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Streams and caustics: the fine-grained structure of ΛCDM halos

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"Milky Way" halo
z = 1.5
N_{200} = 750 x 10^6
Aquarius-A-1
Springel et al 2008

Cold Dark Matter at high redshift (e.g. $z \sim 10^5$)

Well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, their distribution function is

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x})]\, N\,[\{\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x})\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,

$\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$

and N is standard normal with $\sigma^2 << \langle|\mathbf{V}|^2\rangle$

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

$Df/Dt = 0$ → only a 3-D subspace is occupied at later times. Nonlinear evolution leads to a complex, multi-stream structure.

Evolution of CDM structure

Consequences of $Df / Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least 1 sheet must pass through every point $\mathbf{x}$
- In nonlinear objects there are typically many sheets at each $\mathbf{x}$
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain f = const.
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

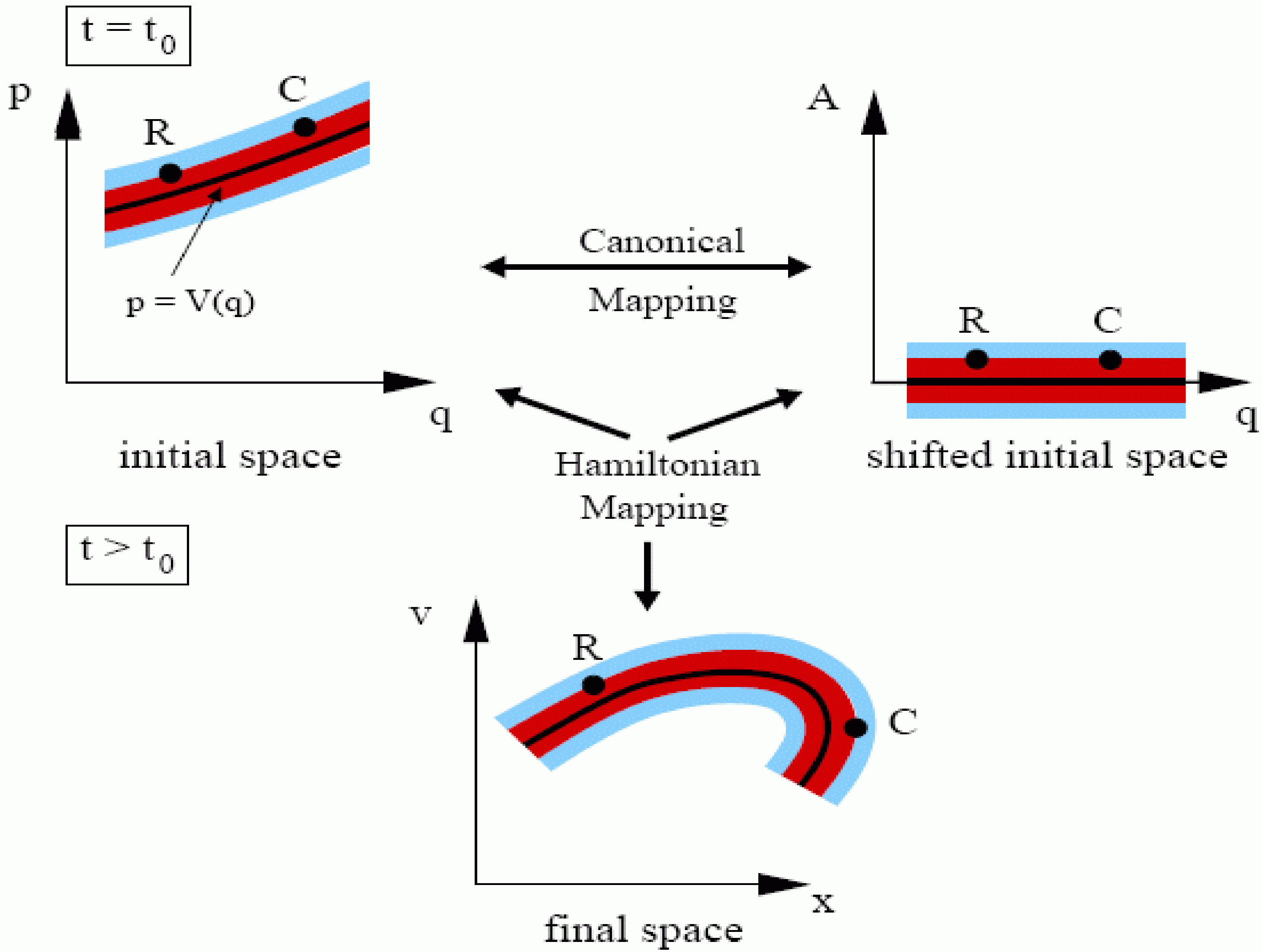
Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot\delta X\,;\;\; T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

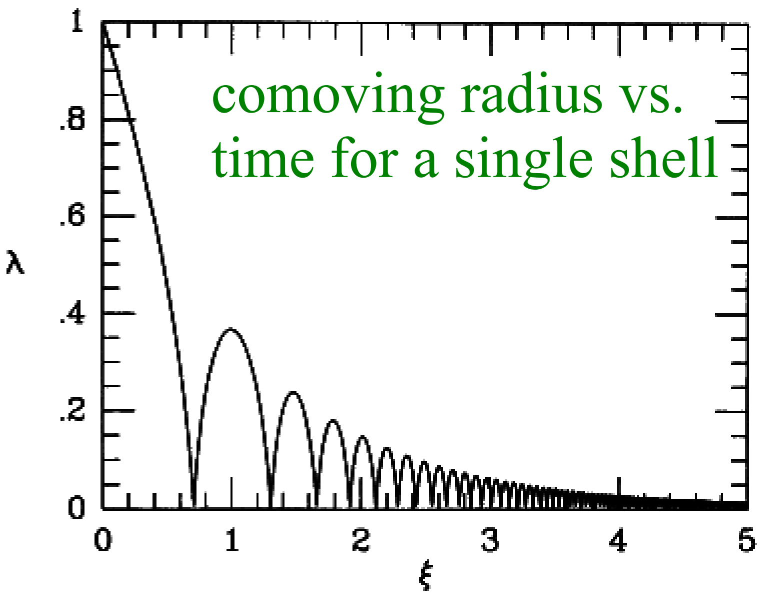
$$\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix}\cdot D \quad \text{with } D_0 = I$$

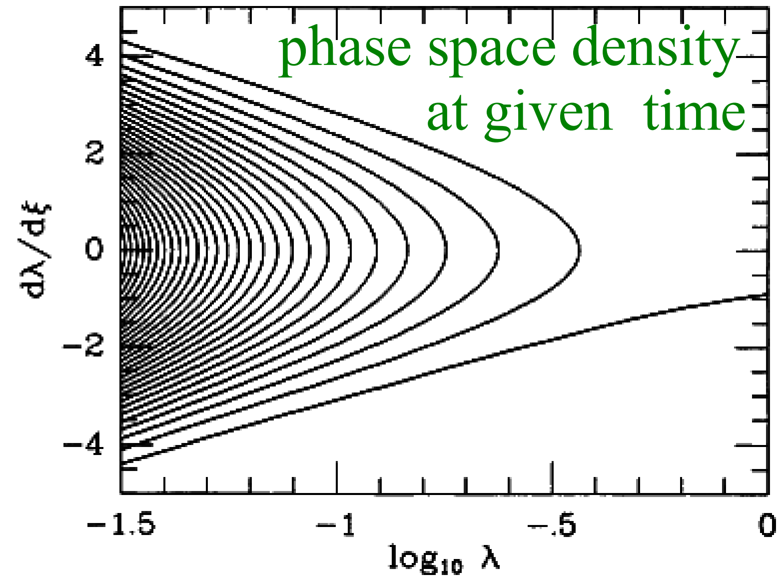
- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- det(D) = 1 at all times by Liouville's theorem
- For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

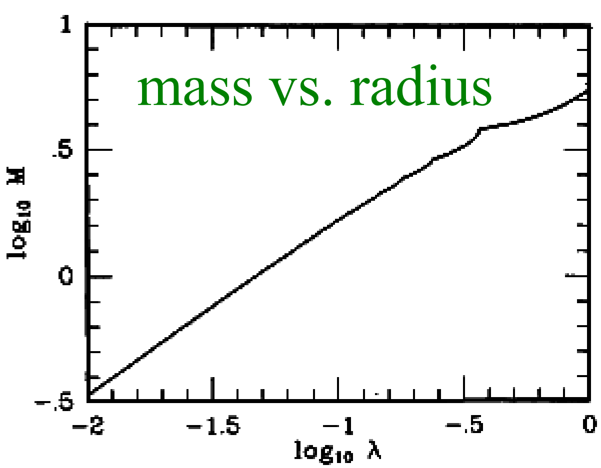

t = t0
p
R
C
p = V(q)
q
initial space
Canonical
Mapping
A
R
C
q
shifted initial space
Hamiltonian
Mapping
t > t0
v
R
C
x
final space

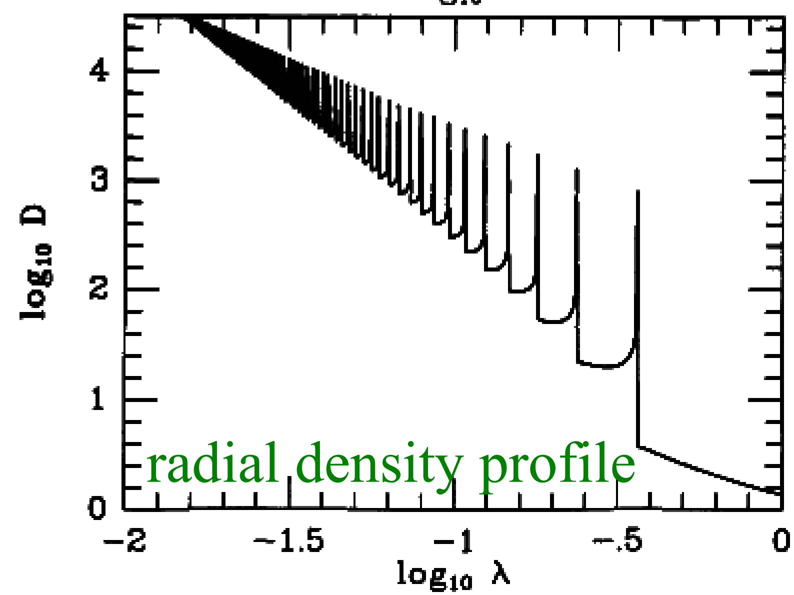
Similarity solution for spherical collapse in CDM

Bertschinger 1985



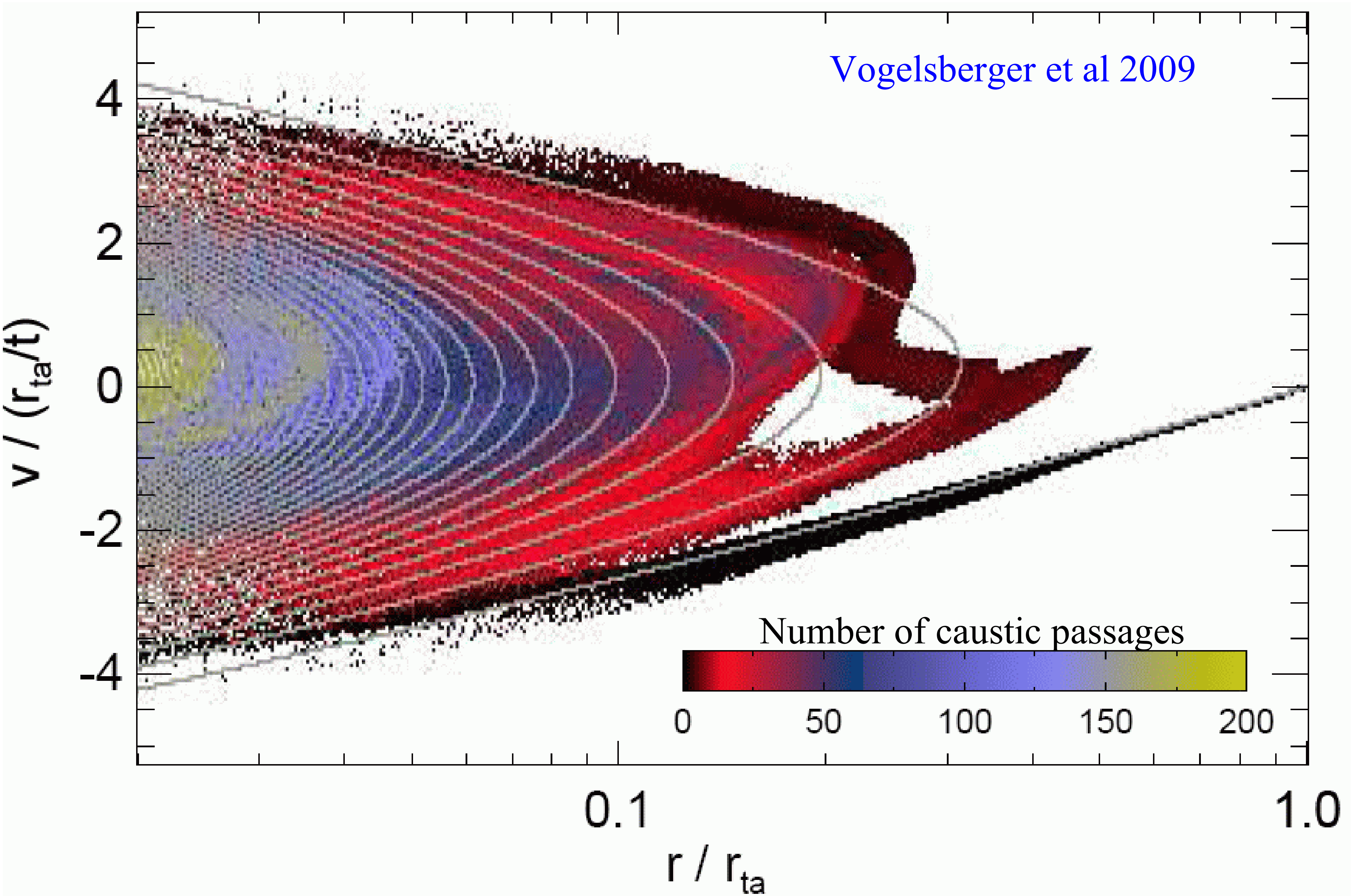




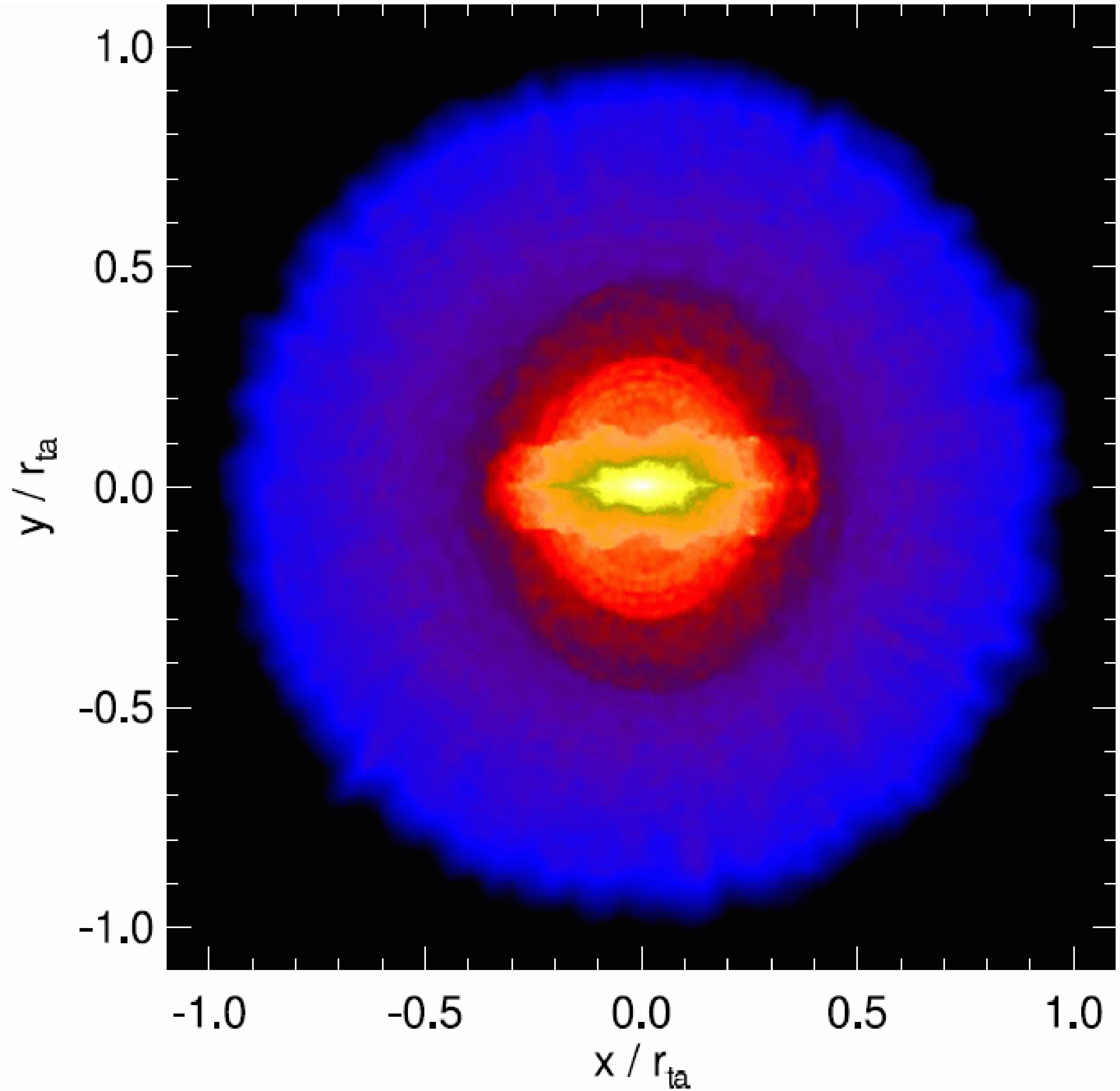


Simulation from self-similar spherical initial conditions

Geodesic deviation equation → phase-space structure local to each particle



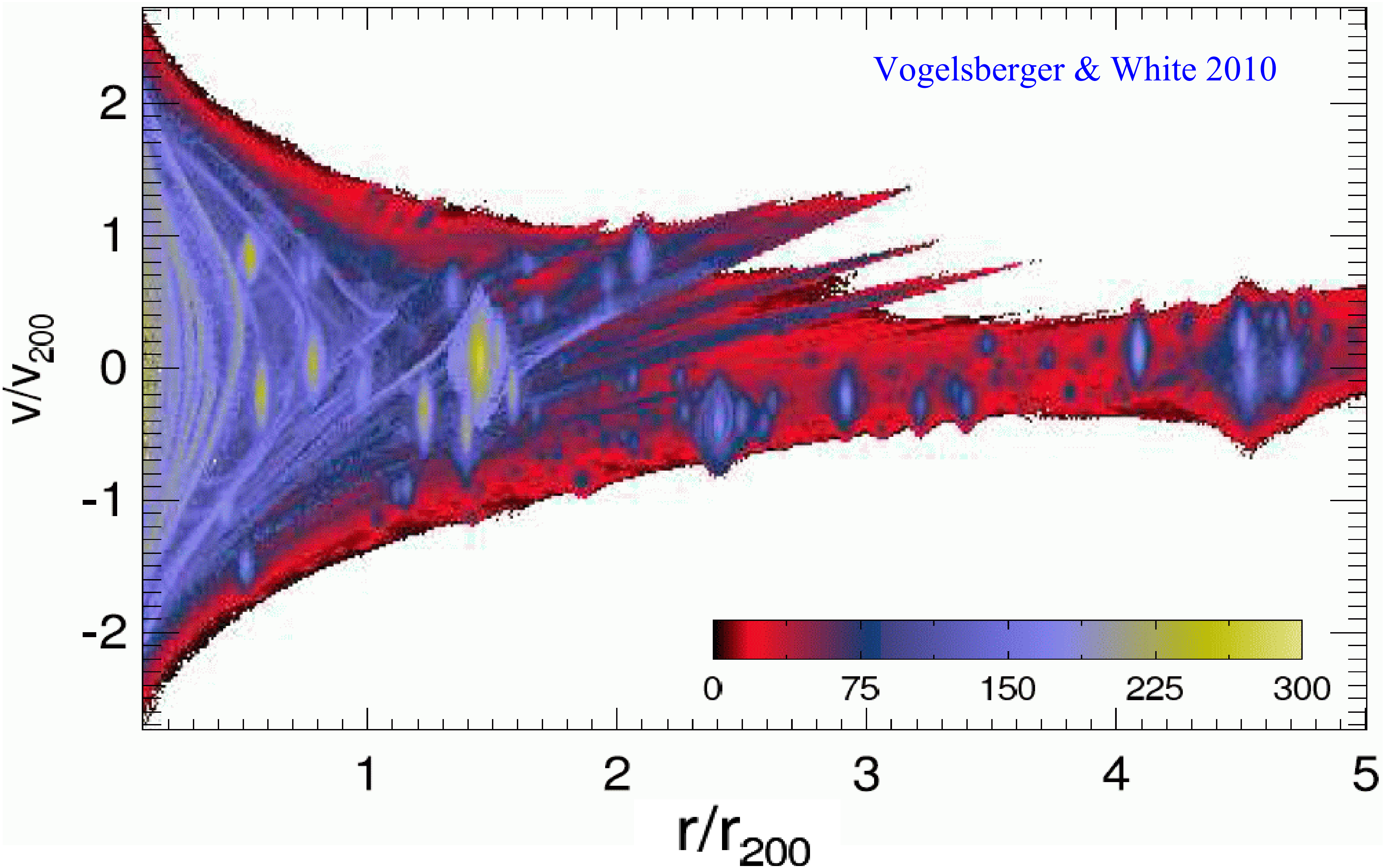
Simulation from self-similar spherical initial conditions



Vogelsberger et al 2009

The radial orbit instability leads to a system which is strongly prolate in the inner nonlinear regions

Caustic crossing counts in a ΛCDM Milky Way halo



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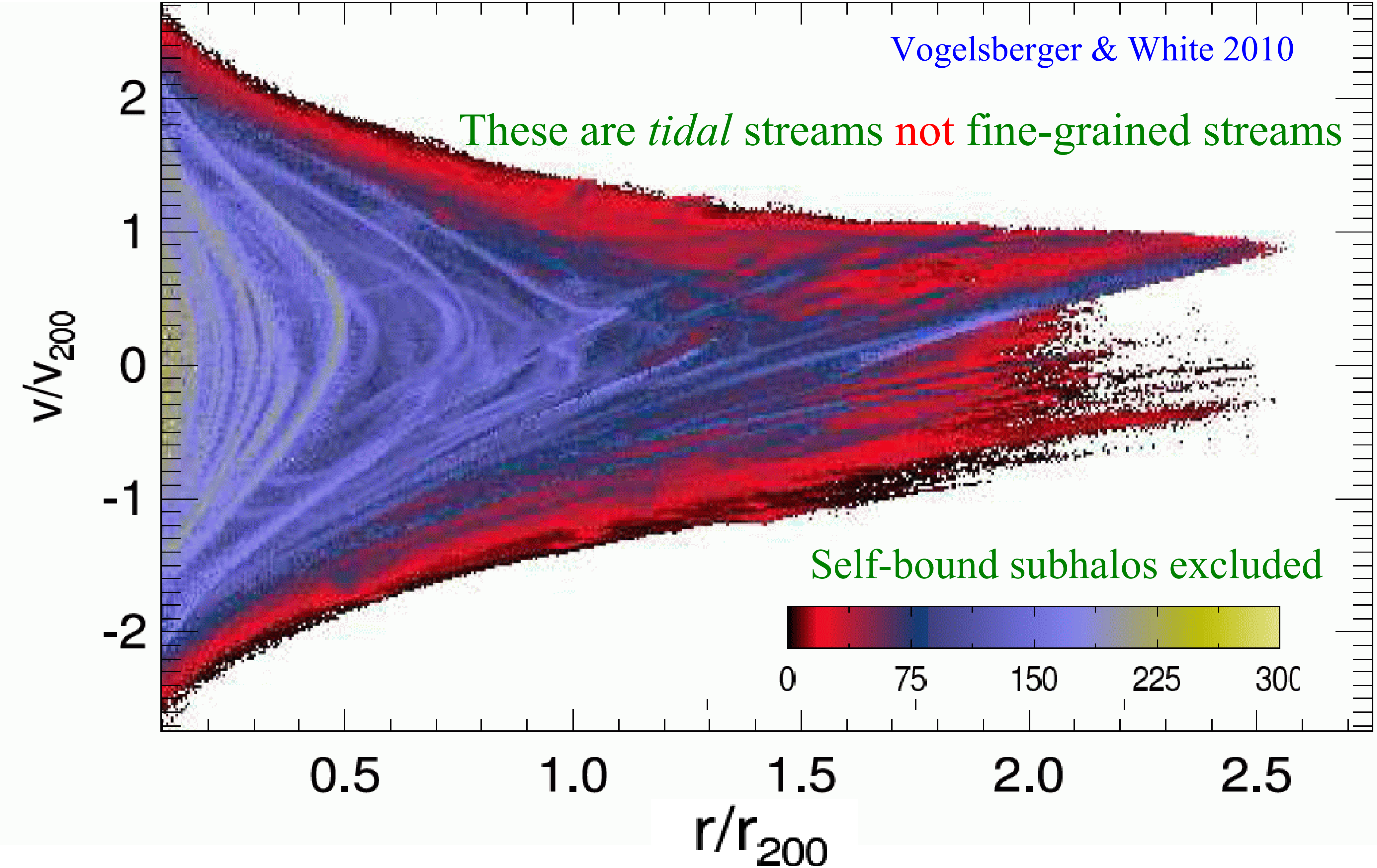
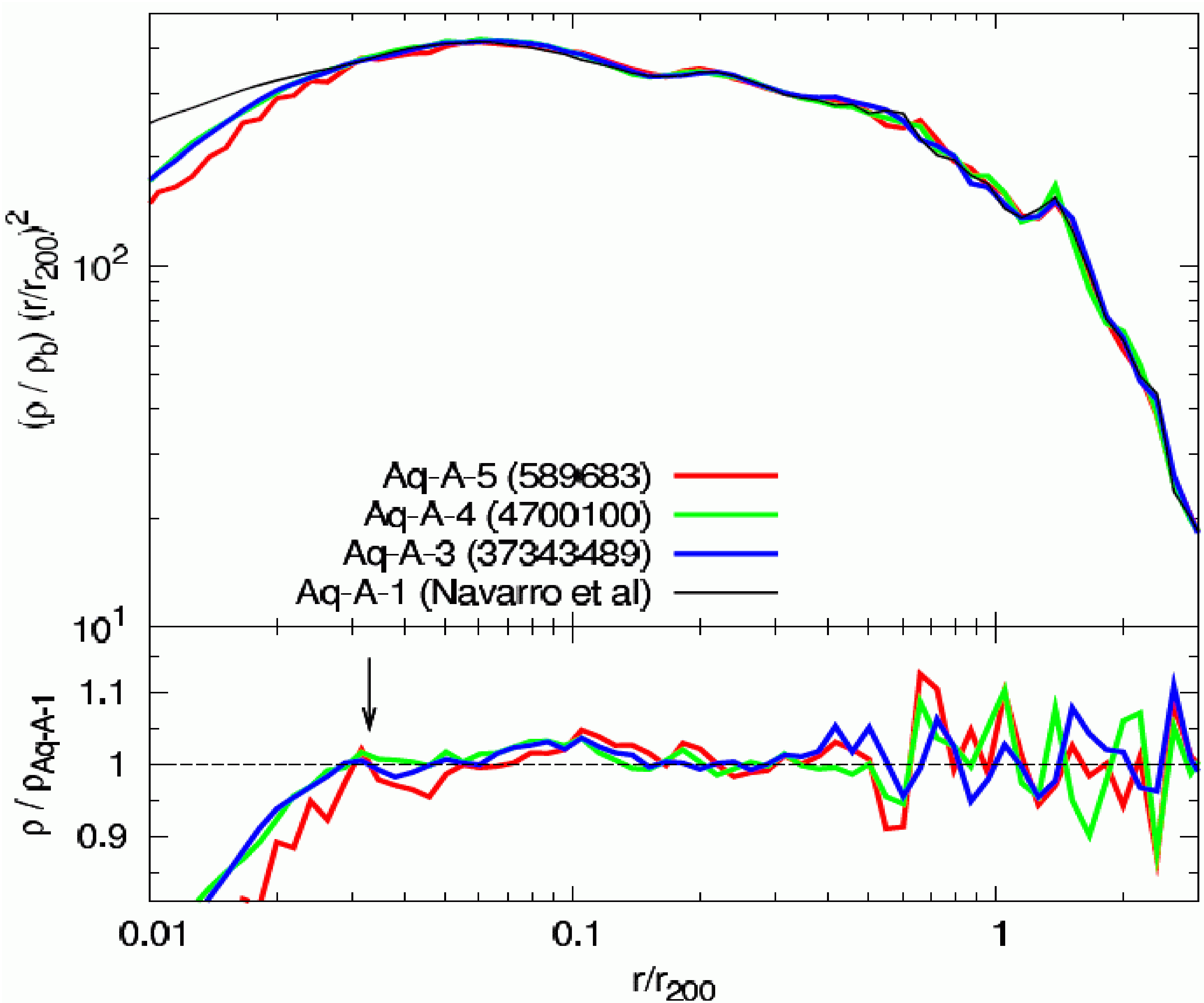


Convergence of density profiles for Aquarius halos

Vogelsberger & White 2010



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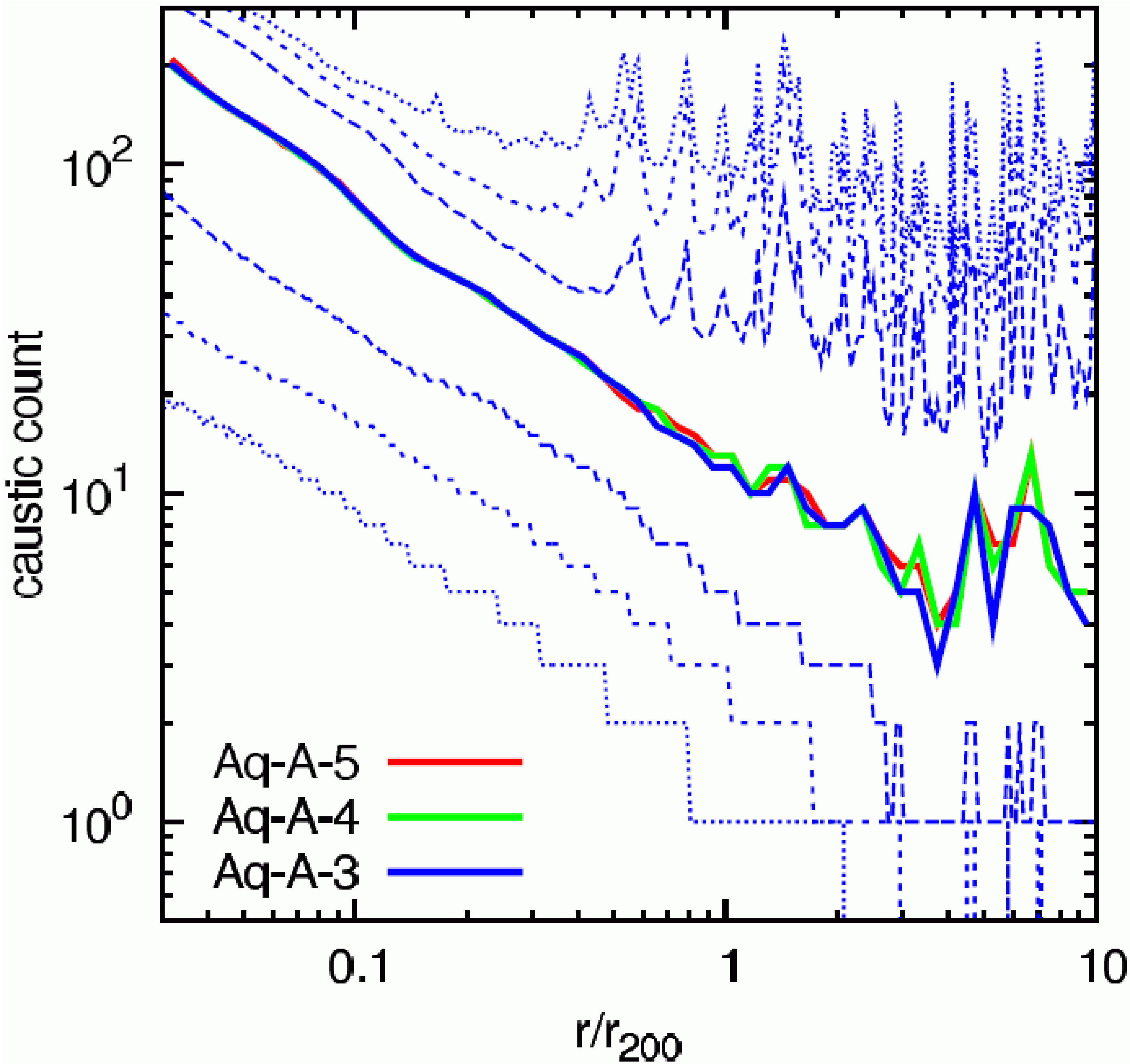
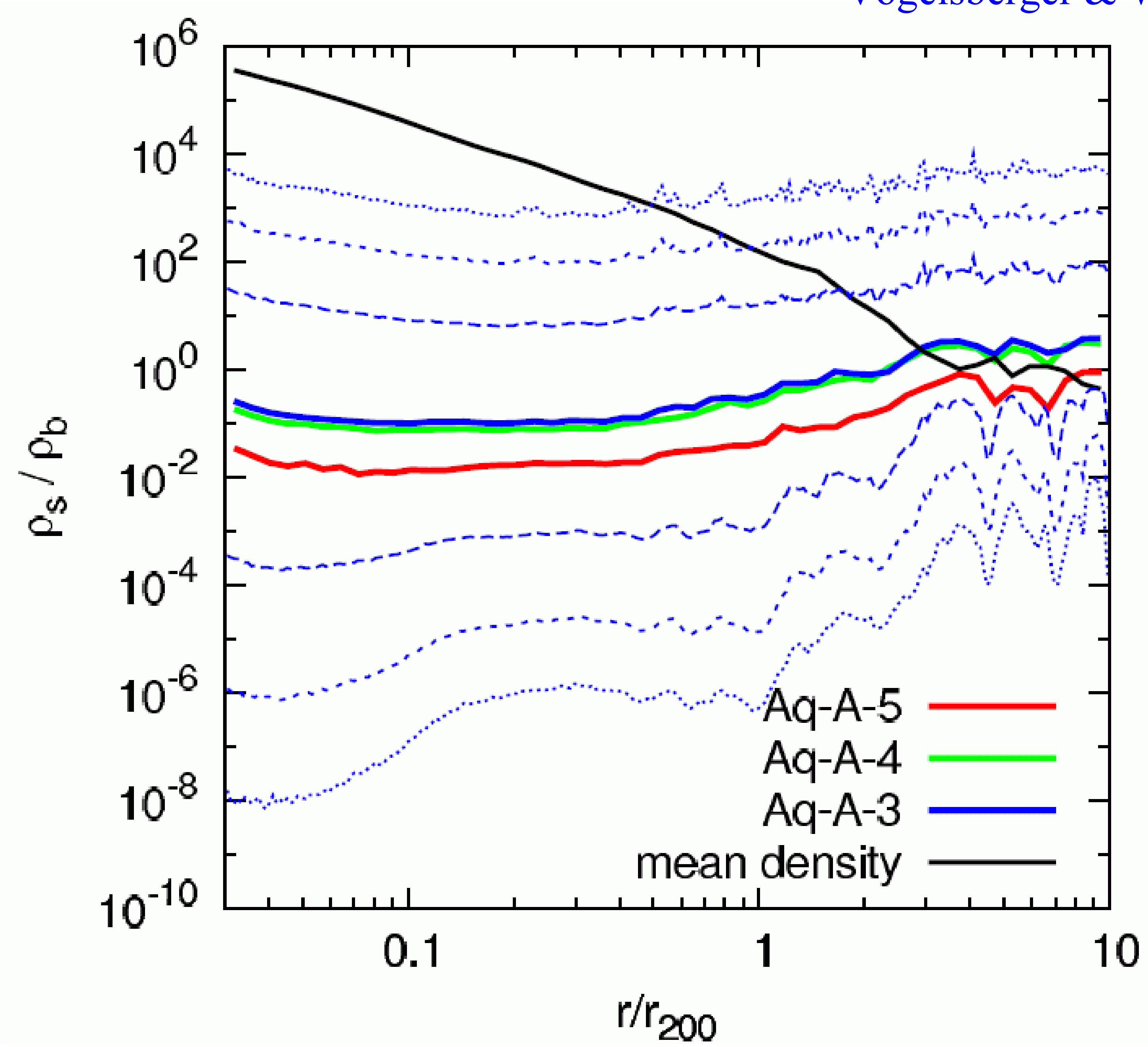


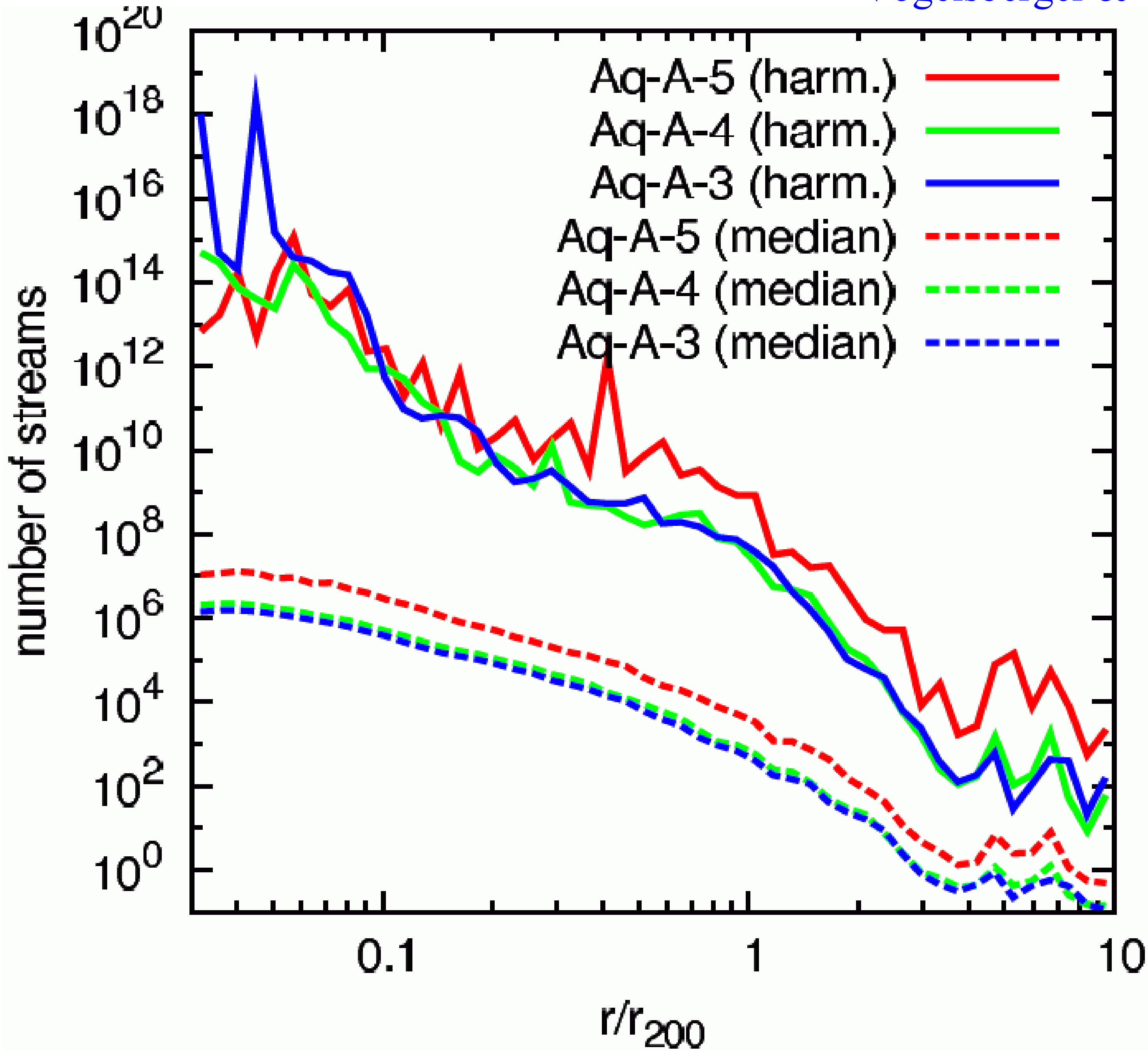
Stream density distribution in Aquarius halos

Vogelsberger & White 2010



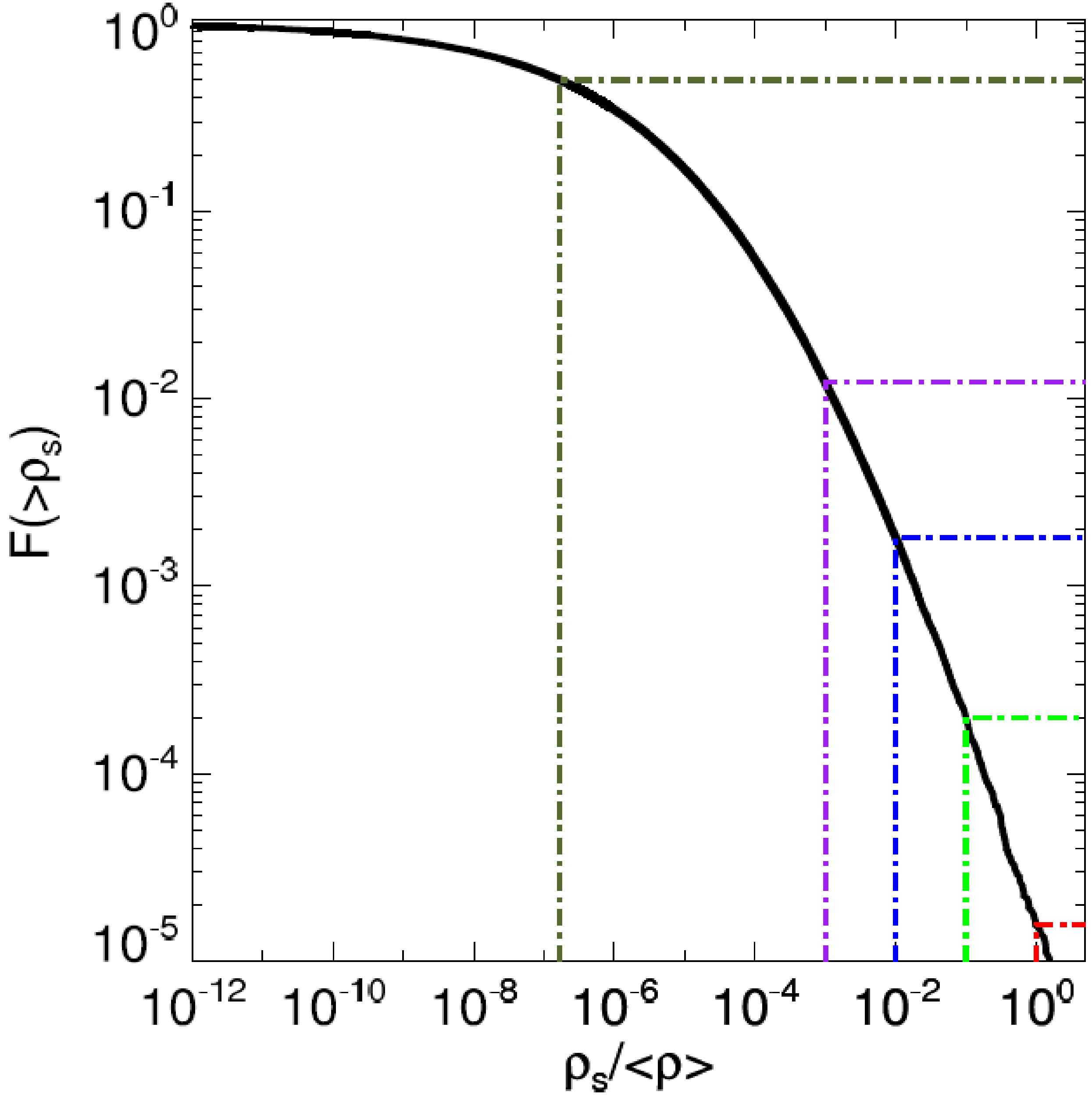
Stream number profiles for Aquarius halos

Vogelsberger & White 2010



Stream density distribution at the Sun

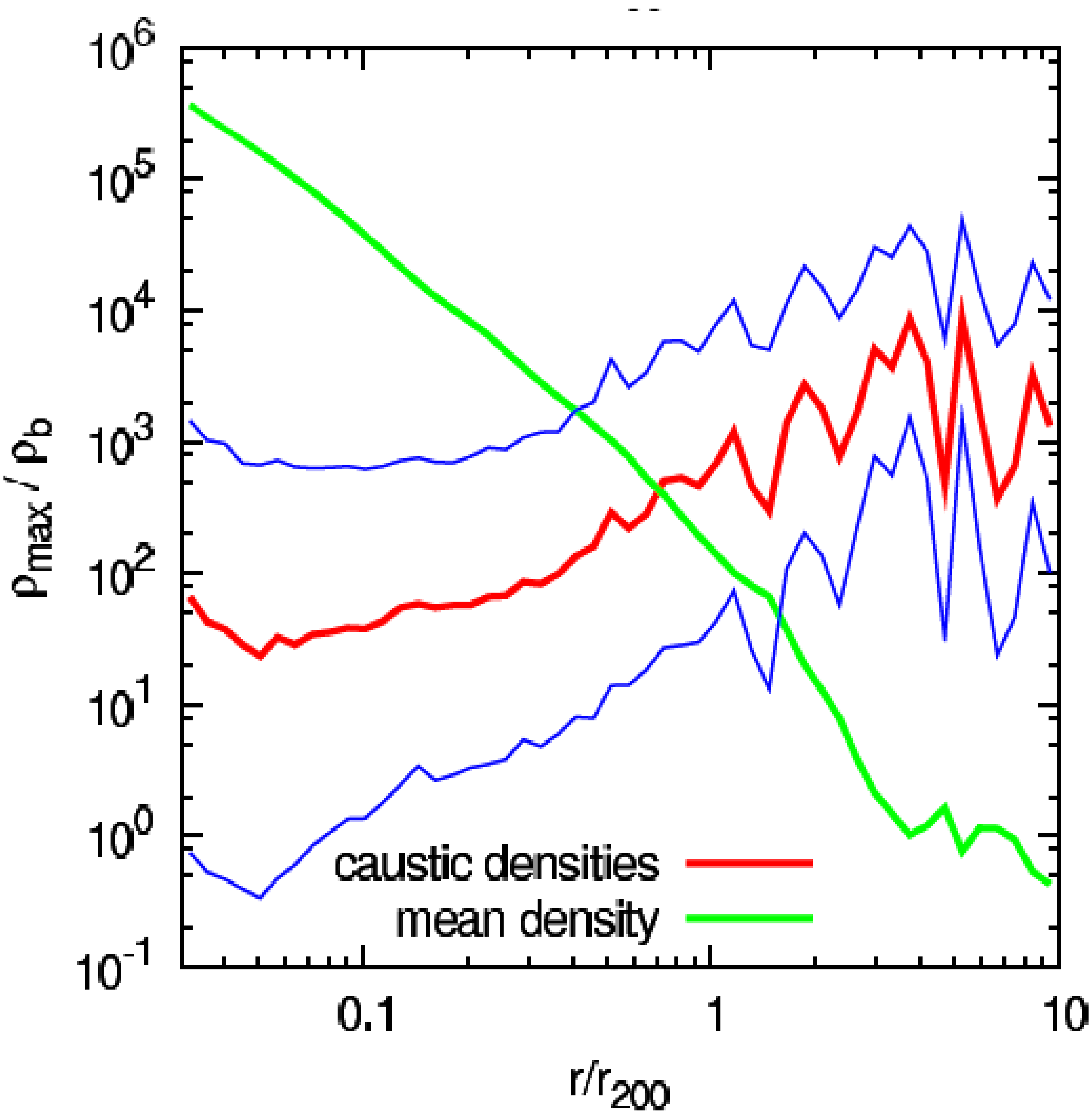
Vogelsberger & White 2010



Cumulative stream density distribution for particles with
7 kpc < r < 13 kpc

Radial distribution of peak density at caustics

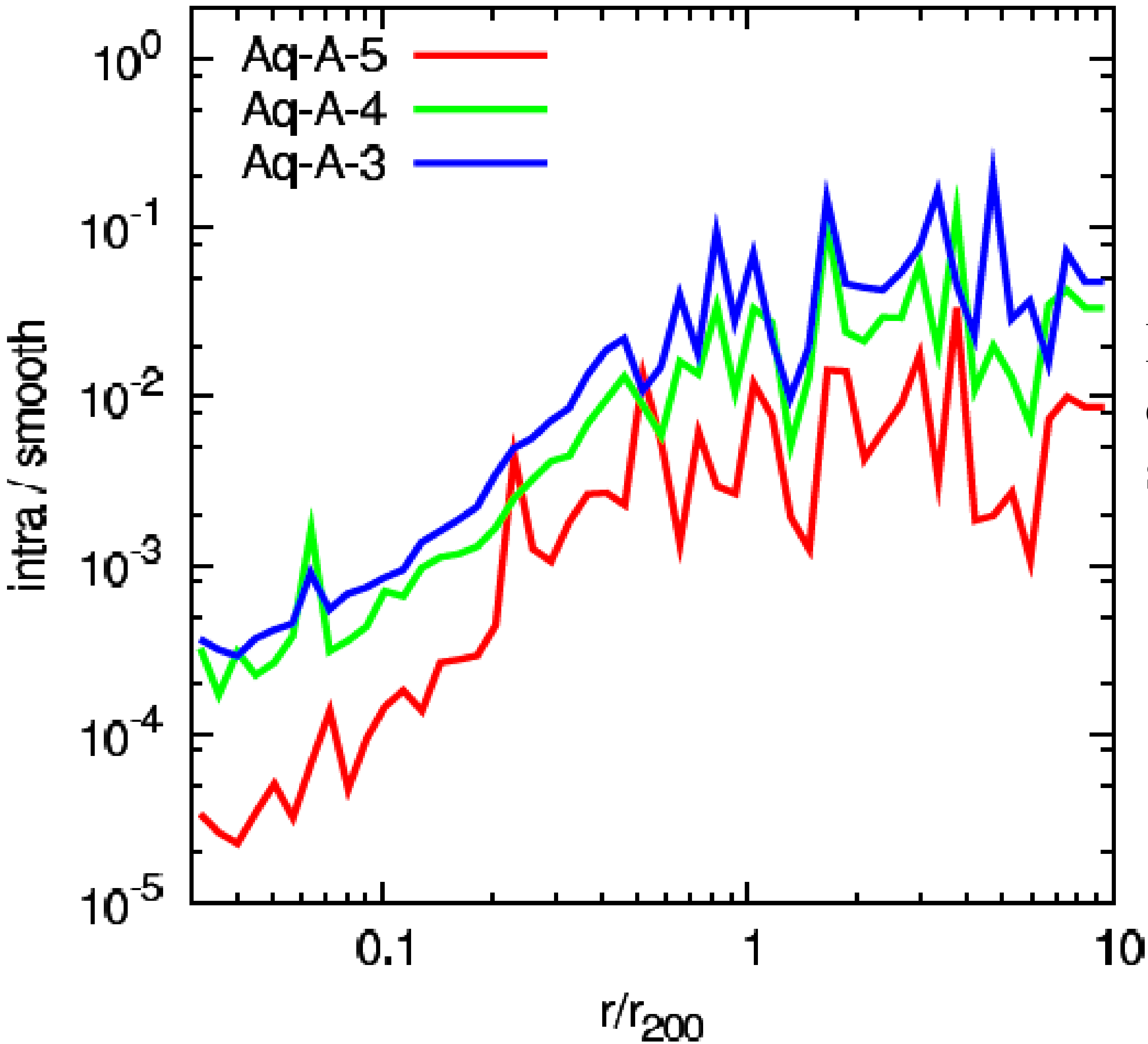
Vogelsberger & White 2010



Initial velocity dispersion assumes a standard WIMP with $m = 100\ GeV/c^2$

Fraction of annihilation luminosity from caustics

Vogelsberger & White 2010



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- Integration of the GDE can augment the ability of ΛCDM simulations to resolve fine-grained structure by 15 to 20 orders of magnitude

- Fine-grained streams and their associated caustics will have no significant effect on direct and indirect Dark Matter detection experiments

- The most massive stream at the Sun should contain roughly 0.001 of the local DM density and so might be detectable in an axion detector