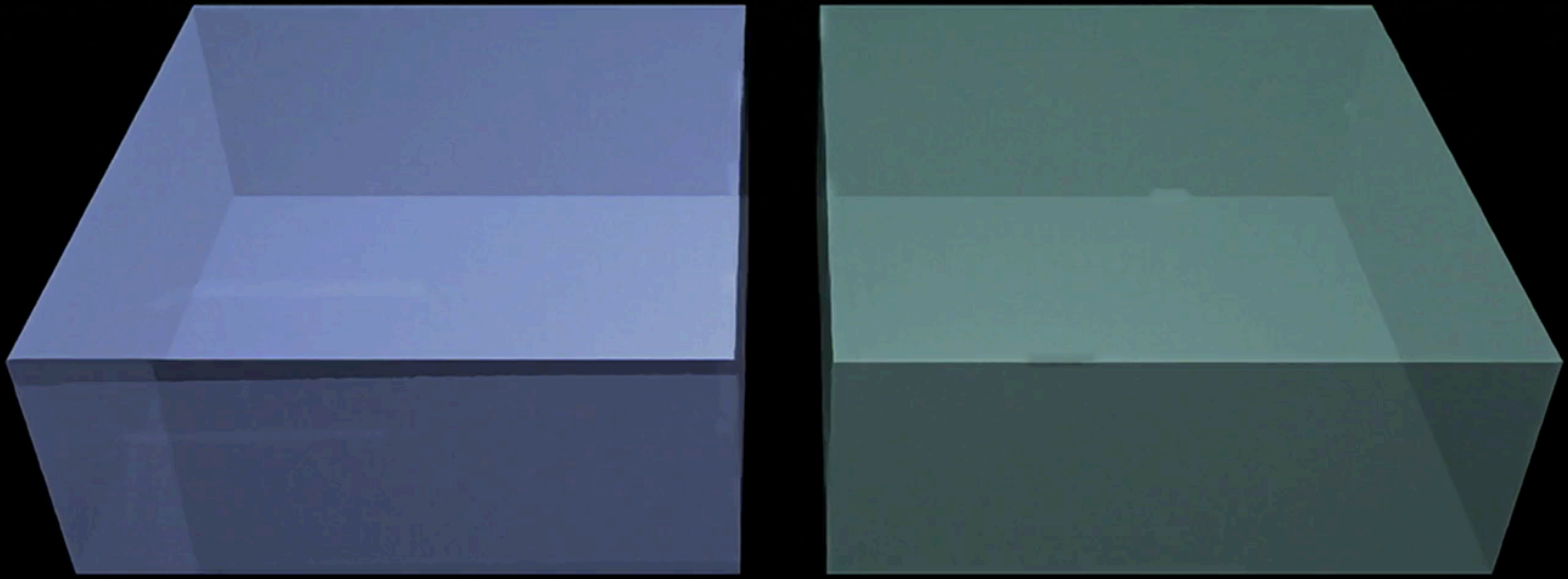
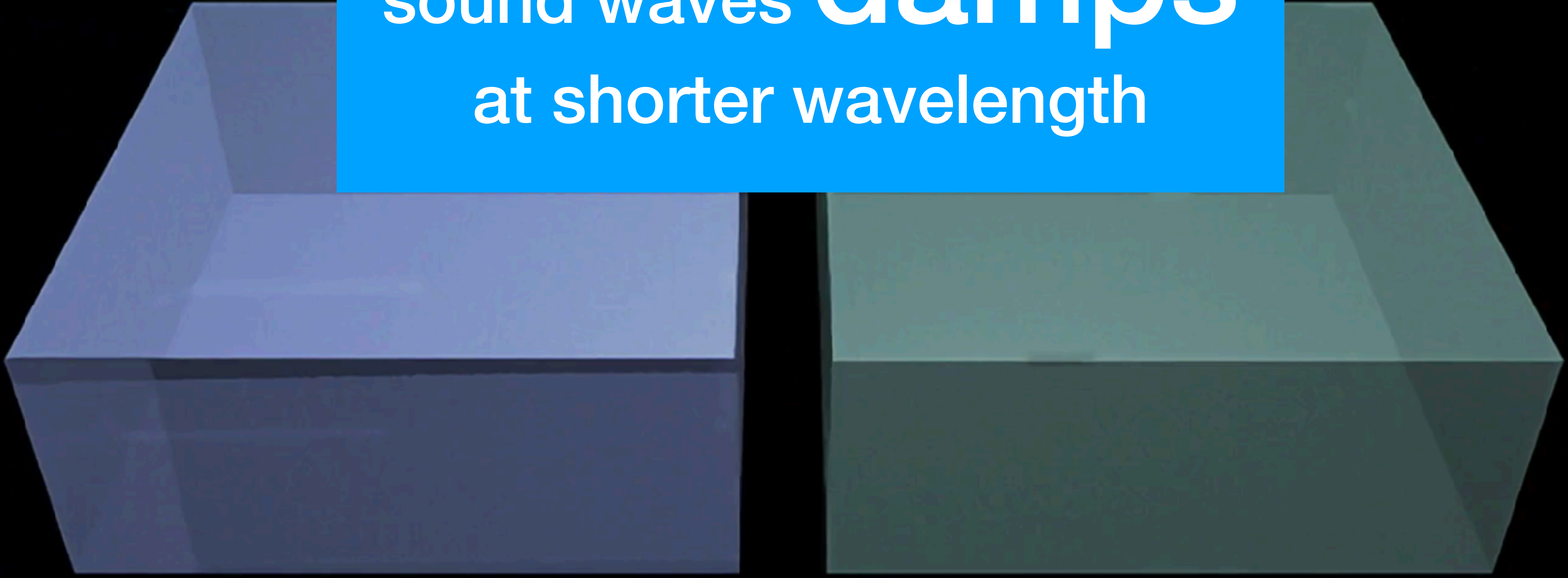
Lecture 5: Sound Waves in the Fireball Universe

Die kosmische Miso-Suppe

- When matter and radiation were hotter than 3000 K, matter was completely ionised. The Universe was filled with plasma, which behaves just like a soup
- Think about a Miso soup (if you know what it is). Imagine throwing Tofus into a Miso soup, while changing the density of Miso
- And imagine watching how ripples are created and propagate throughout the soup



This is a **viscous** fluid,
in which the amplitude of
sound waves **damps**
at shorter wavelength



Part I: Basics of Sound Waves

Getting a wave out of conservation equations

Surprisingly easy!

- What do you imagine when you hear the word “sound”?
  - In this lecture, the “sound wave” refers to a longitudinal (pressure) wave.
- The sound wave arises from two most important conservation equations in physics: **mass and momentum conservation**.
- Let’s work out the simplest possible case as a warm up.
  - Ideal fluid (no viscosity)
  - Non-relativistic
  - No gravity, no expansion of space

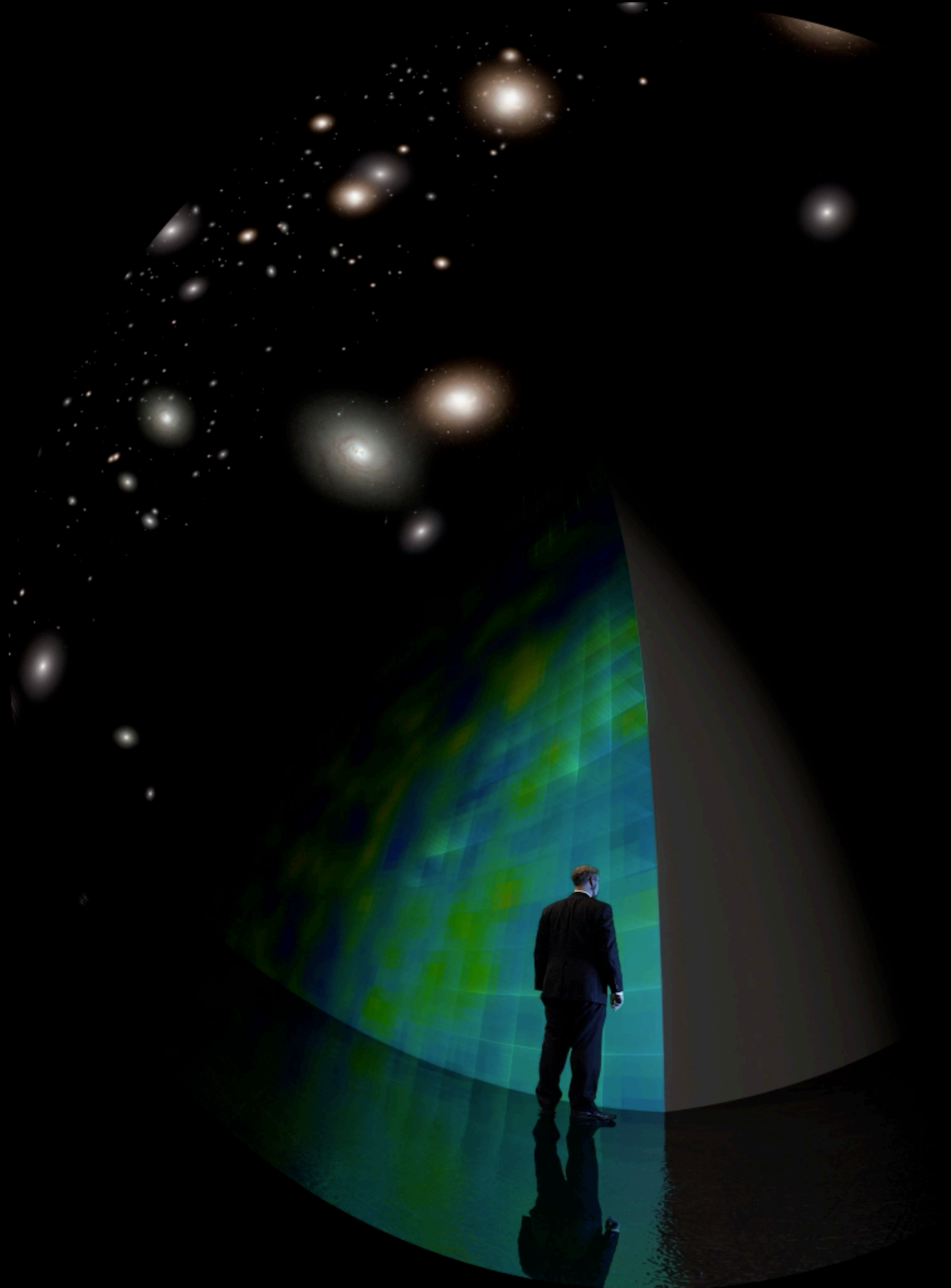
The mass and momentum conservation

The most important conservation equations in all physics

- Mass density and bulk velocity of a fluid element: $\rho = \rho(t, \mathbf{x}), \quad \mathbf{v} = \mathbf{v}(t, \mathbf{x})$
- Mass conservation (continuity equation) $\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$
- Momentum conservation (Euler equation) $\rho \frac{d\mathbf{v}}{dt} = \rho \left(\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\boxed{\nabla P}$

Pressure gradient

- Now, let the fun begin: linear perturbation analysis.
  - We perturb (displace) the fluid from its equilibrium configuration, in which ρ=const and **v**=0.



Linear perturbation analysis

The most powerful technique in all physics

- Let the system be in its equilibrium.
- Add a perturbation: How does the system respond to a perturbation?
    - Will it remain stable, or become unstable?
    - How does the (un)stable solution behave?
- *Advice*: When you start dealing with **any** physical system for the first time, do the linear perturbation analysis. This gives you a lot of physical insight into the system, and often allows you to write a (influential) paper!
    - E.g., Magneto-rotational instability (a.k.a. Balbus-Hawley instability)

Linear perturbation analysis of a perfect fluid

Without gravity

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = \rho \left(\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla P$$

- Add perturbations:

$$\rho = \bar{\rho} + \delta\rho(t, \mathbf{x})$$

$$\mathbf{v} = \nabla \delta u(t, \mathbf{x})$$ [δu: velocity potential]

We do not consider vorticity, which arises at second-order in perturbation.

$$P = \bar{P} + \delta P(t, \mathbf{x})$$

- **Keep the terms up to linear order in perturbation. Then…**

Linear perturbation analysis of a perfect fluid

Equation of state and the speed of sound

$$\delta\dot{\rho} + \bar{\rho}\nabla^2\delta u = 0$$

$$\bar{\rho}\delta\dot{u} = -\delta P$$

- We now need “equation of state”, i.e., how is pressure related density?
- For a “barotropic fluid”, $P = P(\rho)$, which is a useful approximation.
- Then,

$$\delta P = \frac{dP}{d\rho}\delta\rho \equiv c_s^2\delta\rho \quad \text{[}c_s\text{: speed of sound]}$$

- **Now, combine these 3 equations!**

Linear perturbation analysis of a perfect fluid

Tadaa - the sound wave!

$$\delta\ddot{\rho} - c_s^2\nabla^2\delta\rho = 0$$

- This is a wave equation!
  - To see this, Fourier transform $\delta\rho(t,\mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\delta\rho_{\mathbf{q}}(t)\exp(i\mathbf{q}\cdot\mathbf{x})$

$$\delta\ddot{\rho}_{\mathbf{q}} + c_s^2 q^2 \delta\rho_{\mathbf{q}} = 0$$

- **The solution:** $\delta\rho_{\mathbf{q}}(t) = A_{\mathbf{q}}\cos(qc_s t) + B_{\mathbf{q}}\sin(qc_s t)$

With gravity

Add a Newtonian potential gradient to the Euler equation

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = \rho\left(\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla P - \rho \boxed{\nabla \Phi}$$

Potential gradient

- The potential can be provided by either:
  1. The fluid itself (self-gravitating system), or
  2. Some external force

With gravity

Add a Newtonian potential gradient to the Euler equation

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

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Potential gradient

- The potential can be provided by either:

1. **The fluid itself (self-gravitating system):** $\nabla^2 \Phi = 4\pi G \delta\rho$

2. Some external force

With gravity

Add a Newtonian potential gradient to the Euler equation

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$$

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Potential gradient

- The potential can be provided by either:

1. The fluid itself (self-gravitating system), or
2. **Some external force:** $\nabla^2 \Phi = 4\pi G \delta\rho_{\text{other}}$

This is the most relevant case for this lecture because the gravitational force is dominated by cold dark matter, which does not form a sound wave.

Nonetheless, let's study #1 first because it is a very famous example.

Jeans (1902)

The Jeans Instability

PHILOSOPHICAL TRANSACTIONS.

I. *The Stability of a Spherical Nebula.*

By J. H. JEANS, *B.A., Fellow of Trinity College, and Isaac Newton Student in the University of Cambridge.*

Communicated by Professor G. H. DARWIN, *F.R.S.*

Received June 15,—Read June 20, 1901. Revised February 28, 1902.

INTRODUCTION.

§ 1. THE object of the present paper can be best explained by referring to a sentence which occurs in a paper by Professor G. H. DARWIN.* This is as follows:—

"The principal question involved in the nebular hypothesis seems to be the stability of a rotating mass of gas; but, unfortunately, this has remained up to now an untouched field of mathematical research. We can only judge of probable results from the investigations which have been made concerning the stability of a rotating mass of liquid."

The Jeans Instability

Instability of a self-gravitating cloud of gas

$$\delta\dot{\rho} + \bar{\rho}\nabla^2\delta u = 0$$

$$\bar{\rho}\delta\dot{u} = -\delta P - \bar{\rho}\Phi$$

$$\delta P = \frac{dP}{d\rho}\delta\rho \equiv c_s^2\delta\rho$$

$$\nabla^2\Phi = 4\pi G\delta\rho$$

- **OK, let's go!**

The Jeans Instability

Instability of a self-gravitating cloud of gas

$$\delta\dot{\rho} + \bar{\rho}\nabla^2\delta u = 0$$

$$\bar{\rho}\delta\dot{u} = -\delta P - \bar{\rho}\Phi$$

$$\delta P = \frac{dP}{d\rho}\delta\rho \equiv c_s^2\delta\rho$$

$$\nabla^2\Phi = 4\pi G\delta\rho$$

$$\delta\ddot{\rho} + \left(-c_s^2\nabla^2 - 4\pi G\bar{\rho}\right)\delta\rho = 0$$

Fourier transform

$$\delta\rho(t, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\delta\rho_{\mathbf{q}}(t)\exp(i\mathbf{q}\cdot\mathbf{x})$$

$$\delta\ddot{\rho}_{\mathbf{q}} + \left(c_s^2 q^2 - 4\pi G\bar{\rho}\right)\delta\rho_{\mathbf{q}} = 0$$

- This is an important minus sign: **Instability for $c_s^2 q^2 < 4\pi G\rho$**

The Jeans Instability

Instability of a self-gravitating cloud of gas

$$\delta\ddot{\rho}_{\mathbf{q}} + \left(c_s^2 q^2 - 4\pi G\bar{\rho}\right)\delta\rho_{\mathbf{q}} = 0$$

- This is an important minus sign:
  Instability for $c_s^2 q^2 < 4\pi G\rho$

- Two regimes:

1. Short wavelength (large q): Stability, with

   Sound wave: oscillating solutions

   $$\delta\rho_{\mathbf{q}} = A_{\mathbf{q}}\exp(ic_s qt) + B_{\mathbf{q}}\exp(-ic_s qt)$$

2. Long wavelength (small q): Instability, with

   Gravitational collapse: exponential growth

   $$\delta\rho_{\mathbf{q}} = C_{\mathbf{q}}\exp(\omega t) + D_{\mathbf{q}}\exp(-\omega t)$$

   $$\omega \equiv \sqrt{4\pi G\bar{\rho}}$$

Let's include more ingredients!

ads

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QUICK FIELD: Author First Author Abstract Year Fulltext All Search Terms

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author:"^balbus" author:"hawley"

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A Powerful Local Shear Instability in Weakly Magnetized Disks. I. Linear Analysis

Show affiliations

Balbus, Steven A.; Hawley, John F.

A broad class of astronomical accretion disks is presently shown to be dynamically unstable to axisymmetric disturbances in the presence of a weak magnetic field, an insight with consequently broad applicability to gaseous, differentially-rotating systems. In the first part of this work, a linear analysis is presented of the instability, which is local and extremely powerful; the maximum growth rate, which is of the order of the angular rotation velocity, is independent of the strength of the magnetic field. Fluid motions associated with the instability directly generate both poloidal and toroidal field components. In the second part of this investigation, the scaling relation between the instability's wavenumber and the Alfven velocity is demonstrated, and the independence of the maximum growth rate from magnetic field strength is confirmed.

Publication: Astrophysical Journal v.376, p.214

Pub Date: July 1991

Balbus & Hawley (1991)

2.2 *Axisymmetric Dispersion Relation:* $B_R = 0$

Consider an axisymmetric accretion disk of finite vertical extent, not necessarily thin. Set up a standard cylindrical coordinate system (R, ϕ, z) with R being the perpendicular distance from the z-axis. We assume that the equilibrium angular velocity $\Omega(R)$ is constant on cylinders, by tall other flow variables may depend upon R and z if permitted by the magneto-fluid equations. A magnetic field is presumed to be present in the disk, weak enough that in the initially unperturbed state its effect is quite negligible. Differential rotation will cause the equilibrium field to acquire a helical structure, and the presence of a radial component of the magnetic field together with shear will cause the azimuthal component to grow linearly with time. This leads to no great difficulties, but let us nevertheless begin our study with the special case of vanishing radial field component, $B_R = 0$. We then return to treat the more general case by building on the results of this slightly artificial but highly illustrative example.

We denote the azimuthal field component $B_\phi(R, z)\hat{\boldsymbol{\phi}}$, and the vertical component $B_z(R)\hat{\boldsymbol{z}}$. (The notation $\hat{\boldsymbol{\phi}}$, etc. is used to denote a unit vector.) The basic dynamical equations are

You should now be familiar with these equations. The new ingredient is the magnetic field, but the basic concept is the same:
The conservation equations plus extra ingredients

$$\frac{d \ln \rho}{dt} + \nabla \cdot \boldsymbol{v} = 0\,, \tag{2.1a}$$

$$\frac{d\boldsymbol{v}}{dt} + \frac{1}{\rho}\nabla\left(P + \frac{B^2}{8\pi}\right) - \frac{1}{4\pi\rho}(\boldsymbol{B}\cdot\nabla)\boldsymbol{B} + \nabla\Phi = 0\,, \tag{2.1b}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0\,. \tag{2.1c}$$

The notation d/dt indicates the Lagrangian derivative and Φ is the external gravitational potential. Others symbols have their usual meanings.

We consider axisymmetric large-wavenumber Eulerian perturbations with space-time dependence $e^{i(k_R R + k_z z - \omega t)}$. Subscripts refer to vector components. Fourier amplitudes of perturbed flow attributes are denoted as $\delta\rho$, δP, etc. We shall work in the Boussinesq approximation, which is appropriate for the noncompressive disturbances of interest. This eliminates magnetoacoustic waves from consideration, and greatly simplifies the bookkeeping. When written out in component form and only the largest terms retained, the above set of seven equations becomes to linear order

$$k_R\, \delta v_R + k_z\, \delta v_z = 0\,, \tag{2.2a}$$

$$-i\omega\, \delta v_R + \frac{ik_R}{\rho}\,\delta P - 2\Omega\, \delta v_\phi - \frac{\delta\rho}{\rho^2}\frac{\partial P}{\partial R} + \frac{ik_R}{4\pi\rho}(B_\phi\, \delta B_\phi + B_z\, \delta B_z) - \frac{ik_z}{4\pi\rho} B_z\, \delta B_R = 0\,, \tag{2.2b}$$

…equations continue…

We are now done yet

- We need to add a few more ingredients:
  - Expansion of the Universe
  - Viscosity (departure from an ideal fluid)
  - Interaction between electrons and photons (via Thomson scattering)
  - Coulomb interaction between electrons, protons and helium nuclei

- But don't worry: we will not include magnetic fields. If you insist, see:

K. Jedamzik, V. Katalinic, and A. V. Olinto, *Damping of cosmic magnetic fields*, *Phys.Rev.* **D57** (1998) 3264–3284, [astro-ph/9606080].

K. Subramanian and J. D. Barrow, *Magnetohydrodynamics in the early universe and the damping of noninear Alfven waves*, *Phys.Rev.* **D58** (1998) 083502, [astro-ph/9712083].

Part II: Sound Horizon

When do sound waves become important?

Sound horizon

- When would the Sachs-Wolfe approximation (purely gravitational effects) become invalid?
- The key to the answer: **Sound-crossing Time**
- Sound waves cannot alter temperature anisotropy at a given angular scale if there was not enough time for sound waves to propagate to the corresponding distance at the last-scattering surface
    - The distance traveled by sound waves within a given time = **The Sound Horizon**

Photon Horizon and Sound Horizon

The difference is the speed of sound.

- First, the comoving distance traveled by photons is given by setting the space-time distance to be null:

$$ds_4^2 = -c^2 dt^2 + a^2(t) dr^2 = 0$$

$$\Rightarrow \quad r_{\rm photon} = c \int_0^t \frac{dt'}{a(t')}$$

- Then, we replace the speed of light with a time-dependent speed of sound:

$$\Rightarrow \quad r_s = \int_0^t \frac{dt'}{a(t')} c_s(t')$$

- We cannot ignore the effects of sound waves for **$qr_s > 1$**

Sound Speed

- Sound speed of an adiabatic fluid is given by

$$c_s^2 = \delta P / \delta \rho$$

- δP: pressure perturbation
- δρ: density perturbation

- For a baryon-photon system:

$$c_s^2 = \delta P_\gamma / (\delta \rho_\gamma + \delta \rho_B)$$

We can ignore the baryon pressure because it is much smaller than the photon pressure

Sound Speed

- Using the adiabatic relationship between photons and baryons:

$$\delta\rho_B/\bar{\rho}_B = \delta\rho_\gamma/(\bar{\rho}_\gamma + \bar{P}_\gamma) = 3\delta\rho_\gamma/4\bar{\rho}_\gamma$$

[i.e., the ratio of the number densities of baryons and photons is equal everywhere]

and pressure-density relation of a relativistic fluid, **$\delta P_\gamma = \delta\rho_\gamma/3$**, we obtain

$$c_s^2 = \delta P_\gamma/(\delta\rho_\gamma + \delta\rho_B) = 1/3(1 + 3\bar{\rho}_B/4\bar{\rho}_\gamma)$$

- Or equivalently (light speed is c=1)

$$c_s = \frac{1}{\sqrt{3(1+R)}}$$

sound speed is reduced!

where

$$R \equiv 3\bar{\rho}_B/4\bar{\rho}_\gamma$$

The value of R?

- The baryon mass density goes like $[a(t)]^{-3}$, whereas the photon energy density goes like $[a(t)]^{-4}$. Thus, the ratio of the two, R, goes like $a(t)$.
- The proportionality constant is:

$$R = \frac{3\Omega_B}{4\Omega_\gamma}\frac{a}{a_0} = 0.6120\left(\frac{\Omega_B h^2}{0.022}\right)\frac{1091}{1+z}$$

where we used

$$\Omega_\gamma \equiv \frac{8\pi G\rho_{\gamma 0}}{3H_0^2} = 2.471\times 10^{-5}\ h^{-2} \quad \textbf{for}\ T_0 = 2.725\ \mathrm{K}$$

Cosmological Parameters

- Unless stated otherwise, we shall assume a **spatially-flat Λ Cold Dark Matter** (ΛCDM) model with

$$\Omega_B h^2 = 0.022$$ Baryon density

$$\Omega_M h^2 = 0.14$$ Total matter density

$$\Omega_M = 0.3$$

which implies:

Cosmological Constant

$$\Omega_\Lambda = 0.7, \quad \Omega_D h^2 = 0.118, \quad \Omega_B = 0.04714$$

Dark matter density

$$H_0 = 100\ h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}; \quad H_0 = 68.31\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

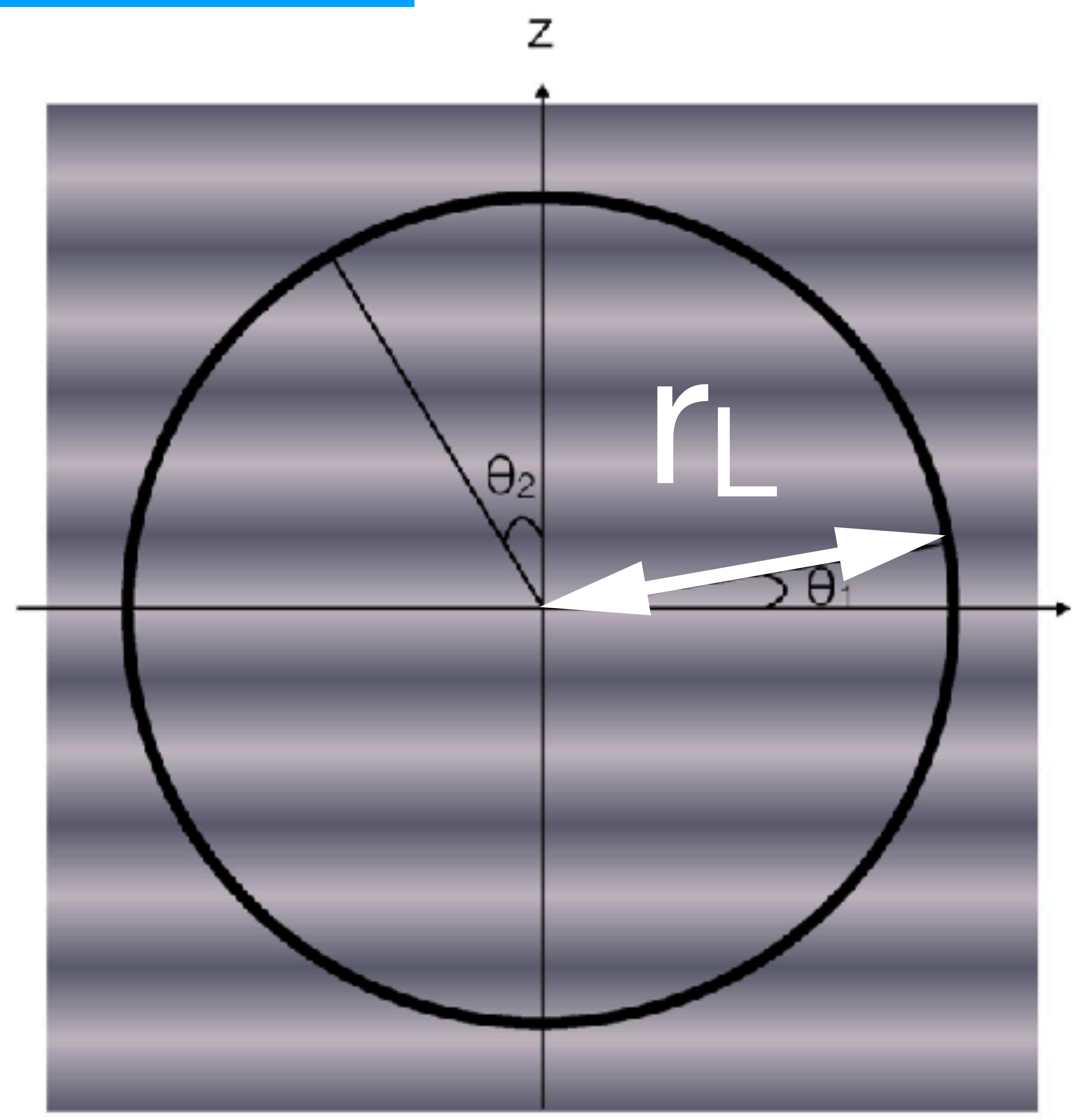
For the last-scattering redshift of z_L=1090
(or last-scattering temperature of T_L=2974 K),

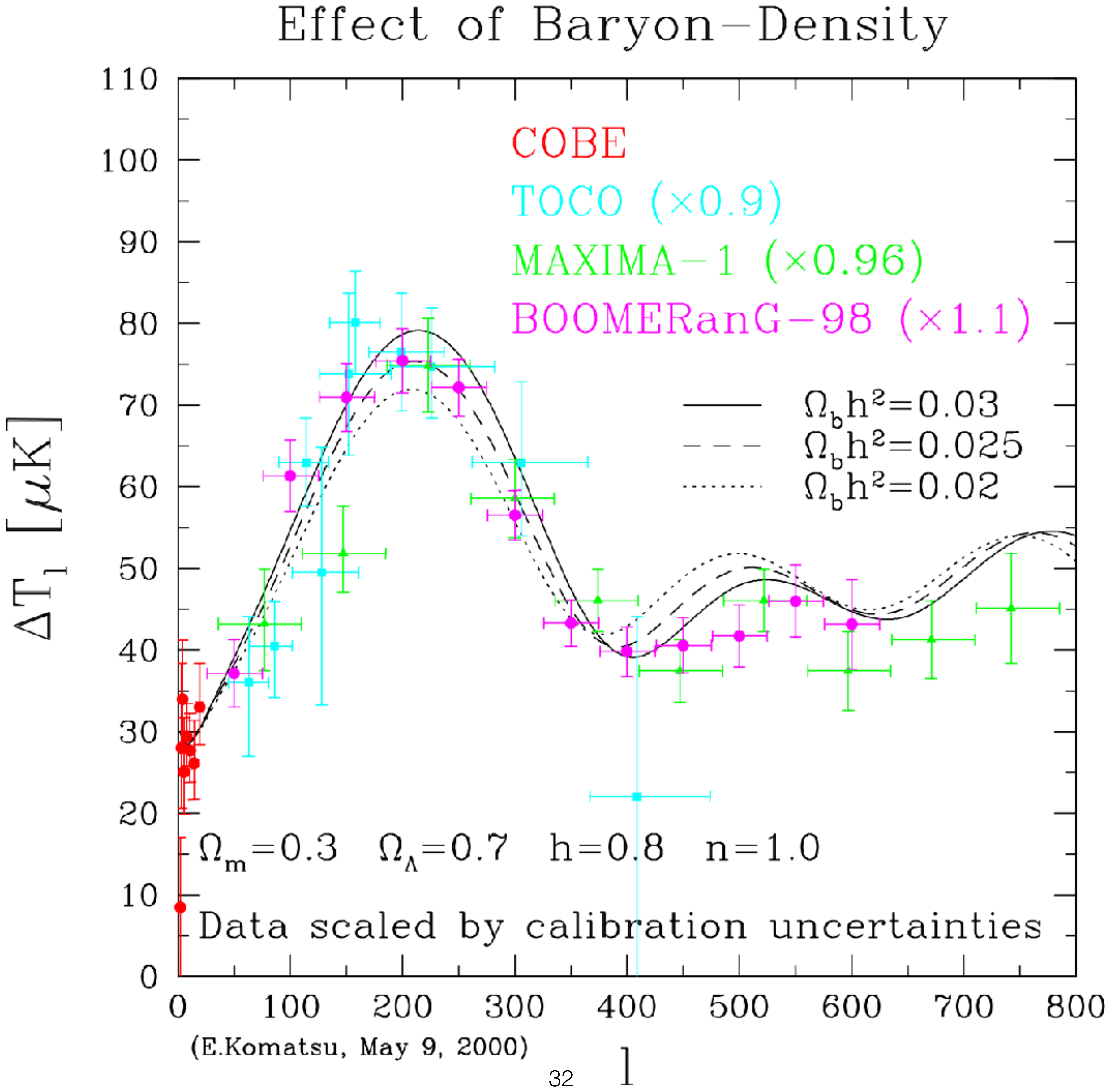
$$a_0 r_s = 145.3 \text{ Mpc}$$

We cannot ignore the effects of sound waves if qr_s>1. Since l~qr_L, this means

$$l > r_L/r_s = 96$$

where we used $a_0 r_L$=13.95 Gpc





Effect of Baryon−Density
COBE
TOCO (×0.9)
MAXIMA−1 (×0.96)
BOOMERanG−98 (×1.1)
Ω_b h²=0.03
Ω_b h²=0.025
Ω_b h²=0.02
Ω_m=0.3 Ω_Λ=0.7 h=0.8 n=1.0
Data scaled by calibration uncertainties
ΔT_l [μK]
l
(E.Komatsu, May 9, 2000)