Testing, testing, and testing theories of Cosmic Inflation

Eiichiro Komatsu (MPA)
MPA Institute Seminar, October 13, 2014

Inflation, defined

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Rightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

- Accelerated expansion during the early universe

- Explaining flatness of our observable universe requires a **sustained** period of acceleration, which requires ε=O(N^{-1}) [or smaller], where N is the number of e-fold of expansion counted from the end of inflation:

$$N \equiv \ln \frac{a_{\mathrm{end}}}{a} = \int_t^{t_{\mathrm{end}}} dt' \; H(t') \approx 50$$

What does inflation do?

- **It provides a mechanism to produce the seeds for cosmic structures, as well as gravitational waves**
- Once inflation starts, it rapidly reduces spatial curvature of the observable universe. Inflation can solve the flatness problem
- But, starting inflation requires a patch of the universe which is homogeneous over a few Hubble lengths, and thus it does not solve the horizon problem (or homogeneity problem), contrary to what you normally learn in class

Nearly de Sitter Space

- When ε<<1, the universe expands quasi-exponentially.

- If ε=0, space-time is exactly de Sitter:

$$ds^2 = -dt^2 + e^{2Ht} d\mathbf{x}^2$$

- *But, inflation never ends if ε=0.* When ε<<1, space-time is nearly, but not exactly, de Sitter:

$$ds^2 = -dt^2 + e^{2\int dt' H(t')} d\mathbf{x}^2$$

Symmetry of de Sitter Space

$$ds^2 = -dt^2 + e^{2Ht} d\mathbf{x}^2$$

- De Sitter spacetime is invariant under 10 isometries (transformations that keep ds^2 invariant):

- Time translation, followed by space dilation

$$t \to t - \lambda/H \,, \quad \mathbf{x} \to e^{\lambda} \mathbf{x}$$

- Spatial rotation, $\mathbf{x} \to R\mathbf{x}$

- Spatial translation, $\mathbf{x} \to \mathbf{x} + c$

- Three more transformations irrelevant to this talk

ε≠0 breaks space dilation invariance

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Consequence: Broken Scale Invariance

- Symmetries of correlation functions of primordial fluctuations (such as gravitational potential) reflect symmetries of the background space-time

- Breaking of spacial dilation invariance implies that correlation functions are not invariant under dilation, either

- To study fluctuations, write the spatial part of the metric as

$$ds_3^2 = \exp\left[2\int H\,dt + 2\zeta(t,\mathbf{x})\right] d\mathbf{x}^2$$

Scale Invariance

- If the background universe is homogeneous and isotropic, the two-point correlation function, $\xi(\mathbf{x},\mathbf{x'})=\langle\zeta(\mathbf{x})\zeta(\mathbf{x'})\rangle$, depends only on the distance between two points, $r=|\mathbf{x}-\mathbf{x'}|$.
- The correlation function of Fourier coefficients then satisfy $\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k'}}^*\rangle=(2\pi)^3\delta(\mathbf{k}-\mathbf{k'})P(k)$
- They are related to each other by

$$\xi(r) = \int \frac{k^2dk}{2\pi^2}P(k)\frac{\sin(kr)}{kr}$$

Scale Invariance

$$\xi(r) = \int \frac{k^2 dk}{2\pi^2} P(k) \frac{\sin(kr)}{kr}$$

- Writing $P(k) \sim k^{ns-4}$, we obtain

$$\xi(r) \propto r^{1-n_s} \int \frac{d(kr)}{2\pi^2} (kr)^{n_s-1} \frac{\sin(kr)}{kr}$$

- Thus, under spatial dilation, $r \to e^{\lambda} r$, the correlation function transforms as

$$\xi(e^{\lambda} r) \to e^{\lambda(1-n_s)} \xi(r)$$

n_s=1 is called the "scale invariant spectrum".

Broken Scale Invariance

- Since inflation breaks spatial dilation by ε which is of order N^{-1}=0.02 (or smaller), n_s is different from 1 by the same order. This is a generic prediction of inflation

- This, combined with the fact that H decreases with time, typically implies that n_s is smaller than unity

✓ This has now been confirmed by WMAP and Planck with more than 5σ! **n_s=0.96: A major milestone in cosmology**

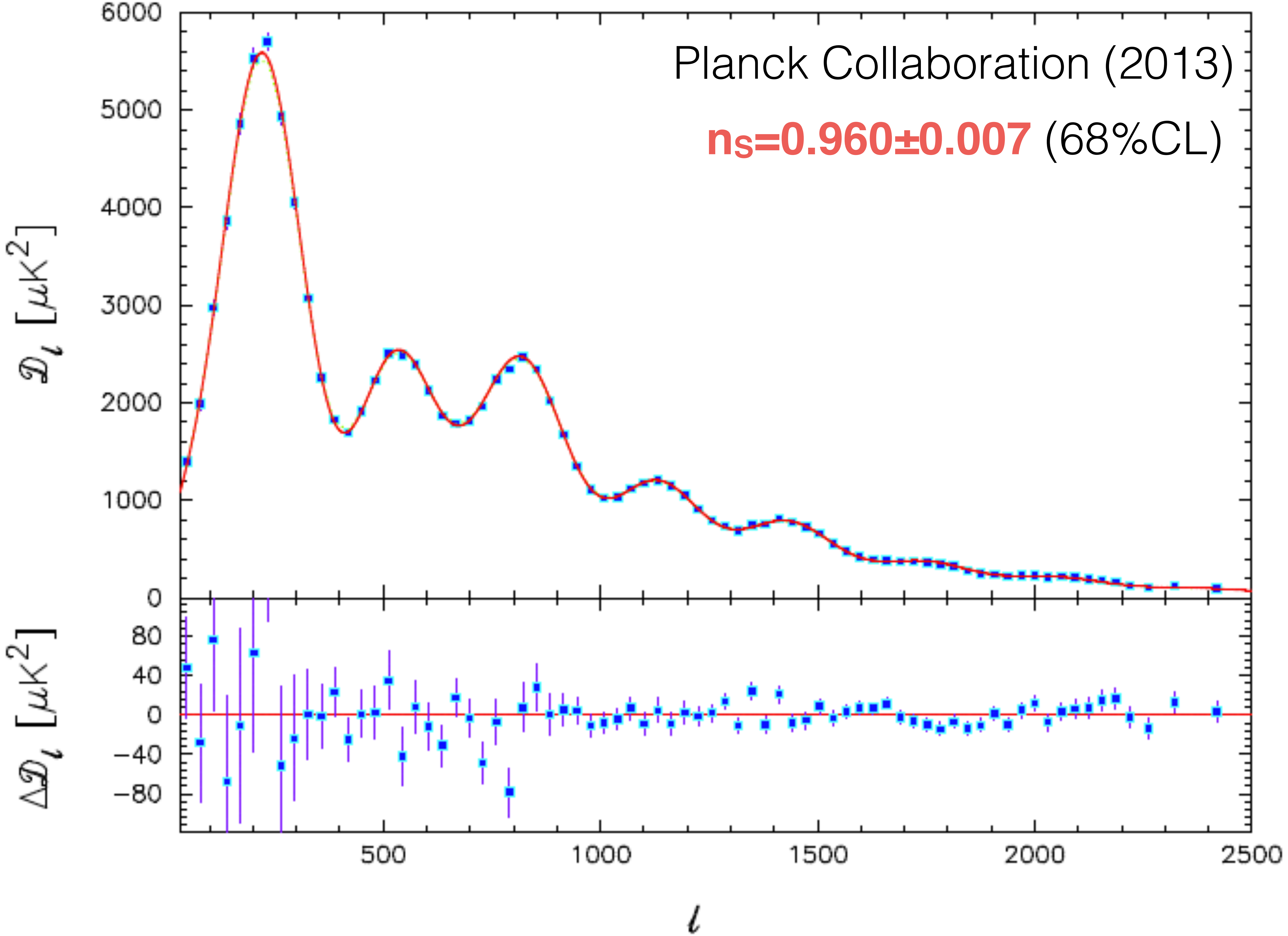
How it was done

- On large angular scales, the temperature anisotropy is related to ζ(**x**) via the Sachs-Wolfe formula as

$$\frac{\Delta T(\hat{n})}{T_0} = -\frac{1}{5}\zeta(\hat{n}r_*)$$

- On smaller angular scales, the acoustic oscillation and diffusion damping of photon-baryon plasma modify the shape of the power spectrum of CMB away from a power-law spectrum of ζ

$$C_\ell = \frac{2}{\pi}\int k^2 dk\ P(k) g_{T\ell}^2\,, \quad \ell(\ell+1)C_\ell \propto \ell^{n_s-1}$$



Planck Collaboration (2013)
ns=0.960±0.007 (68%CL)
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell$ [μK^2]
80
40
0
−40
−80
$\Delta\mathcal{D}_\ell$ [μK^2]
500
1000
1500
2000
2500
ℓ

Gaussianity

- The wave function of quantum fluctuations of an interaction-free field in vacuum is a Gaussian
- Consider a scalar field, ϕ. The energy density fluctuation of this field creates a metric perturbation, ζ. If ϕ is a free scalar field, its potential energy function, $U(\phi)$, is a quadratic function
- If ϕ drives the accelerated expansion, the Friedmann equation gives $H^2=U(\phi)/(3M_P^2)$. Thus, slowly-varying H implies slowly-varying $U(\phi)$.
    - Interaction appears at $d^3U/d\phi^3$. This is suppressed by ε

Gaussianity

- Gaussian fluctuations have vanishing three-point function. Let us define the "bispectrum" as $<\zeta_{\mathbf{k1}}\zeta_{\mathbf{k2}}\zeta_{\mathbf{k3}}>=(2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B(k_1,k_2,k_3)$
- Typical inflation models predict

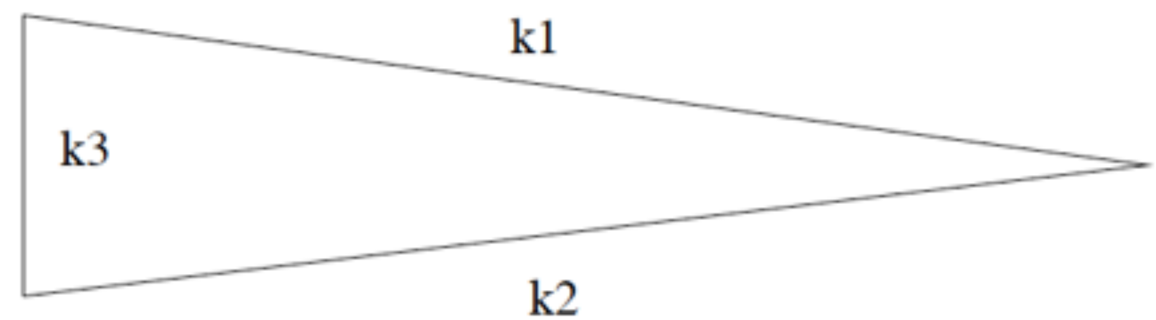
$$\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+\text{cyc.}}=\mathcal{O}(\epsilon)$$

  for any combinations of k_1, k_2, and k_3

- Detection of $B/P^2 >> \varepsilon$ implies more complicated models, or can potentially rule out inflation

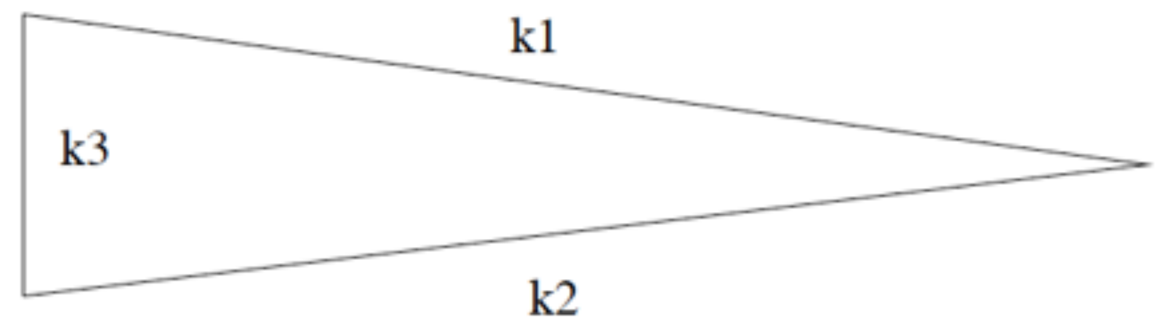
Single-field Theorem

- Take the so-called "squeezed limit", in which one of the wave numbers is much smaller than the other two, e.g., $k_3 << k_1 \sim k_2$



- A theorem exists: **IF**
  - Inflation is driven by a single scalar field,
  - the initial state of a fluctuation is in a preferred state called the Bunch-Davies vacuum, and
  - the inflation dynamics is described by an attractor solution, then…

Single-field Theorem



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  - the initial state of a fluctuation is in a preferred state called the Bunch-Davies vacuum, and
  - the inflation dynamics is described by an attractor solution, then…

$$\frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyc.}} \to \frac{1}{2}(1 - n_s)$$

Detection of $B/P^2 \gg \varepsilon$ in the squeezed limit rules out all single-field models satisfying these conditions

Current Bounds

- Let us define a parameter

$$\frac{6}{5} f_{NL} \equiv \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyc.}}$$

- The bounds in the squeezed configurations are
    - $f_{NL} = 37 \pm 20$ (WMAP9); $f_{NL} = 3 \pm 6$ (Planck2013)
- No detection in the other configurations

Simple single-field models fit the data!

Standard Picture

- Detection of n_s<1 and non-detection of non-Gaussianity strongly support the idea that **cosmic structures emerged from quantum fluctuations generated during a quasi de Sitter phase** in the early universe
- This is remarkable! But we want to test this idea more
- The next major goal is to detect primordial gravitational waves, but I do not talk about that. Instead…

Testing Rotational Invariance

- Kim & EK, PRD 88, 101301 (2013)
- Shiraishi, EK, Peloso & Barnaby, JCAP, 05, 002 (2013)
- Shiraishi, EK & Peloso, JCAP, 04, 027 (2014)
- Naruko, EK & Yamaguchi, to be submitted to JCAP

Rotational Invariance

$$ds^2 = -dt^2 + e^{2Ht}d\mathbf{x}^2$$

- De Sitter spacetime is invariant under 10 isometries (transformations that keep ds² invariant):

- Time translation, followed by space dilation

$$t \to t - \lambda/H\,, \quad \mathbf{x} \to e^{\lambda}\mathbf{x}$$

discovered in 2012/13

- Spatial rotation, $\mathbf{x} \to R\mathbf{x}$

Is this symmetry valid?

- Spatial translation, $\mathbf{x} \to \mathbf{x} + c$

- Three more transformations irrelevant to this talk

Anisotropic Expansion

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-2\beta(t)}dx^2 + e^{2\beta(t)}(dy^2 + dz^2)\right]$$

- How large can $\dot{\beta}/H$ be during inflation?
- In single scalar field theories, Einstein's equation gives

$$\dot{\beta} \propto e^{-3Ht}$$

- But, the presence of anisotropic stress in the stress-energy tensor can source a **sustained** period of anisotropic expansion:

$$T^i_j = P\delta^i_j + \pi^i_j \quad \text{with} \quad \pi^1_1 = -\frac{2}{3}\mathcal{V}, \quad \pi^2_2 = \pi^3_3 = \frac{1}{3}\mathcal{V}$$

$$\Rightarrow \quad \ddot{\beta} + 3H\dot{\beta} = \frac{1}{3}\mathcal{V}$$

Inflation with a vector field

- Consider that there existed a vector field at the beginning of inflation:

$$A_\mu = (0, u(t), 0, 0)$$

A_1: Preferred direction in space at the initial time

- You might ask where A_μ came from. Well, if we have a scalar field and a tensor field (gravitational wave), why not a vector?
- The conceptual problem of this setting is not the existence of a vector field, but that it requires A_1 that is homogeneous over a few Hubble lengths before inflation
  - But, this problem is common with the original inflation, which requires ϕ that is homogeneous over a few Hubble lengths, in order for inflation in occur in the first place!

Coupling ϕ to A_μ

- Consider the action:

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

- A vector field decays in an expanding universe, if "f" is a constant. The coupling pumps energy of ϕ into A_μ, which creates anisotropic stress, and thus sustains anisotropic expansion

$$\pi^1_1 = -\frac{2}{3}\mathcal{V}, \quad \pi^2_2 = \pi^3_3 = \frac{1}{3}\mathcal{V}$$

$$\rho_A = \frac{1}{2}\mathcal{V}, \quad P_A = \frac{1}{6}\mathcal{V}$$

where

$$\mathcal{V} \propto \frac{1}{f^2 e^{4(\alpha+\beta)}}$$

$$\alpha \equiv \int H dt$$

Watanabe, Kanno & Soda (2009,2010)

A Working Example

$$S = \int dx^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

- A choice of **$f=\exp(c\phi^2/2)$** [c is a constant] gives an interesting phenomenology
- [If you wonder: unfortunately, this model does not give you a primordial magnetic field strong enough to be interesting.]
- Let us define a convenient variable I, which is a ratio of the vector and scalar energy densities, divided by ε:

$$I \equiv 4\left(\frac{\partial_\phi U}{U}\right)^{-2}\frac{\rho_A}{U}$$

Slowly-varying function of time

Watanabe, Kanno & Soda (2009,2010)

Sketch of Calculations

- Decompose the metric, ϕ, and A_μ into the background and fluctuations
- There are 15 components (10 metric, 1 ϕ, and 4 A_μ), but only 5 are physical
- 2 of them are gravitational waves, which we do not consider. We are left with **three dynamical degrees of freedom**

$$\delta g_{\mu\nu} = \begin{pmatrix} -2A & e^{2(\alpha-2\beta)}B_x & e^{2(\alpha+\beta)}B_y & 0 \\ e^{2(\alpha-2\beta)}B_x & 2e^{2(\alpha-2\beta)}C & 0 & 0 \\ e^{2(\alpha+\beta)}B_y & 0 & 2e^{2(\alpha+\beta)}C & 0 \\ 0 & 0 & 0 & -2e^{2(\alpha+\beta)}C \end{pmatrix}$$

$$\delta\phi\,, \qquad \delta A_\mu = (\delta A_t\,, 0\,, \delta A_y\,, 0)$$

A, B_x, B_y, and δA_t are non-dynamical

Watanabe, Kanno & Soda (2009,2010)

Sketch of Calculations

- Expand the action

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$

up to second order in perturbations

$S^{(2)}$= [mess]

- This action gives the equations for motion of mode functions of fluctuations. Squaring the mode function of ϕ gives the power spectrum of ζ

Watanabe, Kanno & Soda (2010); Naruko, EK & Yamaguchi (prep)

Observational Consequence 1: Power Spectrum

- Broken rotational invariance makes the power spectrum depend on a direction of wavenumber

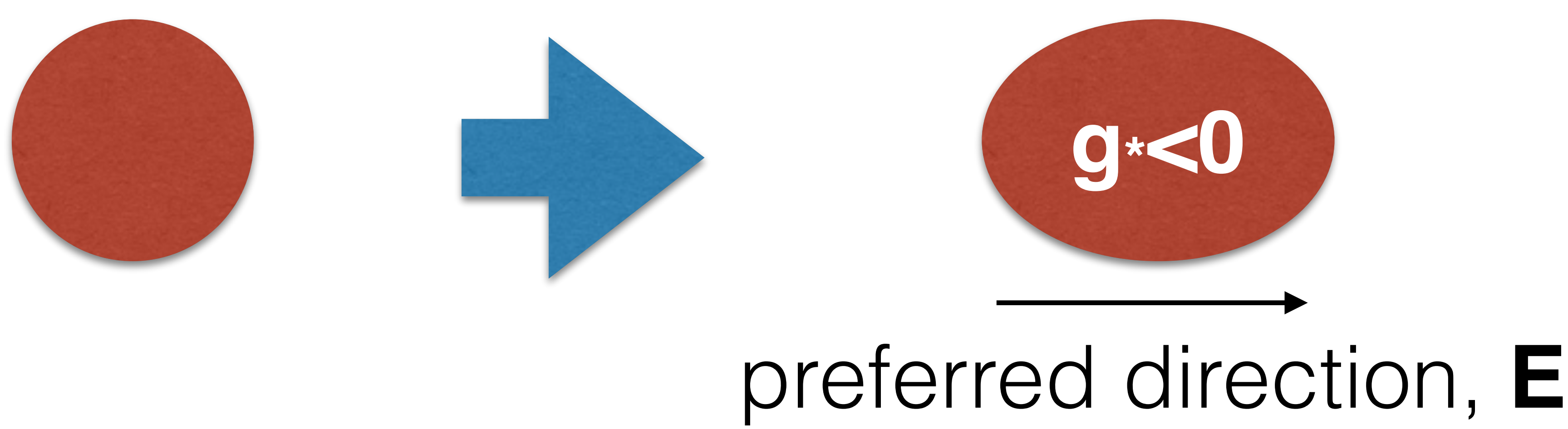
$$P(k) \to P(\mathbf{k}) = P_0(k)\left[1 + g_*(k)(\hat{k}\cdot\hat{E})^2\right]$$

where $\hat{E}$ is a preferred direction in space

- The model predicts: $g_*(k) = -\mathcal{O}(1) \times 24 I_k N_k^2$
- A "natural" (or maximal) value of I_k is O(1); thus, a natural value of $|g_*|$ is **either O(10^5) or zero**

Signatures in CMB

- Quadrupolar modulation of the power spectrum turns a circular hot/cold spot of CMB into an elliptical one



- This is a local effect, rather than a global effect: the power spectrum measured at *any location* in the sky is modulated by $(\hat{k} \cdot \hat{E})^2$

A Beautiful Story

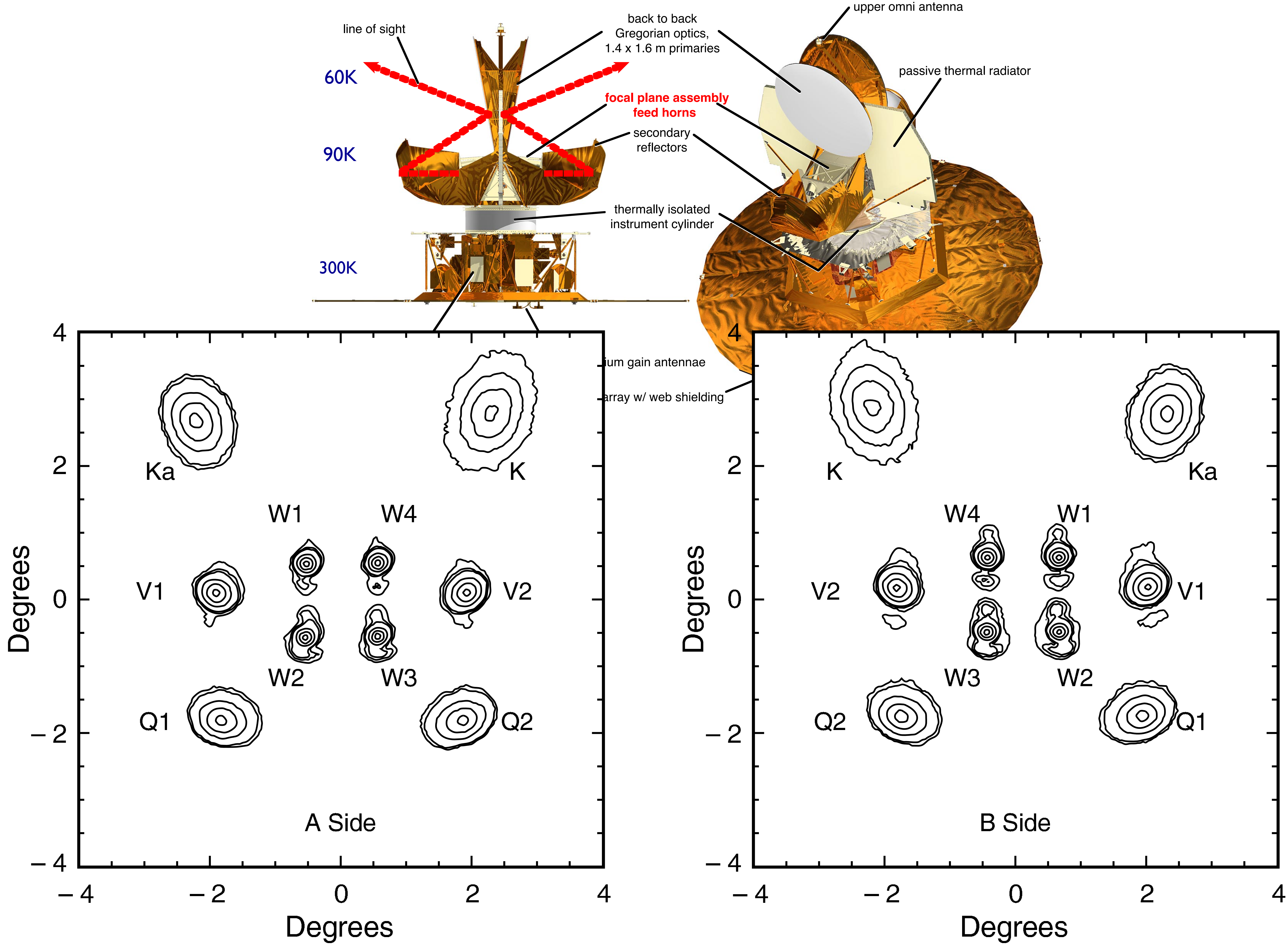
- In 2007, Ackerman, Carroll and Wise proposed g_* as a powerful probe of rotational symmetry

- In 2009, Groeneboom and Eriksen reported a significant detection, $g_*=0.15\pm0.04$, in the WMAP data at 94 GHz

- **Surprise!** And a beautiful story - a new observable proposed by theorists was looked for in the data, and was found

Subsequent Story

- In 2010, Groeneboom et al. reported that the WMAP data at 41 GHz gave the opposite sign: $g_* = -0.18 \pm 0.04$, suggesting instrumental systematics
- The best-fit preferred direction in the sky was the ecliptic pole
- Elliptical beam (point spread function) of WMAP was a culprit!



line of sight
back to back
Gregorian optics,
1.4 x 1.6 m primaries
upper omni antenna
passive thermal radiator
60K
90K
300K
focal plane assembly
feed horns
secondary
reflectors
thermally isolated
instrument cylinder
ium gain antennae
array w/ web shielding
Ka
K
W1
W4
V1
V2
W2
W3
Q1
Q2
A Side
K
Ka
W4
W1
V2
V1
W3
W2
Q2
Q1
B Side
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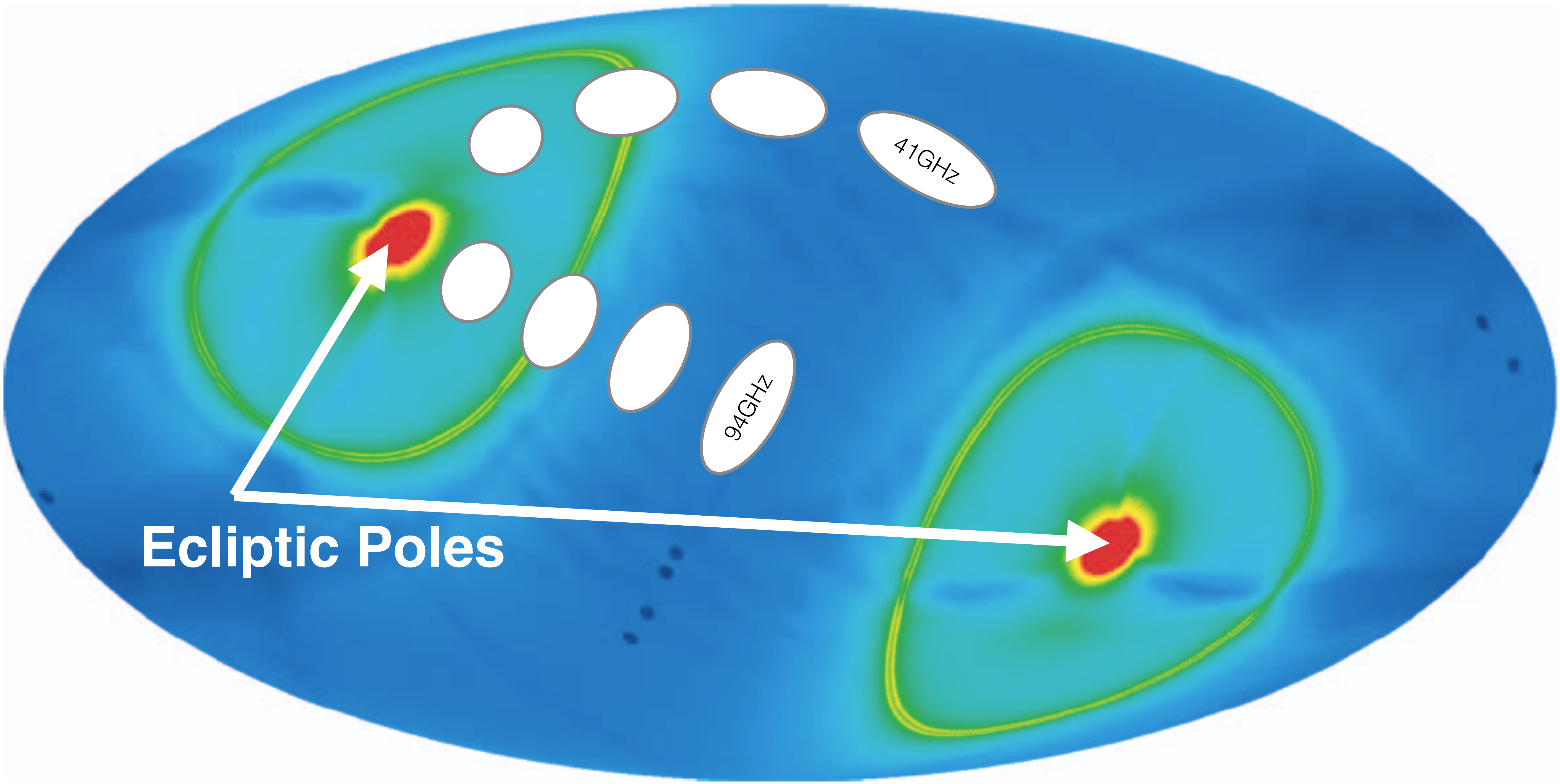
0
2
4

of observations in Galactic coordinates

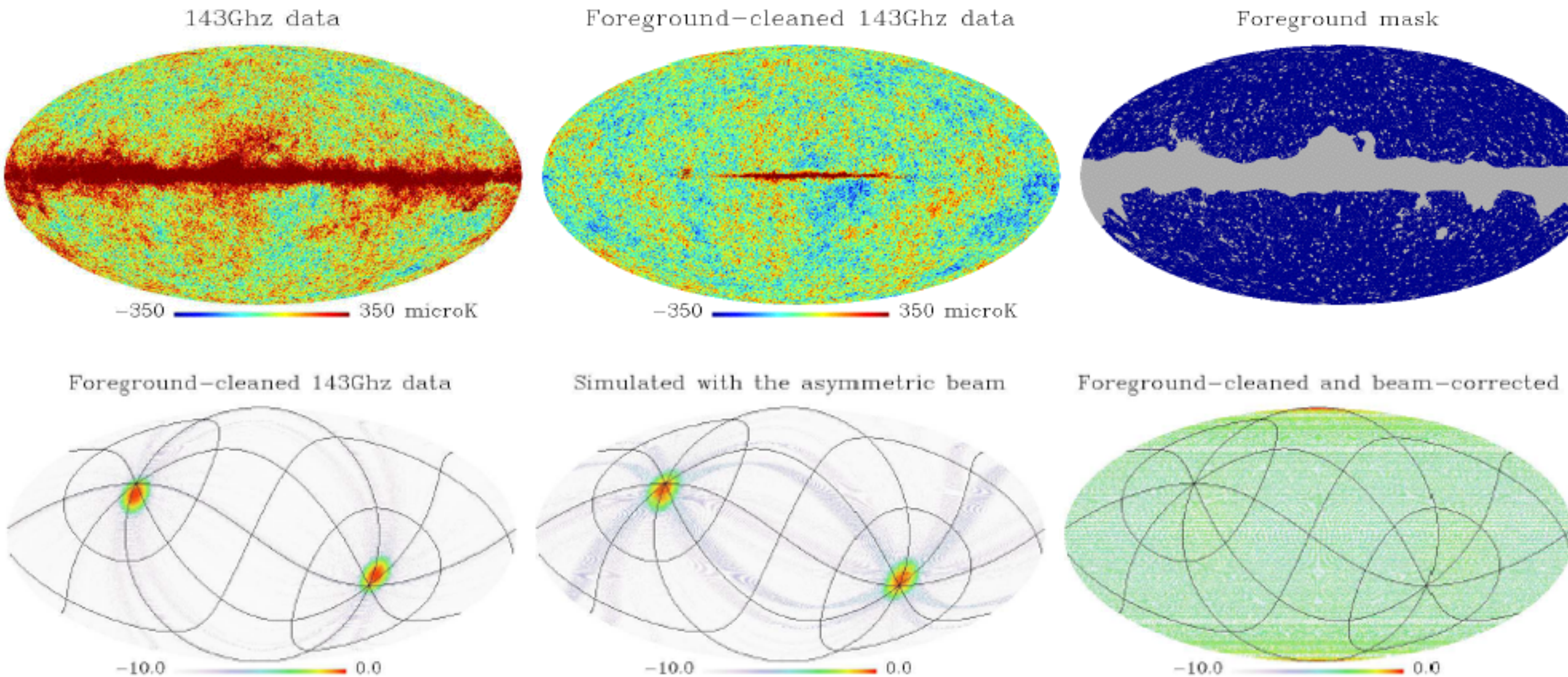


- WMAP visits ecliptic poles from many different directions, circularising beams
- WMAP visits ecliptic planes with 30% of possible angles

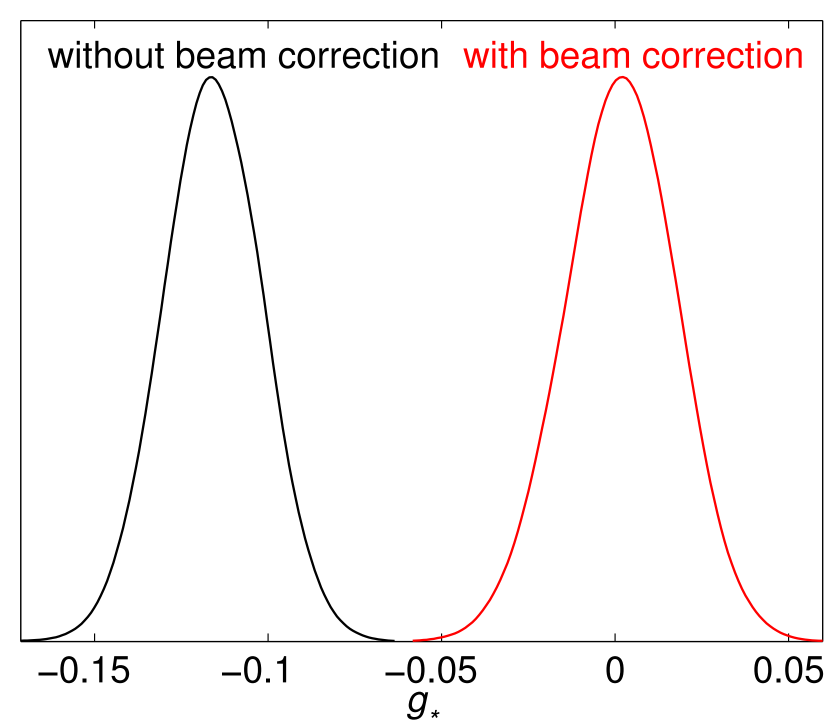
Planck 2013 Data

- With Jaiseung Kim (MPA), we analysed the Planck 2013 temperature data at 143GHz, and found significant $g_*=-0.111\pm0.013$ [after removing the foreground emission]
- This is consistent with what we expect from the beam ellipticity of the Planck data
- After subtracting the effect of beam ellipticities, no evidence for g_* was found

Kim & EK (2013)



g_*(raw)=–0.111±0.013 (68%CL)



g_*=0.002±0.016
(68%CL)

Naruko, EK & Yamaguchi (prep)

Implication for Rotational Symmetry

- g_* is consistent with zero, with 95%CL upper bound of $|g_*|<0.03$
- Comparing this with the model prediction, $|g_*| \sim 24IN^2$, we conclude $I<5\times10^{-7}$
- Thus, $$\frac{\dot{\beta}}{H} \approx \frac{\mathcal{V}}{U} \approx \epsilon I < 5 \times 10^{-9}$$

Breaking of rotational symmetry is tiny, if any!

[cf: "natural" value is either 10^{-2} or $e^{-3N}=e^{-150}$!!]

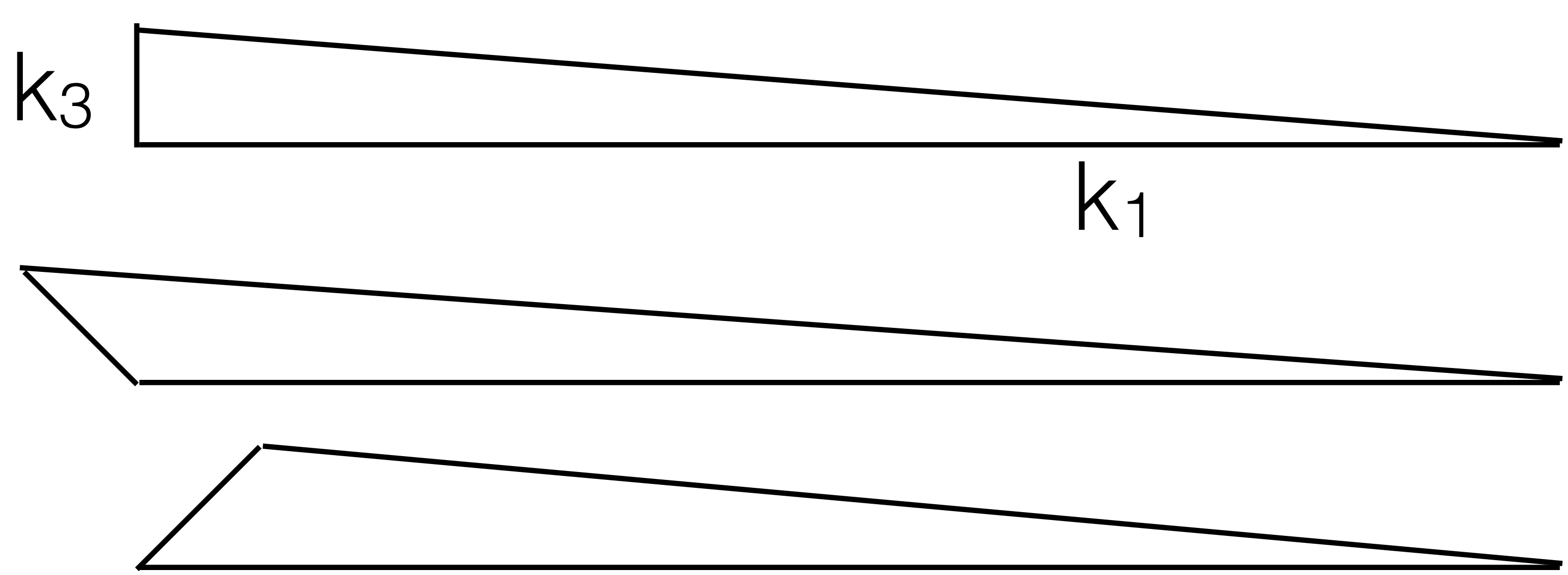
Shiraishi, EK, Peloso & Barnaby (2013)

Observational Consequence 2: Bispectrum

- The bispectrum depends on an angle between two wavenumbers. In the squeezed configuration:

$$B(k_1, k_2, k_3) = [c_0 + c_2 P_2(\hat{k}_1 \cdot \hat{k}_2)]P(k_1)P(k_2) + \text{cyc.}$$

where $P_2(x) = \frac{1}{2}(3x^2 - 1)$ is the Legendre polynomials



Bartolo et al. (2013)

Sketch of Calculations

- Expand the action

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$

up to third order in perturbations

$S^{(3)}$= [huge mess]

- This action gives the bispectrum of ζ, following the standard approach in the literature using the so-called in-in formalism

Shiraishi, EK, Peloso & Barnaby (2013)

Observational Consequence 2: Bispectrum

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$$B(k_1,k_2,k_3)=[c_0+c_2P_2(\hat{k}_1\cdot\hat{k}_2)]P(k_1)P(k_2)+\mathrm{cyc.}$$

- The f^2F^2 model predicts:

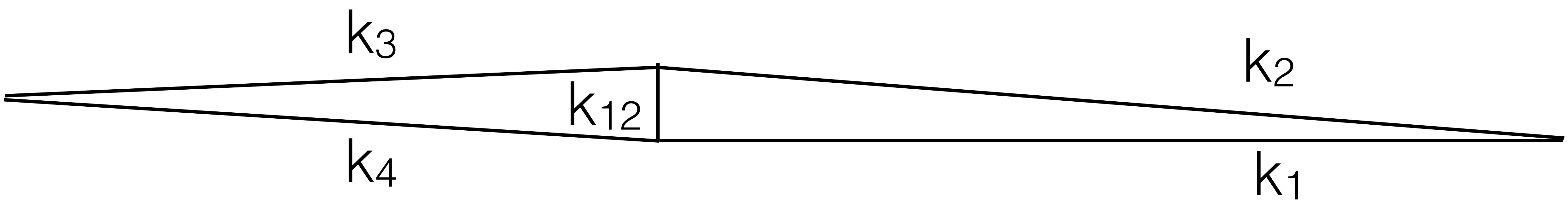
$$c_0=32\,\frac{|g_*(k_1)|}{0.1}\,\frac{N_{k_3}}{60}\quad,\quad c_2=\frac{c_0}{2}$$

- The Planck team finds: $\mathbf{c_2 = 4 \pm 28}$ [note: $c_0=6f_{NL}/5$]

Shiraishi, EK & Peloso (2014)

Observational Consequence 3: Trispectrum

- We can even consider the four-point function: $\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T(k_1,k_2,k_3,k_4,k_{12})$



$$T=\left\{3d_0+d_2\left[P_2(\hat{k}_1\cdot\hat{k}_3)+P_2(\hat{k}_1\cdot\hat{k}_{12})+P_2(\hat{k}_3\cdot\hat{k}_{12}\right]\right\}P(k_1)P(k_3)P(k_{12})$$

$$+23\ \mathrm{perm}$$

- The f^2F^2 model predicts: $d_2=2d_0\approx 14|g_*|N^2$

No constraints obtained yet

Summary

testing, testing [2003–2013]

- Anticipated broken scale invariance [hence broken time translational invariance] of order 10^{-2} finally found! Non-Gaussianity strongly constrained
    - These results support the quantum origin of structures in the universe

and testing [2013–present]

- Rotational invariance is respected during inflation with precision better than **5×10^{-9}**
    - Three- and four-point functions can also be used to test rotational invariance

Outlook

- **Testing the remaining predictions of inflation**
  - *Primordial gravitational waves*
    - Evidence reported in March by the BICEP2 team is pretty much gone now. We will keep searching!
  - *Spatial translation invariance*
    - No one cared to look for it in the data yet, but some theoretical work is being done (by others)