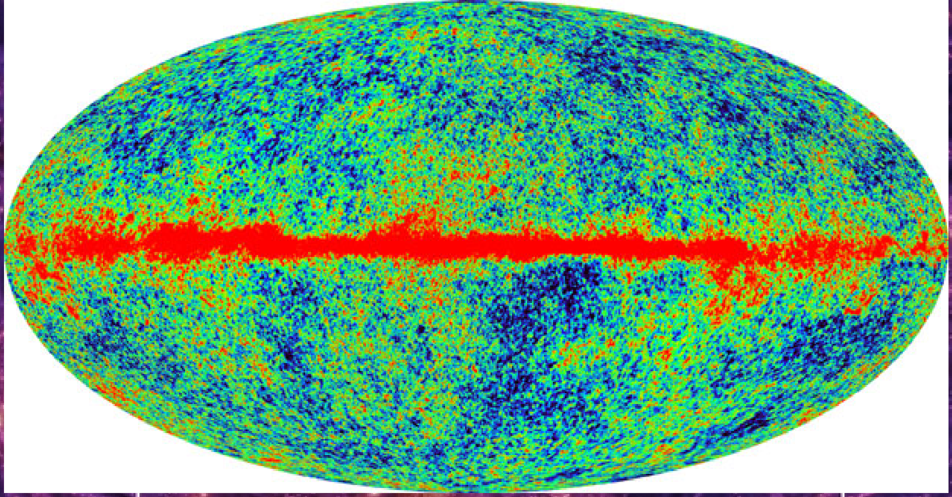
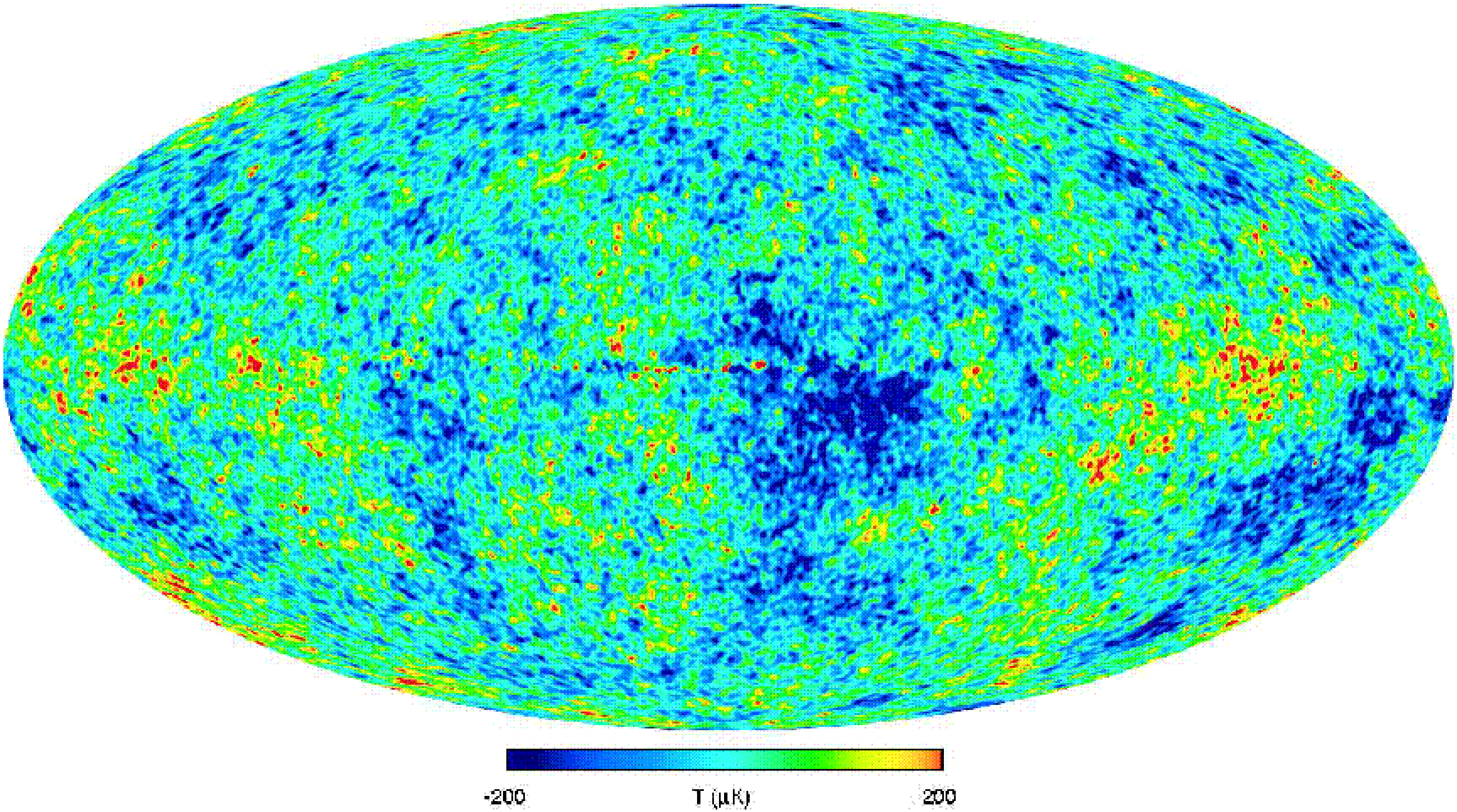


Brouwer Lecture
April 2008

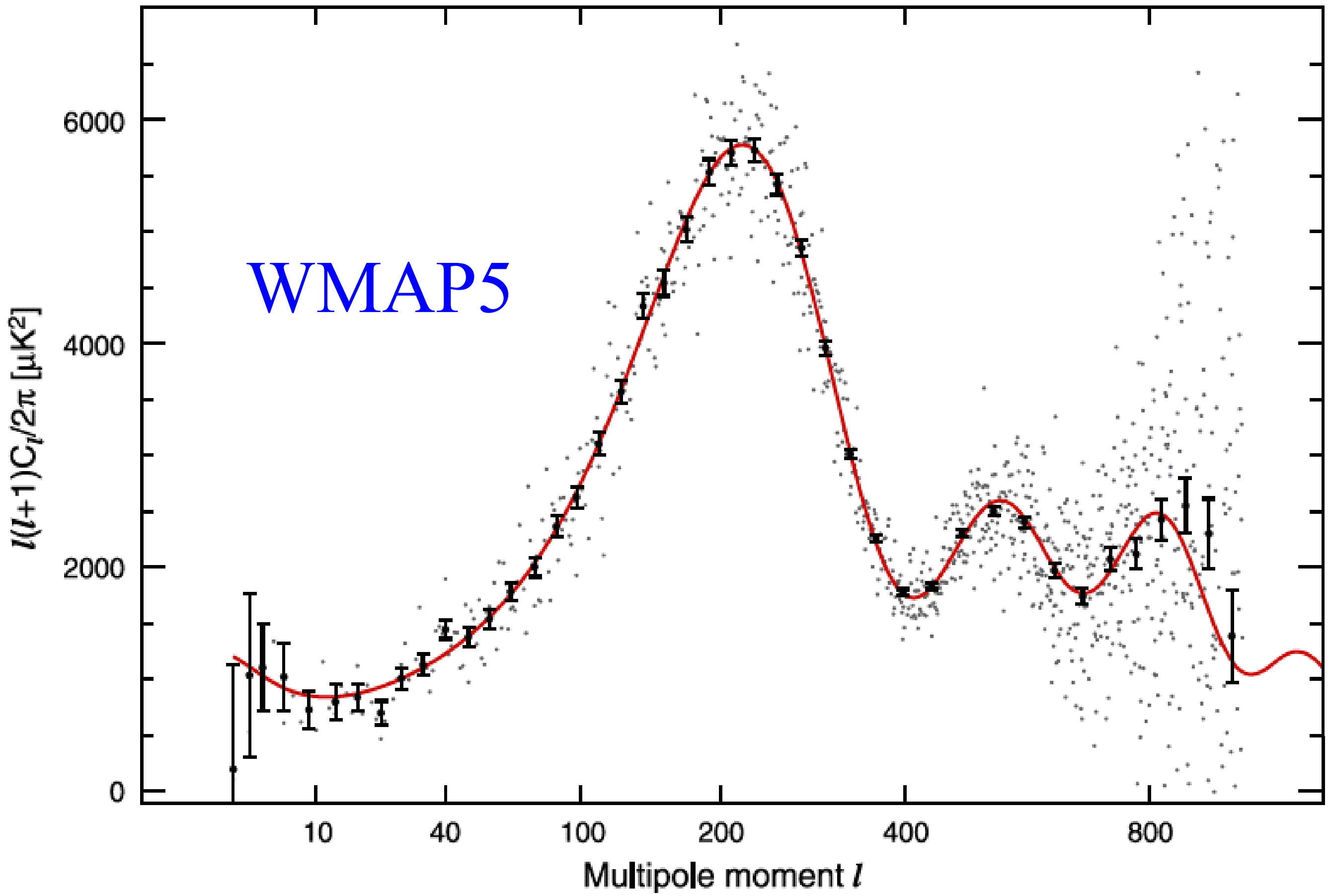
Galaxy halos at (very) high resolution

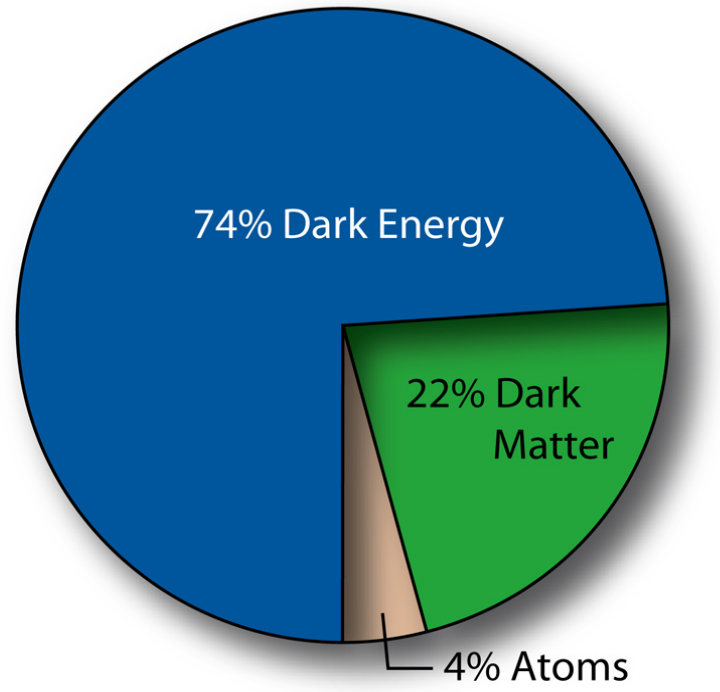
Simon White
Max Planck Institute for Astrophysics

The *WMAP* of the whole CMB sky



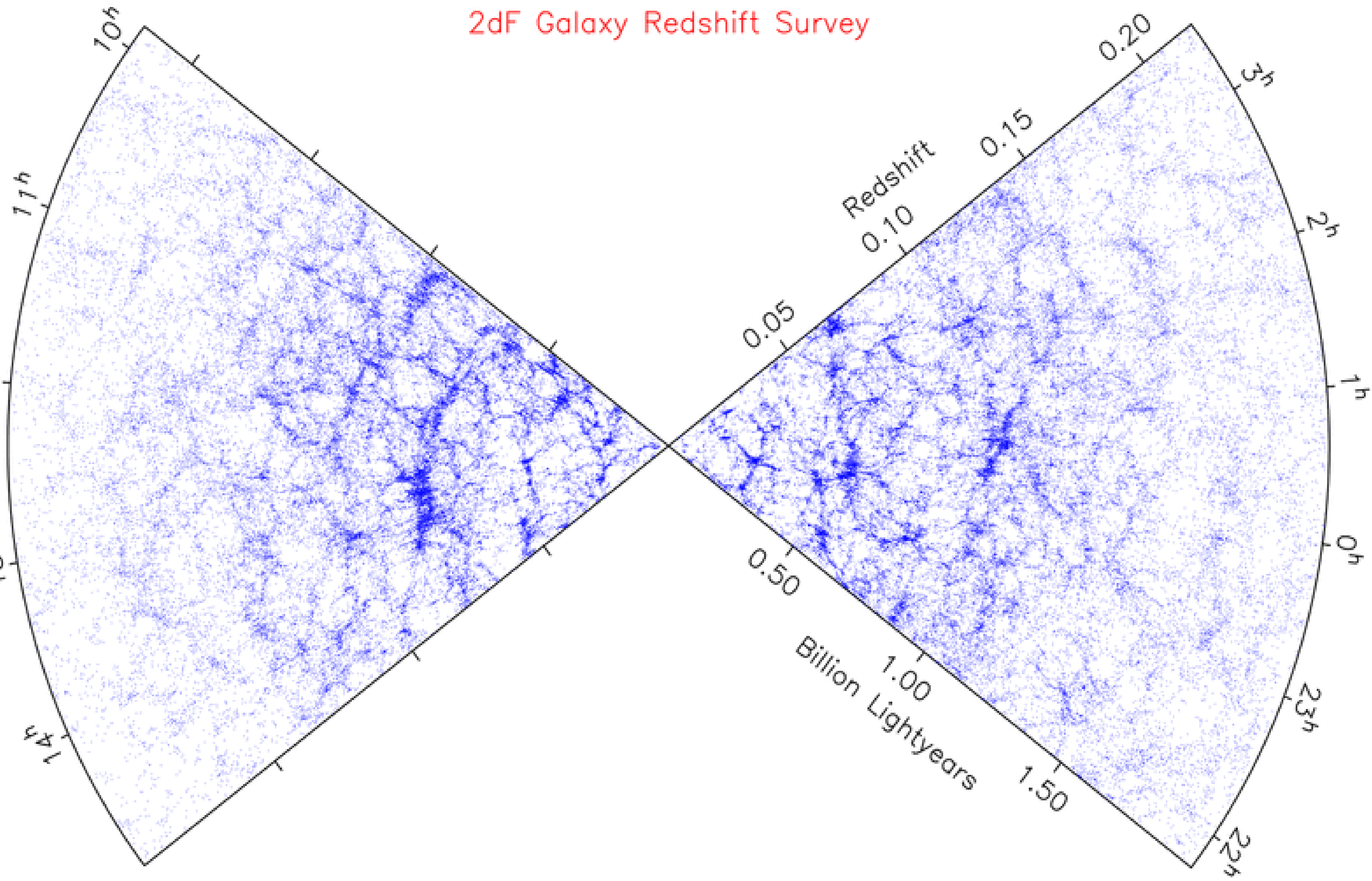
Bennett et al 2003



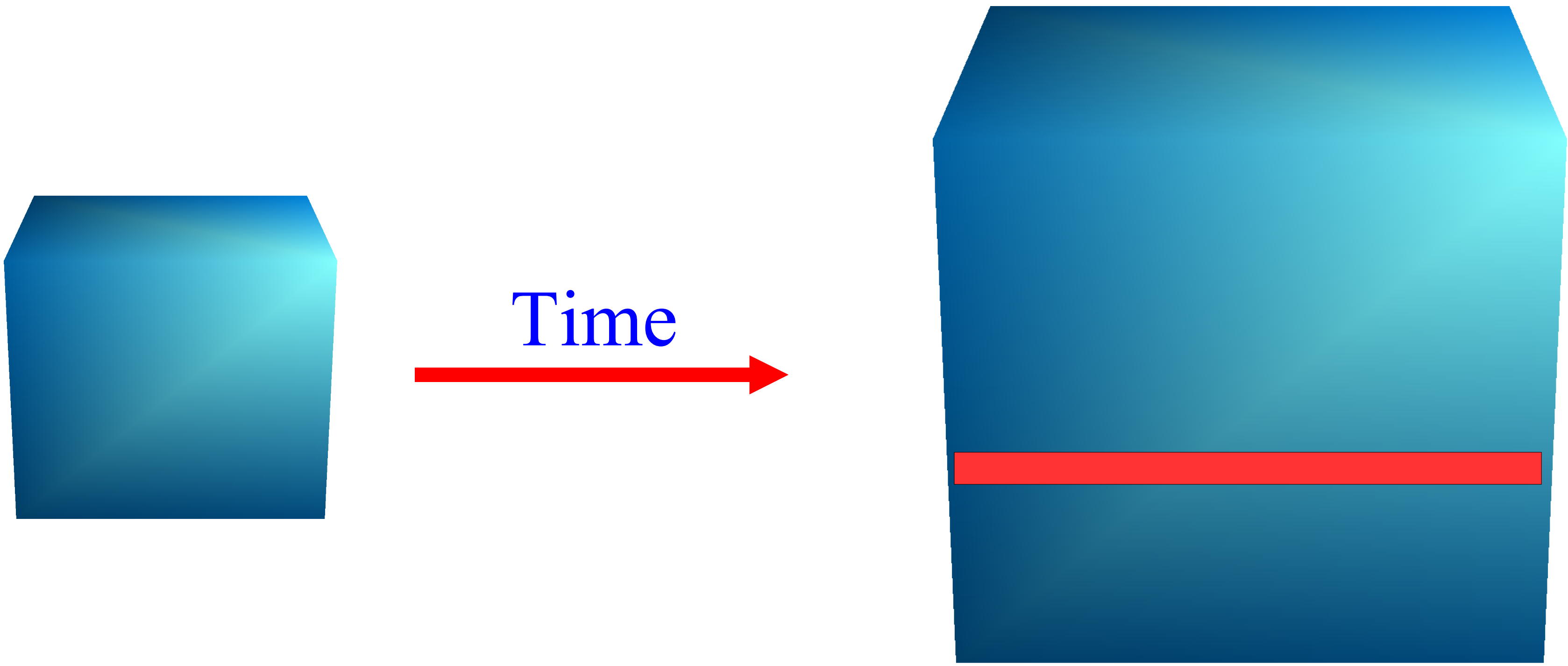


parameter	symbol	WMAP-5 alone	WMAP-5 + BAO + SNe	comment
CMB temperature	T_{CMB}	2.728 ± 0.004 K	–	from (Fixsen *et al.* 1996)
total matter density	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064	
matter density	Ω_{m0}	0.258 ± 0.03	0.279 ± 0.015	assuming spatial flatness
baryon density	Ω_{b0}	0.0441 ± 0.0030	0.0462 ± 0.0015	here and below
cosmological constant	$\Omega_{\Lambda 0}$	0.742 ± 0.03	0.721 ± 0.015	
Hubble constant	h	$0.719^{+0.026}_{-0.027}$	0.701 ± 0.013	
power-spectrum normalisation	σ_8	0.796 ± 0.036	0.817 ± 0.026	
age of the Universe in Gyr	t_0	13.69 ± 0.13	13.73 ± 0.12	
decoupling redshift	z_{dec}	1087.9 ± 1.2	1088.2 ± 1.1	
reionisation optical depth	τ	0.087 ± 0.017	0.084 ± 0.016	
spectral index	n_{s}	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$	

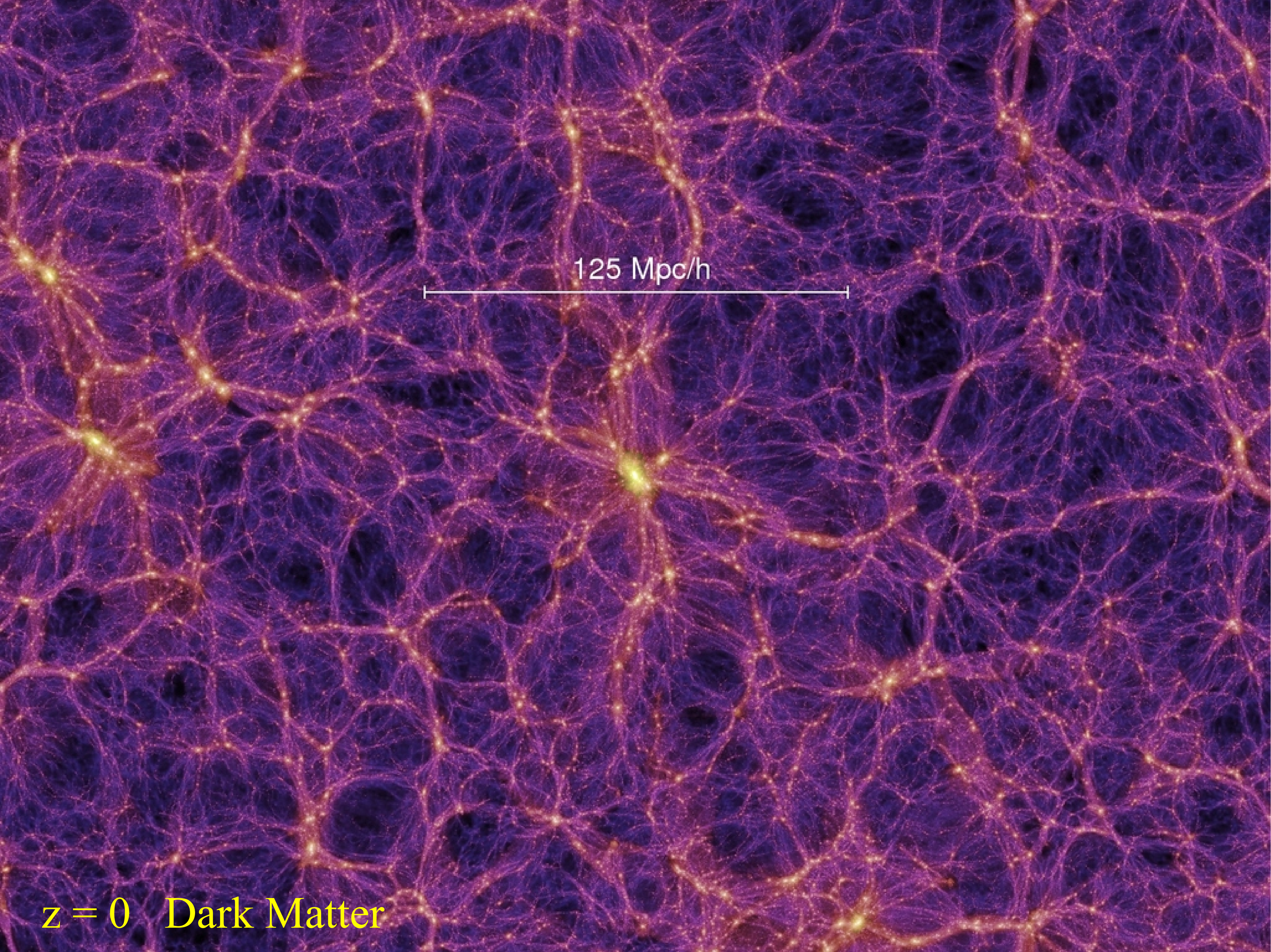
Nearby large-scale structure



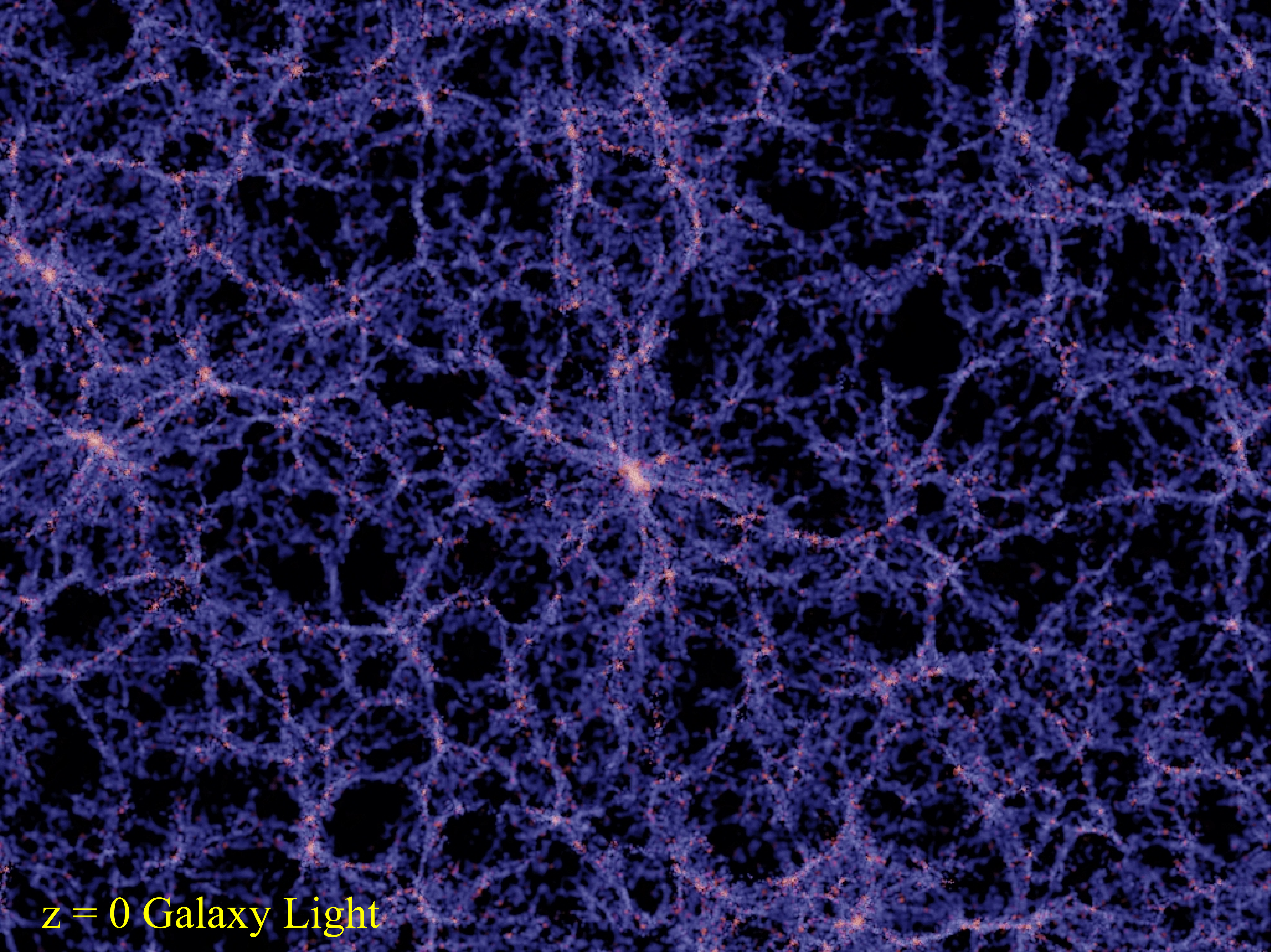
Evolving the Universe in a computer

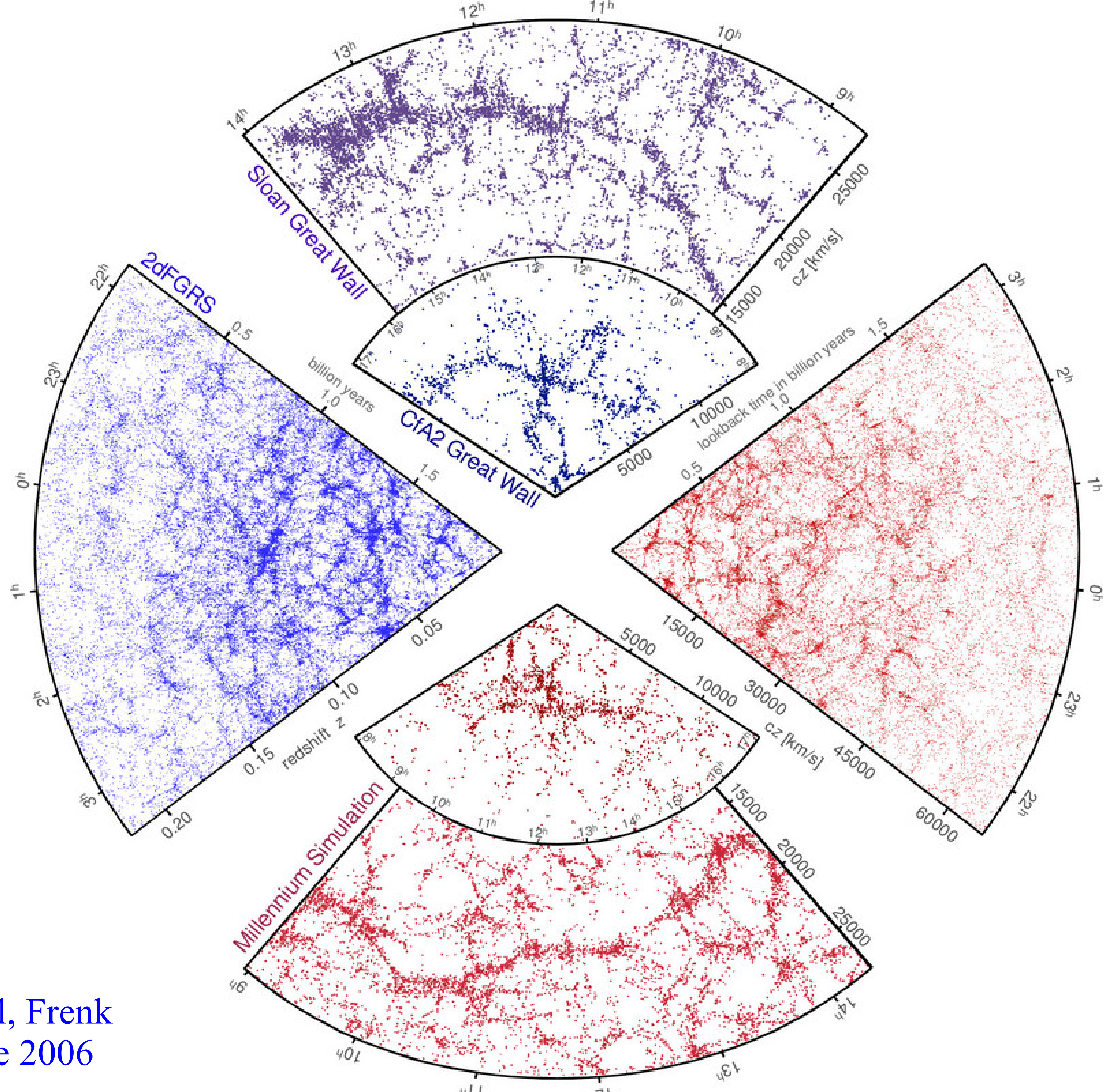


- Follow the matter in an expanding cubic region
- Start 400,000 years after the Big Bang
- Match initial conditions to the observed Microwave Background
- Calculate evolution forward to the present day



125 Mpc/h
z = 0 Dark Matter


z = 0 Galaxy Light

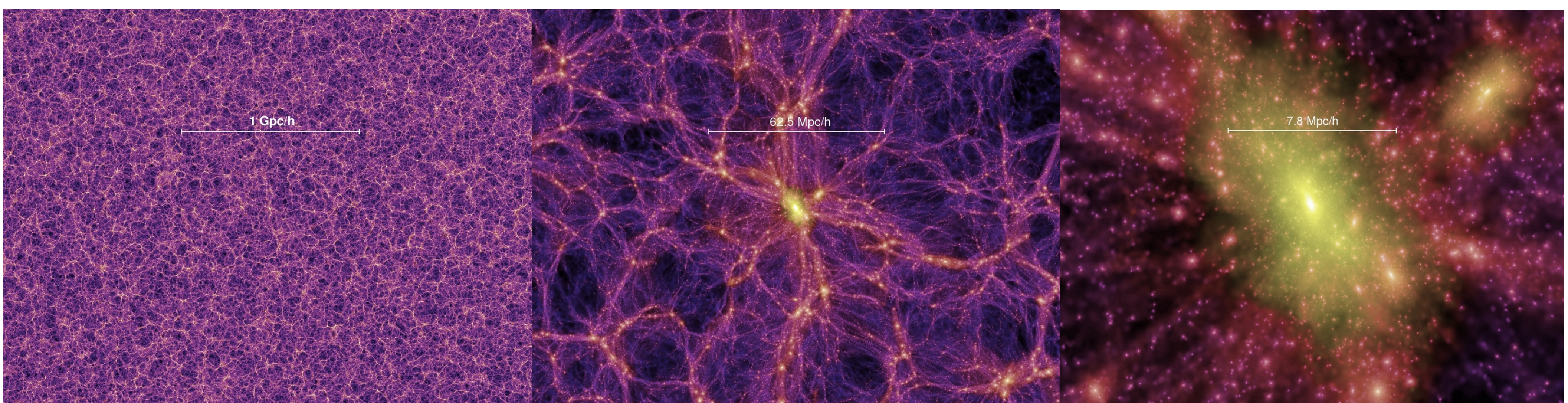


Springel, Frenk
& White 2006

Visualizing Darkness

- The smooth becomes rough with the passing of time

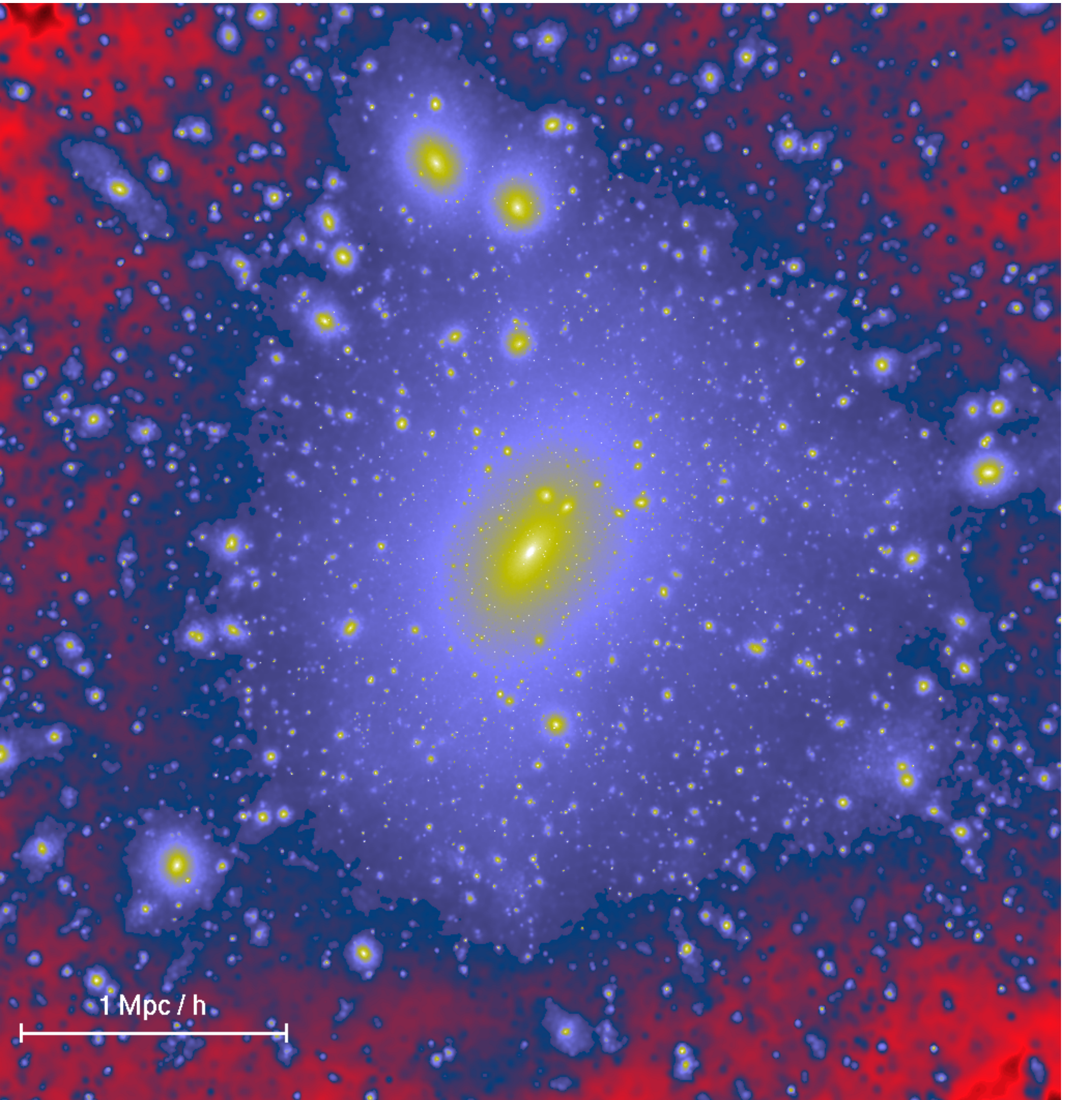
- Uniformity, filamentarity, hierarchy – it all depends on scale

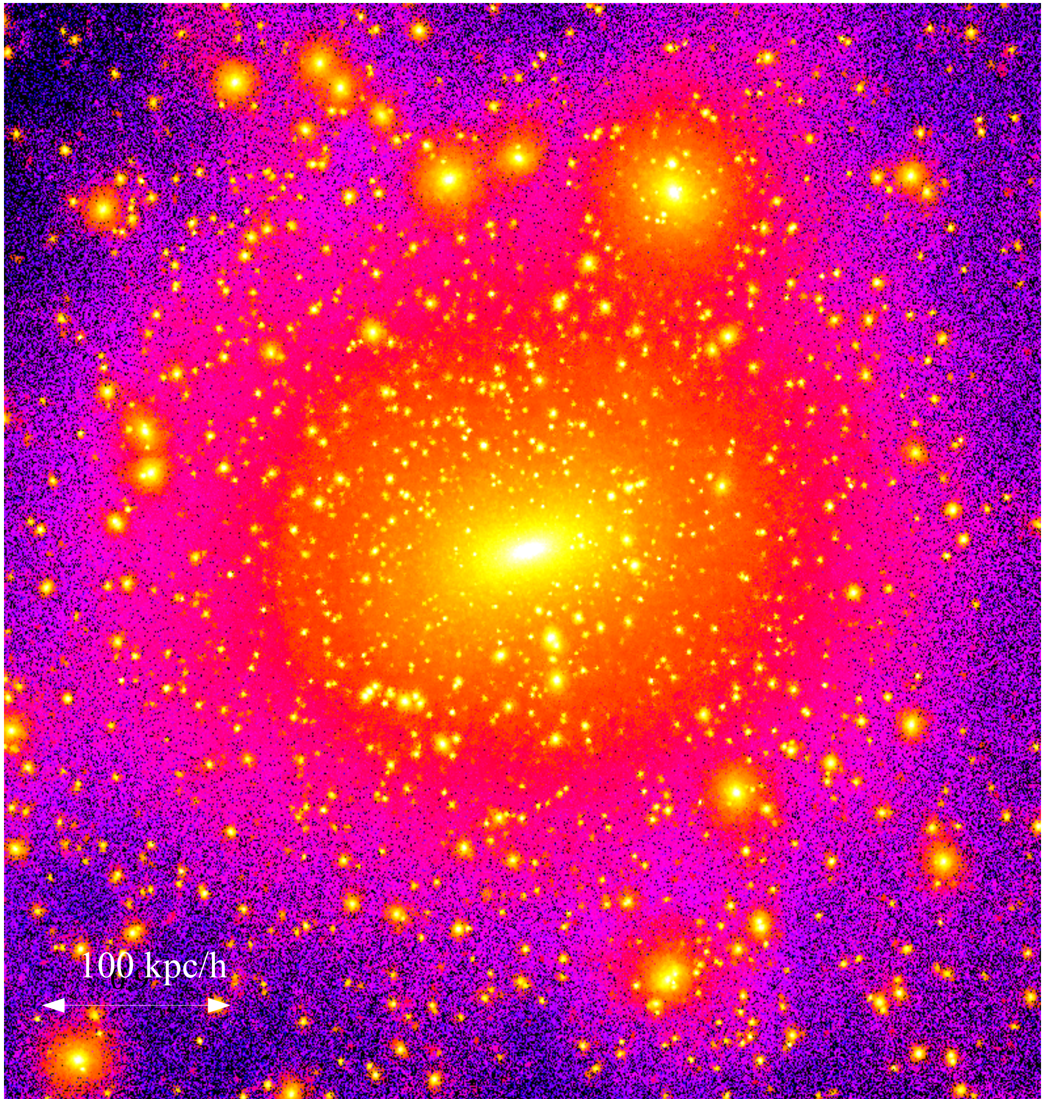


The dark matter structure of ΛCDM halos

A rich galaxy cluster halo
Springel et al 2001

A 'Milky Way' halo
Power et al 2002





ΛCDM galaxy halos (without galaxies!)

- Halos extend to ~10 times the 'visible' radius of galaxies and contain ~10 times the mass in the visible regions

- Halos are not spherical but approximate triaxial ellipsoids
  - -- more prolate than oblate
  - -- axial ratios greater than two are common

- "Cuspy" density profiles with outwardly increasing slopes
  - -- $d \ln \varrho / d \ln r = \gamma$ with $\gamma < -2.5$ at large r

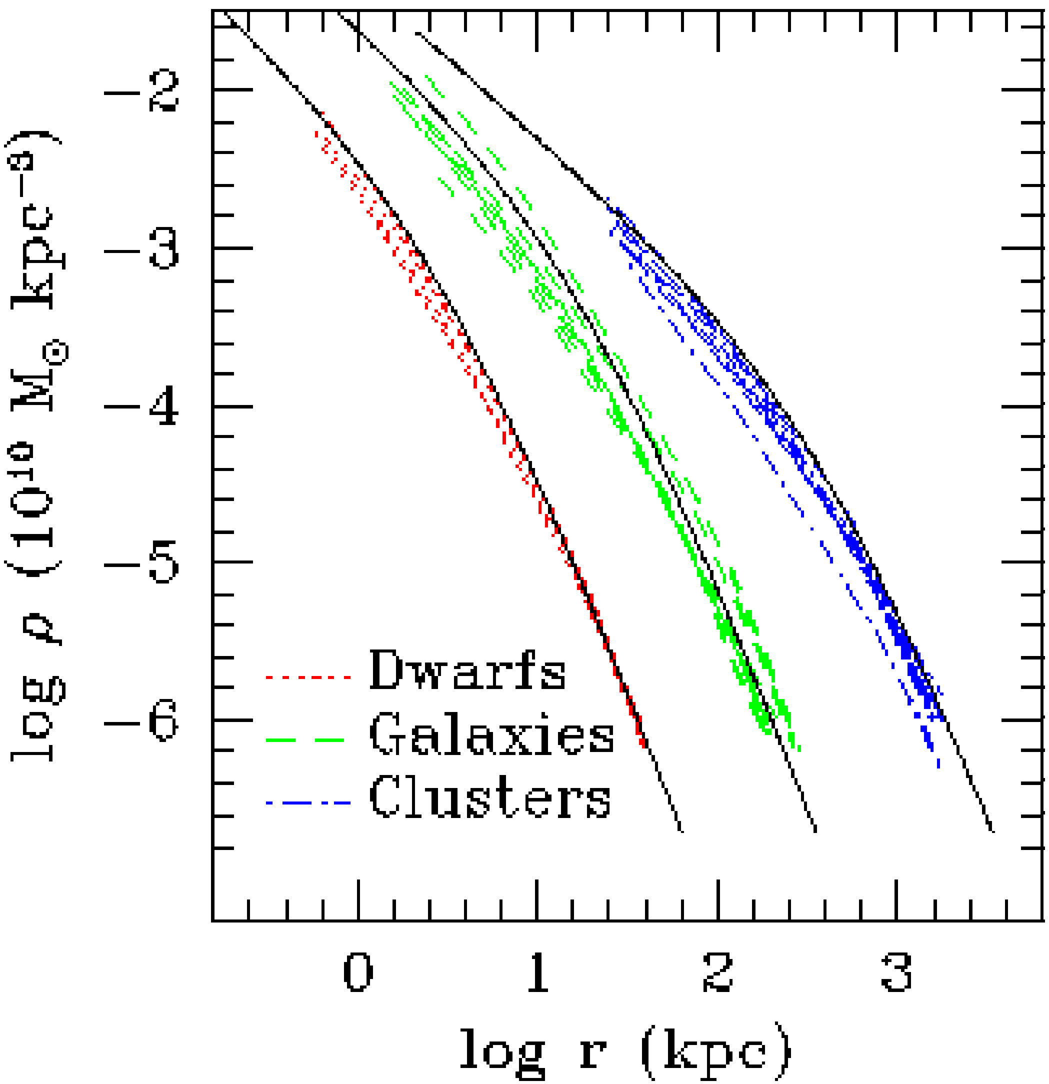
    $\gamma > -1.2$ at small r

- Substantial numbers of self-bound subhalos contain ~10% of the halo's mass and have $dN / dM \sim M^{-1.8}$

→ Most substructure mass is in most massive subhalos

Density profiles of dark matter halos

Navarro, Frenk & White 1996



The average dark matter density of a dark halo depends on distance from halo centre in a very similar way in halos of all masses at all times

-- a universal profile shape --

$$\rho(r)/\rho_{crit} \approx \delta \; r_s \,/\, r(1 + r/r_s)^2$$

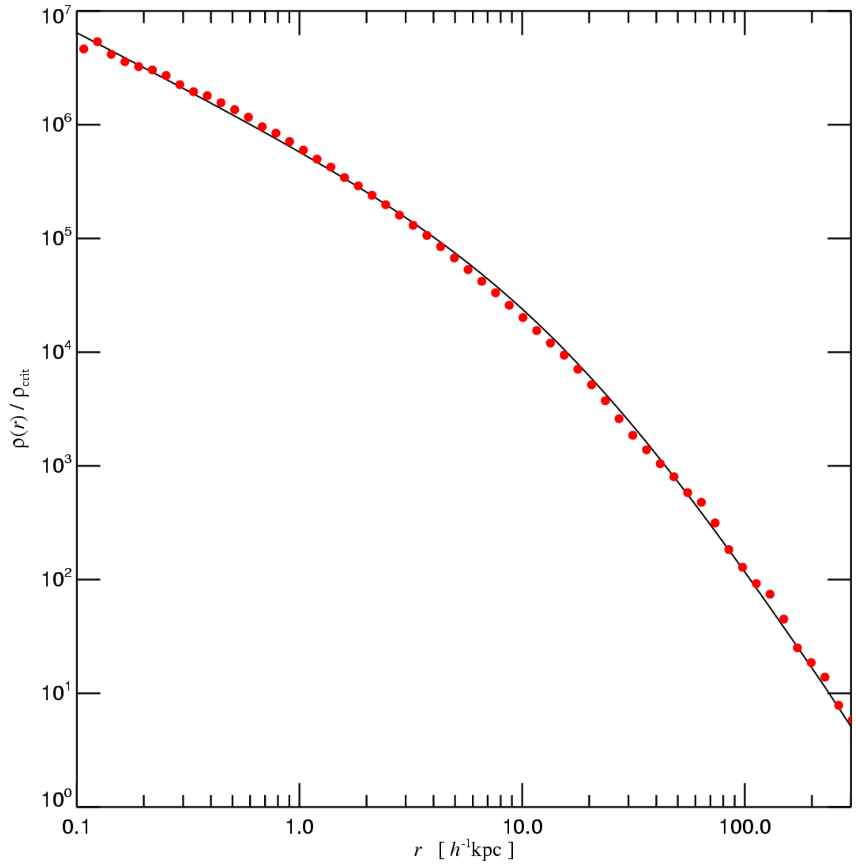
More massive halos and halos that form earlier have higher densities (bigger δ)

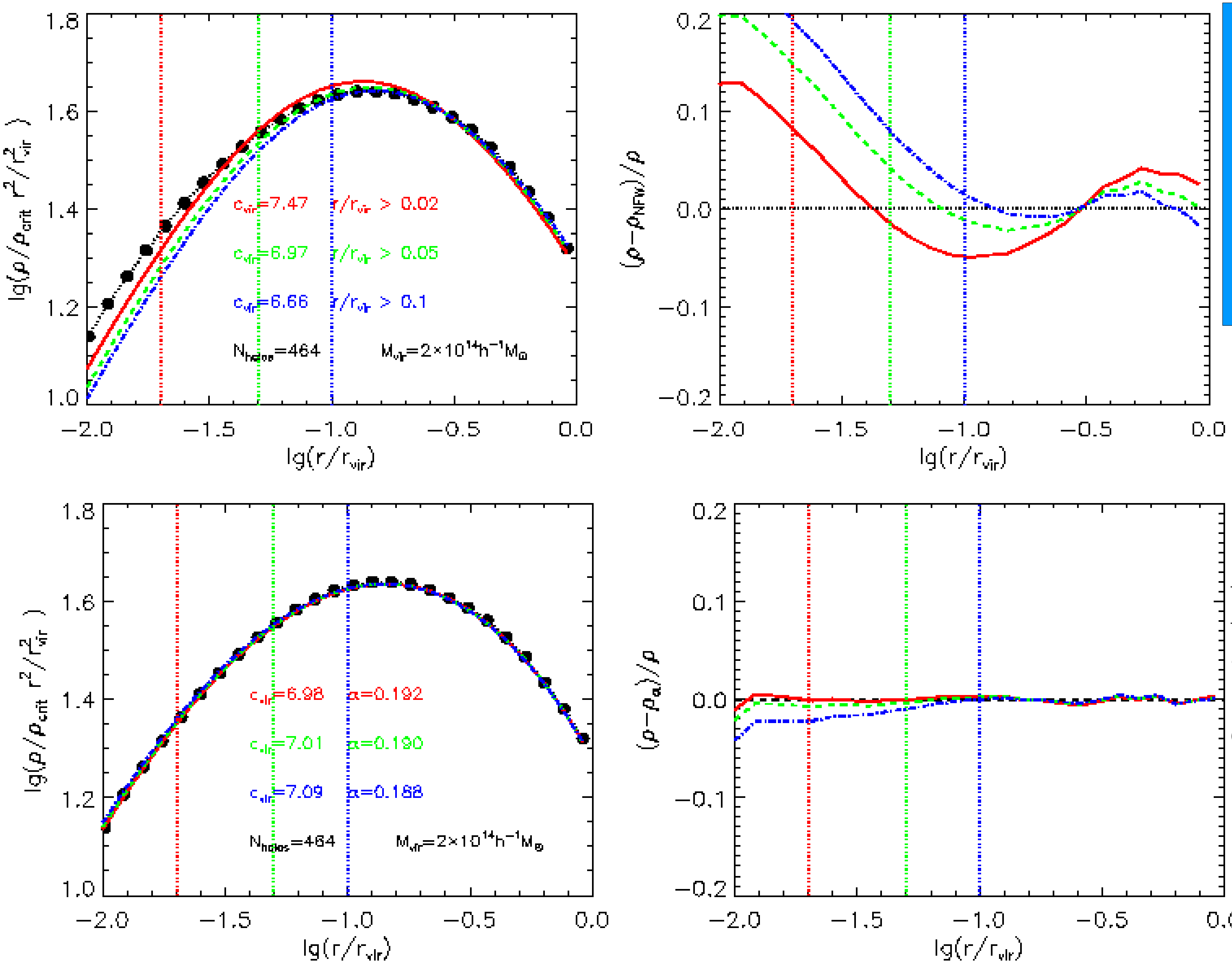
A high-resolution Milky Way halo

600 kpc

Navarro et al 2006

$N_{200} \sim 3 \times 10^7$





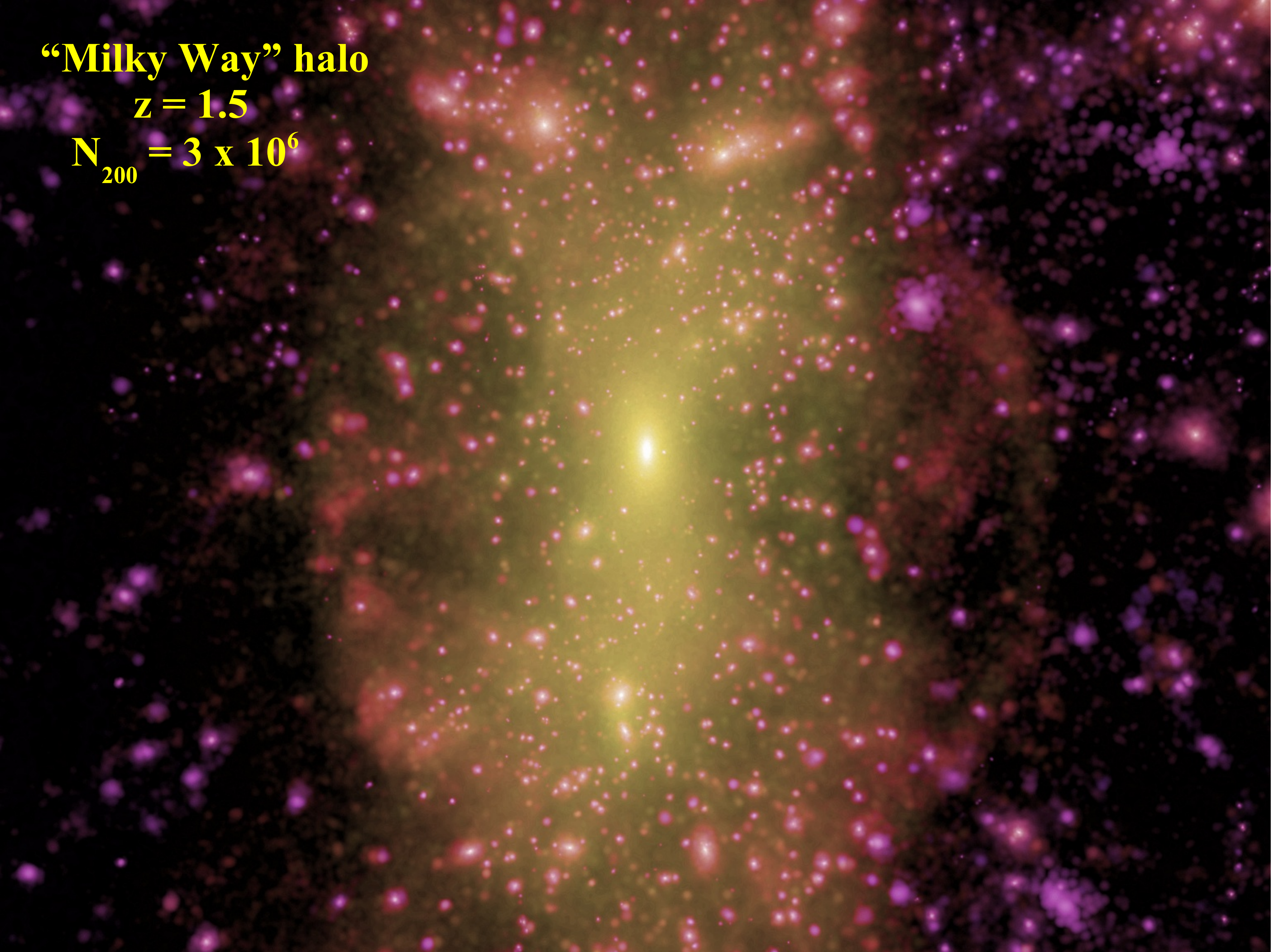
Einasto fits better than NFW!

Gao et al 2007

Mean density profile for 464 cluster halos of similar mass

In 1963 Einasto suggested modelling the Galactic spheroid with

$$\ln\left[\varrho(r)/\varrho_{-2}\right] = -2/\alpha\ \left[\left(r/r_{-2}\right)^{\alpha} - 1\right] \rightarrow \underline{\text{shape}}\ \text{parameter},\ \alpha$$

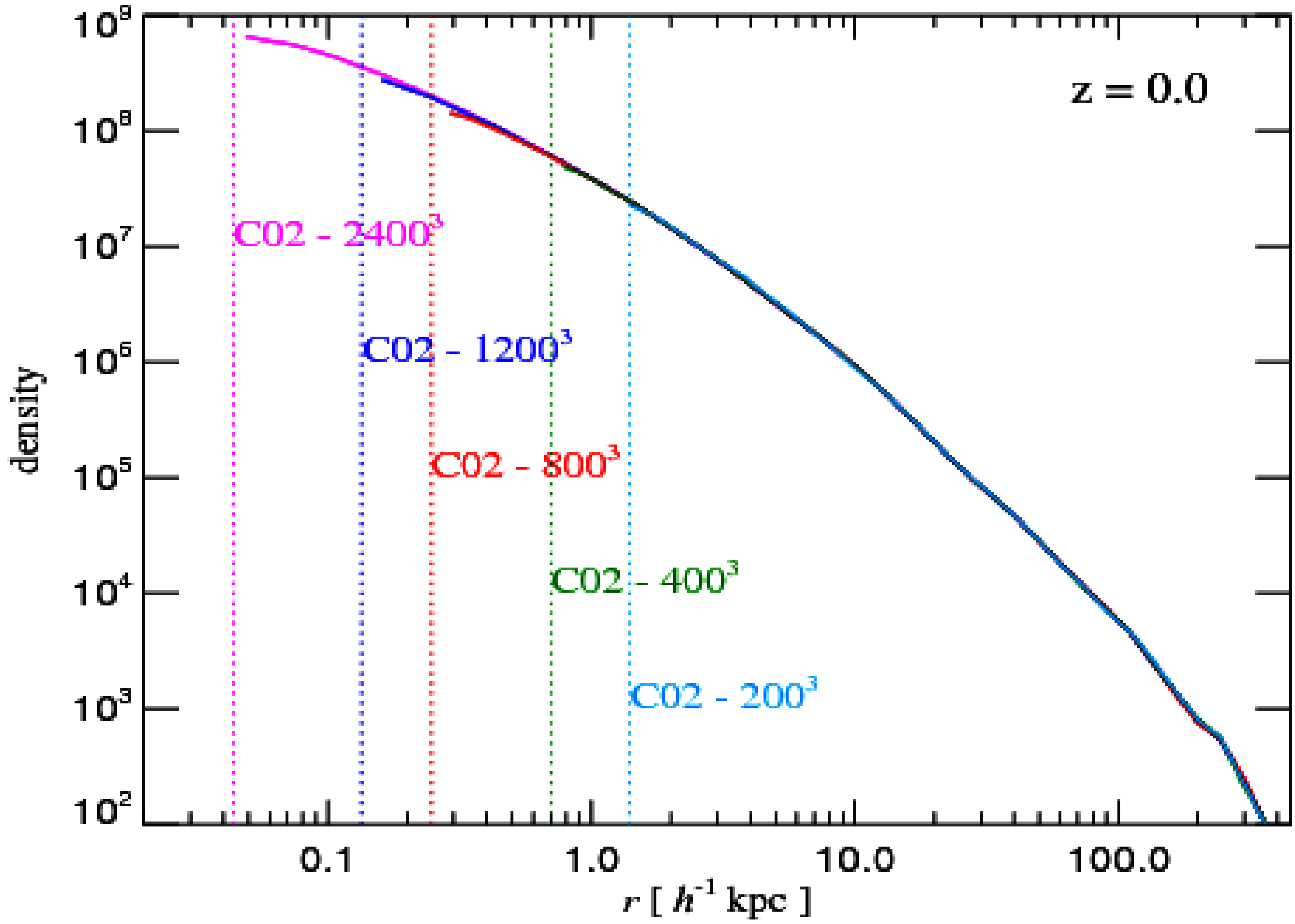

"Milky Way" halo
z = 1.5
N200 = 3 x 10^6


"Milky Way" halo
z = 1.5
N200 = 94 x 10^6


"Milky Way" halo
z = 1.5
N200 = 750 x 10^6

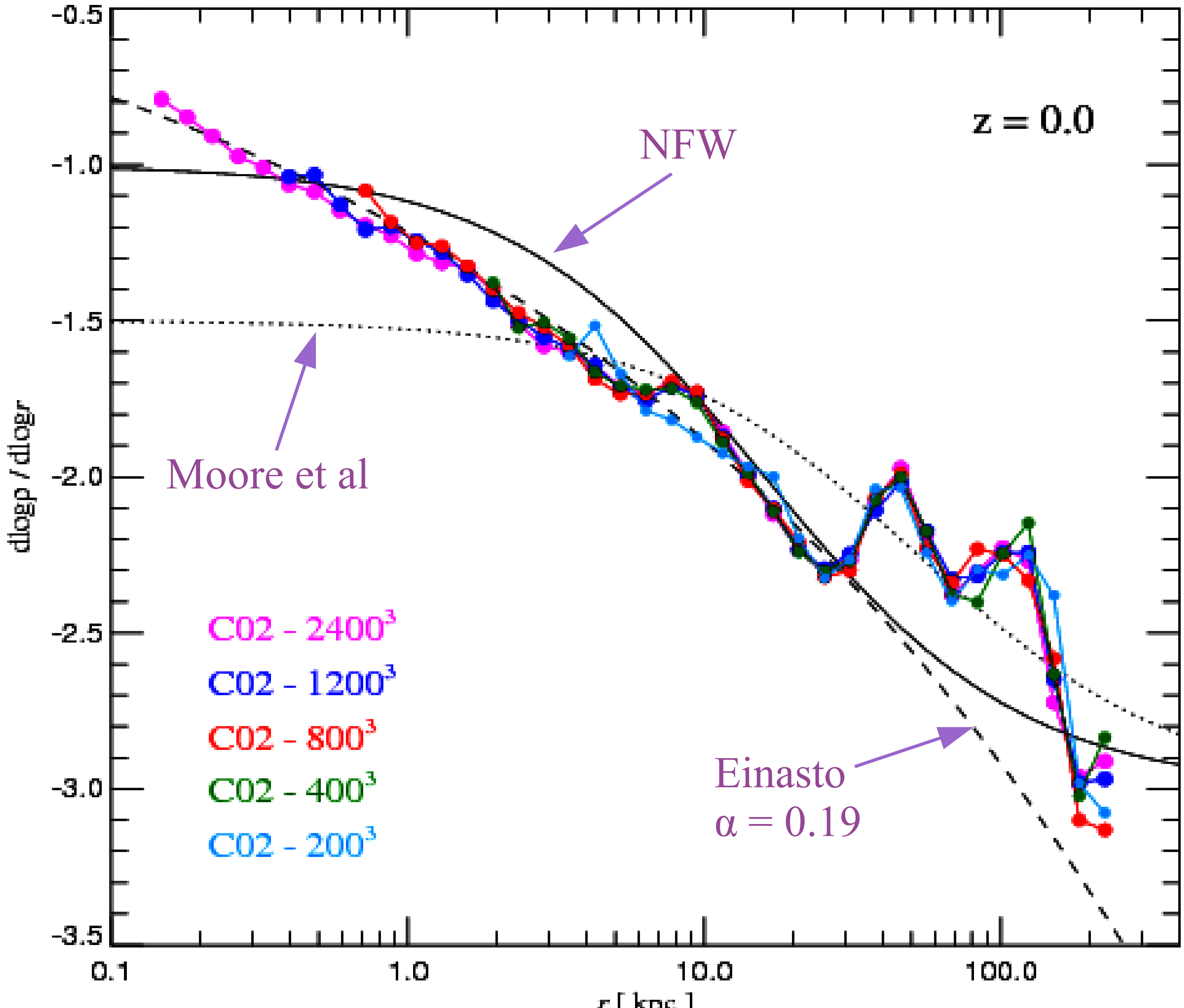
How well do density profiles converge?

Virgo Consortium 2008



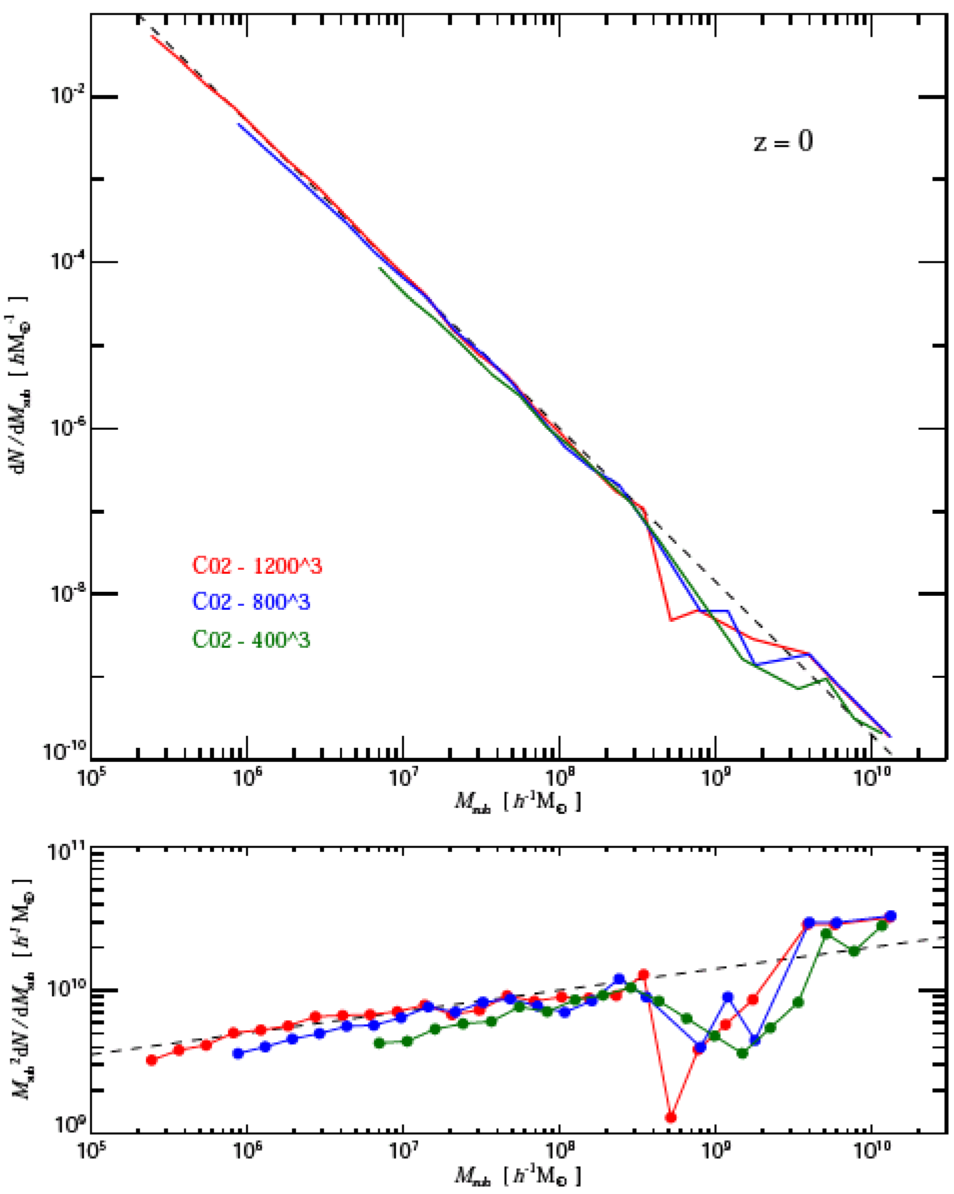
How well do density profiles converge?

Virgo Consortium 2008



How well does substructure converge?

Virgo Consortium 2008



Small-scale structure of the CDM distribution

- Direct detection involves bolometers/cavities of meter scale which are sensitive to particle momentum
  -- what is the density structure between m and kpc scales?
  -- how many streams intersect the detector at any time?

- Intensity of annihilation radiation depends on

$$\int \rho^2(\boldsymbol{x}) \langle \sigma v \rangle \, dV$$

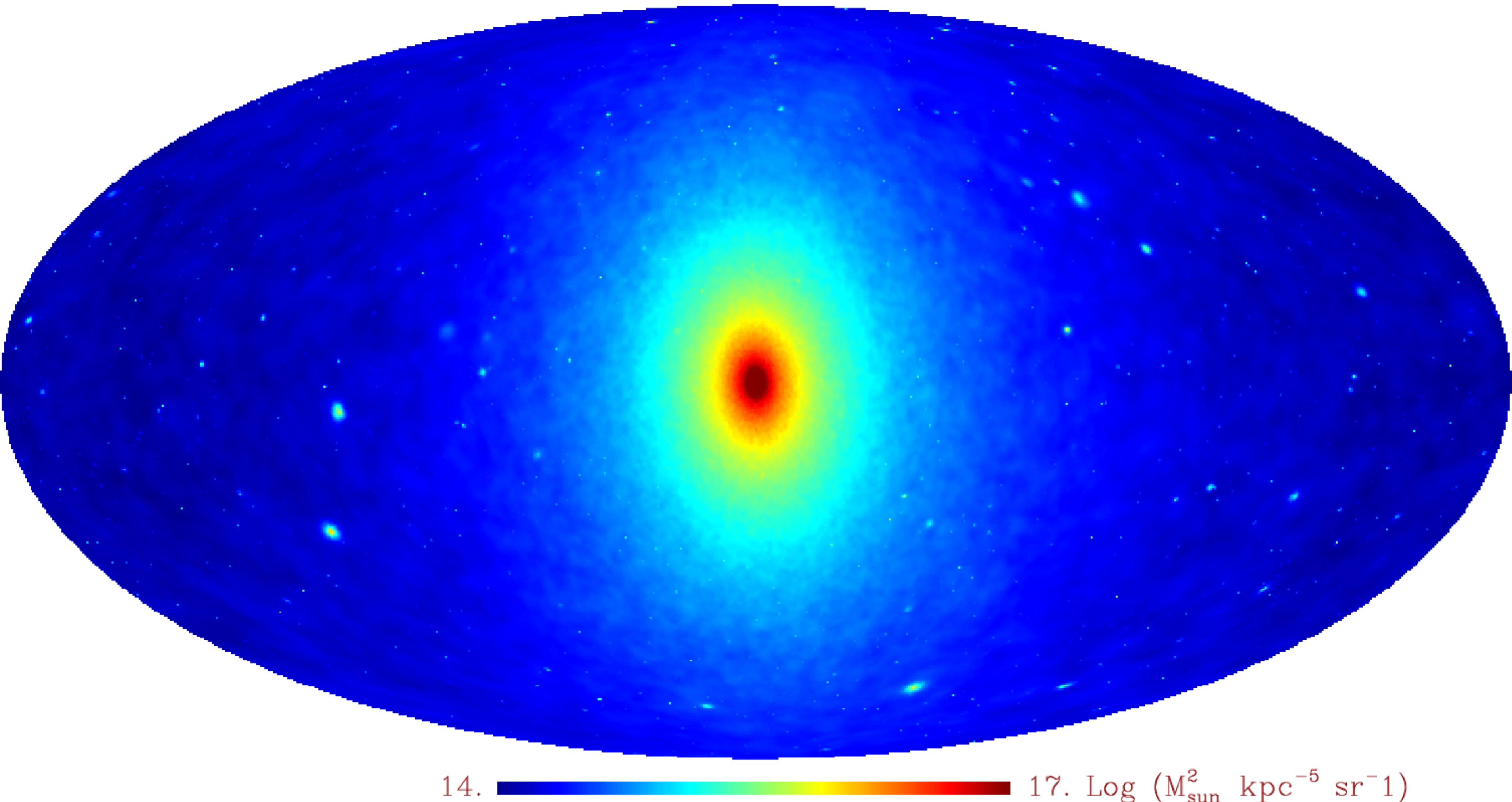
  -- what is the density distribution around individual CDM particles on the annihilation interaction scale?

Predictions for detection experiments depend on the CDM distribution on scales <u>far</u> below those accessible to simulation

→ We require a good theoretical understanding of mixing

Milky Way halo seen in DM annihilation radiation

Aquarius simulation: $N_{200} = 190{,}000{,}000$



Cold Dark Matter at high redshift (e.g. $z \sim 10^5$)

Well *after* CDM particles become nonrelativistic, but *before* they dominate the cosmic density, their distribution function is

$$f(\boldsymbol{x}, \boldsymbol{v}, t) = \rho(t)\,[1 + \delta(\boldsymbol{x})]\, N\,[\{\boldsymbol{v} - \boldsymbol{V}(\boldsymbol{x})\}/\sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\boldsymbol{x})$ is a Gaussian random field with finite variance $\ll 1$,

$\boldsymbol{V}(\boldsymbol{x}) = \nabla\psi(\boldsymbol{x})$ where $\nabla^2\psi(\boldsymbol{x}) \propto \delta(\boldsymbol{x})$

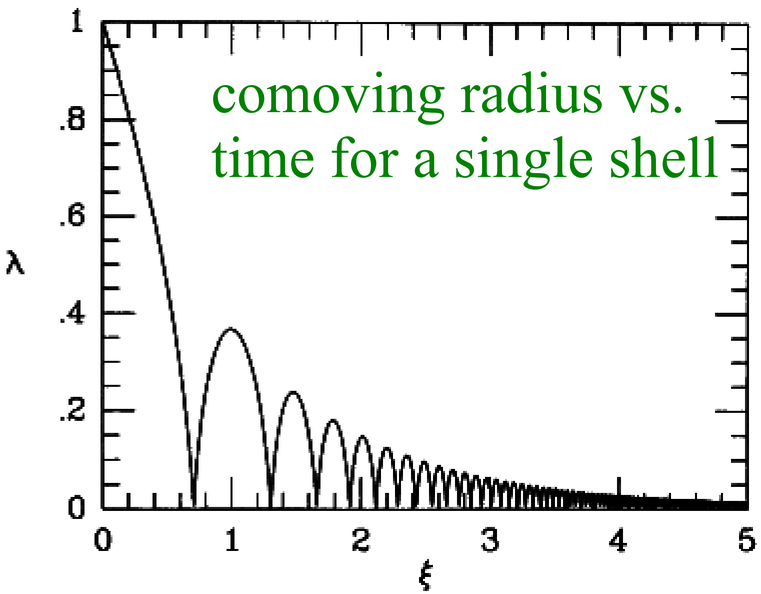
and N is standard normal with $\sigma^2 << \langle|\mathbf{V}|^2\rangle$

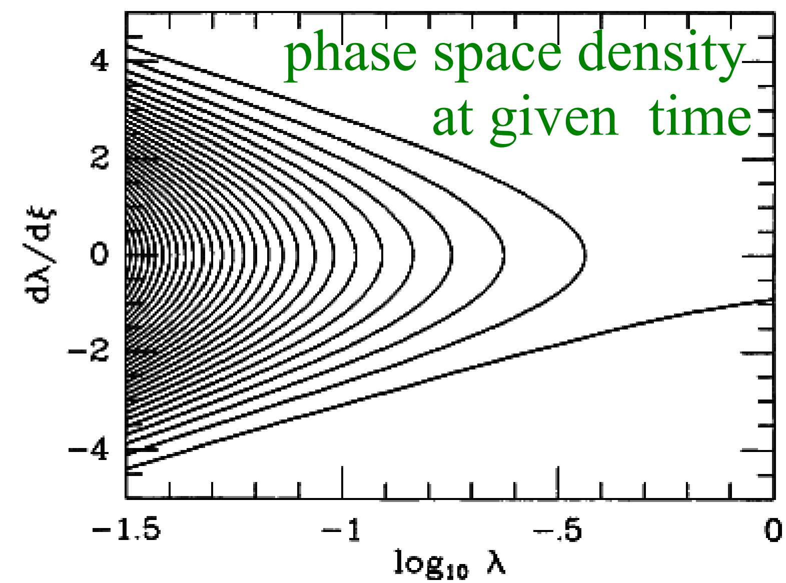
CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto **x**-space is near-uniform.

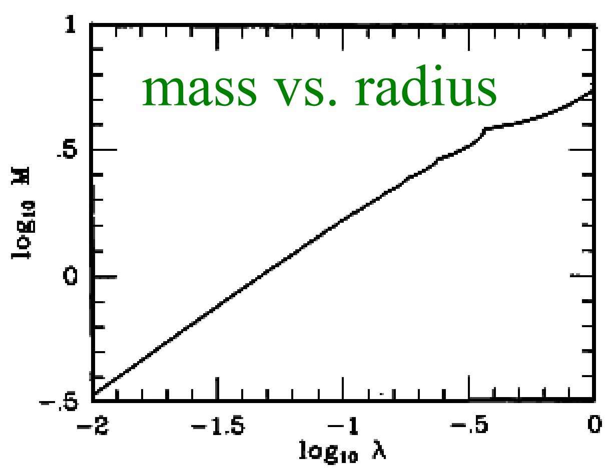
$Df / Dt = 0$ → only a 3-D subspace is occupied at later times.
Nonlinear evolution leads to a complex, multi-stream structure.

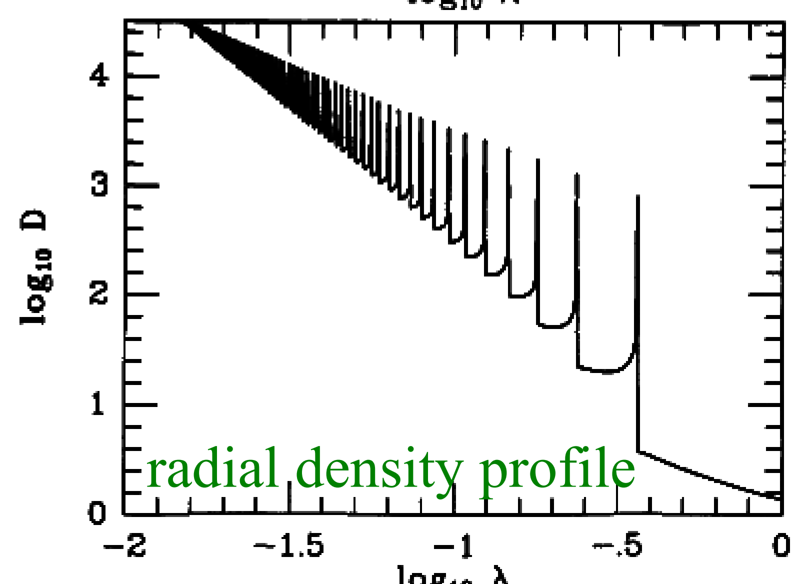
Similarity solution for spherical collapse in CDM

Bertschinger 1985









Evolution of CDM structure

Consequences of $Df / Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least 1 sheet must pass through every point $\mathbf{x}$
- In nonlinear objects there are typically many sheets at each $\mathbf{x}$
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f =$ const.
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ T\cdot\delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix} \cdot \delta X\,;\;\; T = -\nabla(\nabla\phi)$

Write $\delta X(t) = D(X_0, t)\cdot\delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & I \\ T & 0 \end{bmatrix} \cdot D \quad \text{with } D_0 = I$$

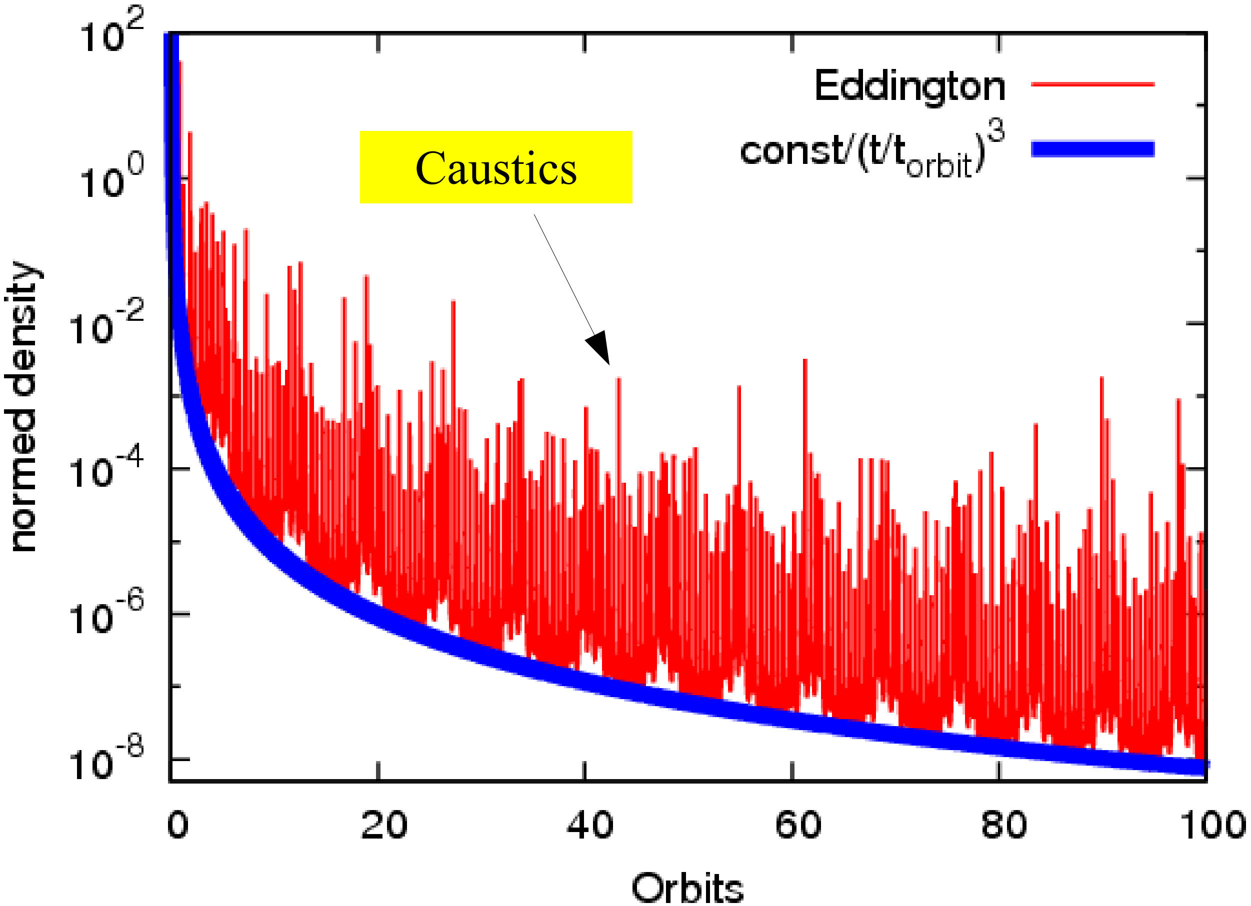
- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- det(D) = 1 at all times by Liouville's theorem
- For CDM, $1/|\det(D_{xx})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

Static highly symmetric potentials

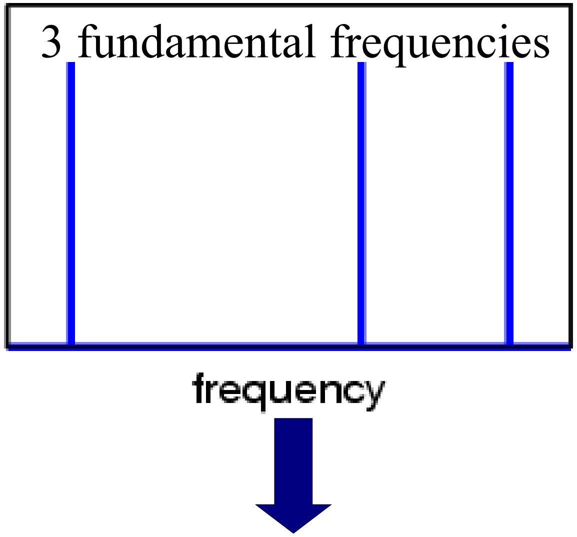
Mark Vogelsberger, Amina Helmi, Volker Springel

Axisymmetric Eddington potential

$$\Phi(r,\theta) = v_{\rm h}^2 \log\left(r^2 + d^2\right) + \frac{\beta^2 \cos^2\theta}{r^2}$$



Spectral analysis of orbit:

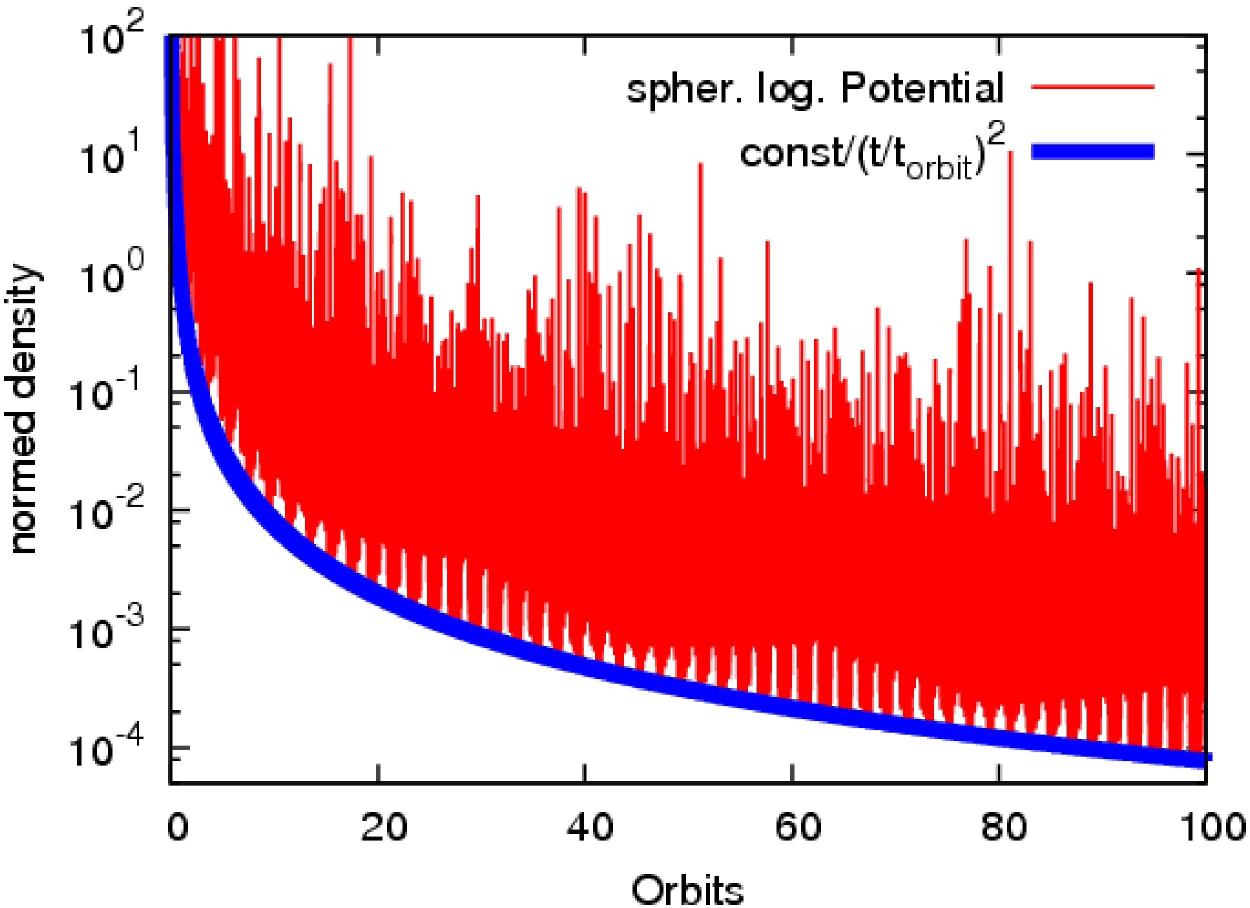


density decreases like $1/t^3$

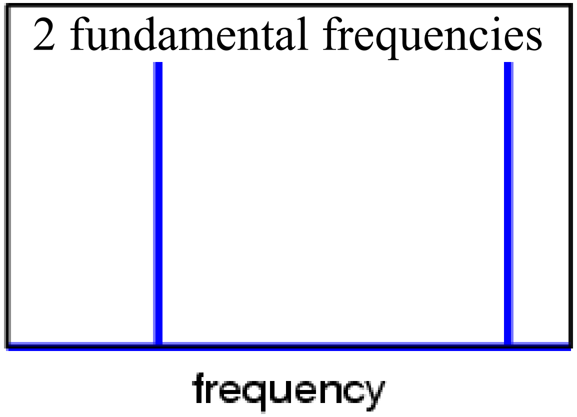
Changing the number of frequencies

Spherical logarithmic potential

$$\Phi(r,\theta) = v_h^2 \log\left(r^2 + d^2\right)$$



Spectral analysis of orbit:



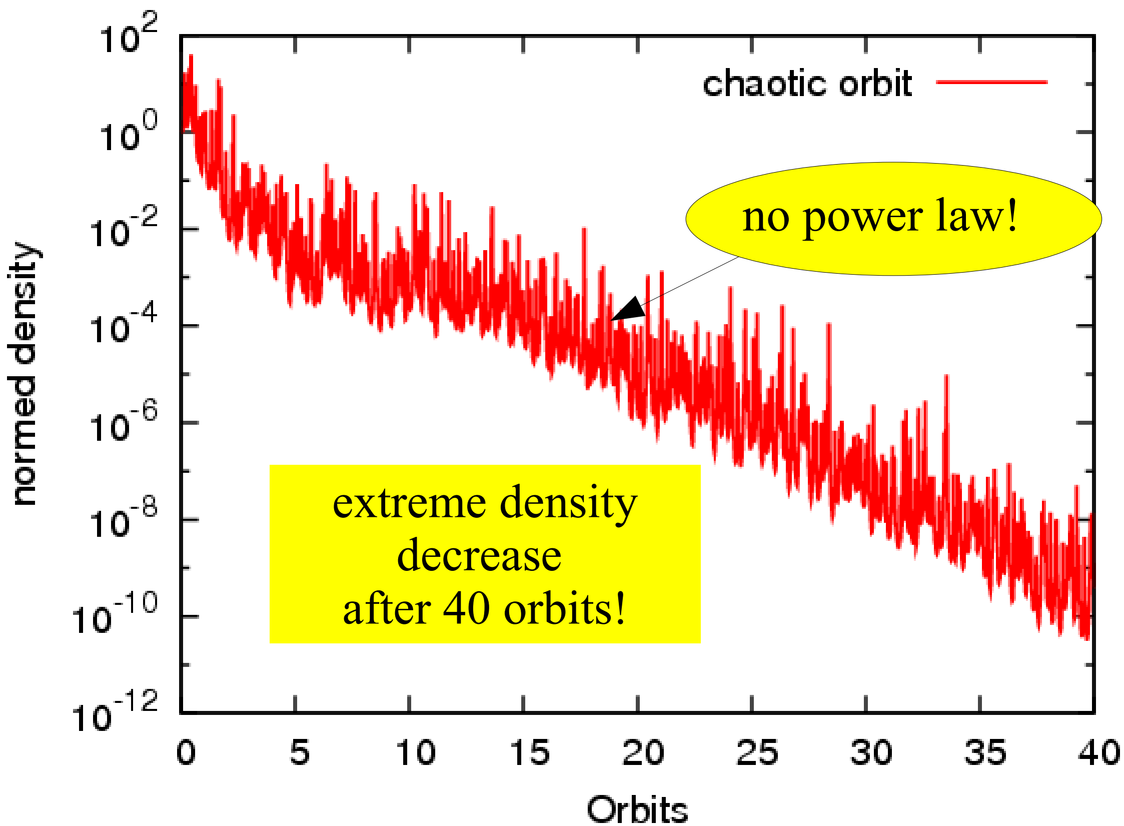
density decreases like $1/t^2$

Number of fundamental frequencies dictates the density decrease of the stream

Chaotic mixing

chaotic motion implies a **rapid stream density decrease** → **rapid mixing**

density decrease is **not** like a power law anymore

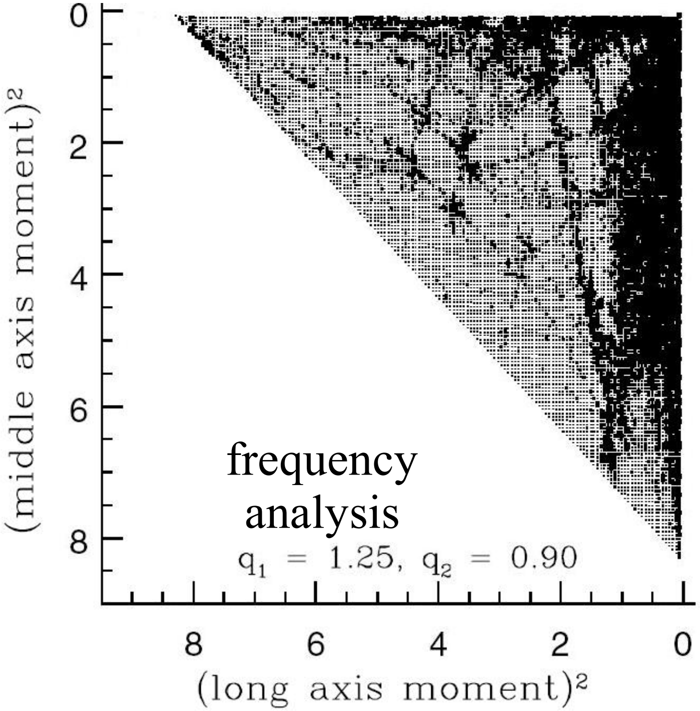


how to find chaotic regions in phase space?

Common method:

- Lyapunov exponents
- frequency analysis (NAFF)
- ...

Compare frequency analysis results with geodesic deviation equation results

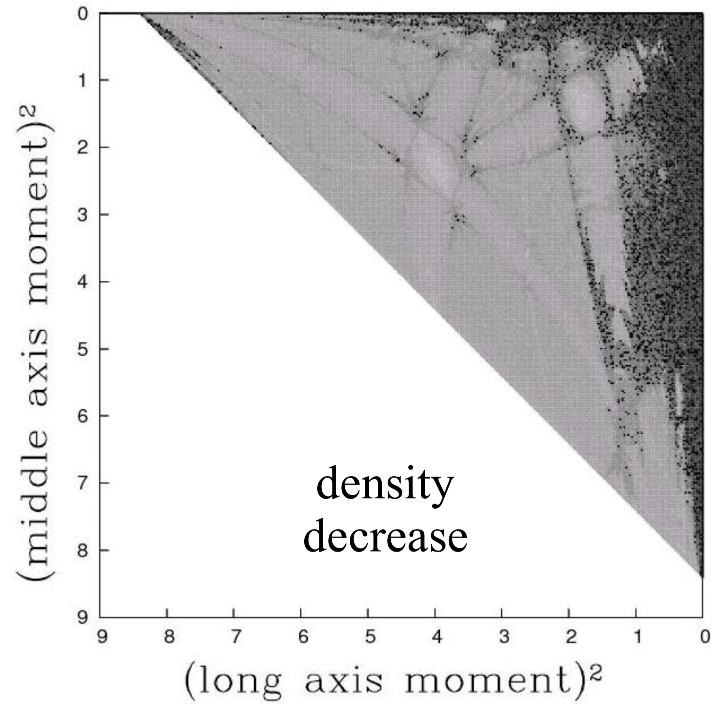


moderate triaxiality

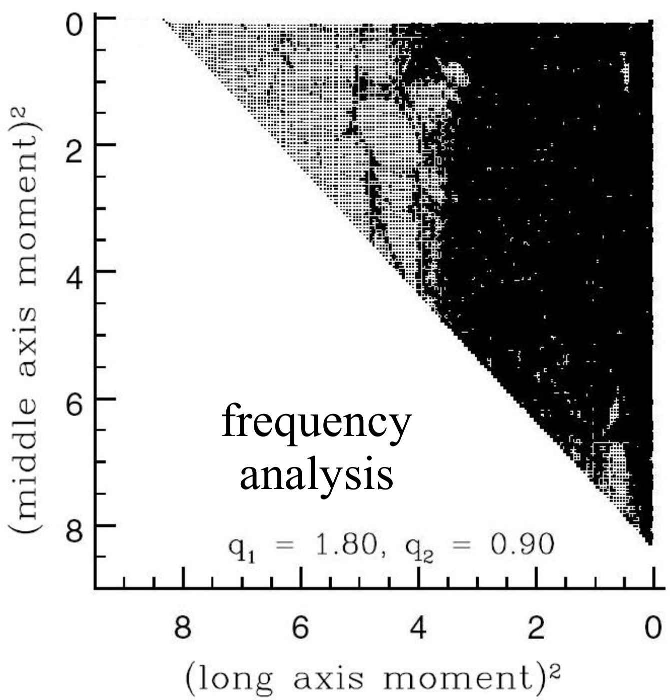
↓

small fraction of chaotic orbits

stream density mostly decaying like a power law



Papaphilippou & Laskar 1998



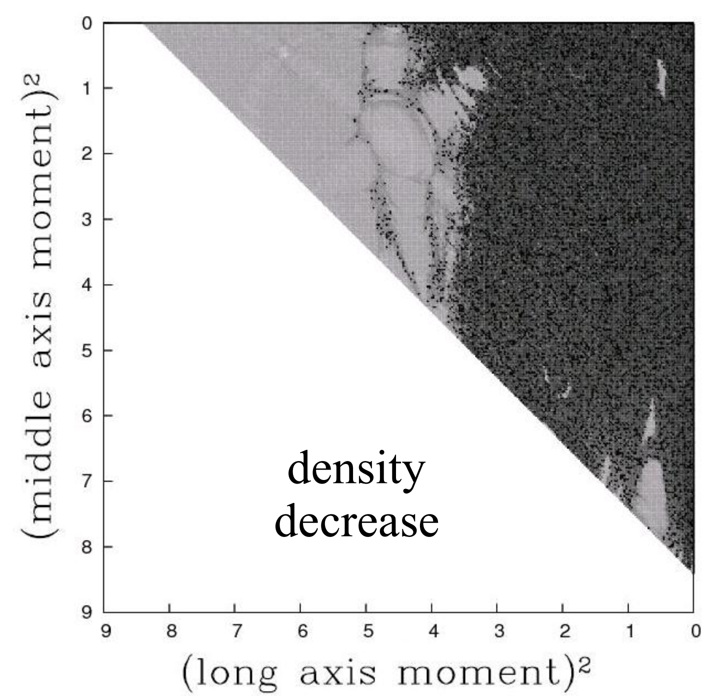
high triaxiality

↓

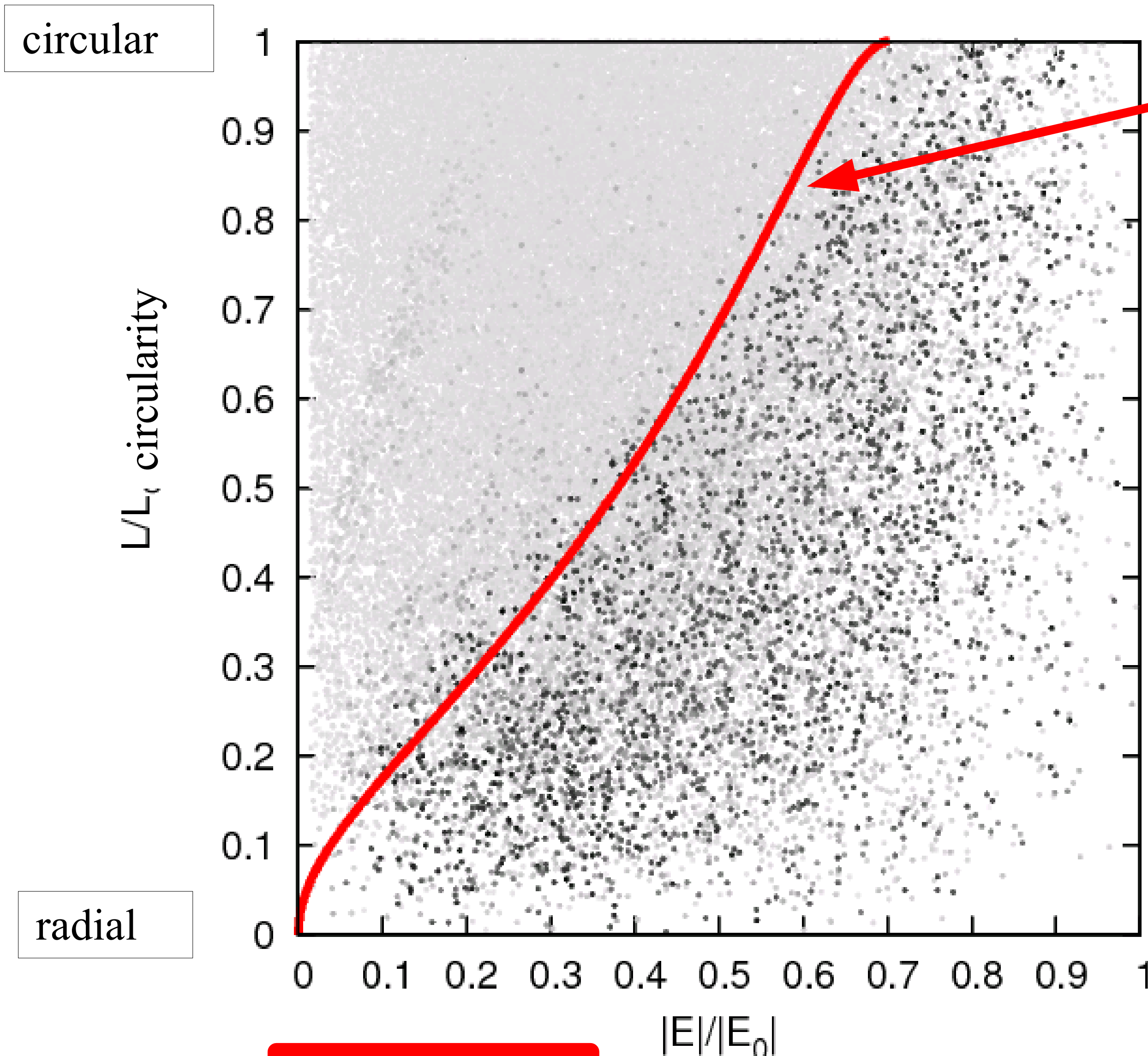
large fraction of chaotic orbits

stream density mostly decaying much faster than a power law

integrate 10^5 different orbits



Chaotic mixing in a triaxial NFW?



constant pericentre:
~1/3 NFW scale radius

~40.000 orbits

Orbits reaching inner part of the halo are affected by the triaxiality

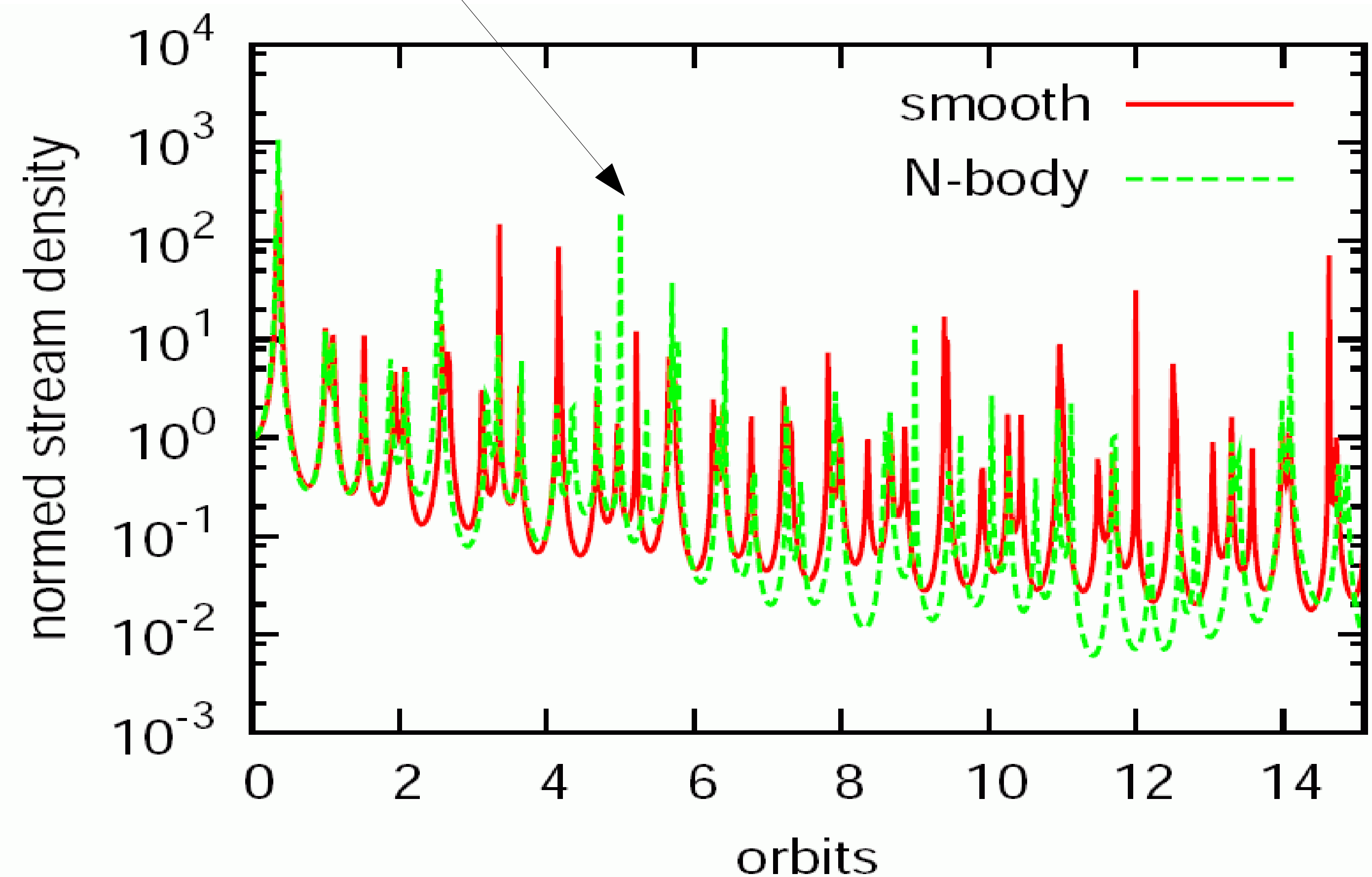
Chaotic mixing

A particle orbit in a live Halo

spherical Hernquist density profile

$$\rho(r) = \frac{M}{2\pi} \frac{a}{r} \frac{1}{(r+a)^3}$$

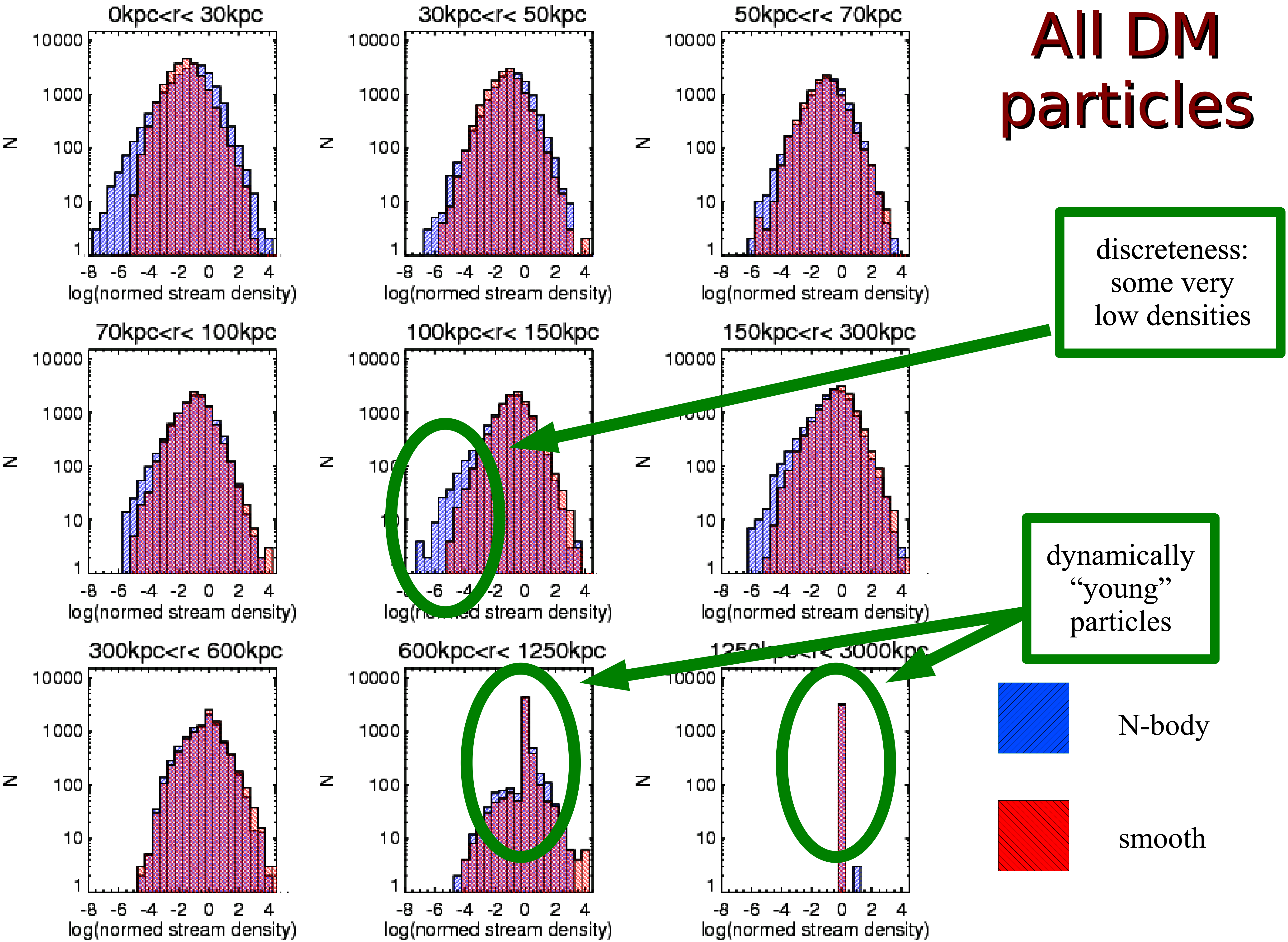
caustics resolved in N-body live halo!



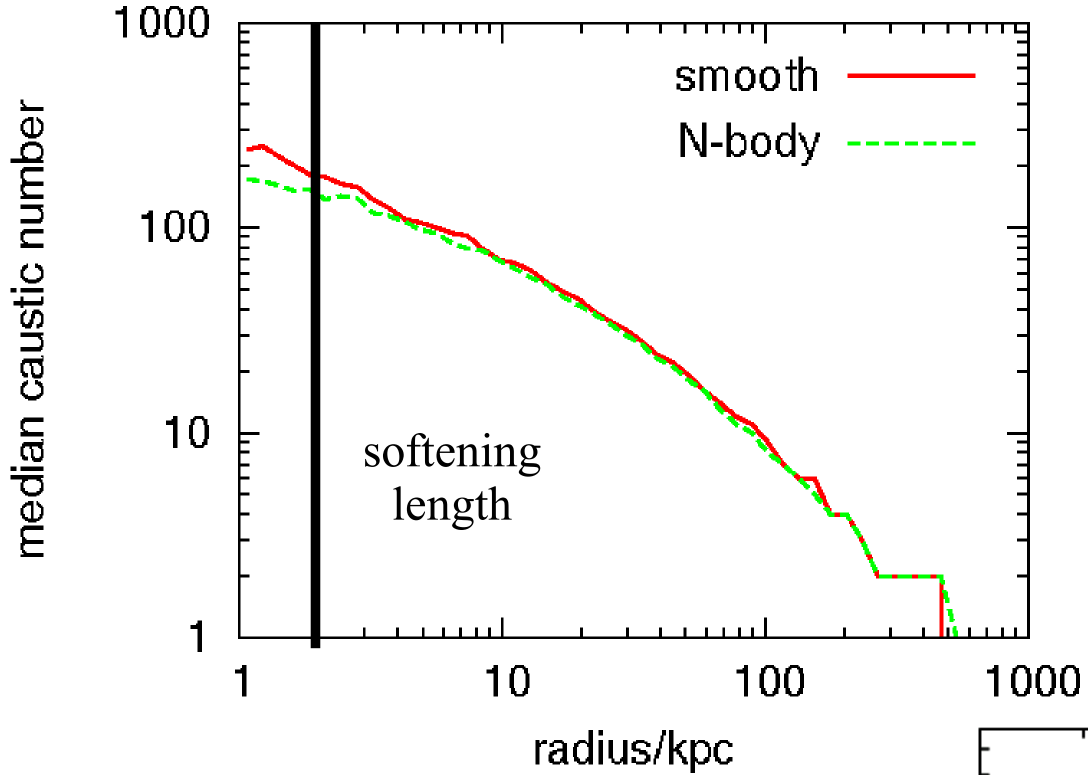
general shape and caustic spacing/number very similiar!

phase-space density conservation: 10^{-8}

All DM particles



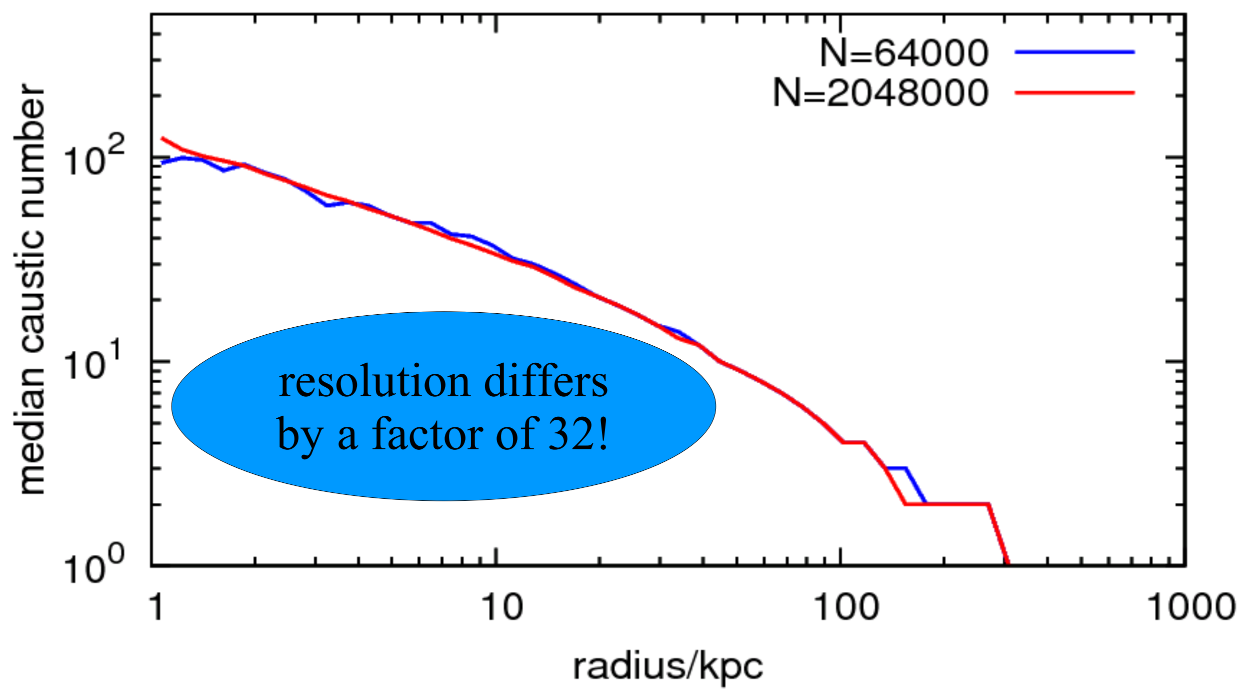
Number of Caustic Passages

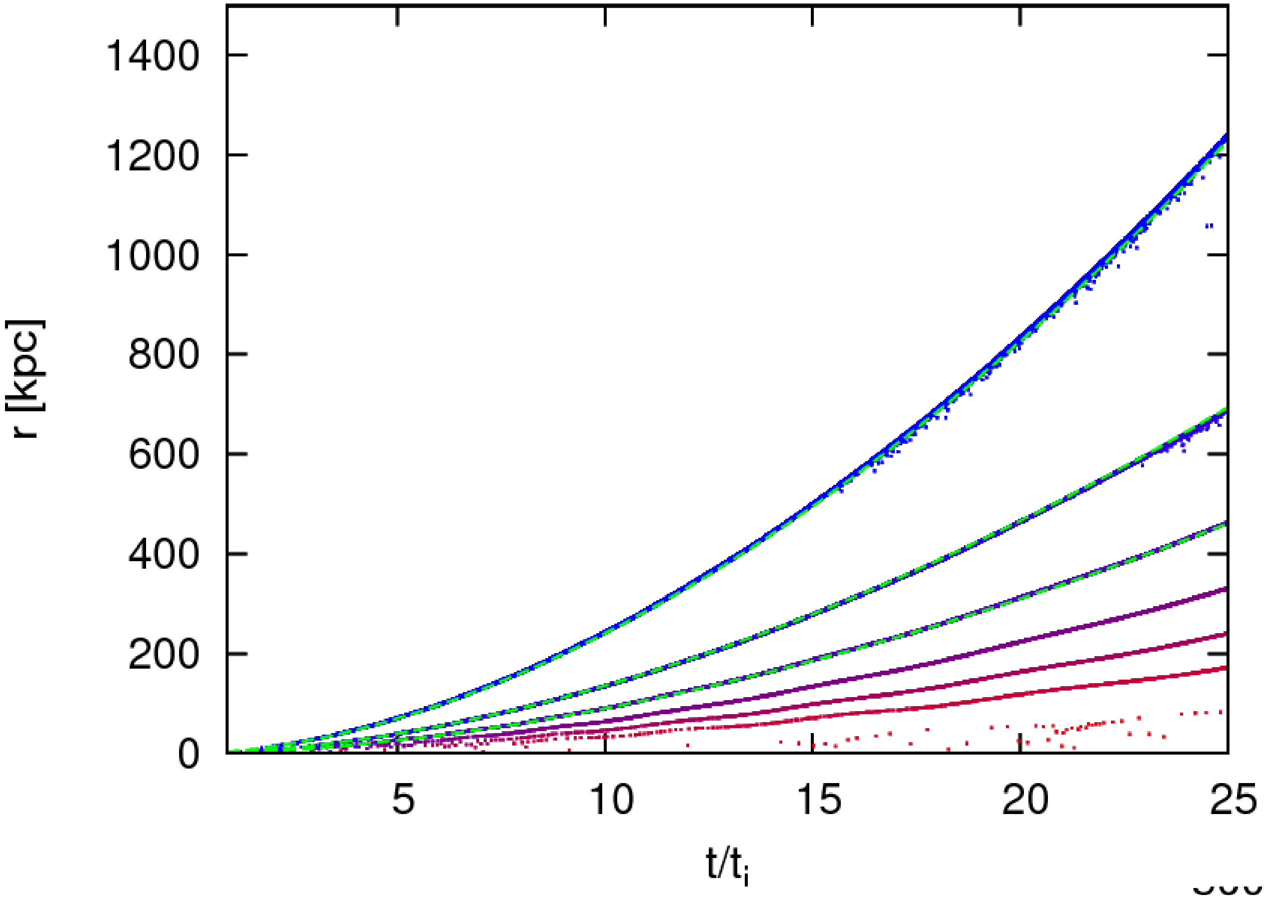


analytic and N-body results nearly the same!

Annihilation boost factor estimates due to caustics should be very robust!

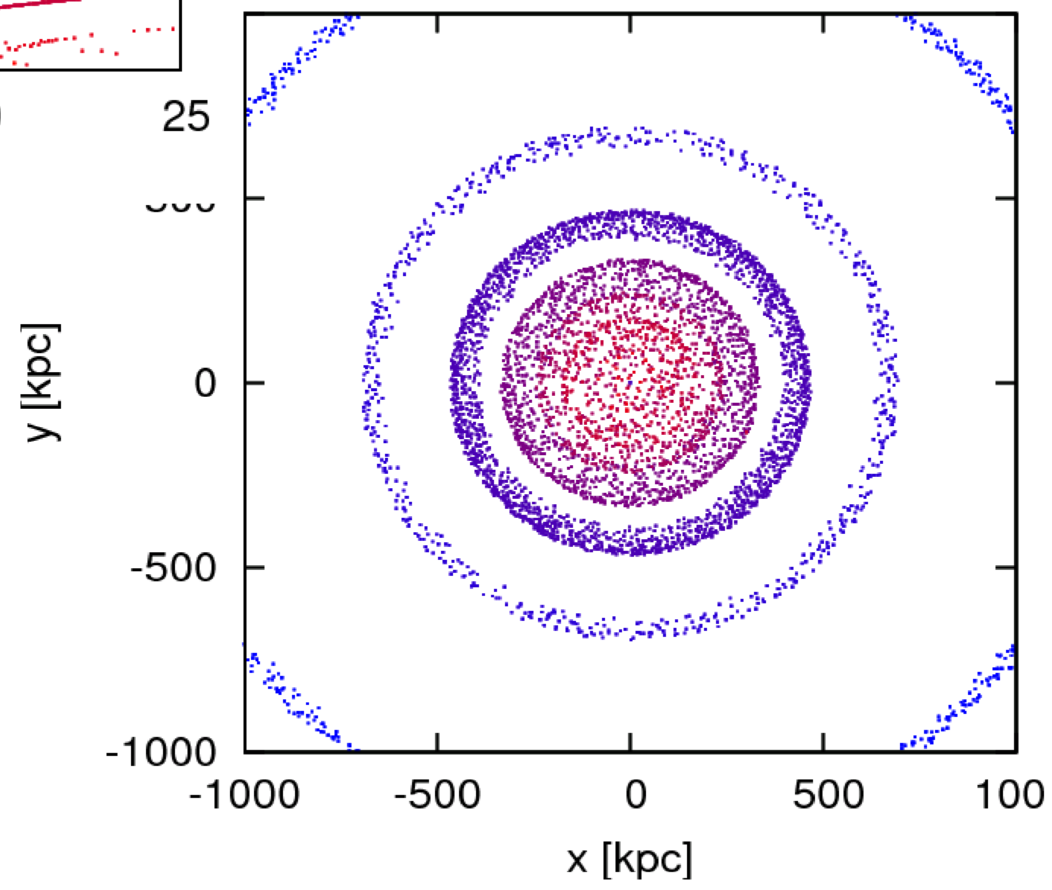
Very stable against particle number and softening length!

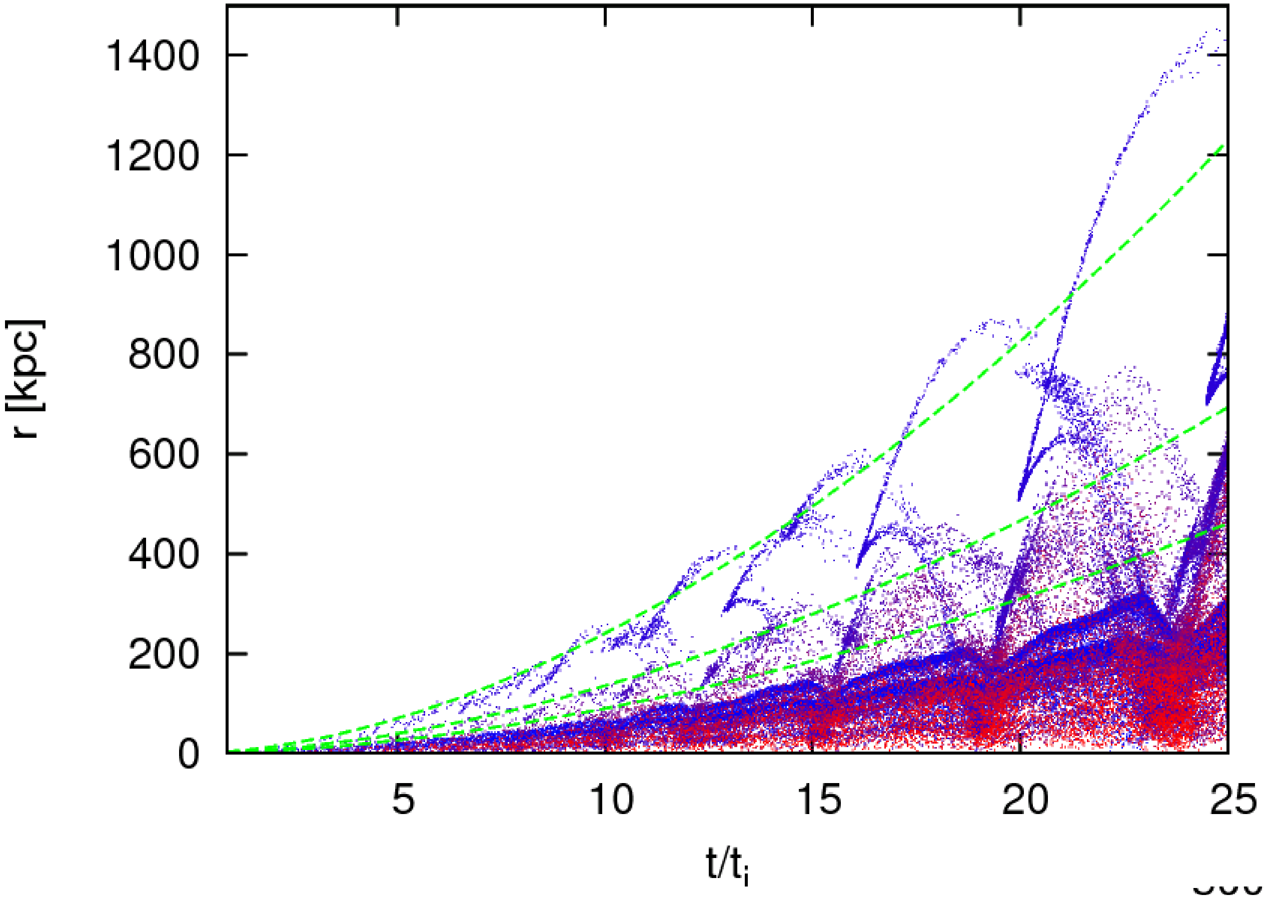




Caustic structure and evolution for growth from spherical self-similar IC's

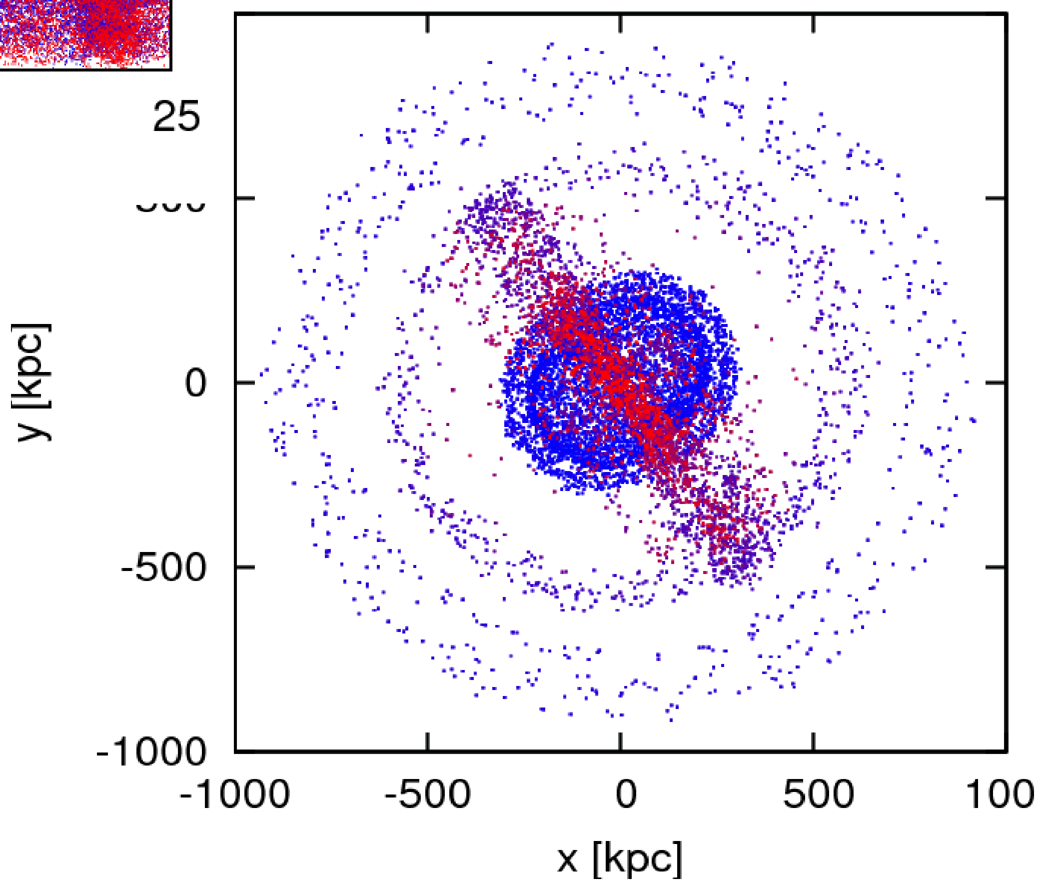
1-D (spherical) gravity





Caustic structure and evolution for growth from spherical self-similar IC's

Fully 3-D gravity



Conclusions (so far)

- GDE robustly identifies caustic passages and gives fair stream density estimates for particles in fully 3-D CDM simulations

- Many streams are present at each point well inside a CDM halo (at least 100,000 at the Sun's position)

  → quasi-Gaussian signal in direct detection experiments

- Caustic structure is more complex in realistic 3-D situations than in matched 1-D models but the caustics are weaker

  → negligible boosting of annihilation signal due to caustics

- Boost due to small substructures still uncertain but appears modest