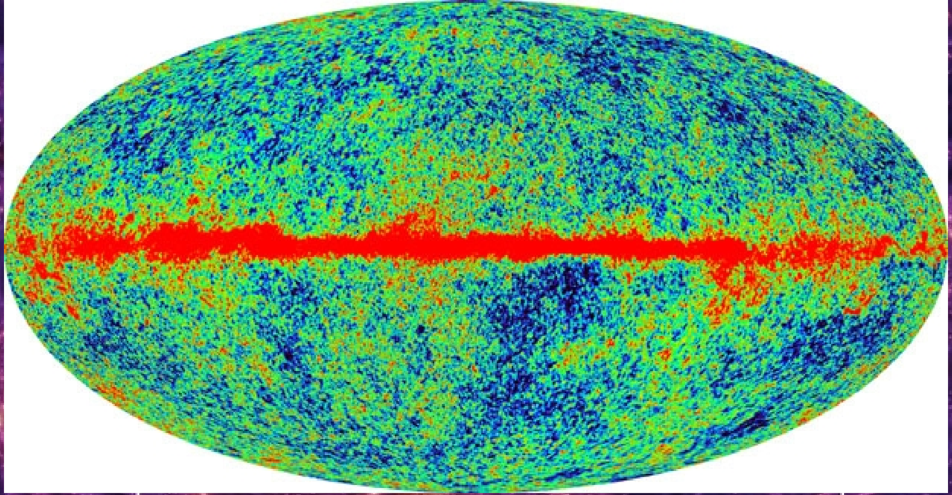


CITA seminar
November 2007

Galaxy halos at (very) high resolution

Simon White & Mark Vogelsberger
Max Planck Institute for Astrophysics

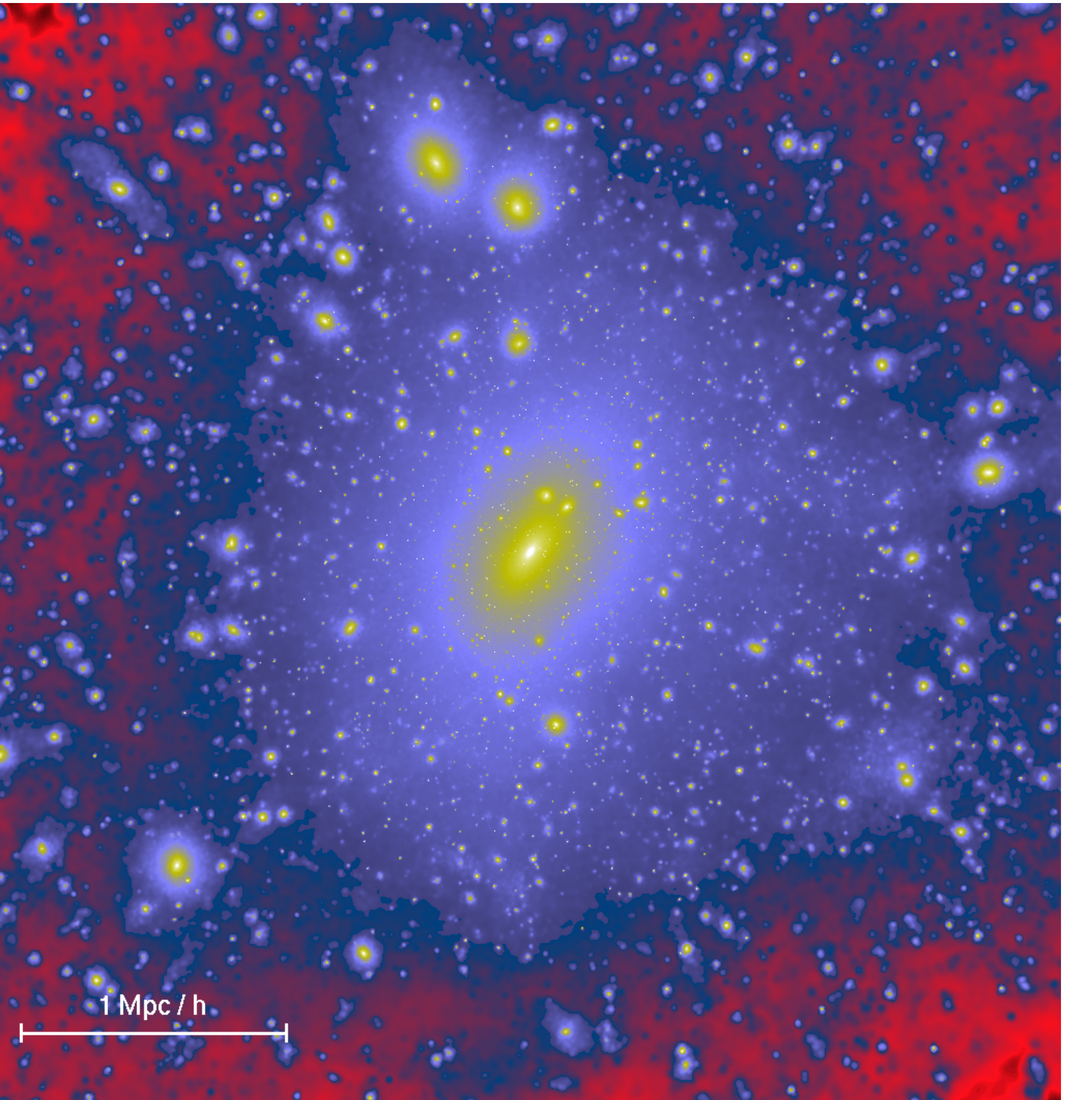
Visualizing Darkness

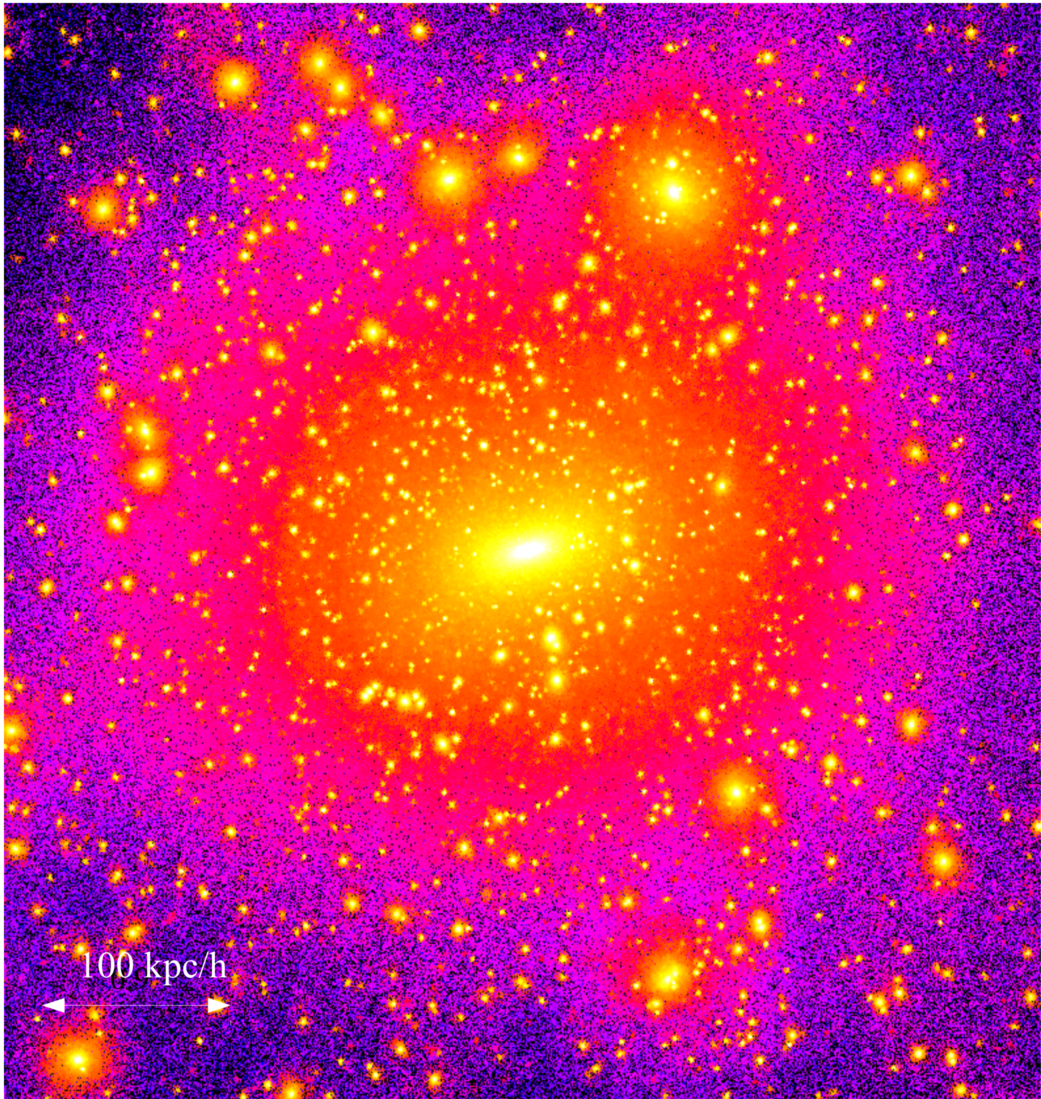
- Uniformity, filamentarity, hierarchy – it all depends on scale
- The smooth becomes rough with the passing of time
- The Milky Way hums with memories of its past

Small-scale structure in ΛCDM halos

A rich galaxy cluster halo
Springel et al 2001

A 'Milky Way' halo
Power et al 2002





ΛCDM galaxy halos (without galaxies!)

- Halos extend to ~10 times the 'visible' radius of galaxies and contain ~10 times the mass in the visible regions

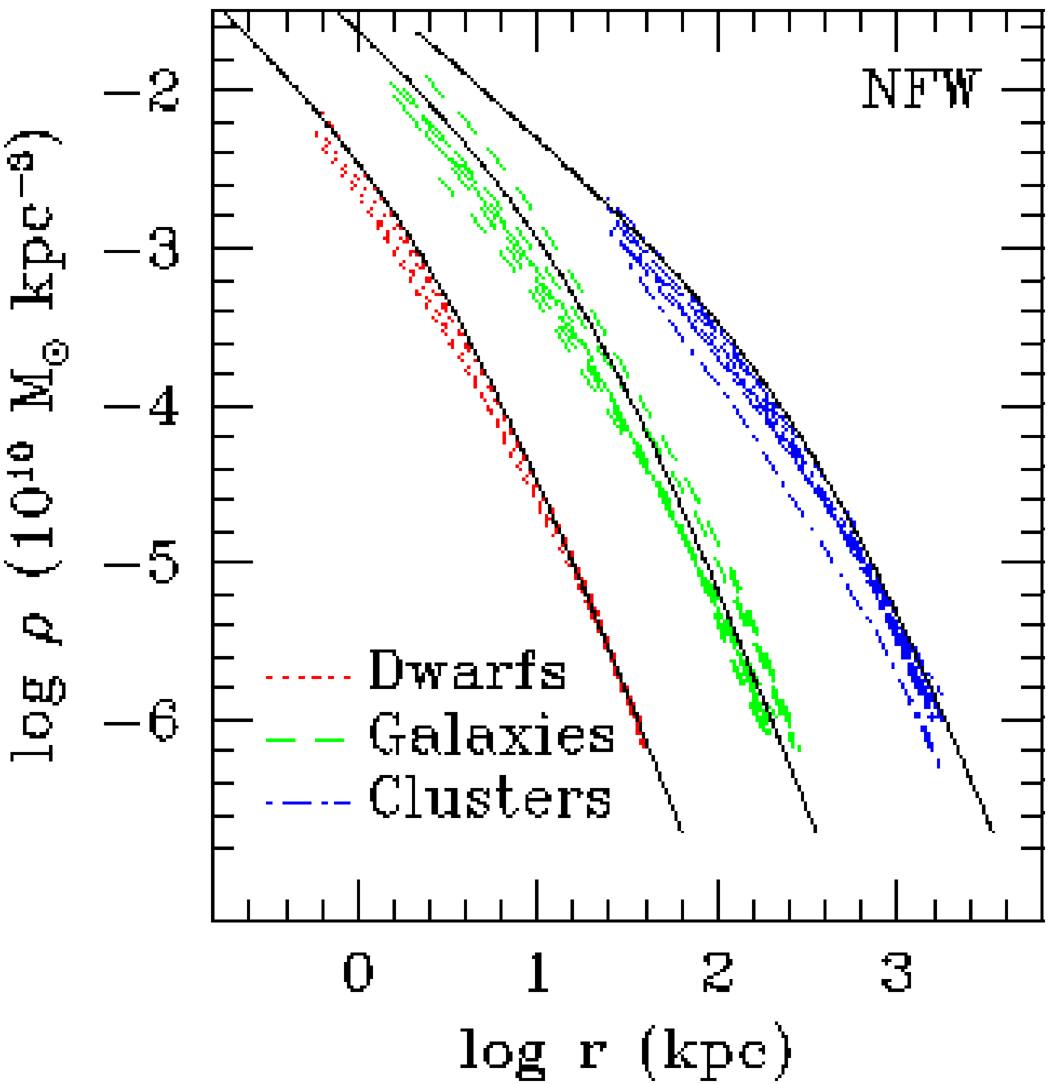
- Halos are not spherical but approximate triaxial ellipsoids
  - -- more prolate than oblate
  - -- axial ratios greater than two are common

- "Cuspy" density profiles with outwardly increasing slopes
  -- $d \ln \varrho / d \ln r = \gamma$ with $\gamma < -2.5$ at large r
  $\gamma > -1.2$ at small r

- Substantial numbers of self-bound subhalos contain ~10% of the halo's mass and have $d N / d M \sim M^{-1.8}$

→ Most substructure mass is in most massive subhalos

Density profiles of dark matter halos



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times

-- a universal profile shape --

$$\rho(r)/\rho_{crit} \approx \delta \; r_s \,/\, r(1 + r/r_s)^2$$

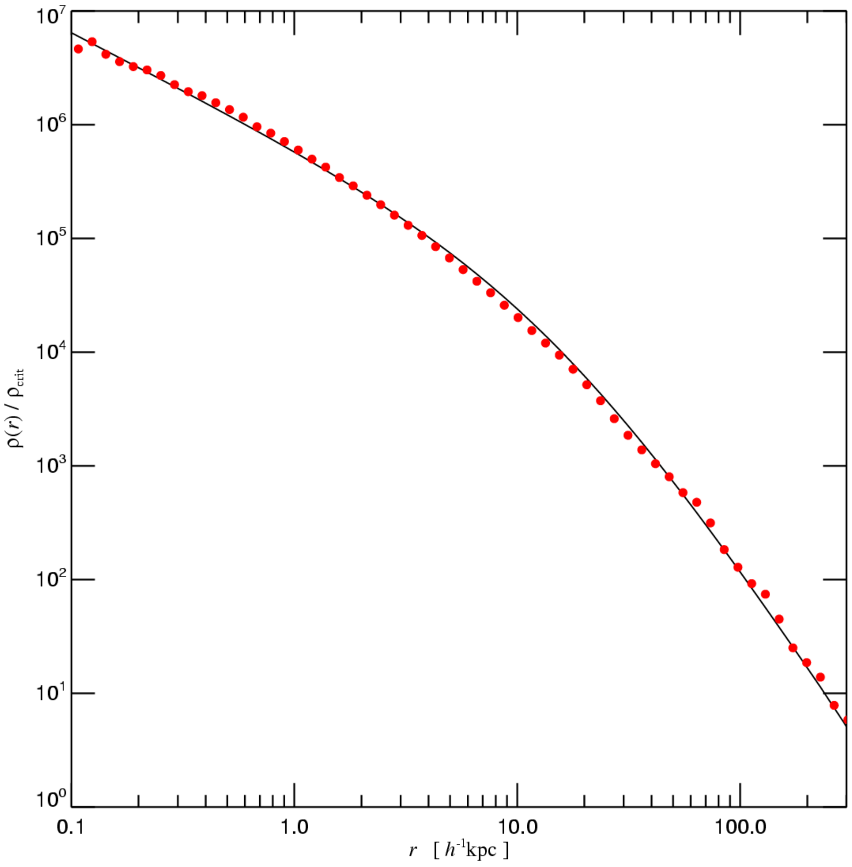
More massive halos and halos that form earlier have higher densities (bigger δ)

A high-resolution Milky Way halo

600 kpc

Navarro et al 2006

$N_{200} \sim 3 \times 10^7$

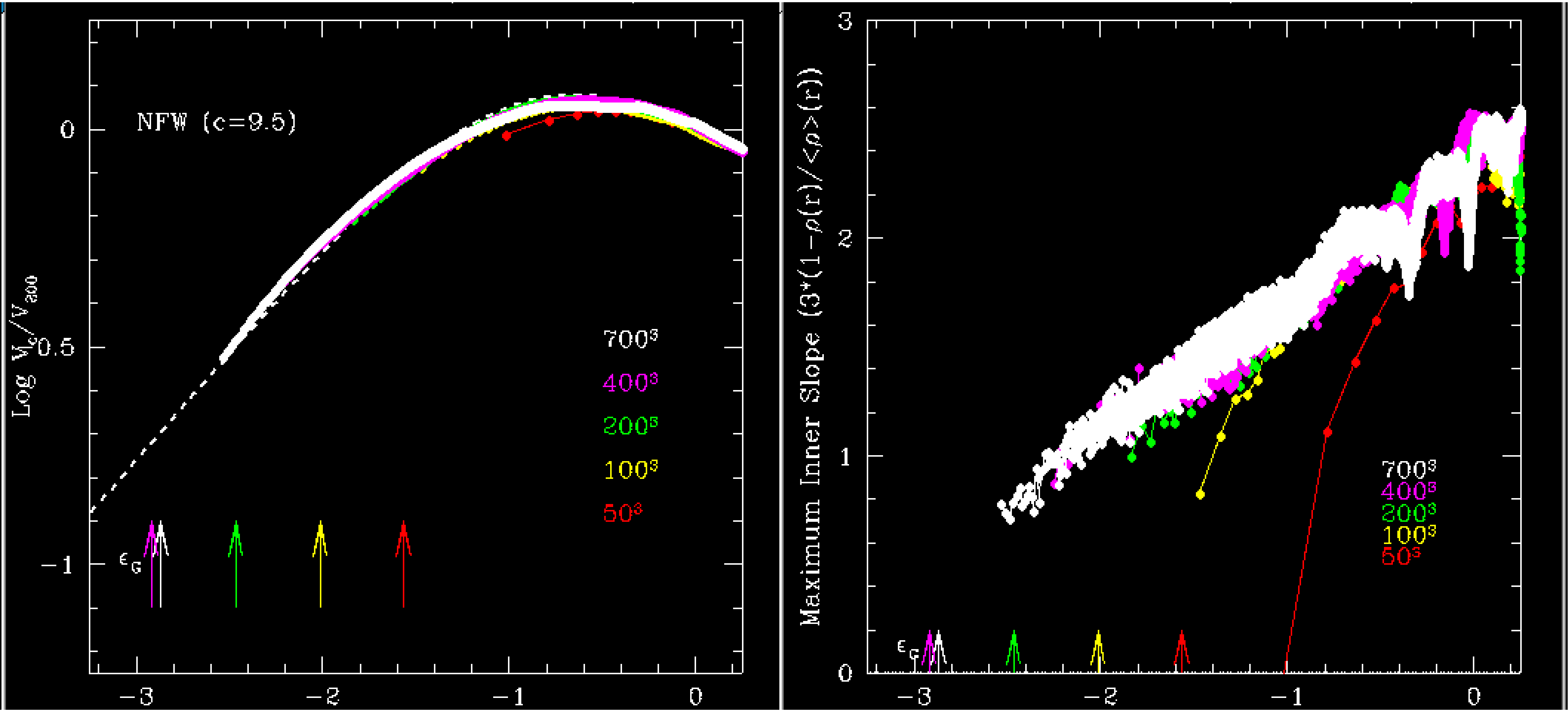


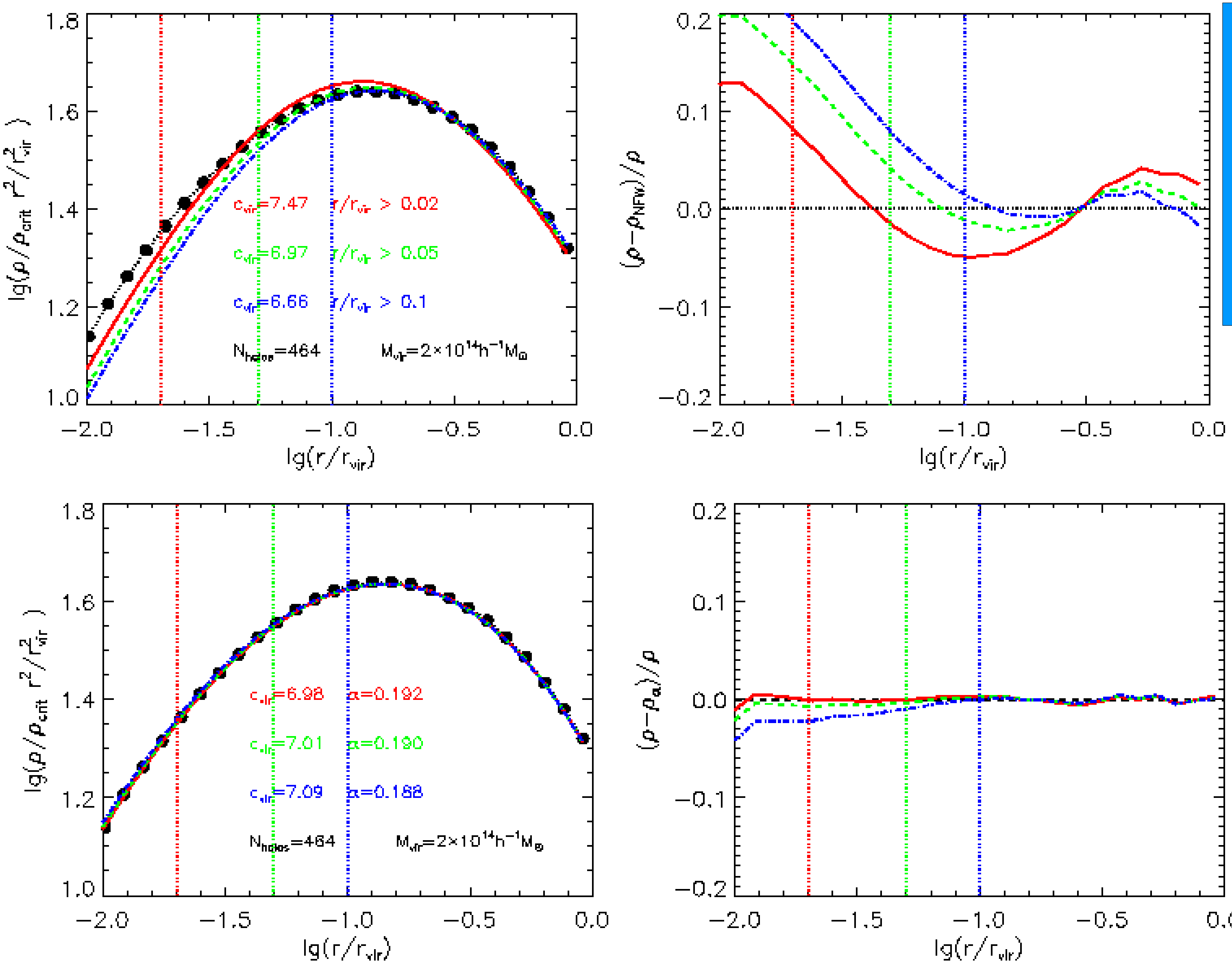
Convergence tests on density profile shape

Navarro et al 2006

DM profiles are converged to a few hundred parsecs

The inner asymptotic slope must be shallower than – 0.9





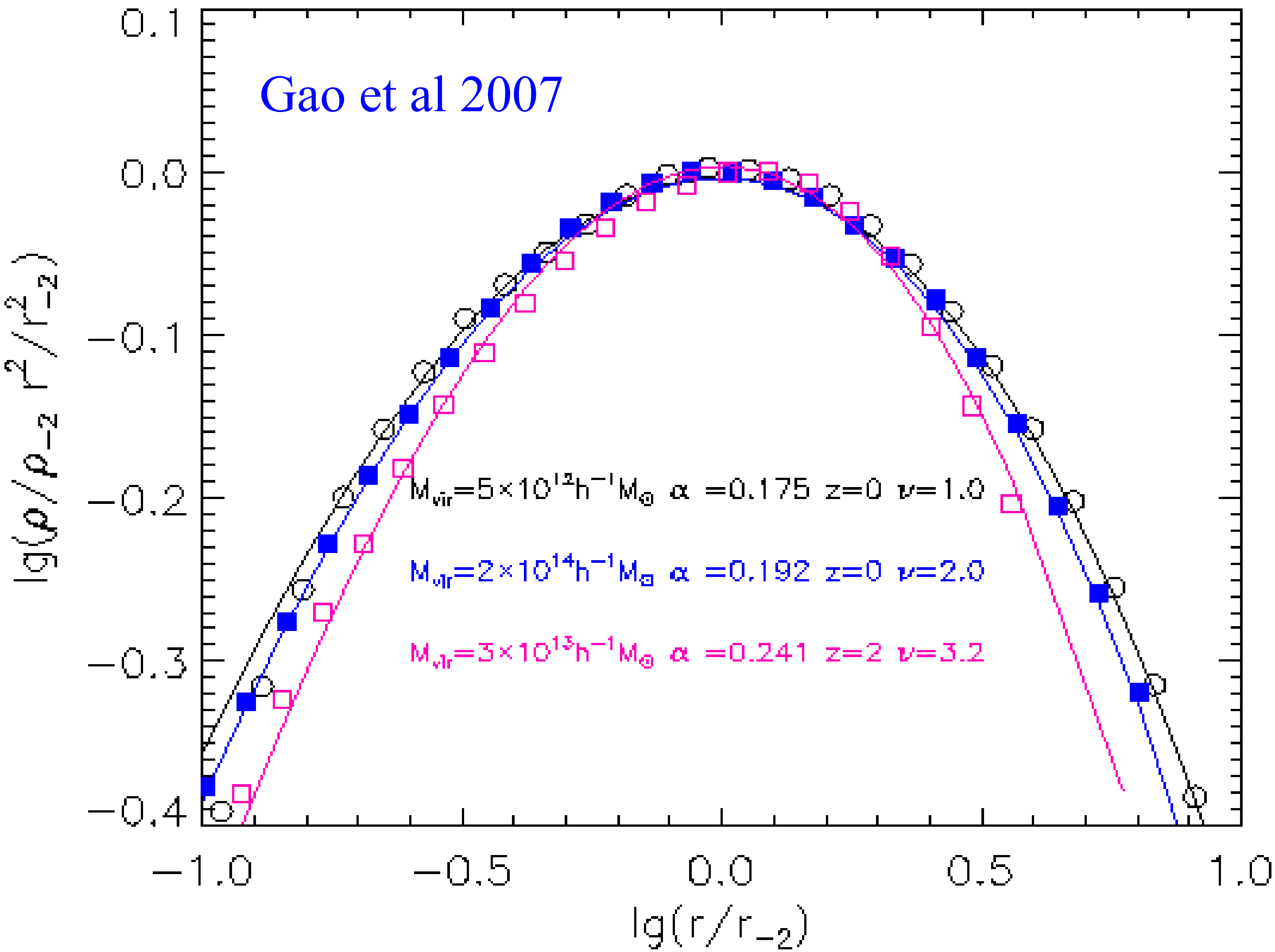
Einasto fits better than NFW!

Gao et al 2007

In 1963 Einasto suggested modelling the Galactic spheroid with

$$\ln\left[\varrho(r)/\varrho_{-2}\right] = -\,2/\alpha\,\left[\left(r/r_{-2}\right)^{\alpha} - 1\right] \rightarrow \text{shape parameter, } \alpha$$

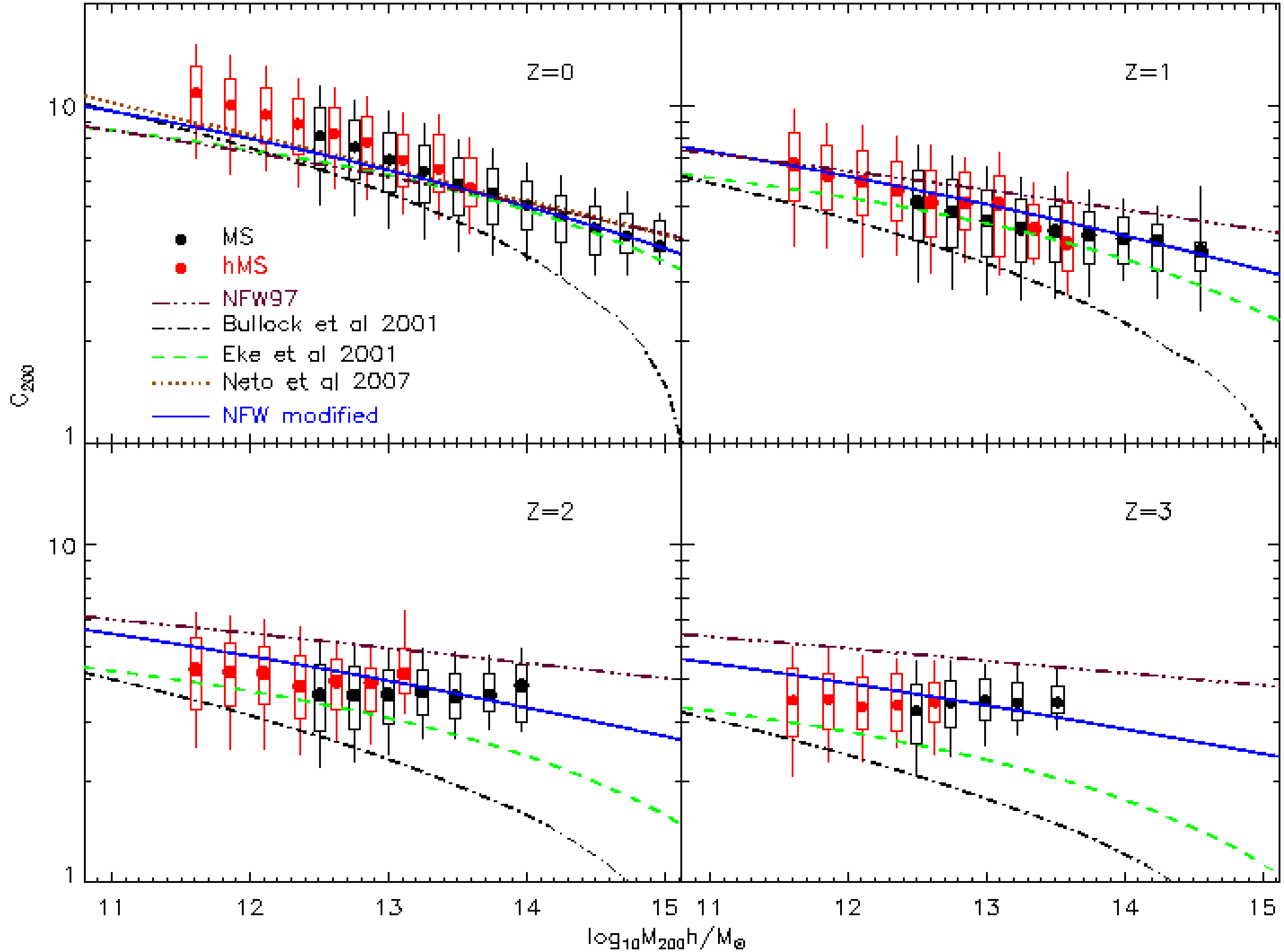
Profile shape depends on halo mass

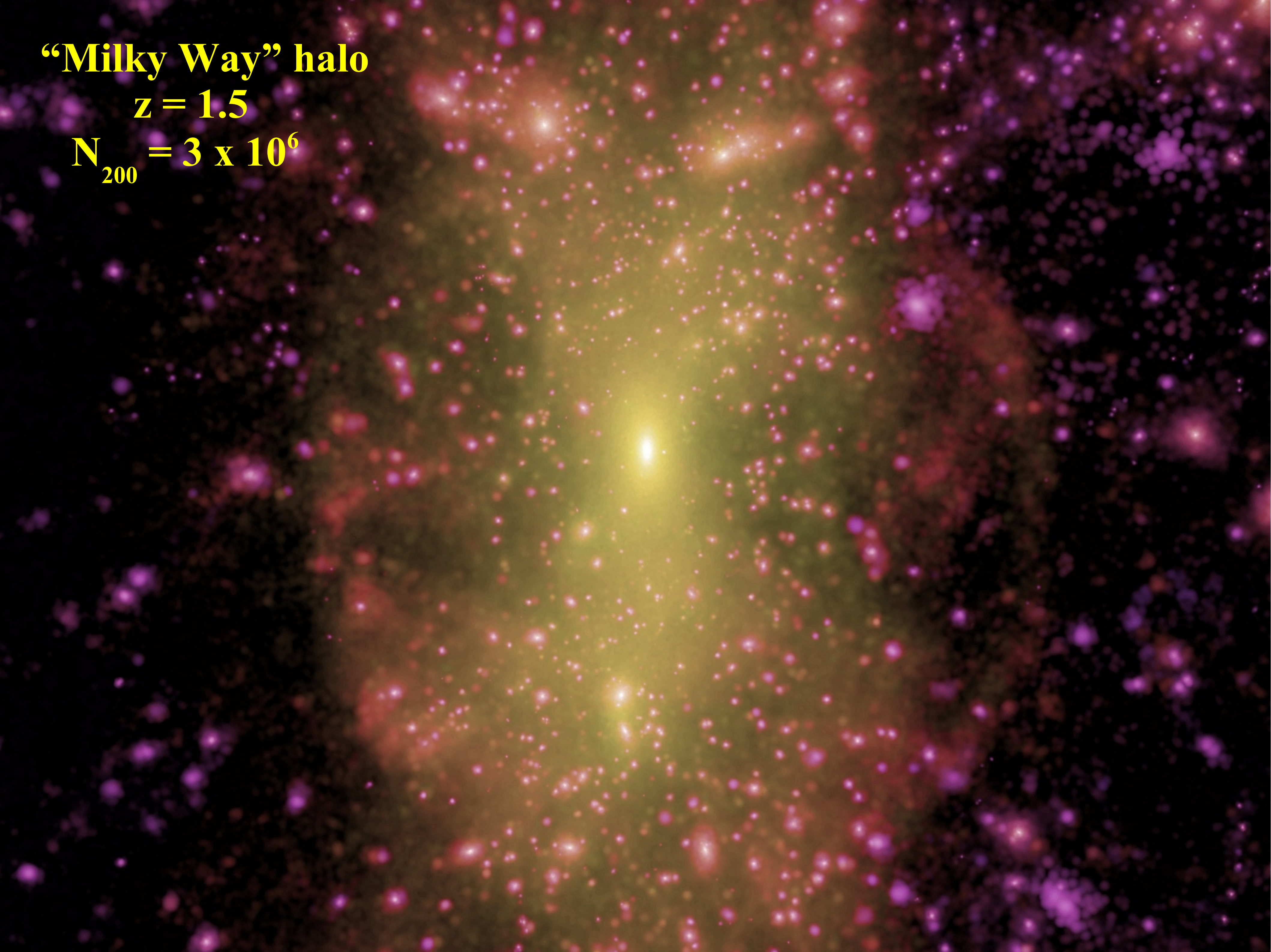


The Einasto shape parameter α depends on the dimensionless "peak height" parameter $\nu(M, z)$

Published relations don't fit the measured c(M, z)

Gao et al 2007



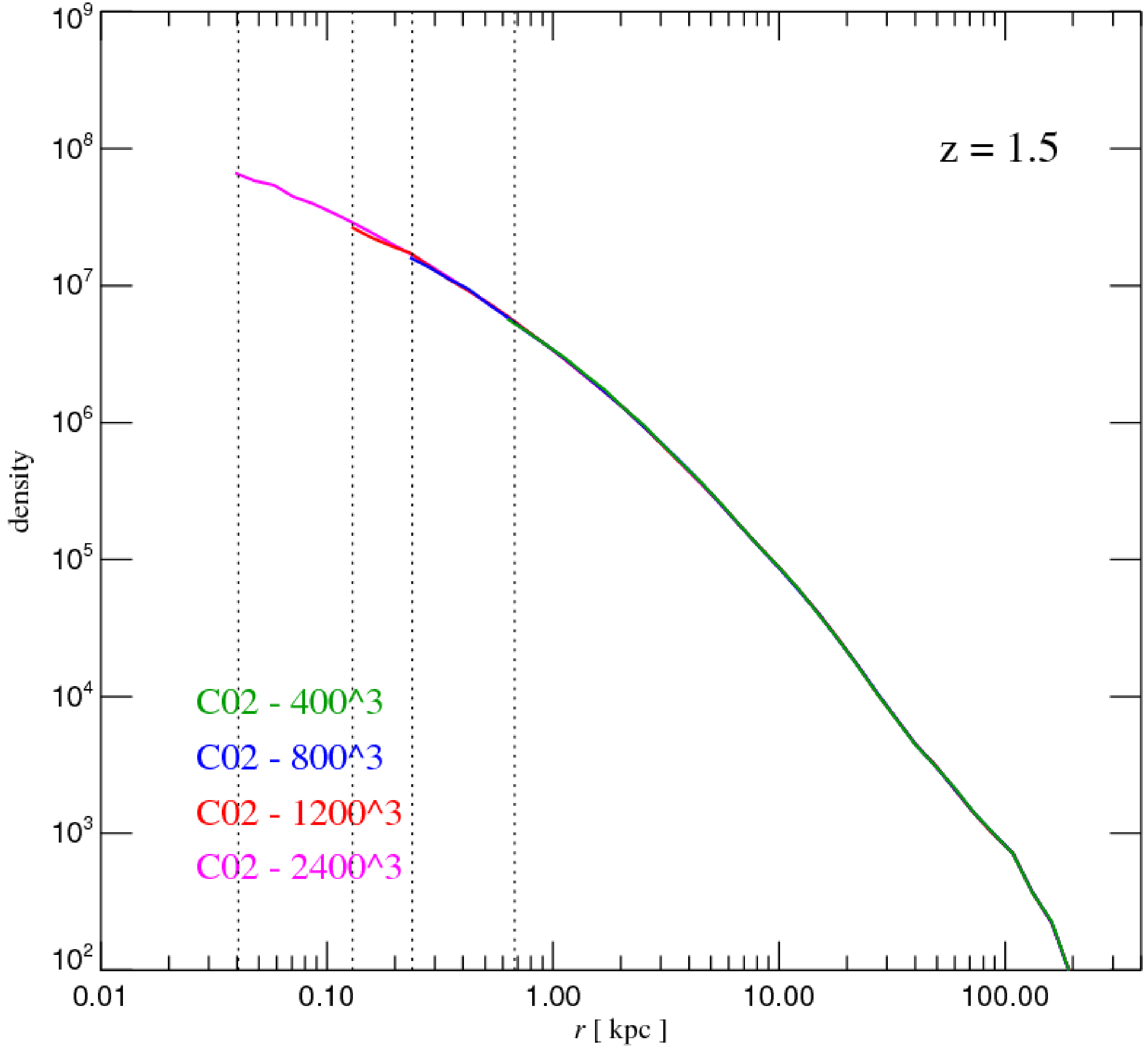

"Milky Way" halo
z = 1.5
N200 = 3 x 10^6


"Milky Way" halo
z = 1.5
N200 = 94 x 10^6


"Milky Way" halo
z = 1.5
N200 = 750 x 10^6

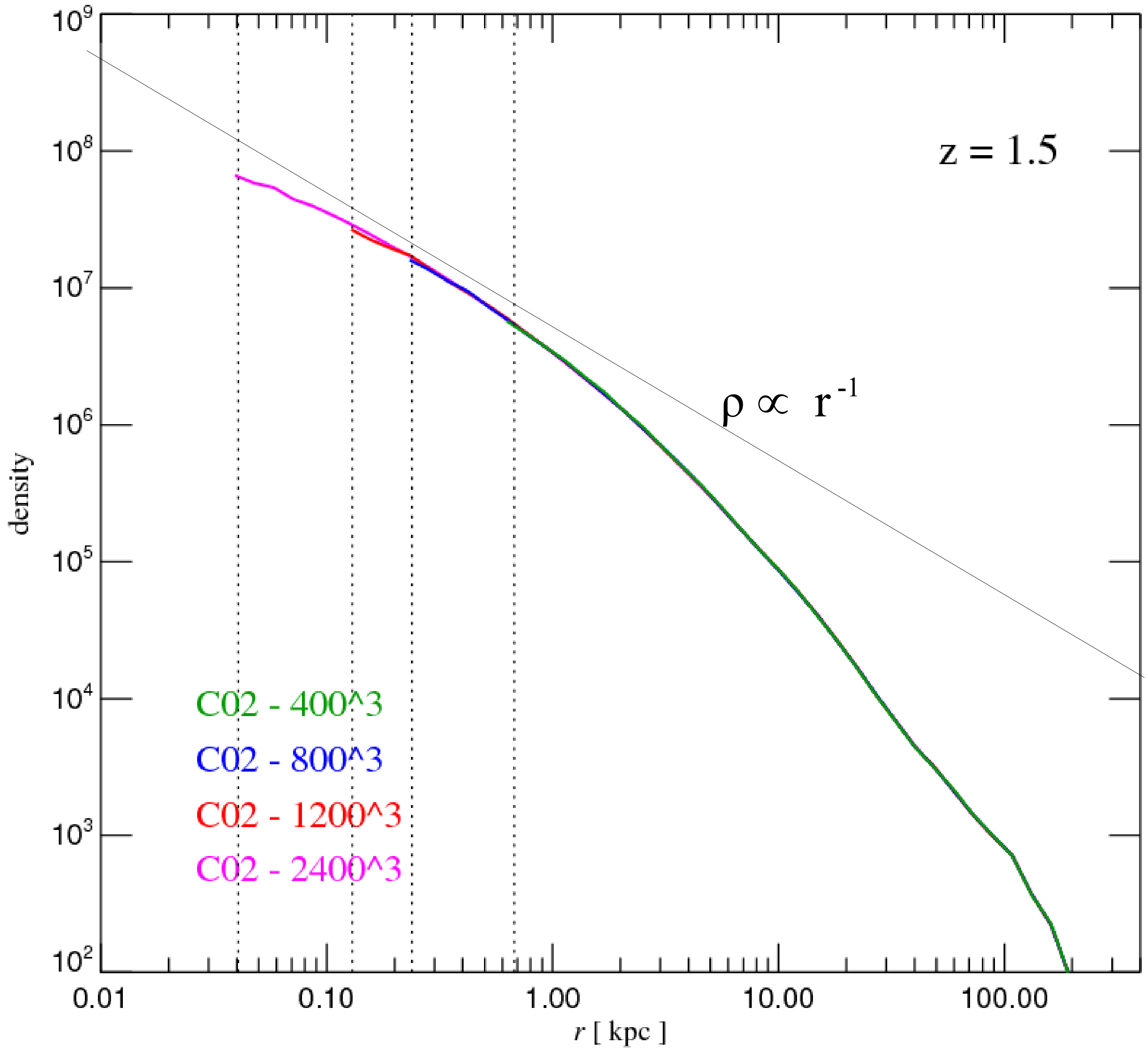
How well do density profiles converge?

Virgo Consortium 2007



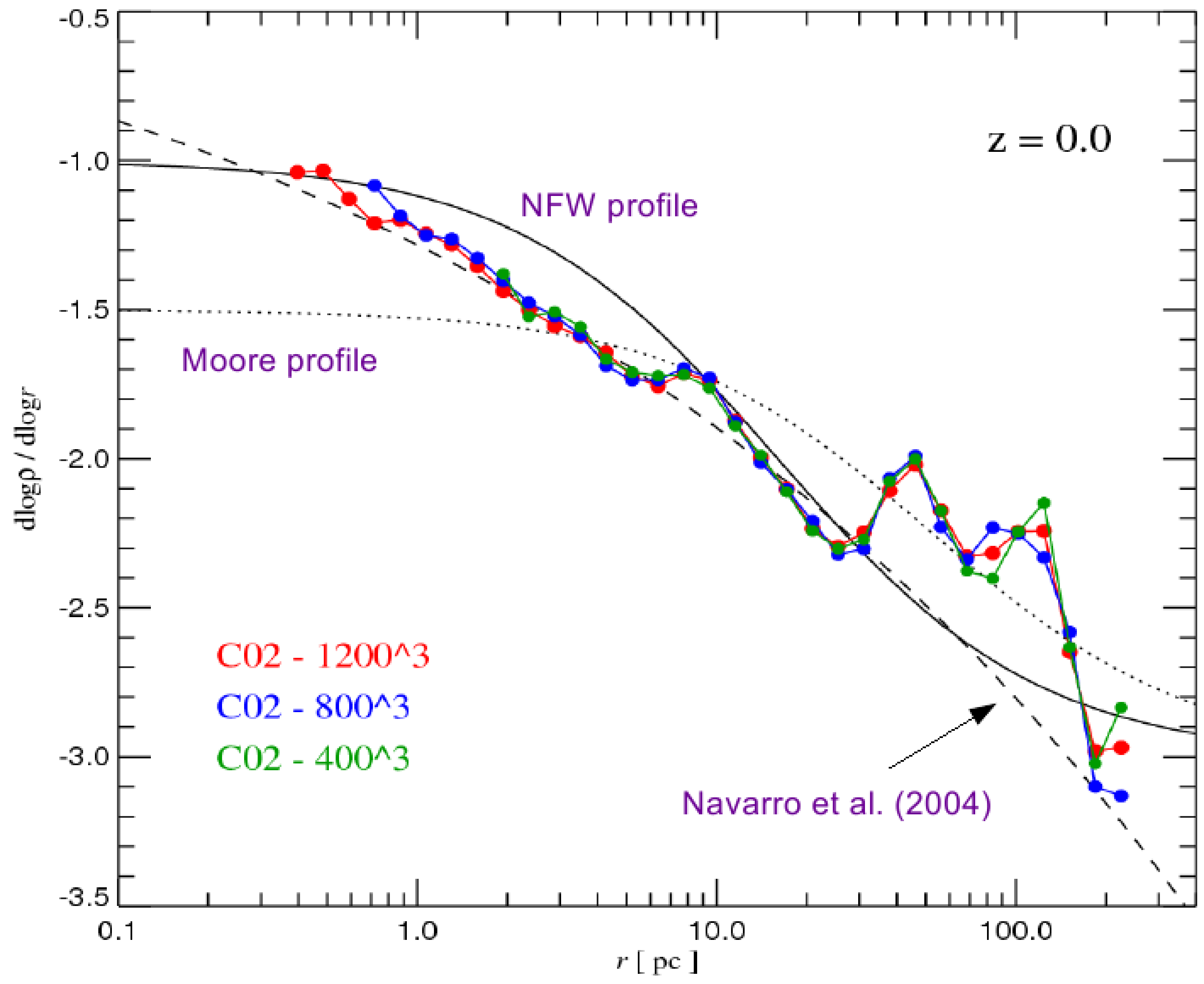
How well do density profiles converge?

Virgo Consortium 2007



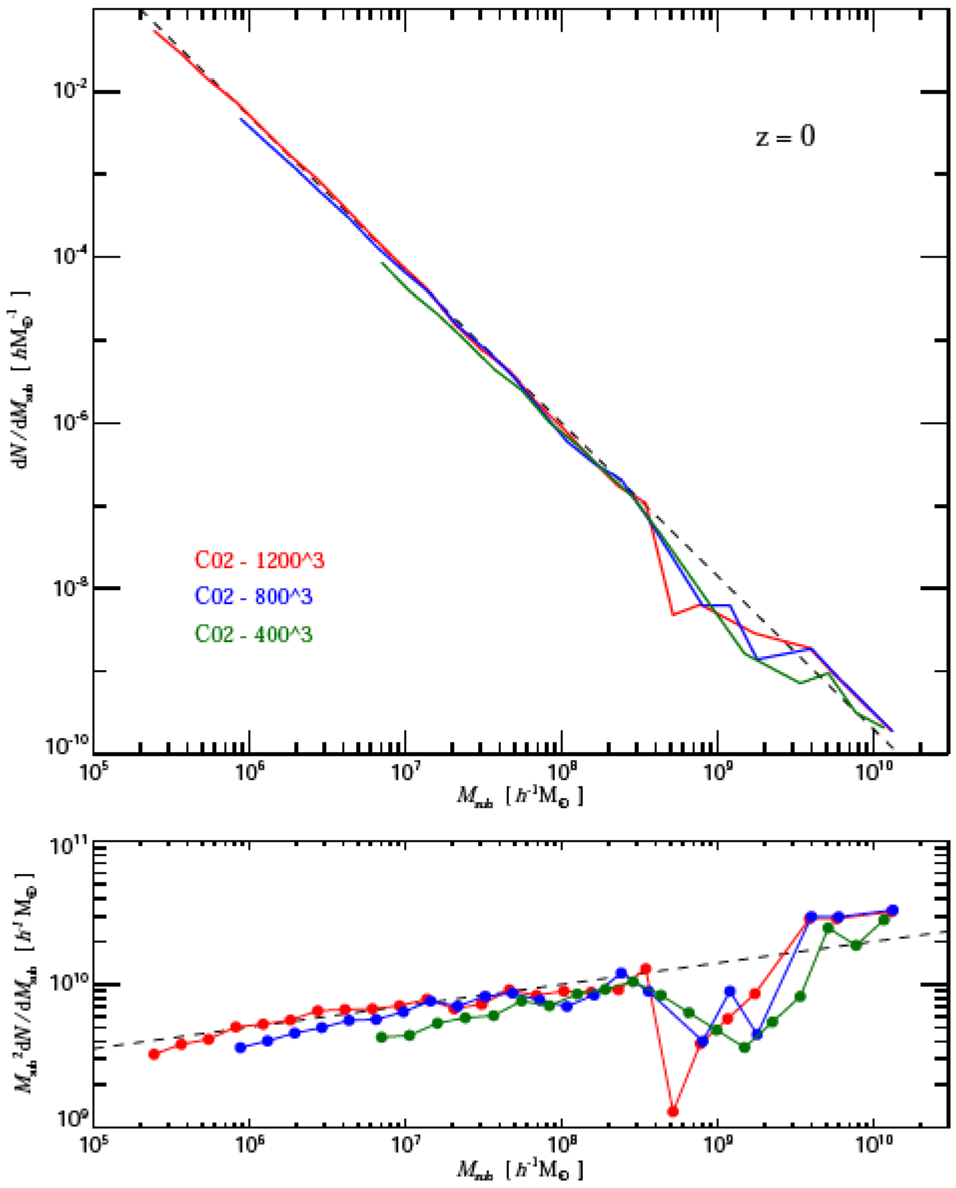
How well do density profiles converge?

Virgo Consortium 2007



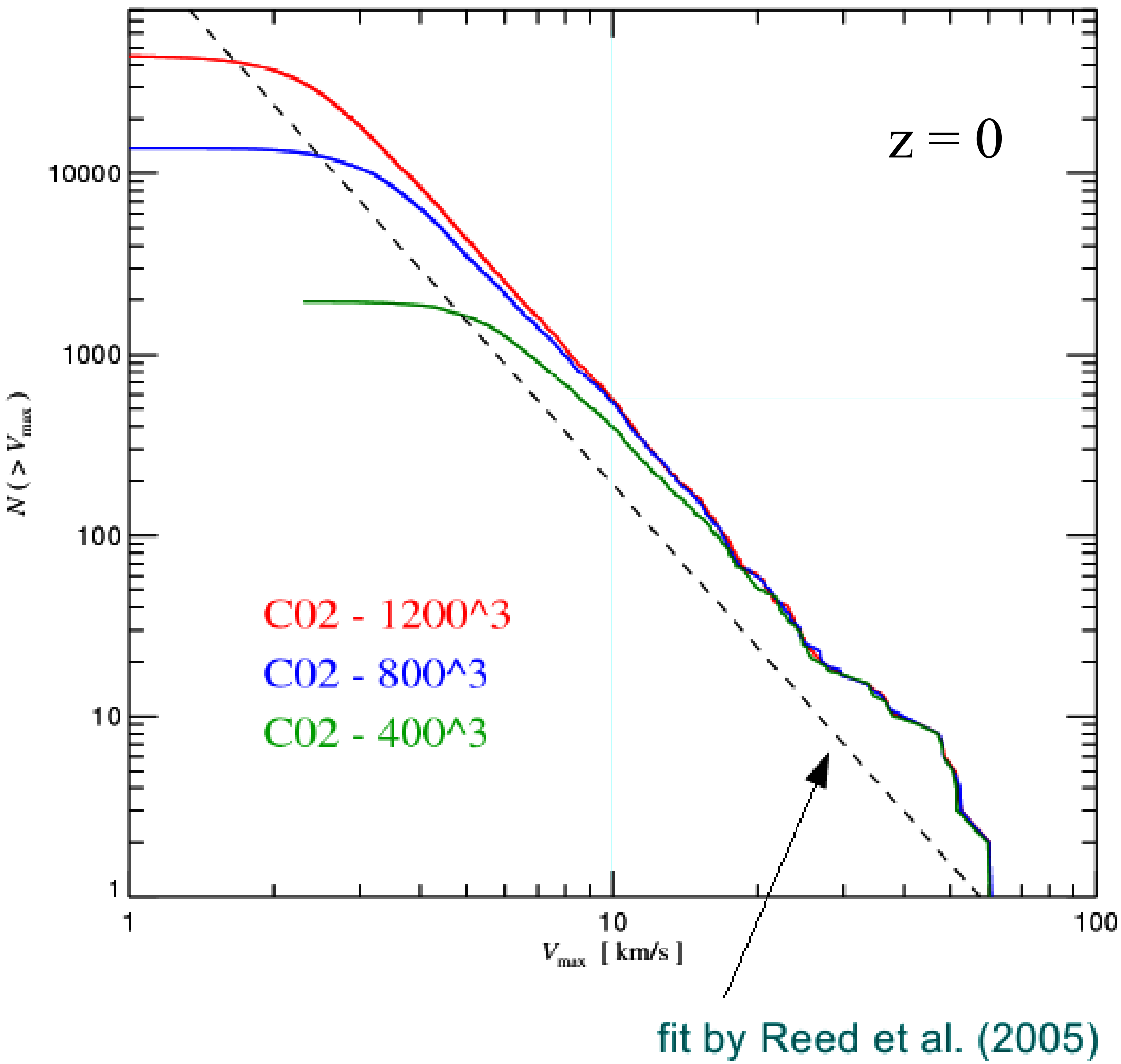
How well does substructure converge?

Virgo Consortium 2007

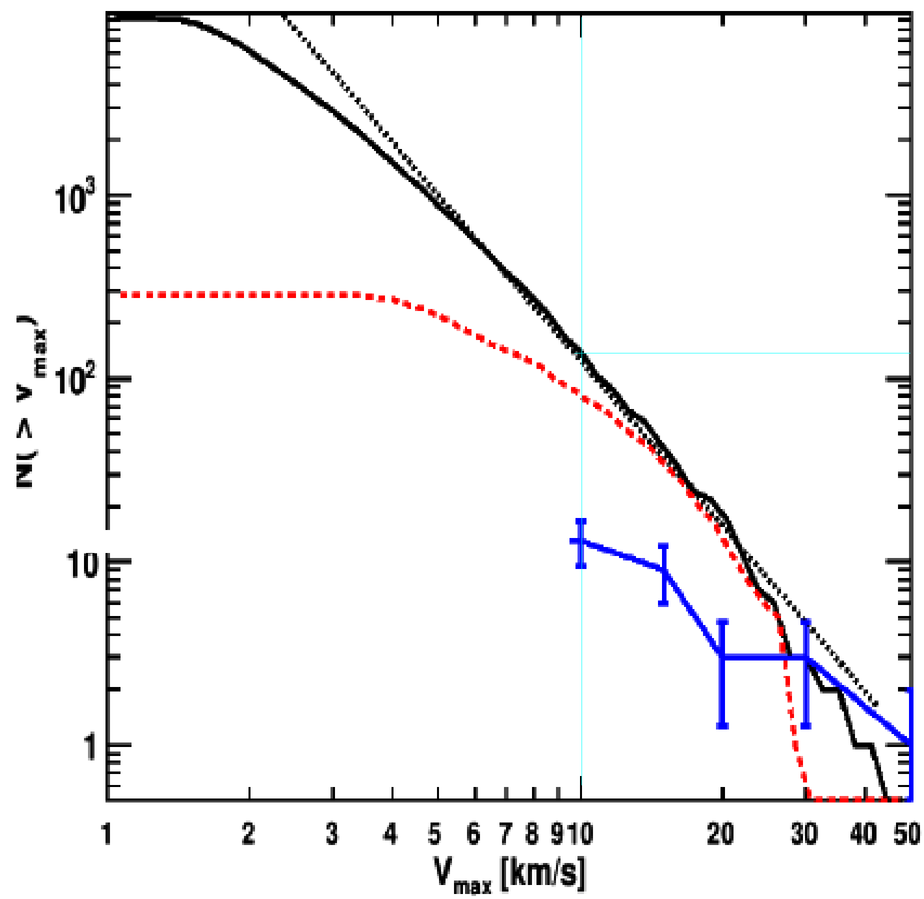


How well does substructure converge?

Virgo Consortium 2007



Diemand, Kuhlen & Madau (2007)



Cold Dark Matter at high redshift (e.g. $z \sim 10^6$)

At epochs well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, the inflationary model for the origin of structure predicts the distribution function:

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x})]\,\delta_D(\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x}))$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,

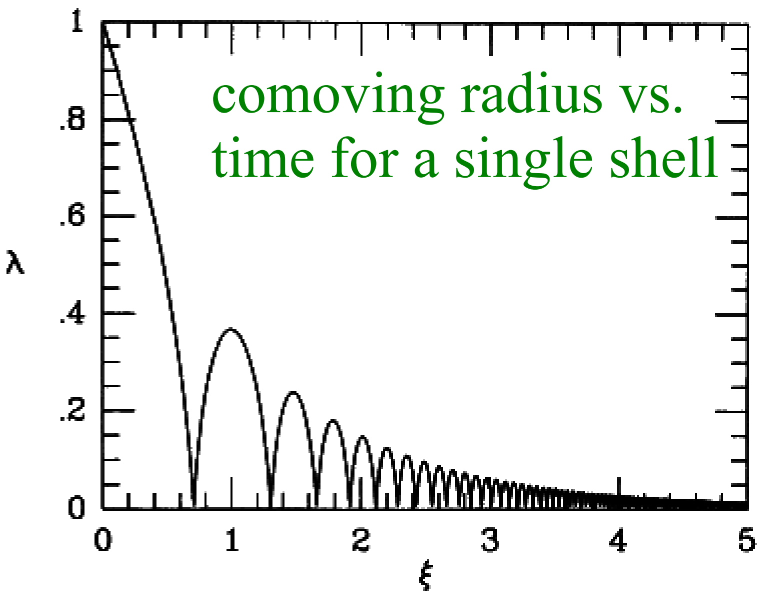
and $\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$

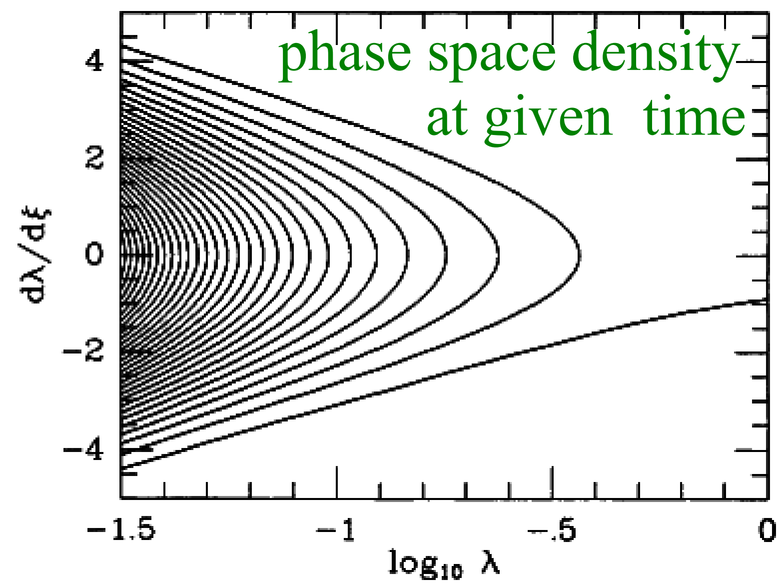
The phase density of CDM occupies a 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

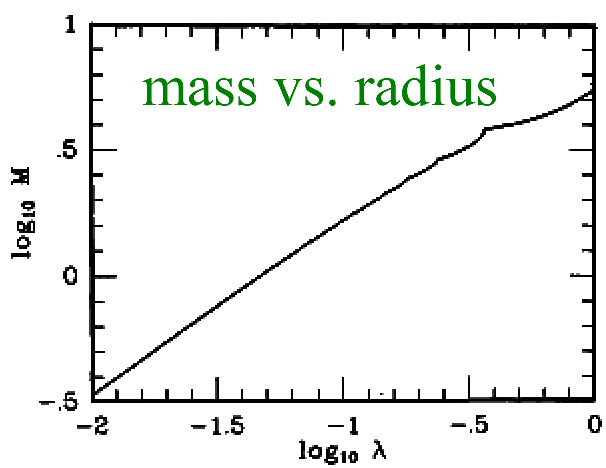
$Df / Dt = 0 \longrightarrow$ only a 3-D subspace is occupied at later times.
Nonlinear evolution leads to a complex, multi-stream structure.

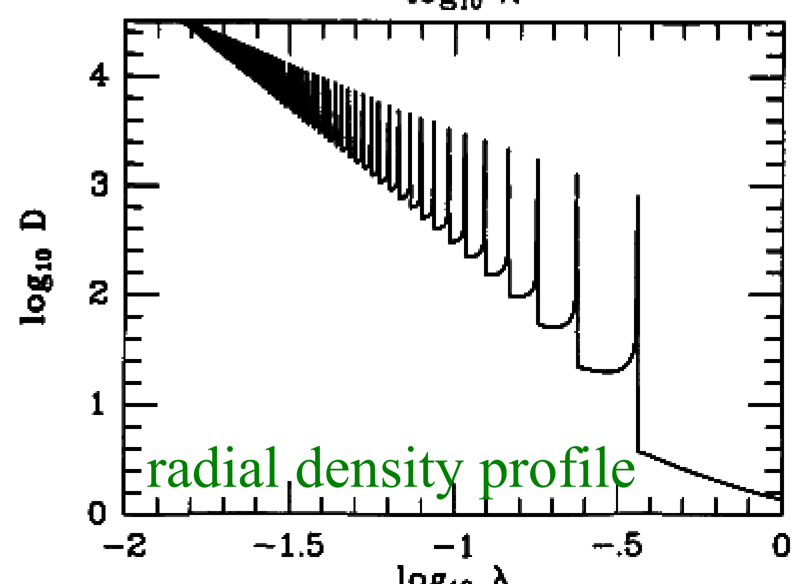
Similarity solution for a 1-D collapse in CDM

Bertschinger 1985









Small-scale structure of the CDM distribution

- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
  - -- what is the density structure between m and kpc scales?
  - -- how many streams intersect the detector at any time?
- Intensity of annihilation radiation depends on

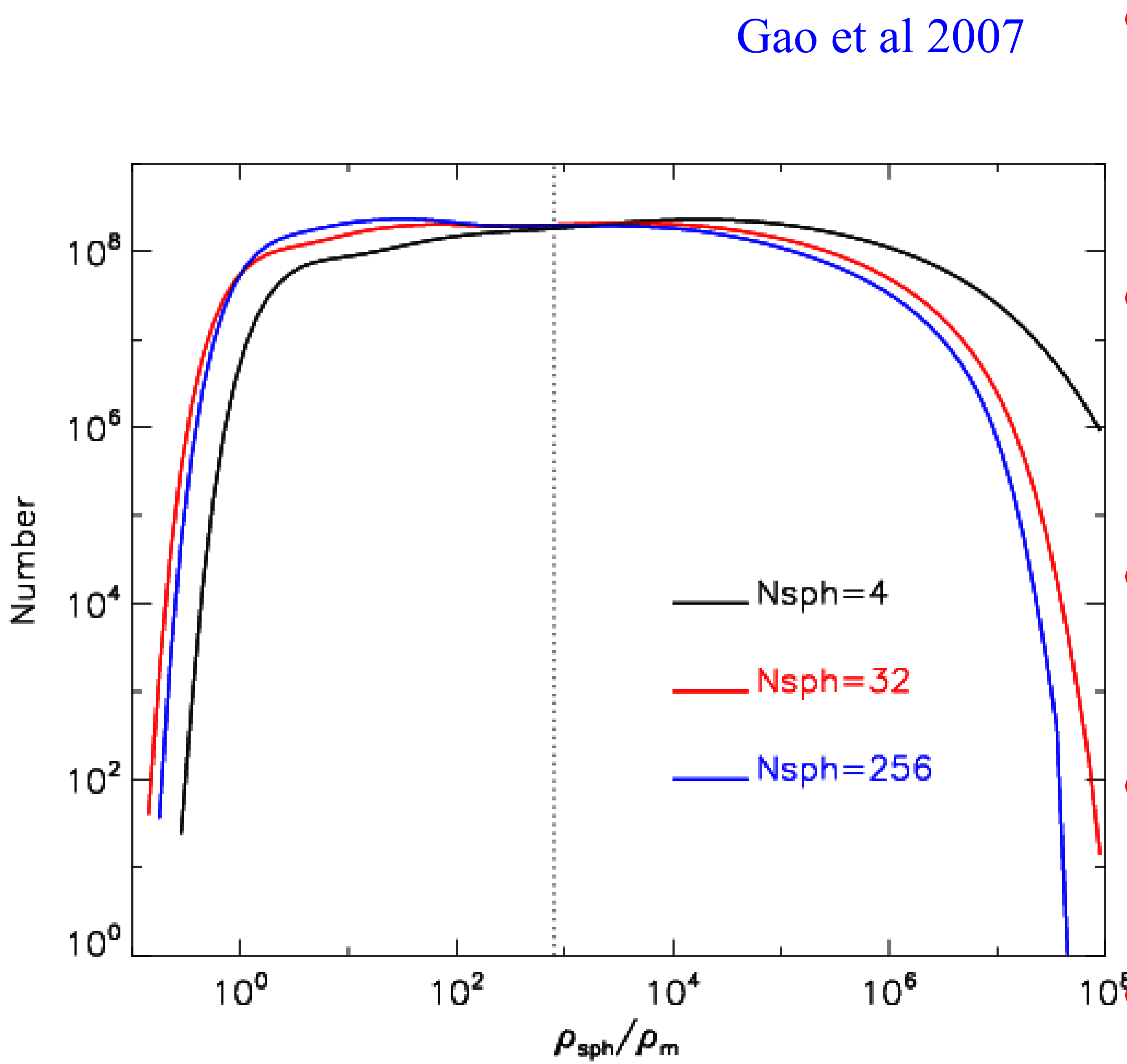
$$\int \rho^2(\boldsymbol{x}) \langle \sigma\, v \rangle \, dV$$

  - -- what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales <u>far</u> below those accessible to simulation

→ We require a good theoretical understanding of mixing

Lagrangian DM density at the present day



- Lagrangian smoothing gives density today on given *mass* scale
- Distribution function is flat over at least 6 orders of magnitude
- It is very far from lognormal
- No clear linear-nonlinear transition
- No clear convergence as resolution improves

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot\delta X\,;\ \ T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\boxed{\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = I}$$

- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- det(D) = 1 at all times by Liouville's theorem
- For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

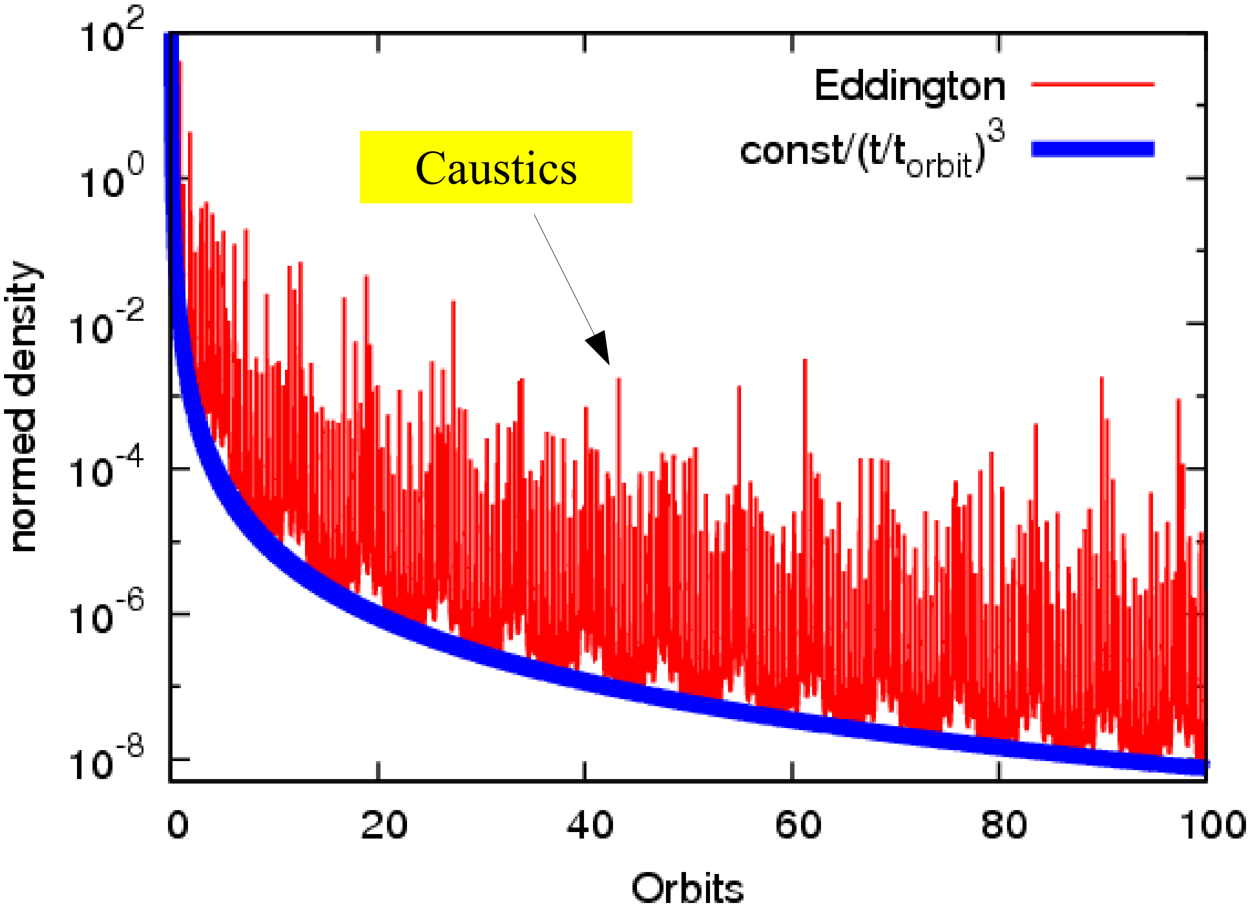
Static highly symmetric potentials

Code *DaMaFlow* developed for static potentials:

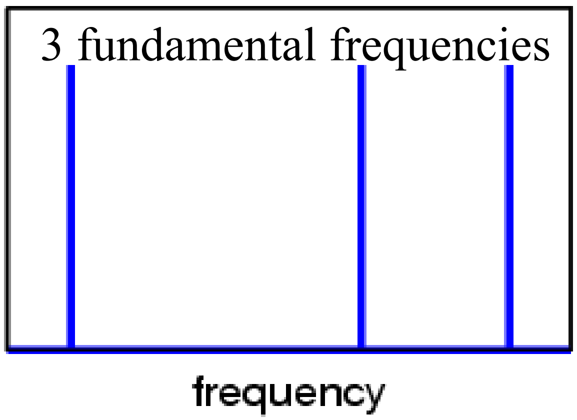
- orbit + geodesic deviation integrator (symplectic DKD/KDK Leapfrog + DOPRI853)
- modular design allows large variety of potentials to be analyzed
- precise spectral analysis on the fly (NAFF algorithm, $1/T^4$ accuracy) with integer programming to get the fundamental frequencies of motion
- automated stream density fitting

$$\Phi(r,\theta) = v_{\mathrm{h}}^2 \log\left(r^2 + d^2\right) + \frac{\beta^2 \cos^2\theta}{r^2}$$

Axisymmetric Eddington potential



Spectral analysis of orbit:

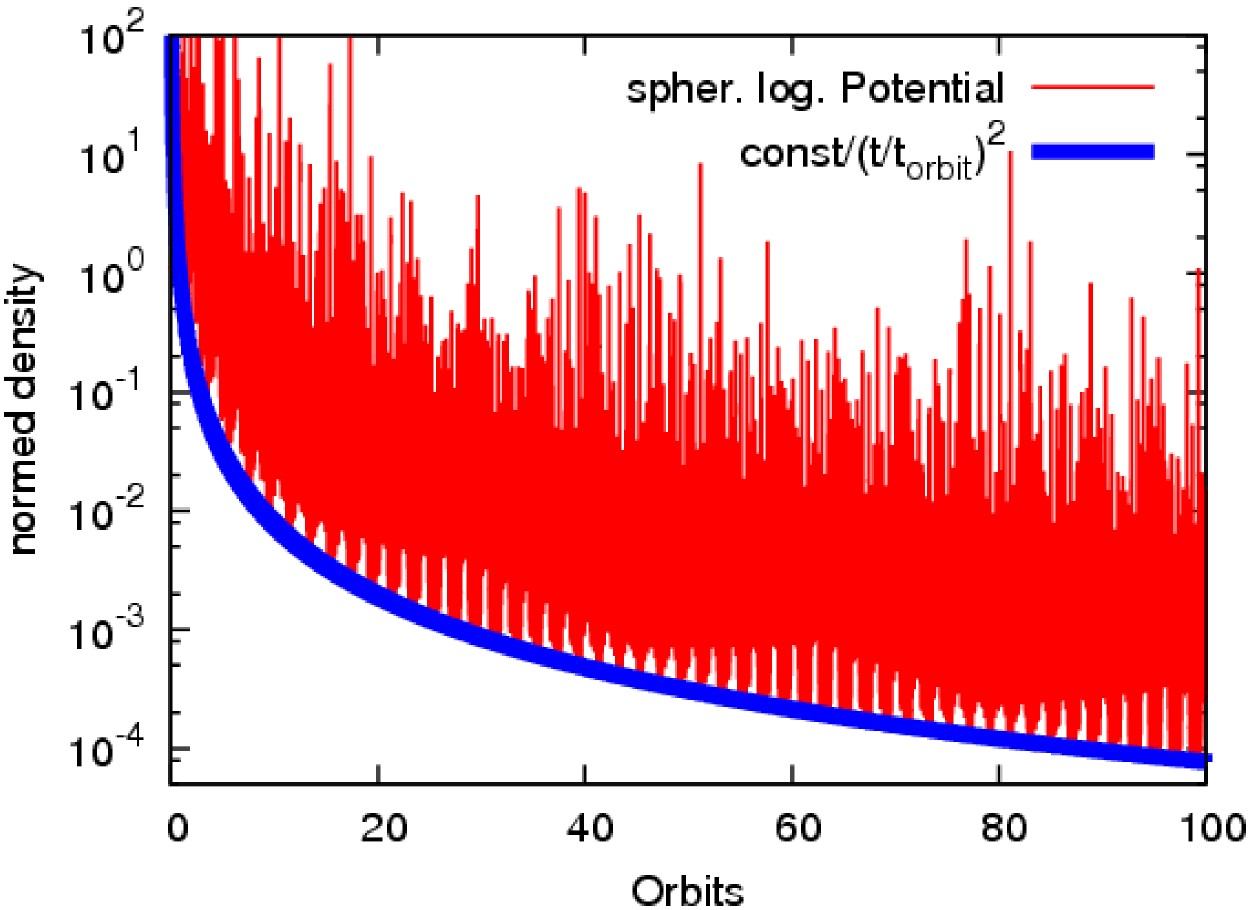


density decreases like $1/t^3$

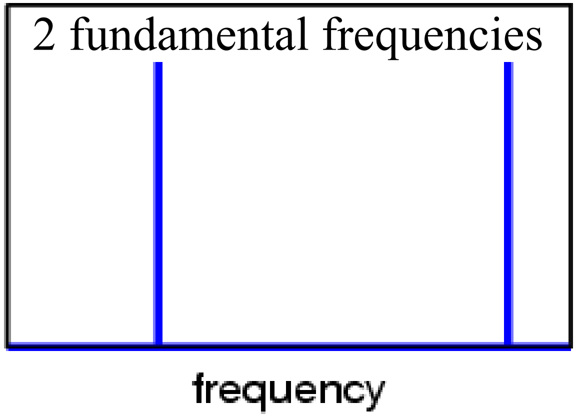
Changing the number of frequencies

Spherical logarithmic potential

$$\Phi(r,\theta) = v_{\mathrm{h}}^2 \log\left(r^2 + d^2\right)$$



Spectral analysis of orbit:



density decreases like $1/t^2$

Number of fundamental frequencies dictates the density decrease of the stream

What about non-trivial potentials?

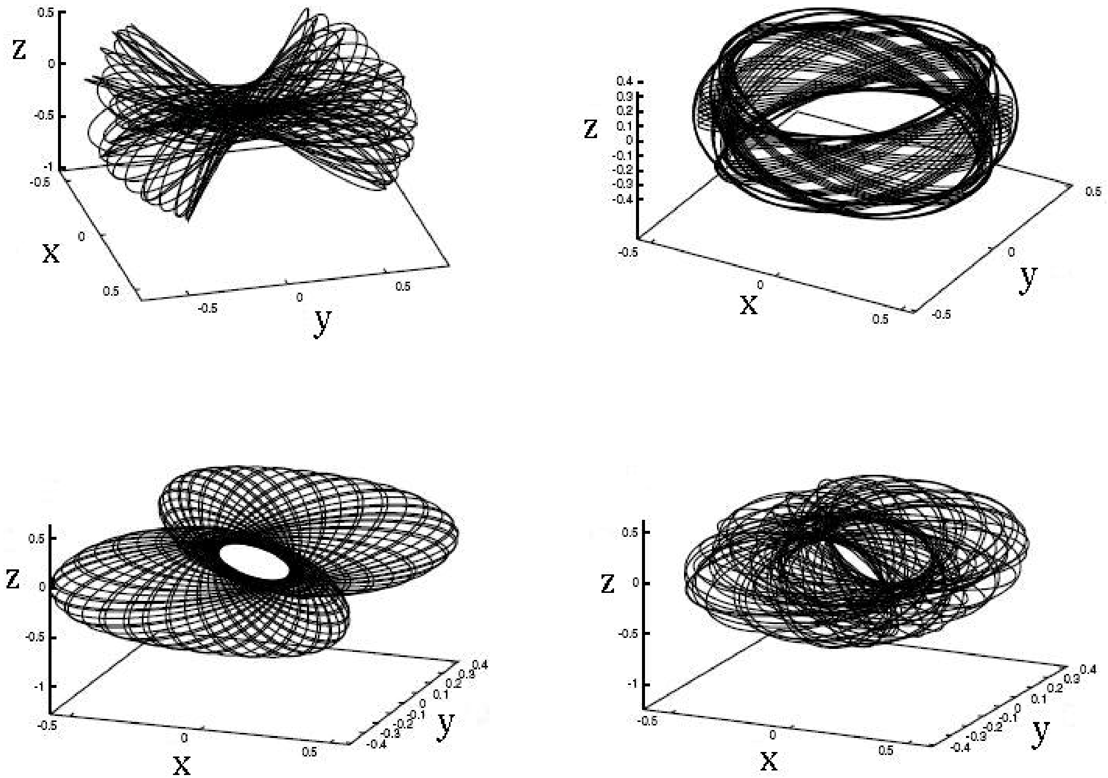
integrable systems give only rise to **regular motion**

<u>non integrable (more realistic) systems:</u>

have more complicated phase space structure, possibly with **chaotic regions**

this has an impact on dark matter stream density

Try to get more insights with our new approach!



<u>Example:</u>

triaxial logarithmic potential with core

$$H = \frac{1}{2}\left(X^2 + Y^2 + Z^2\right) + \ln\left(x^2 + \frac{y^2}{q_1^2} + \frac{z^2}{q_2^2} + R_c^2\right)$$

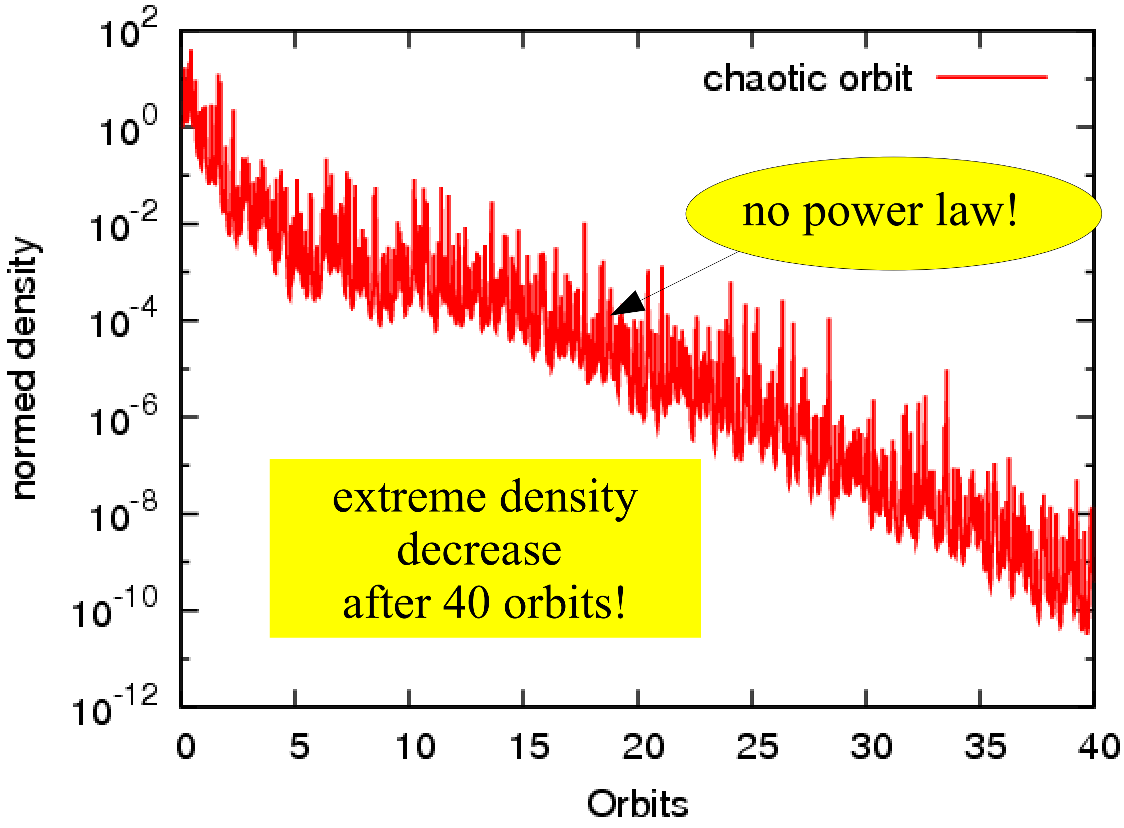
regular motion

box and tube orbits; density decreasing like a <u>power law</u> in time for regular motion

Chaotic mixing

chaotic motion implies a **rapid stream density decrease** → **rapid mixing**

density decrease is **not** like a power law anymore

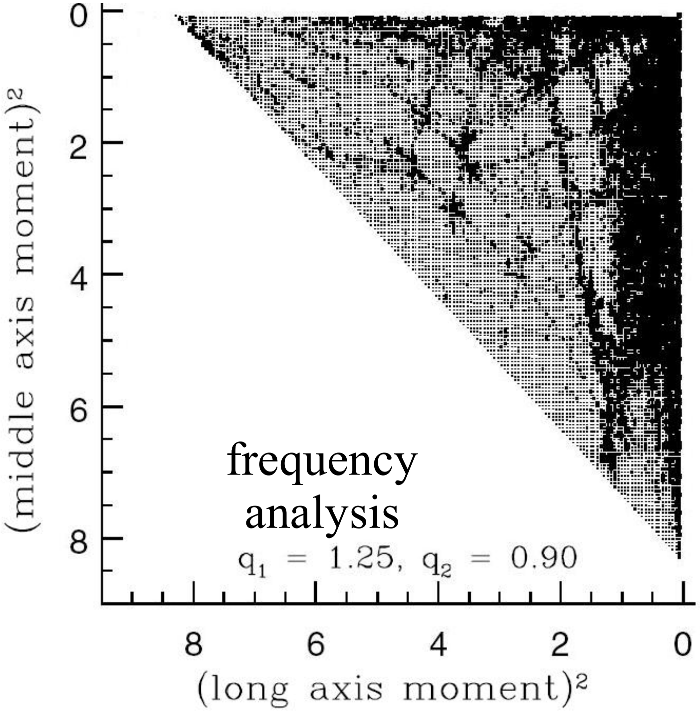


how to find chaotic regions in phase space?

Common method:

- Lyapunov exponents
- frequency analysis (NAFF)
- ...

Compare frequency analysis results with geodesic deviation equation results

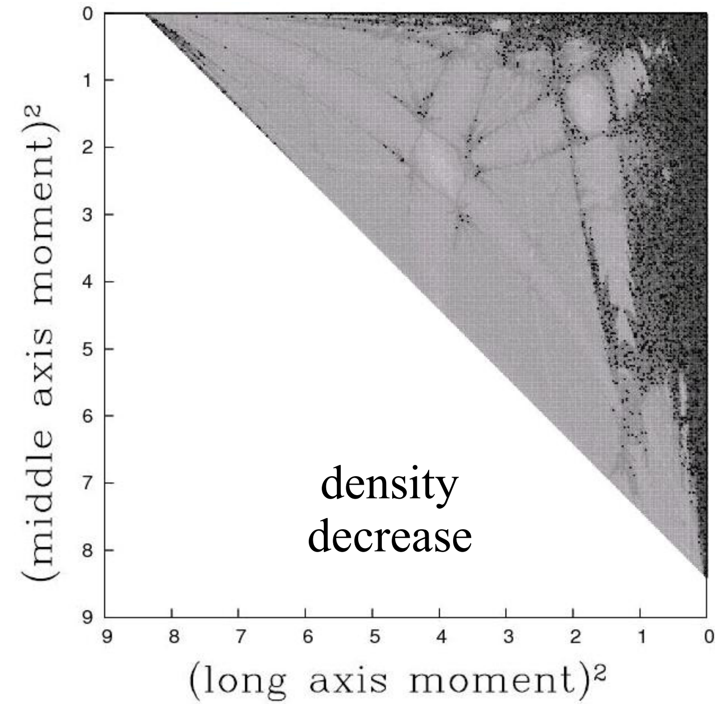


moderate triaxiality

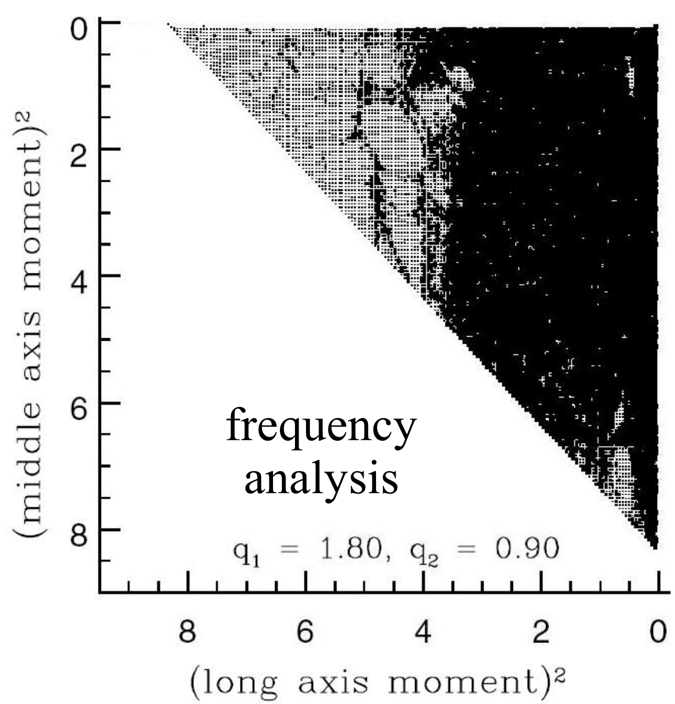
↓

small fraction of chaotic orbits

stream density mostly decaying like a power law



Papaphilippou & Laskar 1998



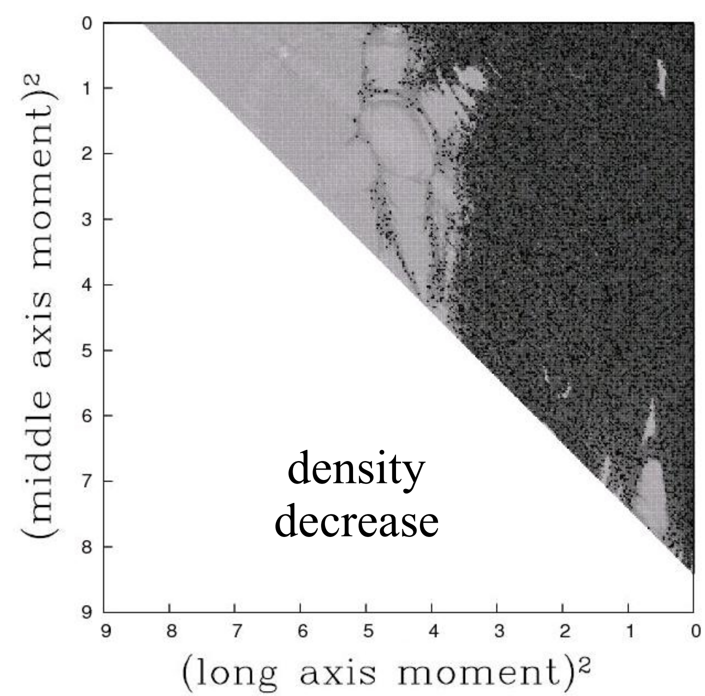
high triaxiality

↓

large fraction of chaotic orbits

stream density mostly decaying much faster than a power law

integrate 10^5 different orbits



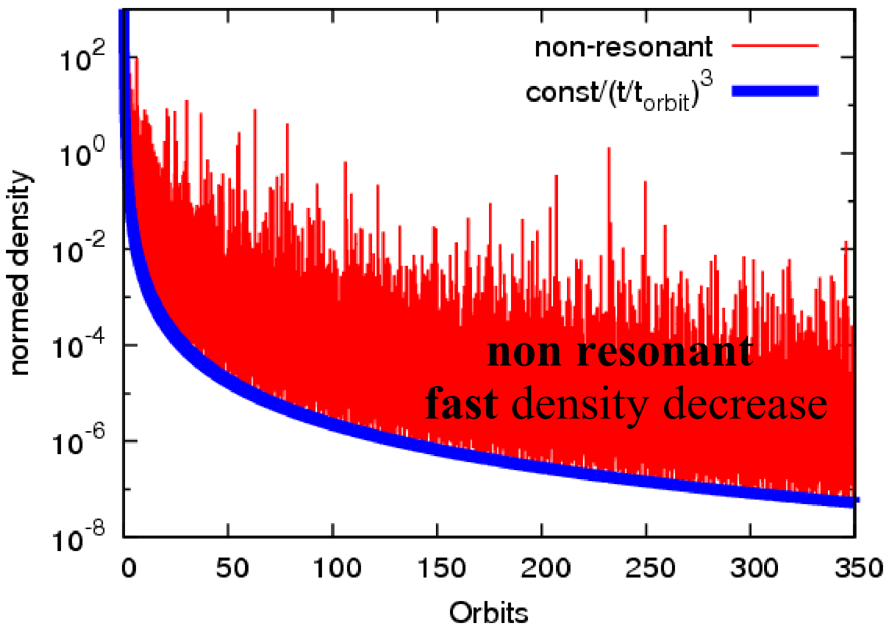
Resonances in phase space

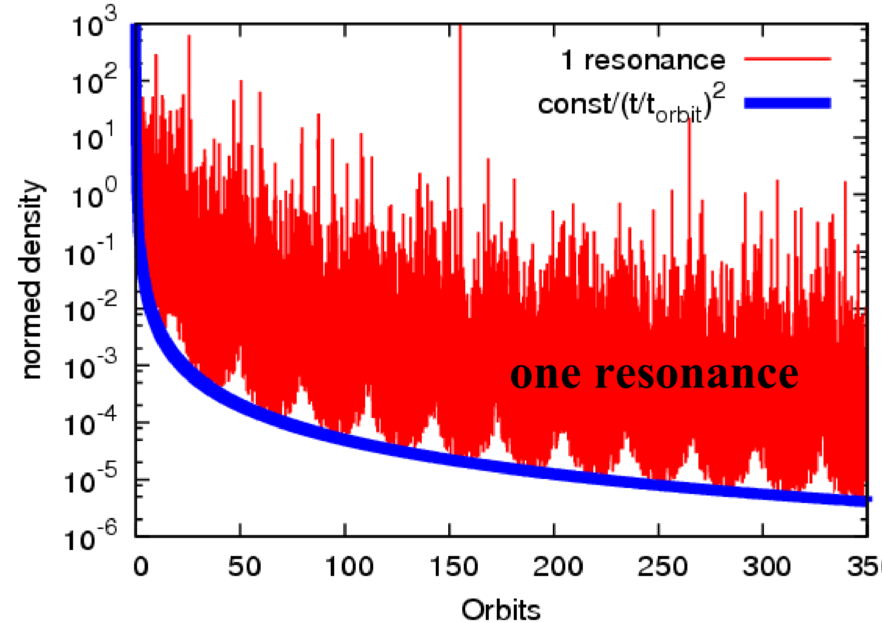
motion might be resonant in certain portions of phase space

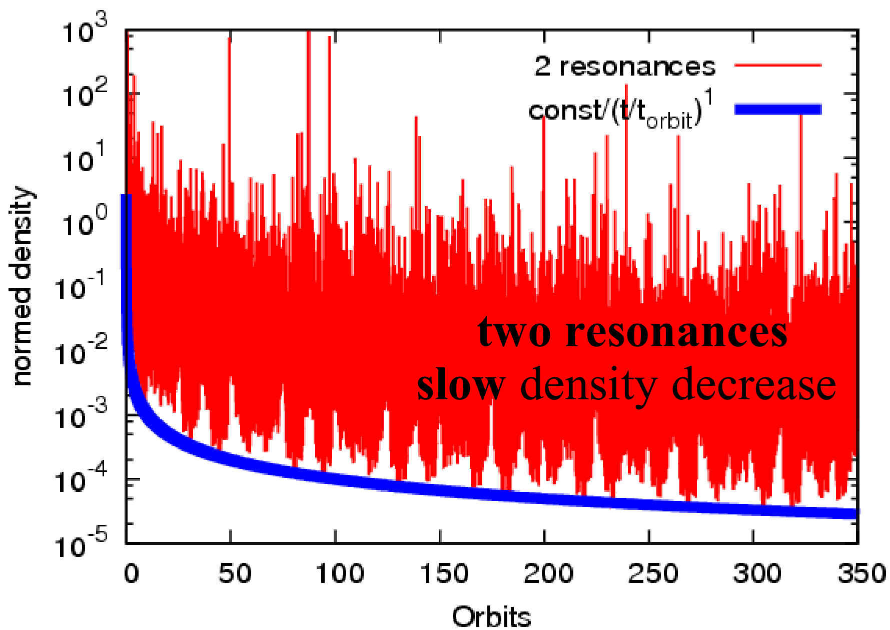
$$m_1\omega_1+m_2\omega_2+m_3\omega_3=0$$

KAM torus not completely covered

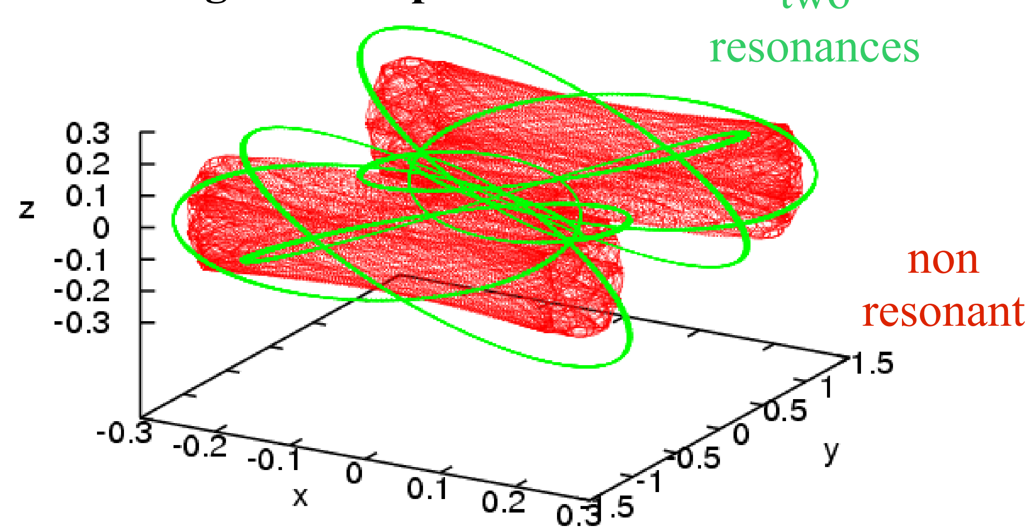
stream density decreases slower



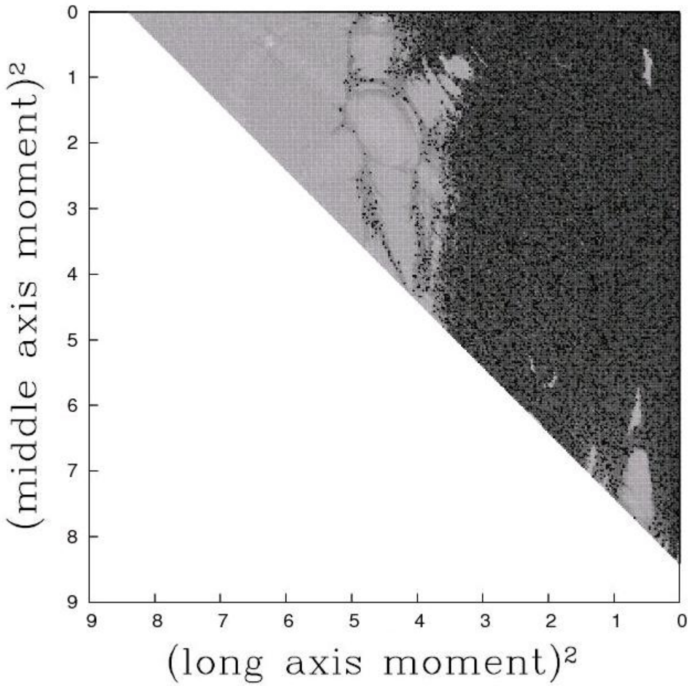




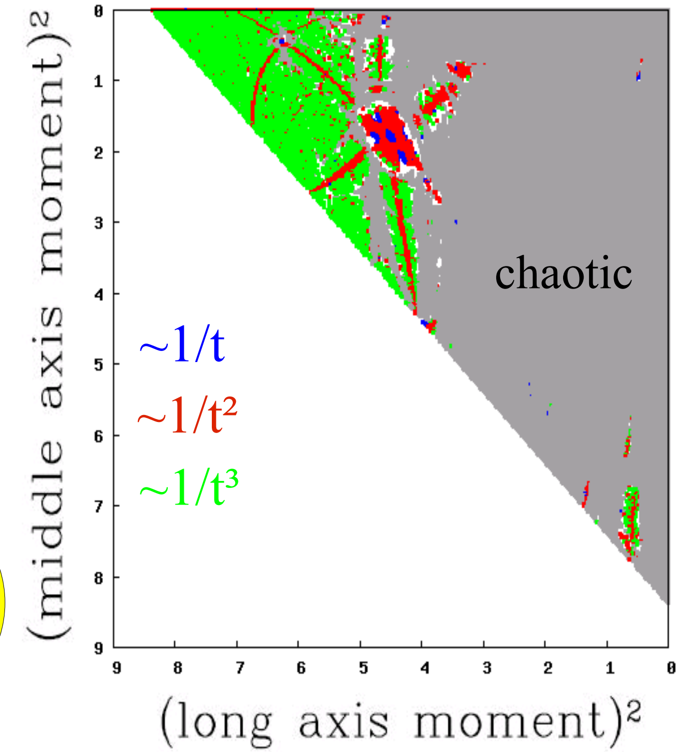
Configuration space



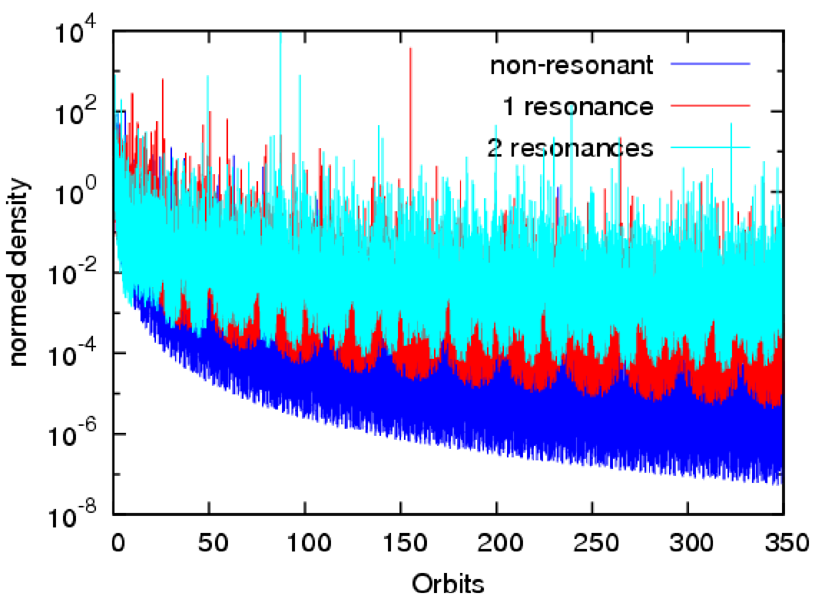
Resonances: scanning phase space



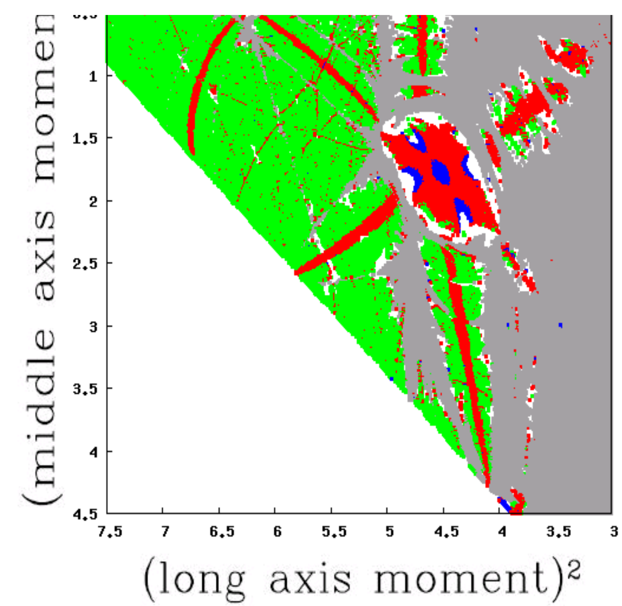
fitting / binning
density decrease



phase space
of non-integrable
systems highly
complex



stream density has
very different
evolution

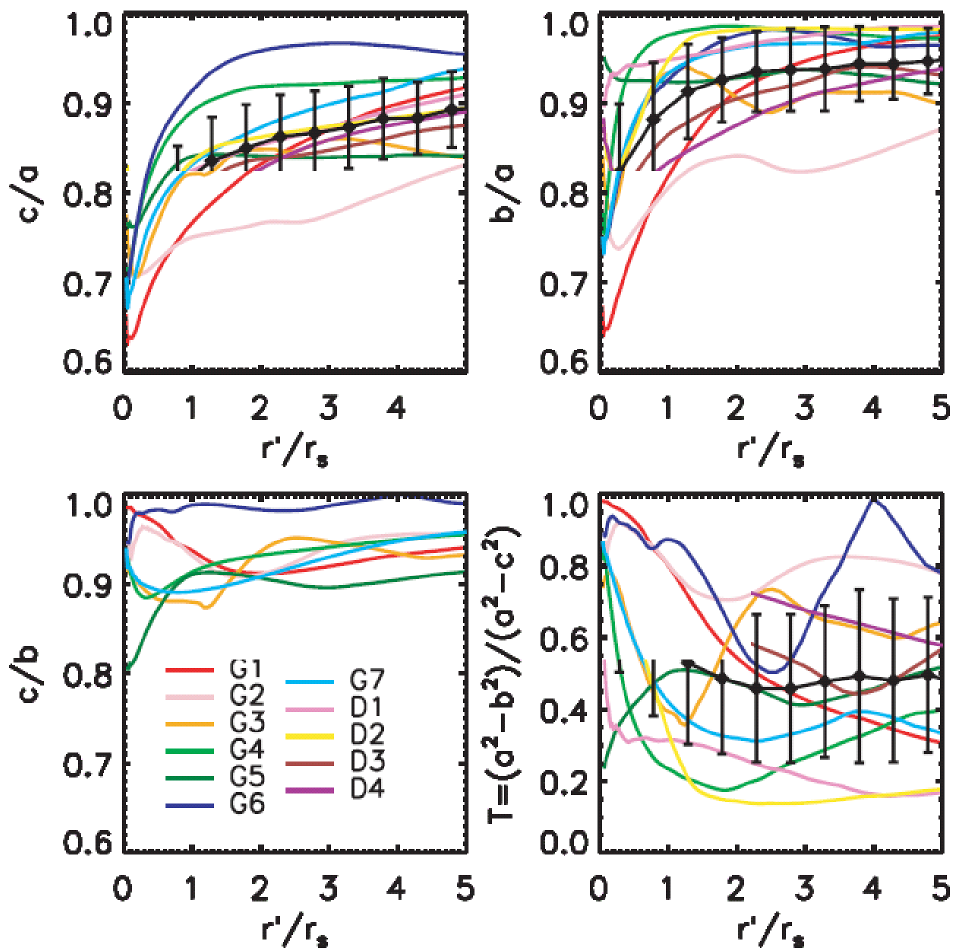


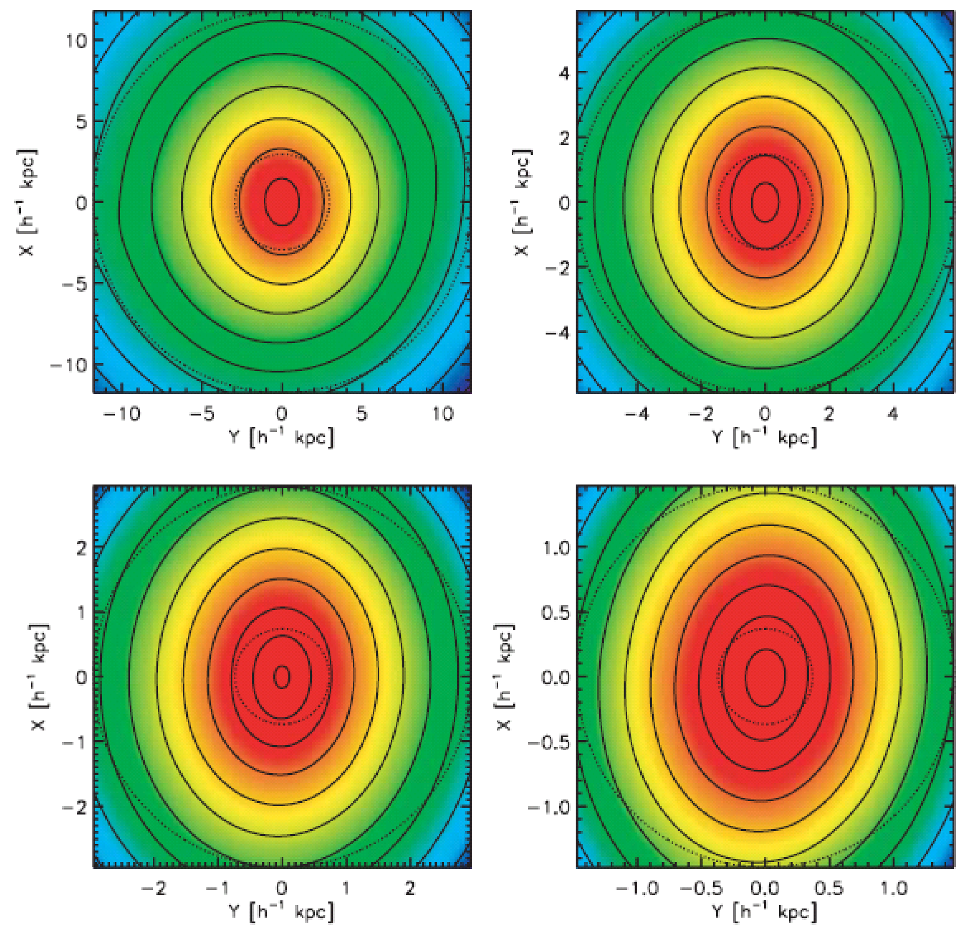
Realistic dark matter halo potentials

Cosmological simulations:

Hayashi et al, 2007

- outer regions *spherical*
- inner regions *aspherical*
- principal axes well aligned over radius
- halos tend to become more prolate near center





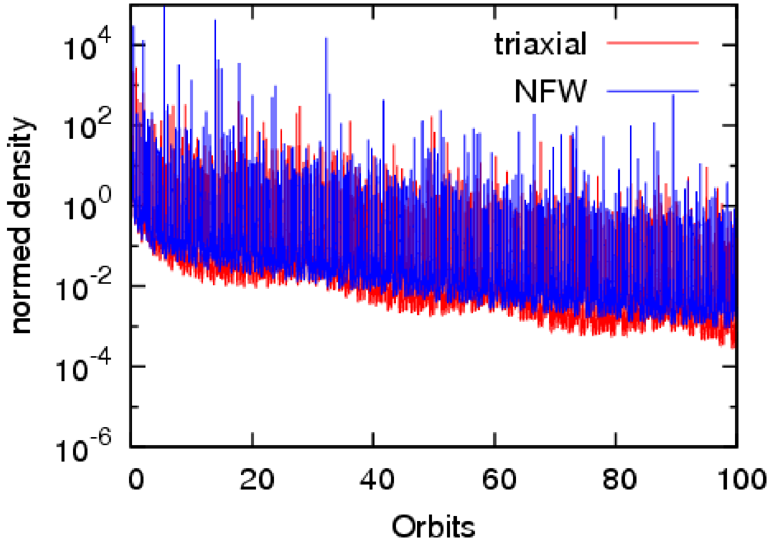
shape of potential very relevant for **stream dynamics**

build a model for the potential with similar qualitative behaviour

Dark matter streams in a triaxial NFW

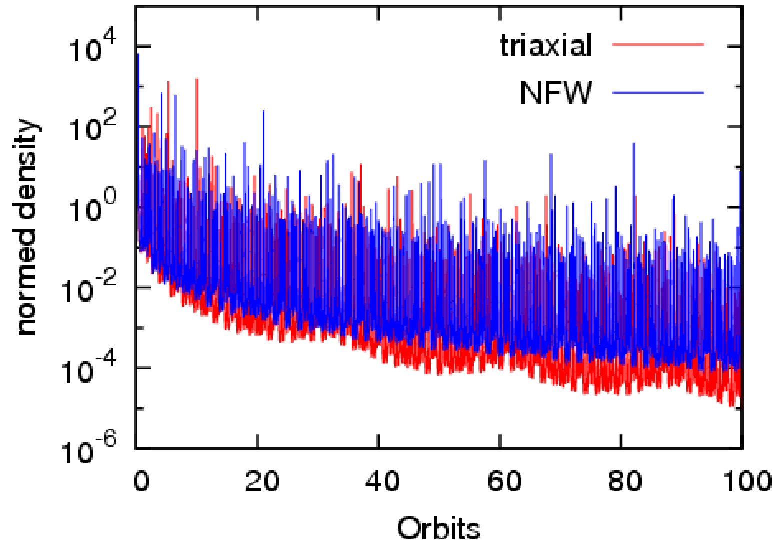
How does the radial shape variation influence the stream density evolution?

outer orbit: similar stream behaviour



Apo:8.9r_s

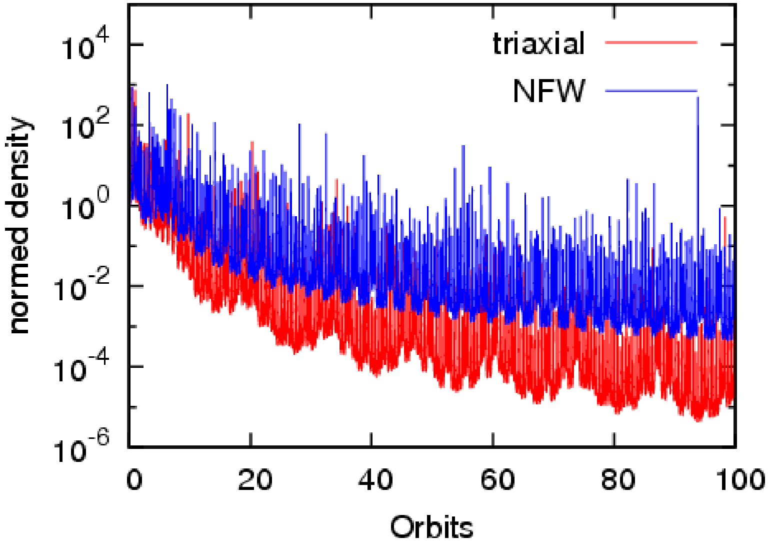
Peri:6.1r_s



Apo:8.7r_s

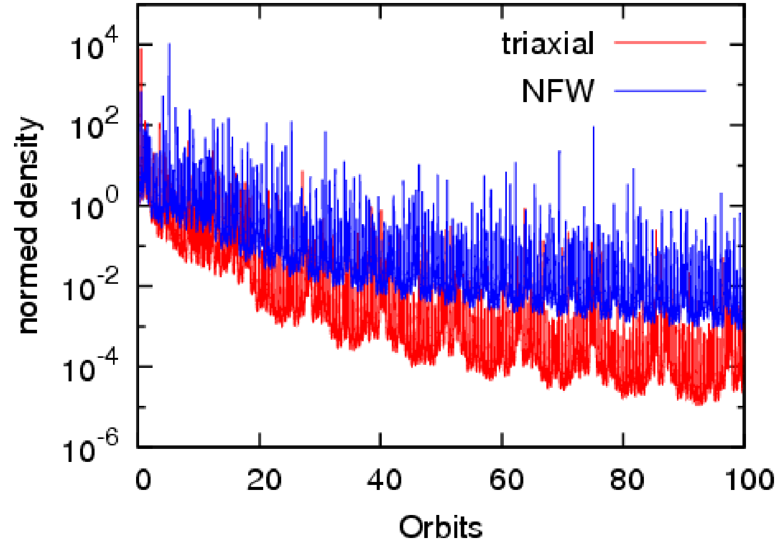
Peri:4.8r_s

inner orbits: ~100 times lower stream density after 100 orbits



Apo:2.2r_s

Peri:0.5r_s

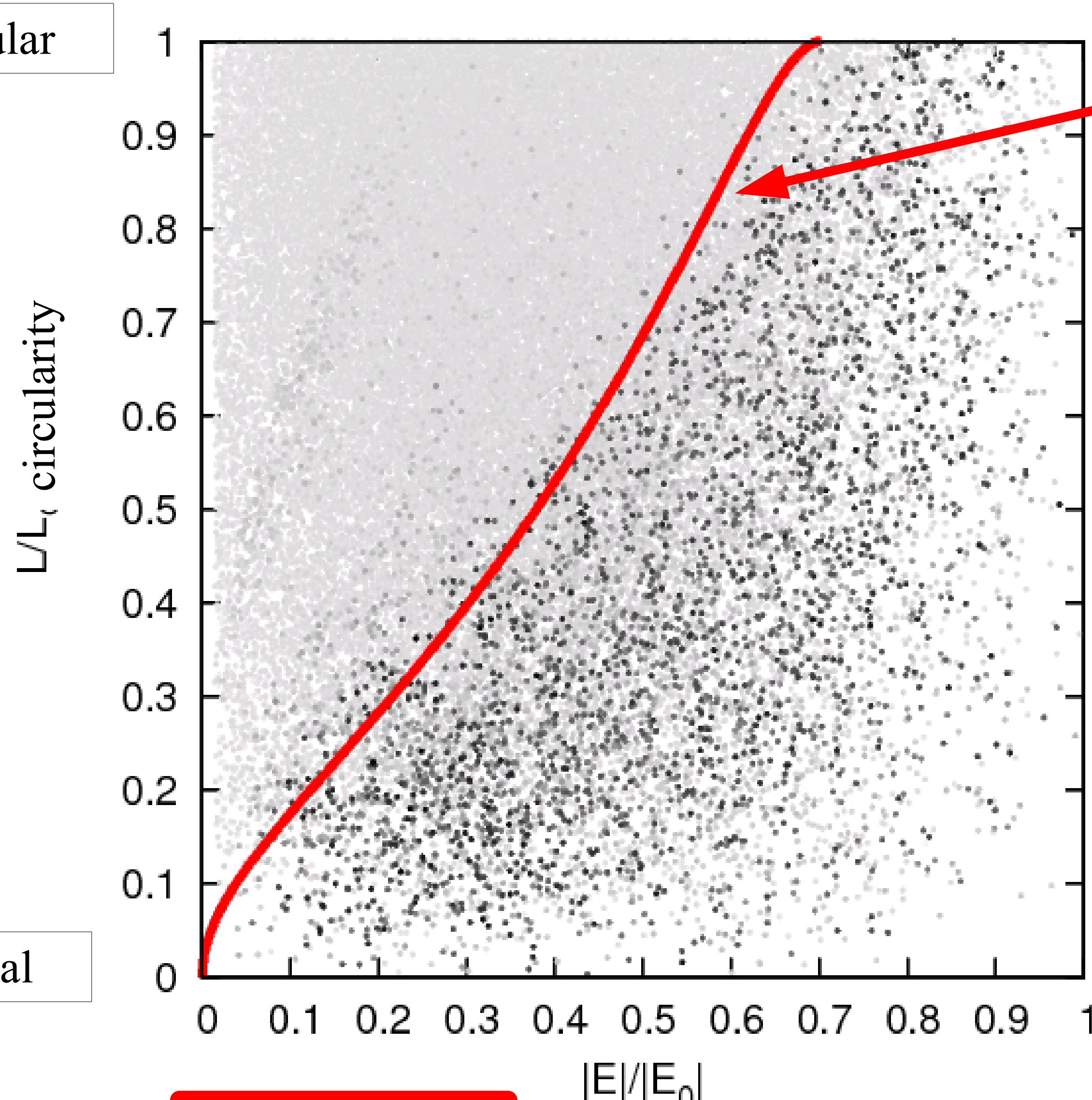


Apo:1.2r_s

Peri:0.4r_s

Chaotic mixing in a triaxial NFW?

circular



radial

constant pericentre:
~1/3 NFW scale radius

~40.000 orbits

Orbits reaching inner part of the halo are affected by the triaxiality

Chaotic mixing

Fine-grained phase-space in Cosmological Simulations?

main advantage of the GDE approach → **implementation in N-body codes straightforward**

→ **Accessing the fine-grained phase-space in cosmological simulations for the first time**

Some details:

- calculate (softened) tidal field for DM particles (in tree and mesh code)
- integrate geodesic deviation equation parallel to equation of motion for DM particles
- automatic caustic identification for every DM particle
- projection to configuration-space

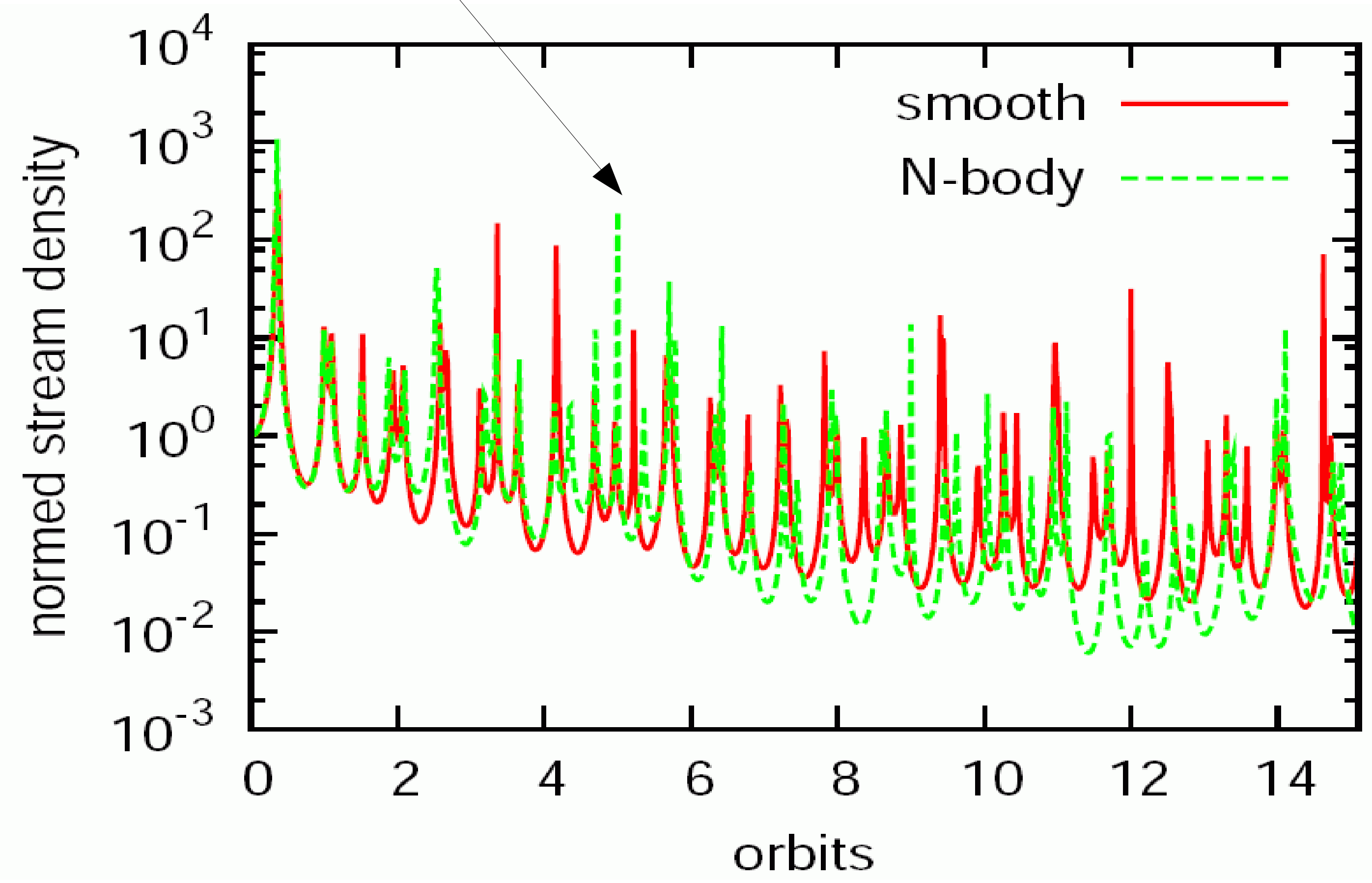
→ **Implementation in cosmological simulation code GADGET (Springel,2005)**

A particle orbit in a live Halo

caustics resolved in N-body live halo!

spherical Hernquist density profile

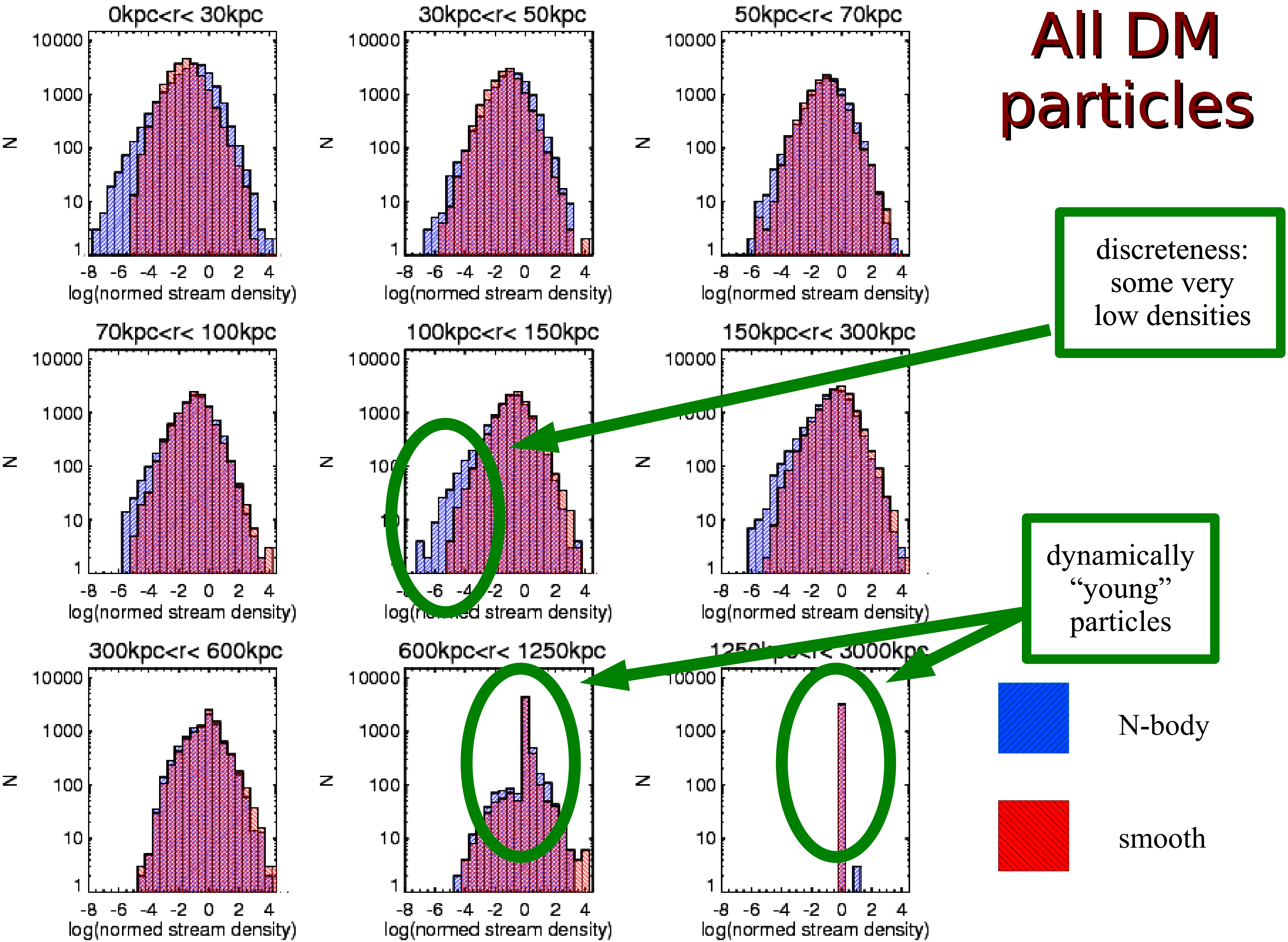
$$\rho(r) = \frac{M}{2\pi}\frac{a}{r}\frac{1}{(r+a)^3}$$



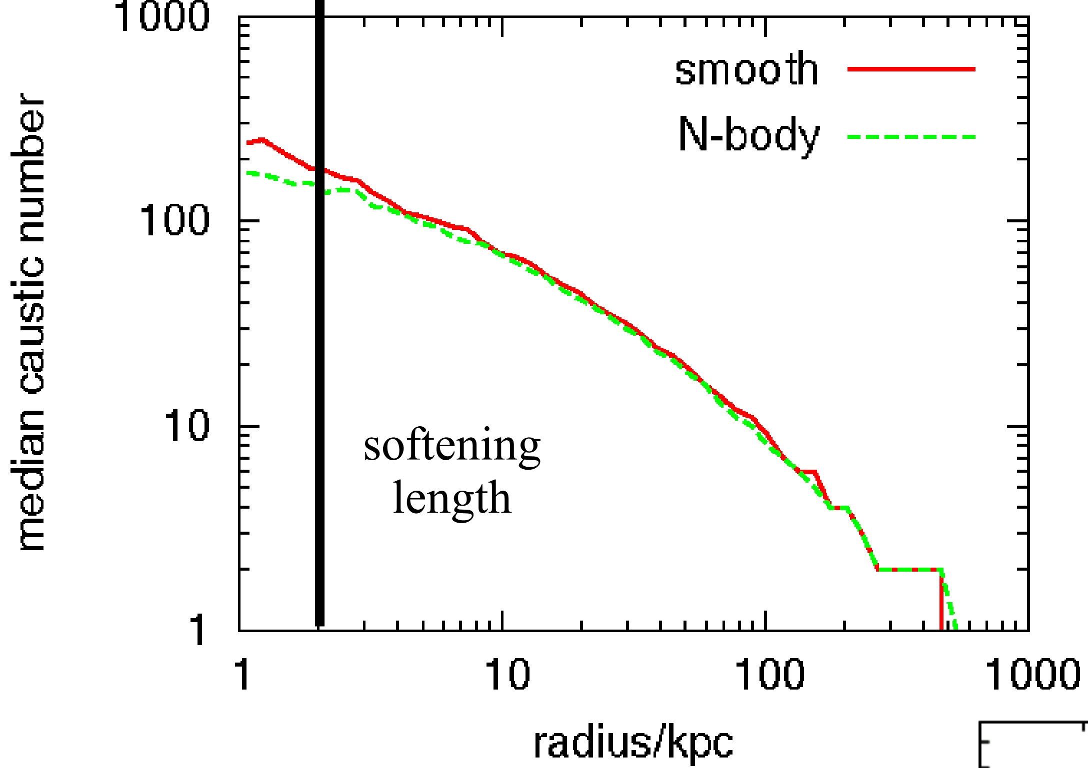
general shape and caustic spacing/number very similiar!

phase-space density conservation: 10^{-8}

All DM particles



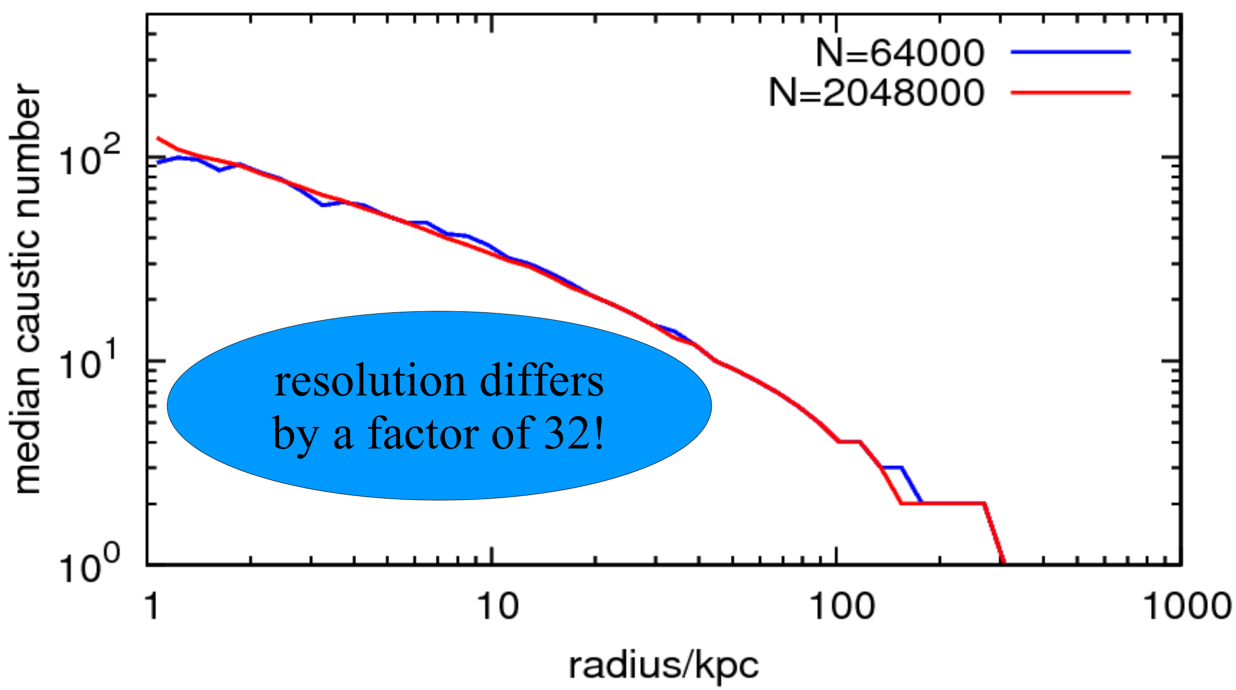
Number of Caustic Passages



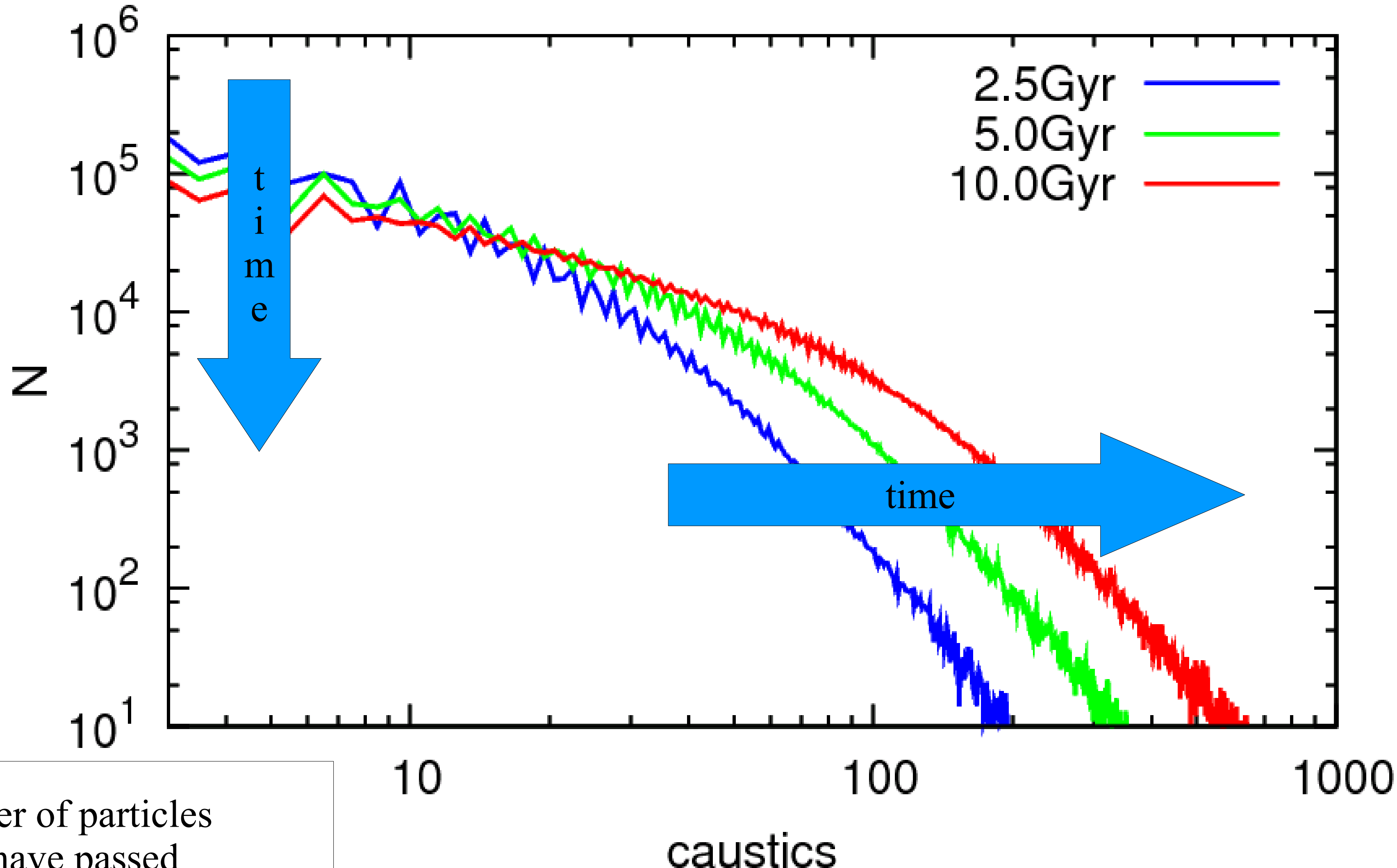
analytic and N-body results nearly the same!

Annihilation boost factor estimates due to caustics should be very robust!

Very stable against particle number and softening length!



Distribution of the number of Caustic Passages



Number of particles that have passed through a given number of caustics after some given time

Summary

- **cosmological simulations** can help direct and indirect **detection experiments**
- resolving the required scales is currently **impossible** with **standard techniques**

The GDE is a completely general and new technique for calculating the fine-grained phase-space structure!

Static potentials:

- **chaos** and **resonance** structure can be reproduced
- **triaxial NFW** halo:
  - **near the Sun** most CDM particles should belong to streams with **very low density**
  - a **smooth** velocity distribution

Applying the geodesic deviation equation method within N-body codes:

- **stream density distributions** can be reproduced
- **caustics** can be resolved
- **number of caustics** is **very** robust

→ **resolve small-scale structure in cosmological simulations**