$D_A(z)$ from Strong Lenses

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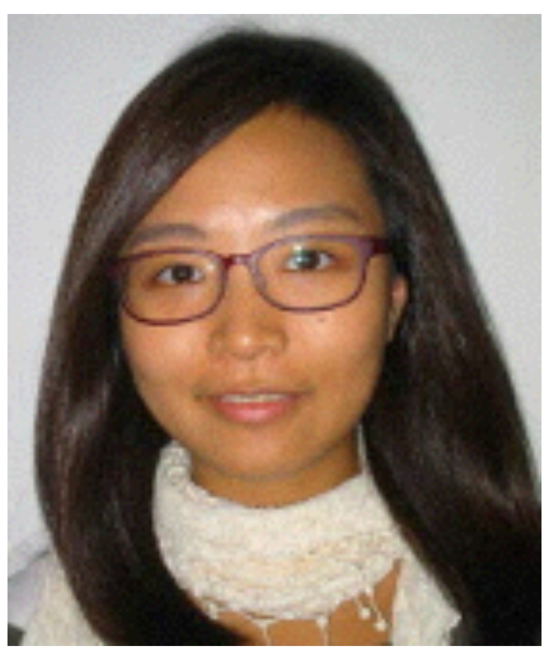
This presentation is based on:

- "Measuring angular diameter distances of strong gravitational lenses," **Inh Jee**, EK, and **Sherry Suyu**, in preparation

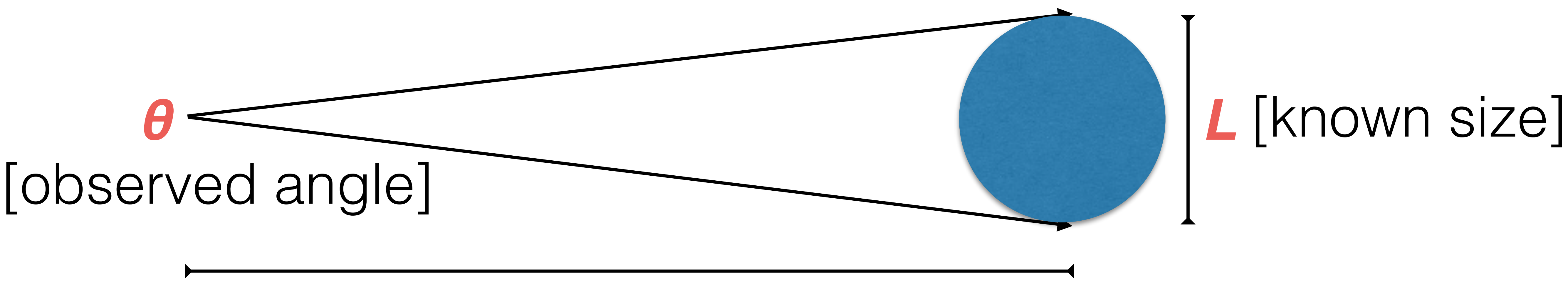


Inh Jee (MPA)

Sherry Suyu (ASIAA)

Motivation

- We wish to measure angular diameter distances!



$$D_A = \frac{L}{\theta}$$

How do we know the intrinsic physical size?

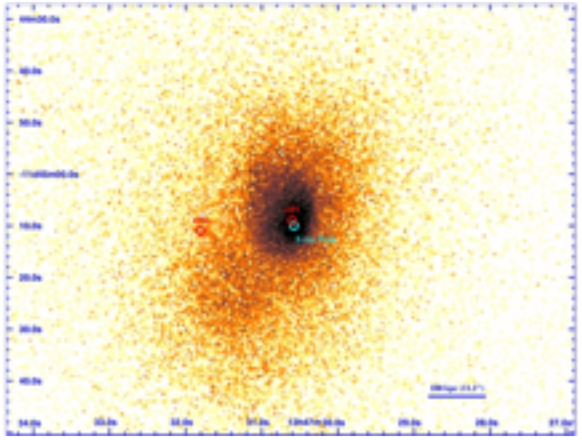
- Two methods:

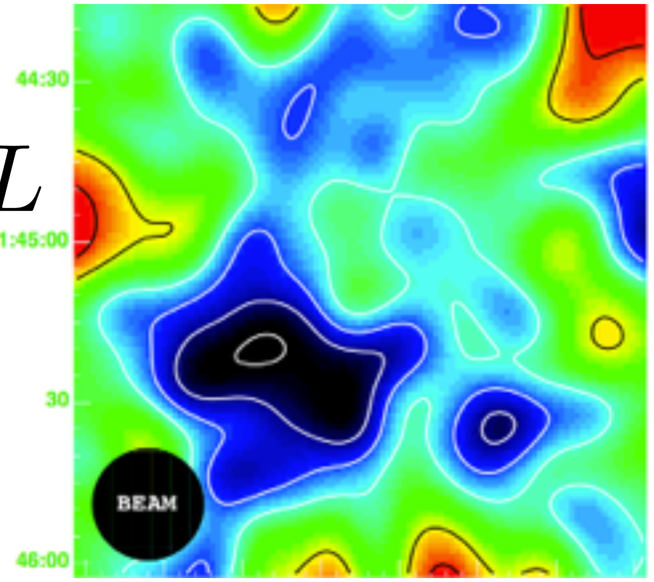
  1. To estimate the physical size of an object from observations

  2. To use the “standard ruler”

Silk & White (1978)

Example #1: Galaxy Clusters





- X-ray intensity

$$I_X = \int dl\ n_e^2 \Lambda(T_e) \approx n_e^2 \Lambda(T_e) L$$

- Sunyaev-Zel'dovich intensity

$$I_{SZ} = \frac{g_\nu \sigma_T k_B}{m_e c^2} \int dl\ n_e T_e \approx \frac{g_\nu \sigma_T k_B}{m_e c^2} n_e T_e L$$

- Combination gives the LOS extension

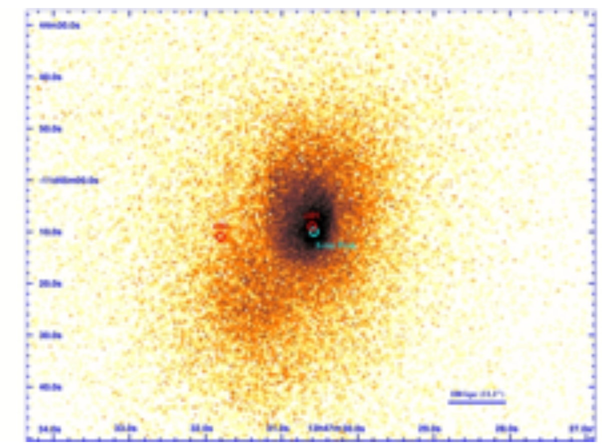
$$L \propto \frac{I_{SZ}}{I_X} \frac{\Lambda(T_e)}{T_e^2} \frac{\langle n_e^2 \rangle}{\langle n_e \rangle^2}$$

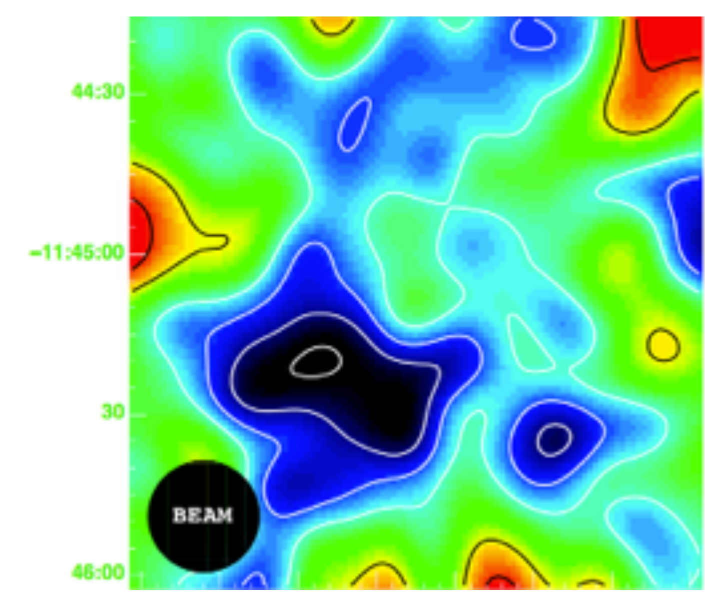
Silk & White (1978)

Example #1: Galaxy Clusters

- Combination gives the LOS extension

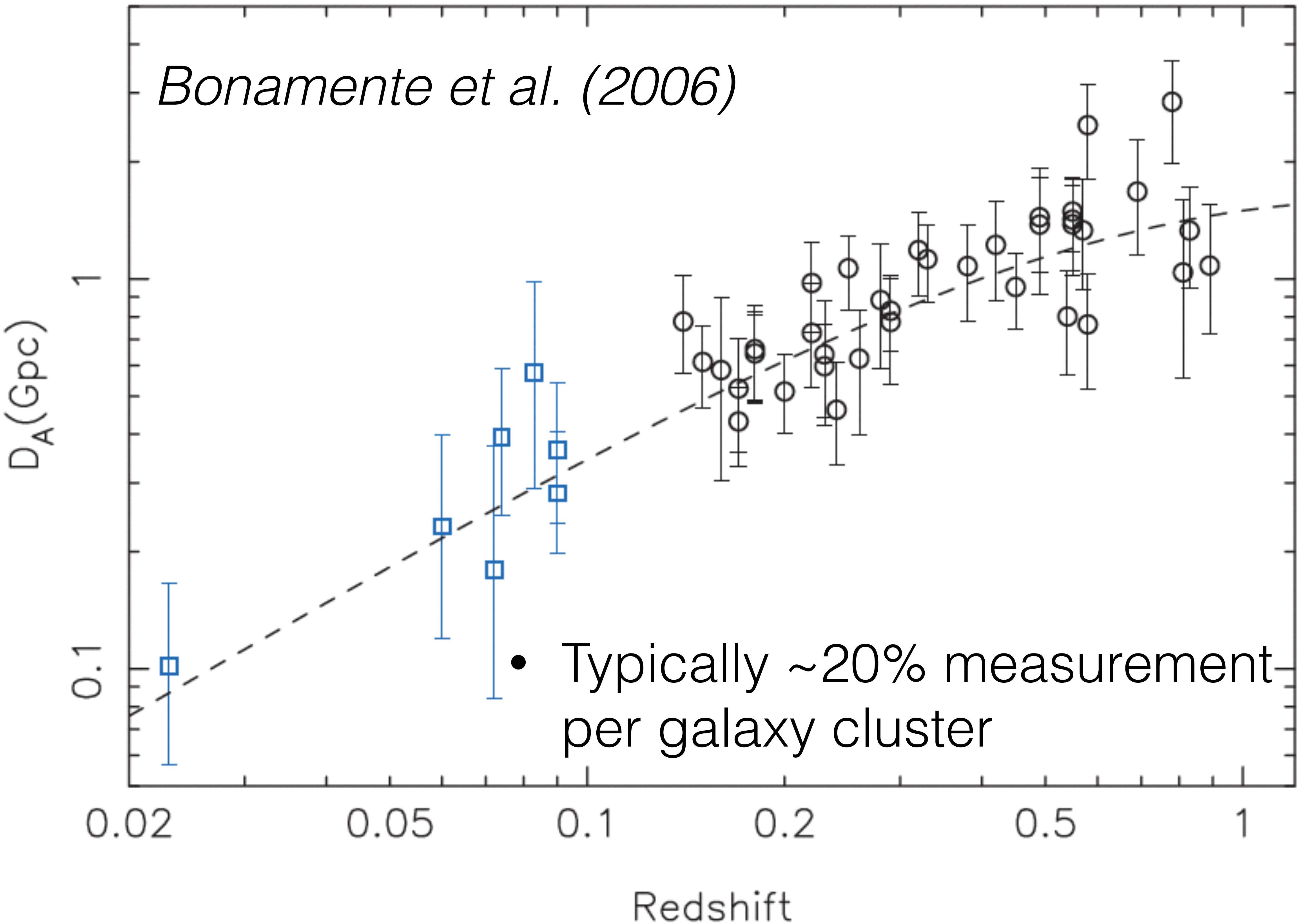
$$L \propto \frac{I_{SZ}}{I_X} \frac{\Lambda(T_e)}{T_e^2} \frac{\langle n_e^2 \rangle}{\langle n_e \rangle^2}$$





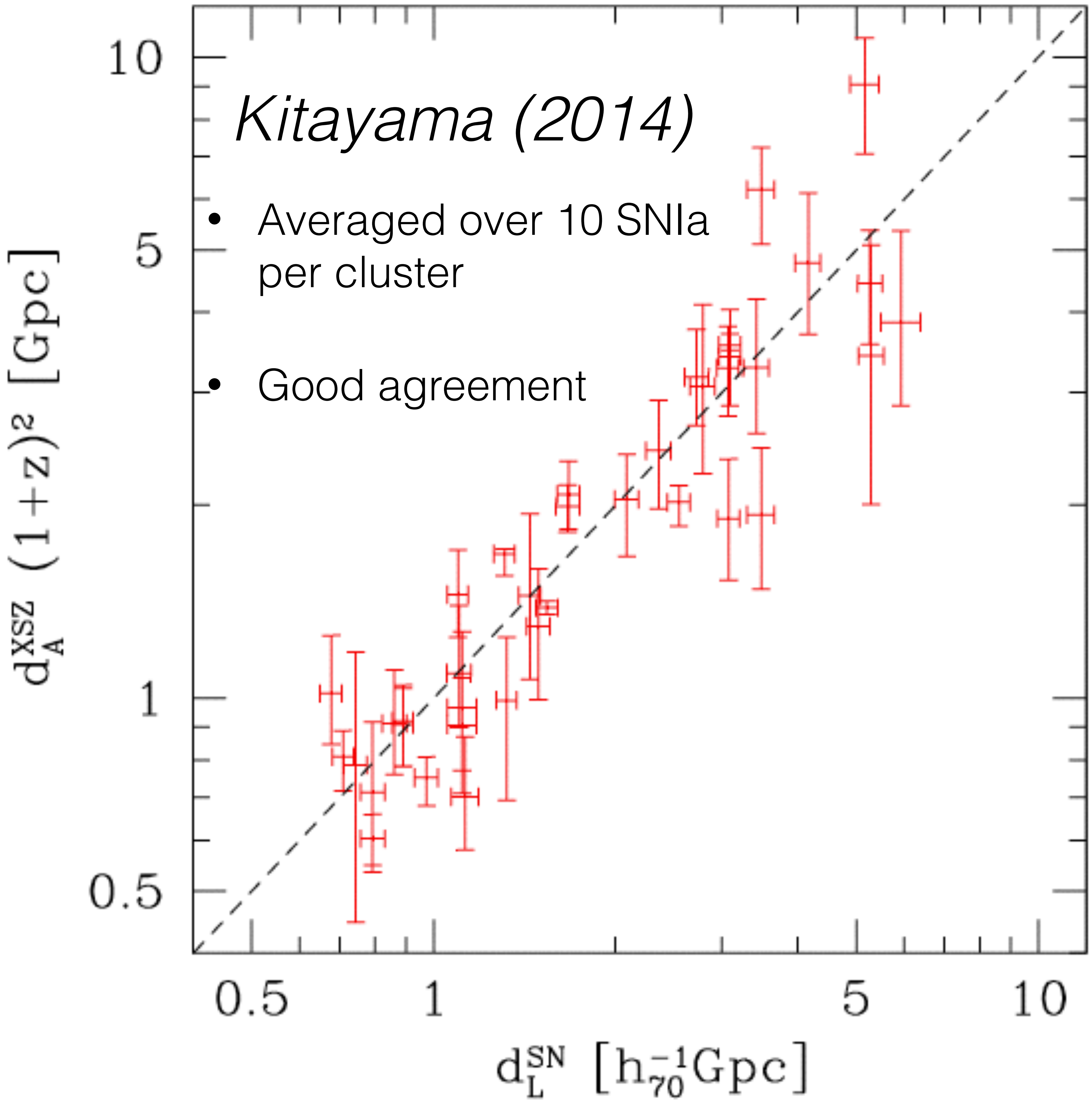
- Assuming spherical symmetry and using the measured angular extension, we get D_A

Galaxy Cluster Hubble Diagram



- Typically ~20% measurement per galaxy cluster

Galaxy Clusters vs Type Ia SN



Kitayama (2014)

- Averaged over 10 SNIa per cluster
- Good agreement

Example #2: Baryon Acoustic Oscillation

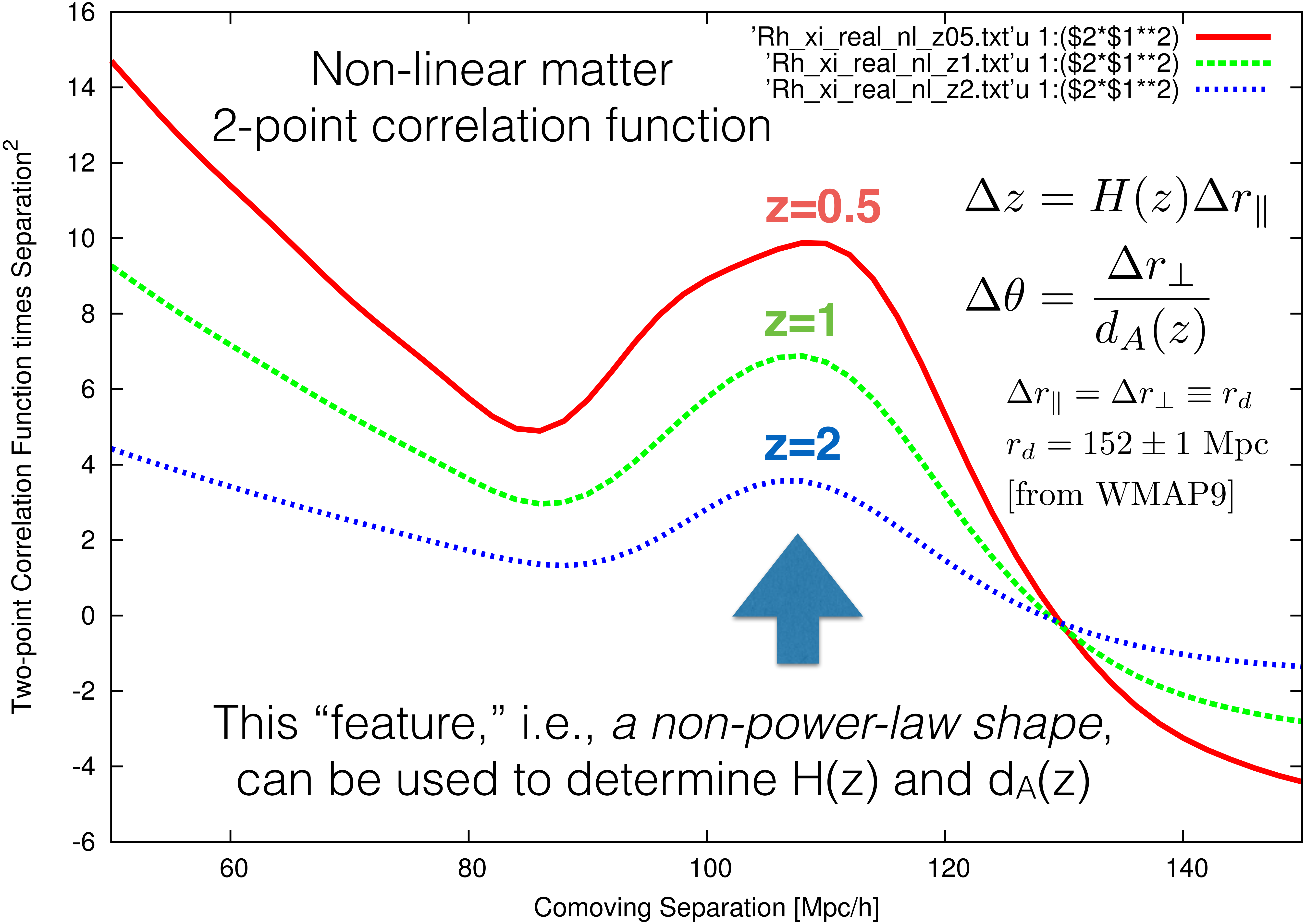
- **Standard ruler method applied to correlation functions of galaxies**

- Use known, well-calibrated, specific features in the 2-point correlation function of matter in angular and redshift directions

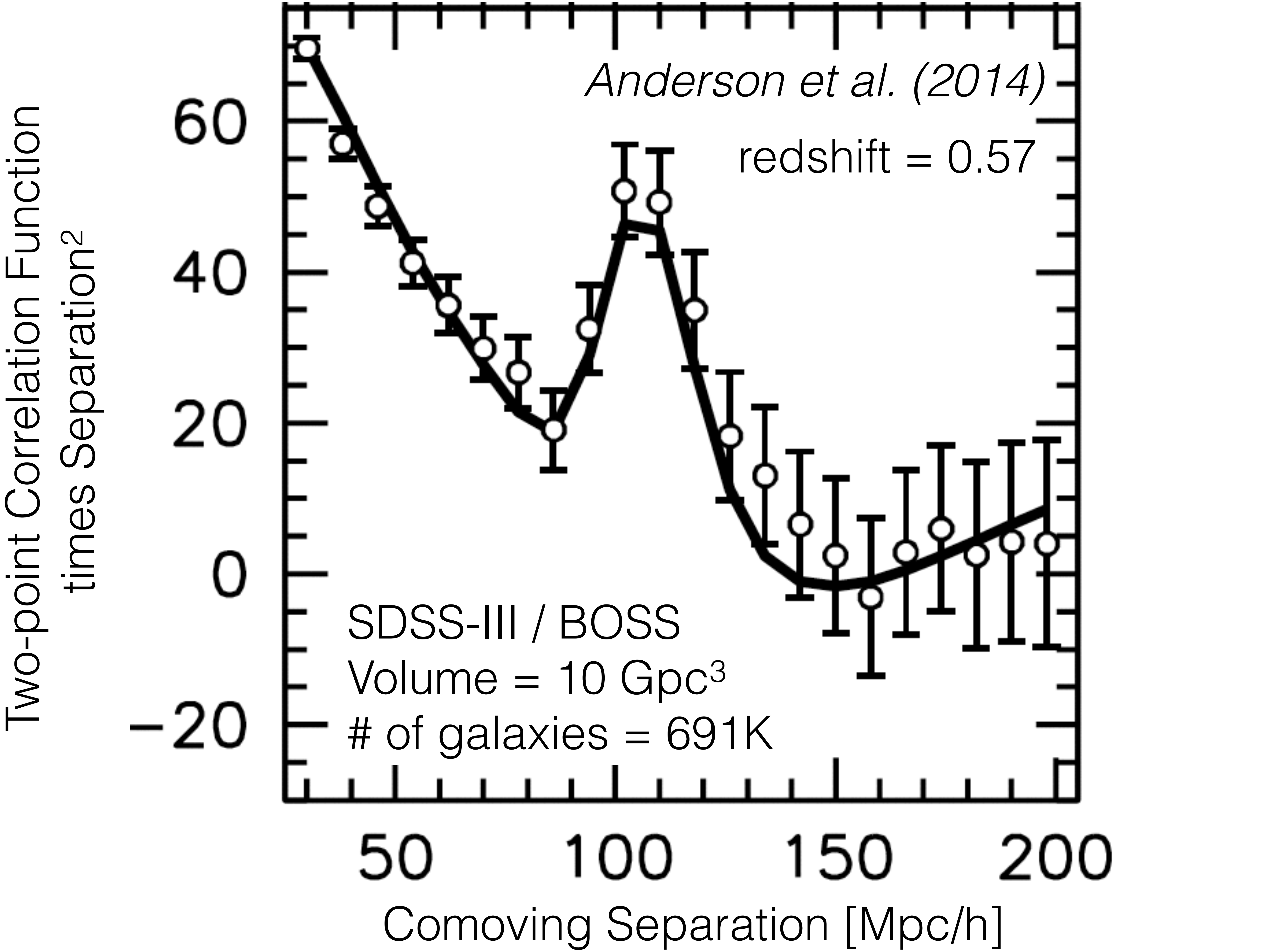
- Mapping the observed separations of galaxies to the comoving separations:

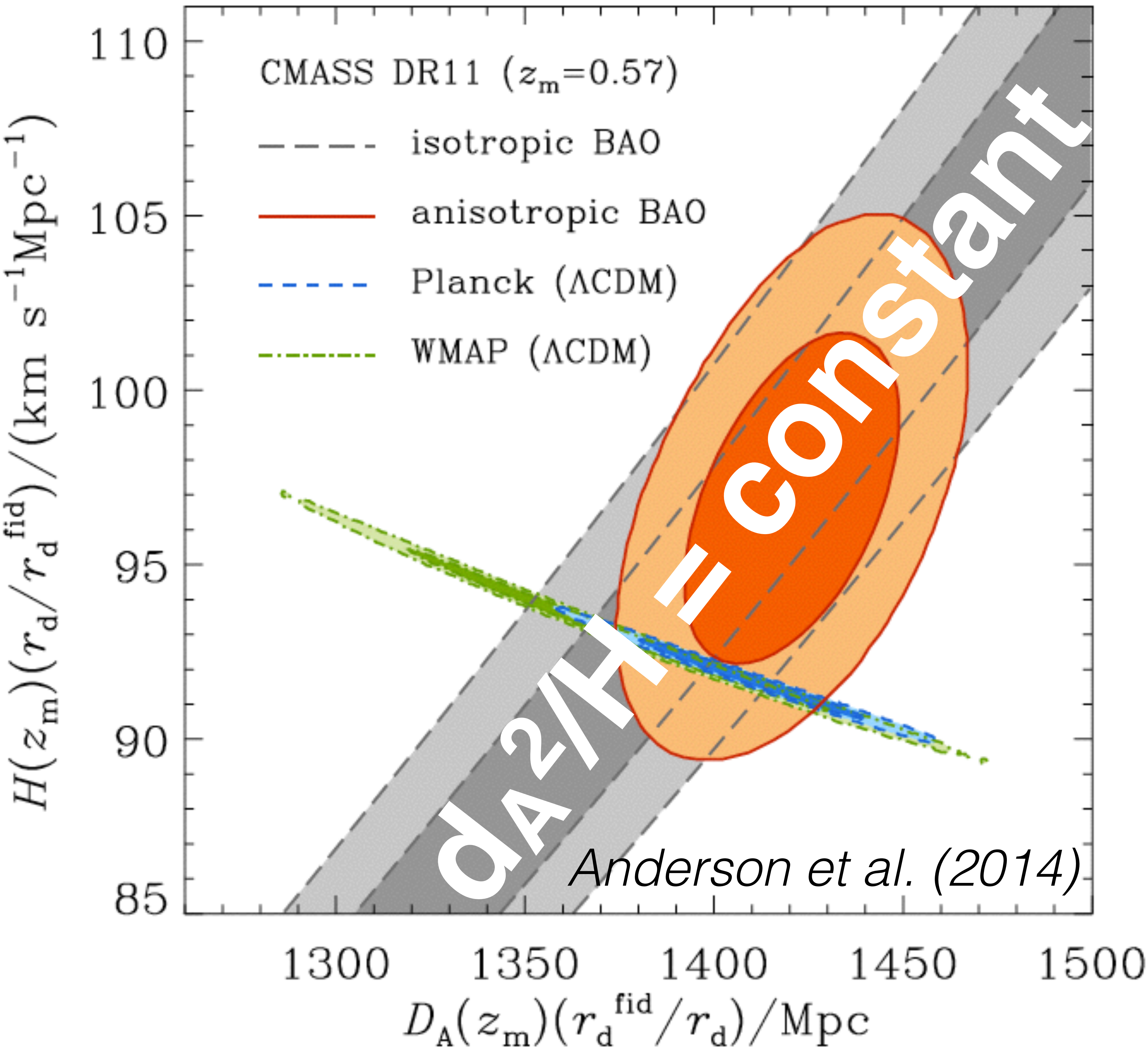
$$\Delta z = H(z)\Delta r_{\|} \quad \text{[Line-of-sight direction]}$$

$$\Delta\theta = \frac{\Delta r_{\perp}}{d_A(z)} \quad \text{[Angular directions]} \quad d_A = \int_0^z \frac{dz'}{H(z')}$$

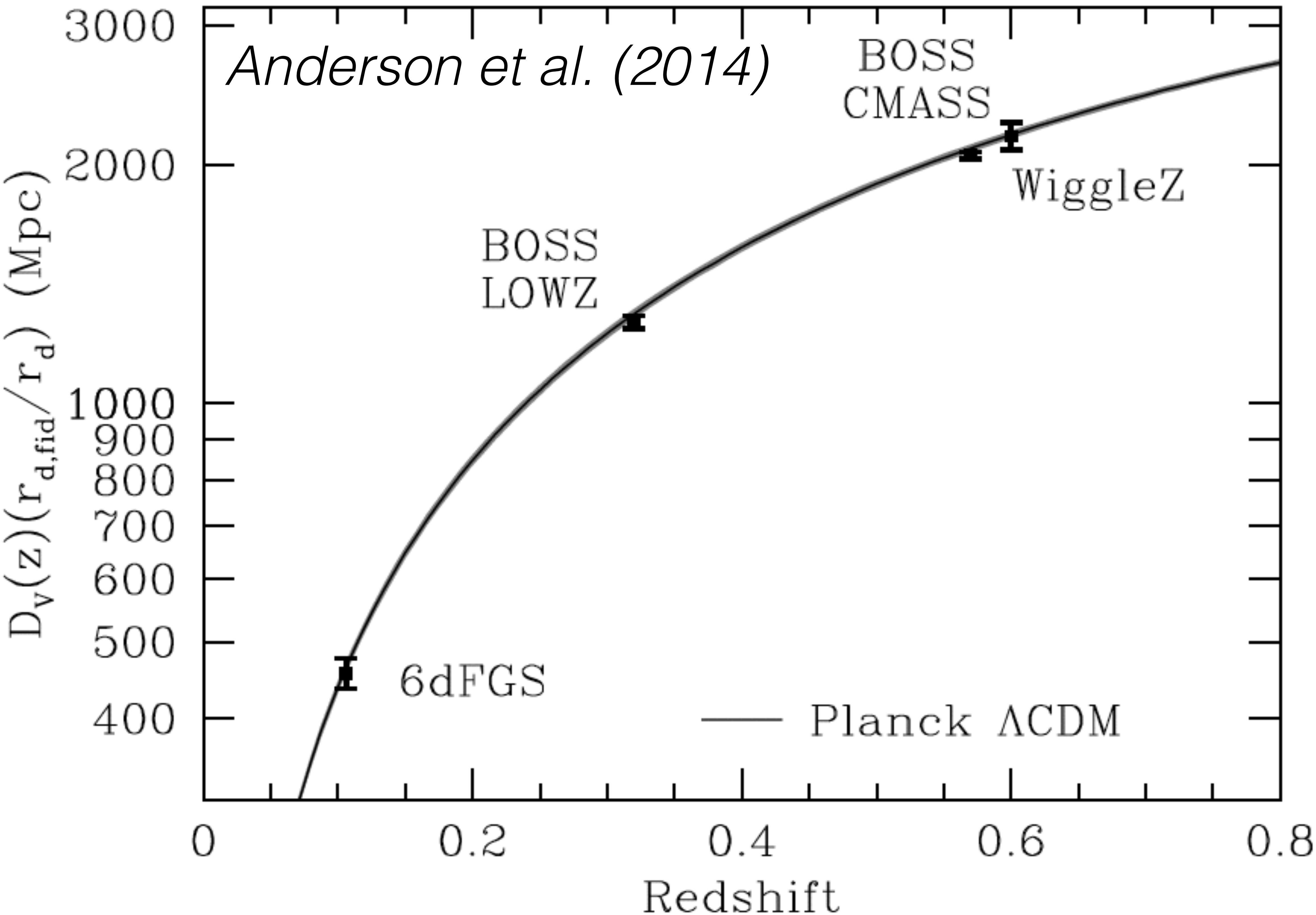


Non-linear matter
2-point correlation function
'Rh_xi_real_nl_z05.txt'u 1:($2*$1**2)
'Rh_xi_real_nl_z1.txt'u 1:($2*$1**2)
'Rh_xi_real_nl_z2.txt'u 1:($2*$1**2)
z=0.5
z=1
z=2
$\Delta z = H(z)\Delta r_{\parallel}$
$\Delta\theta = \frac{\Delta r_{\perp}}{d_A(z)}$
$\Delta r_{\parallel} = \Delta r_{\perp} \equiv r_d$
$r_d = 152 \pm 1$ Mpc
[from WMAP9]
This "feature," i.e., *a non-power-law shape*,
can be used to determine H(z) and d_A(z)
Two-point Correlation Function times Separation^2
Comoving Separation [Mpc/h]
16
14
12
10
8
6
4
2
0
-2
-4
-6
60
80
100
120
140

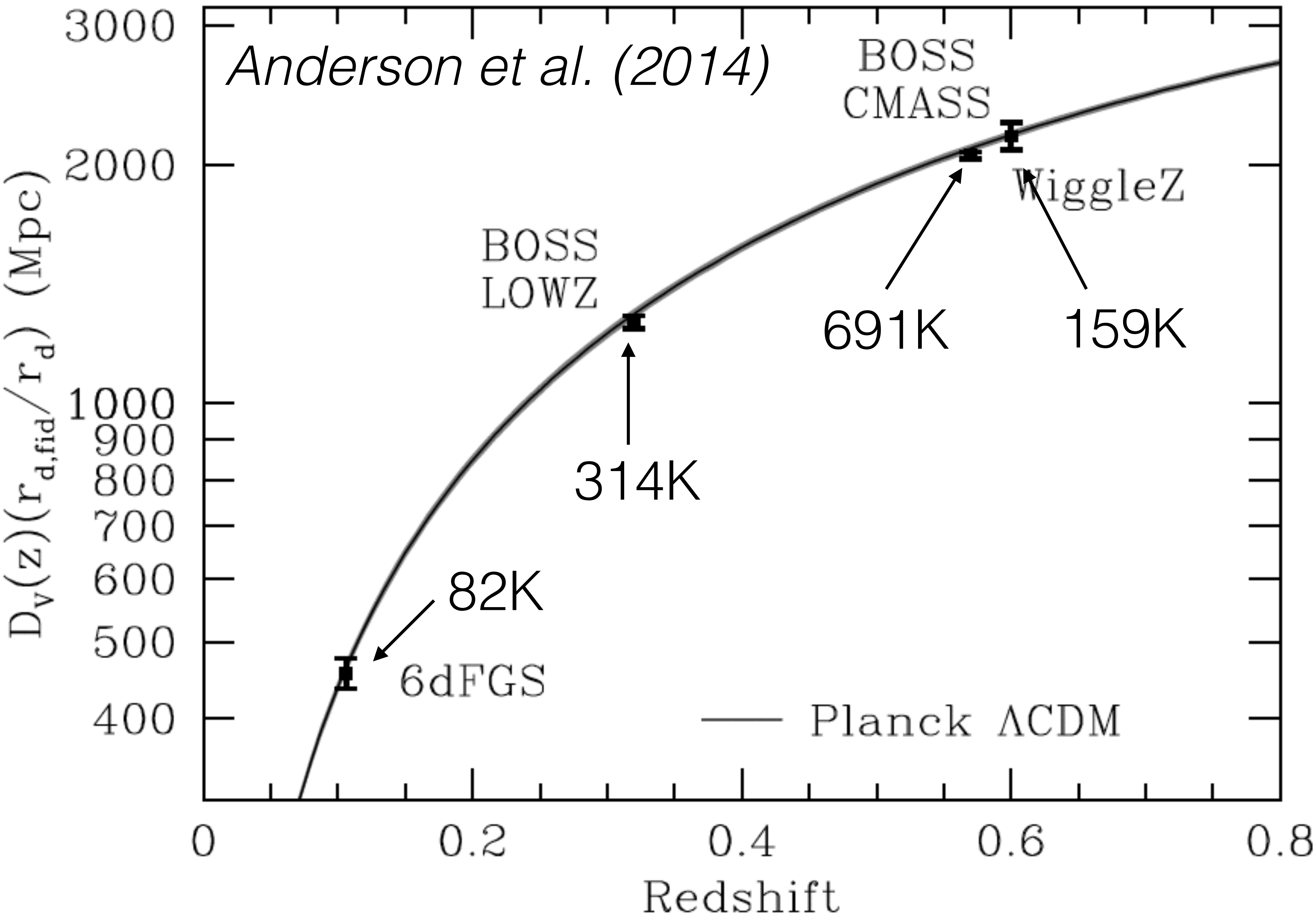

Anderson et al. (2014)
redshift = 0.57
SDSS-III / BOSS
Volume = 10 Gpc^3
# of galaxies = 691K
Two-point Correlation Function times Separation^2
Comoving Separation [Mpc/h]
60
40
20
0
−20
50
100
150
200


CMASS DR11 (z_m=0.57)
isotropic BAO
anisotropic BAO
Planck (ΛCDM)
WMAP (ΛCDM)
d_A^2/H = constant
Anderson et al. (2014)
110
105
100
95
90
85
1300
1350
1400
1450
1500
$H(z_m)(r_d/r_d^{fid})/(\mathrm{km\ s^{-1}Mpc^{-1}})$
$D_A(z_m)(r_d^{fid}/r_d)/\mathrm{Mpc}$

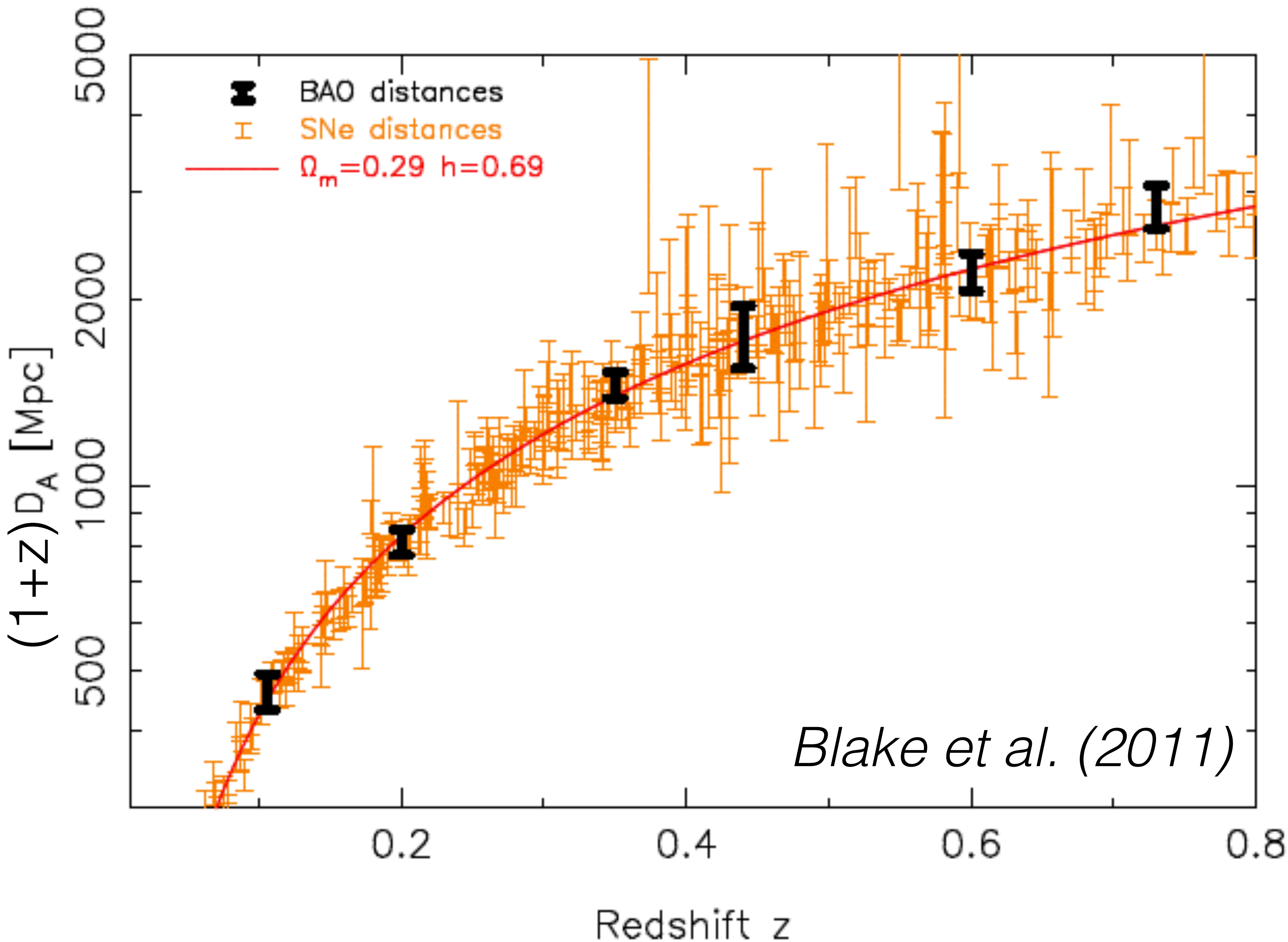
BAO Hubble Diagram



of galaxies used



BAO vs Type Ia SN



D_A: Current Situation

- X-ray + SZ: already systematics limited per cluster
  - Departure from spherical symmetry
  - Gas clumpiness, $\langle n^2 \rangle / \langle n \rangle^2$
- BAO: precise measurements, but requires a huge number of galaxies to average over per redshift bin, and each BAO project takes more than ten years from the construction to the completion

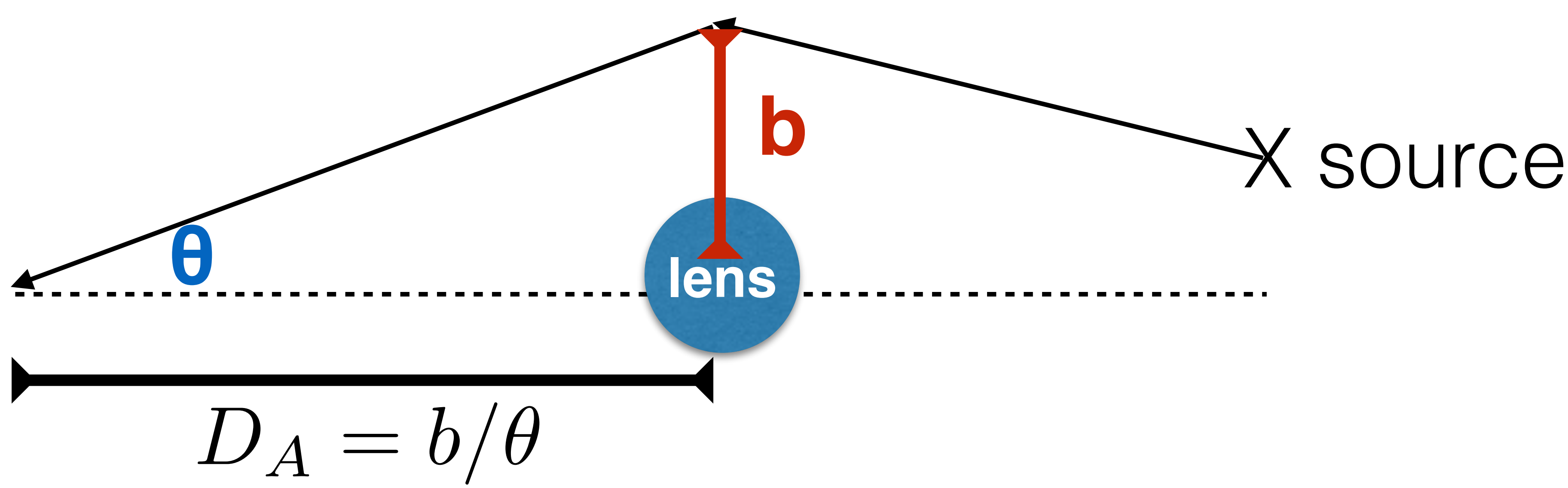
Refined Motivation

- We wish to measure D_A to ~10% precision per redshift, over many redshifts
  - Better than galaxy clusters per object
  - Less demanding than BAO measurements [depending on how you look at them]
- **We propose to use strong lenses to achieve this**
  - Goal: "*One [or two] distance per graduate student*"
  - With the existing facilities

Strong Lens -> D_A: Logic

- If we know the “physical size of the lens”, we can estimate D_A from the observed image separations

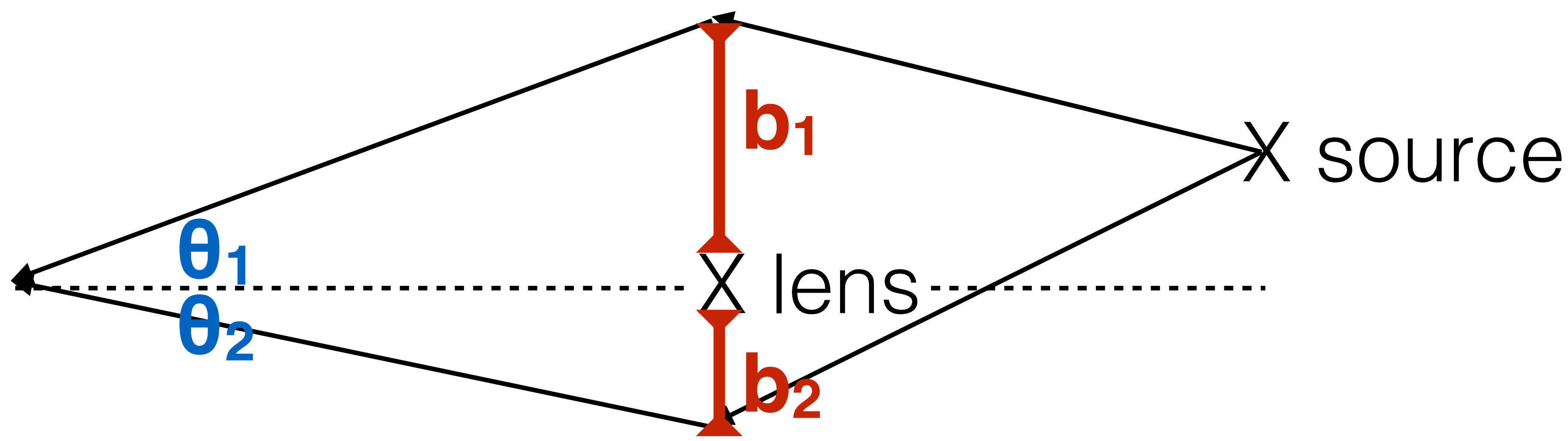
  - To simplify the logic, let us equate the “physical size of the lens” with the “impact parameter of a photon path,” **b** [i.e., the distance of the closest approach to the lens]



[Simplified] Physical Picture

- Three observables
  - Image positions, $\theta = b/D_A$
  - Stellar velocity dispersion, $\sigma^2 \sim GM/b$
  - Time delay, $\tau \sim GM$
- Thus, we can predict the impact parameter, b, from the stellar velocity dispersion and the time delay, and the image positions give a direct estimate of D_A!

[Geometric] Time Delay

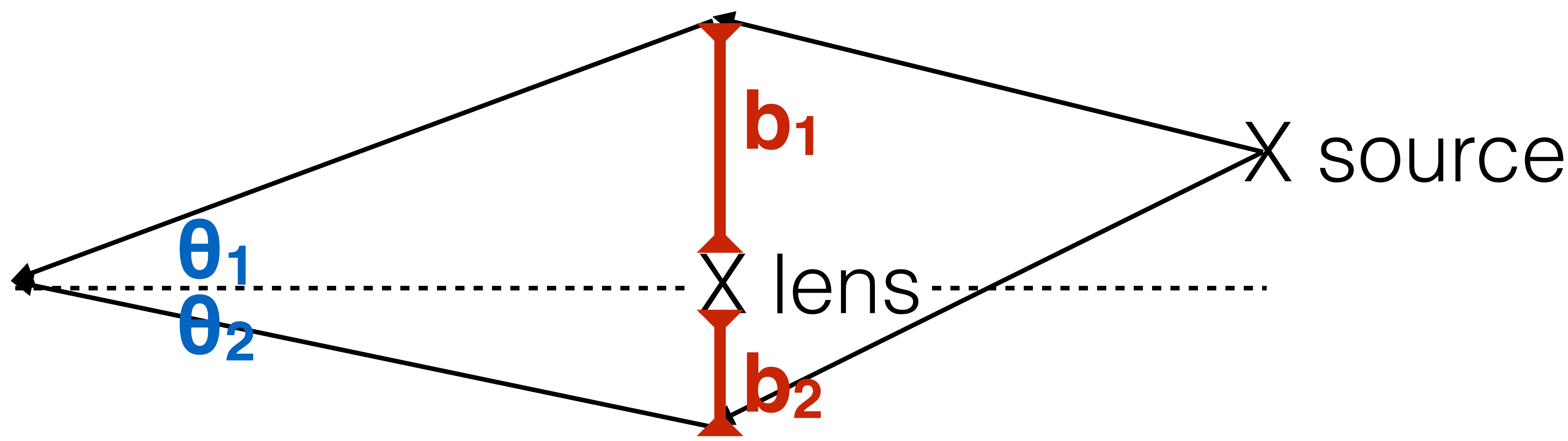


- For a <u>point mass lens</u>, the difference between time delays due to the difference in light paths is given by

$$[\tau_1 - \tau_2]_{\text{geometry}} = 4GM(1+z)\frac{b_1^2 - b_2^2}{b_1 b_2}$$

*we need an asymmetric system, $b_1 \neq b_2$

[Potential] Time Delay

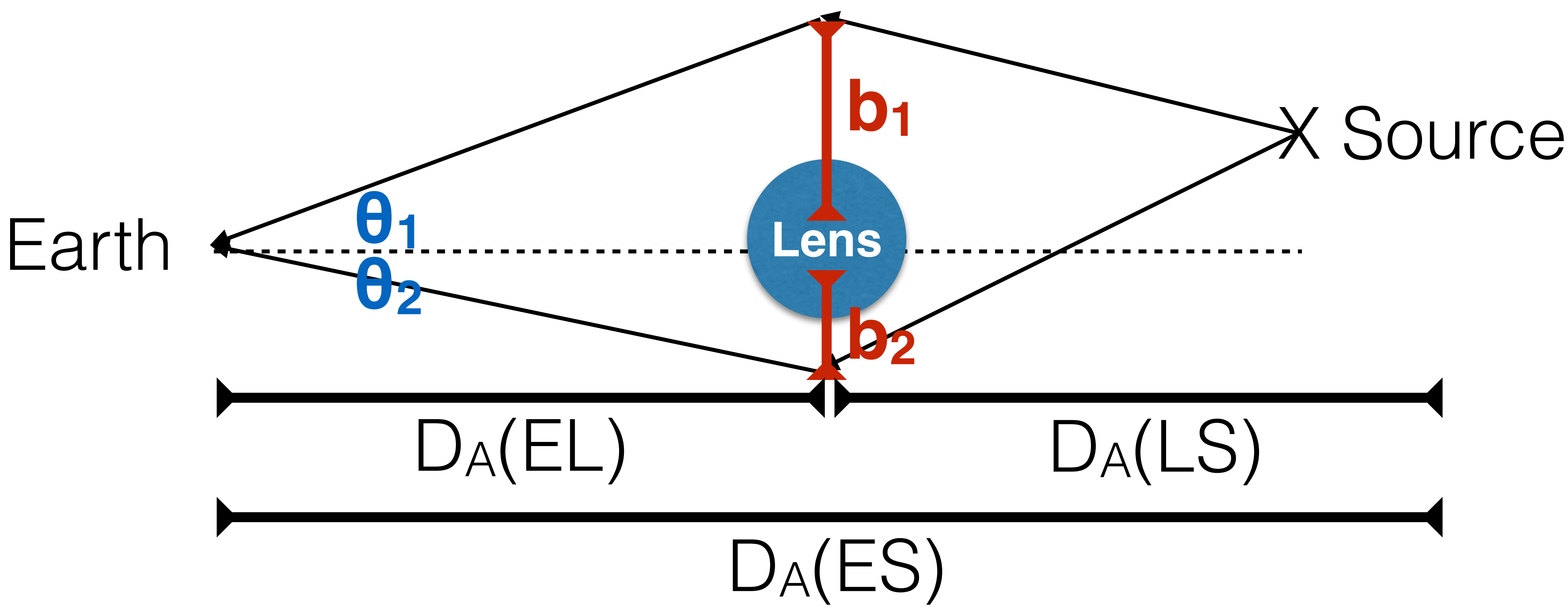


- For a point mass lens, the difference between time delays due to the difference in potential depths is given by

$$[\tau_1 - \tau_2]_{\text{potential}} = 4GM(1+z)\ln\left(\frac{b_2}{b_1}\right)$$

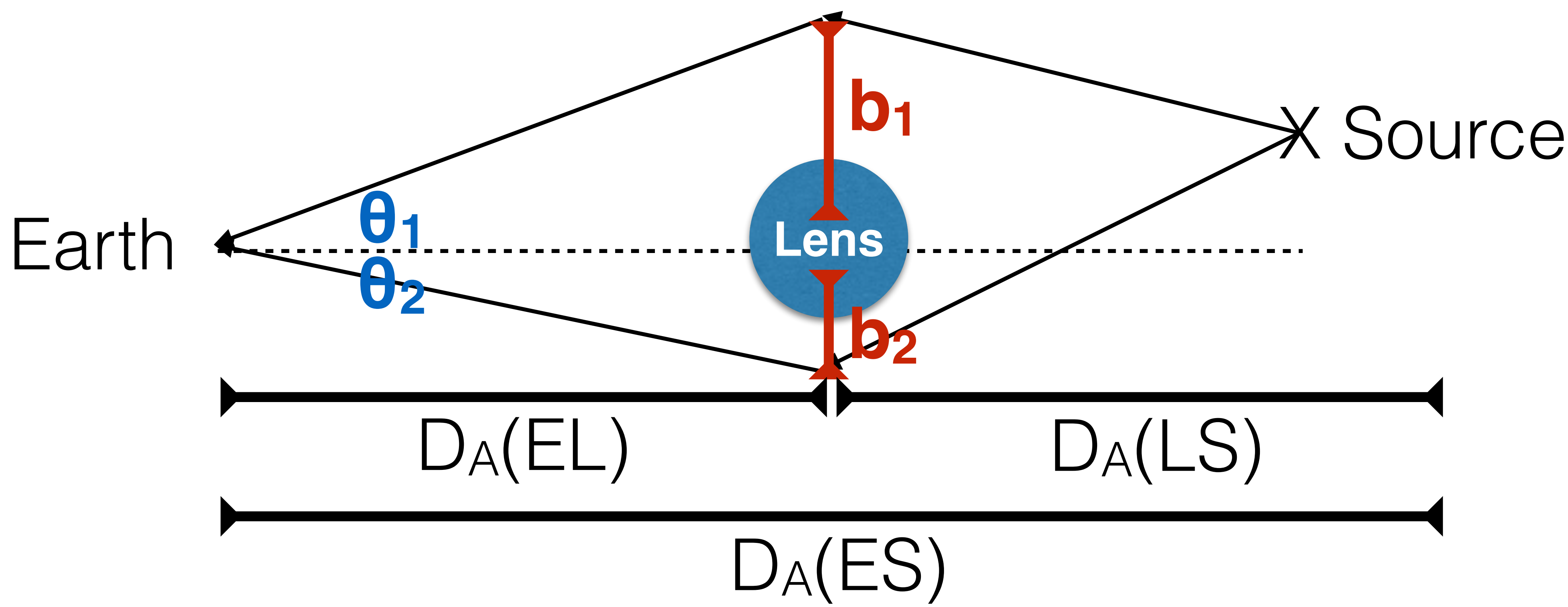
*we need an asymmetric system, $b_1 \neq b_2$

An Extended Lens: SIS



- As the first concrete calculation, let us study a singular isothermal sphere (SIS), $\rho(r) \sim r^{-2}$

An Extended Lens: SIS



Velocity disp: $\sigma^2 = \frac{\theta_1 + \theta_2}{8\pi} \frac{D_A(ES)}{D_A(LS)}$

Time-delay diff: $\tau_1 - \tau_2 = \frac{1}{2}(1 + z_L)\frac{D_A(EL)D_A(ES)}{D_A(LS)}(\theta_1^2 - \theta_2^2)$

Paraficz & Hjorth (2009)

An Extended Lens: SIS

Velocity disp: $\sigma^2 = \frac{\theta_1 + \theta_2}{8\pi} \frac{D_A(ES)}{D_A(LS)}$

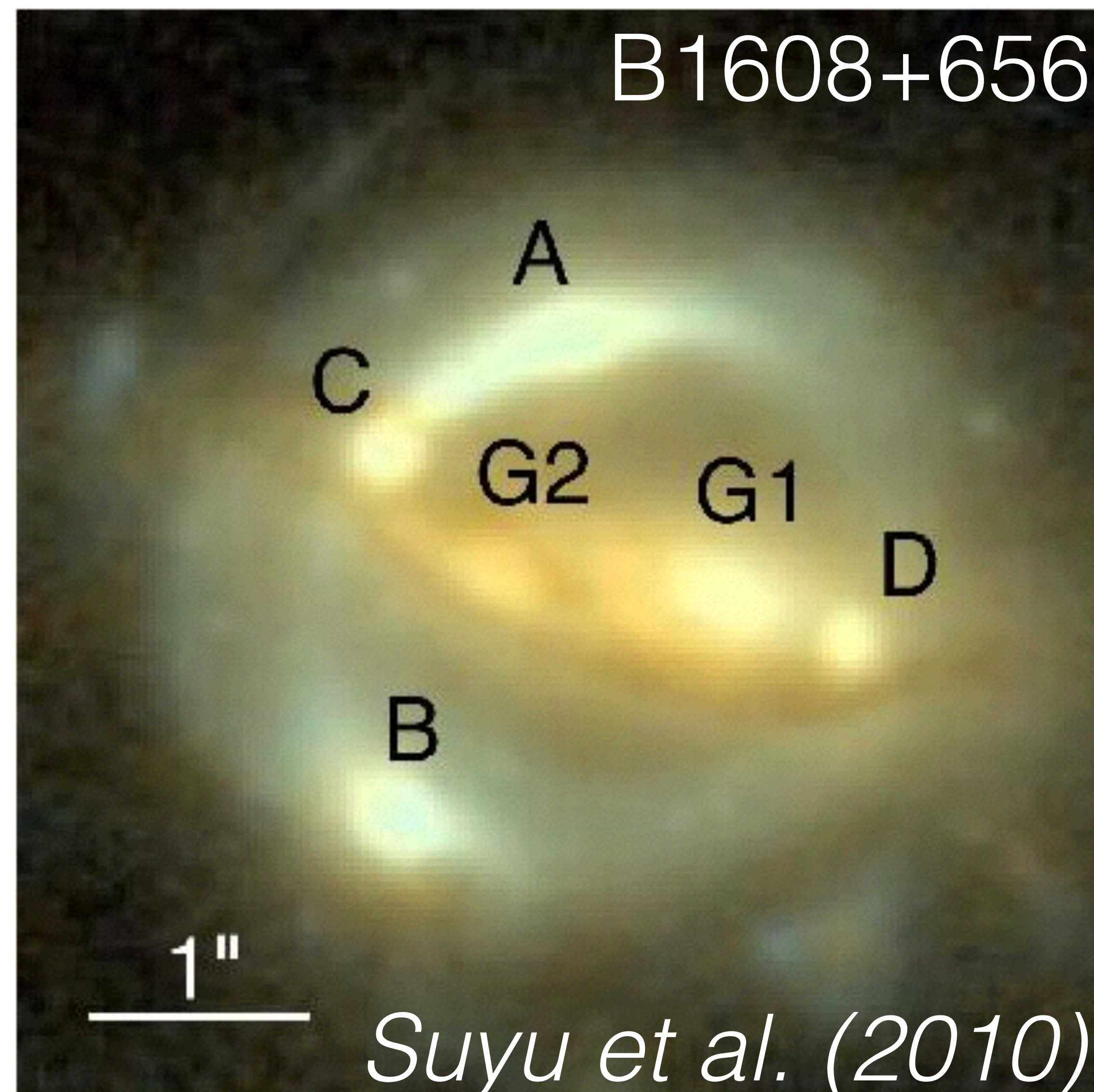
Time-delay diff: $\tau_1 - \tau_2 = \frac{1}{2}(1 + z_L) \frac{D_A(EL) D_A(ES)}{D_A(LS)} (\theta_1^2 - \theta_2^2)$

$$(\theta_1 - \theta_2) D_A(EL) = \frac{\tau_1 - \tau_2}{4\pi\sigma^2(1 + z_L)}$$

Expected Uncertainty in D_A

- Ignoring uncertainties in image positions [which are small], the uncertainty in D_A is the quadratic sum of the uncertainties in the time delay and σ^2

- For example, B1608+656:
  - $\mathrm{Err}[\sigma^2]/\sigma^2 = 12\%$
  - $\mathrm{Err}[\Delta\tau]/\Delta\tau = 3\text{–}6\%$



Thus, we expect the uncertainty in the velocity dispersion to dominate the uncertainty in D_A [of order 10%]

Jee, EK & Suyu (in prep)

More Realistic Analysis

- We extend the SIS results of Paraficz & Hjorth to include:
    - Arbitrary power-law spherical density, $\rho \sim r^{-\gamma}$
        - Hence, radius-dependent stellar velocity dispersion, $\sigma^2(r)$
    - External convergence
    - Anisotropic stellar velocity dispersion

Jee, EK & Suyu (in prep)

More Realistic Analysis

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    - Hence, radius-dependent stellar velocity dispersion, $\sigma^2(r)$
  - **External convergence**
  - **Anisotropic stellar velocity dispersion**

External Convergence

- So far, the analysis assumes that the observed lensed images are caused entirely by the lens galaxy

- However, in reality there are extra masses, which are not associated with the lens galaxy, along the line of sight

  - This is the so-called “external convergence”, κ_{ext}

  - Taking this account **reduces the contribution from the lens galaxy to the total deflection by $1-\kappa_{ext}$**, modifying the relationship between the time delay and the lens mass

Effect of κ_{ext} due to a uniform mass sheet

- The difference between time delays between two images is caused by the lens mass only. The additional contribution from a uniform mass sheet does not contribute to the time-delay *difference*:

$$\tau_1 - \tau_2 = \frac{1}{2}\underline{(1-\kappa_{\text{ext}})}(1+z_L)\frac{D_A(EL)D_A(ES)}{D_A(LS)}(\theta_1^2 - \theta_2^2)$$

Effect of κ_{ext} due to a uniform mass sheet

- The observed stellar velocity dispersion is solely due to the lens mass distribution, while the observed image separations contain the contributions from the lens galaxy and a mass sheet:

$$\sigma^2 = (1 - \kappa_{\mathrm{ext}}) \frac{\theta_1 + \theta_2}{8\pi} \frac{D_A(ES)}{D_A(LS)}$$

Effect of κ_{ext} due to a uniform mass sheet

- Therefore, remarkably, the inferred angular diameter distance is independent of κ_{ext} from a uniform mass sheet:

$$(\theta_1 - \theta_2) D_A(EL) = \frac{\tau_1 - \tau_2}{4\pi\sigma^2(1+z_L)}$$

- **This property is not particular to SIS, but is generic**

Anisotropic Velocity Dispersion

- We use the measured stellar velocity dispersion to determine the mass enclosed within lensed images

- However, this relation depends on anisotropy of the velocity dispersion, such that

$$\frac{1}{\rho_*(r)}\frac{d(\rho_*\sigma_r^2)}{dr} + 2\beta(r)\frac{\sigma_r^2(r)}{r} = -\frac{GM(<r)}{r^2}$$

- where

$$\beta(r) \equiv 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)}$$

σ_r: radial dispersion
σ_t: transverse dispersion

Anisotropic Velocity Dispersion

- We parametrize the anisotropy function, β(r), following Merritt (1985) [also Osipkov (1979)]

$$\beta(r) \equiv 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)} = \frac{r^2}{r^2 + (n r_{\rm eff})^2}$$

  - $r_{\rm eff}$ is the effective radius of the lens galaxy, and
  - n is a free parameter to marginalize over [0.5,5]

- Smaller n -> Smaller total kinetic energy [given σ_r] -> Shallower gravitational potential

- Since GM is fixed, a smaller GM/b implies a larger physical size of the lens -> Larger D_A

Stellar Density Distribution

- For the stellar density distribution, we take Hernquist's profile:

$$\rho_*(r) \propto \frac{1}{r(r+a)^3}$$

- where a=0.551R_{eff}. With this distribution, we calculate the observable, i.e., the projected line-of-sight velocity dispersion at a projected radius of R:

$$\sigma_s^2(R) \equiv 2[I(R)]^{-1} \int_R^\infty dr \left[1 - \beta(r)\frac{R^2}{r^2}\right] \frac{\rho_*(r)\sigma_r^2(r)r}{\sqrt{r^2 - R^2}}$$

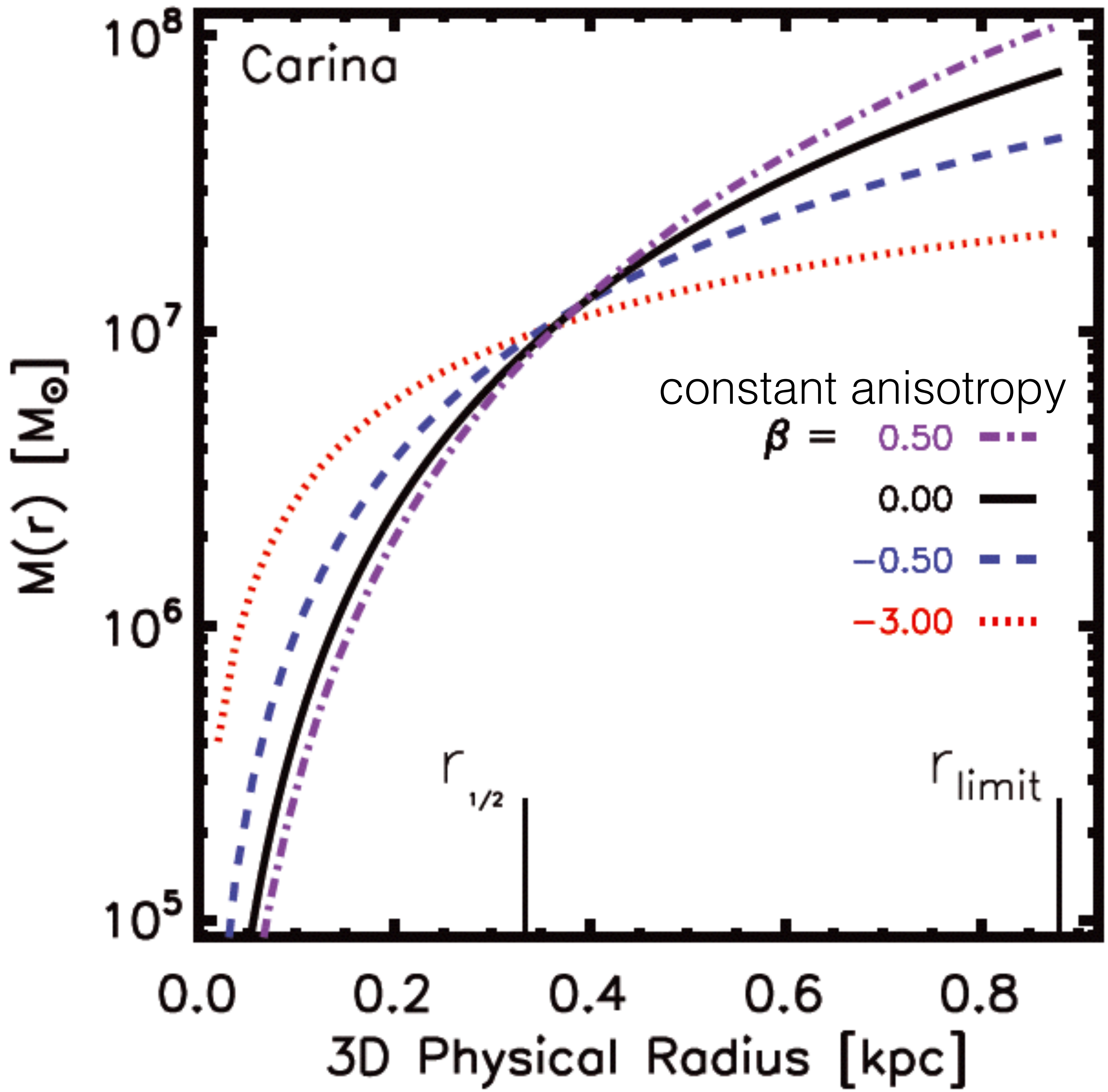
- where I(R) is the projected Hernquist profile

Which R to measure σ_s?

- The mass estimate given the observed projected velocity dispersion, $\sigma_s(R)$, is heavily affected by anisotropy. At first sight, this may seem to ruin a whole thing…

- However, Wolf et al. (2010) show that the estimate of the mass enclosed within the 3-d half-light radius, $r_{1/2}$, is insensitive to anisotropy. This is a great news!
    - The 2-d projected effective radius is $R_{eff} \sim (3/4) r_{1/2}$
    - This is true for systems with σ^2 ~ constant over radii

Wolf et al.'s Mass Estimate



Even Better: “Sweet-spot Radius”

- Lyskova et al. (2014) [also Churazov et al. (2010)] show that the radius at which the effect of anisotropic velocity dispersion is minimised depends on the local slope of the stellar surface brightness profile
  - Specifically, they compute the “sweet-spot radius”, R_{sweet}, at which the local surface brightness profile is $I(R) \sim R^{-2}$. R_{sweet} is $0.78R_{eff}$ for Hernquist’s profile
  - This is an improvement over Wolf et al. (2010)
- We use both Wolf et al. (2010) and the sweet-spot radius to calculate the expected uncertainties in D_A

System 1: B1608+656

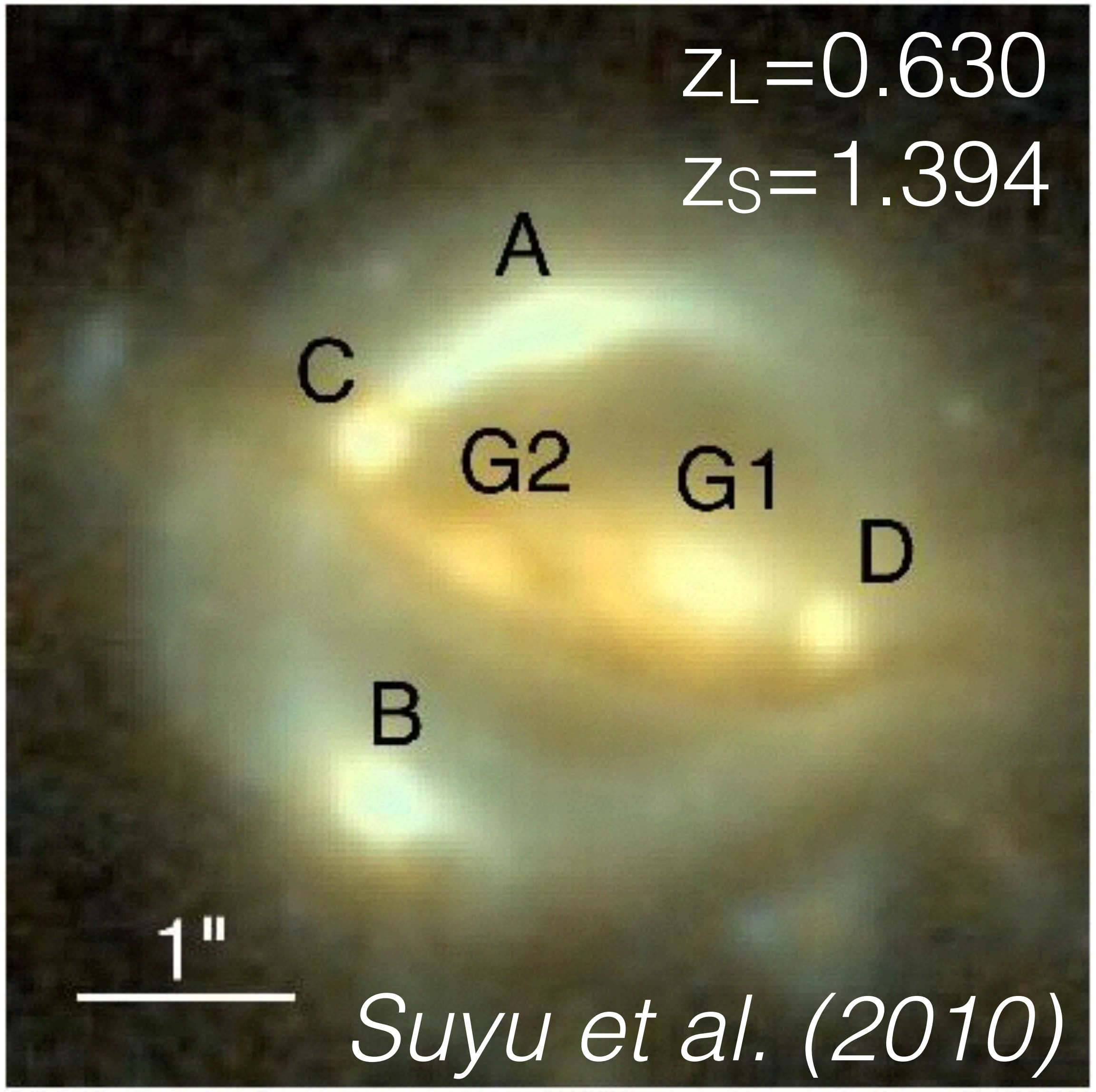
- The power-law mass density slope is $\rho \sim r^{-2.08\pm0.03}$ [G1]
- R_{eff}=0.58 arcsec
- σ_s[G1; **averaged over 0.84"**] = 260 ± 15 km/s
- Time delays:

$$\Delta t_{AB} = 31.5^{+2.0}_{-1.0}\ days$$

$$\Delta t_{CB} = 36.0^{+1.5}_{-1.5}\ days$$

$$\Delta t_{DB} = 77.0^{+2.0}_{-1.0}\ days$$

$$\Delta t_{CD} = \Delta t_{CB} - \Delta t_{DB} = -41.0^{+2.5}_{-1.8}\ days$$



We will use only CD [for now]

Approximate Likelihood of D_A

$$P(D_A|\mathrm{data}) \propto \int_0^\infty d\sigma_s \int_{0.5}^{50} dn \ \exp\left[-\frac{(\sigma_s - 260)^2}{2\mathrm{Var}(\sigma_s)}\right] \times \delta[D_A - D_A^{\mathrm{model}}(\sigma_{\mathrm{iso}})] \times \delta[\sigma_{\mathrm{iso}} - \sigma_s(n)]$$

- Assumptions:
  - We ignore the sub-dominant uncertainties in the density slope, γ, the time delays, and the image positions
  - The current velocity dispersion measurement is the aperture-averaged value, rather than at R_{eff} or R_{sweet}; however, we pretend that it is at R_{eff} or R_{sweet}, i.e., it is a forecast rather than the measurement. [We will also investigate what the current data can tell us]

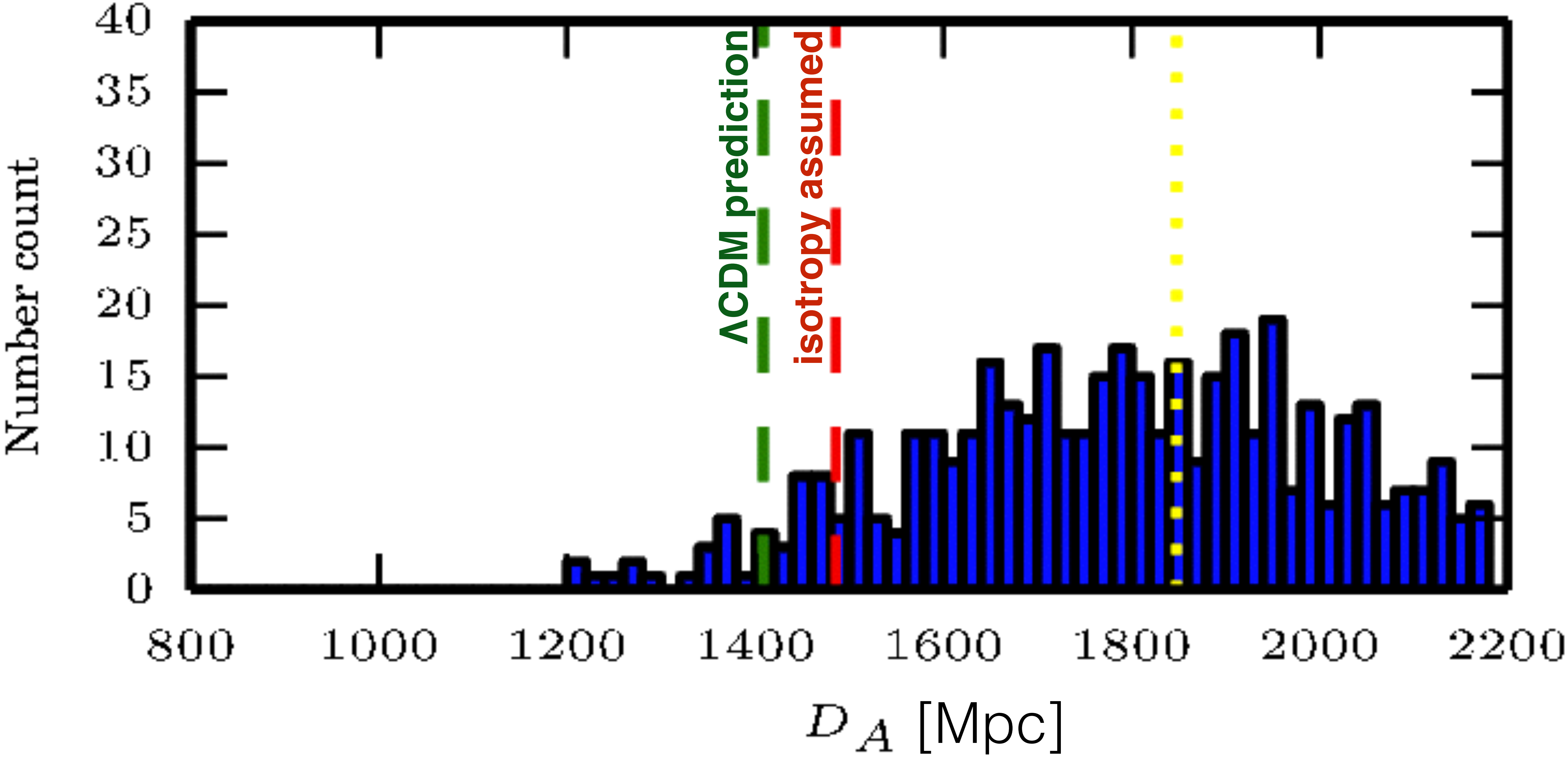
Procedures

- We first assume that B1608+656 has an anisotropic velocity profile with a certain value of n
- We then compute the posterior probability of D_A, marginalising over n=[0.5,50]
- We compare the results with the ΛCDM prediction

$\sigma_s(R_{eff})$ is used

$n_{input} = 0.5$

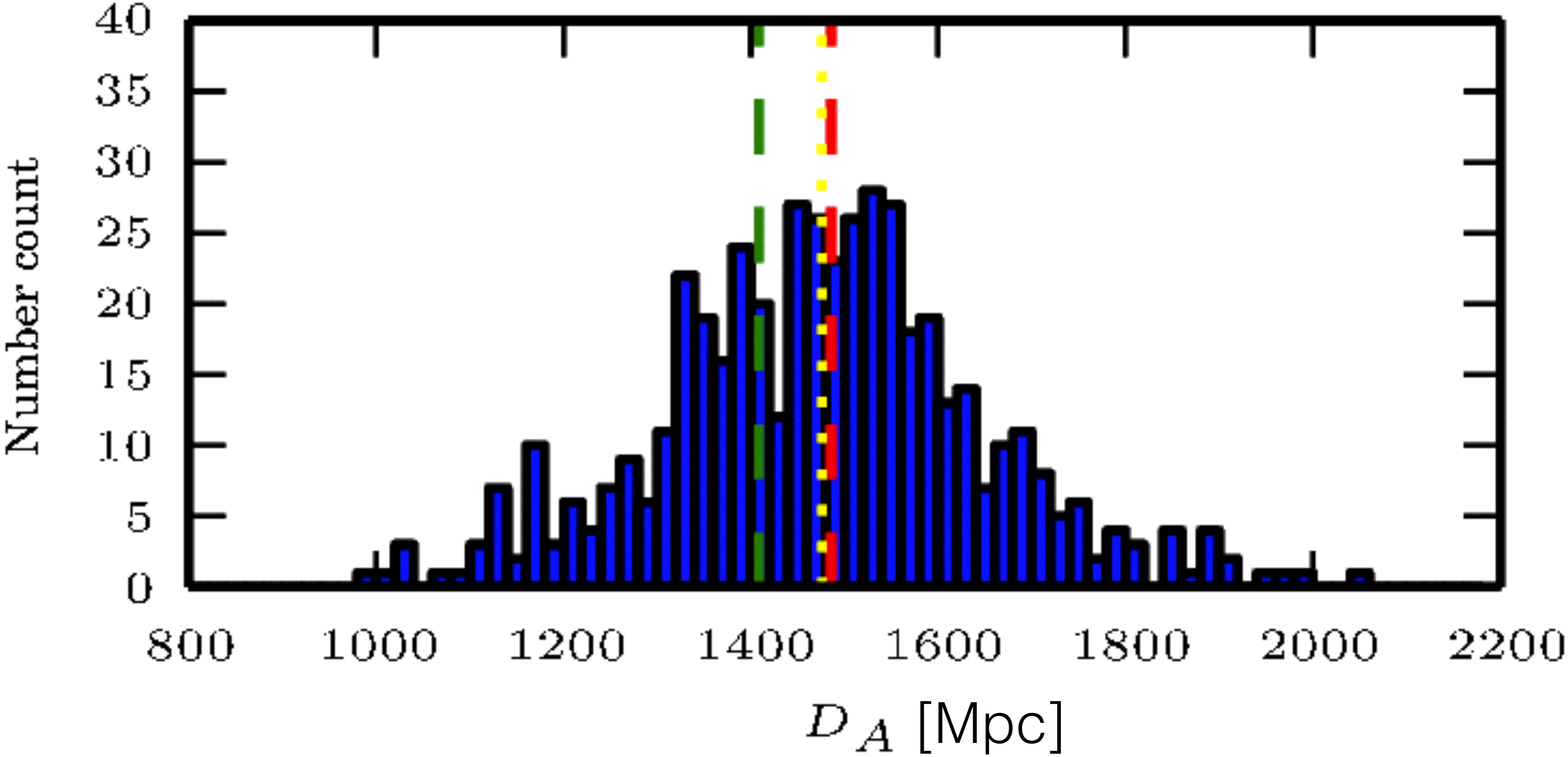
$D_A = 1848 \pm 273$ Mpc



$\sigma_s(R_{eff})$ is used

$n_{input} = 1$

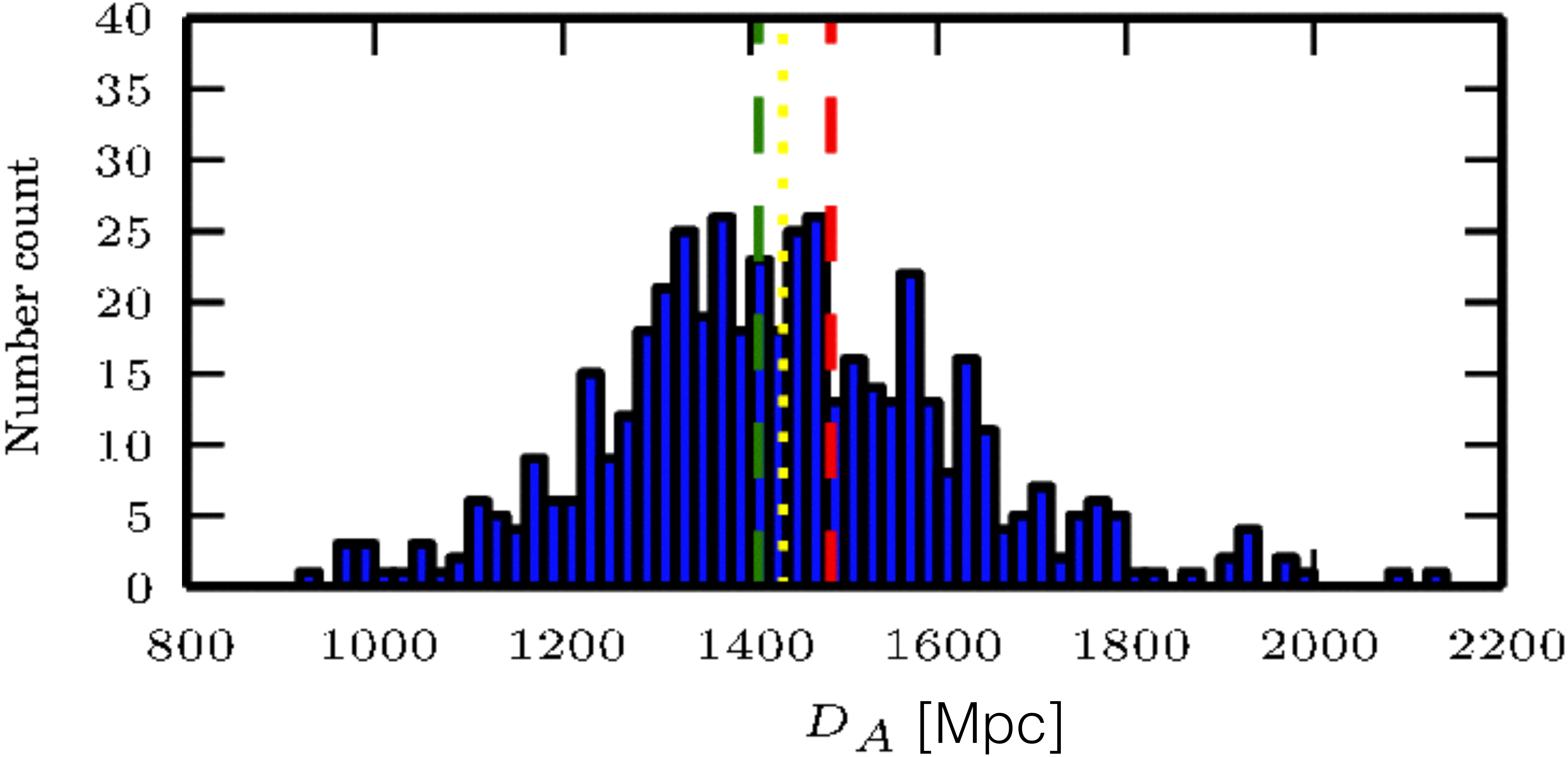
$D_A = 1476 \pm 173$ Mpc



$\sigma_s(R_{eff})$ is used

$n_{input} = 2$

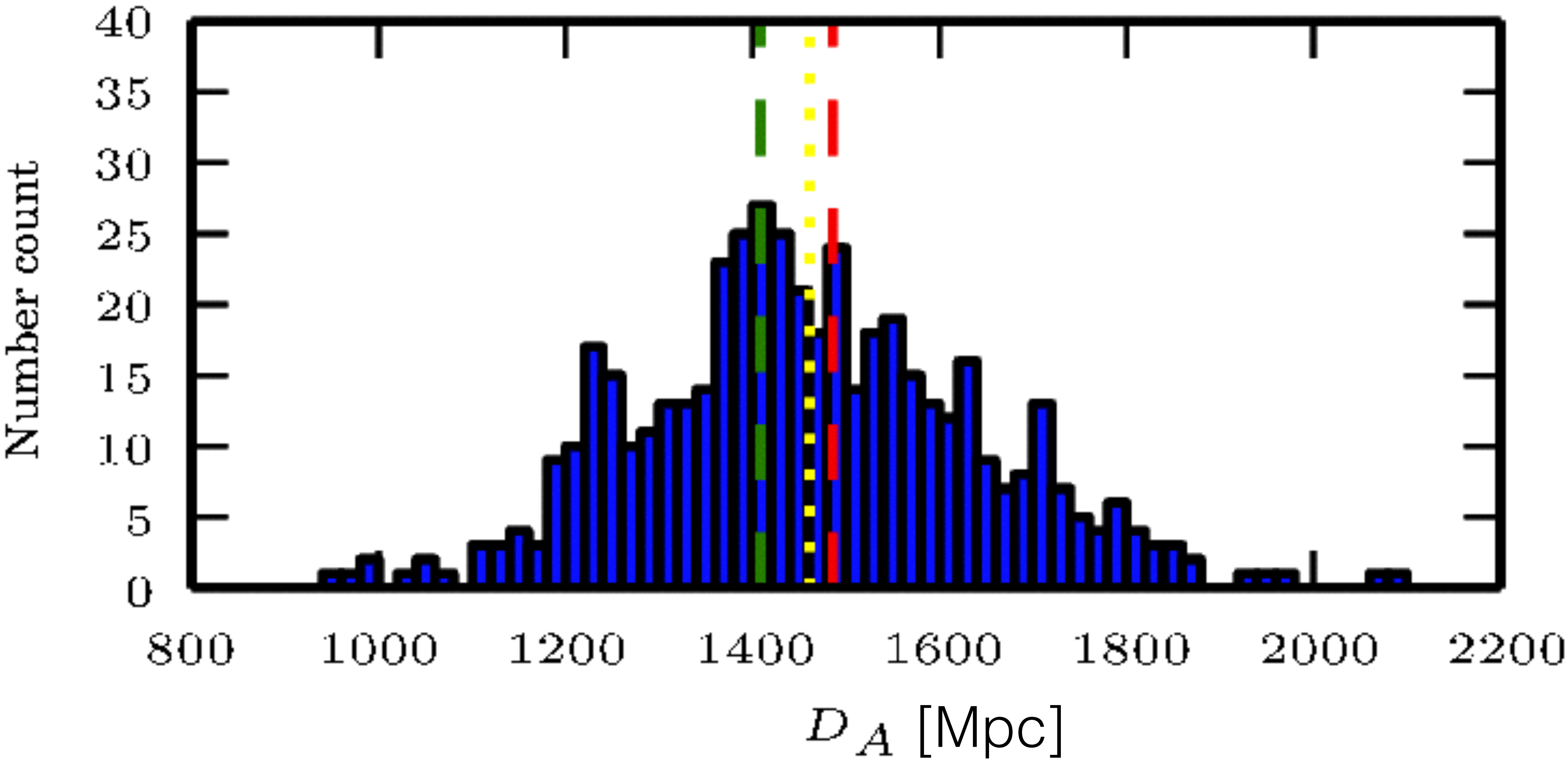
$D_A = 1382 \pm 164$ Mpc



$\sigma_s(R_{eff})$ is used

$n_{input} = 3$

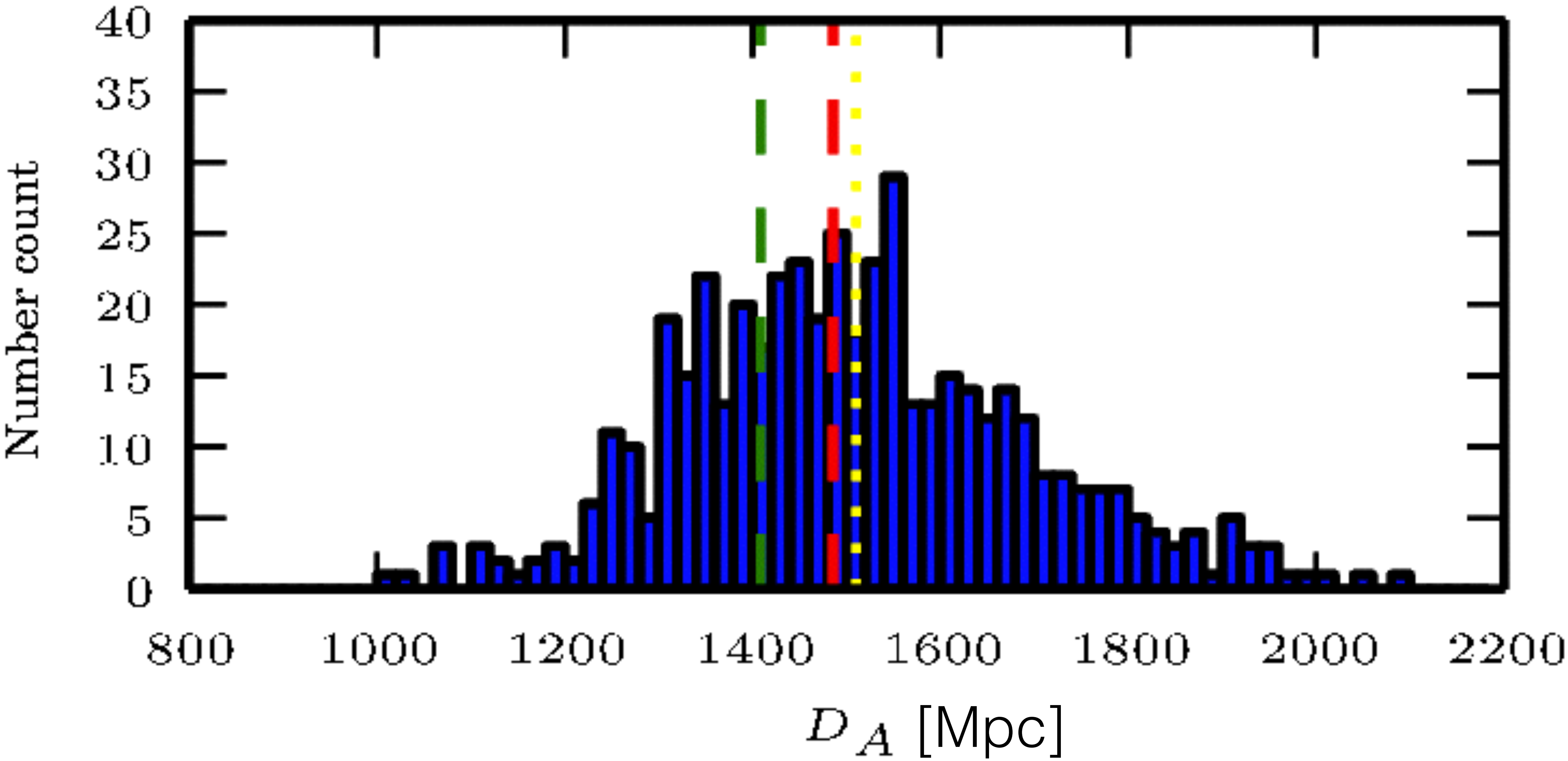
$D_A = 1461 \pm 186$ Mpc



$\sigma_s(R_{eff})$ is used

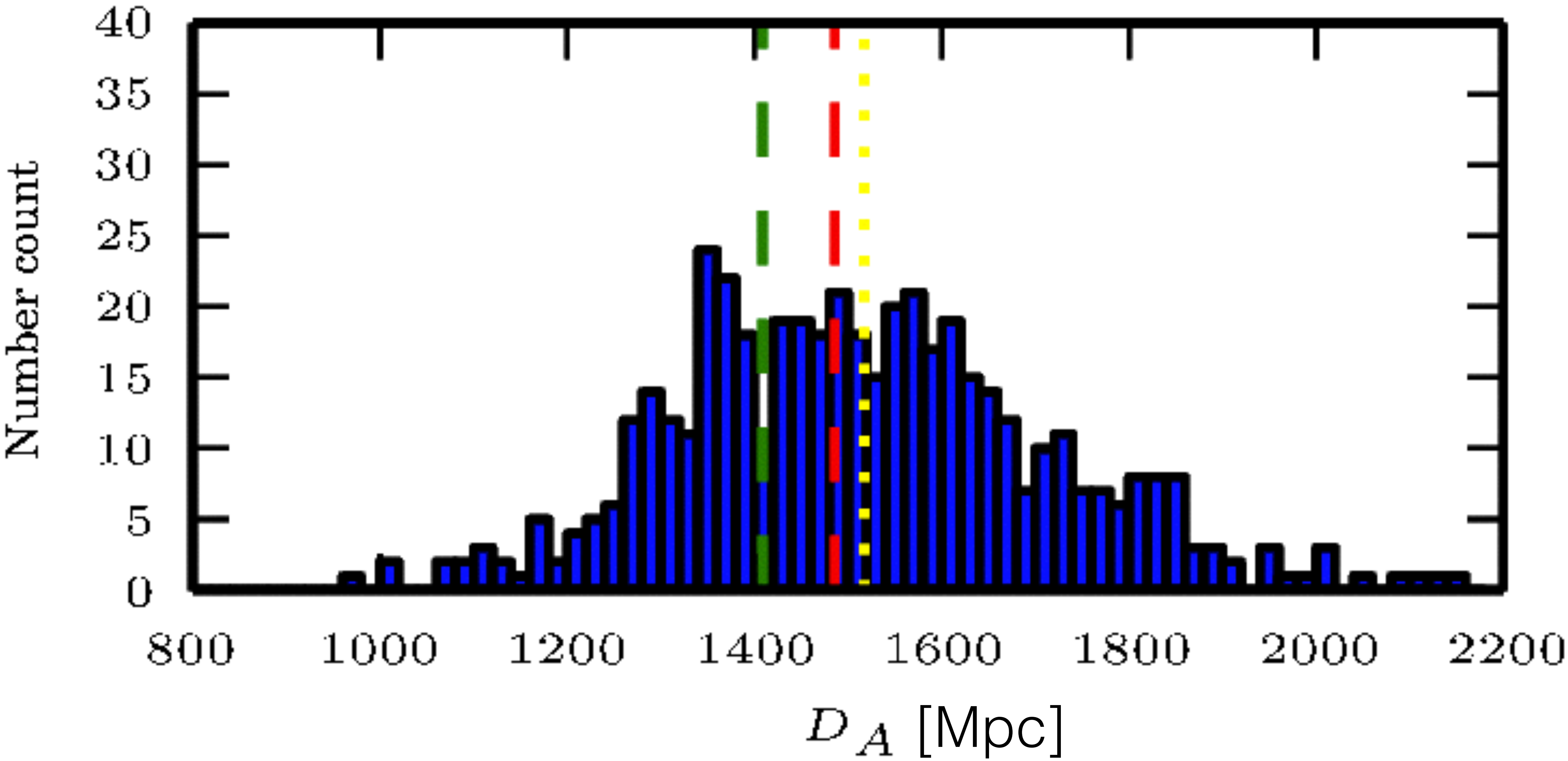
$n_{input} = 4$

$D_A = 1510 \pm 190$ Mpc



$\sigma_s(R_{eff})$ is used

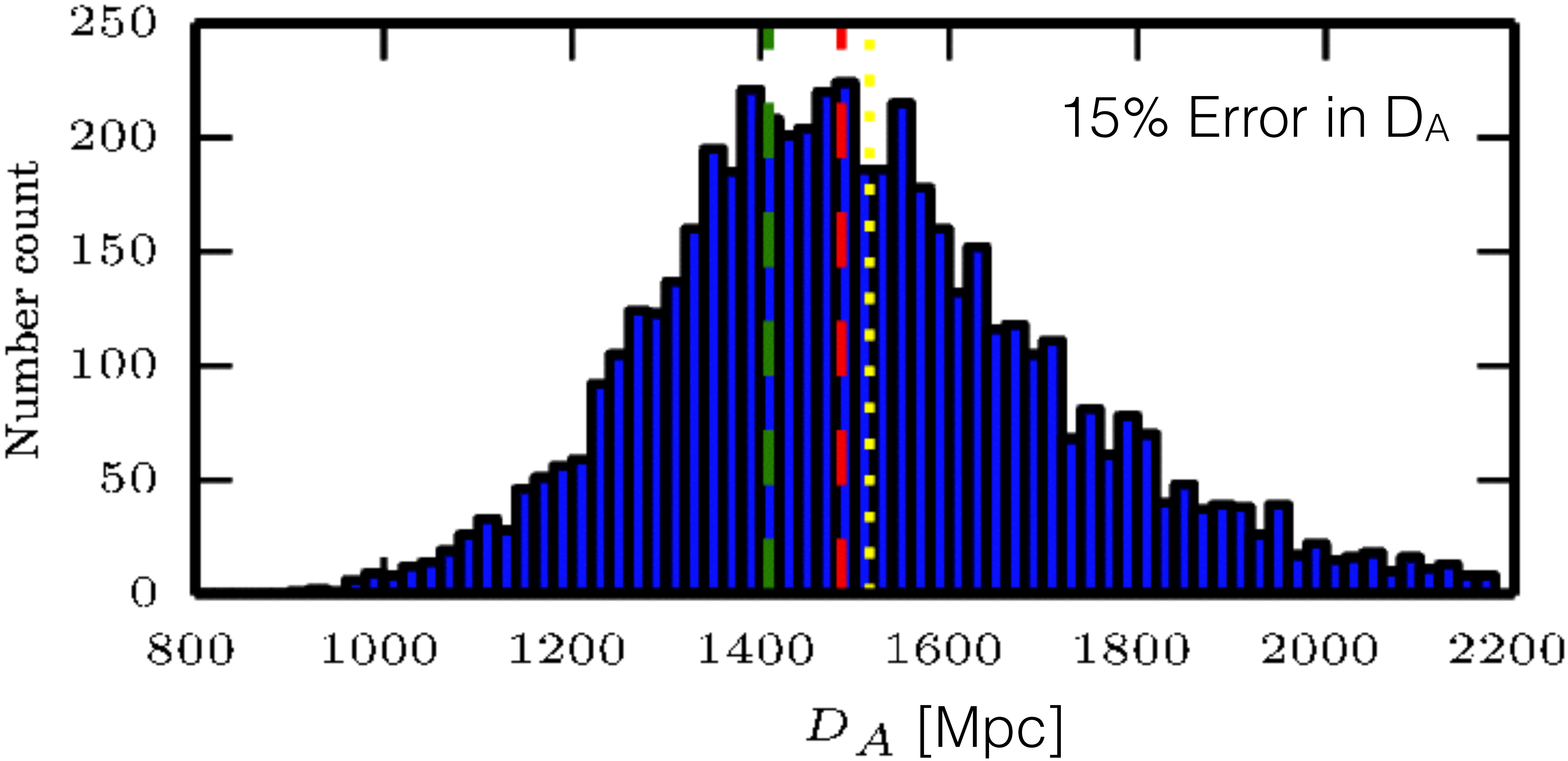
$n_{input} = 5$
$D_A = 1517 \pm 199$ Mpc



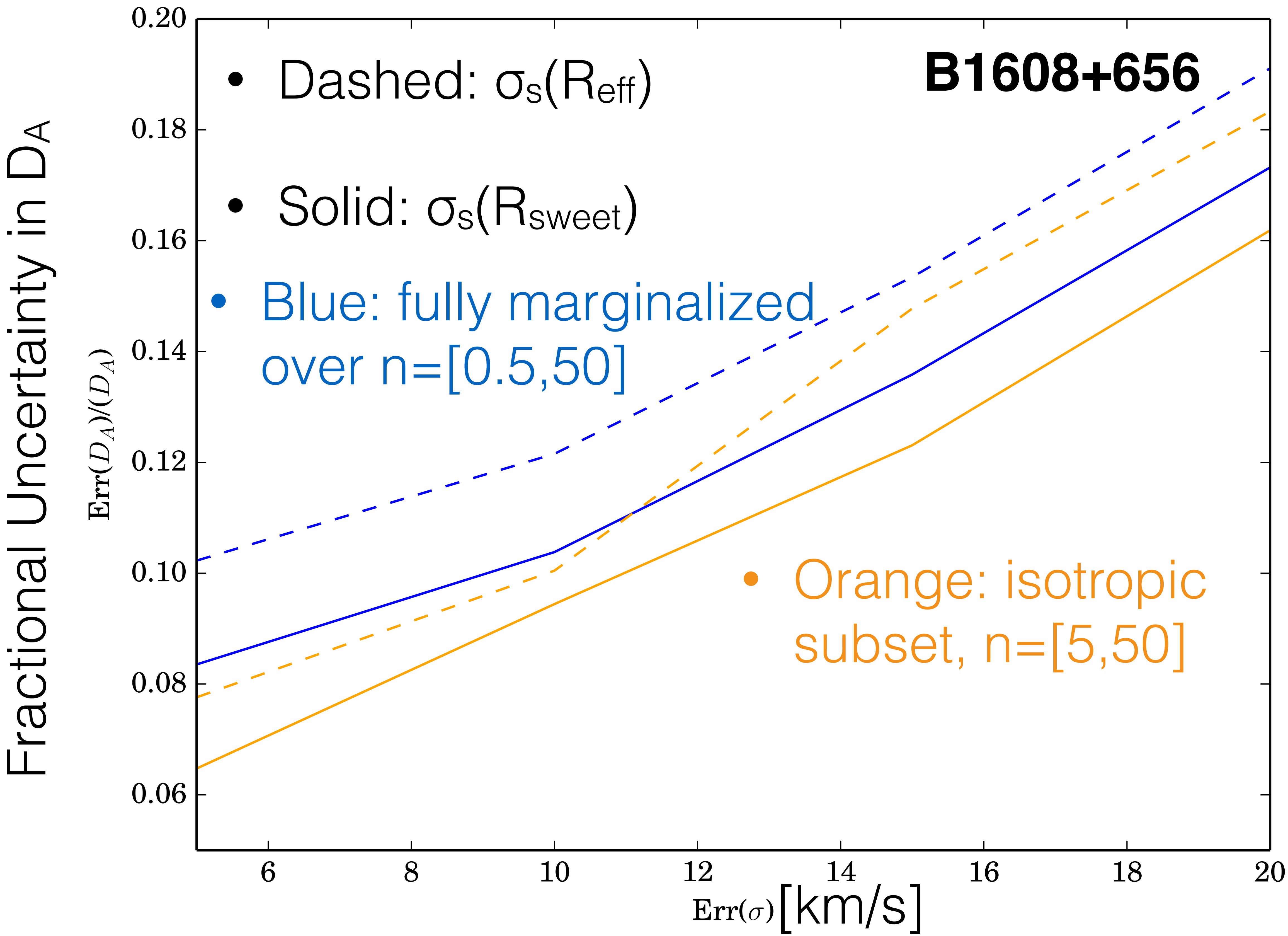
$\sigma_s(R_{eff})$ is used

All Stacked

D_A = 1515±233 Mpc



Uncertainties: D_A vs σ_s

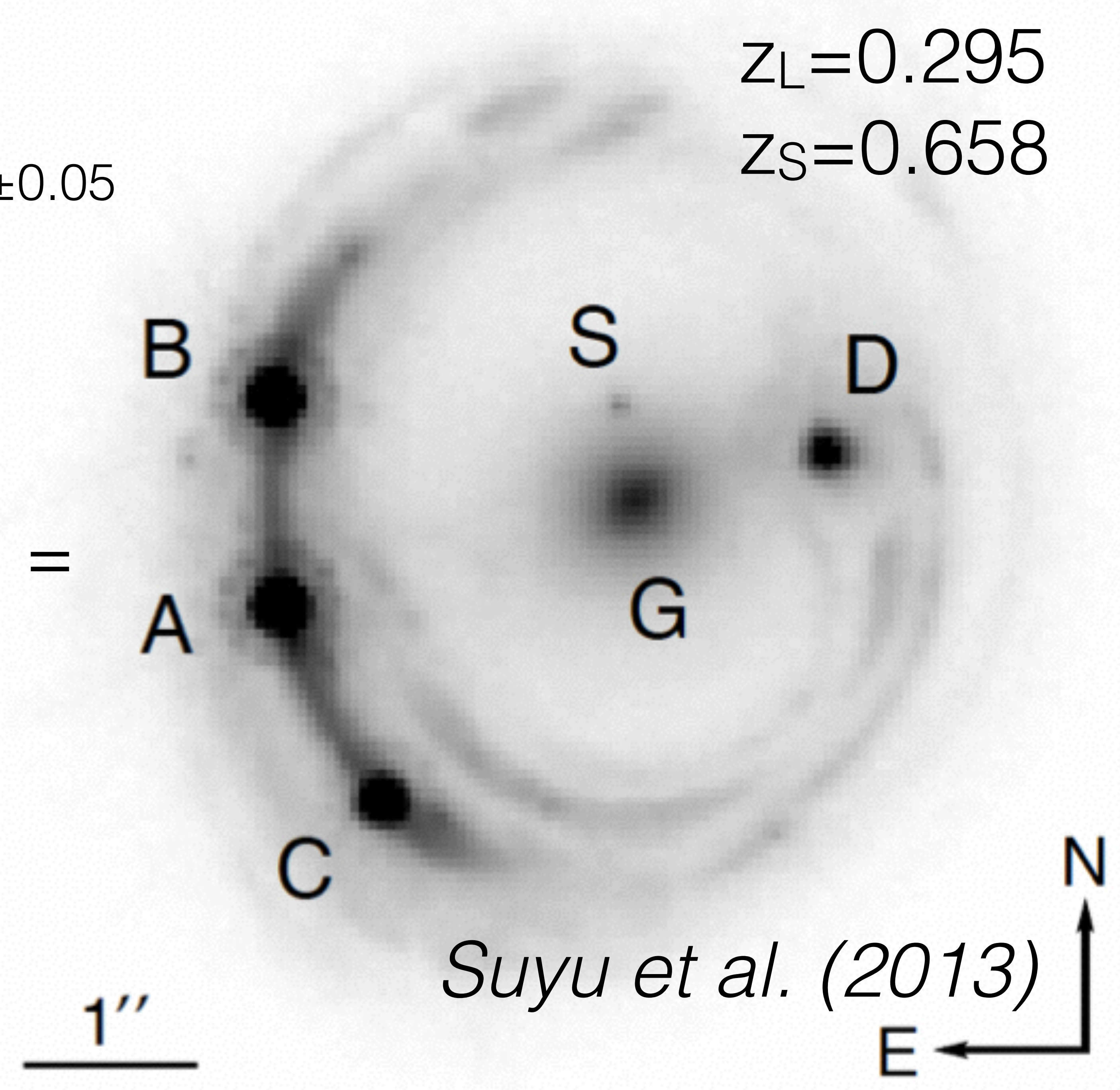


System 2: RXJ1131–1231

- The power-law mass density slope is $\rho \sim r^{-1.95\pm0.05}$
- R_{eff}=1.85 arcsec
- σ_s[averaged over 0.81”] = 323 ± 20 km/s
- Time delays:

$$\Delta t_{AB} = 0.7 \pm 1.4\ days$$
$$\Delta t_{DB} = 91.4 \pm 1.5\ days$$
$$\Delta t_{AD} = \Delta t_{AB} - \Delta t_{DB} = -90.7 \pm 2.1$$

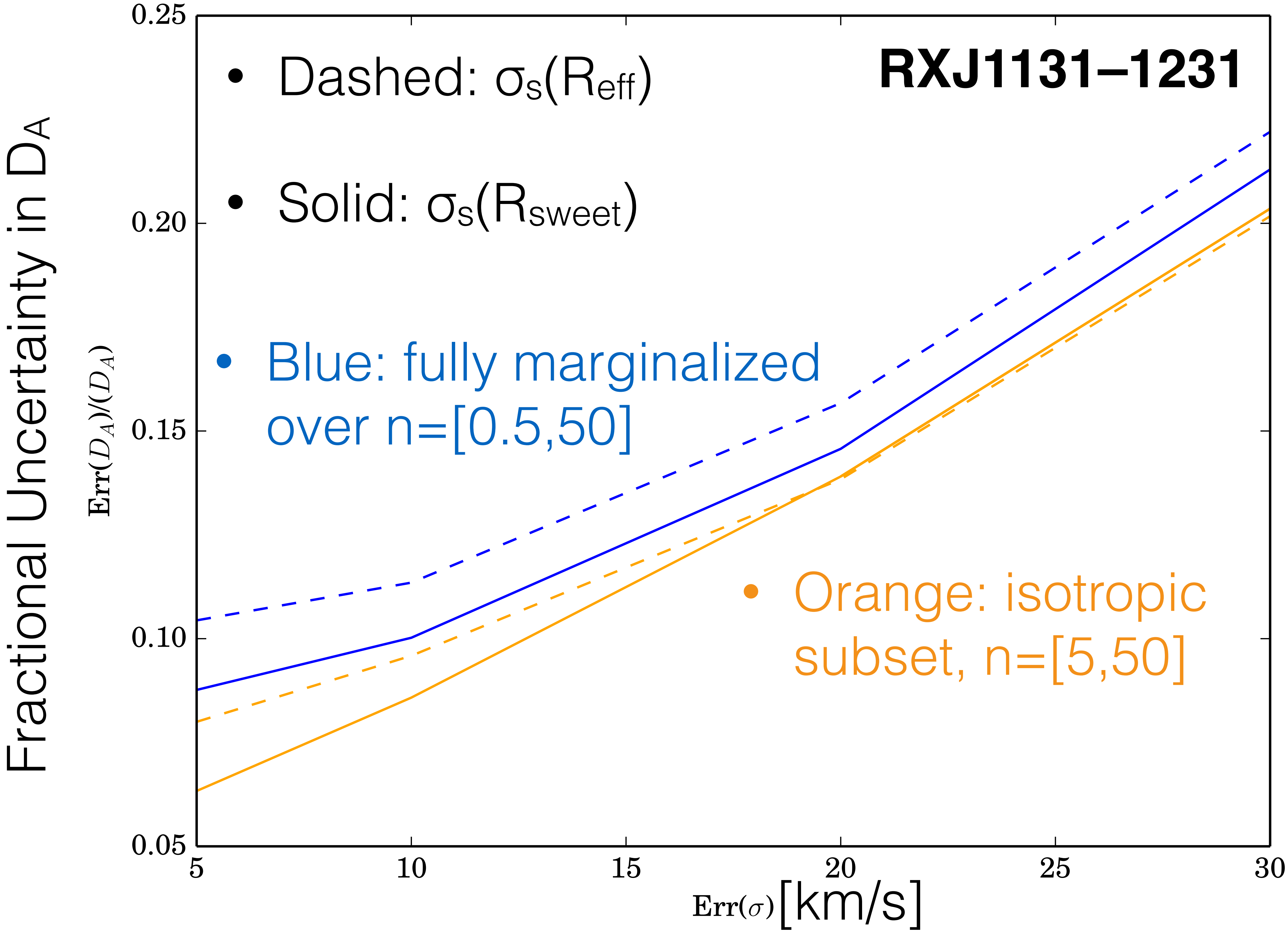
We will use only AD [for now]



z_L=0.295
z_S=0.658

Suyu et al. (2013)

Uncertainties: D_A vs σ_s



Running through Sherry's code: RXJ1131–1231

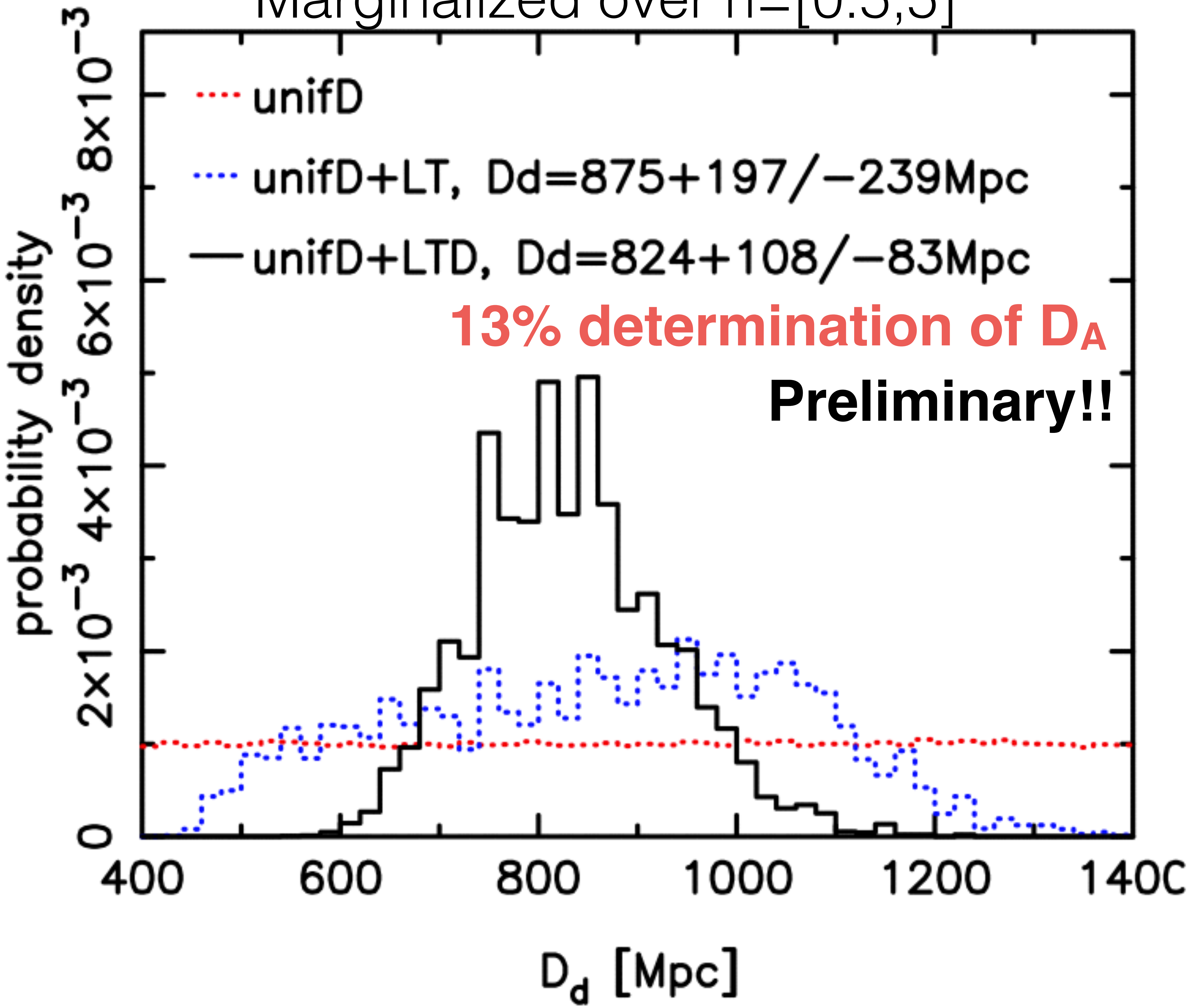
- So far, our analysis was simplified: only the uncertainties in the velocity dispersions were propagated, and a subset of images were used
  - We also assumed spherical lens mass distribution
- It turns out that the measurement of D_A is possible with a minimal modification to Sherry Suyu's code [about which you will hear more about on Thursday] which was extensively used for determining the time-delay distances to strong lens systems

Sherry’s code

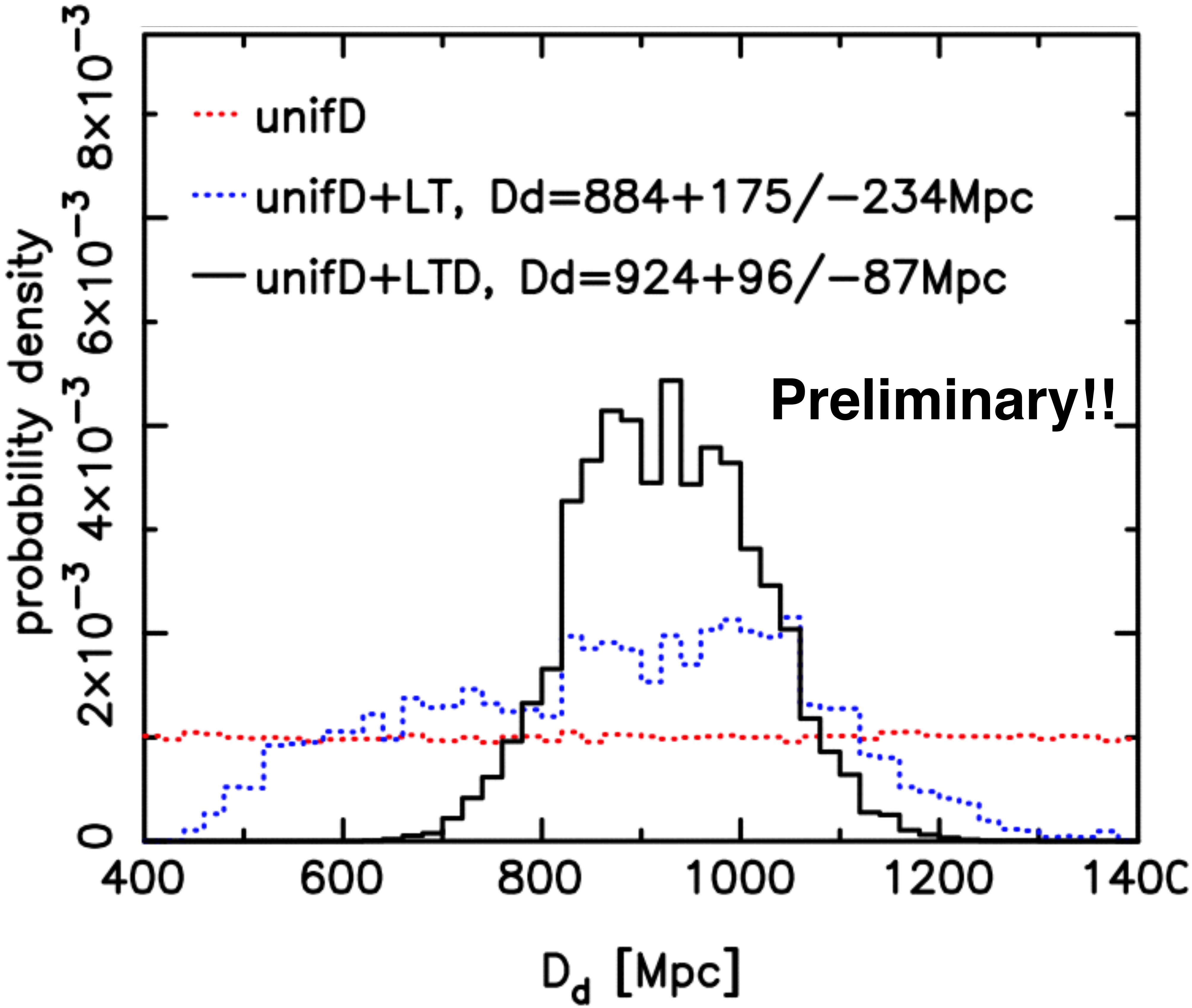
- Elliptical power-law mass distribution
- Use all images and time delays
- Marginalized over the power-law index, external convergence, and velocity anisotropy [with Osipkov-Merritt form]
- Sherry’s code shows that the inferred D_A’s are indeed independent of the external convergence due to a uniform mass sheet!

RXJ1131−1231

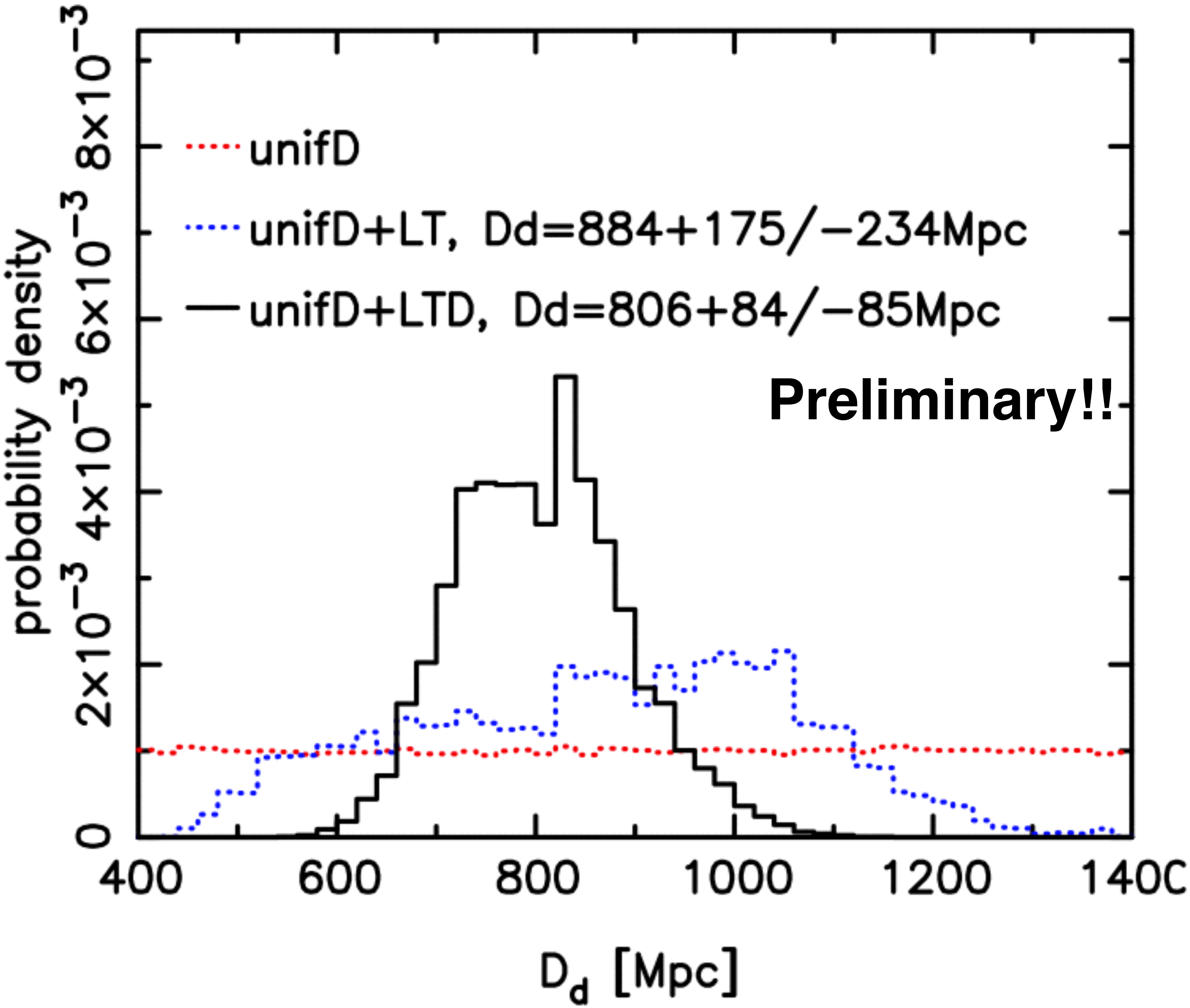
Marginalized over n=[0.5,5]



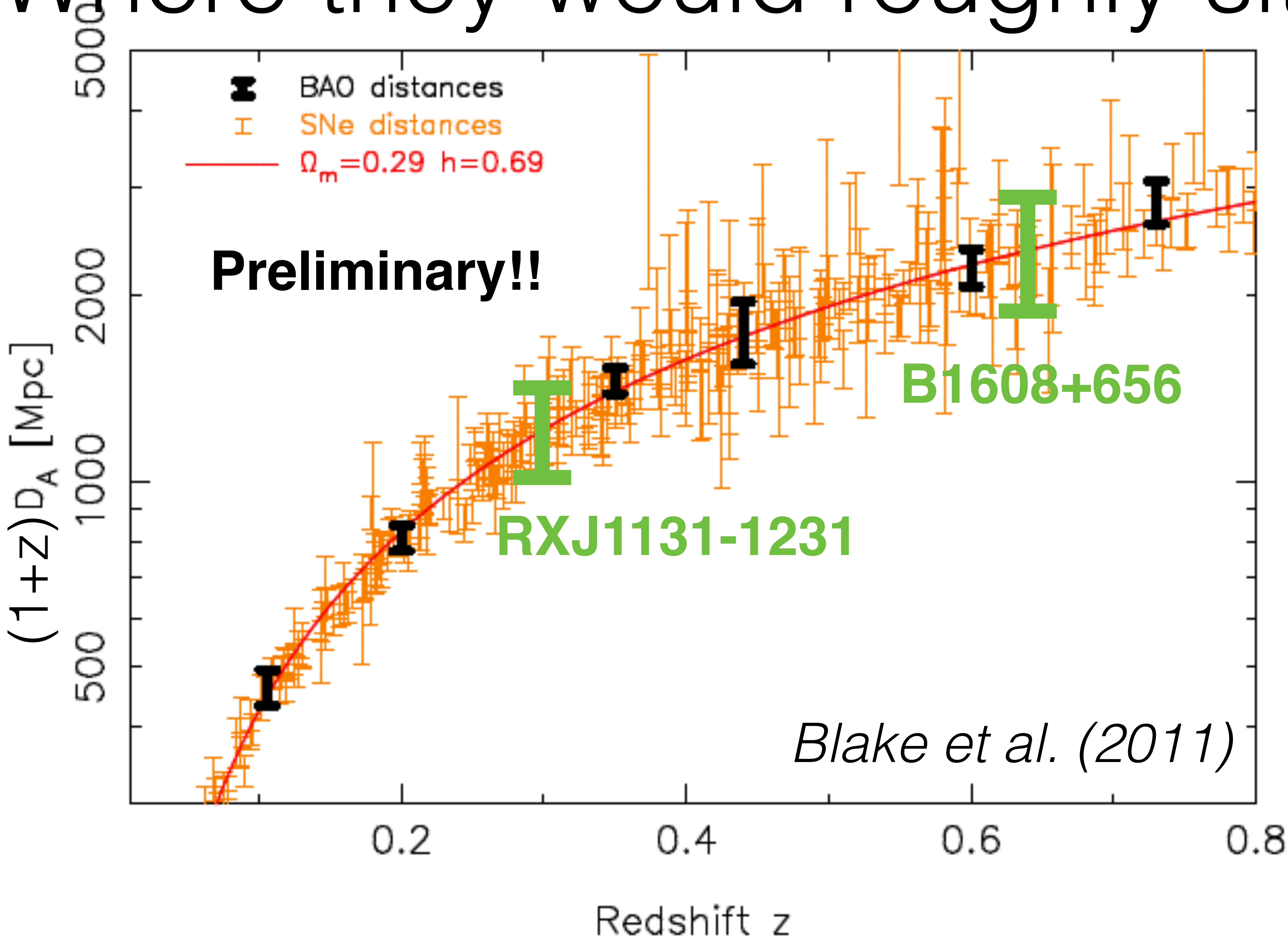
Marginalized over n=[0.5,1]



Marginalized over n=[2.5,5]



Strong Lenses: Where they would roughly sit



Summary

- Strong lenses can be used to measure the angular diameter distances!
  - D_A is independent on the external convergence
  - D_A is sensitive to anisotropy in the velocity dispersion, which must be marginalised over
- The current data (RXJ1131-1231 and B1608+656) can provide ~15% measurements of D_A at z=0.295 and 0.63
  - We can reduce the uncertainties in D_A by reducing the uncertainties in the velocity dispersion. E.g., ~10% precision is possible by halving Err[σ_s]

Discussion Topics

- Is it still interesting to determine D_A accurately up to $z{\sim}1$?

- How accurately can we determine the velocity dispersion? [Is 5 km/s possible?]

- How accurately can we determine the velocity **profile**?

- Is there a better way to reduce the uncertainty due to anisotropic velocity dispersion?