Polarization of the CMB: Toward an **Observational** Proof of Cosmic Inflation

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Has inflation happened?

- If anyone asks you this question, your answer must always be:
  - “We don’t know yet.”
Flatness? Homogeneity?

• Aren’t flatness and homogeneity of the universe the proof of inflation?
  • **No.** Inflation was invented to solve the flatness and homogeneity problems.
The Key Predictions of Inflation

- Fluctuations we observe today originated from quantum fluctuations generated during inflation

- There should also be ultra-long-wavelength gravitational waves originated from quantum (or classical) fluctuations generated during inflation
We are measuring distortions in space

- A distance between two points in space
  \[ dl^2 = a^2(t)e^{2\zeta(x,t)}[e^h]_{ij}dx^idx^j \]
  \[ = a^2(t)[1+2\zeta(x,t)+...][\delta_{ij}+h_{ij}(x,t)+...]dx^idx^j \]

- \( \zeta(x,t) \): “curvature perturbation” (scalar mode)
- \( h_{ij}(x,t) \): “gravitational waves” (tensor mode)
- Area-conserving anisotropic stretching of space: \( \text{det}[e^h] = 1 \)
We are measuring distortions in space

• A distance between two points in space

\[
dl^2 = a^2(t)e^{2\zeta(x,t)}[e^h]_{ij}dx^i dx^j
\]

\[
= a^2(t)[1+2\zeta(x,t)+...][\delta_{ij}+h_{ij}(x,t)+...]dx^i dx^j
\]

• \(\zeta(x,t)>0\): more (isotropic) stretching of space

• More redshift \(\rightarrow\) colder photons

• The Sachs-Wolfe formula gives \(dT/T = -\zeta/5\)
We are measuring distortions in space

- A distance between two points in space

\[ dl^2 = a^2(t)e^{2\zeta(x,t)}[e^h]_{ij}dx^i dx^j \]

\[ = a^2(t)[1+2\zeta(x,t)+...][\delta_{ij}+h_{ij}(x,t)+...]dx^i dx^j \]

- \( h_{ij}(x,t) \): *anisotropic* stretching of space
Gravitational waves are coming toward you... What do you do?

- Gravitational waves stretch space, causing particles to move.
Two Polarization States of GW

- This is great - this will automatically generate quadrupolar anisotropy around electrons!
From GW to temperature anisotropy
From GW to temperature anisotropy
Scalar Mode

• Inflation predicts “nearly scale-invariant spectrum”
  • which means, for $P_\zeta(k) = \langle |\zeta_k|^2 \rangle \sim k^{n_s-4}$, $n_s$ is close to unity

• Inflation predicts “nearly Gaussian fluctuations”
  • which means, for $f_{NL} \sim \langle \zeta_{k1} \zeta_{k2} \zeta_{k3} \rangle / [P_\zeta(k_1)P_\zeta(k_2) + \text{cyc.}]$, $f_{NL}$ is much less than unity*

*for single-field canonical models
Scalar Mode: Current Status

- $n_s < 1$ is discovered at last (i.e., by more than $5\sigma$!)
  - WMAP9+ACT+SPT+BAO: $n_s = 0.958 \pm 0.008$ (68% CL)
  - Beautifully confirmed by Planck+WMAP9 polarization: $n_s = 0.960 \pm 0.007$ (68% CL)
- Remarkably tight limit on $f_{NL}^{\text{local}} = 2.7 \pm 5.8$ (68% CL) by Planck
  - A massive (a factor of 3.4) improvement from WMAP9

Single-field, canonical inflation models agree with all the data:

$1 - n_s \approx f_{NL} \approx O(\text{slow-roll parameter}) = O(10^{-2})$
Yet

• Neither $n_s<1$ nor $f_{NL}<1$ proves that inflation happened!

• We need to detect long-wavelength, scale-invariant primordial gravitational waves to definitively prove inflation observationally
Energy density spectrum of primordial GW from inflation

$E_{\text{inflation}} = 10^{16}$ GeV

CMB scale

no $v$ free-streaming, $g_*$=const.
Tool

• CMB Polarization!
CMB Polarization

- CMB is (very weakly) polarized
“Stokes Parameters”

Q<0; U=0

Q<0, U=0
Q=0, U<0
Q=0, U>0
Q>0, U=0

θ
φ

North
East

Q<0; U=0
23 GHz [polarized]

Stokes Q

Stokes U

WMAP
23 GHz [polarized]
33 GHz [polarized]

Stokes Q

Stokes U
41 GHz [polarized]

Stokes Q

Stokes U
61 GHz [polarized]
94 GHz [polarized]
How many components?

1. **CMB**: $T_\nu \sim \nu^0$

2. **Synchrotron** (electrons going around magnetic fields): $T_\nu \sim \nu^{-3}$

3. **Free-free** (electrons colliding with protons): $T_\nu \sim \nu^{-2}$

4. **Dust** (heated dust emitting thermal emission): $T_\nu \sim \nu^2$

5. **Spinning dust** (rapidly rotating tiny dust grains): $T_\nu$-complicated

You need at least **THREE** frequencies to separate them!
Physics of CMB Polarization

- CMB Polarization is created by a local temperature \textbf{quadrupole} anisotropy.
Principle

- Polarization direction is parallel to “hot.”
Stacking Analysis

• Stack polarization images around temperature hot and cold spots.

• Outside of the Galaxy mask (not shown), there are 11536 hot spots and 11752 cold spots.
Radial and Tangential Polarization Patterns around Temp. Spots

- All hot and cold spots are stacked
- “Compression phase” at θ=1.2 deg and “slow-down phase” at θ=0.6 deg are predicted to be there and we observe them!
- The 7-year overall significance level: 8σ
Quadrupole From Velocity Gradient (Large Scale)

\[ a = -\partial \Phi \]

\[ a > 0 \quad = 0 \]

Potential \( \Phi \)

Acceleration

Velocity in the rest frame of electron

Polarization

Radial  None

Velocity

Velocity gradient

Stuff flowing \textit{in}

Sachs-Wolfe: \( \Delta T/T = \Phi/3 \)

The left electron sees colder photons along the plane wave

\[ \Delta T \]

Stuff flowing \textit{in}
Quadrupole From Velocity Gradient (Small Scale)

\[ a = -\partial \Phi - \partial P \]

- Potential \( \Phi \)
- Acceleration \( a > 0, < 0 \)
- Velocity
- Velocity in the rest frame of electron
- Polarization
  - Radial
  - Tangential

Compression increases temperature

Stuff flowing in

Pressure gradient slows down the flow

Velocity gradient
• The 9-year overall significance level: 10σ
Planck Data!

Planck Collaboration I (2013)
E-mode and B-mode

• Gravitational potential can generate the E-mode polarization, but not B-modes.

• **Gravitational waves** can generate both E- and B-modes!
Two Polarization States of GW

• This is great - this will automatically generate quadrupolar anisotropy around electrons!
From GW to CMB Polarization
From GW to CMB Polarization
Gravitational waves can produce both E- and B-mode polarization
No detection of B-mode polarization yet.

**B-mode is the next holy grail!**
“Tensor-to-scalar Ratio,” $r$

$r = \frac{\text{[Power in Gravitational Waves]}}{\text{[Power in Curvature Perturbation]}}$

$= \frac{\langle h_{ij,k0}h_{ij,k0}^* \rangle}{\langle |\zeta_{k0}|^2 \rangle}$ at $k_0=0.002$ Mpc$^{-1}$

Inflation predicts $r \sim 1$
WMAP 9-year results
(Hinshaw, Larson, Komatsu, et al. 2012)

$r < 0.12$ (95% CL)
Planck Collaboration XXII (2013) confirms our results.

$R^2$ Inflation

$H_Z$

$r < 0.12$ (95% CL)
Next Step

- A bench-mark model: Starobinsky’s $R^2$ inflation (1980)
  - $n_s=0.96$ and $r=0.005$.
  - $n_s$ confirmed. *Can we ever reach $r=O(10^{-3})$?*
• Lensing contamination forces us to go to \textit{large angular scales}
How low should noise be?

- Due to lensing, an experiment with noise < 5uK arcmin is equivalent to the “noiseless” experiment.
To quantify the precision on $r$, it is convenient to use the variance, $\sigma_r^2$, given by the second moment of the likelihood:

$$\sigma_r^2 = \int_0^\infty dr \mathcal{L}(r) r^2 - \left[ \int_0^\infty dr \mathcal{L}(r) r \right]^2. \quad (5)$$

- Lensing severely limits the precision with which we can determine the value of $r$.
- No foreground is included yet here.

*Katayama & Komatsu (2011)* Maximum Multipole, $I_{\text{max}}$
Curse you, FG, I curse you...

- Even in the science channel (100GHz), foreground is a few orders of magnitude bigger in power at $l < \sim 30$
Gauss will help you

- Don’t be scared too much: the power spectrum captures only a fraction of information.
- Yes, CMB is very close to a Gaussian distribution. But, foreground is highly non-Gaussian.
- CMB scientist’s best friend is this equation:

\[-2\ln L = ([data]_i - [stuff]_i)^T (C^{-1})_{ij} ([data]_j - [stuff]_j)\]

where “C_{ij}” describes the two-point correlation of CMB and noise
WMAP’s Simple Approach

\[ \text{[data]} = [Q', U'](v) = \frac{[Q, U](v) - \alpha_S(v)[Q, U](v = 23 \text{ GHz})}{1 - \alpha_S(v)} \]

- Use the 23 GHz map as a tracer of synchrotron.
- Fit the 23 GHz map to a map at another frequency (with a single amplitude \( \alpha_S \)), and subtract.
- After correcting for “CMB bias,” this method removes foreground completely, provided that:
  - Spectral index (“\( \beta \)” of \( T \sim v^\beta \); e.g., \( \beta \sim -3 \) for synchrotron) does not vary across the sky.

\[ \text{[data]} = [Q', U'](v) = \frac{[Q, U](v) - \alpha_S(v)[Q, U](v = 23 \text{ GHz})}{1 - \alpha_S(v)} \]
Limitation of the simplest approach

- The index $\beta$ does vary a lot for synchrotron!
- We don’t really know what $\beta$ does for dust (just yet)
Nevertheless...

• Let’s try and see how far we can go with the simplest approach. The biggest limitation of this method is a position-dependent index.

• And, obvious improvements are possible anyway:
  • Fit multiple coefficients to different locations in the sky
  • Use more frequencies to constrain the index
We describe the data (=CMB+noise+PSMv1.6.2) by

- Amplitude of the B-mode polarization: $r$ [this is what we want to measure at the level of $r \approx 10^{-3}$]

- Amplitude of the E-mode polarization from gravitational potential: $s$ [which we wish to marginalize over]

- Amplitude of synchrotron: $\alpha_{\text{Synch}}$ [which we wish to marginalize over]

- Amplitude of dust: $\alpha_{\text{Dust}}$ [which we wish to marginalize over]
**Methodology:** we simply maximize the following likelihood function estimating $r$, $s$, and $\alpha_i$:

\[
\mathcal{L}(r, s, \alpha_i) \propto \frac{\exp \left[-\frac{1}{2} \mathbf{x}'(\alpha_i)^T \mathbf{C}^{-1}(r, s, \alpha_i) \mathbf{x}'(\alpha_i)\right]}{\sqrt{|\mathbf{C}(r, s, \alpha_i)|}}, \tag{9}
\]

where

\[
\mathbf{x}' = \left[Q, U\right](v) - \sum_i \alpha_i(v)\left[Q, U\right](v_i^\text{template}) \quad \frac{1}{1 - \sum \alpha_i(v)} \tag{10}
\]

is a template-cleaned map. This is a generalization of Equation (6) for a multi-component case. In this paper, $i$ takes on “S” and “D” for synchrotron and dust, respectively, unless noted otherwise. For definiteness, we shall choose

\[
v = 100 \text{ GHz},
\]

\[
v^\text{template}_S = 60 \text{ GHz},
\]

\[
v^\text{template}_D = 240 \text{ GHz}.
\]
\[ L \propto \exp \left[ -\frac{1}{2} x'(\alpha_i)^T C^{-1}(r, s, \alpha_i) x'(\alpha_i) \right] \sqrt{|C(r, s, \alpha_i)|} \]

\[ C(r, s, \alpha_i) = r c^{\text{tensor}} + s c^{\text{scalar}} + \frac{N_1 + N_2}{(1 - \sum_i \alpha_i)^2} \]

\text{signal part} (after correcting for CMB bias)

\text{noise part}

\text{SIGNAL COVARIANCE MATRIX}

Given power spectra, \(c_i^{BB}\) and \(c_i^{EE}\), the components of the signal covariance matrix for \(Q\) and \(U\) can be computed analytically. We have

\[ c(\hat{n}, \hat{n}') = \begin{pmatrix} c_{QQ}(\hat{n}, \hat{n}') & c_{QU}(\hat{n}, \hat{n}') \\ c_{UQ}(\hat{n}, \hat{n}') & c_{UU}(\hat{n}, \hat{n}') \end{pmatrix}, \]

where

\[ c_{QQ}(\hat{n}, \hat{n}') = \sum_l c_i^{EE} w_i^2 \sum_m W_{lm}(\hat{n}) W^*_{lm}(\hat{n}') + \sum_l c_i^{BB} w_i^2 \sum_m X_{lm}(\hat{n}) X^*_{lm}(\hat{n}') \]

\[ c_{QU}(\hat{n}, \hat{n}') = \sum_l c_i^{EE} w_i^2 \sum_m [-W_{lm}(\hat{n}) X^*_{lm}(\hat{n}')] + \sum_l c_i^{BB} w_i^2 \sum_m X_{lm}(\hat{n}) W^*_{lm}(\hat{n}') \]

\[ c_{UQ}(\hat{n}, \hat{n}') = \sum_l c_i^{EE} w_i^2 \sum_m [-X_{lm}(\hat{n}) W^*_{lm}(\hat{n}')] + \sum_l c_i^{BB} w_i^2 \sum_m W_{lm}(\hat{n}) X^*_{lm}(\hat{n}') \]

\[ c_{UU}(\hat{n}, \hat{n}') = \sum_l c_i^{EE} w_i^2 \sum_m X_{lm}(\hat{n}) X^*_{lm}(\hat{n}') + \sum_l c_i^{BB} w_i^2 \sum_m W_{lm}(\hat{n}) W^*_{lm}(\hat{n}'). \]

and

\[ W_{lm}(\hat{n}) = (-1)^l Y_{lm}(\hat{n}) + \frac{1}{2} Y_{lm}(\hat{n}) \]

\[ X_{lm}(\hat{n}) = (-1)^l Y_{lm}(\hat{n}) - \frac{1}{2} Y_{lm}(\hat{n}) \]
Here goes $O(N^3)$

$$\mathcal{L}(r, s, \alpha_i) \propto \exp \left[ -\frac{1}{2} x'(\alpha_i)^T C^{-1}(r, s, \alpha_i) x'(\alpha_i) \right] \frac{1}{\sqrt{|C(r, s, \alpha_i)|}}$$

- A numerical challenge: for each set of $r, s, \alpha_{\text{Synch}}$ and $\alpha_{\text{Dust}}$, we need to invert the covariance matrix.
- For this study, we use low-resolution Q&U maps with 3072 pixels per map (giving a $6144 \times 6144$ matrix).
We target the low-$l$ bump

- This is a semi-realistic configuration for a future satellite mission targeting the B-modes from inflation.
Two Masks and Choice of Regions for Synch Index

(a) 48 $\alpha_S$ regions with the P06 mask ($f_{\text{sky}} = 73\%$) for Method I

(b) 12 $\alpha_S$ regions with extended mask ($f_{\text{sky}} = 50\%$) for Method II

“Method I”

“Method II”
• It works quite well!

• For dust-only case (for which the index does not vary much): we observe no bias in the B-mode amplitude, as expected.

• For Method I (synch+dust), the bias is $\Delta r = 2 \times 10^{-3}$

• For Method II (synch+dust), the bias is $\Delta r = 0.6 \times 10^{-3}$
OK, it is unbiased, but

- What about the error bar (precision) on $r$?
• Foreground does inflate the error bars on $r$.
• For $r=0.001$ with lensing, the error bar is inflated by a factor of two.
• The inflation of error bars seems unavoidable: the bias can be eliminated, but it comes with the expense...
Conclusion

- The biggest obstacle toward an observational proof of inflation using B-mode polarization is Galactic foreground.
- The simplest approach is already quite promising.
  - Using just 3 frequencies gets the bias down to $\Delta r < 10^{-3}$
- The bias is totally dominated by the spatial variation of the synchrotron index.
- How to improve further? We can use 4 frequencies: two frequencies for synchrotron to constrain the index.
  - The biggest worry: we do not know much about the dust index variation (yet; until next year). Perhaps we should have two frequencies for the dust index as well.
- The minimum number of frequencies = 5