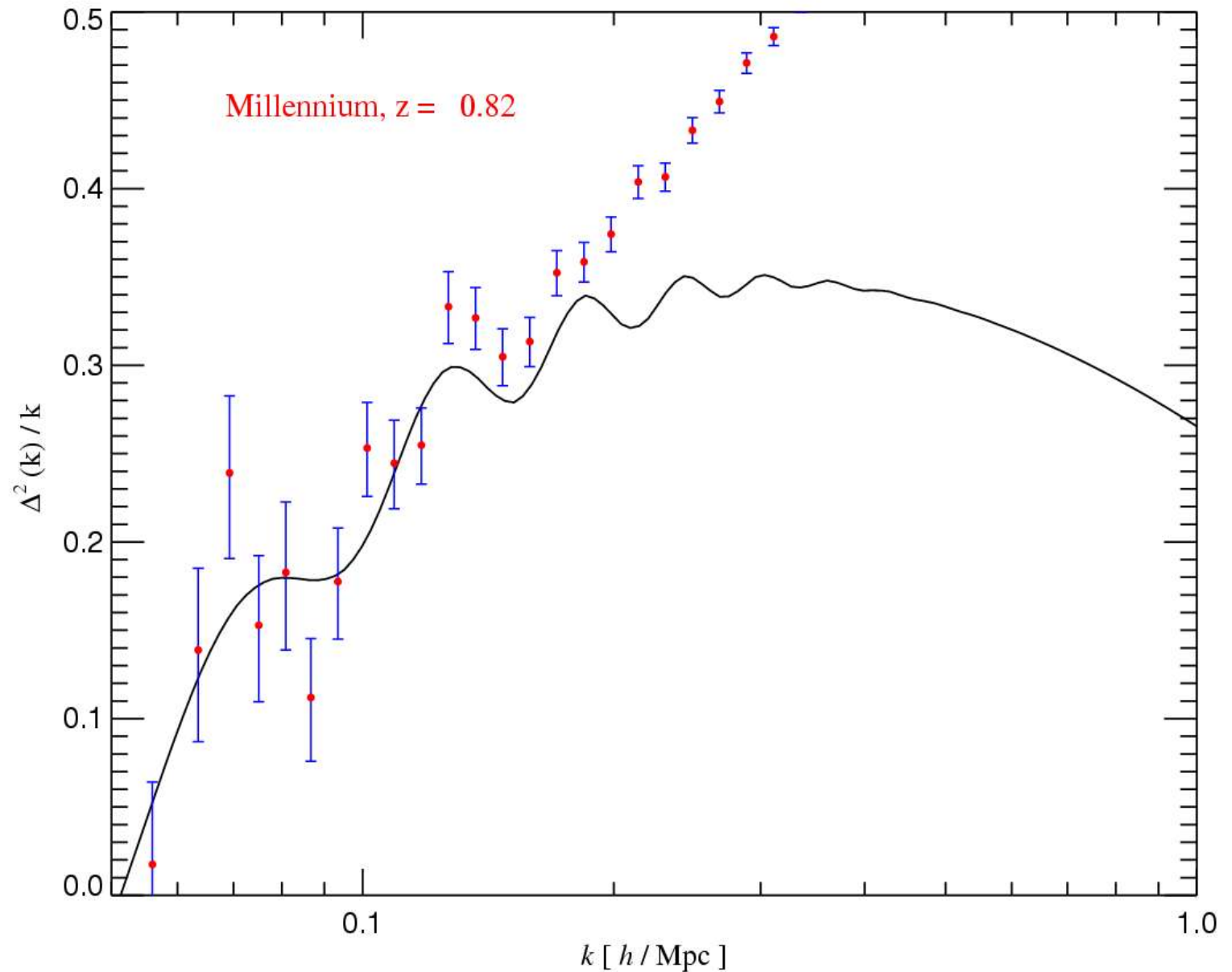
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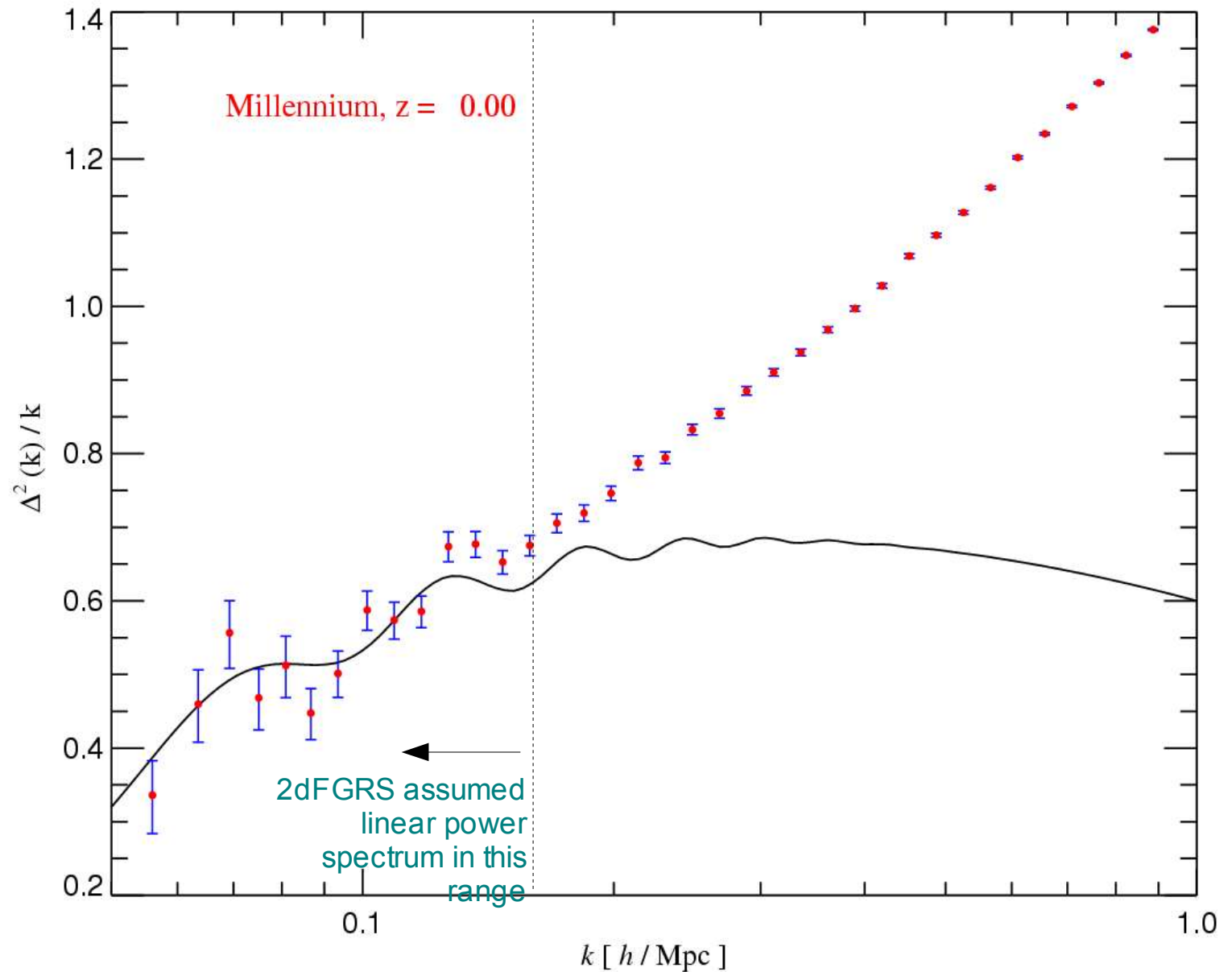
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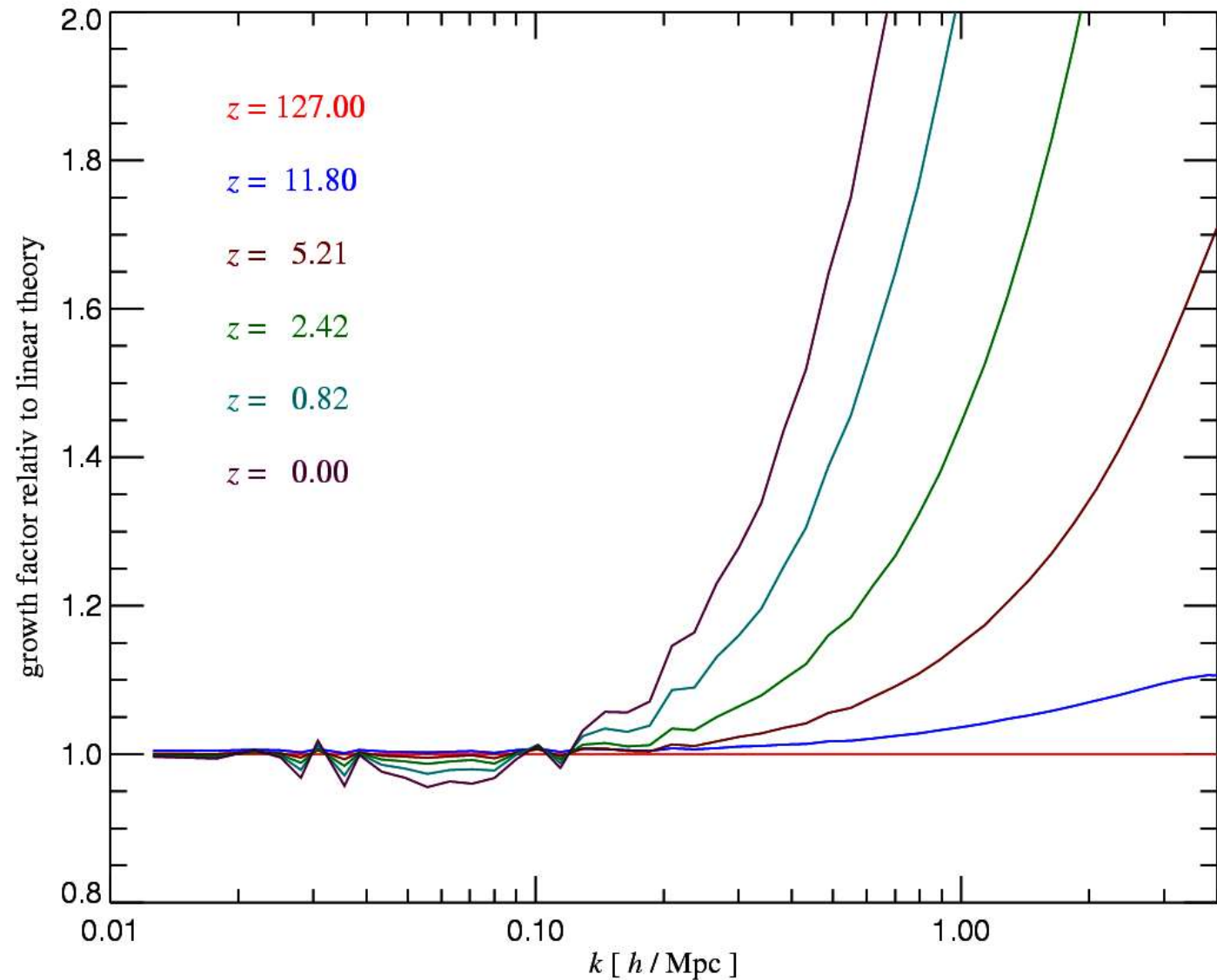
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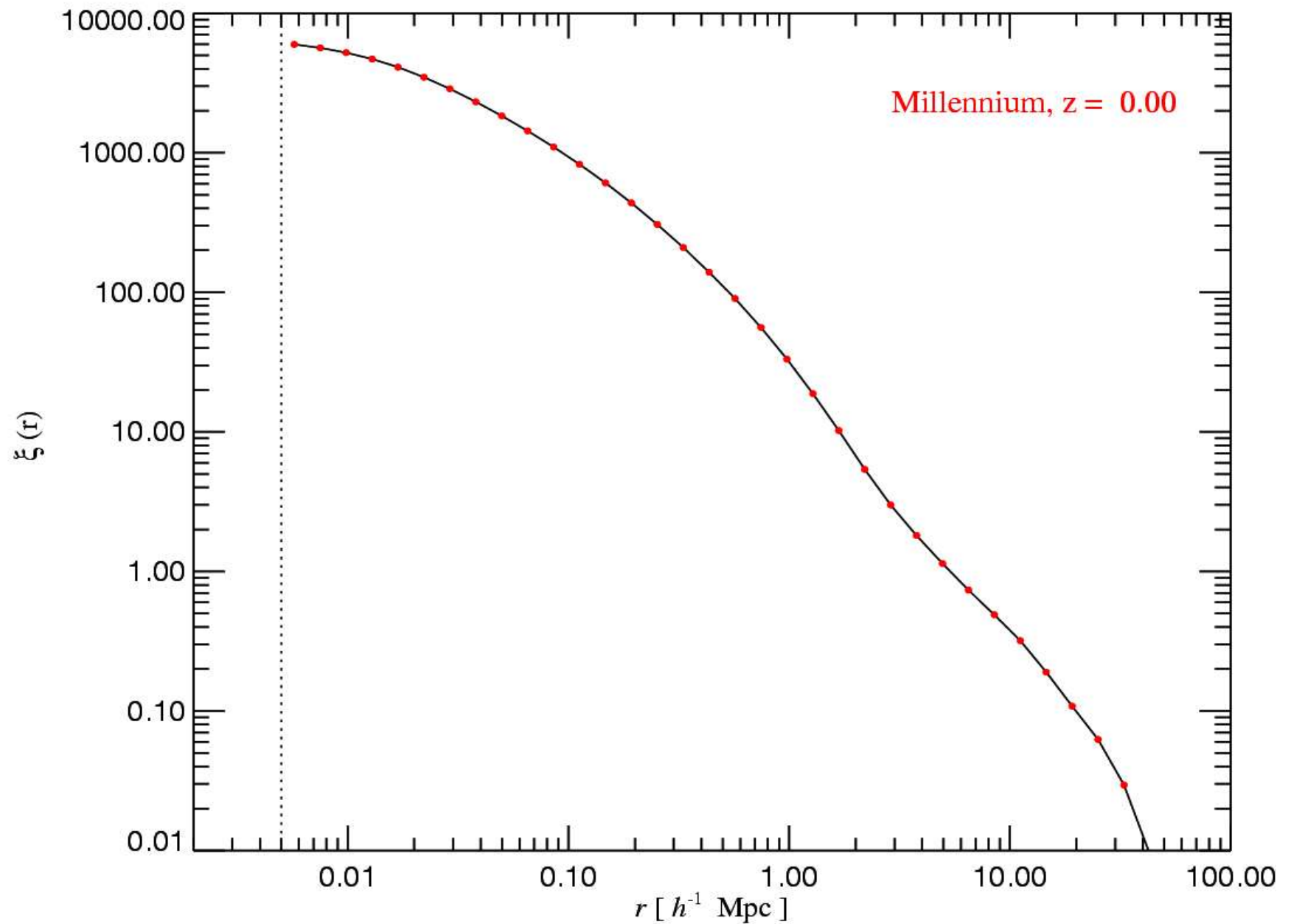
The Evolution of the Baryonic 'Wiggles'



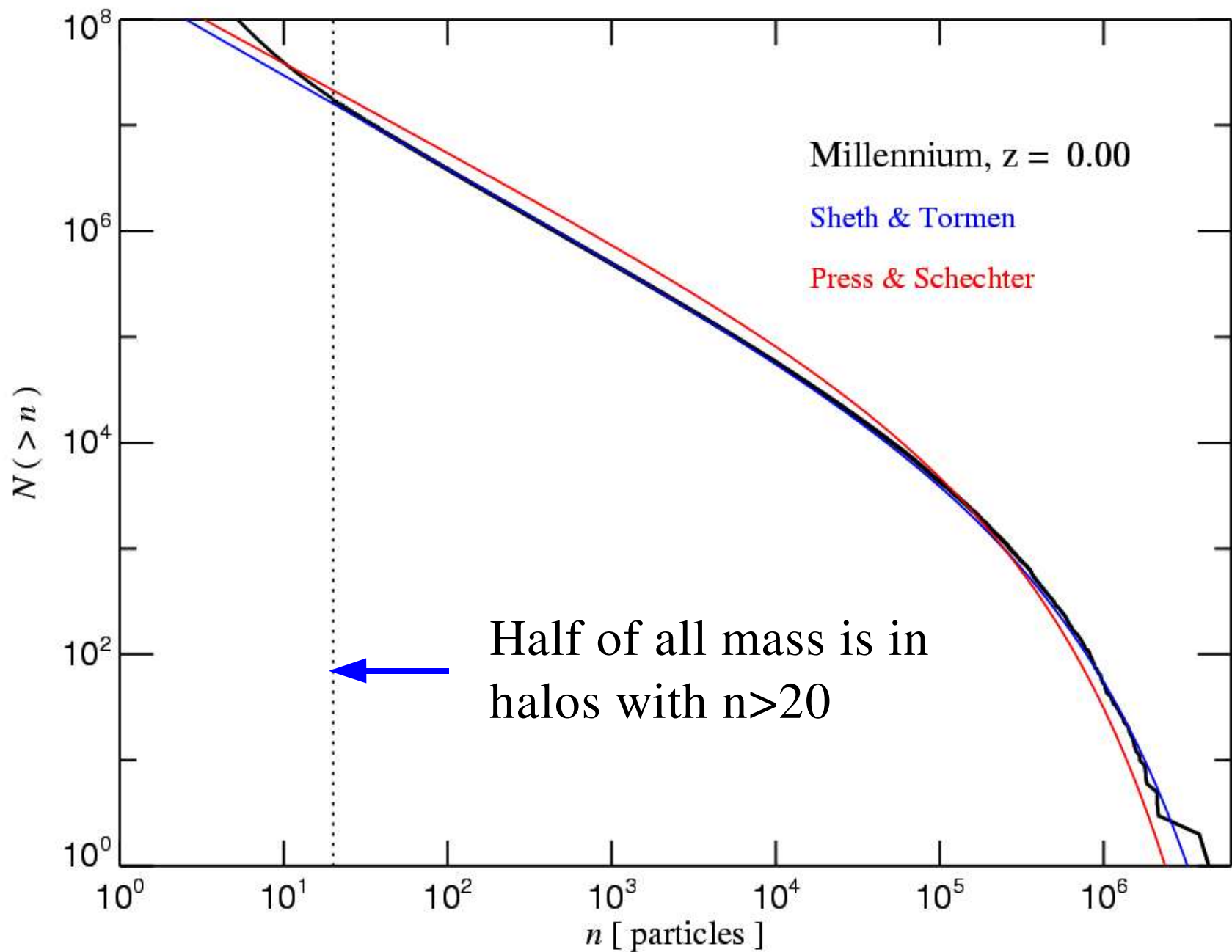
Growth relative to linear as a function of scale



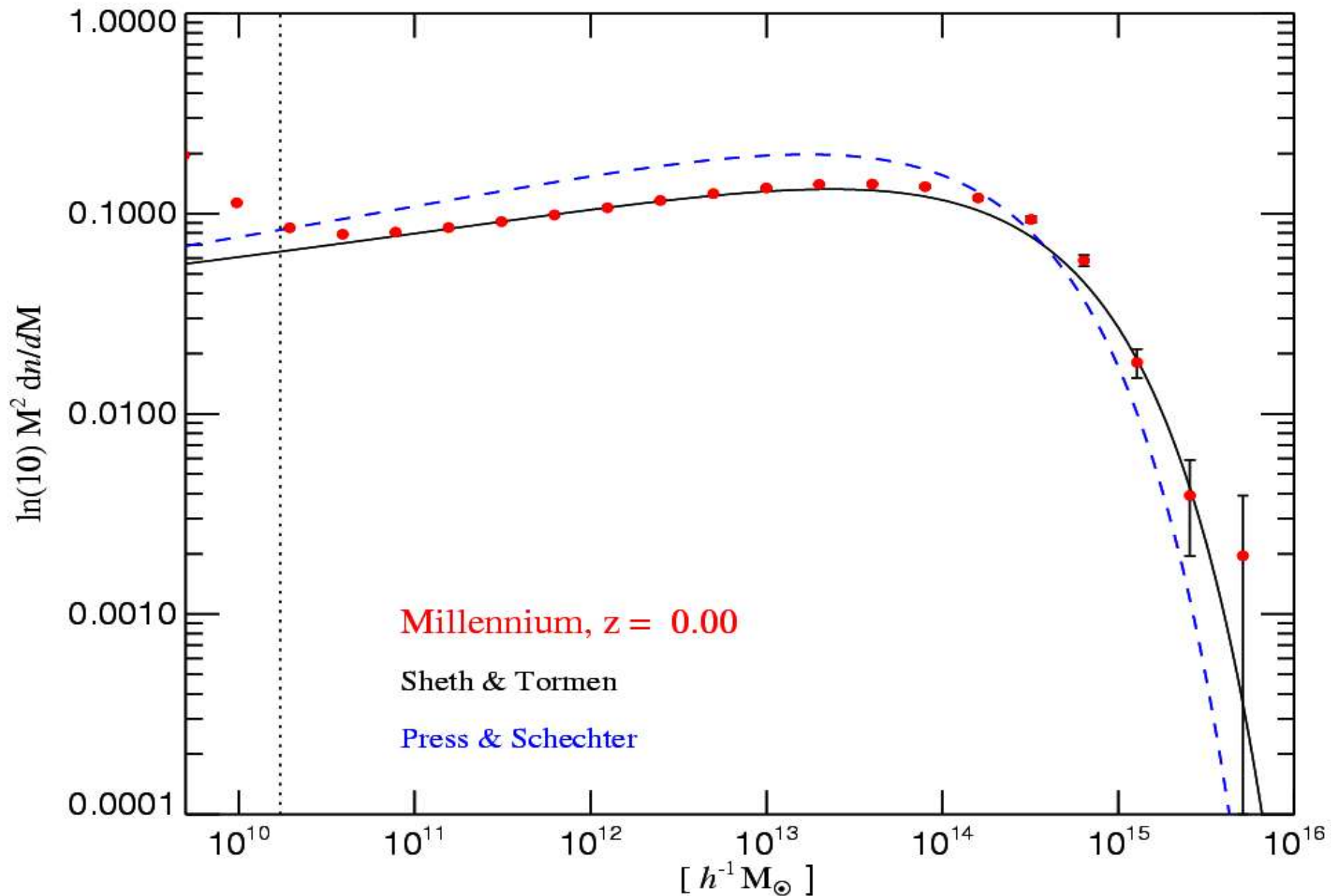
Mass autocorrelation function



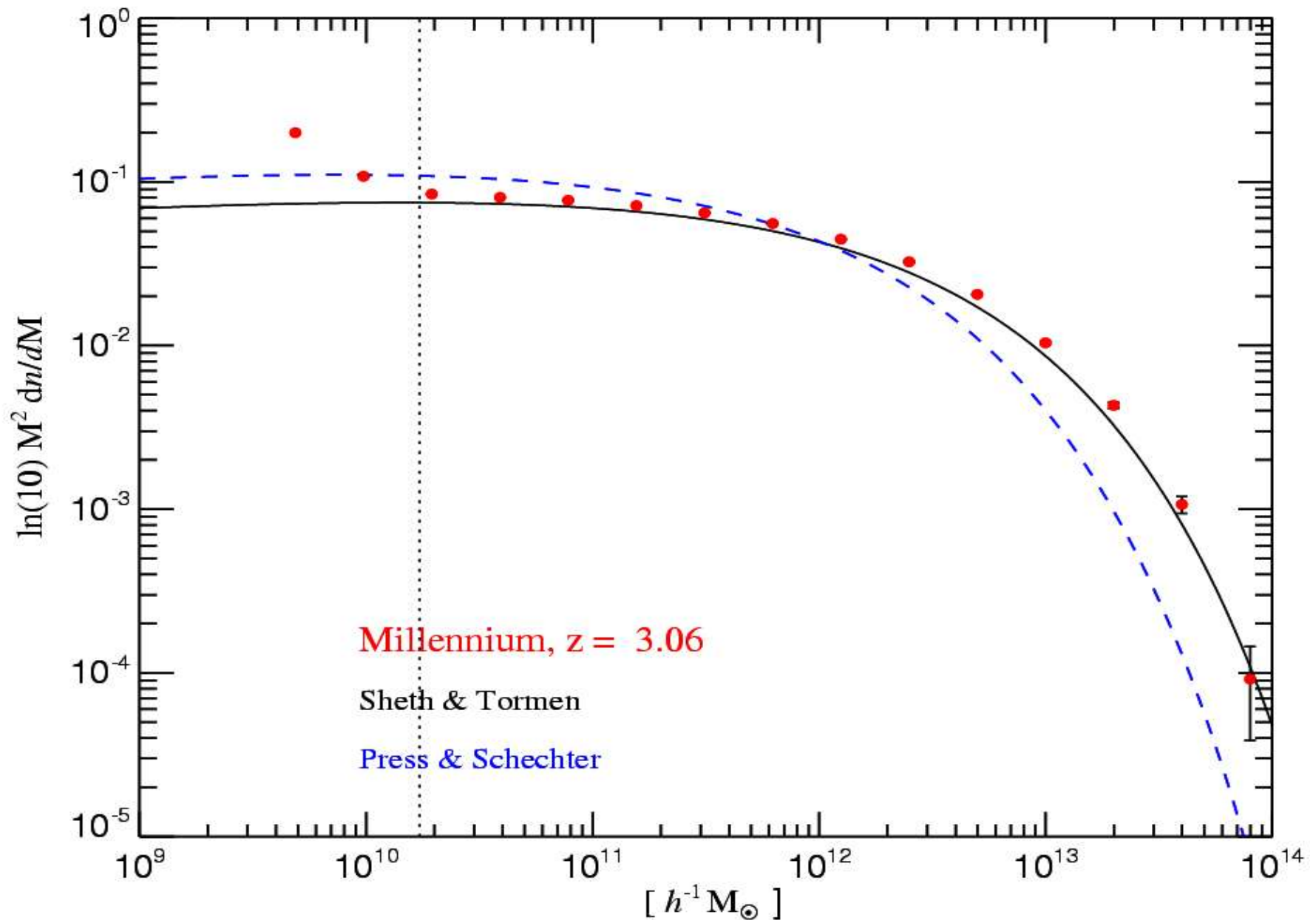
Cumulative halo mass function



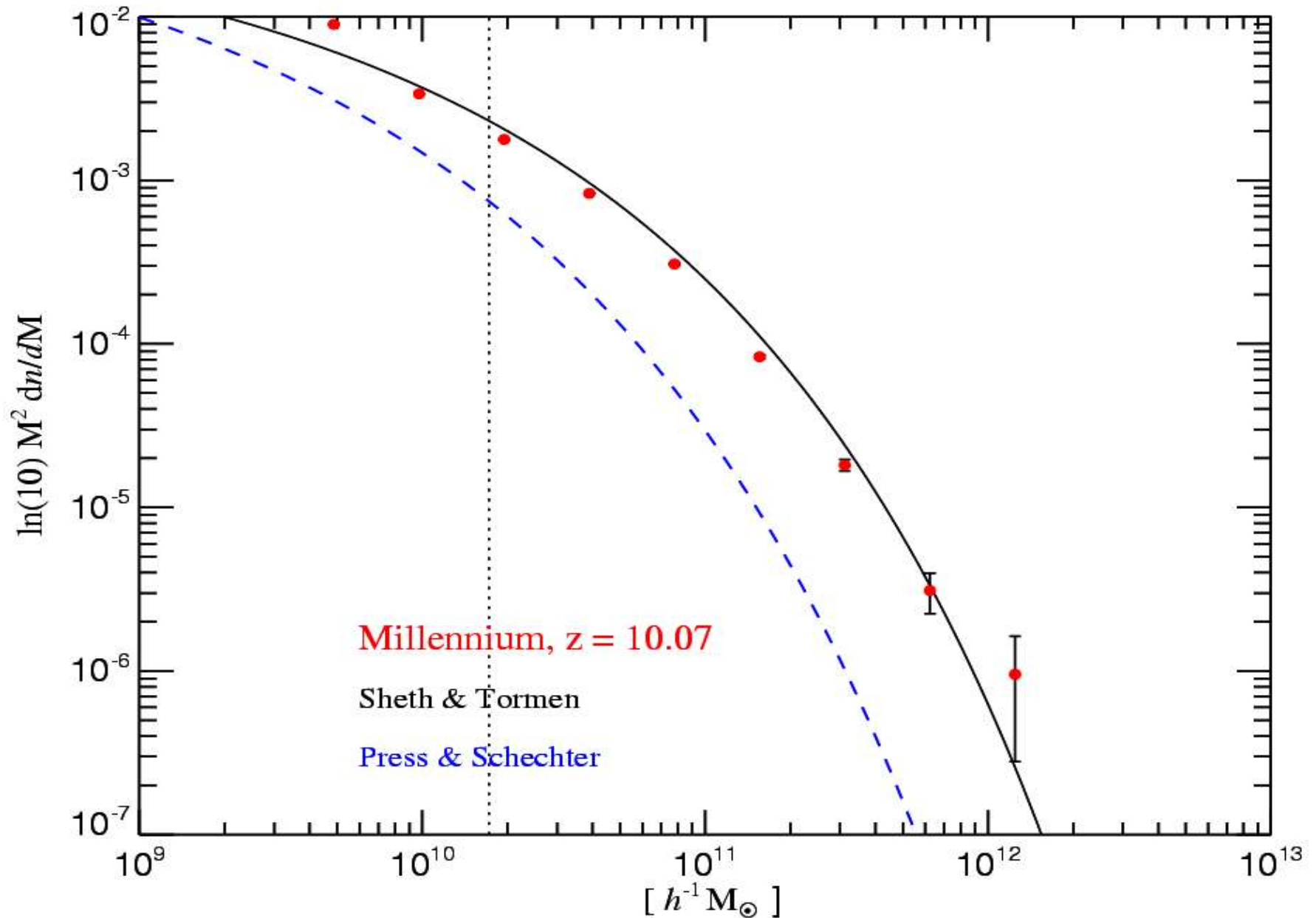
Differential halo mass function



Differential halo mass function



Differential halo mass function

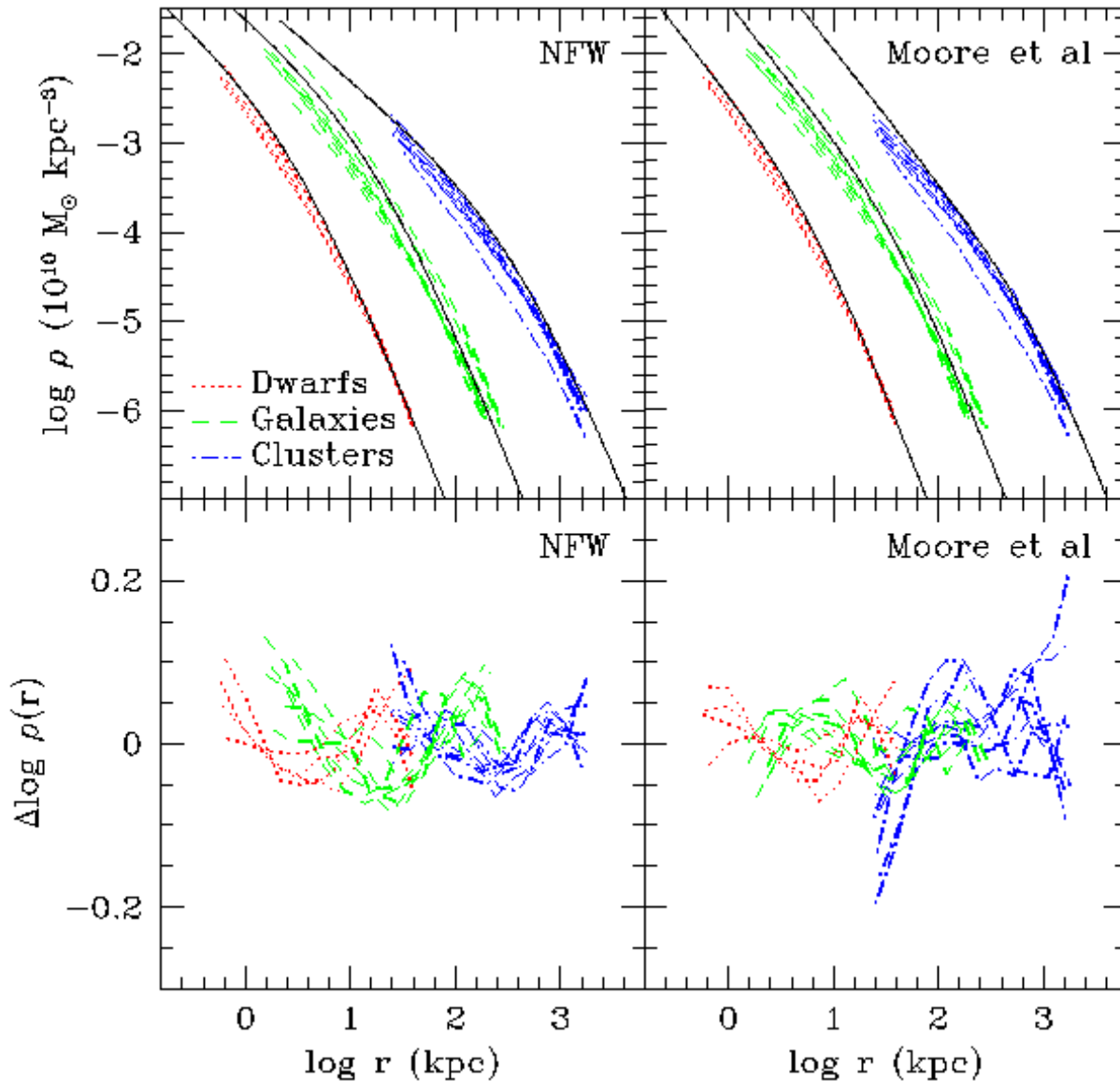


Science from halo (cluster) cores

- Initial velocities of DM (cold, warm, hot...)
- Interactions of DM (self-interacting, interactions with baryons)
- Small scale power in the initial power spectrum (tilt, break...)
- Baryon accumulation effects (assembly sequence...)

Profiles from high-resolution simulations

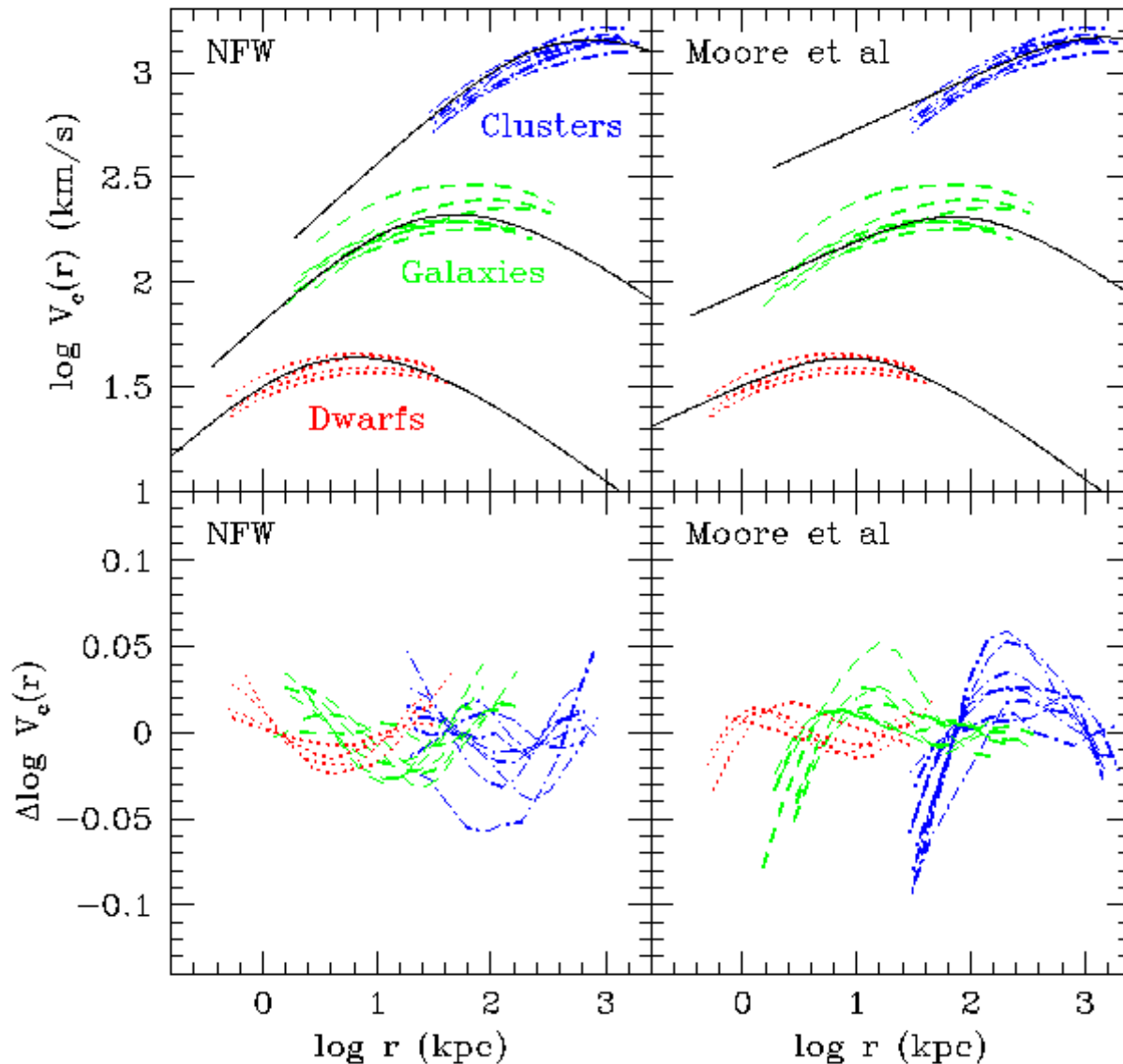
Navarro et al 2004



- Λ CDM halos simulated individually with high resolution -- $N_{200} > 10^6$
- Least square fit to NFW and Moore profiles
- Systematic deviations in inner regions in both cases, particularly for clusters

Profiles from high-resolution simulations

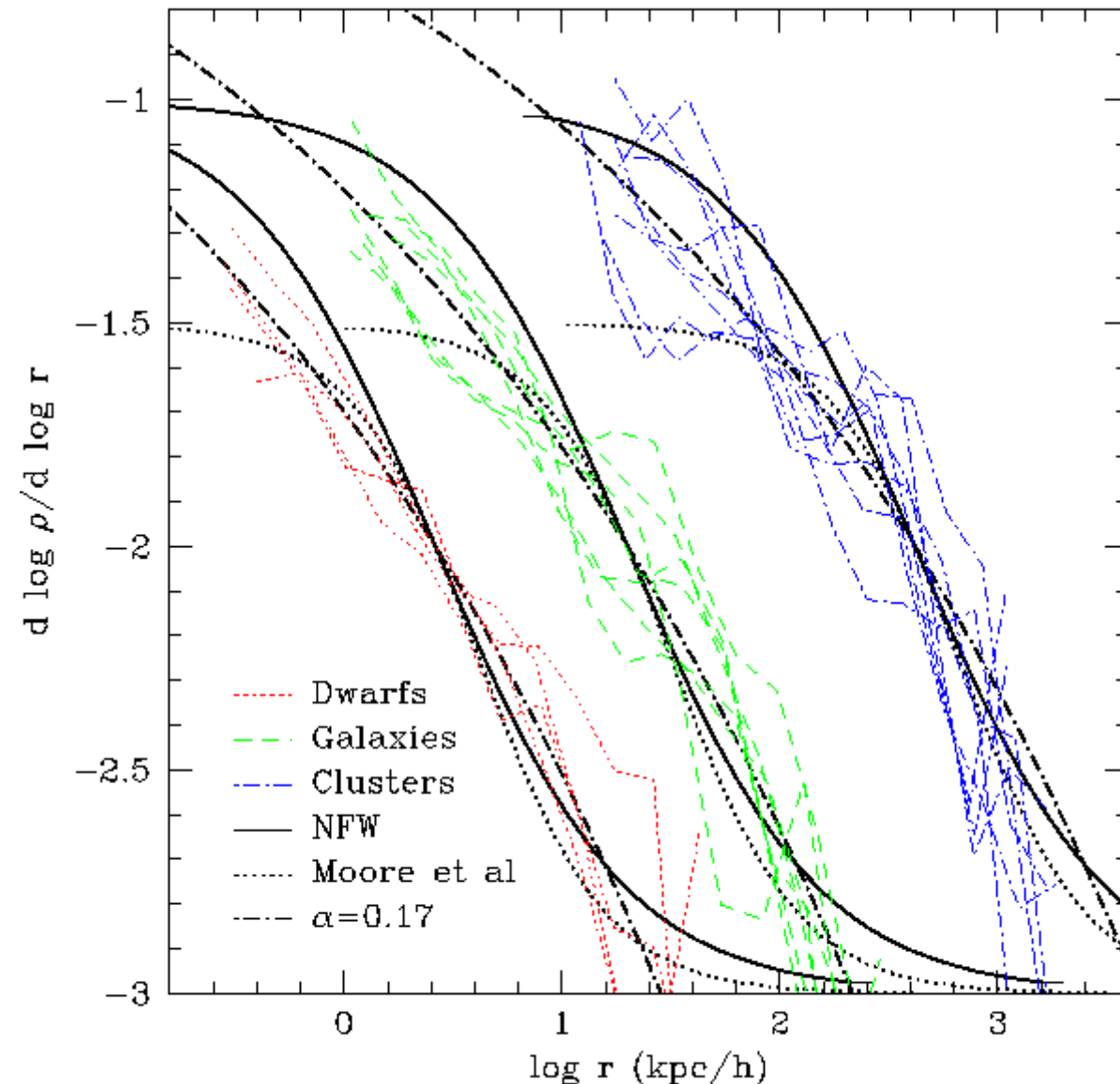
Navarro et al 2004



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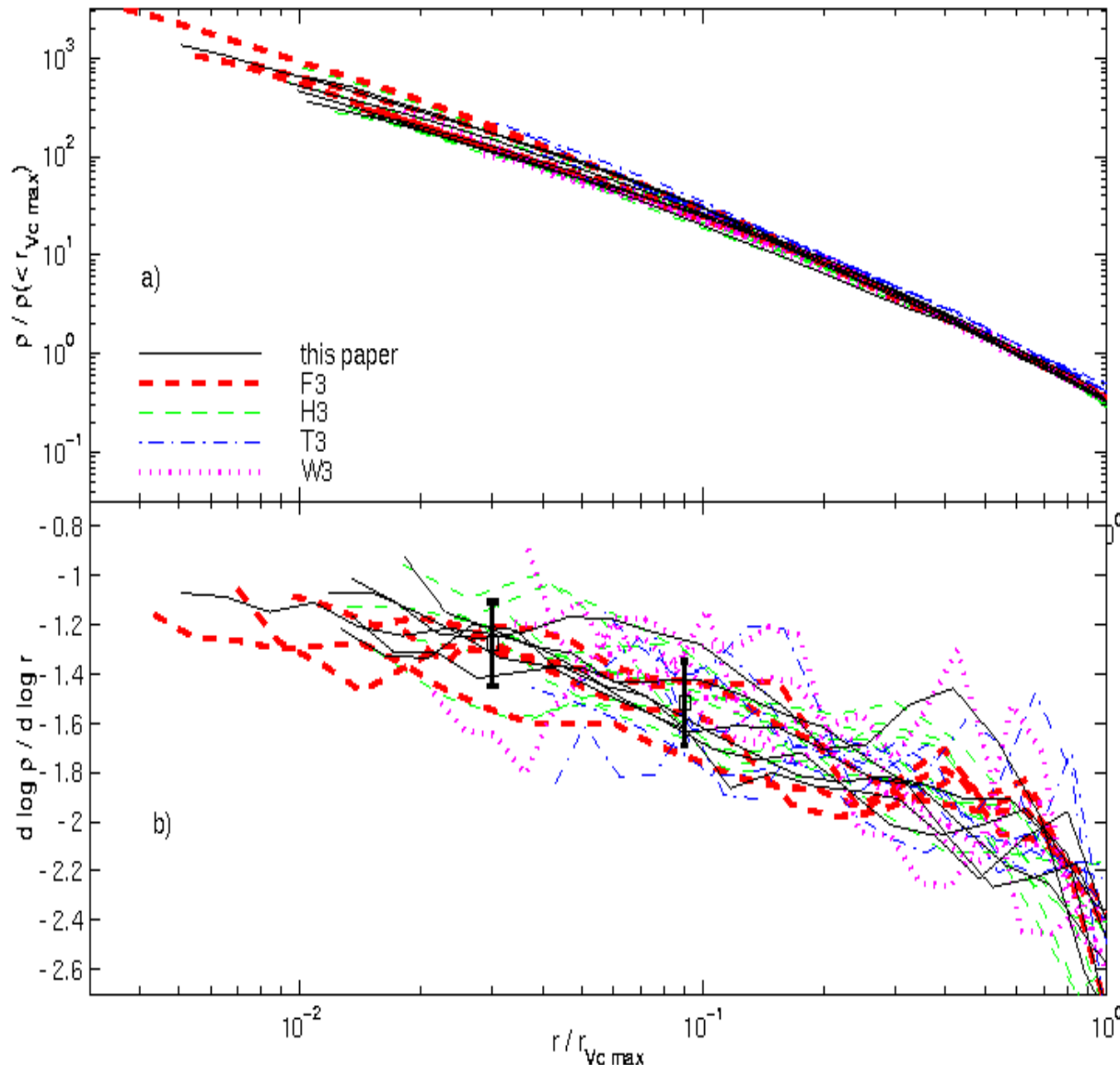
Navarro et al 2004



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- Density profile slopes vary more gradually than Moore or NFW profiles
- No sign of converging to *any* asymptotic inner slope

Profiles from high-resolution simulations

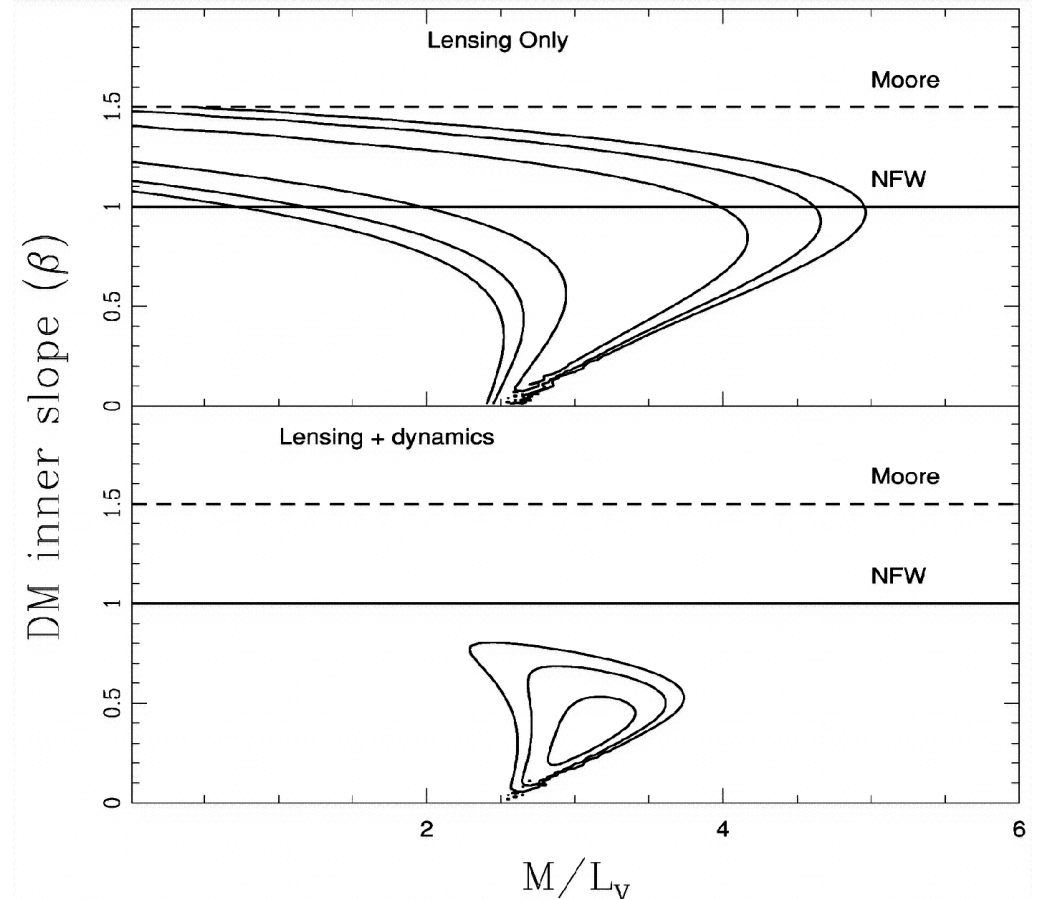
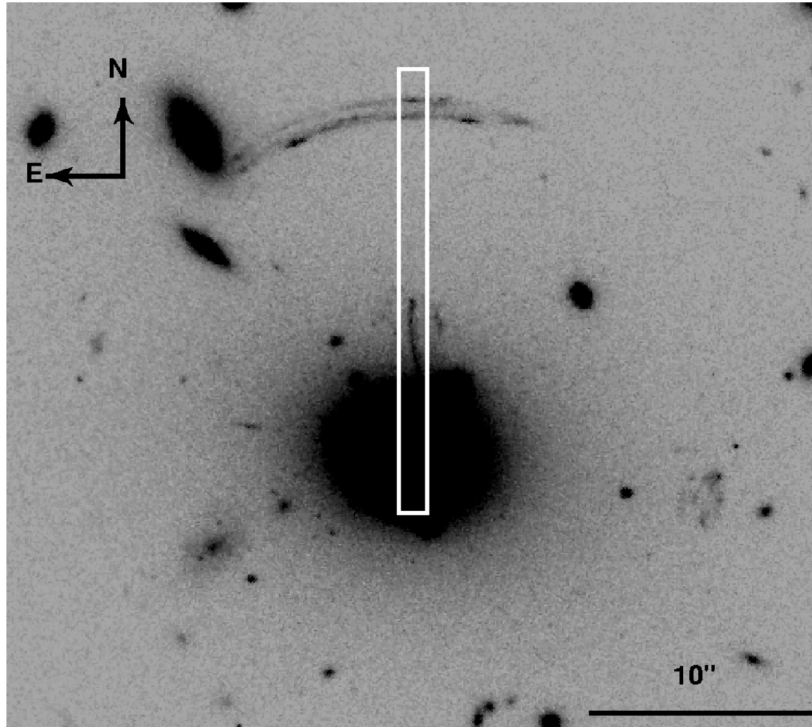
Diemand, Moore & Stadel 2004



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- Density profile slopes vary more gradually than Moore or NFW profiles
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Constraining DM properties with strong lensing ?

Sand, Treu & Ellis 2002

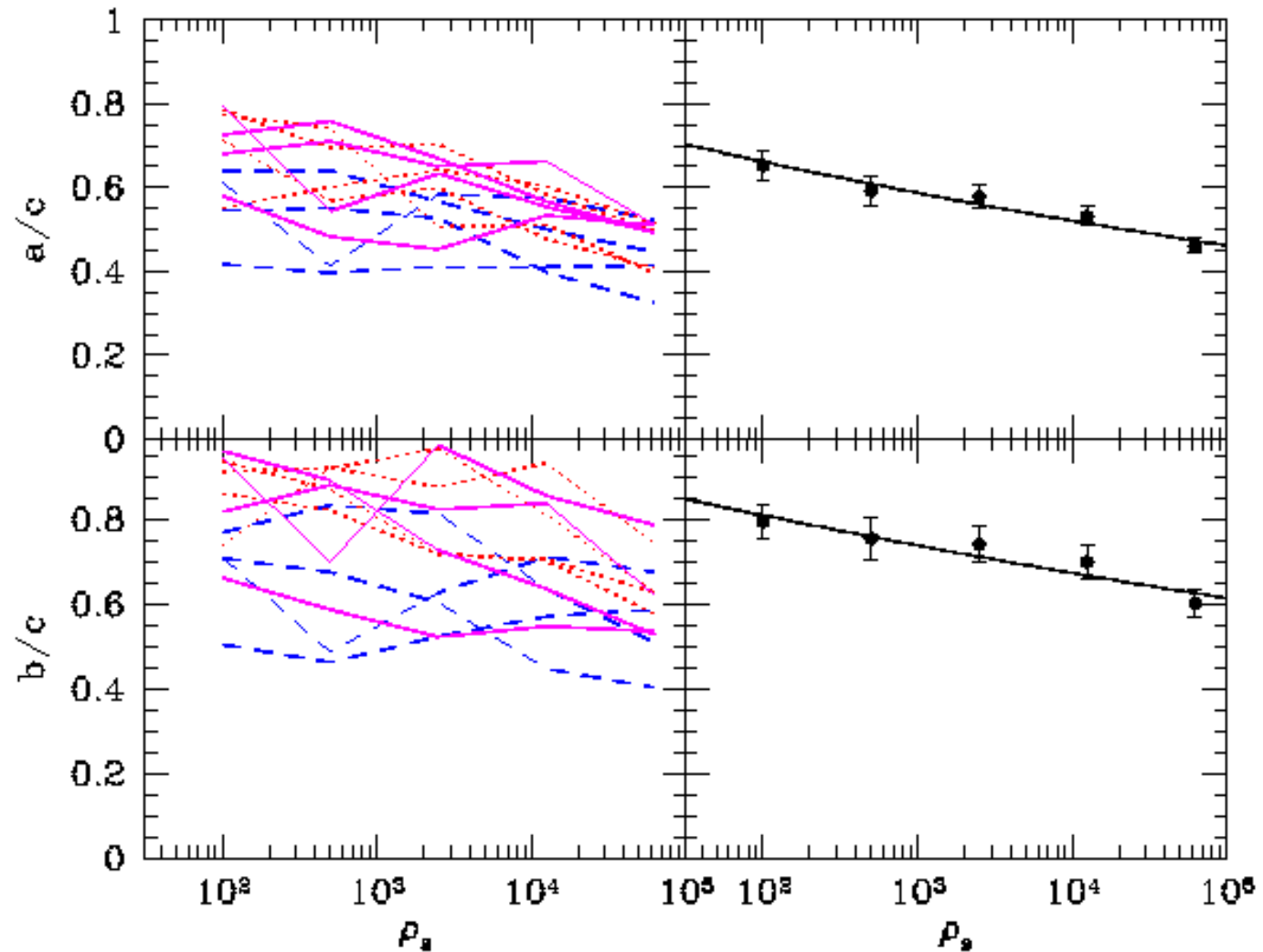


- Model potential as power law DM + galaxy with constant M/L
- Consistency with radial arc, tangential arc & velocity dispersion profile
→ inner slope of DM profile shallower than NFW
- Constraint is substantially weakened if the inner DM distribution can be significantly flattened ([Bartelmann & Meneghetti 2004](#), [Dalal & Keaton 2004](#))

Flattening of Λ CDM dark halos

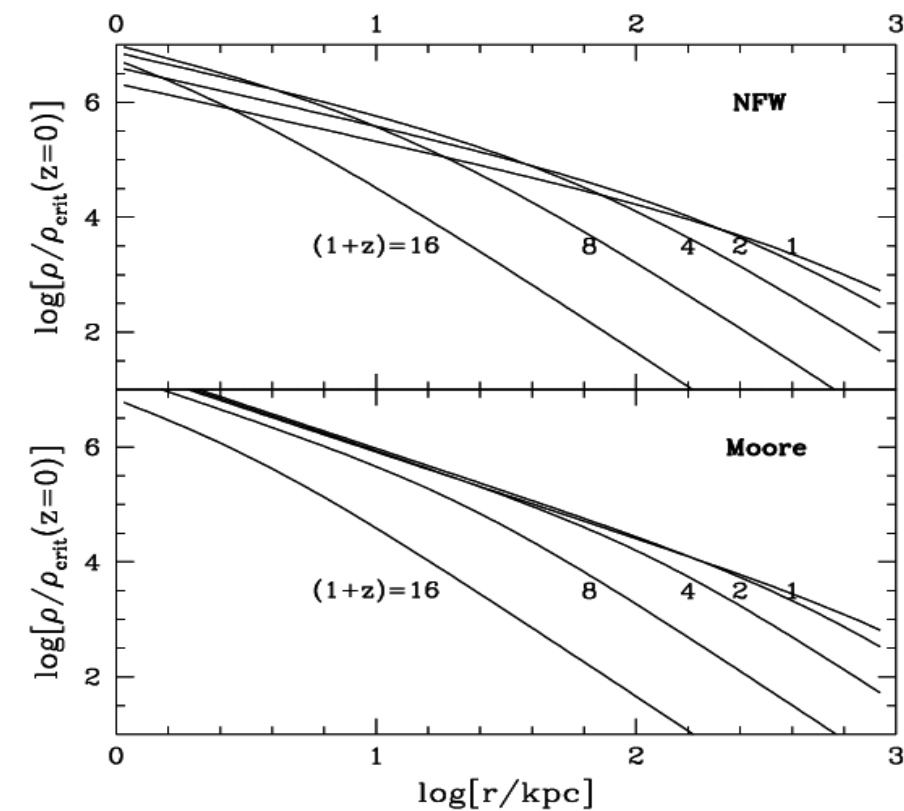
Jing & Suto 2002

Axial ratios of
equidensity
surfaces

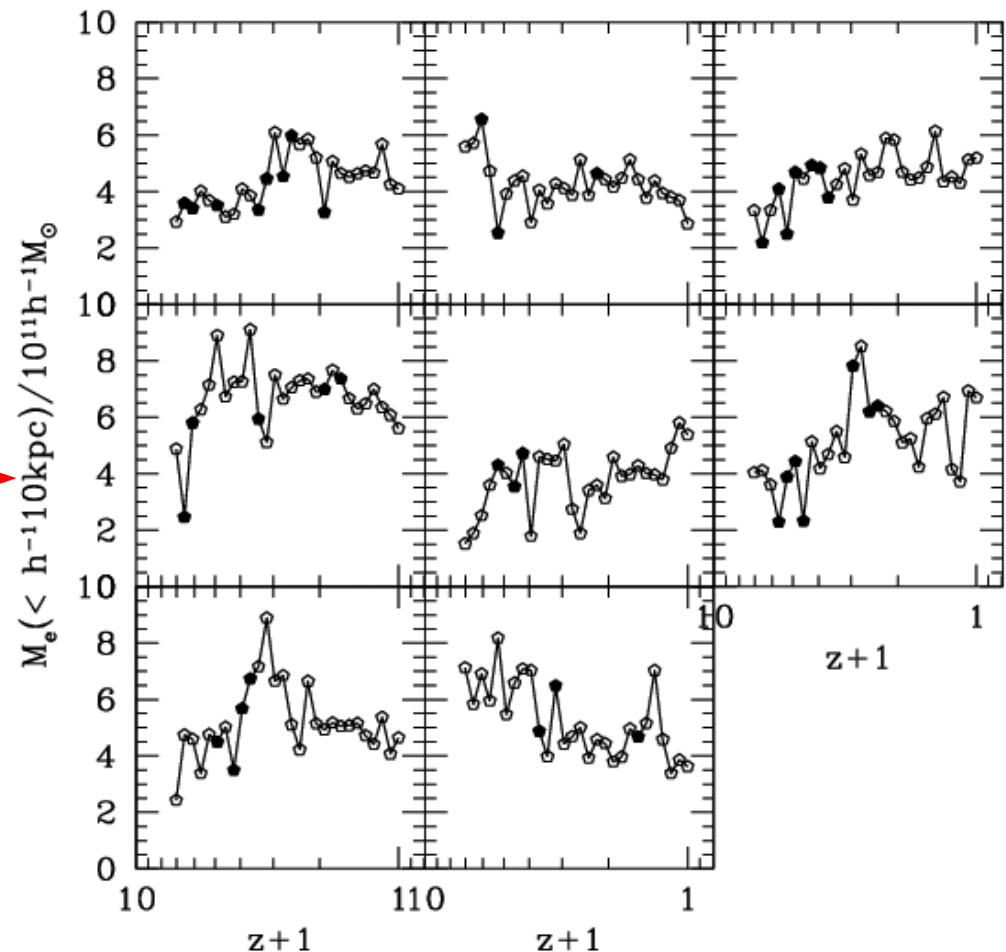


When was the inner cluster core assembled?

Gao, Loeb, Peebles, White & Jenkins 2004



Both analytic and simulation results suggest that the inner mass structure of cluster halos has been stable since $z \sim 6$



Cluster structure in Λ CDM

- 'Concordance' cosmology
- Final cluster mass $\sim 10^{15} M_{\odot}$
- DM within 20kpc at $z = 0$ is shown black

2.5 Mpc/h

$z = 0.00$

Cluster structure in Λ CDM

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$z = 1.00$

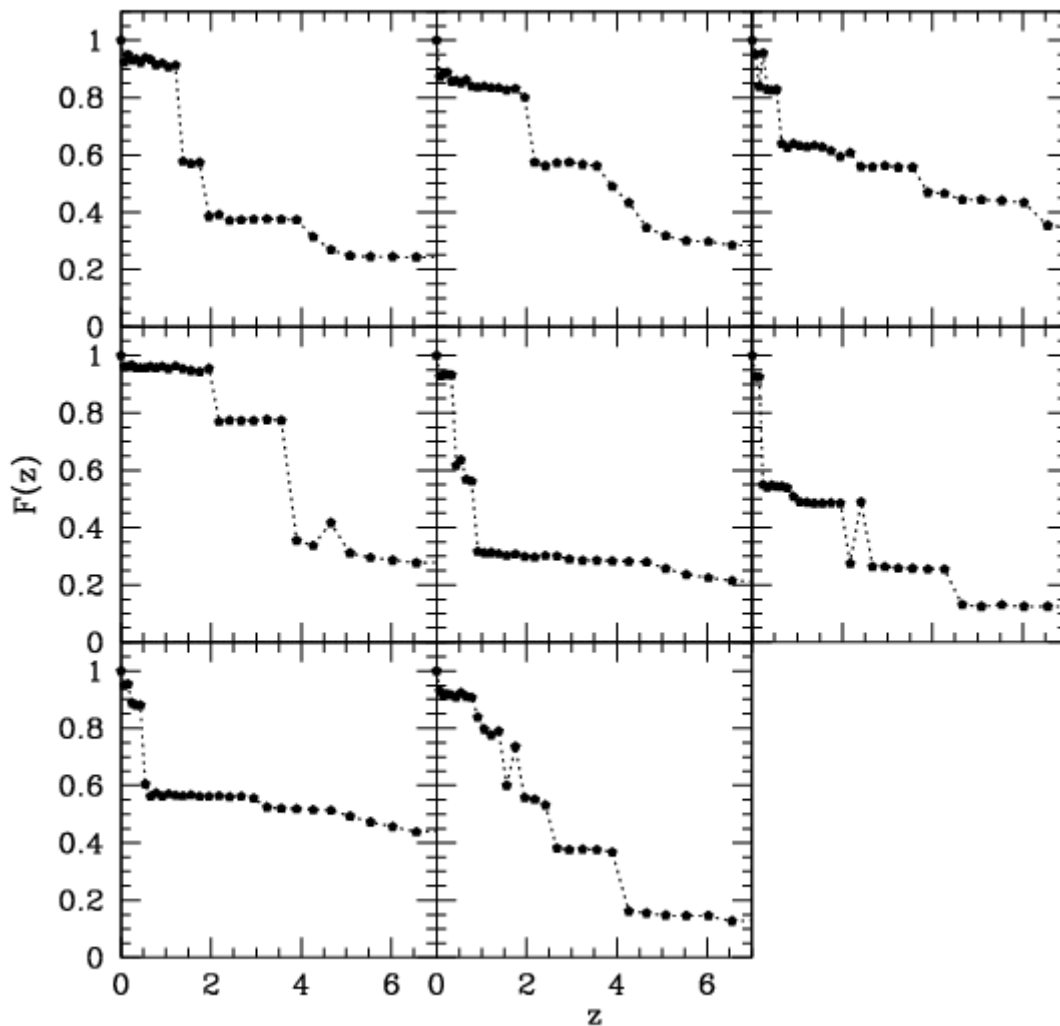
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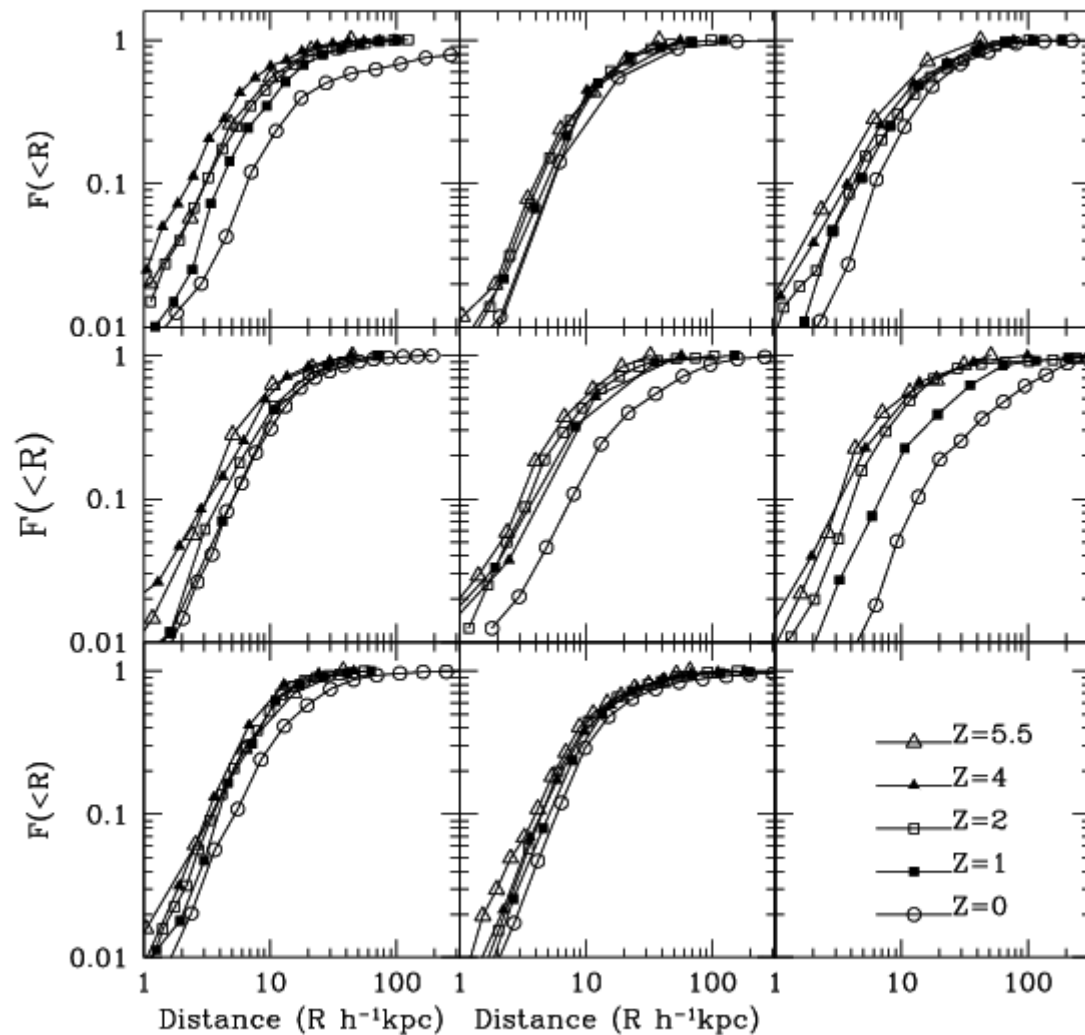


2.5 Mpc/h

$z = 3.00$



Fraction of final inner
core mass (< 15 kpc)
in a single object at
earlier times

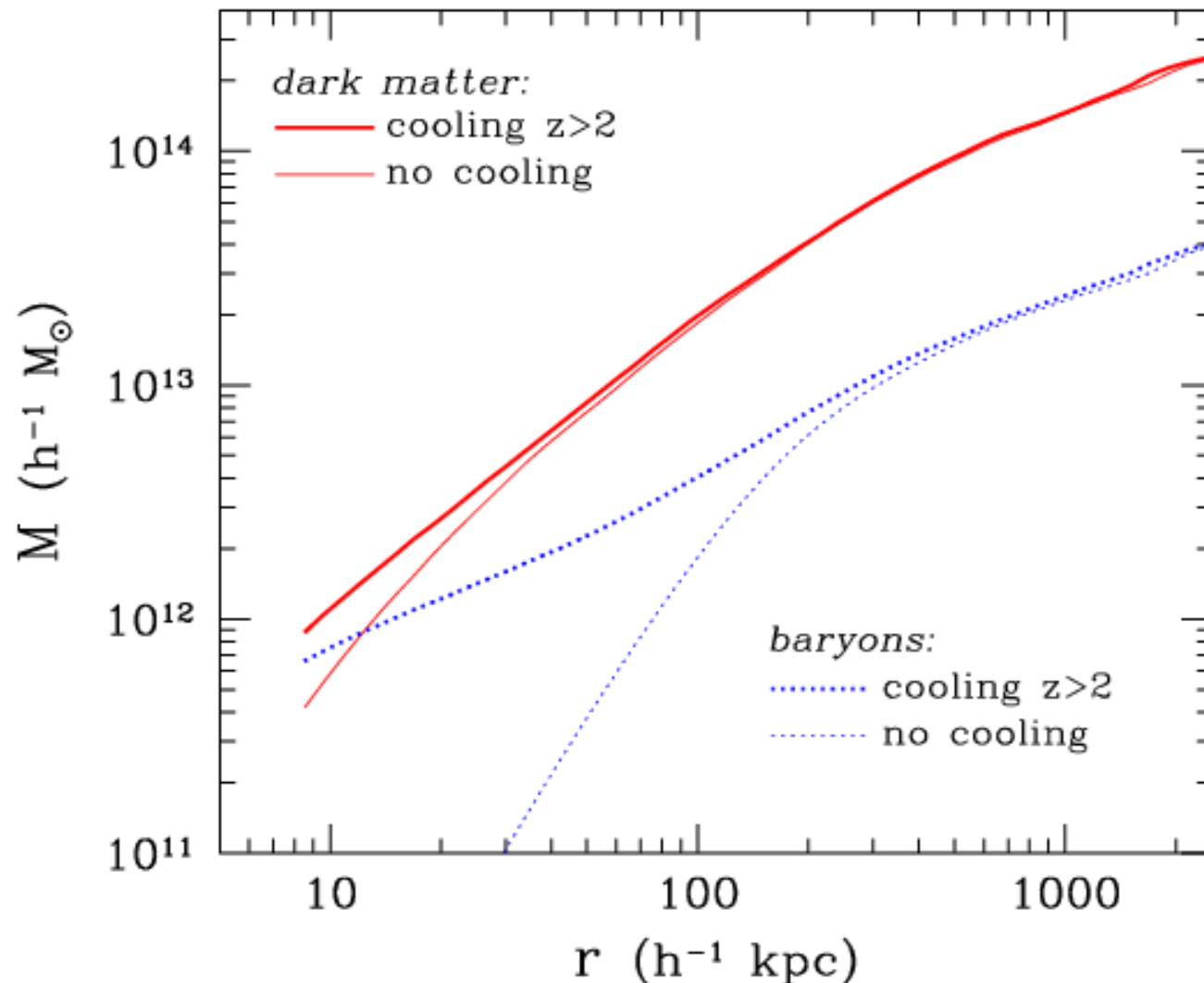


What happened to the mass which was in the inner core at $z=6$?

It was pushed outwards as new material was added.

Does the *total* mass profile converge to NFW?

Gnedin, Kravtsov, Klypin & Nagai 2004

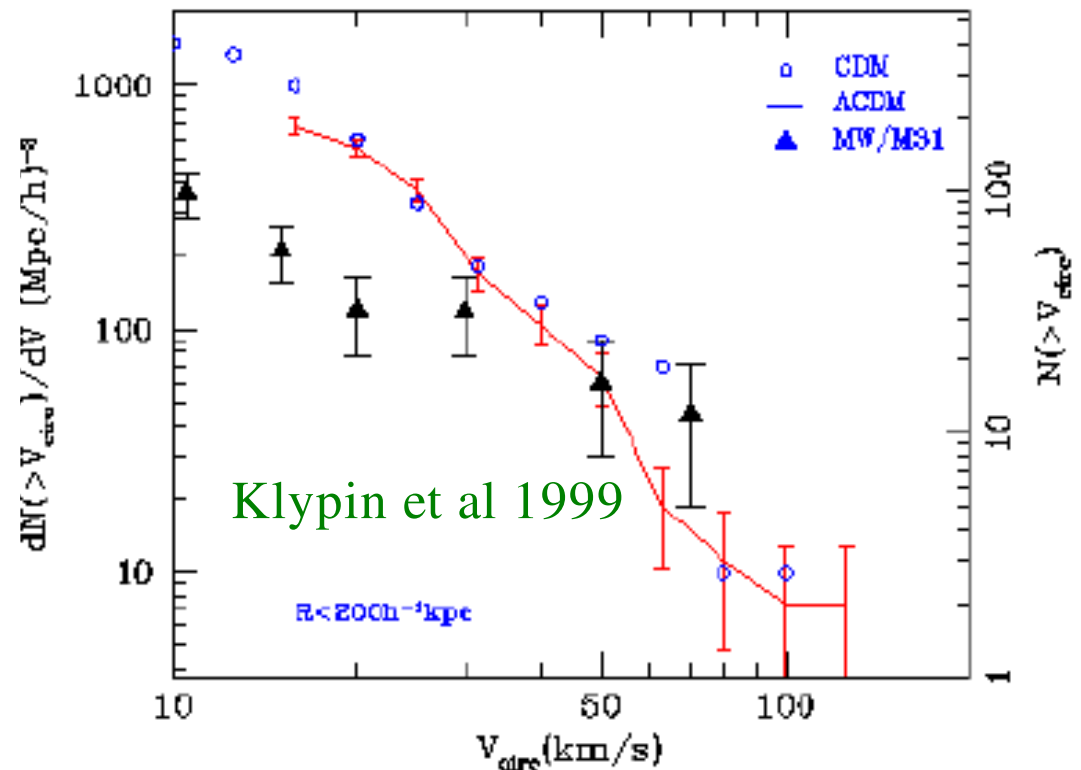
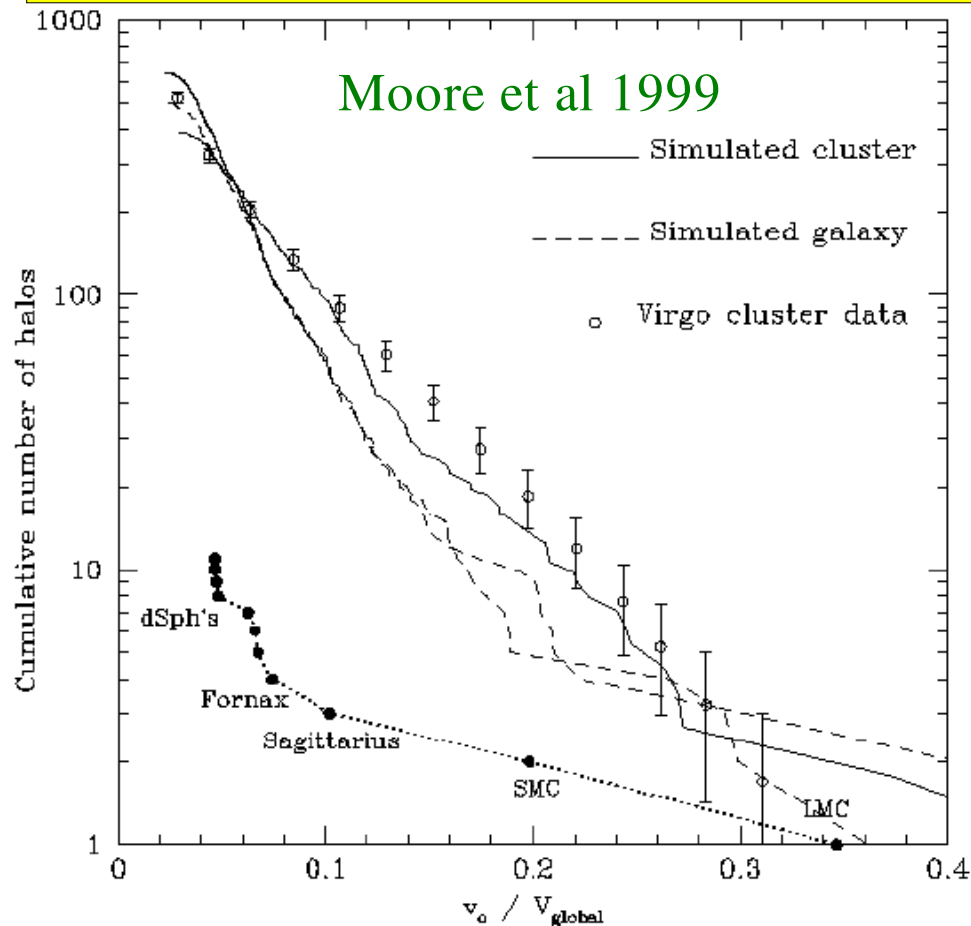


- Two simulations of the formation of a cluster including gas and with identical initial cond'ns
- No cooling in one: cooling/star-formation at $z > 2$ in the other
- Several mergers occur in the core at $z < 2$
- The DM distribution is still more concentrated in the model with stars

Science from halo substructures

- Initial velocities of DM (cold, warm, hot...)
- Interactions of DM (self-interacting, interactions with baryons)
- Small scale power in the initial power spectrum (tilt, break...)
- Baryon accumulation effects (assembly sequence...)
- Tidal effects as a function of environment history

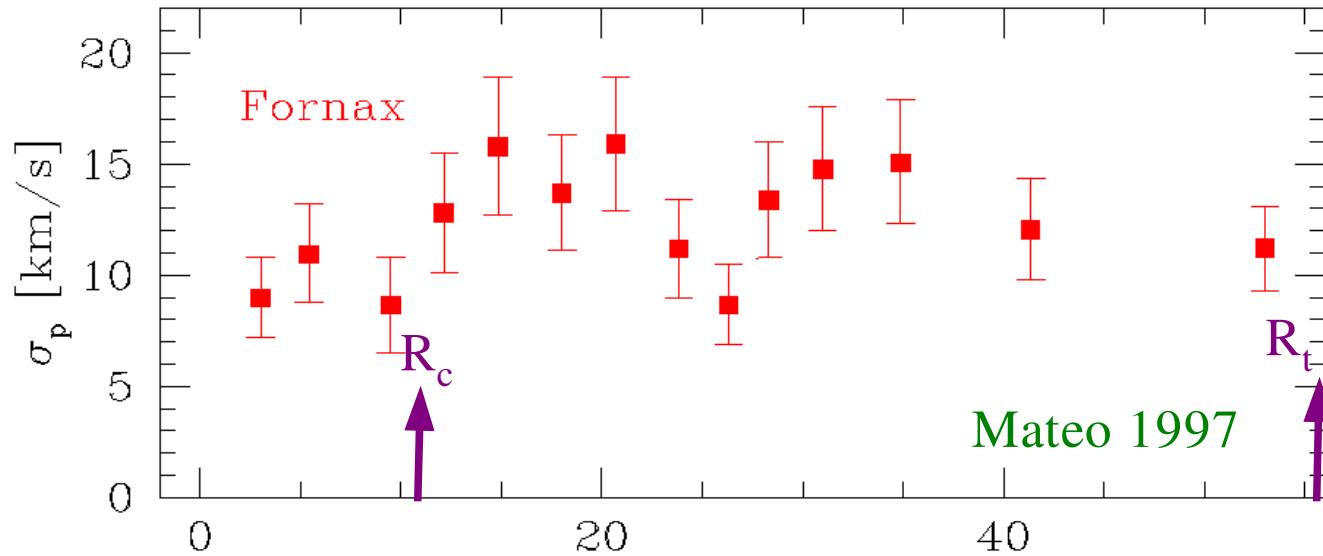
Is the kinematics of the Milky Way's satellites inconsistent with Λ CDM substructure?



- Number of observed satellites was *claimed* to be $\sim 1/10$ the number of Λ CDM satellites with the same max. circular velocity $V_c = (GM/r)^{1/2}$
- But the MW data are plotted at the *incorrect* values of V_c for this test!

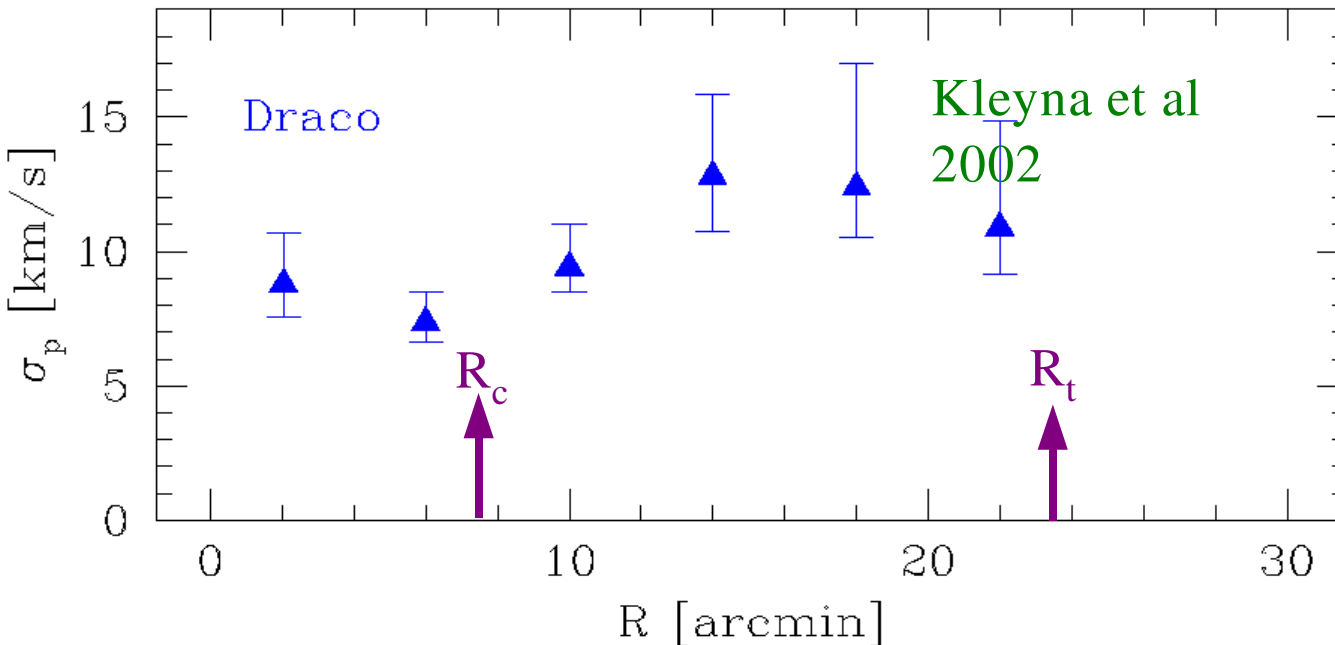
Stoeckl et al 2002

Dark Matter within Satellites



- Flat stellar velocity dispersion out to the tidal radius
→ *rising* V_c curve

- Extended DM halos?



- High DM phase density? ~~WDM~~ ?

- $V_{c,\max} \gtrsim 25$ km/s ?

- Critical observation: extratidal stars?

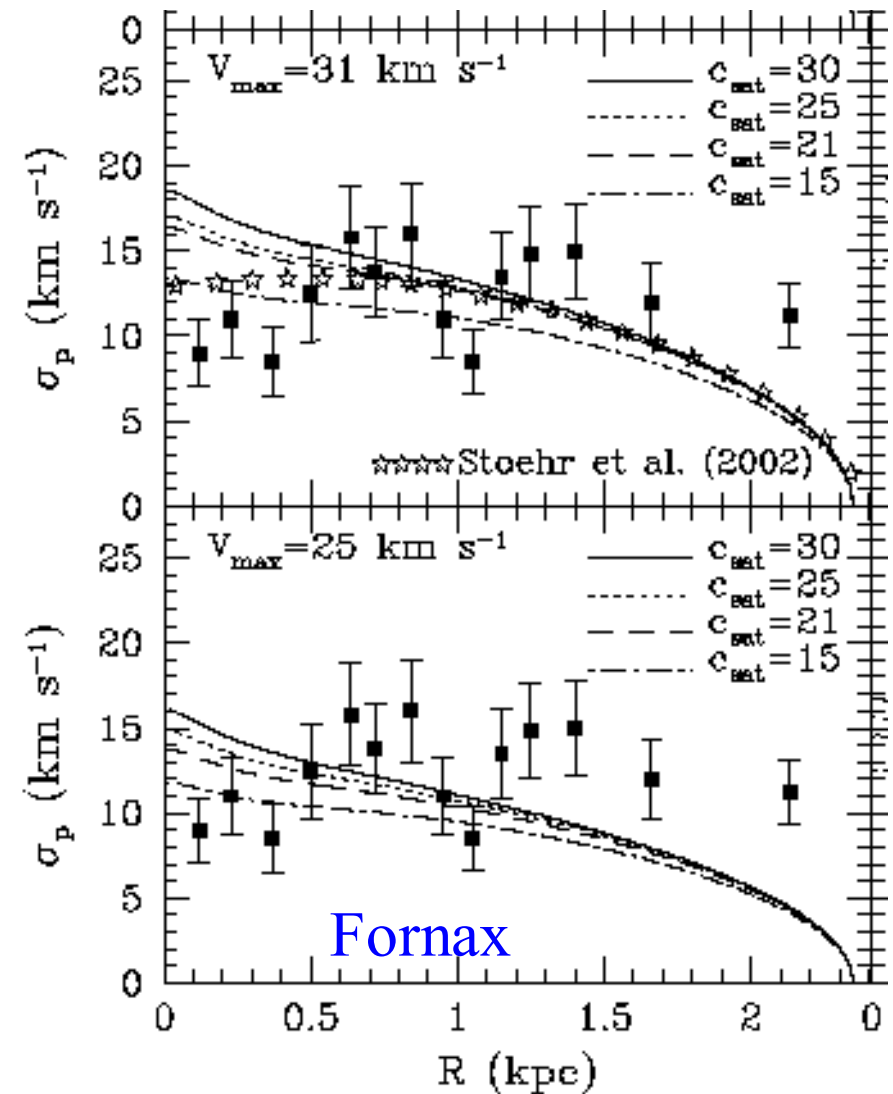
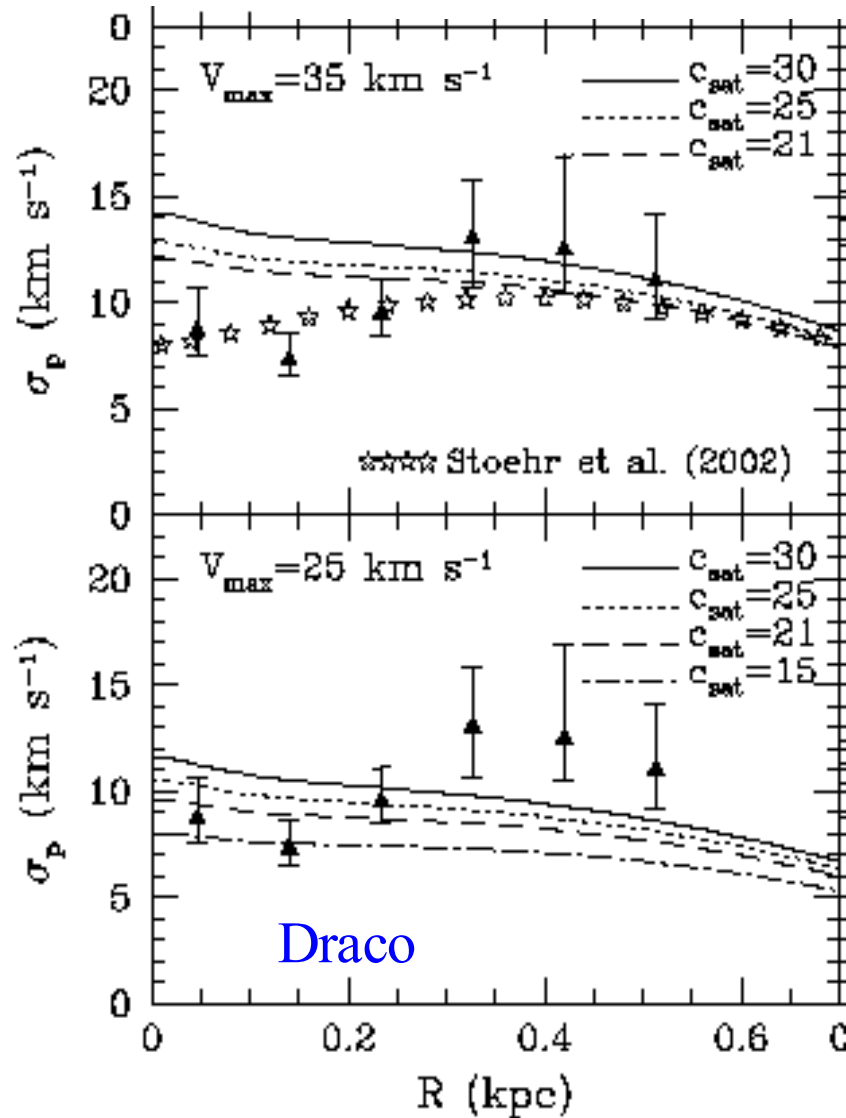
DENSITY PROFILES OF COLD DARK MATTER SUBSTRUCTURE: IMPLICATIONS FOR THE MISSING-SATELLITES PROBLEM

2004 (ApJ in press)

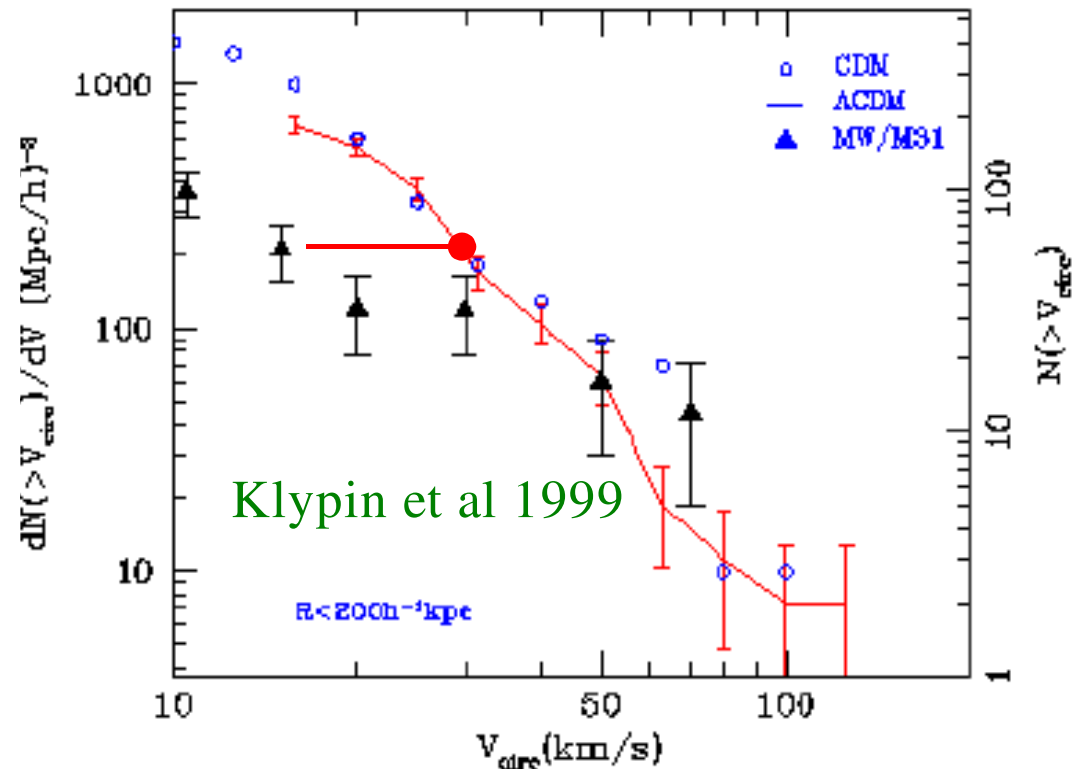
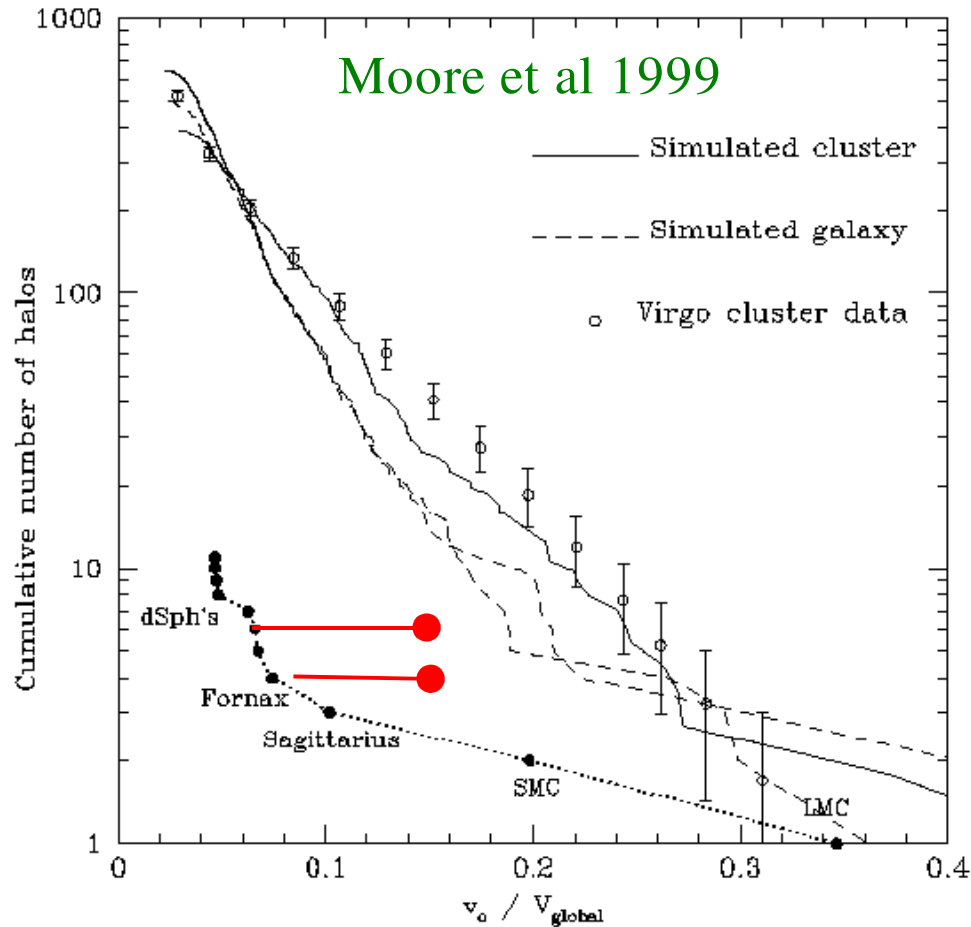
STELIOS KAZANTZIDIS¹, LUCIO MAYER, CHIARA MASTROPIETRO, JÜRGE DIEMAND, JOACHIM
STADEL, AND BEN MOORE

Motivated

by the structure of our stripped satellites, we compare the predicted velocity dispersion profiles of Fornax and Draco to observations, assuming that they are embedded in CDM halos. We demonstrate that models with isotropic and tangentially anisotropic velocity distributions for the stellar component fit the data only if the surrounding dark matter halos have maximum circular velocities in the range $20 - 35 \text{ km s}^{-1}$.



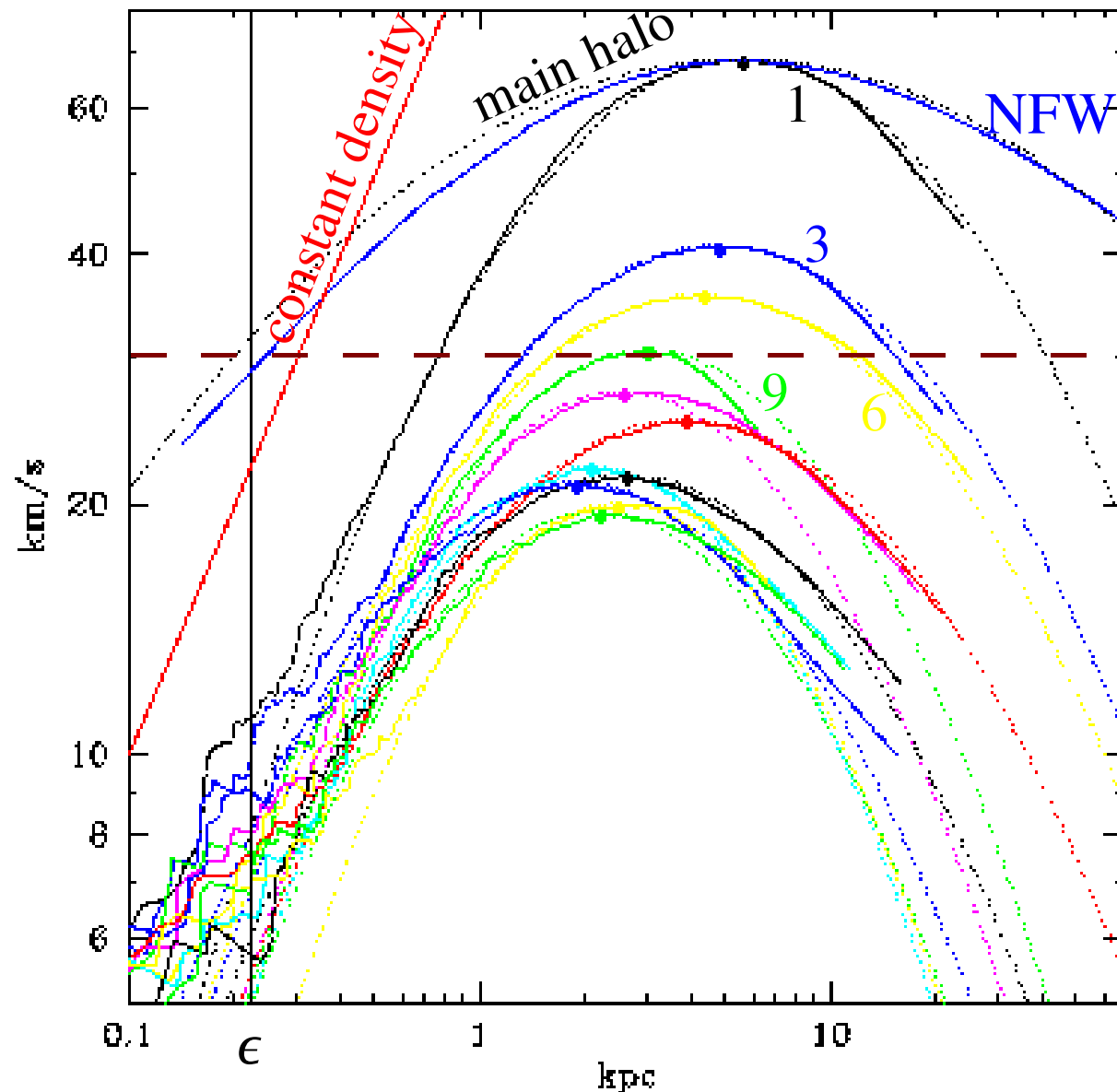
Inconsistency with observed satellite kinematics?



- Inconsistency is much less dramatic when one uses the *limiting* circular velocity inferred from the velocity dispersion profiles
- The *maximum* of the DM circular velocity profile may be outside the visible galaxy and still larger (plots show shift to $V_{\text{max}} = 30 \text{ km/s}$)

Satellite circular velocity curves

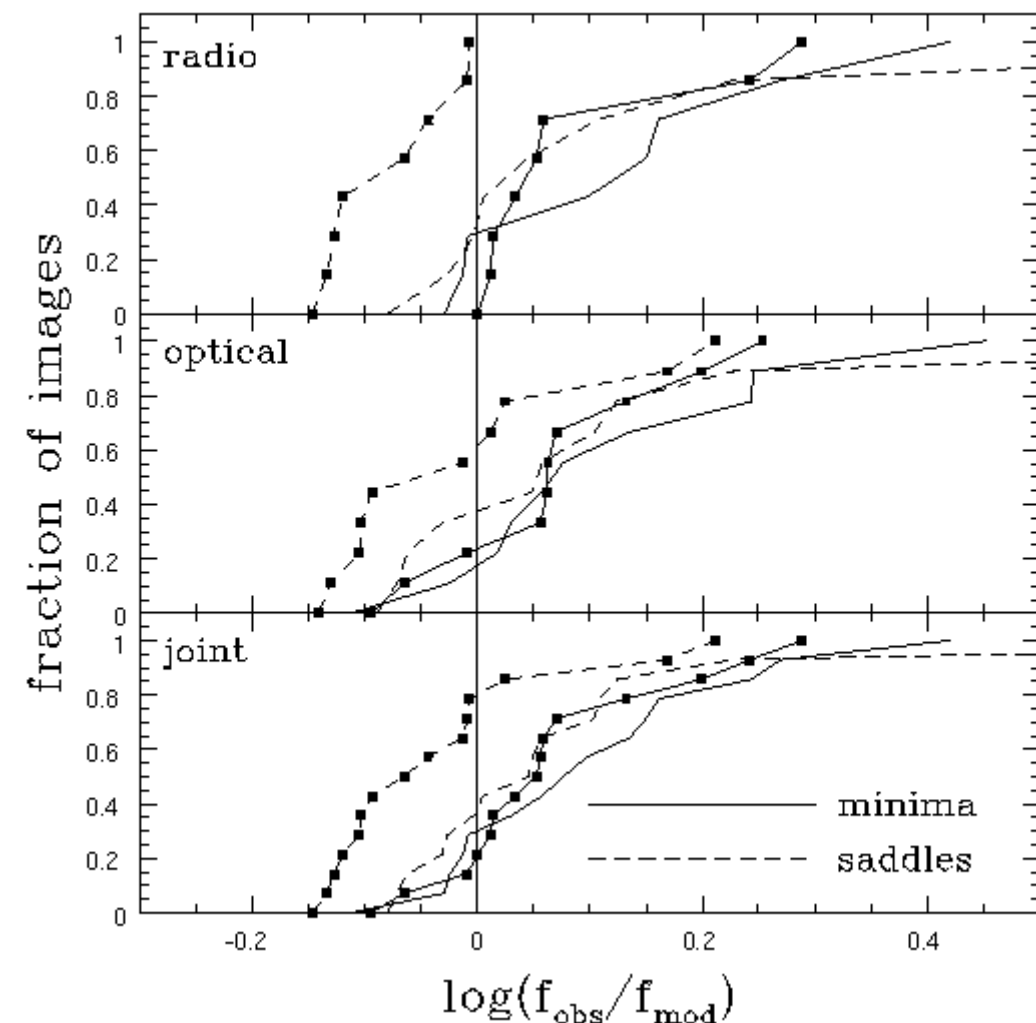
Stoehr et al 2003



- Circular velocity curves for 11 of the 30 most massive subhalos in a 10^7 particle 'Milky Way' halo
- The NFW and 'main halo' curves are scaled to the (r_m, V_m) of largest subhalo
- All curves are narrower than NFW or 'main halo'
- The maximum circular velocities are at radii well outside observed satellites
- ***The MOST MASSIVE of these potentials could host the observed satellites***

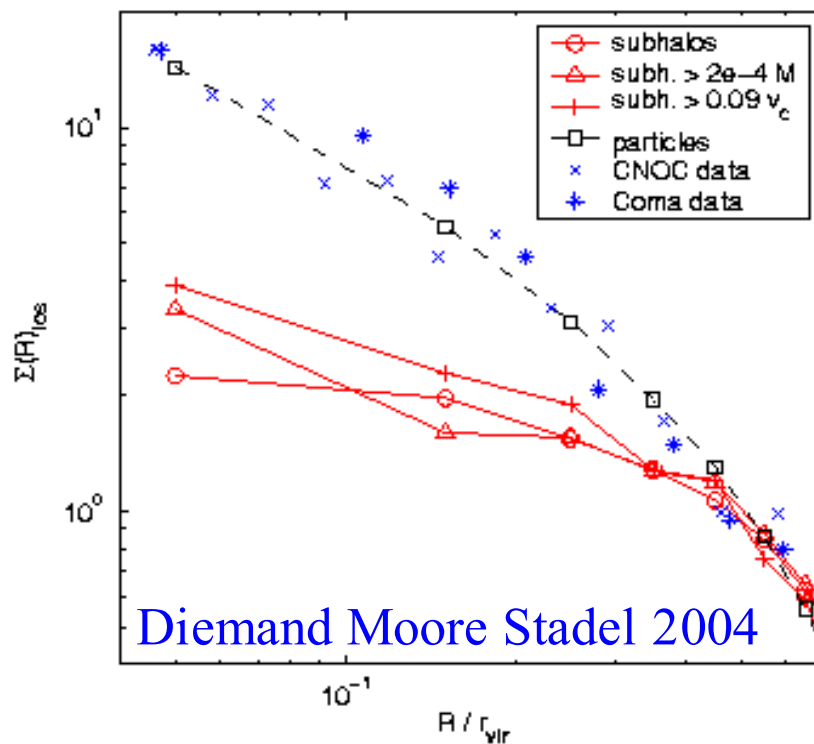
Detection of Λ CDM substructure?

Dalal & Kochanek 2002

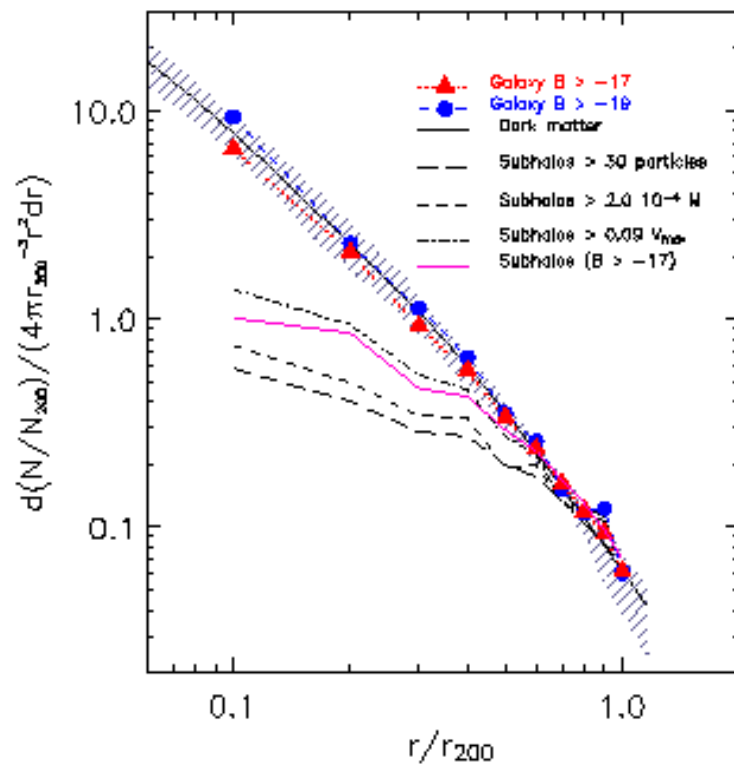


- In 4-image lensed quasars, the image *geometry* allows image classification into minima/saddles and brighter/fainter of each type
- Smooth lens models which fit the image positions usually *fail* to fit their relative brightness
- The brightest saddle image is preferentially dimmed, as expected for perturbation by fine structure
- This *cannot* be due to propagation effects, e.g. in the ISM of the lens
- It *cannot* be due to microlensing as radio images are too big
- 5 - 10% of lens mass must be in substructure but it might be just *projected* on the lens (Metcalf 2004)

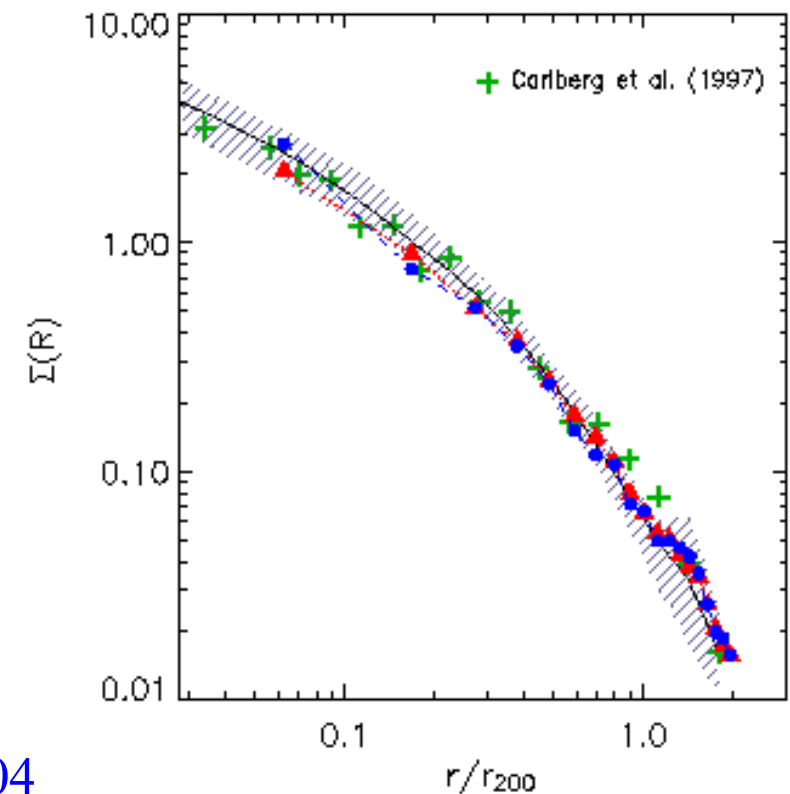
Λ CDM may have too little substructure?



- Radial density profile of substructure is much less concentrated than that of the DM as a whole
- too little substructure projected on the centre to produce anomalies? or to produce cluster galaxy profiles?



Gao et al 2004

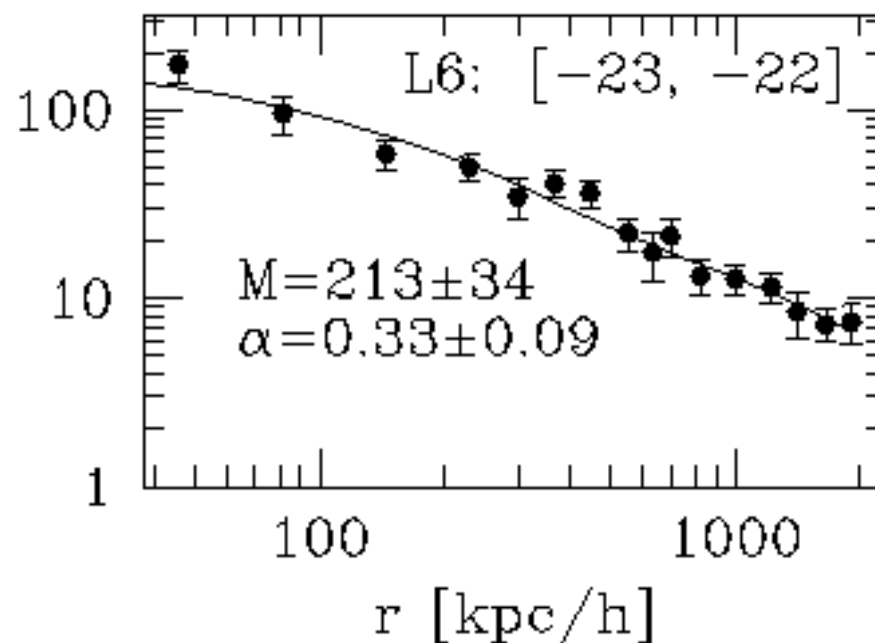
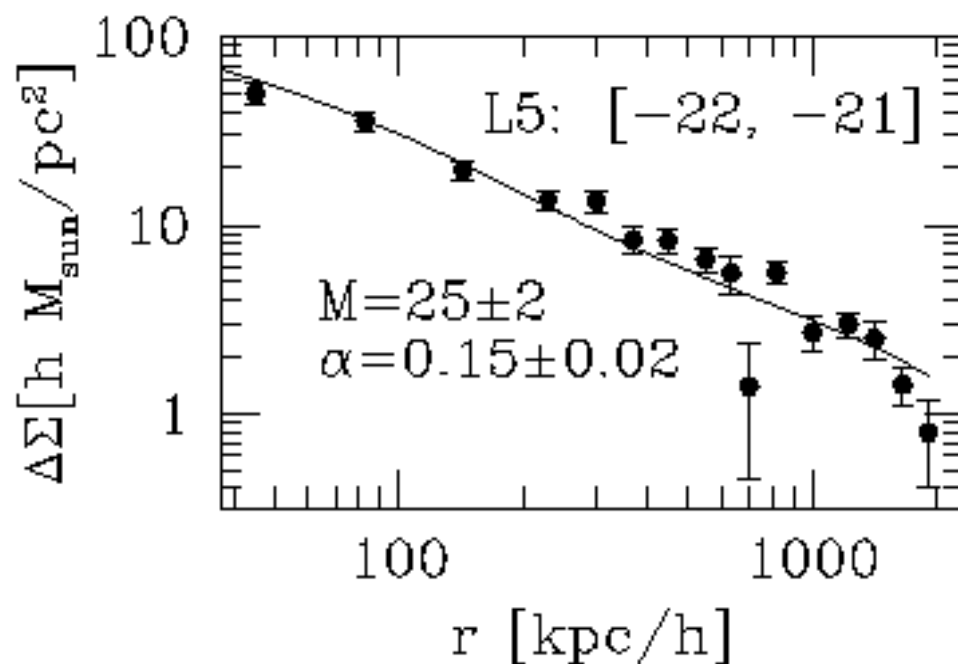
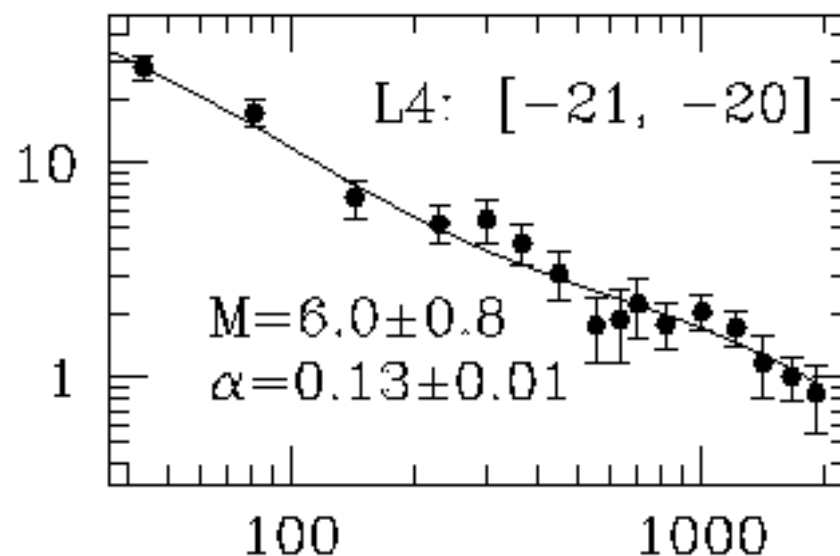
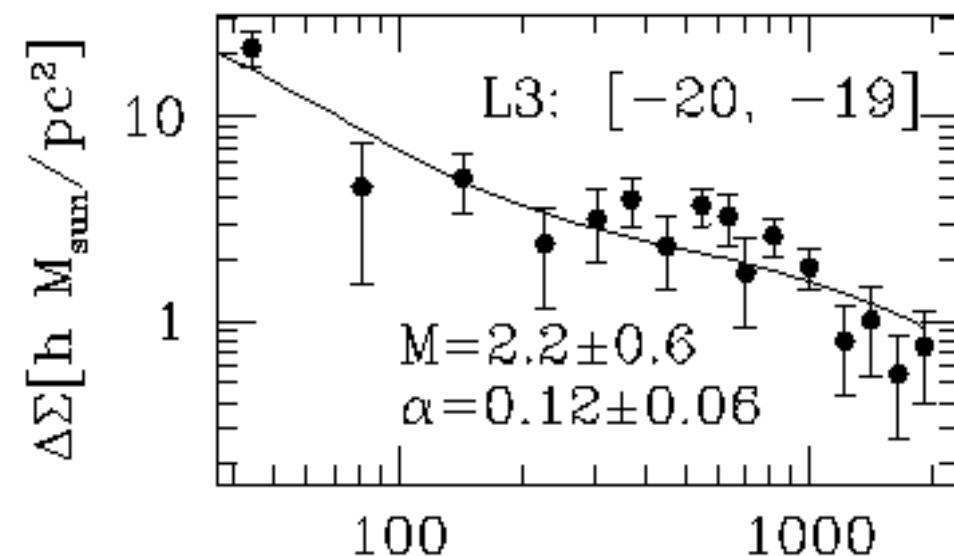


Science from DM/galaxy correlations

- Halo shapes and correlation with galaxy orientations
- Halo mass and extent as a function of galaxy properties
luminosity, morphology, SFR
- Halo mass as a function of environment
tidal truncation and its relation to morphology/SFR evolution
- Galaxy bias as a function of galaxy properties -- relation to formation history

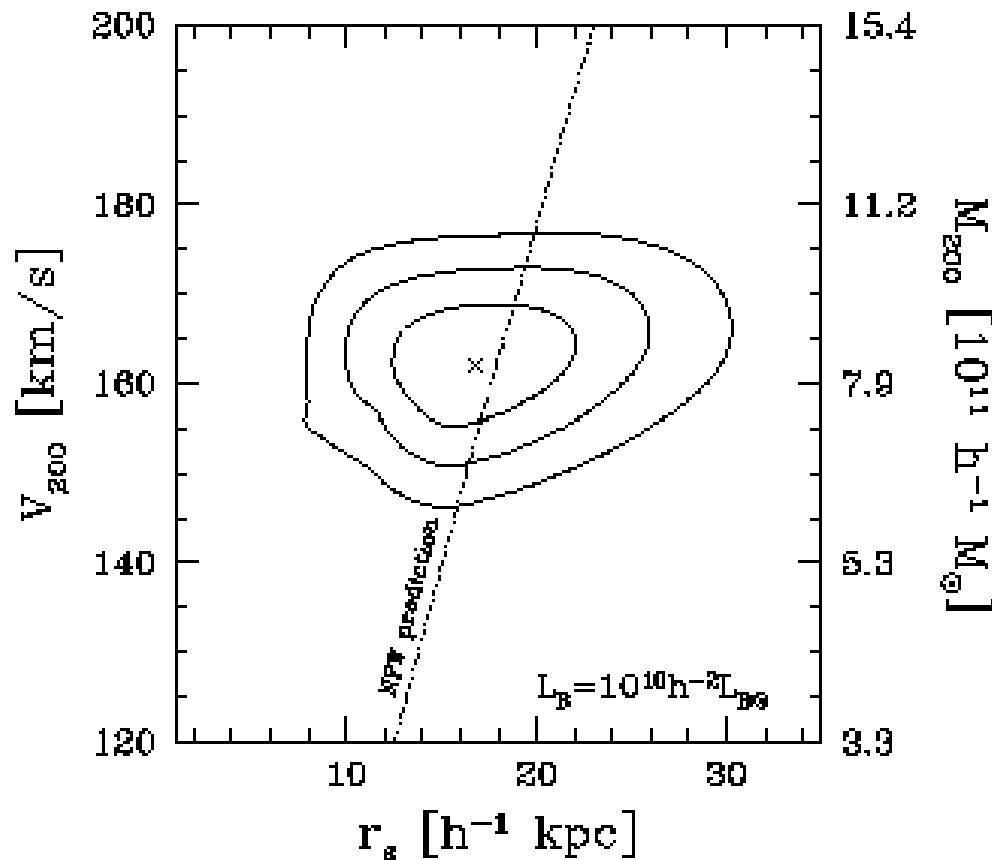
Weak lensing measures of halo mass profiles

Seljak et al 2004: from SDSS

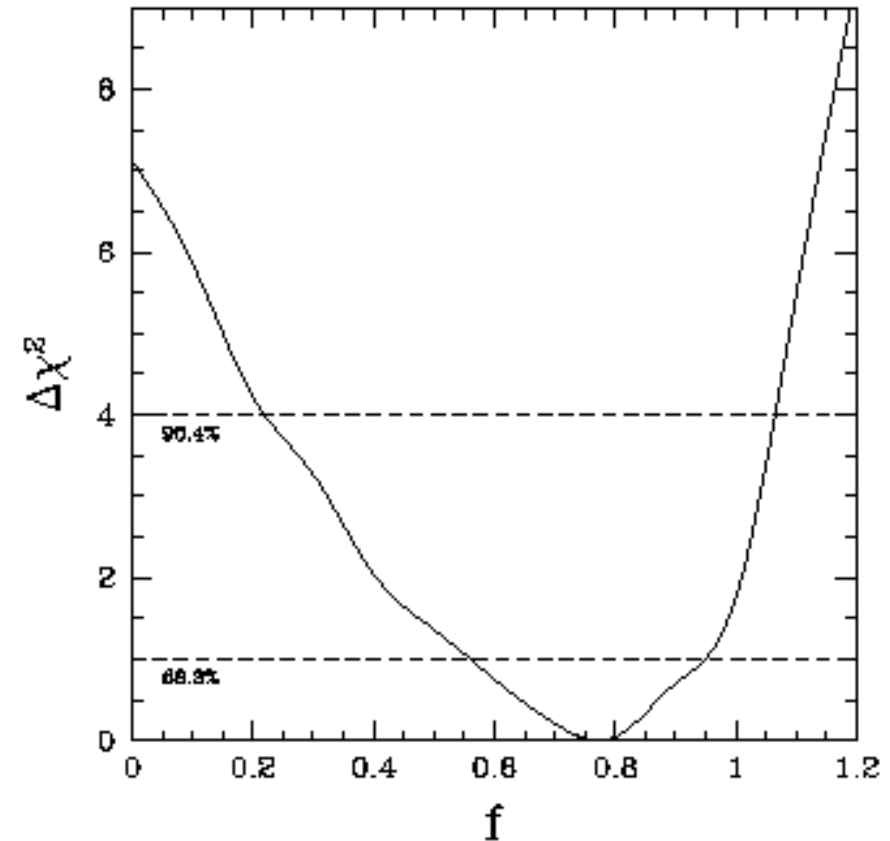


NFW confirmed + detection of halo flattening

Hoekstra, Yee & Gladders 2004



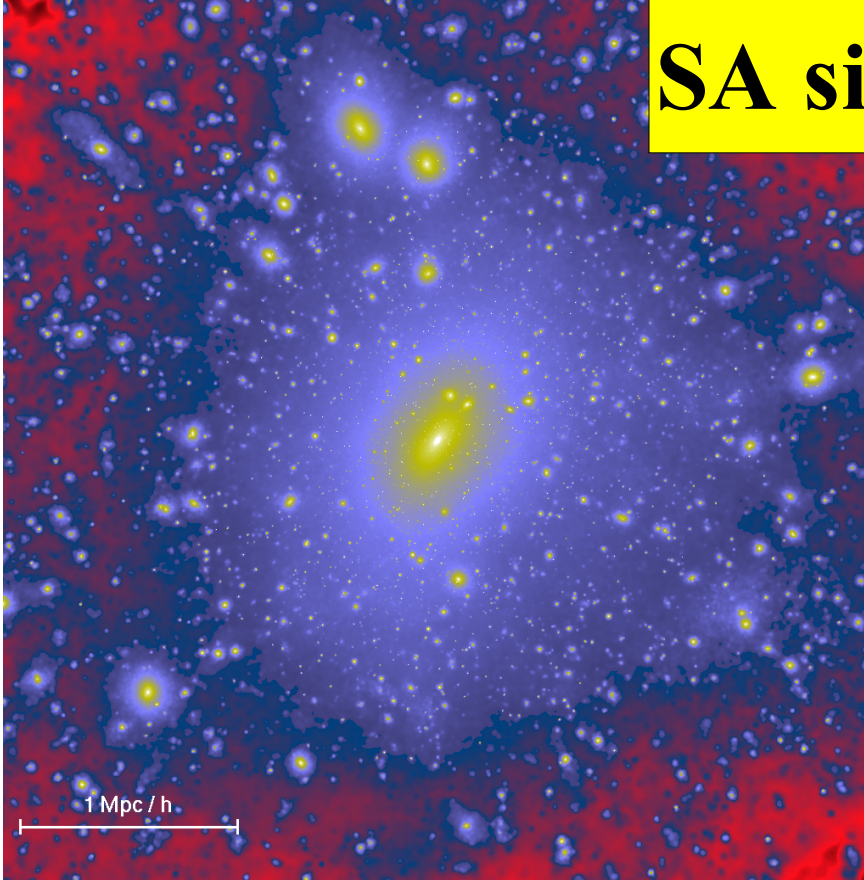
Halo profile is good fit to NFW
with the expected parameters



Ellipticity of halo assumed to be
 $f \times$ that of the central galaxy

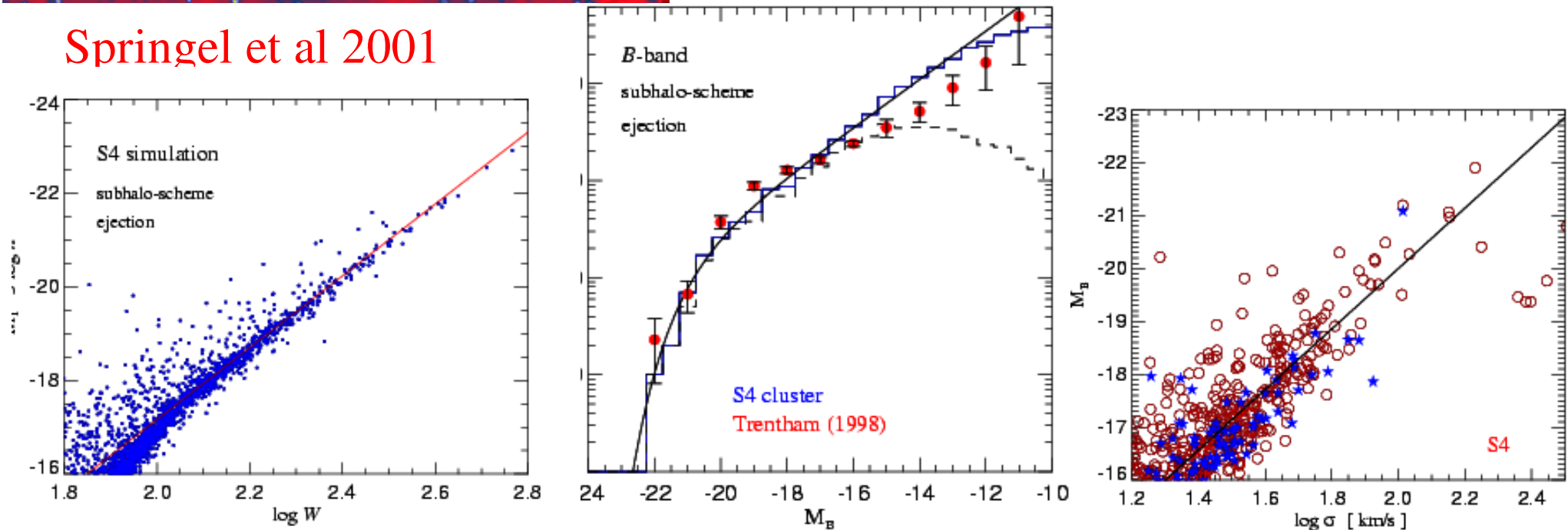
→ $\langle b/a \rangle = 0.67$

SA simulation of cluster formation



- Semi-analytic methods allow the simulation of a Coma cluster following all galaxies with $M_B < -12$
- Nearly all galaxies with $M_B < -16$ retain their own dark halos
- Protocluster can be analysed at high z

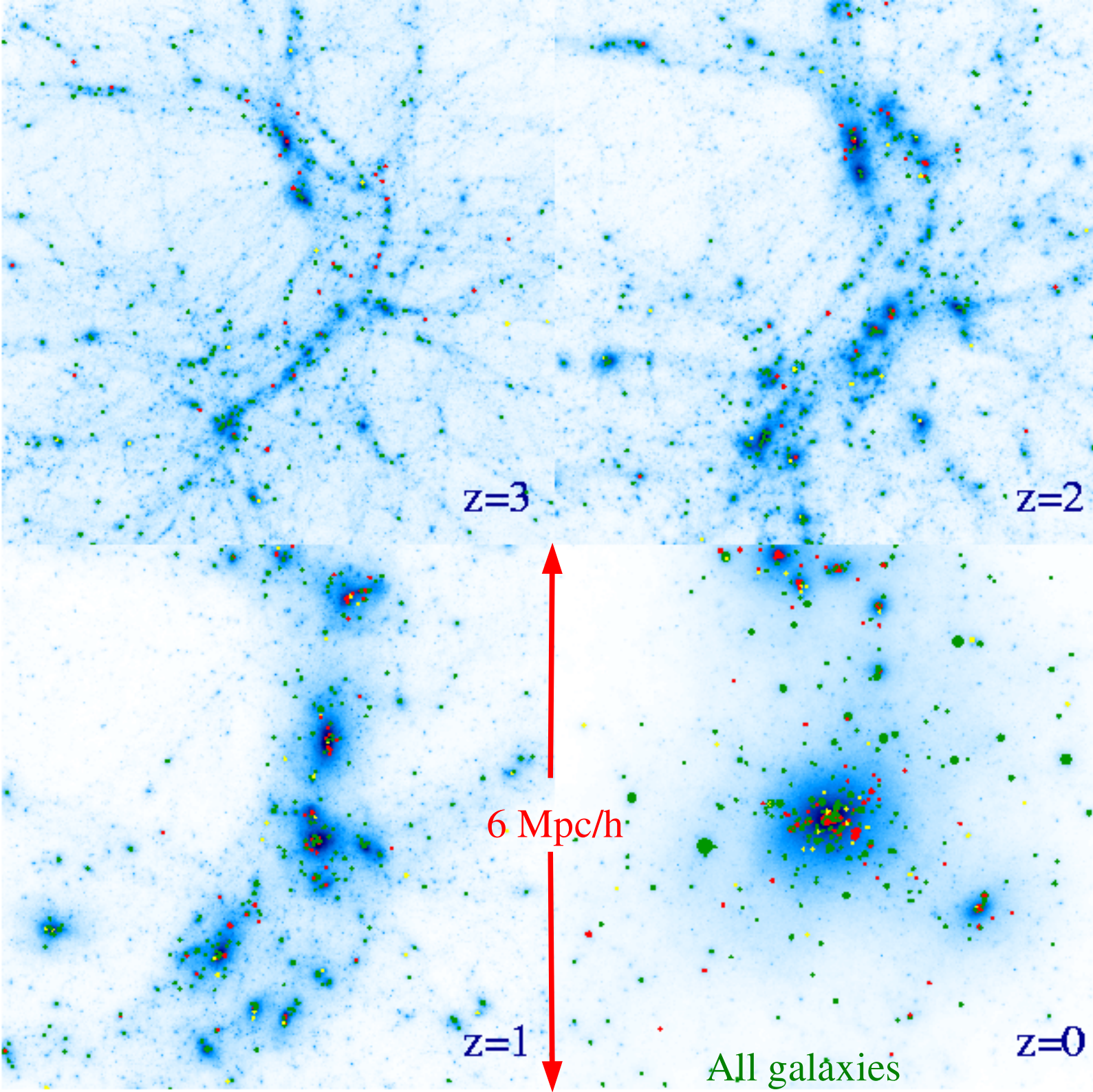
Springel et al 2001



Evolution of the galaxy population in a Coma-like cluster

Springel et al 2001

- Formation of the galaxies tracked within evolving (sub)halos
- Luminosity and mass of galaxies is uncertain
- Positions and velocities are followed well



$z=3$

$z=2$

6 Mpc/h

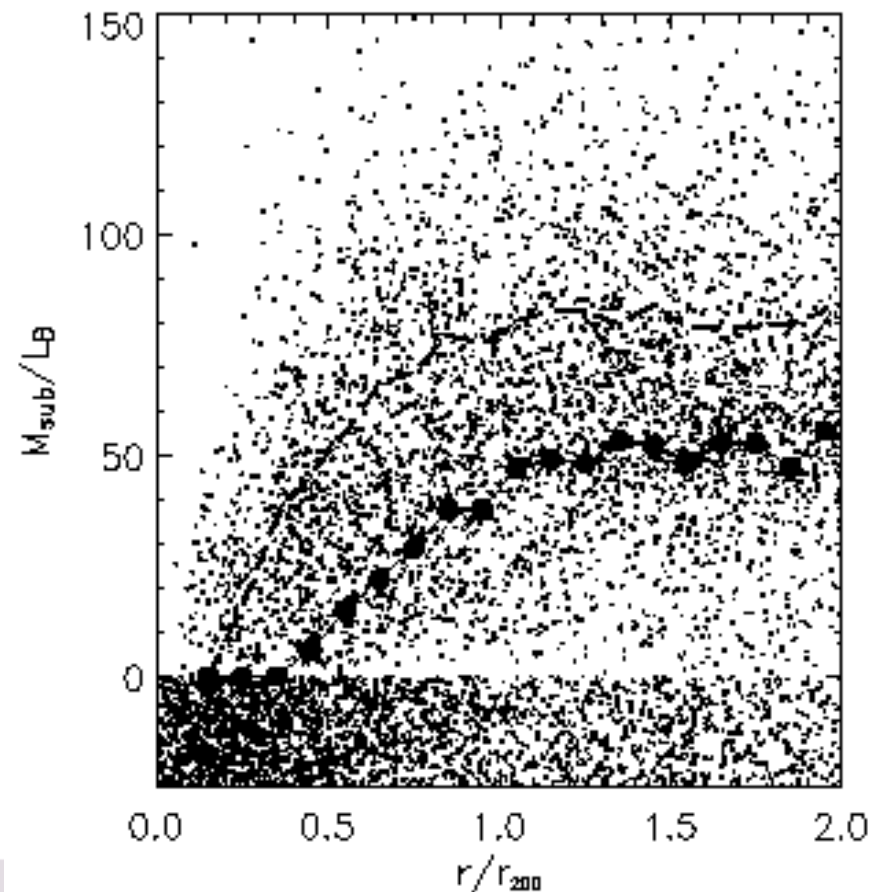
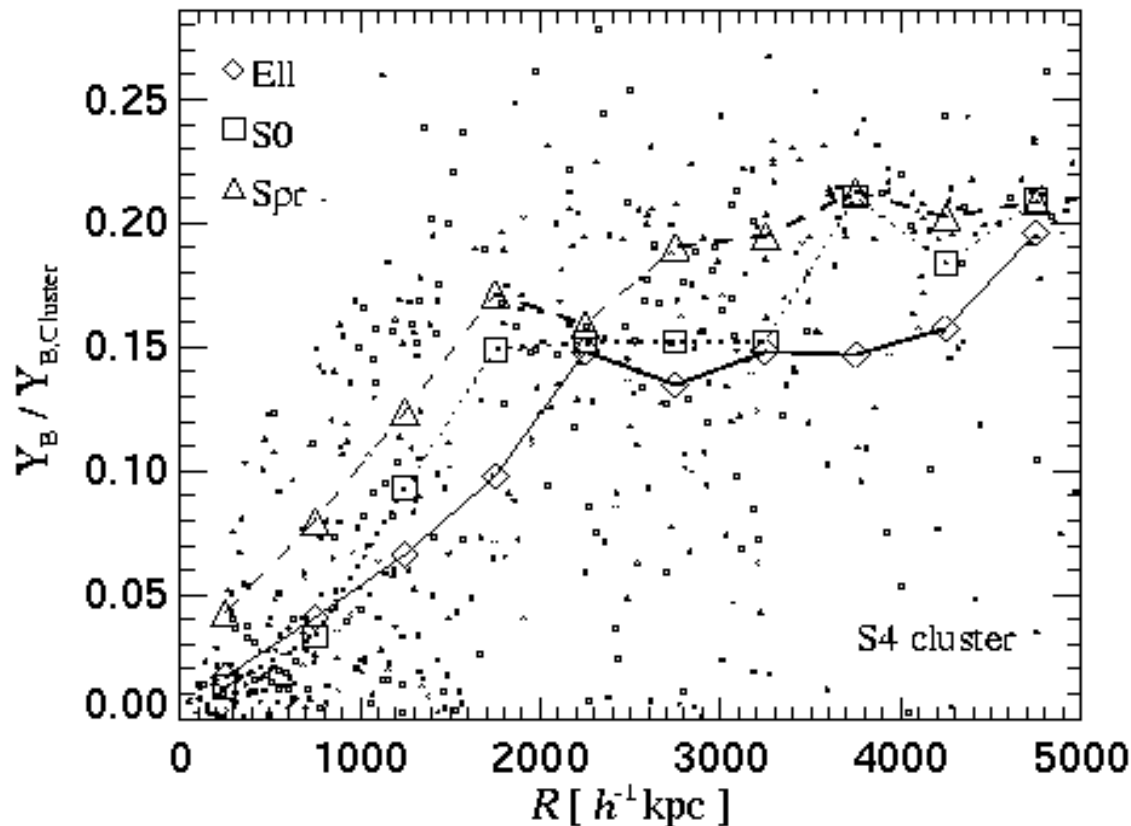
$z=1$

All galaxies

$z=0$

Halos of galaxies in clusters

- The halos of cluster galaxies are less massive at smaller radii
- E's have smaller halo masses than disk galaxies of the same L
- Many galaxies have almost all their halo (and some stars?) stripped

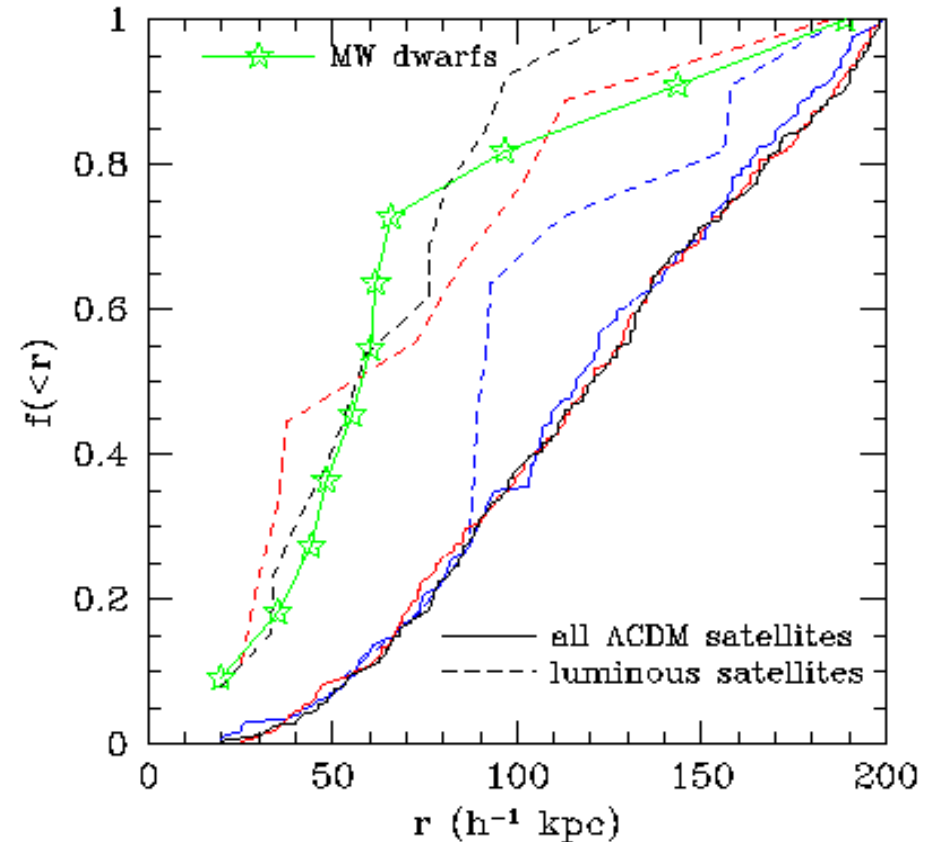
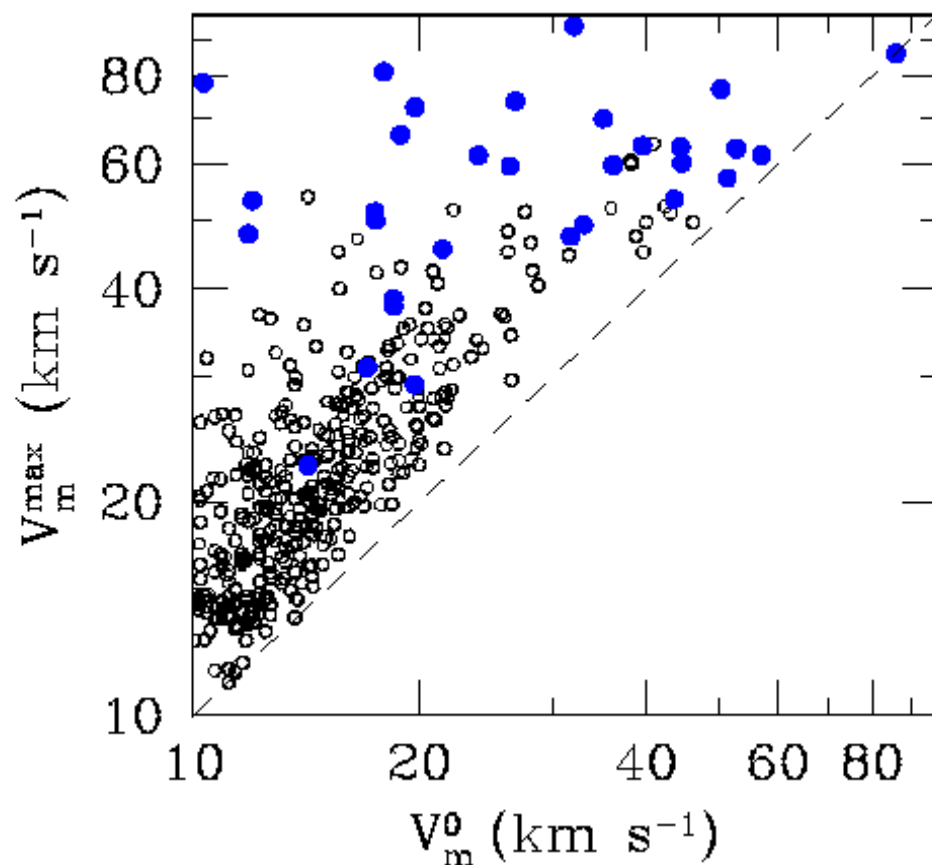


Springel et al (2001) All resolved subhaloes

Gao et al (2004) All galaxies $M < -17$

Milky Way satellites with SF modelling

Kravtsov, Gnedin & Klypin 2004



- Semi-analytic model in a high resolution simulation suggests that stars are preferentially in the satellites which had maximum past mass. These are more concentrated to MW centre than average

SGY

Coma

$z=0$

A constrained realisation of the Local Universe

Mathis et al 2002

Mass resolution
four times worse
than in the
Millennium Run

Centaurus

Virgo

Pisces Perseus

Pavo Indus

SGX

SGY

Opt

A constrained realisation of the Local Universe

Mathis et al 2002

Mass resolution
four times worse
than in the
Millennium Run

Modelling here
gives all galaxies
with $M < -17.5$

MR will give all
galaxies with
 $M < -16$

SGX

Questions for Galaxy/DM simulations

- Shape and extent of dark matter halos
- Orientation of galaxies within halos
- Correlation of halo and galaxy properties
- Relation of halo properties to larger scale structure
- Truncation of halos within larger structures
- Line-of-sight effects along cosmological light-cones (Sachs-Wolfe, CMB-lensing/galaxy distribution cross-correlations, higher-order shear and shear-galaxy correlations)

Precision cosmology will require precise (galaxy) simulations!