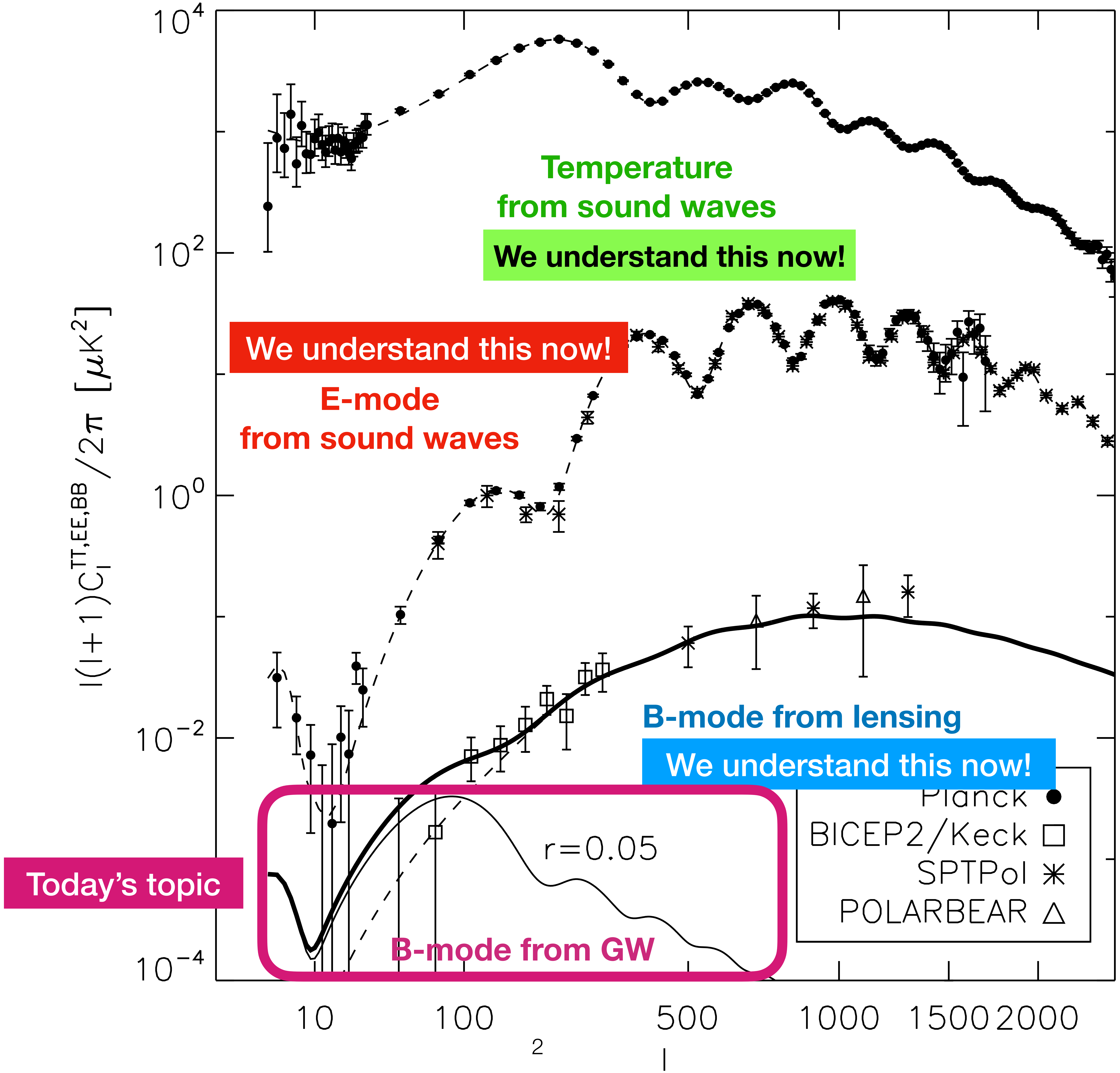
The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Lecture 10: Gravitational Waves

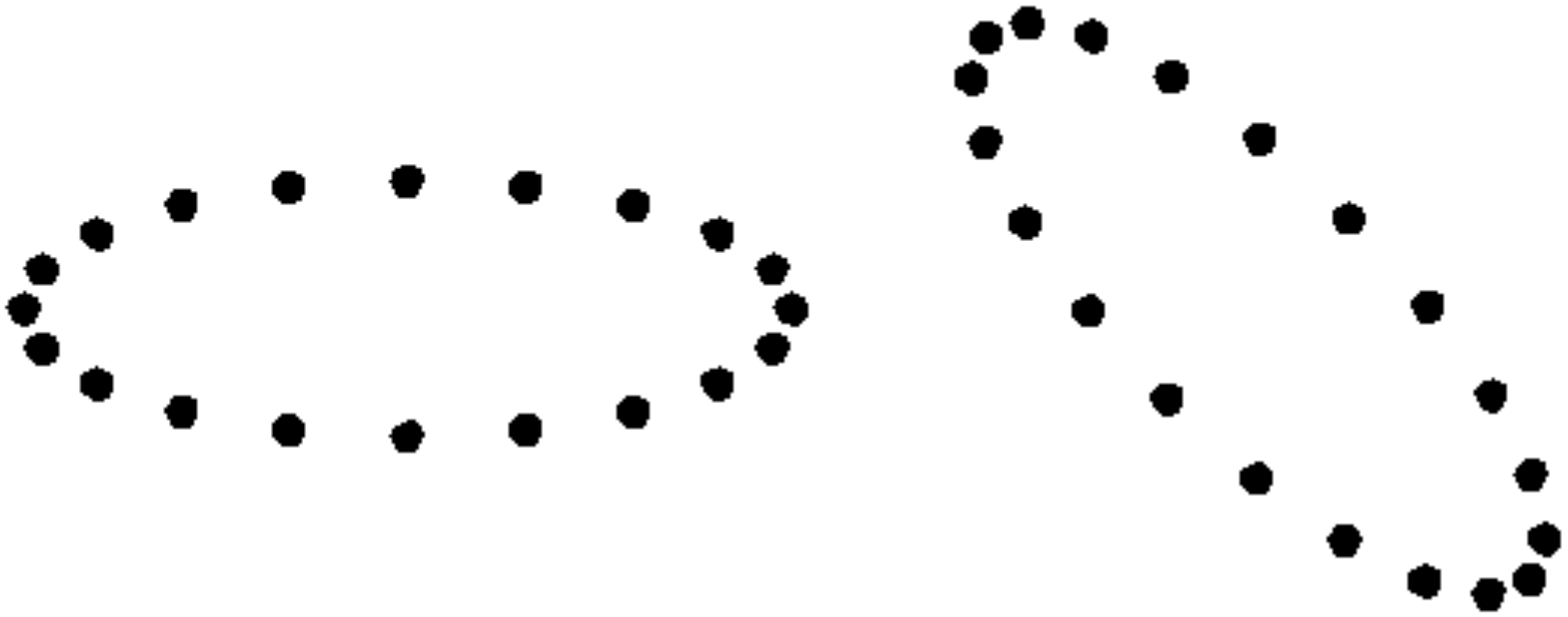


Temperature
from sound waves
We understand this now!
We understand this now!
E-mode
from sound waves
B-mode from lensing
We understand this now!
Today's topic
B-mode from GW
r=0.05
Planck
BICEP2/Keck
SPTPol
POLARBEAR
10
100
500
1000
1500
2000

Part I: Basics of the Gravitational Waves

Gravitational waves are coming towards you!

To visualise the waves, watch motion of test particles.



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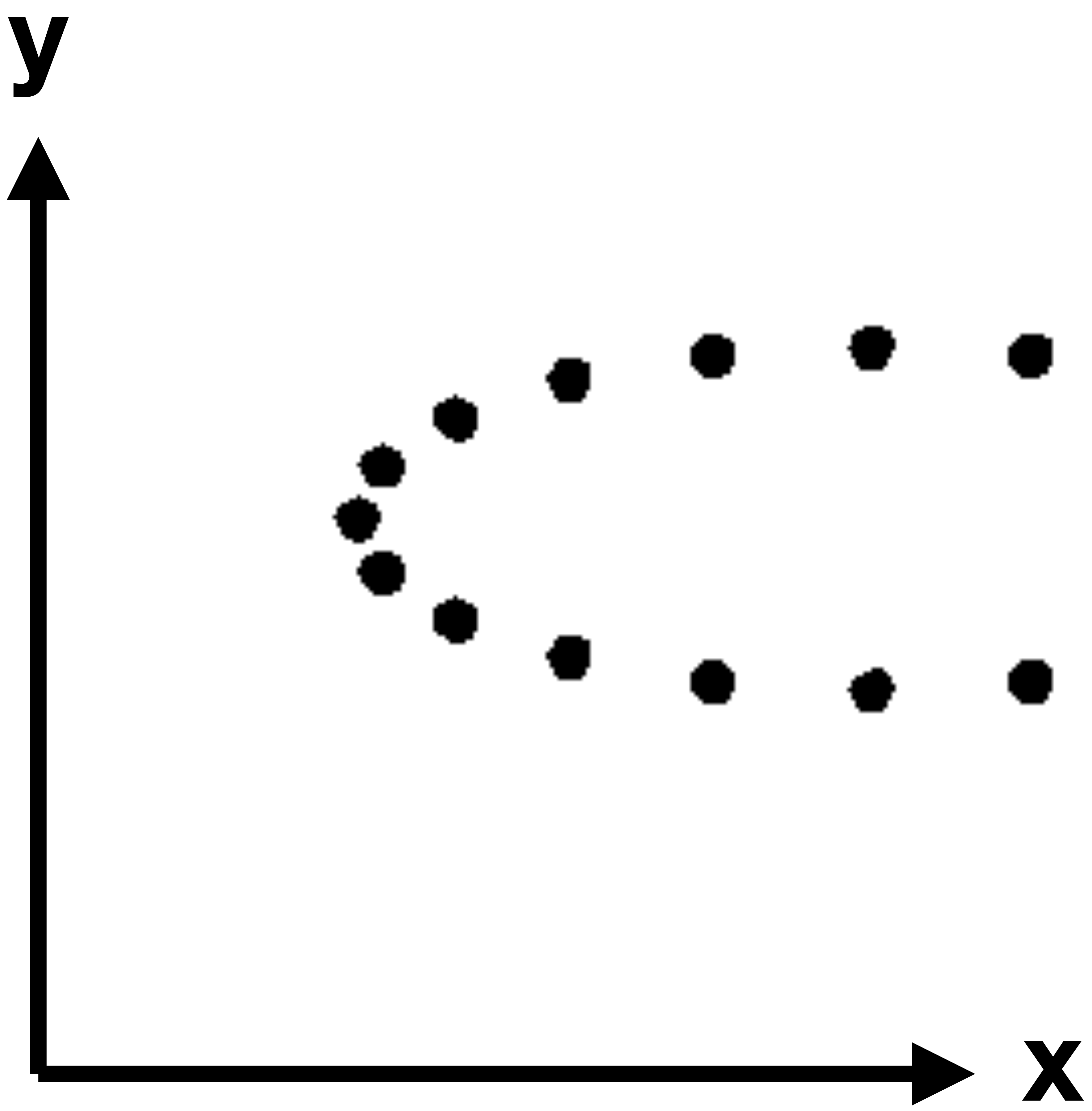
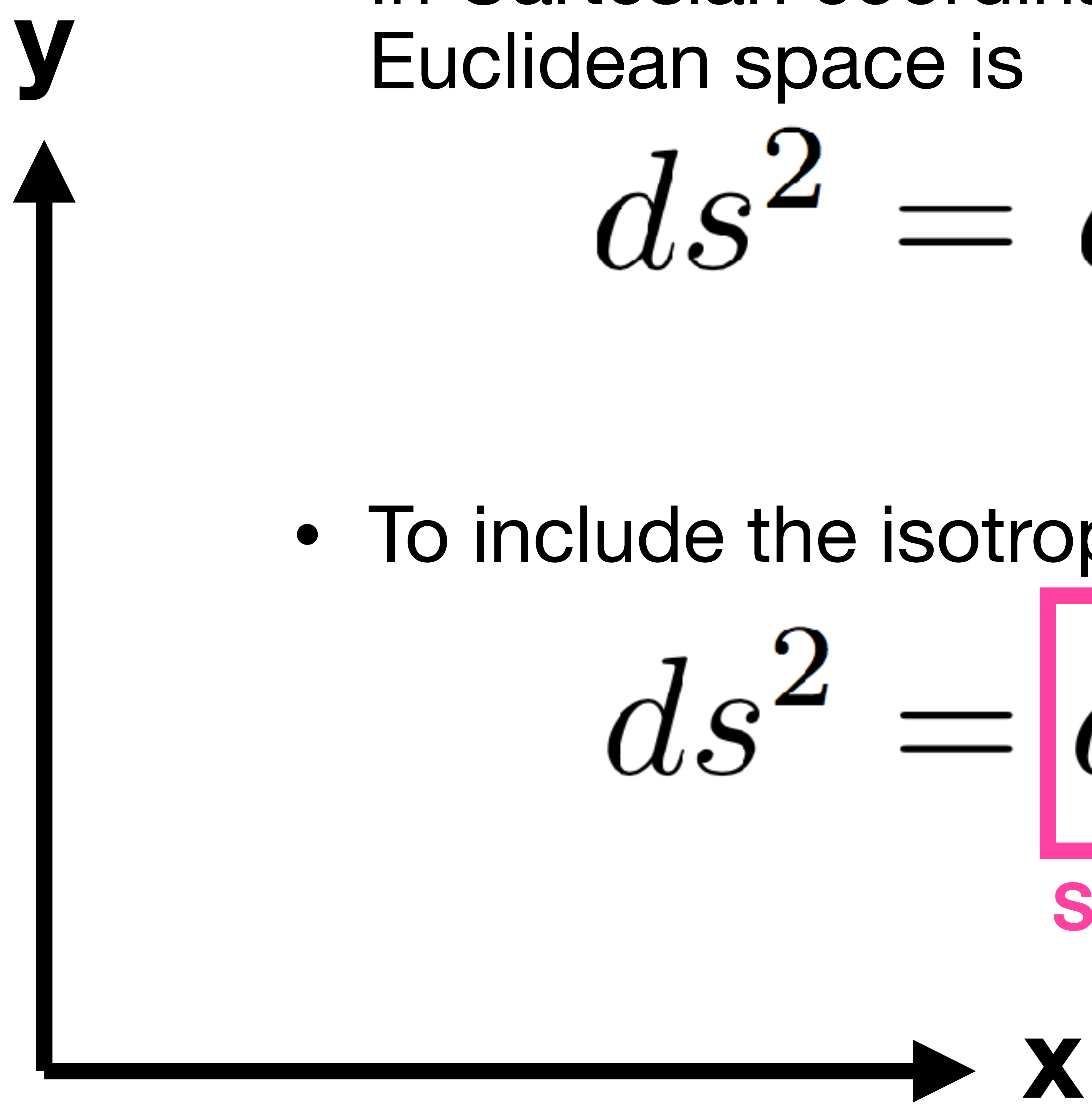


Distance between two points



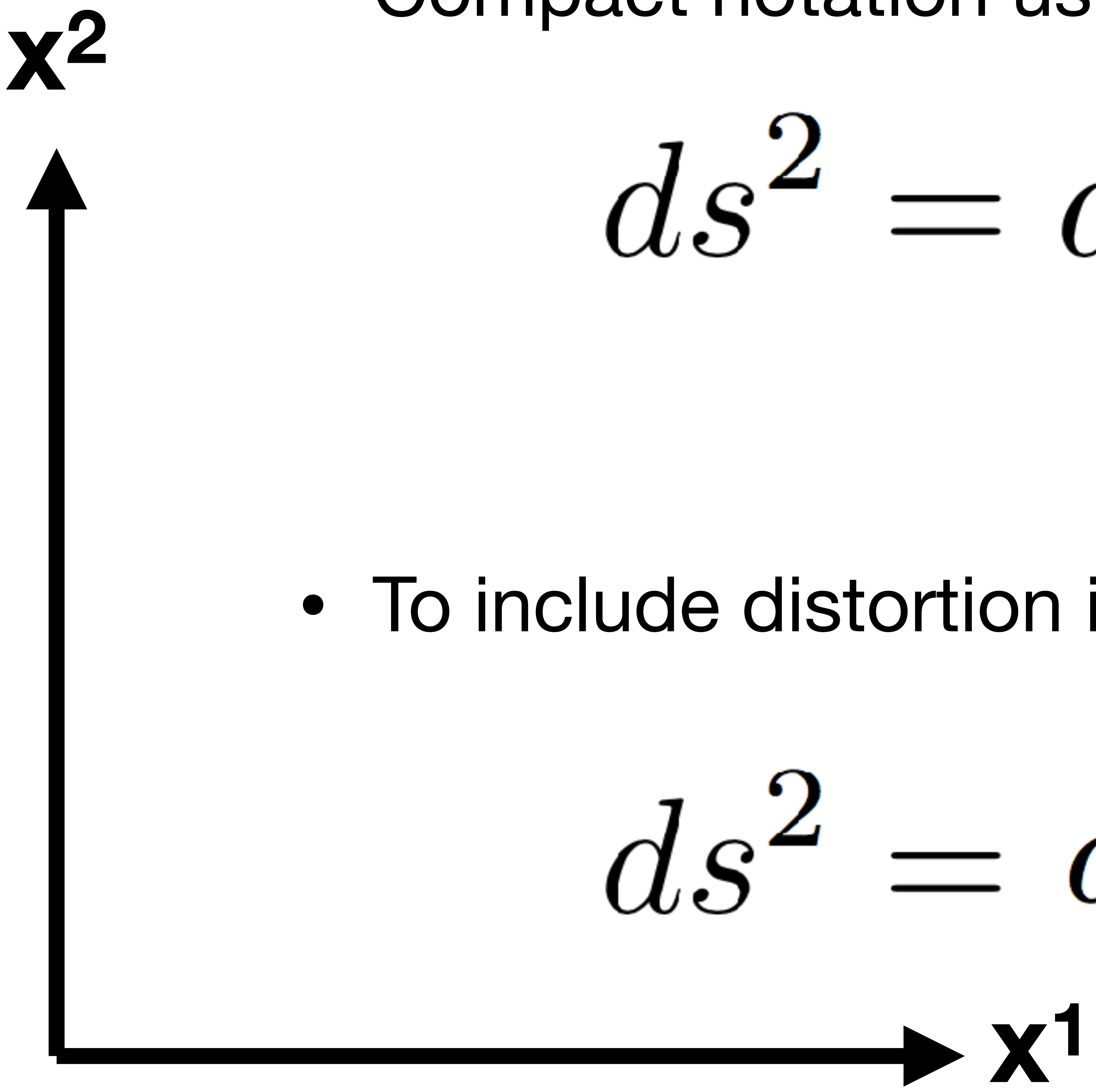
- In Cartesian coordinates, the distance between two points in Euclidean space is

$$ds^2 = dx^2 + dy^2 + dz^2$$

- To include the isotropic expansion of space,

$$ds^2 = \underbrace{a^2(t)}_{\text{Scale Factor}}(dx^2 + dy^2 + dz^2)$$

Distortion in space



- Compact notation using Kronecker's delta symbol:

$$ds^2 = a^2(t) \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij}\, dx^i\, dx^j$$

$\boldsymbol{x} = (x, y, z)$

δ_{ij} = 1 for i=j;
δ_{ij} = 0 otherwise

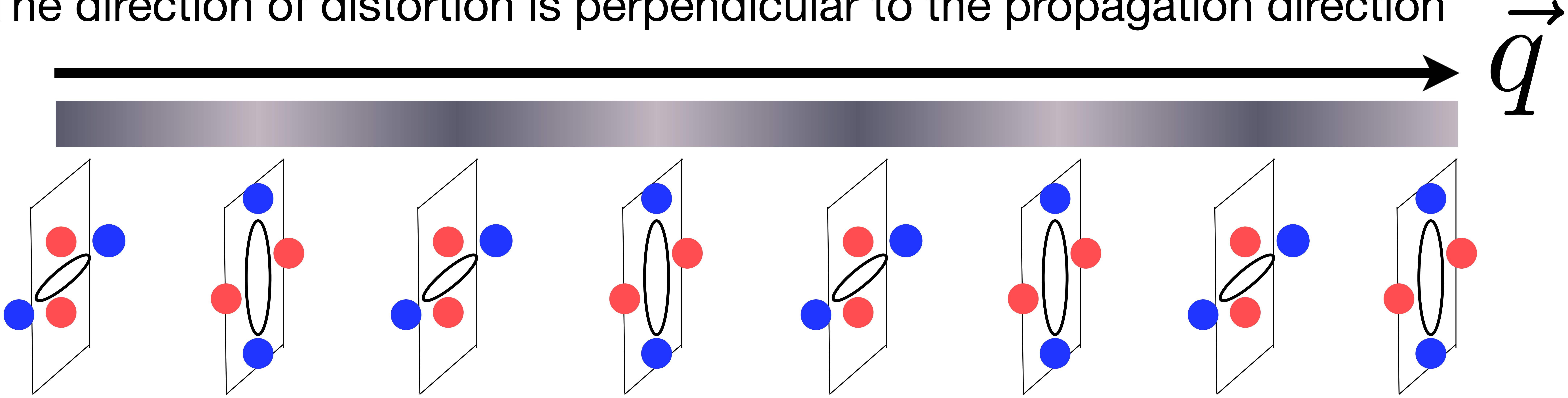
- To include distortion in space,

$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + \boxed{D_{ij}})\, dx^i\, dx^j$$

Distortion in space!

Four conditions for gravitational waves

- **The gravitational wave shall be <u>transverse.</u>**
  - The direction of distortion is perpendicular to the propagation direction

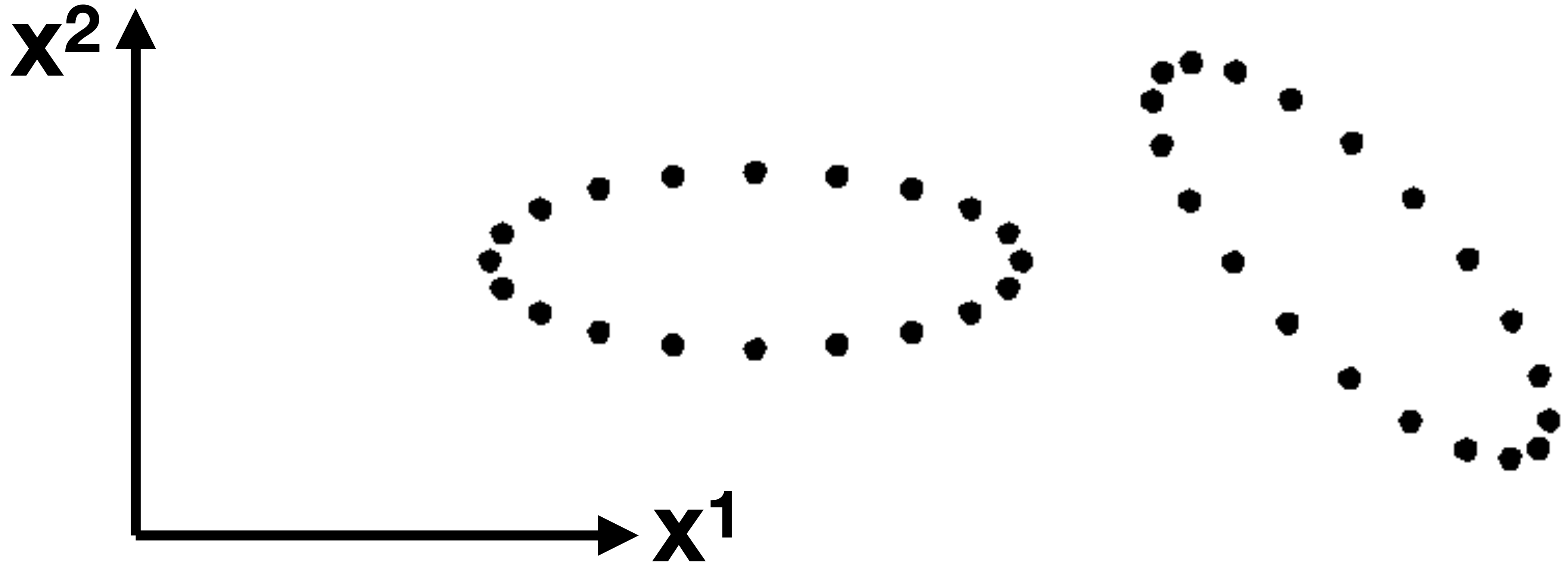


Thus, $$\sum_{i=1}^{3} q^i D_{ij} = 0$$ **3 conditions for D_{ij}**

Four conditions for gravitational waves

- **The gravitational wave shall <u>not change the area</u>**
  - The determinant of $\delta_{ij}+D_{ij}$ is 1

$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + D_{ij}) dx^i dx^j$$



Thus, $$\sum_{i=1}^{3} D_{ii} = 0$$

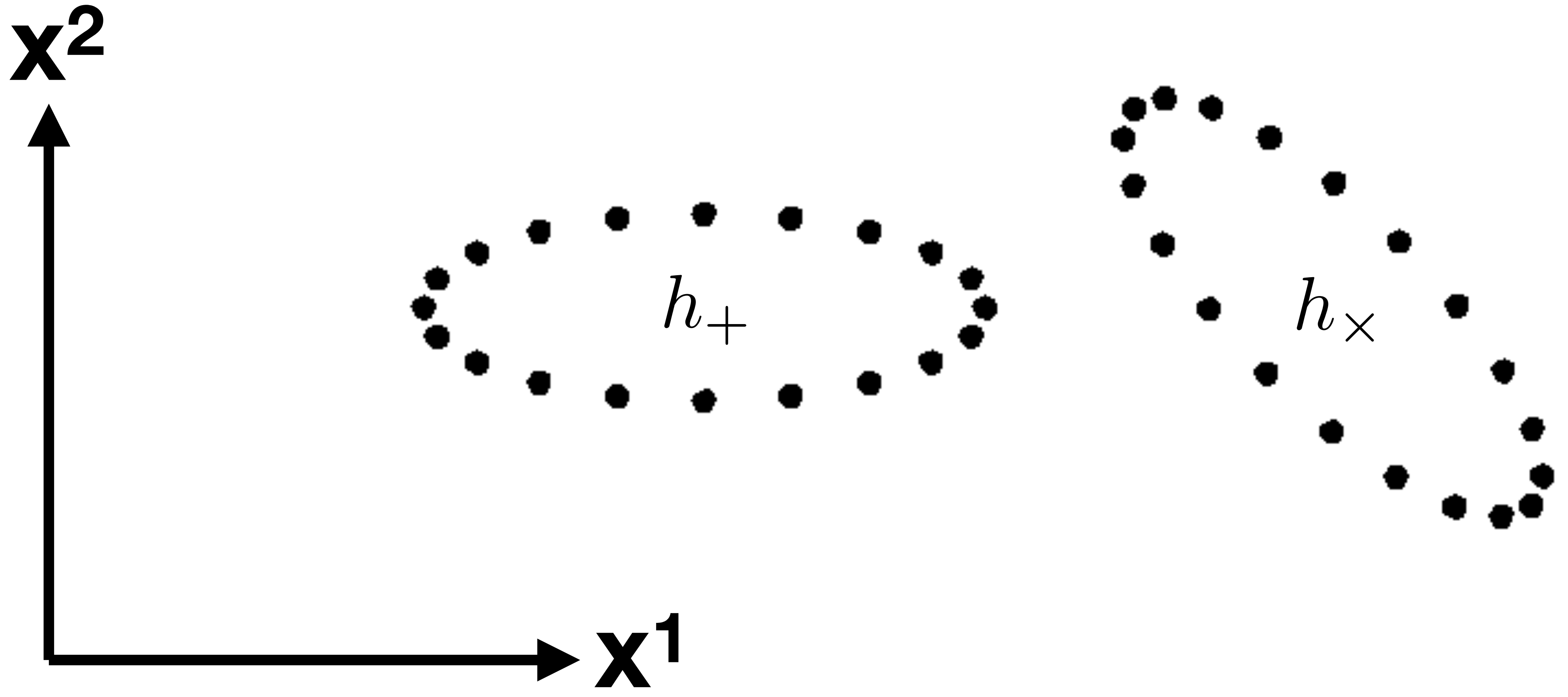
1 condition for D_{ij}

6 – 4 = 2 degrees of freedom for GW

We call them “plus” and “cross” modes

- The symmetric matrix D_{ij} has 6 components, but there are 4 conditions. Thus, we have two degrees of freedom.
- If the GW propagates in the x^3=z axis, non-vanishing components of D_{ij} are

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Propagation direction of GW $\vec{q}$
z
h+=cos(qz)
hx=cos(qz)

Tensor-to-scalar Ratio, the "r"

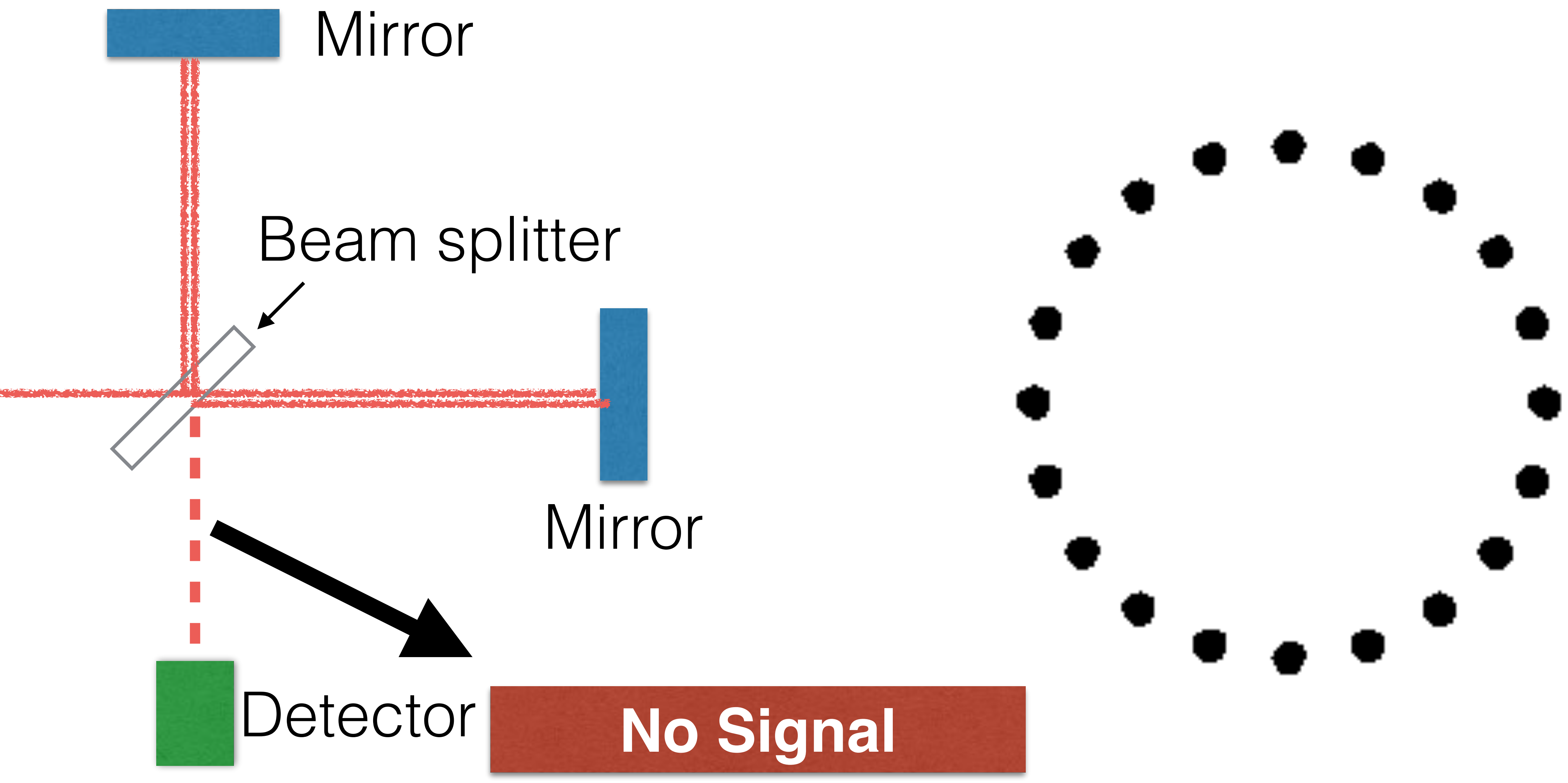
Everyone is after this!

$$r = \frac{\sum_{ij} \langle D_{ij} D_{ij} \rangle}{\langle \zeta^2 \rangle}$$

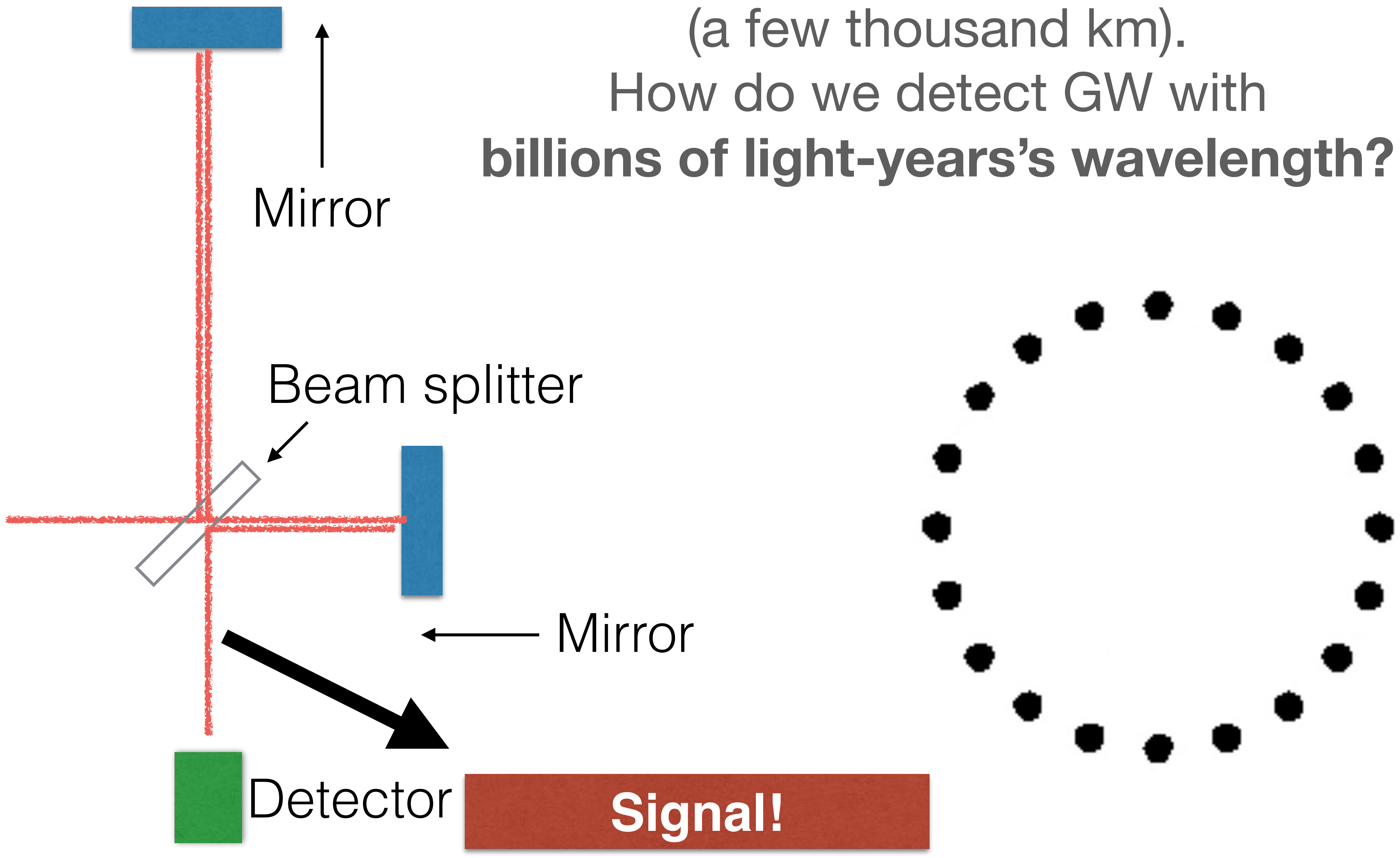
- The current upper bound is r < 0.044 (95%CL) [Tristram et al., arXiv:2010.01139]
- We want to find this in the B-mode polarisation of the CMB.

How to detect GW?

Laser interferometer technique, used by LIGO and VIRGO



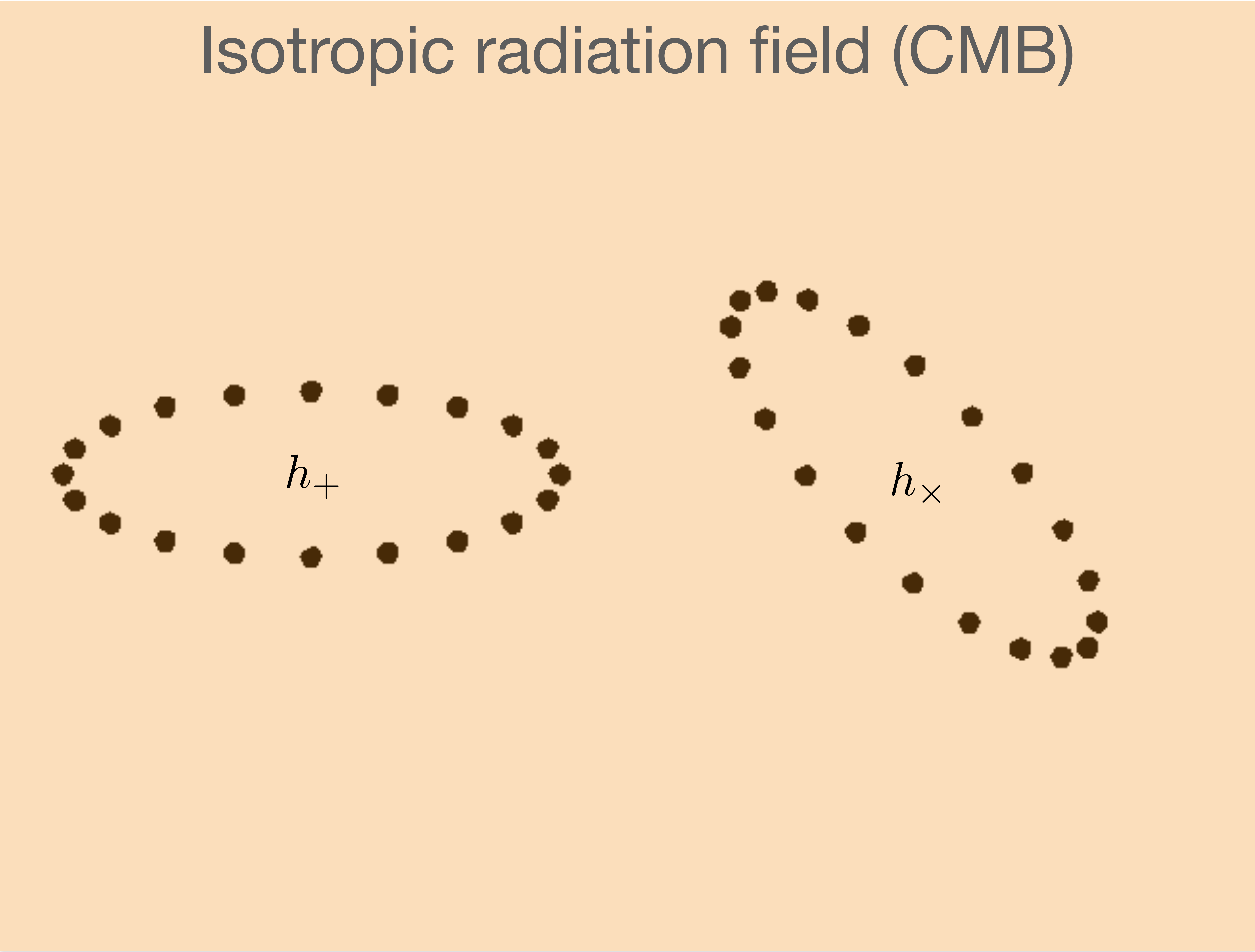
The wavelength of GW detectable by this method is the size of Earth (a few thousand km).
How do we detect GW with **billions of light-years's wavelength?**

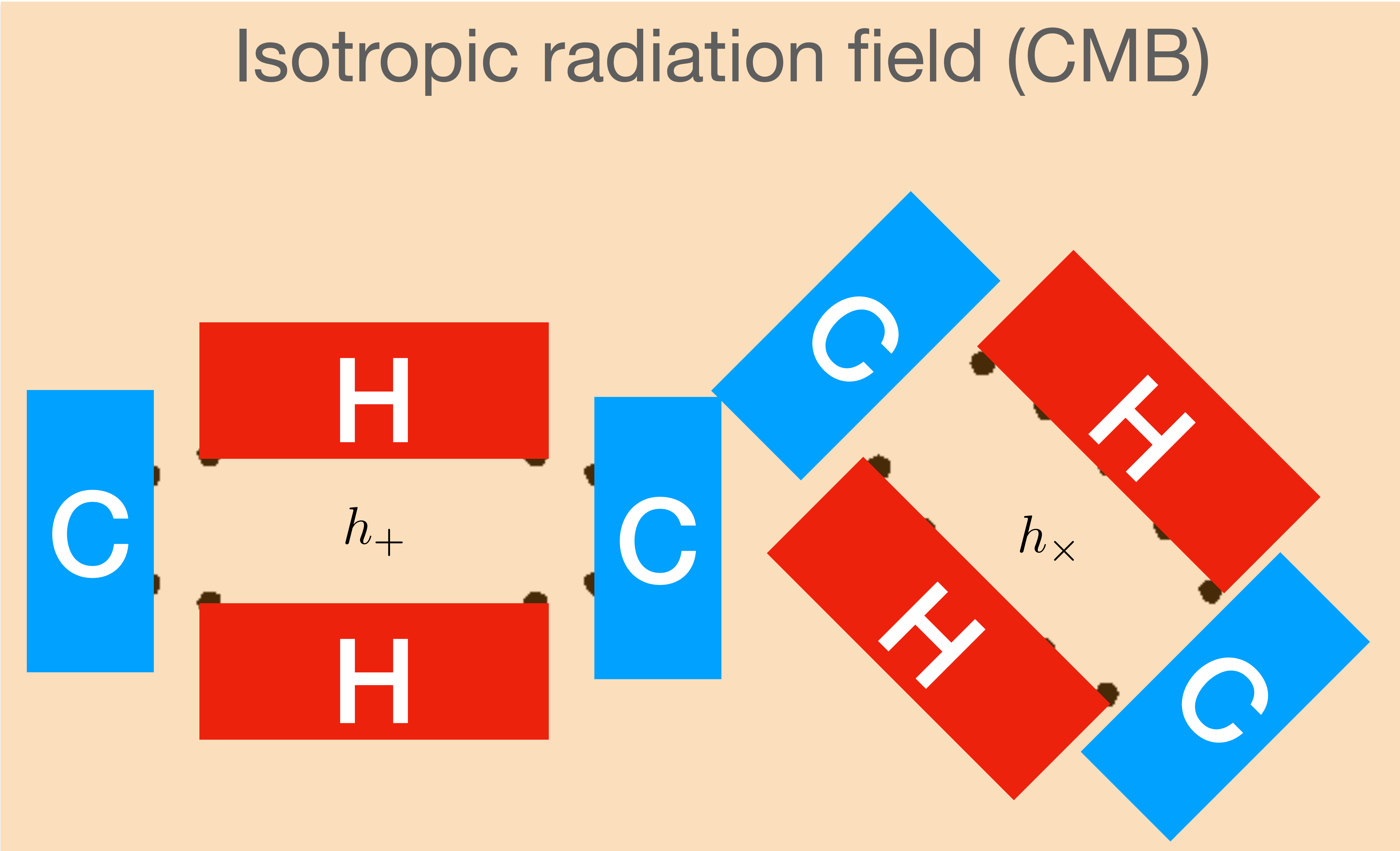


Sachs & Wolfe (1967)

Detecting GW by CMB

Quadrupole temperature anisotropy generated by red- and blue-shifting of photons





From Lecture 2:

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

Detecting GW by CMB

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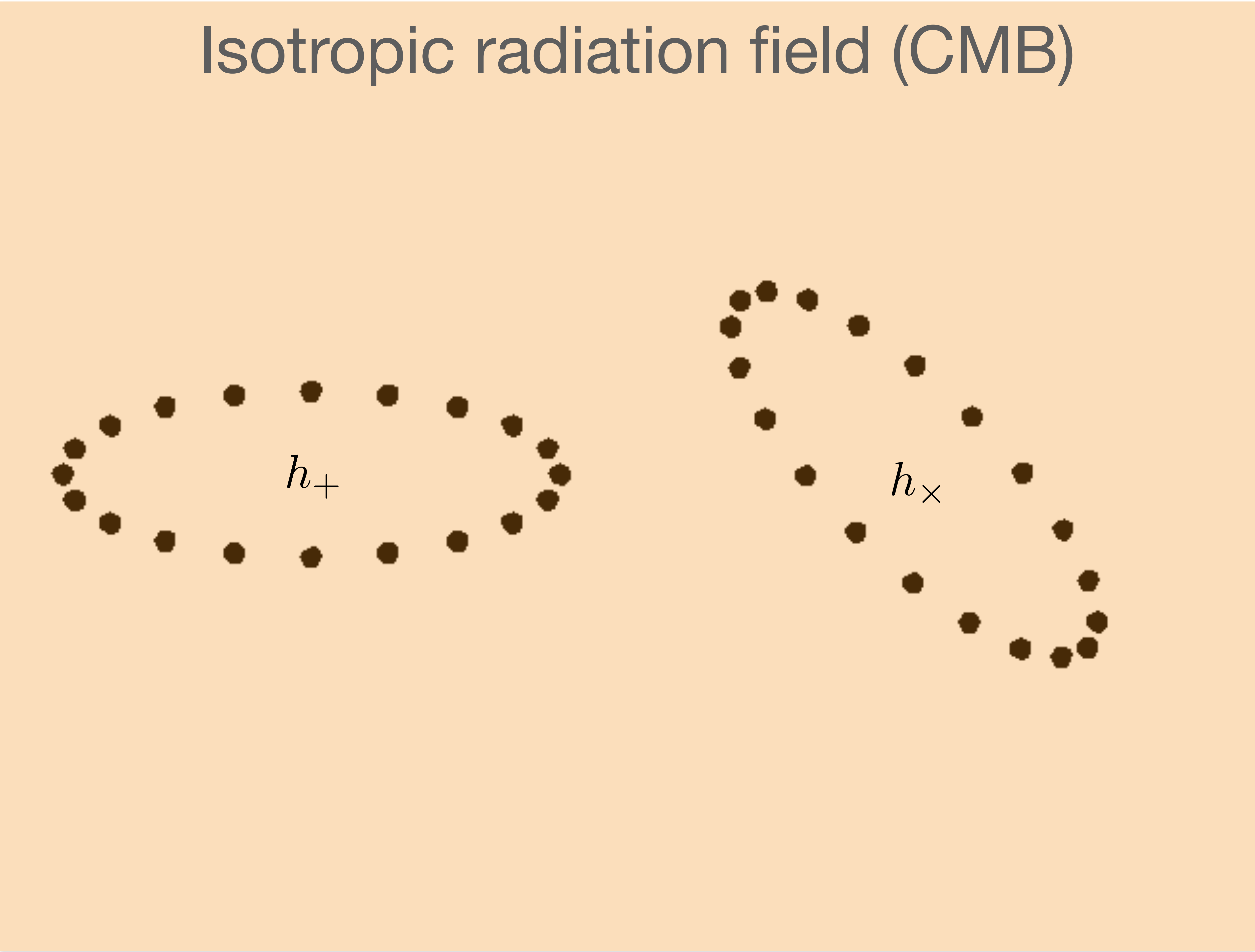
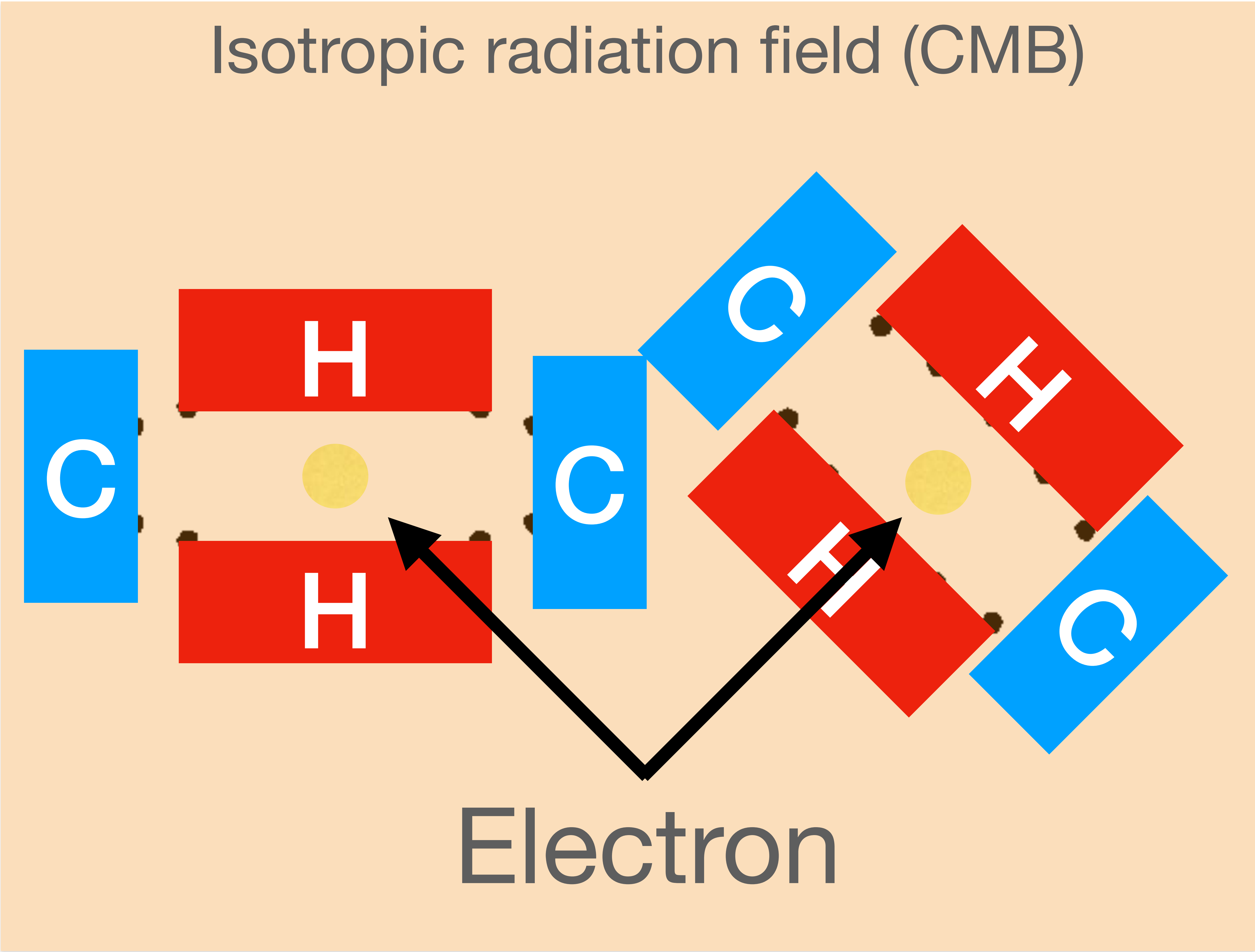
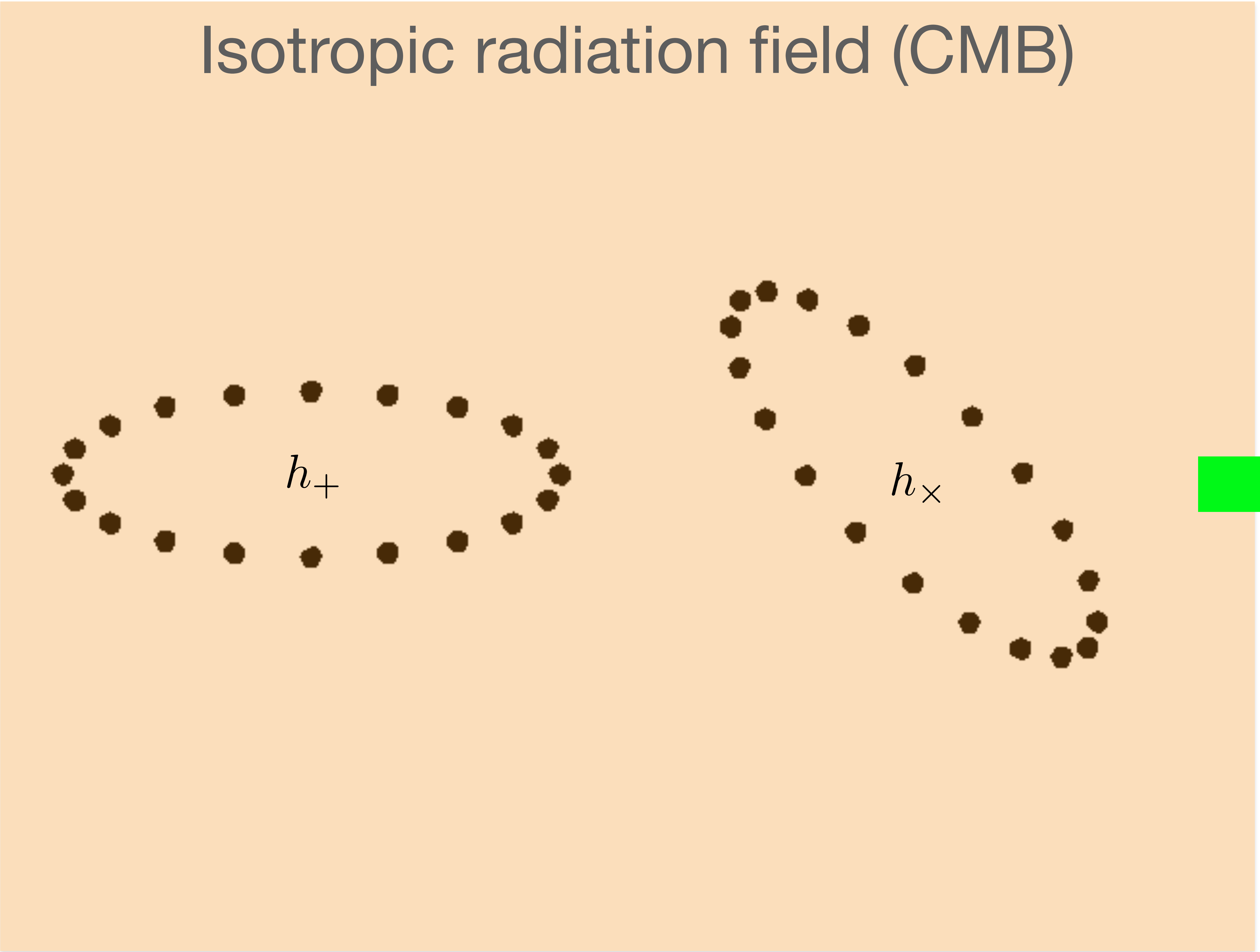


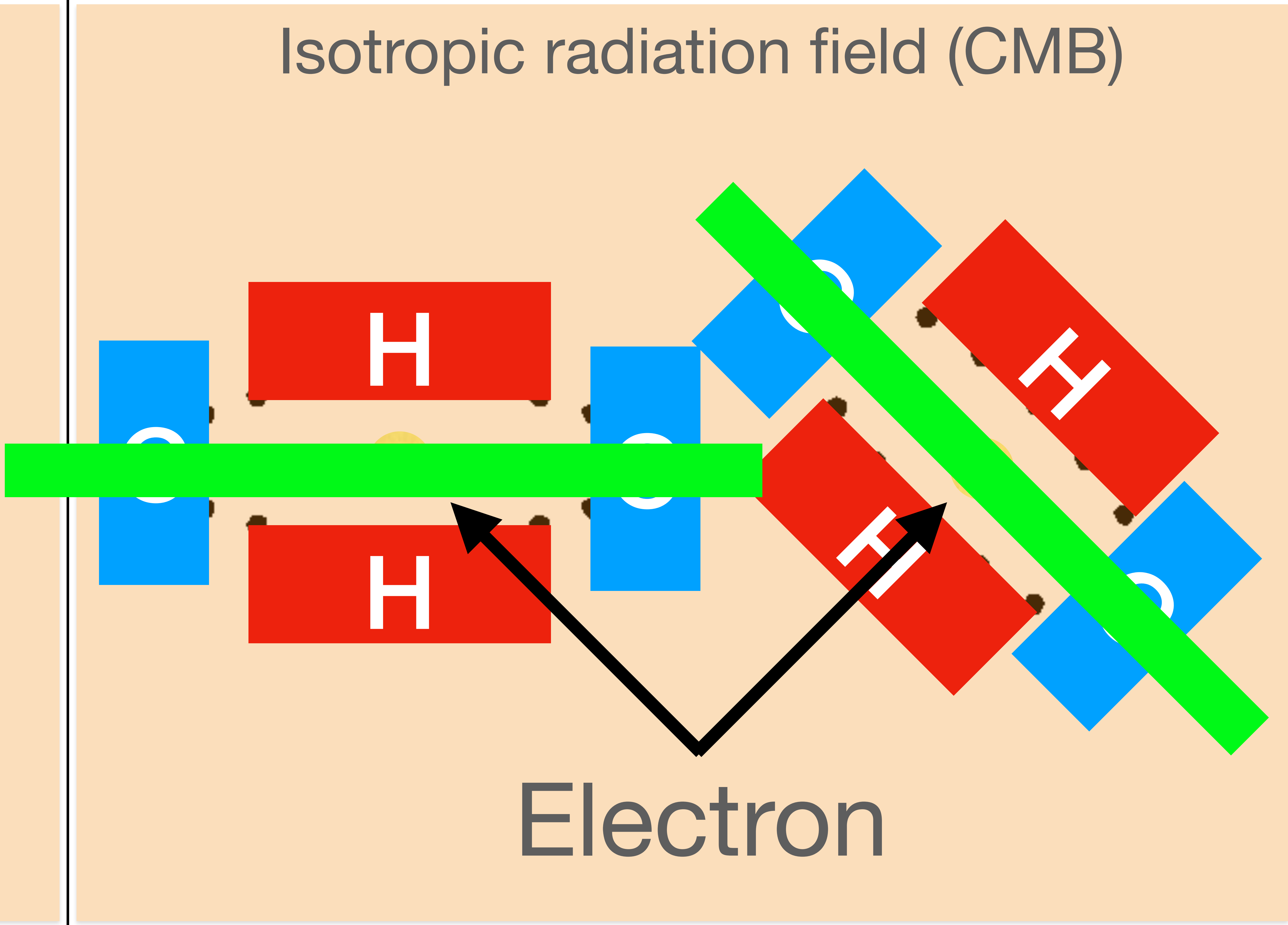


Polnarev (1985)

Detecting GW by CMB *Polarisation*

Quadrupole temperature anisotropy scattered by an electron





Generation and erasure of tensor quadrupole (viscosity)

- Gravitational waves create quadrupole temperature anisotropy (i.e., tensor viscosity of a photon-baryon fluid) gravitationally, **without a velocity potential.**

- Still, tight-coupling between photons and baryons erases the tensor viscosity exponentially before the last scattering.

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\mathrm{ISW}} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

negligible contribution before the last scattering

Part II: Propagation of Gravitational Waves in an Expanding Universe

Equation of Motion of the Gravitational Wave

Wave equation in a non-expanding Universe

- Einstein's equation gives a wave equation for the GW:

$$\Box D_{ij} = -16\pi G T_{ij}^{GW}$$

The stress-energy source of GW

where

$$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2 = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}$$

with

$$\eta^{00} = -1, \quad \eta^{0i} = 0, \quad \eta^{ij} = \delta^{ij}$$

$$\left(ds_4^2 = -dt^2 + d\mathbf{x}^2 = \sum_{\mu\nu} \eta_{\mu\nu} dx^\mu dx^\nu \right)$$

Equation of Motion of the Gravitational Wave

Wave equation in an expanding Universe

- Einstein's equation gives a wave equation for the GW:

$$a^2 \Box D_{ij} = -16\pi G T_{ij}^{GW}$$

The stress-energy source of GW

where

$$\Box \equiv \frac{1}{\sqrt{-g}} \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu} \right)$$

with

$$g^{00} = -1, \quad g^{0i} = 0, \quad g^{ij} = a^{-2}(t)\delta^{ij}, \quad \sqrt{-g} = a^3(t)$$

$$\left(\begin{aligned} ds_4^2 &= -dt^2 + a^2(t) d\mathbf{x}^2 \\ &= \sum_{\mu\nu} g_{\mu\nu} dx^\mu dx^\nu \end{aligned} \right)$$

Equation of Motion of the Gravitational Wave

Wave equation in an expanding Universe

- Einstein's equation gives a wave equation for the GW:

$$a^2 \Box D_{ij} = -16\pi G T_{ij}^{GW}$$

The stress-energy source of GW

where

$$\Box = -\frac{\partial^2}{\partial t^2} - 3\frac{\dot{a}}{a}\frac{\partial}{\partial t} + \frac{1}{a^2}\nabla^2$$

Effect of the expansion of the Universe!

Equation of Motion of the Gravitational Wave

Wave equation in an expanding Universe

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$$a^2 \Box D_{ij} = -16\pi G T_{ij}^{GW}$$

The stress-energy source of GW

where

$$\Box = -\frac{\partial^2}{\partial t^2} - 3\frac{\dot{a}}{a}\frac{\partial}{\partial t} - \frac{q^2}{a^2}$$

Effect of the expansion of the Universe!

In Fourier space

$$\nabla^2 \exp(i\mathbf{q}\cdot\mathbf{x}) = -q^2 \exp(i\mathbf{q}\cdot\mathbf{x})$$

Equation of Motion of the Gravitational Wave

Wave equation in an expanding Universe

- Einstein's equation gives a wave equation for the GW:

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Effect of the expansion of the Universe!

Physical wavenumber, q/a

In Fourier space

$$\nabla^2 \exp(i\mathbf{q}\cdot\mathbf{x}) = -q^2 \exp(i\mathbf{q}\cdot\mathbf{x})$$

Super-horizon Solution (q << aH)

$$\ddot{D}_{ij} + \frac{3\dot{a}}{a}\dot{D}_{ij} = 0$$

$$\Rightarrow D_{ij} = \text{constant} + \text{decaying term}$$

- **Super-horizon tensor perturbation is conserved!**
  - Similar to the conserved scalar perturbation, ζ.
  - Thus, **no ISW temperature anisotropy on super-horizon scales**
- It does not look like “gravitational waves”, but it will start oscillating and behaving like waves once it enters the horizon

Matter-dominated Solution

$$D_{ij,\boldsymbol{q}}(t) = C_{ij,\boldsymbol{q}} \frac{3j_1(qr)}{qr} \propto \frac{1}{a(t)}$$

$$\dot{D}_{ij,\boldsymbol{q}}(t) = -C_{ij,\boldsymbol{q}} \frac{q}{a(t)} \frac{3j_2(qr)}{qr} \propto \frac{1}{a^2(t)}$$

- $\partial D_{ij}/\partial t$ gives the ISW. **It peaks at the horizon crossing, qr~2.**
- The energy density is given by $(\partial D_{ij}/\partial t)^2$, which indeed decays like radiation, a^{-4}.

$$r(t) = c \int_0^t \frac{dt'}{a(t')}$$

$$j_1(qr) = \frac{\sin(qr)}{(qr)^2} - \frac{\cos(qr)}{qr}$$

Cf: Sound wave

$$\frac{\delta\rho_\gamma}{4\bar{\rho}_\gamma} + \Phi = A\cos(qr_s) + B\sin(qr_s) - R\Phi$$

The oscillation of the GW solution is given by the photon horizon, not the sound horizon, as the GW propagates at the speed of light.

Energy density of the gravitational waves

$$\rho_{\rm GW}(t) = \frac{1}{4} M_{\rm pl}^2 \sum_{ij} \langle \dot{D}_{ij}(t,\mathbf{x}) \dot{D}_{ij}(t,\mathbf{x}) \rangle$$

$$= \frac{1}{2} M_{\rm pl}^2 \sum_{\lambda=+,\times} \langle \dot{h}_\lambda^2(t,\mathbf{x}) \rangle$$

$$M_{\rm pl} = (8\pi G)^{-1/2}$$

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- The equation of motion yields

$$\dot{D}_{ij} \propto a^{-2}(t) \quad \Rightarrow \quad \rho_{\rm GW}(t) \propto a^{-4}(t)$$

The GW density redshifts as relativistic particles (radiation)!

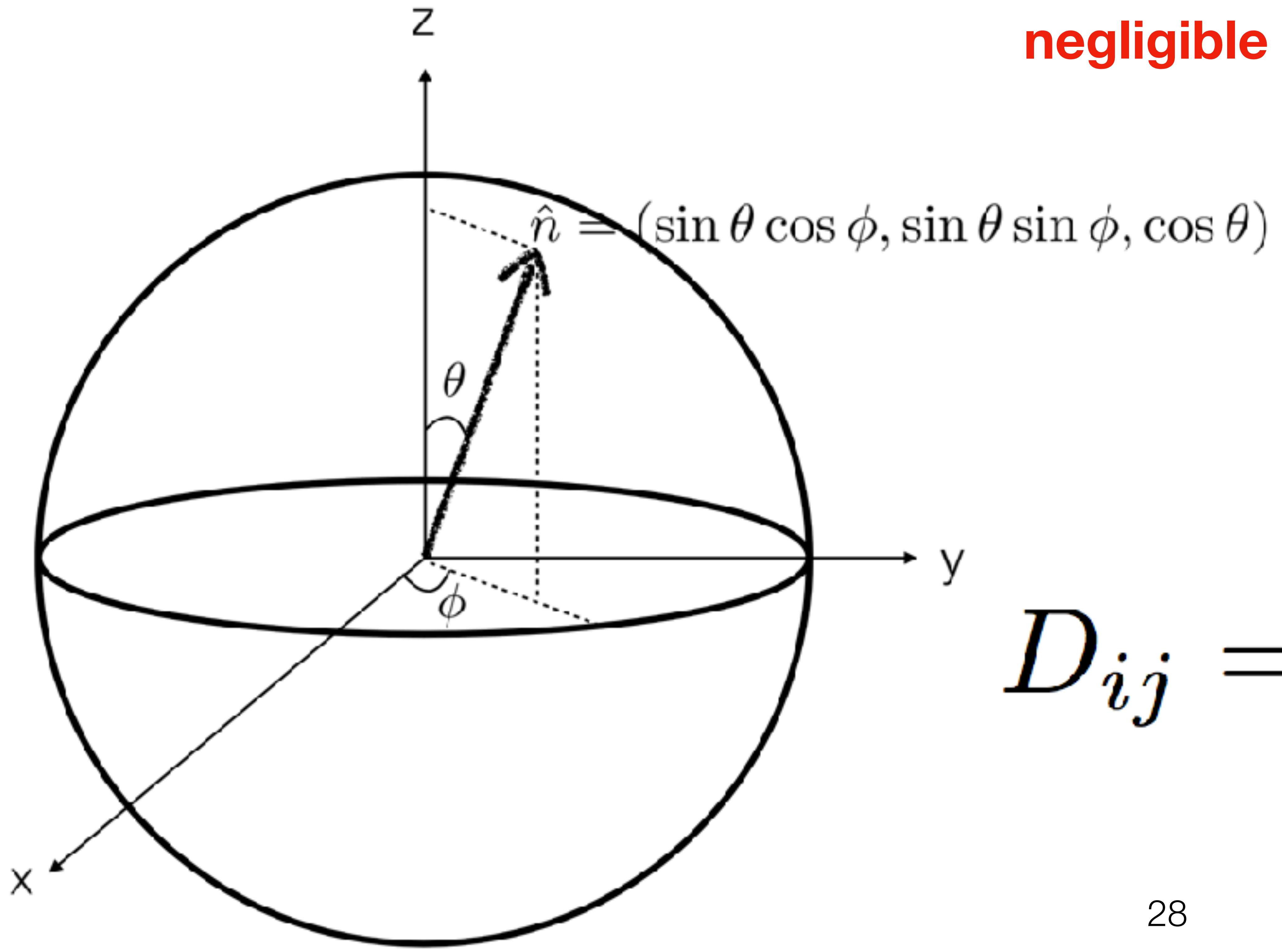
Part III: Temperature Anisotropy from Gravitational Waves

Sachs & Wolfe (1967)

Formal Solution (Tensor)

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\mathrm{ISW}} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

negligible contribution before the last scattering



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Sachs & Wolfe (1967)

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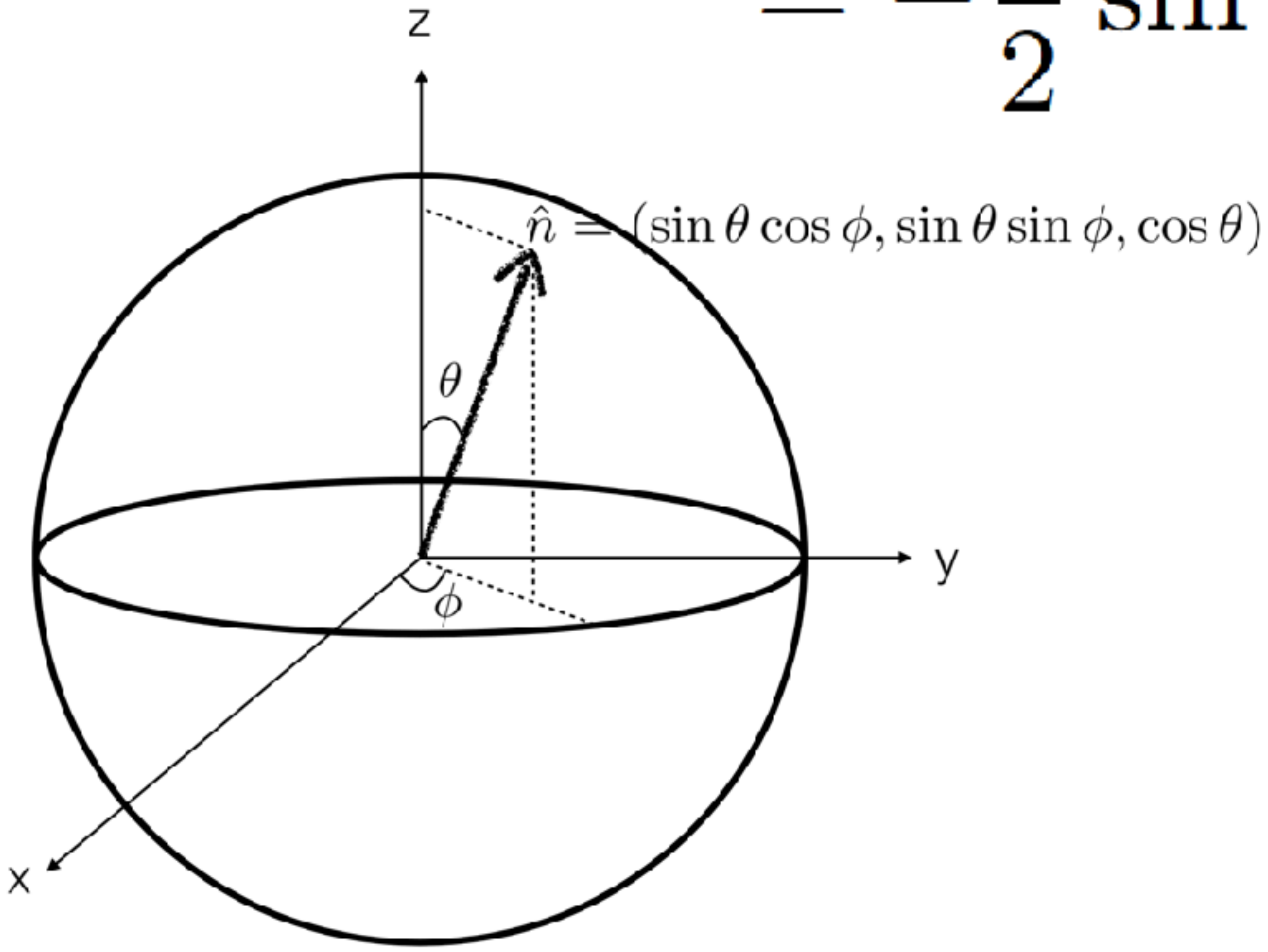
negligible contribution before the last scattering

$$= -\frac{1}{2}\sin^2\theta \int_{t_L}^{t_0} dt\,(\dot{h}_+\cos 2\phi + \dot{h}_\times \sin 2\phi)$$



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Sachs & Wolfe (1967)

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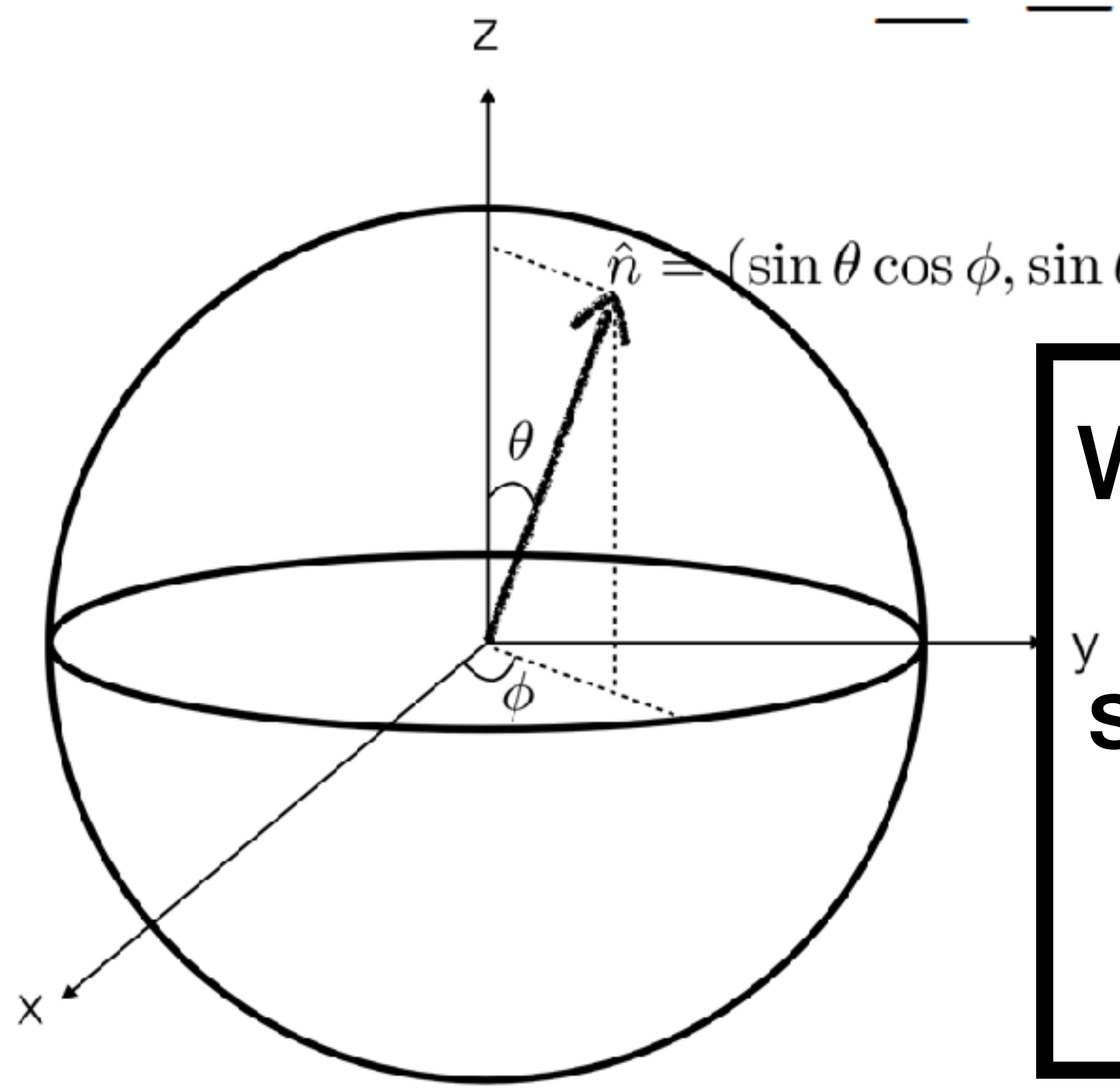

When a plane wave gravitational wave propagates in the z direction, no temperature anisotropy is seen towards the poles (θ=0, π). The anisotropy is maximised on the horizon (θ=π/2) with cos(2ϕ) & sin(2ϕ) modulation in the azimuthal directions.

Sachs & Wolfe (1967)

Formal Solution (Tensor)

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\rm ISW} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

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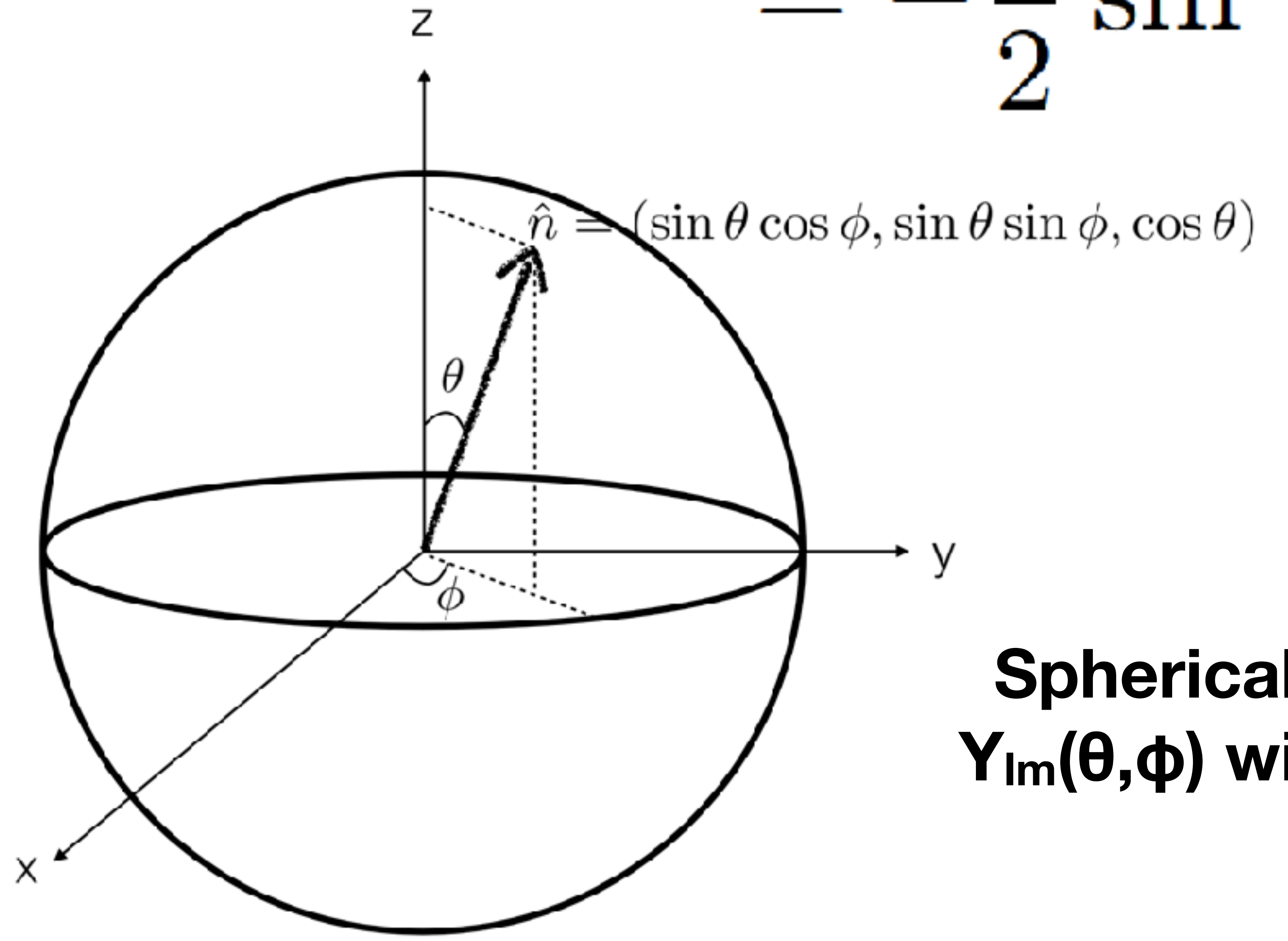
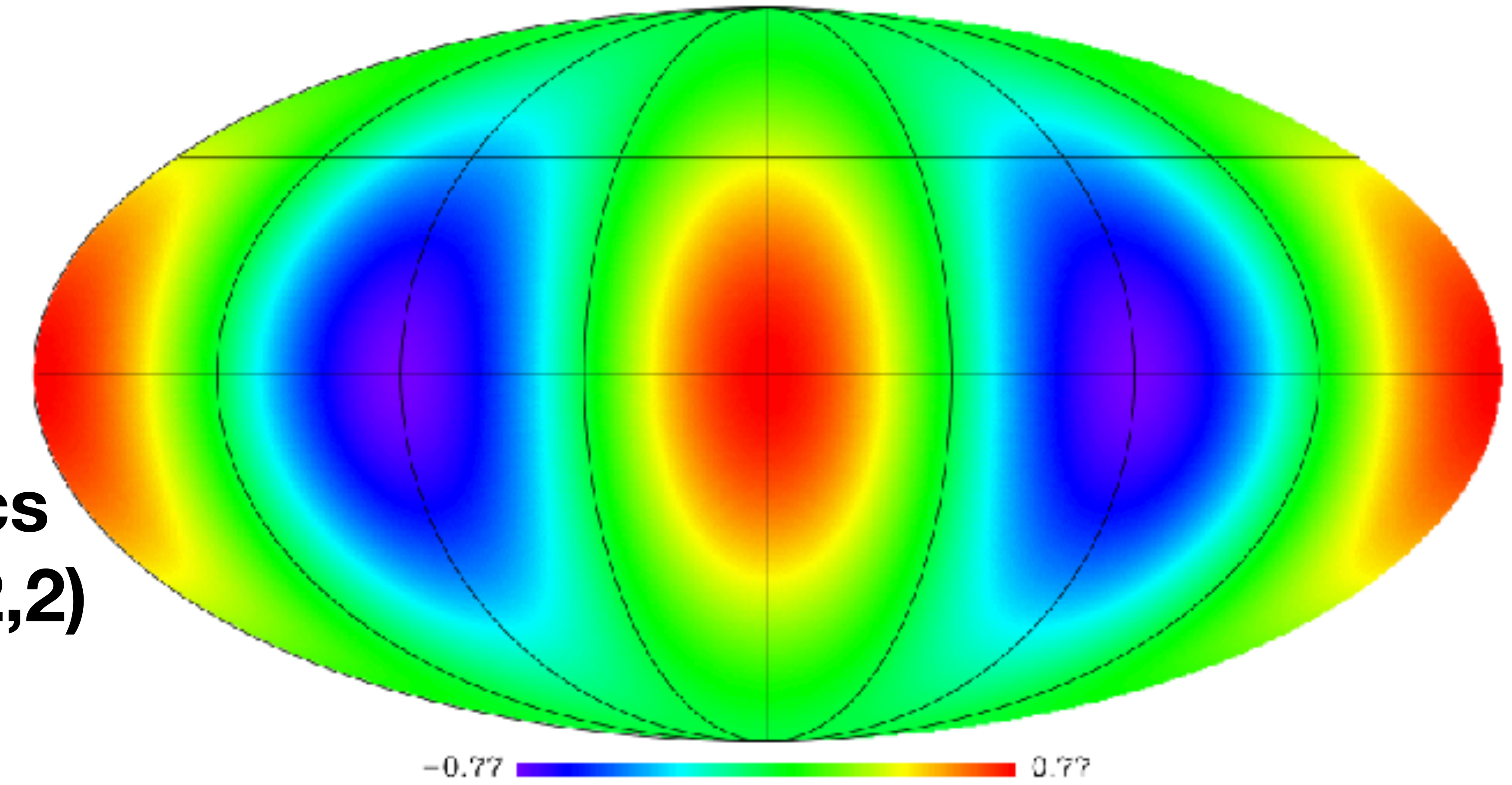
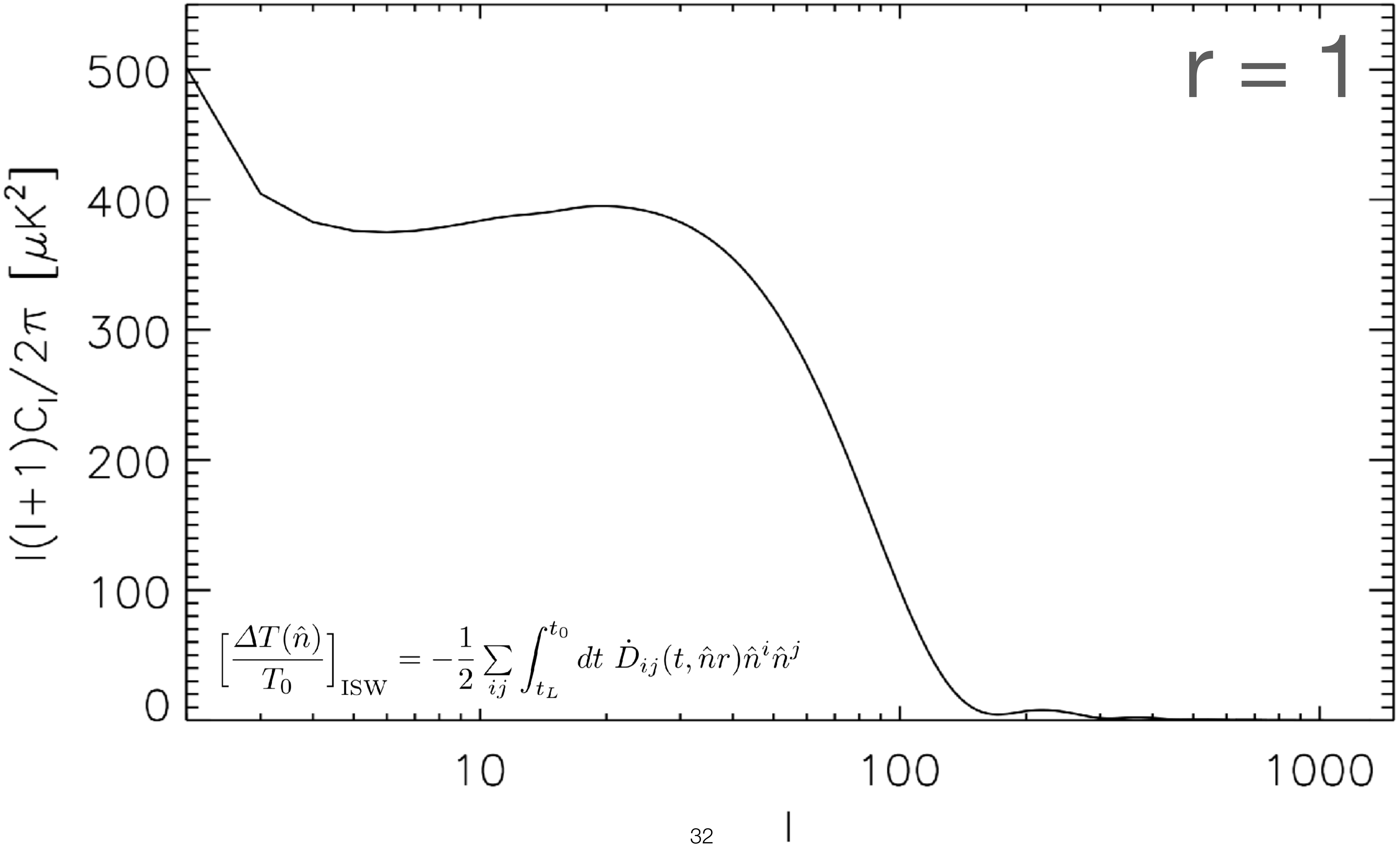


Spherical harmonics
$Y_{lm}(\theta,\phi)$ with (l,m)=(2,2)



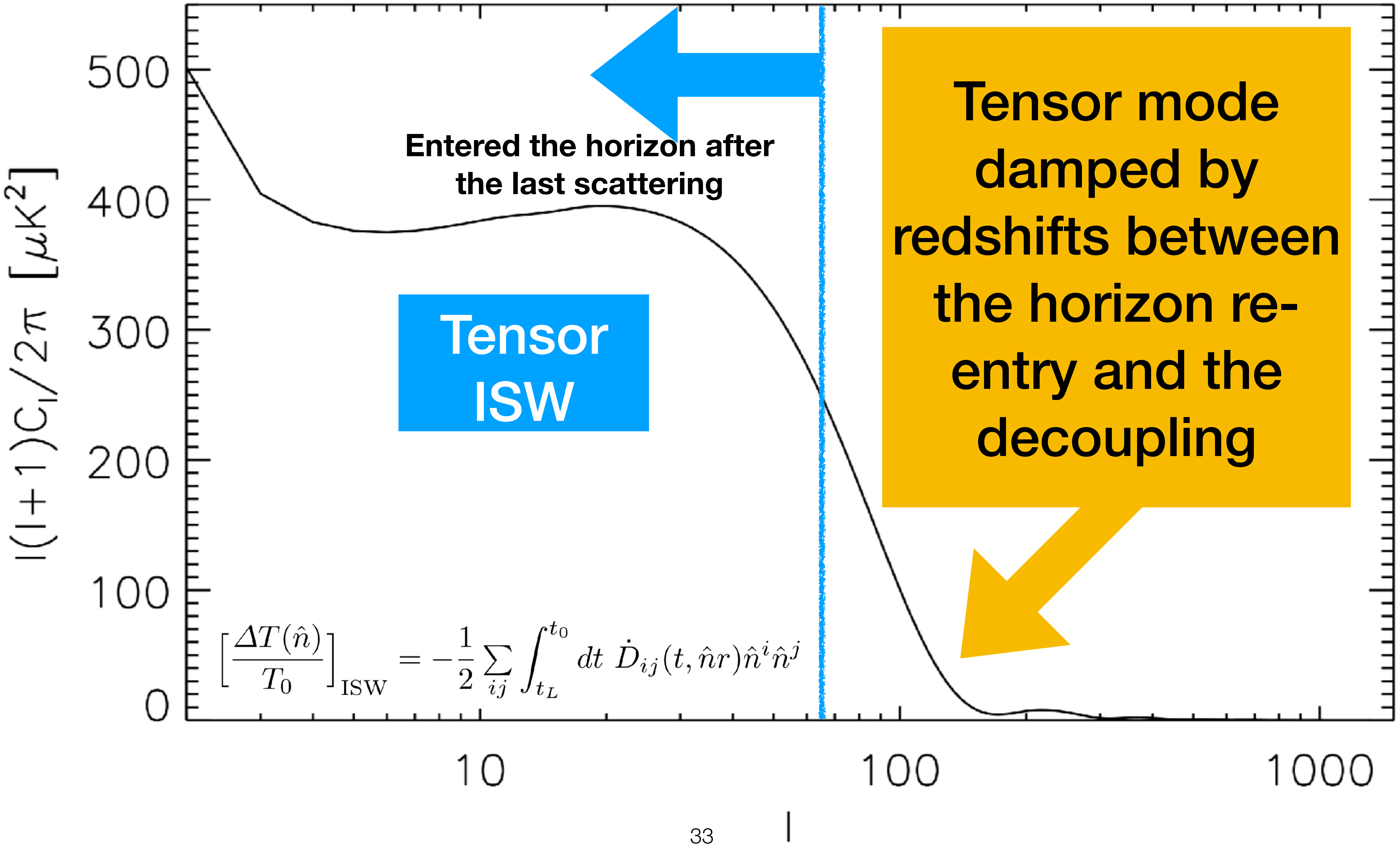
Scale-invariant

Temperature C_l from GW



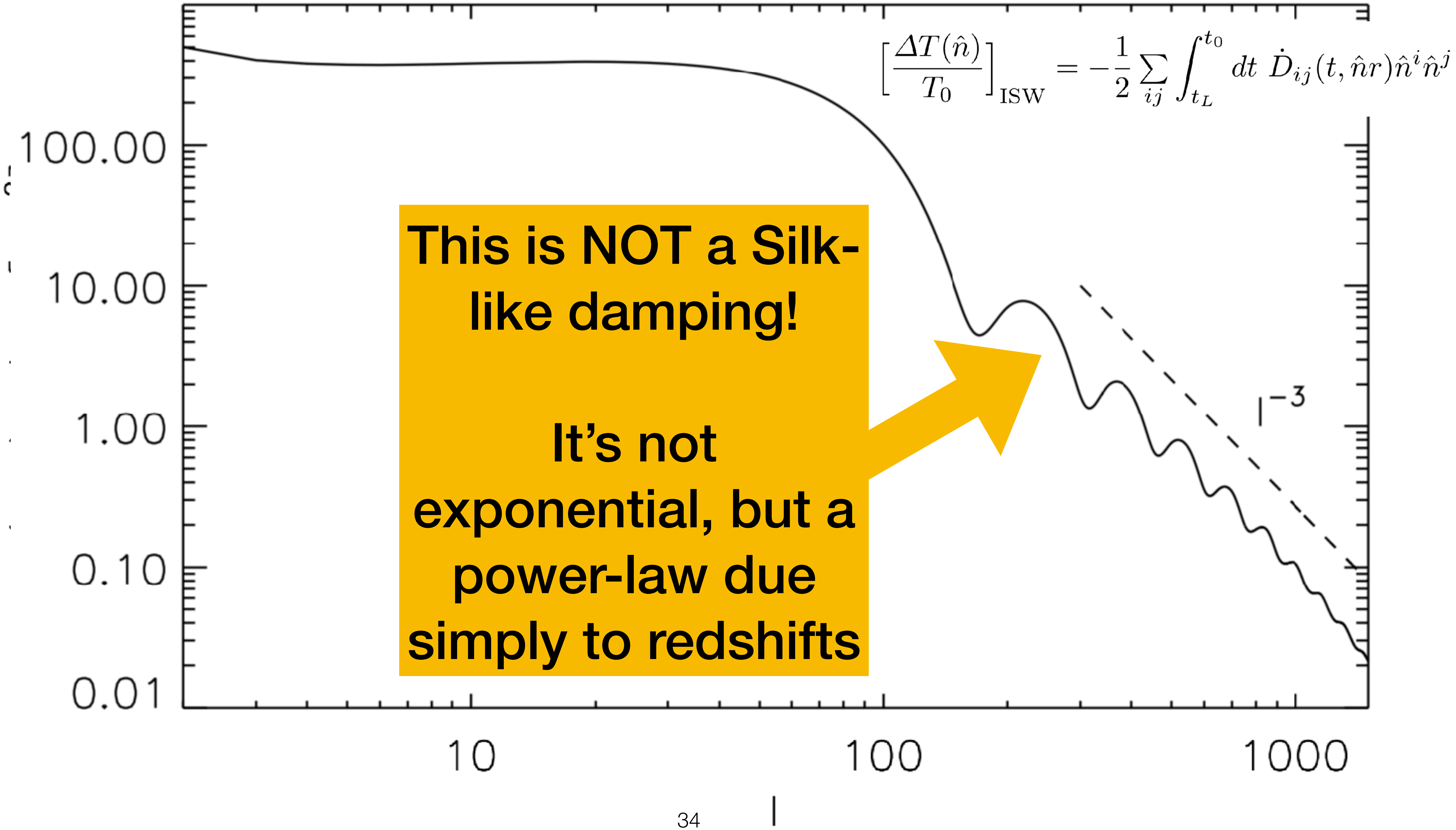
Scale-invariant

Temperature C_l from GW

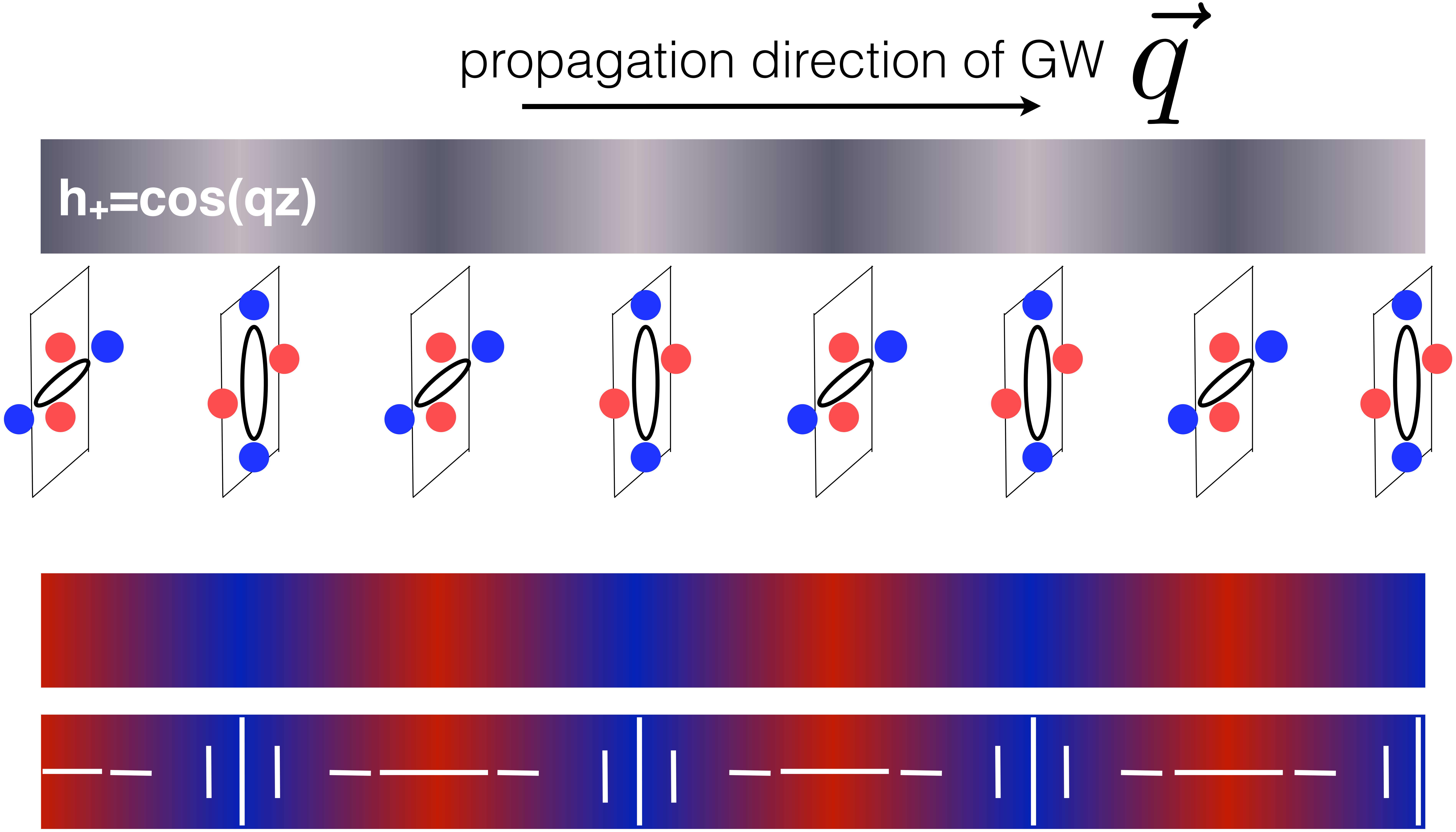


Temperature C_l from GW

Scale-invariant

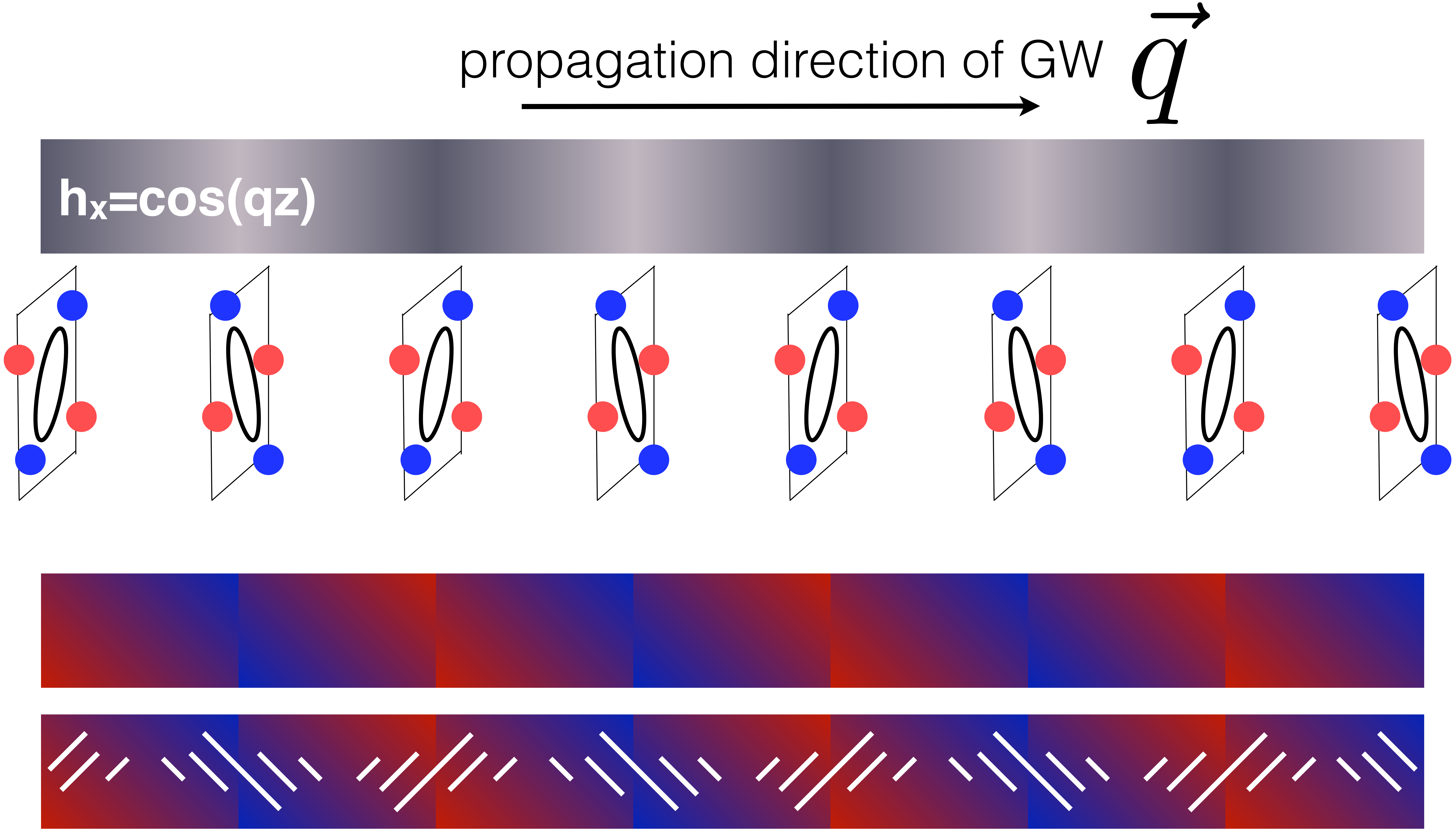


Part IV: E- and B-mode Polarisation from Gravitational Waves



Polarisation directions perpendicular/parallel to the wavenumber vector

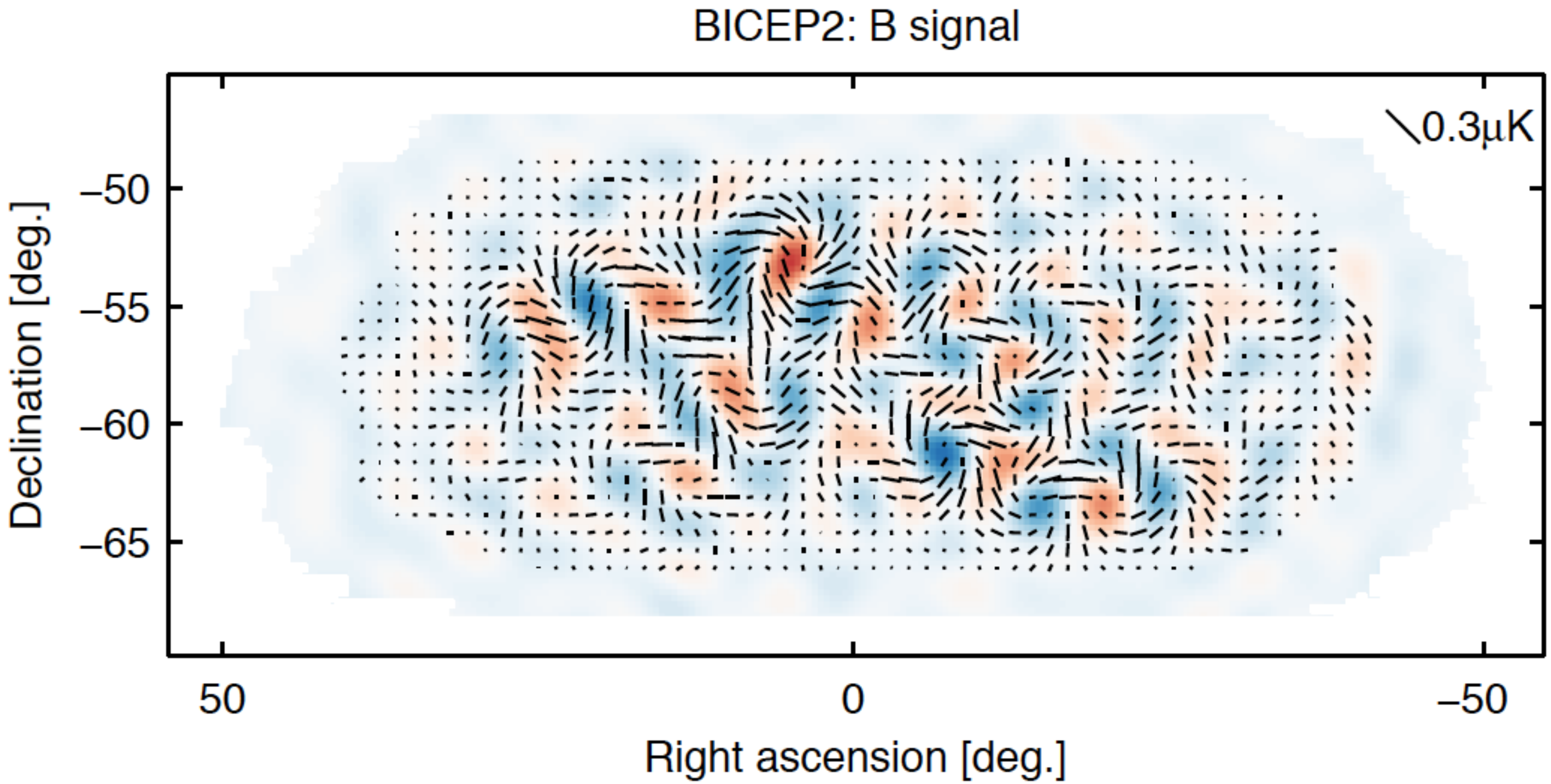
-> **E mode polarisation**



Polarisation directions 45 degrees tilted from to the wavenumber vector

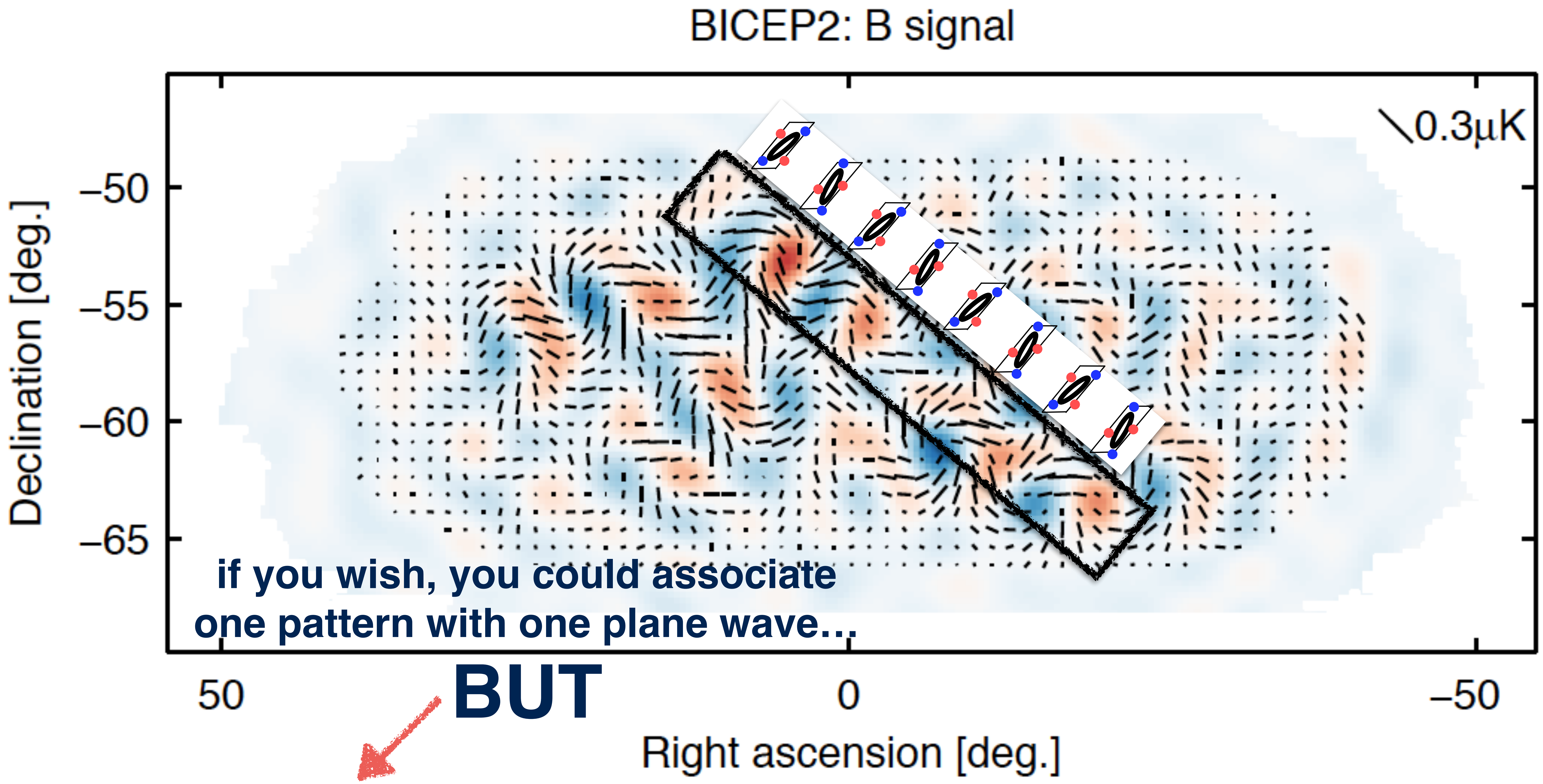
-> **B mode polarisation**

Signature of gravitational waves in the sky [?]



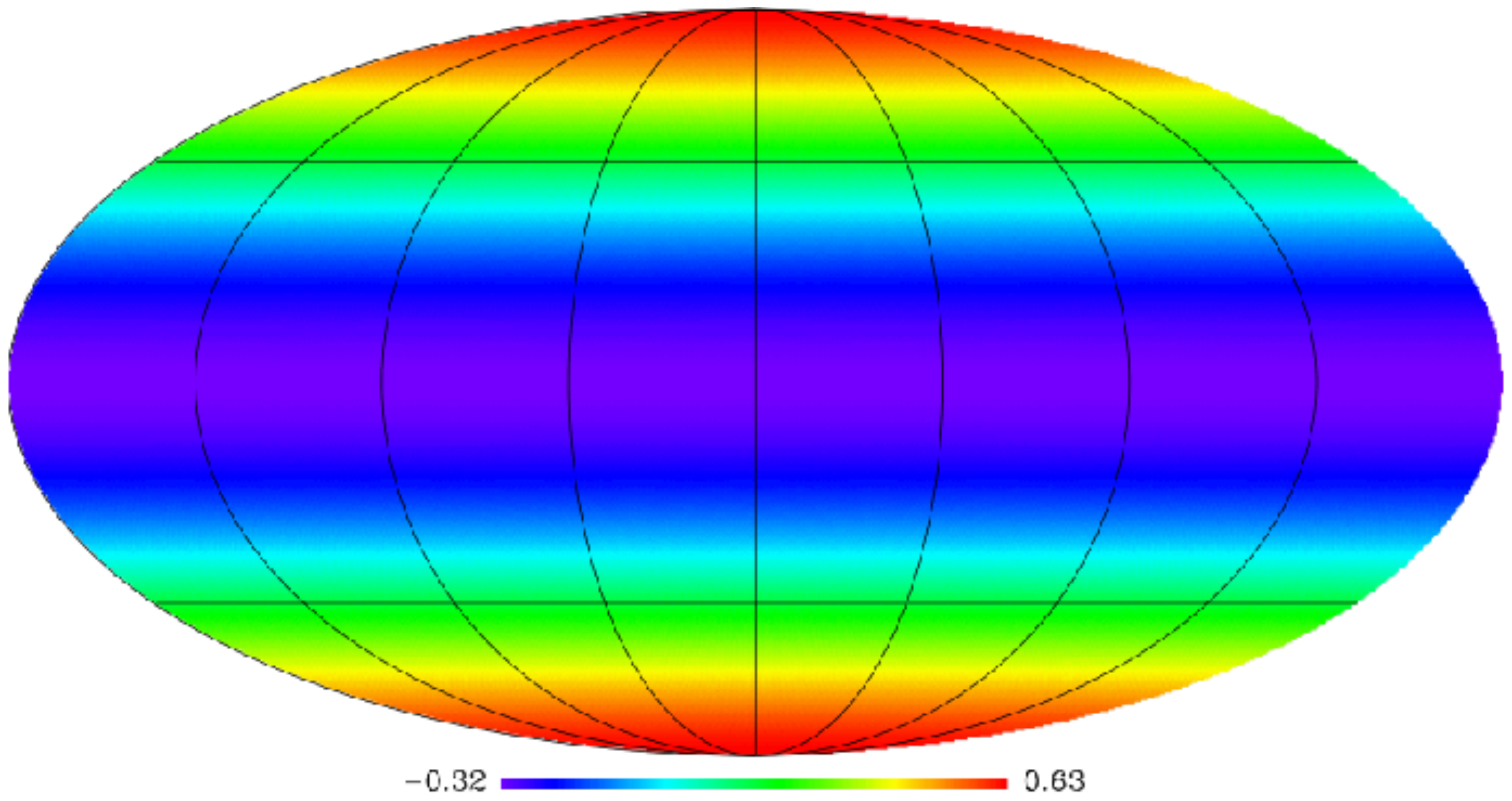
CAUTION: we are NOT seeing a single plane wave propagating perpendicular to our line of sight

Signature of gravitational waves in the sky [?]

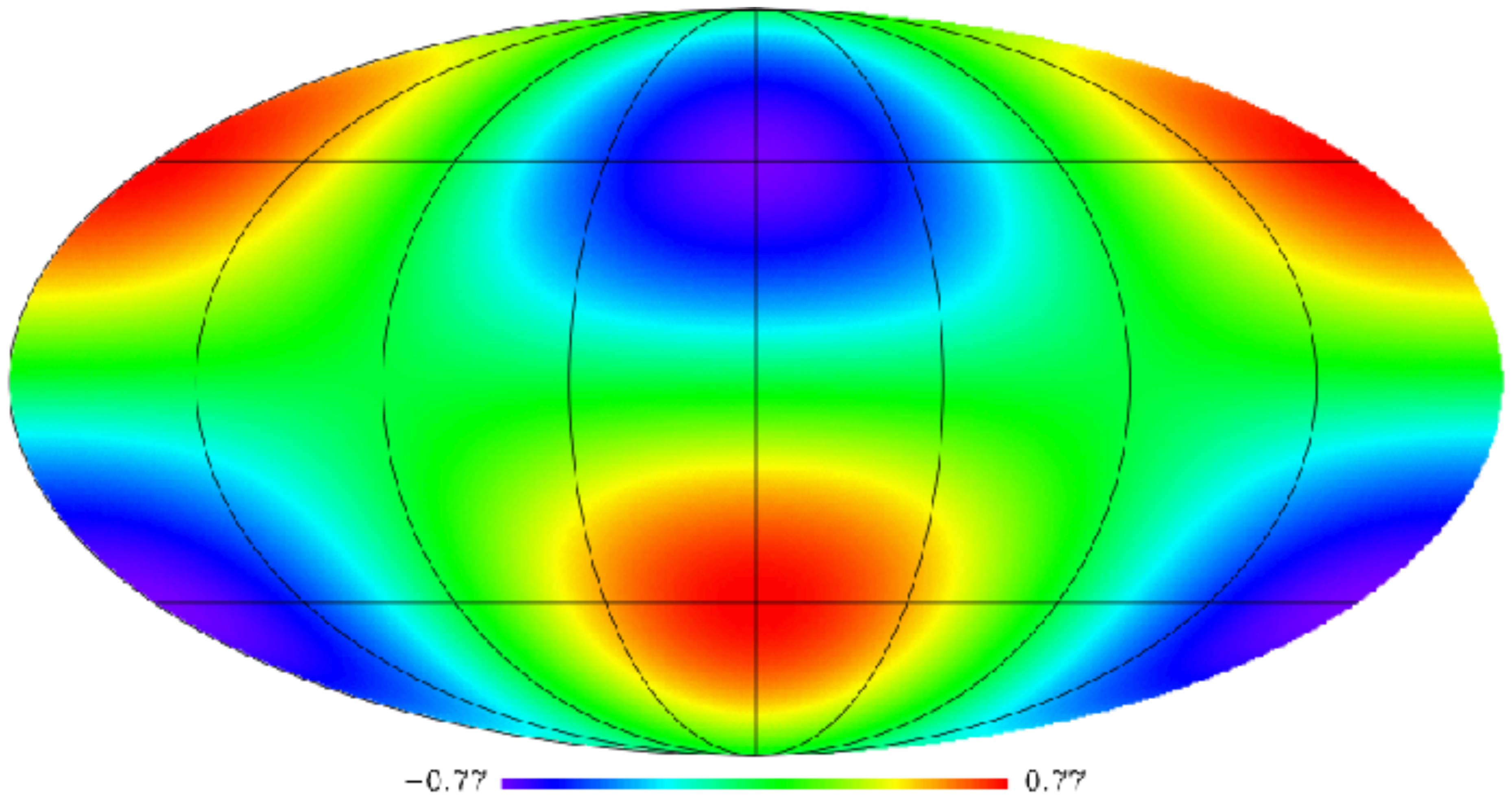


CAUTION: we are NOT seeing a single plane wave propagating perpendicular to our line of sight

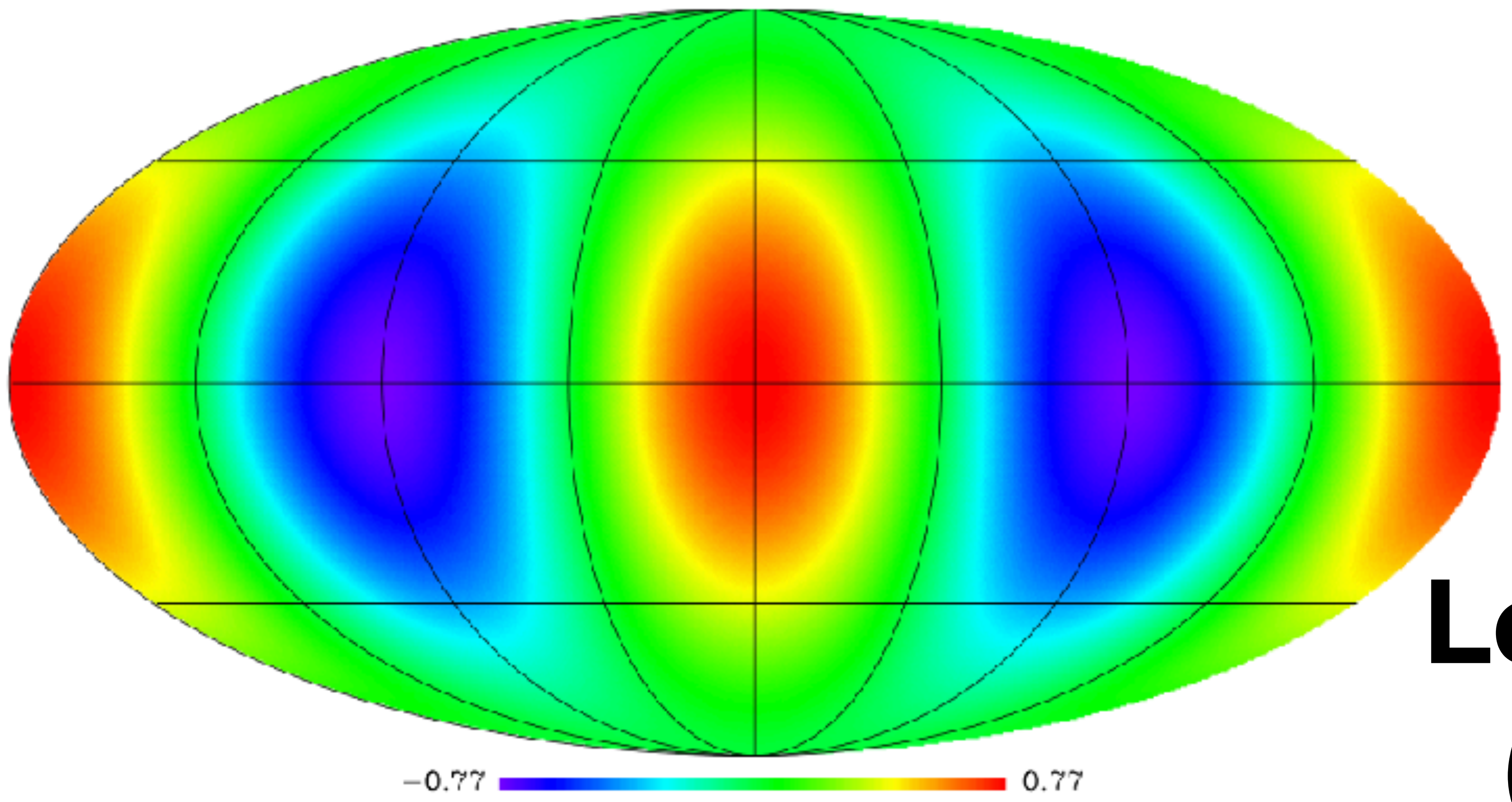
(l,m)=(2,0)

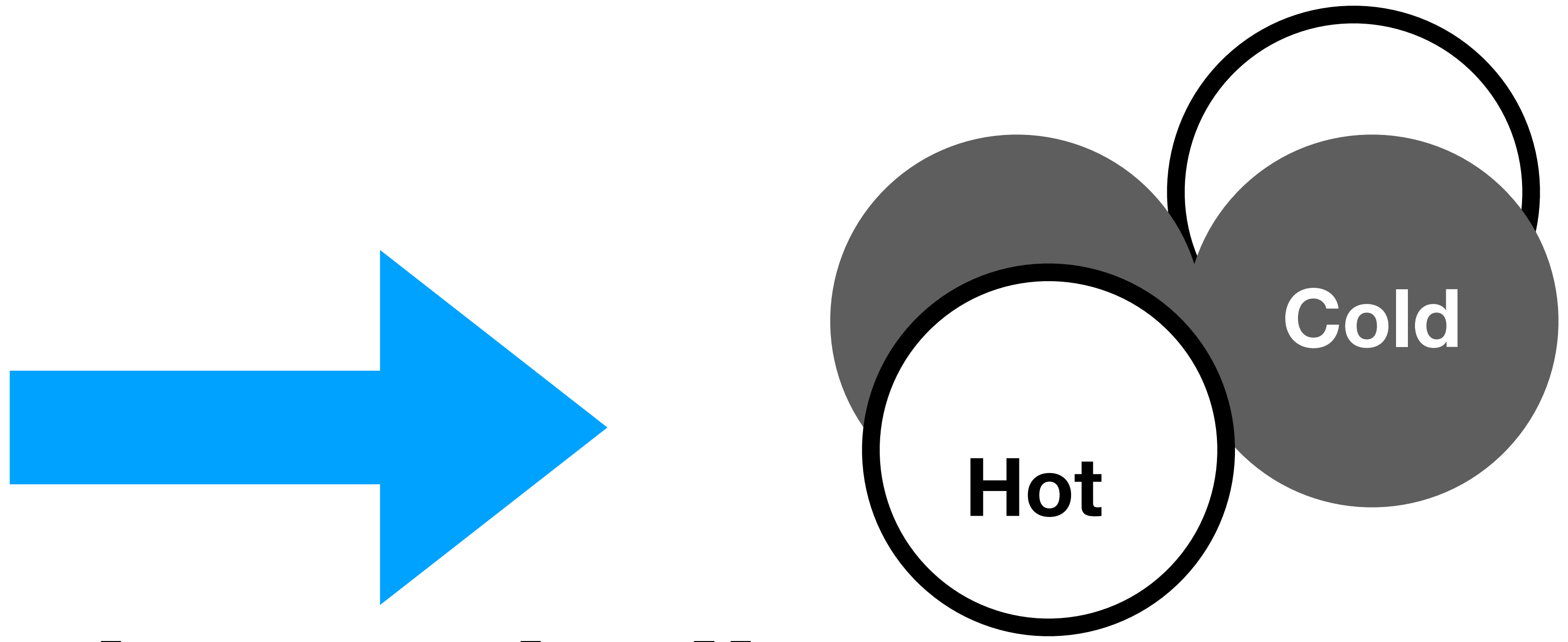


(l,m)=(2,1)

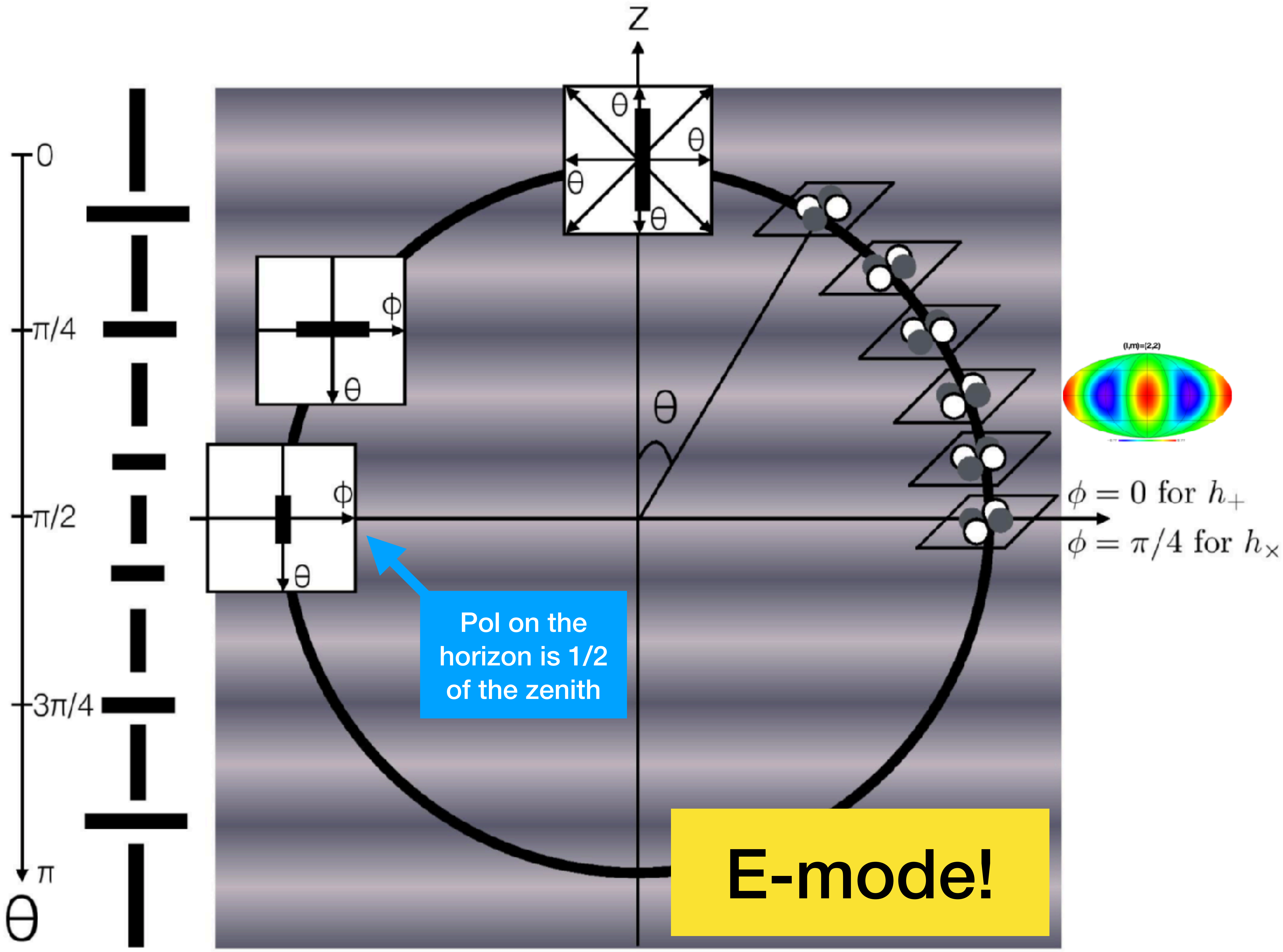


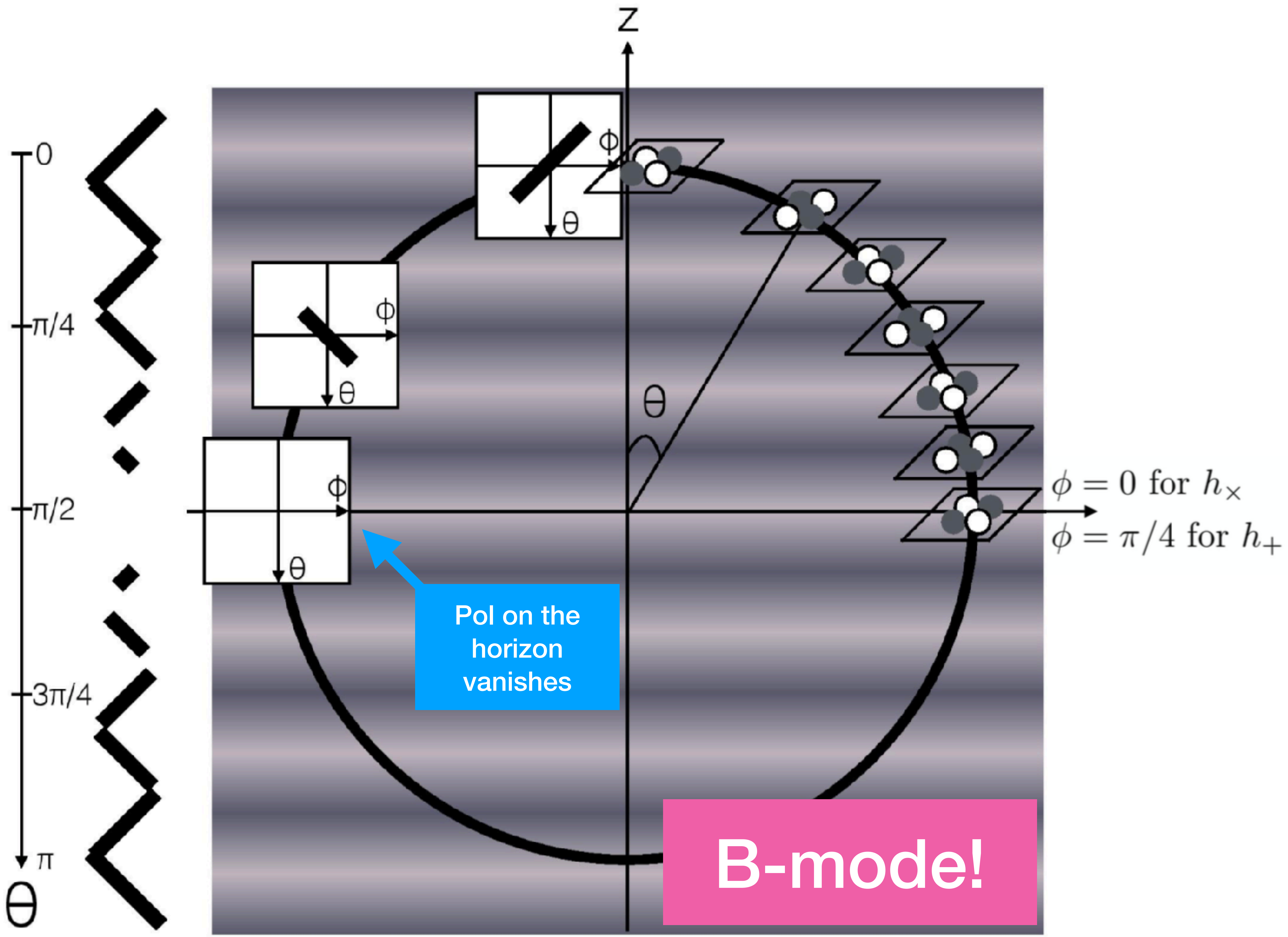
(l,m)=(2,2)



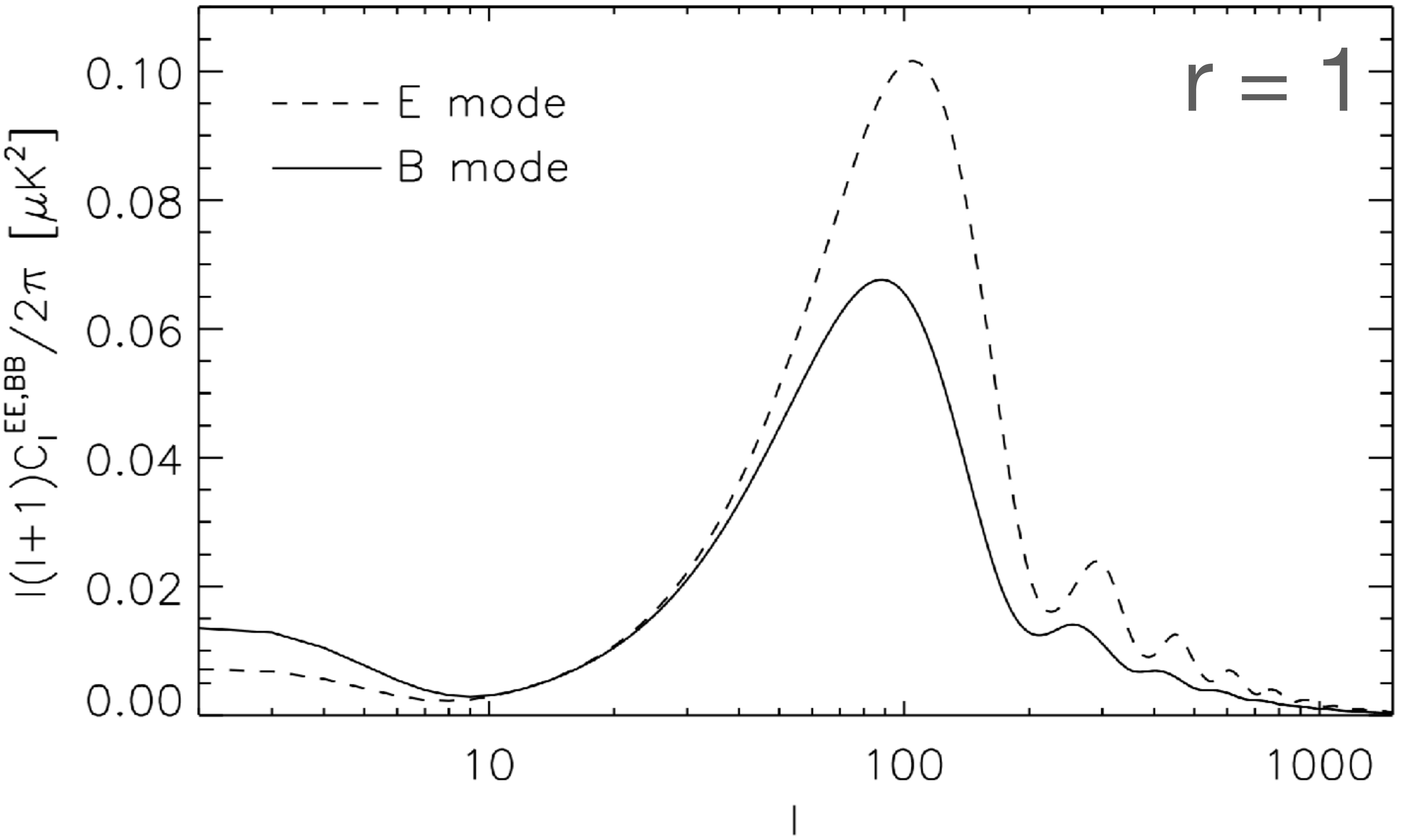


Let’s symbolise (l,m)=(2,2) as

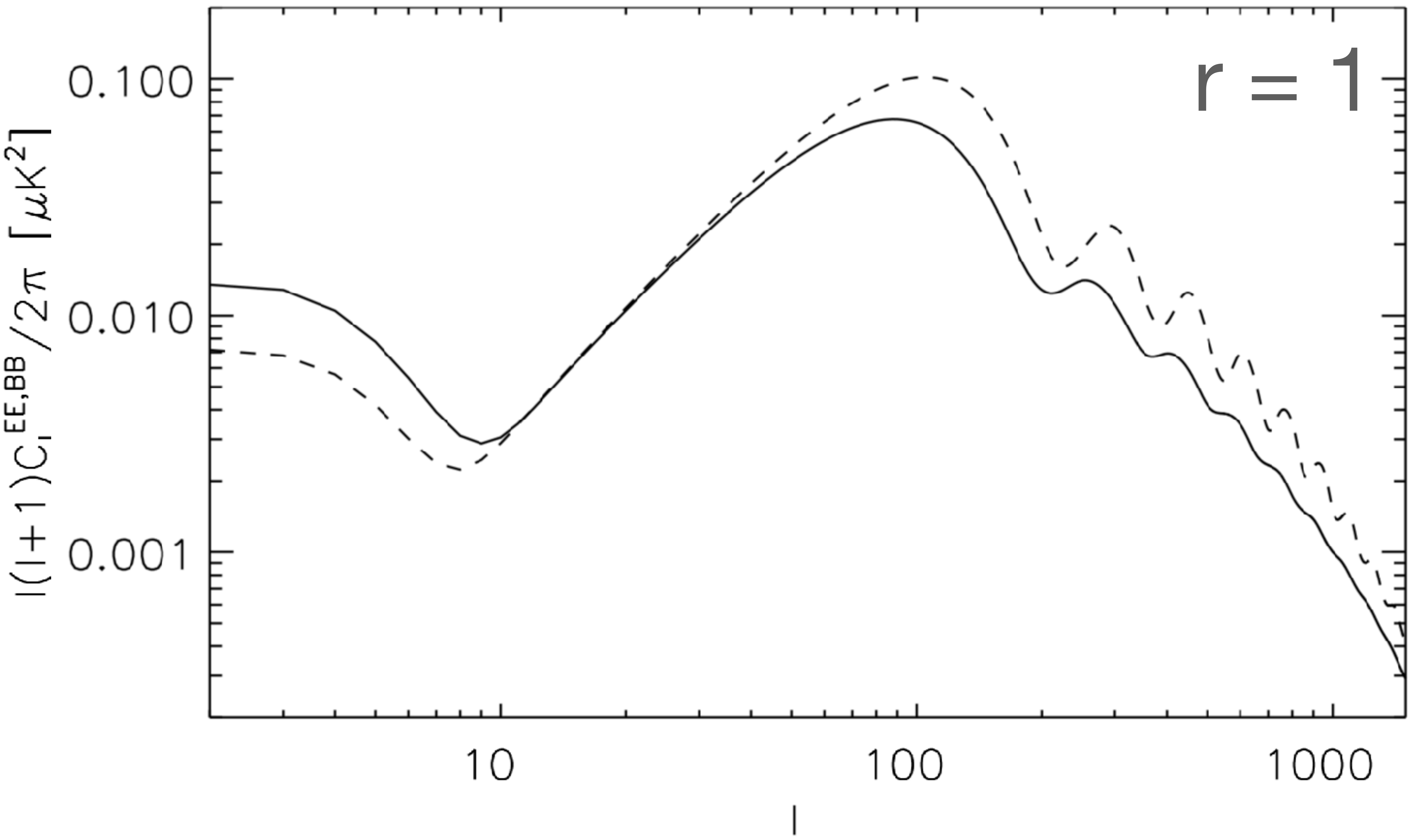

Z
0
π/4
π/2
3π/4
π
θ
φ
θ
$\phi = 0$ for h_+
$\phi = \pi/4$ for $h_\times$
Pol on the horizon is 1/2 of the zenith
E-mode!



z
0
π/4
π/2
3π/4
π
θ
ϕ
θ
$\phi = 0$ for $h_\times$
$\phi = \pi/4$ for h_+
Pol on the horizon vanishes
B-mode!



- E and B modes are produced nearly equally, but on small scales B is smaller than E because B vanishes on the horizon.



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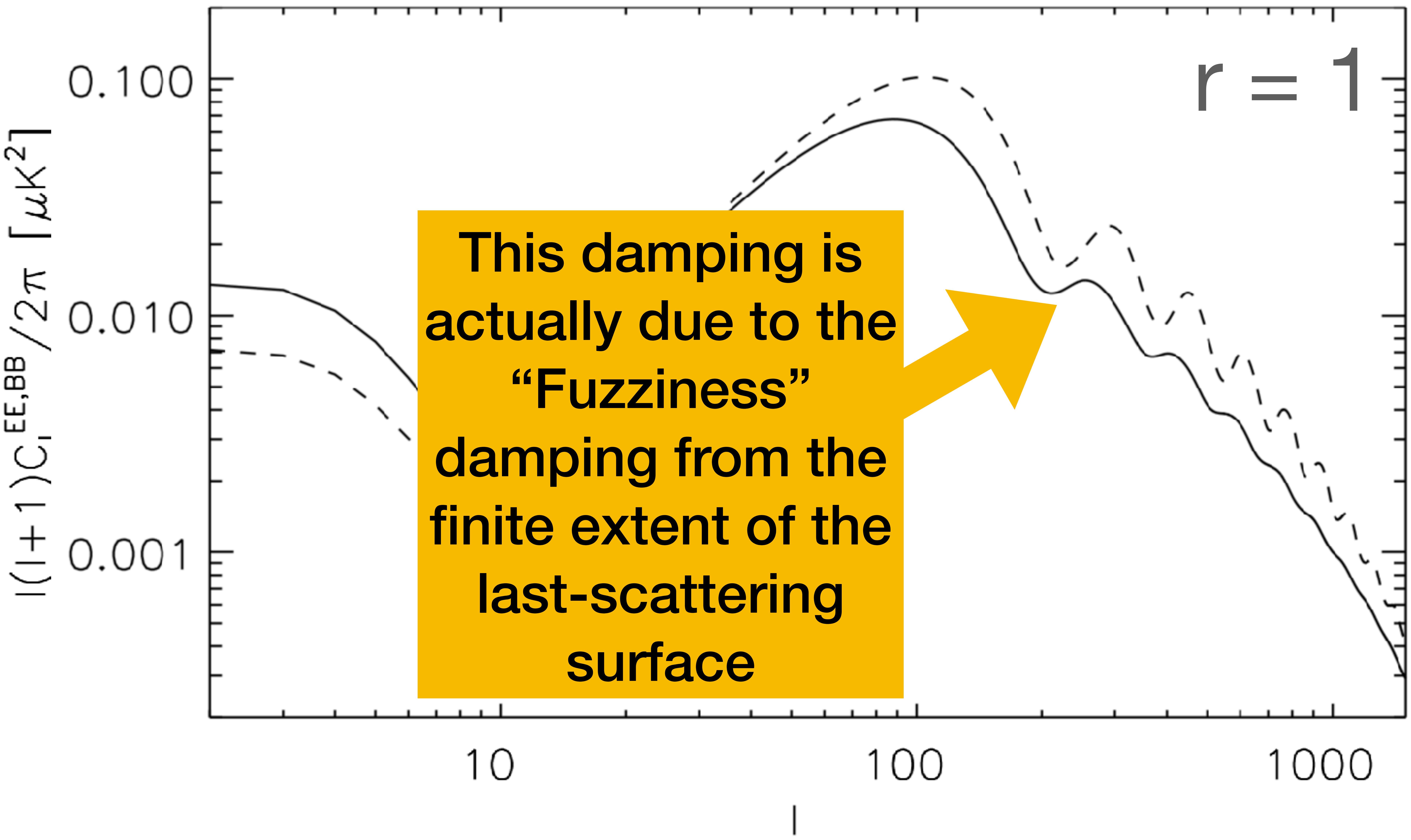

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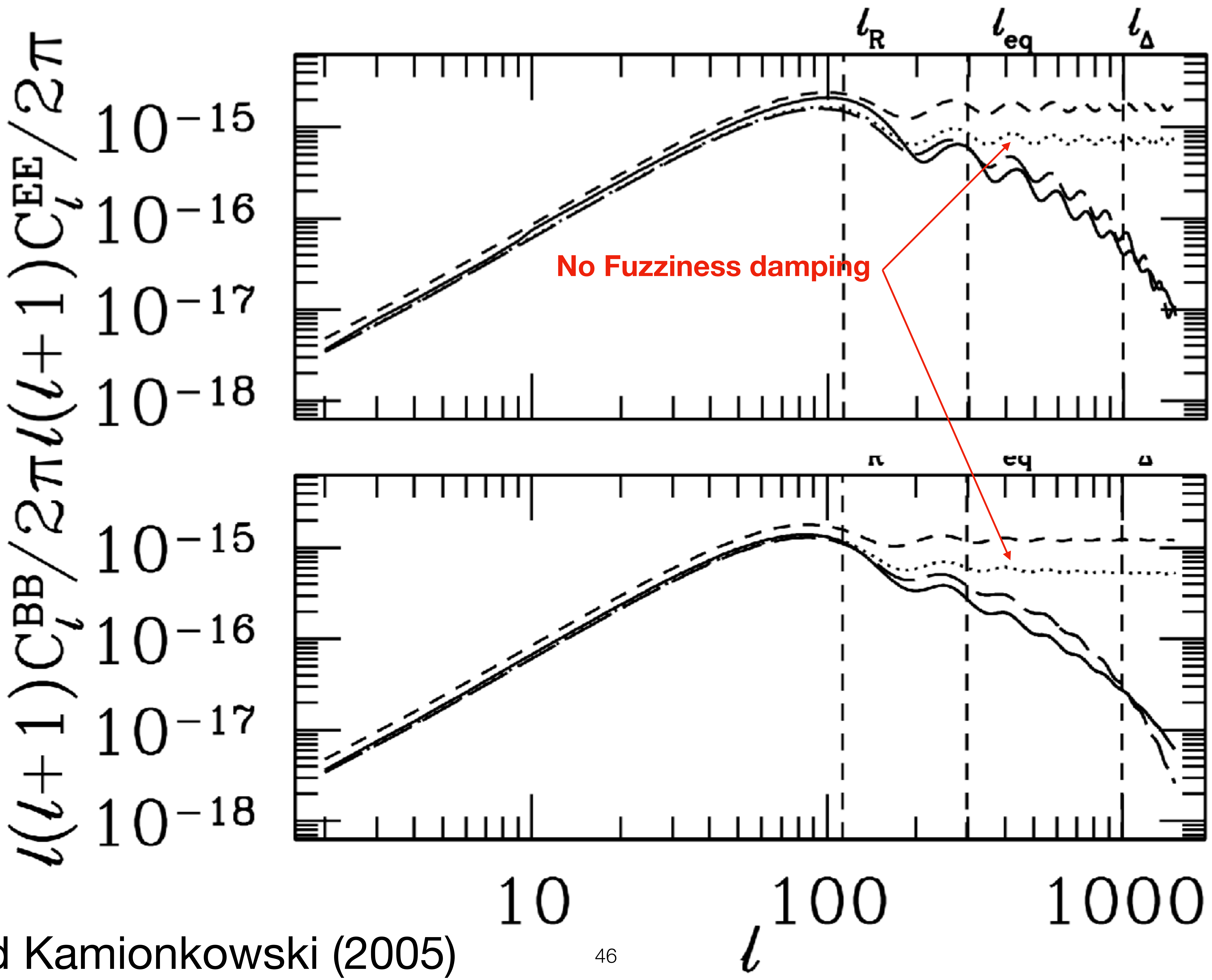
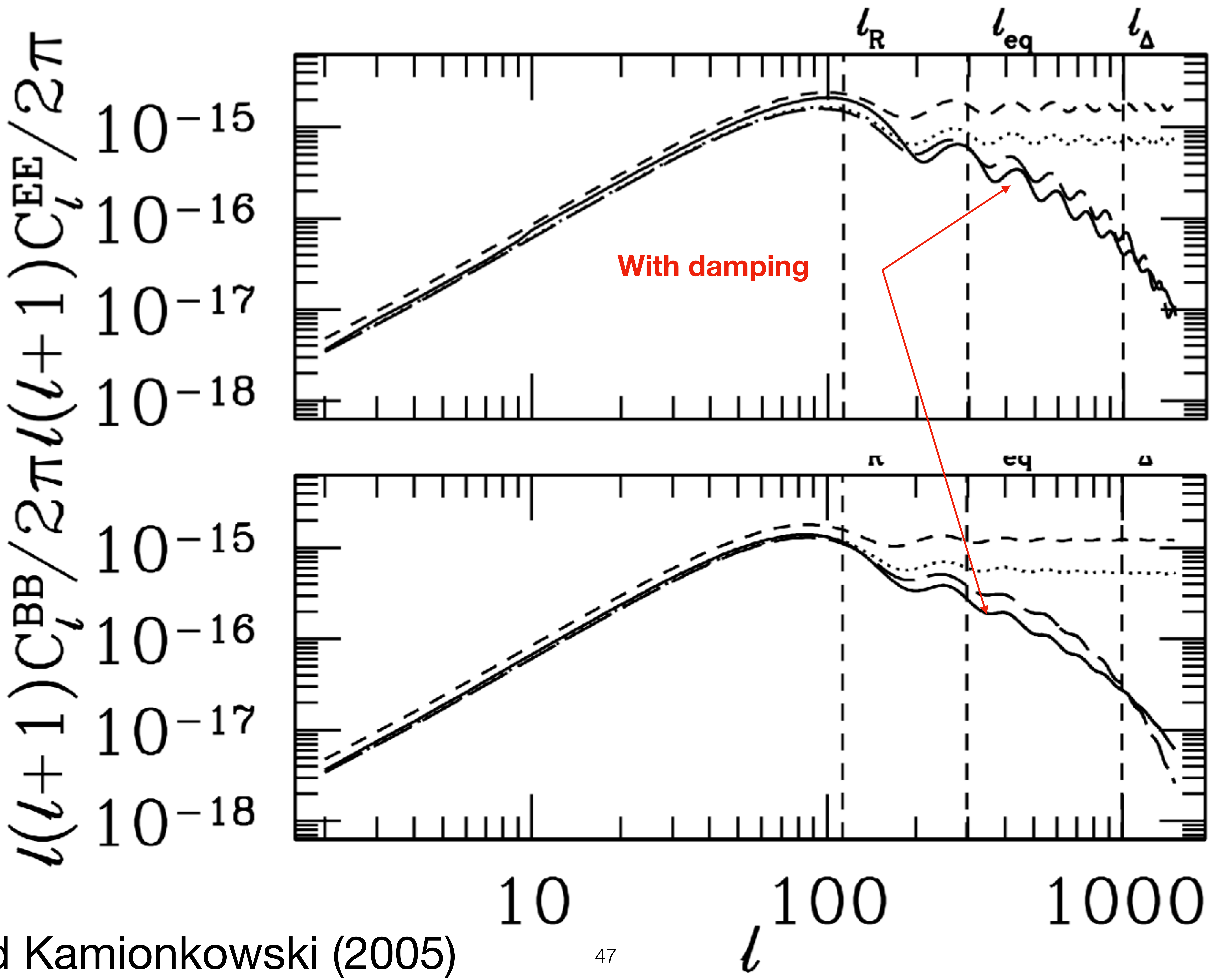
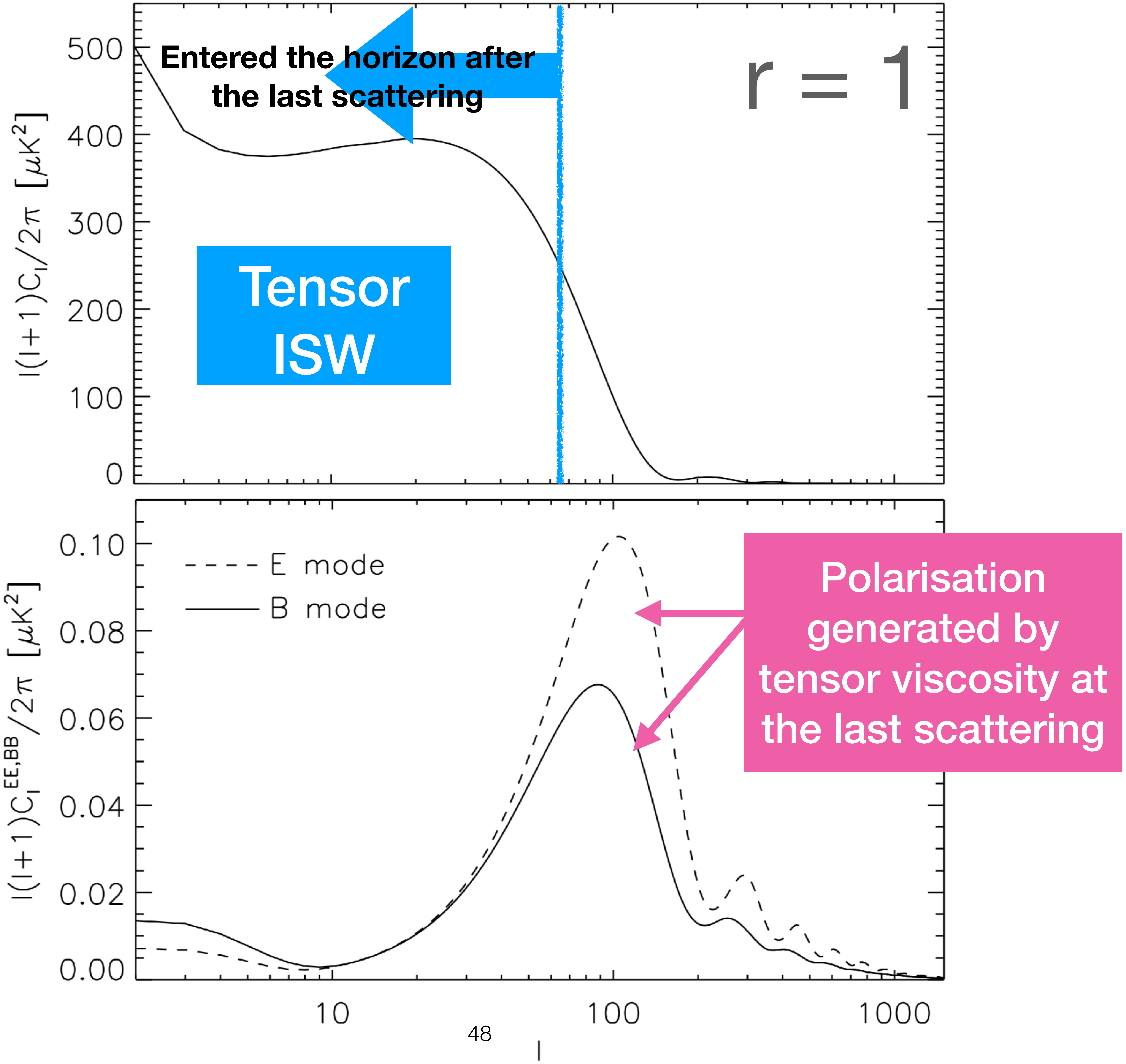
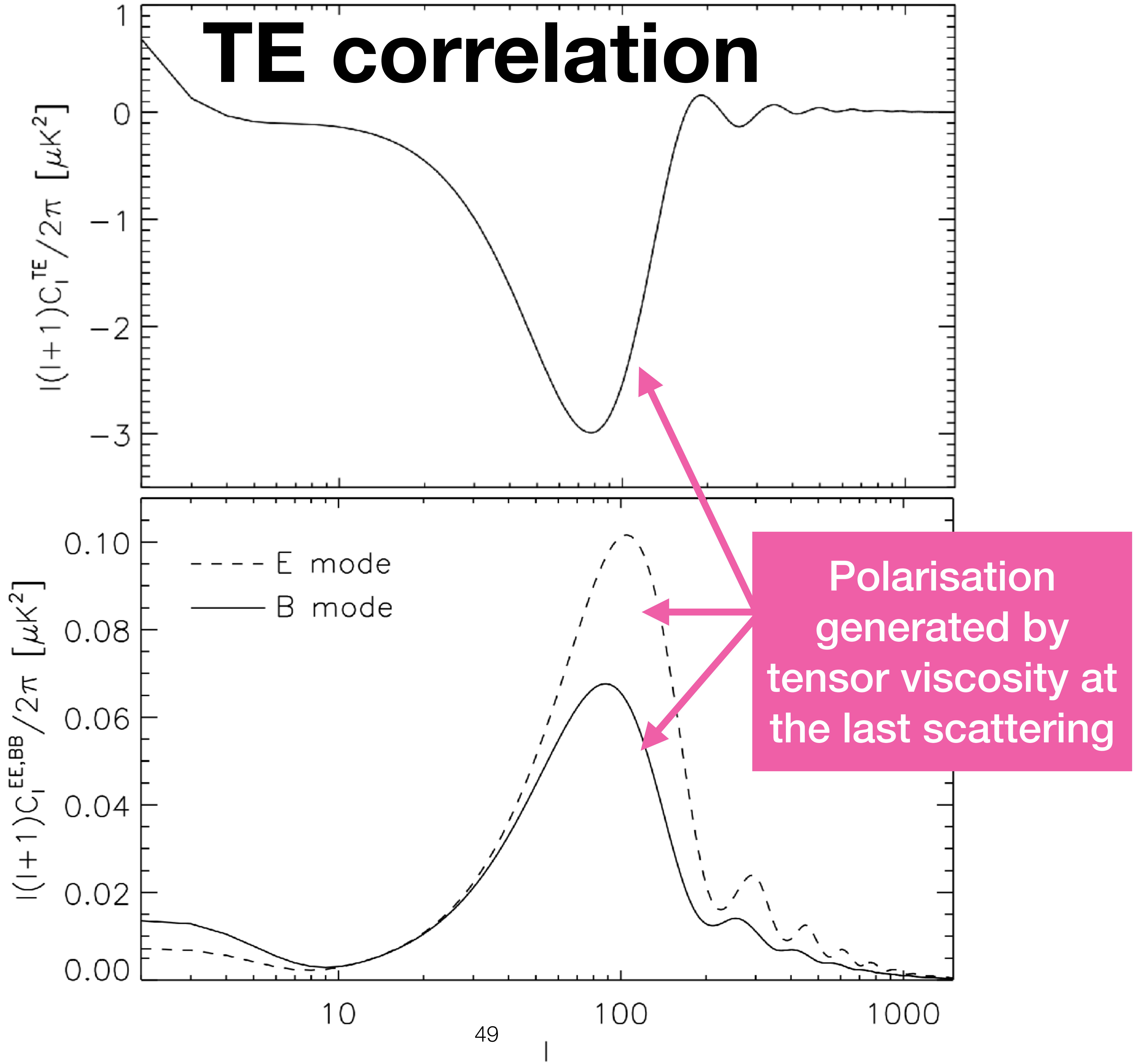


Pritchard and Kamionkowski (2005)

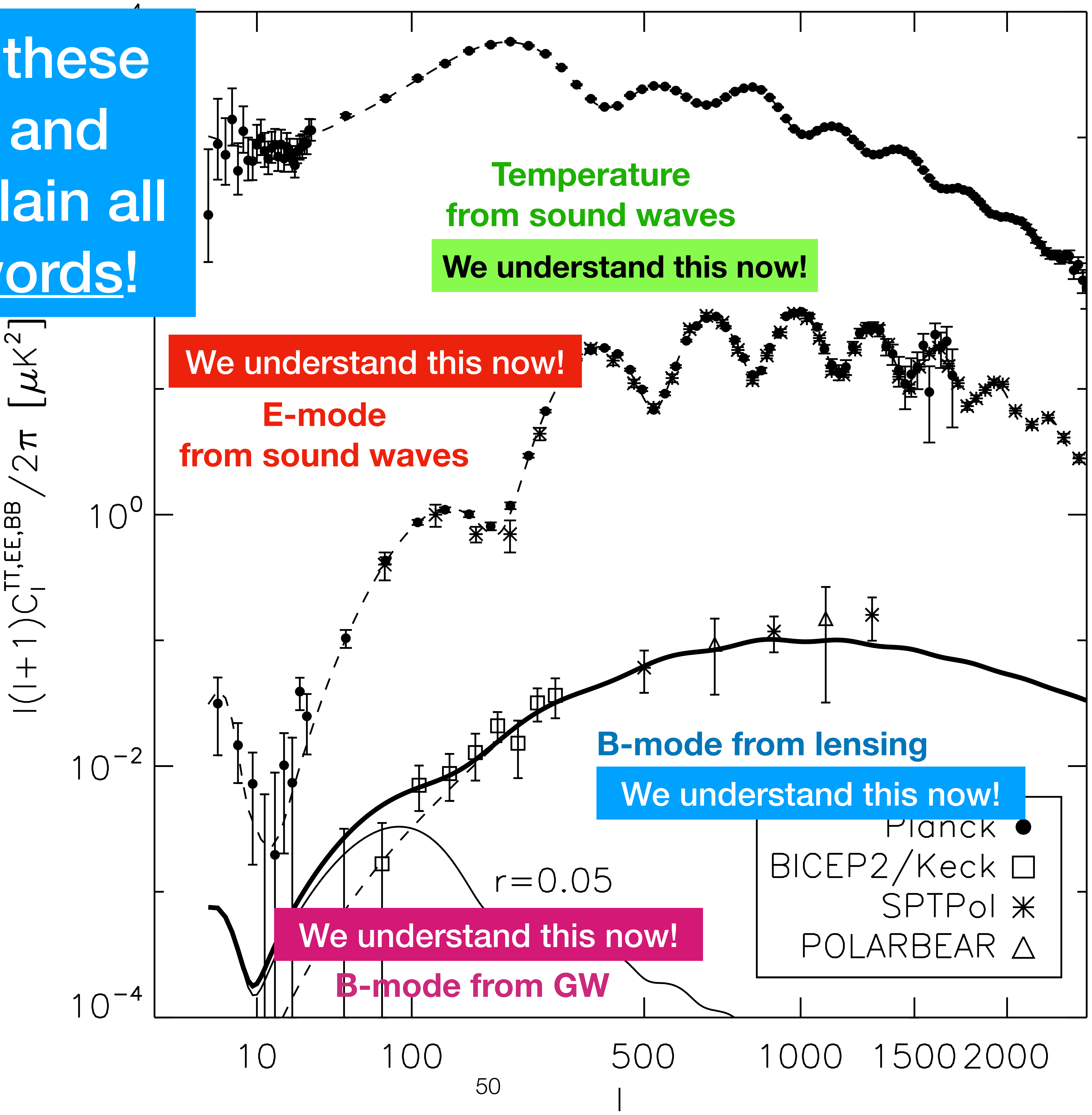


Pritchard and Kamionkowski (2005)

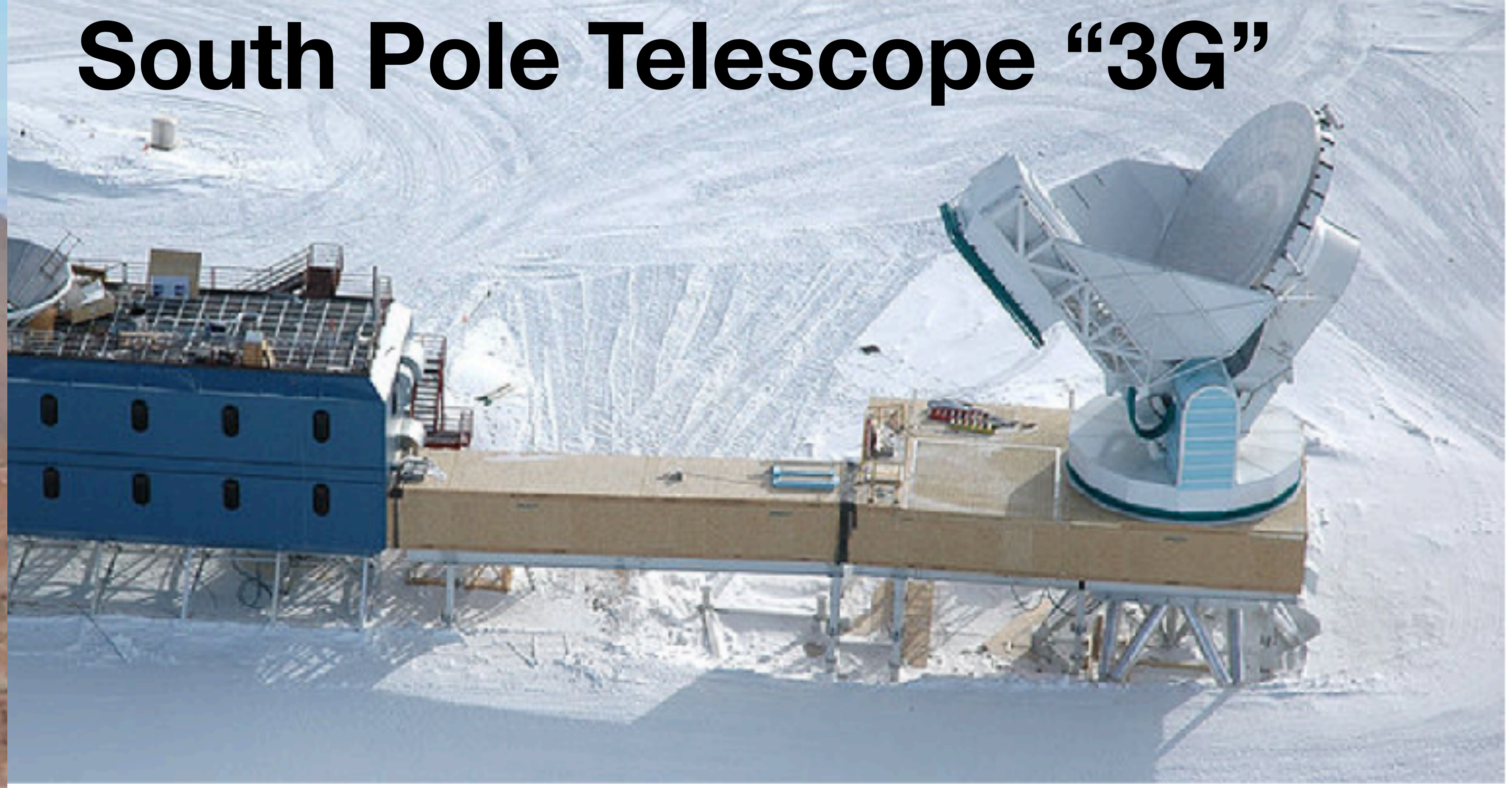

Entered the horizon after the last scattering
r = 1
Tensor ISW
l(l+1)C_l/2π [μK²]
500
400
300
200
100
0
E mode
B mode
Polarisation generated by tensor viscosity at the last scattering
l(l+1)C_l^{EE,BB}/2π [μK²]
0.10
0.08
0.06
0.04
0.02
0.00
10
100
1000
l


TE correlation
Polarisation generated by tensor viscosity at the last scattering
E mode
B mode
l

Enjoy starting at these power spectra, and being able to explain all the features <u>in words</u>!

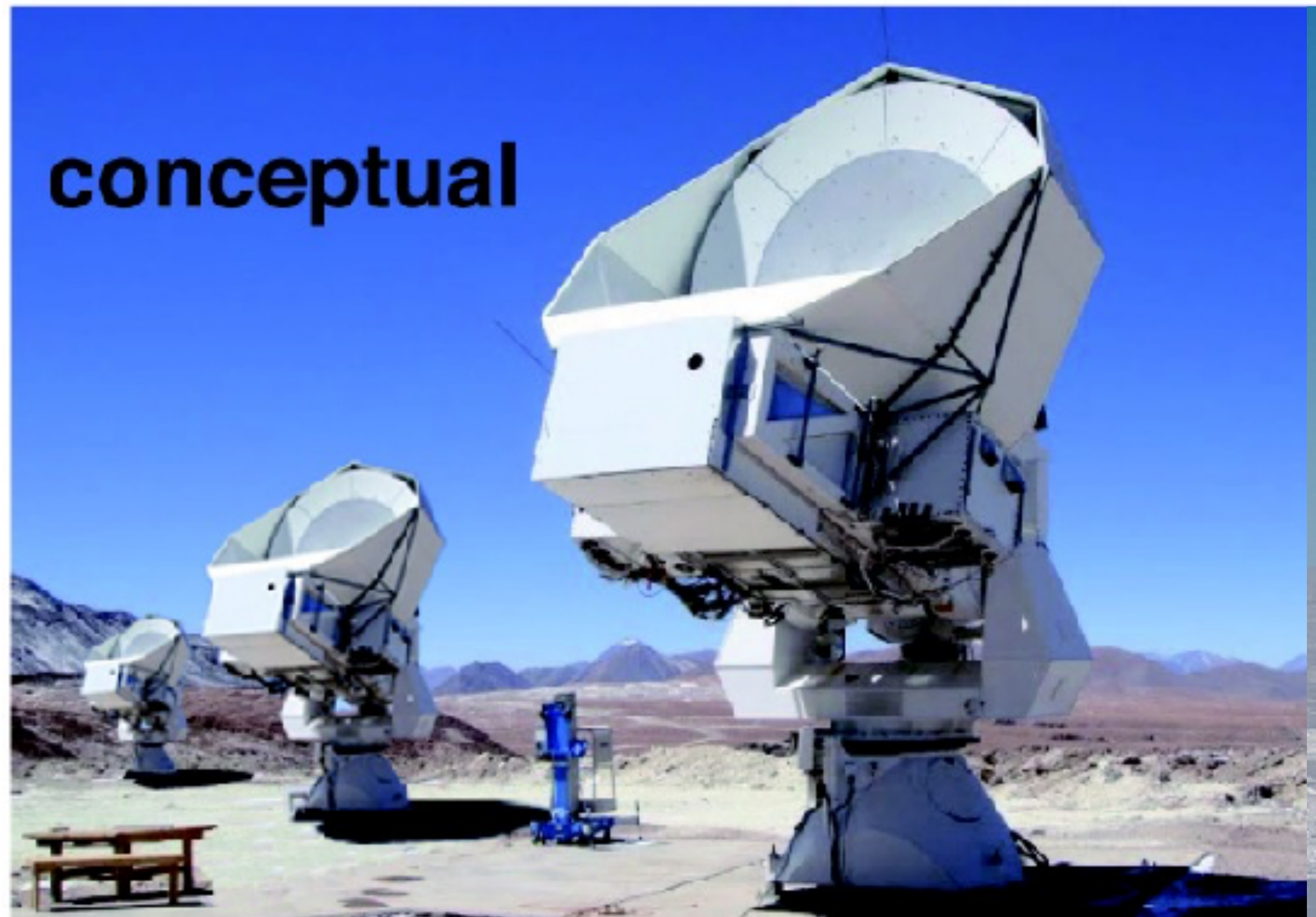


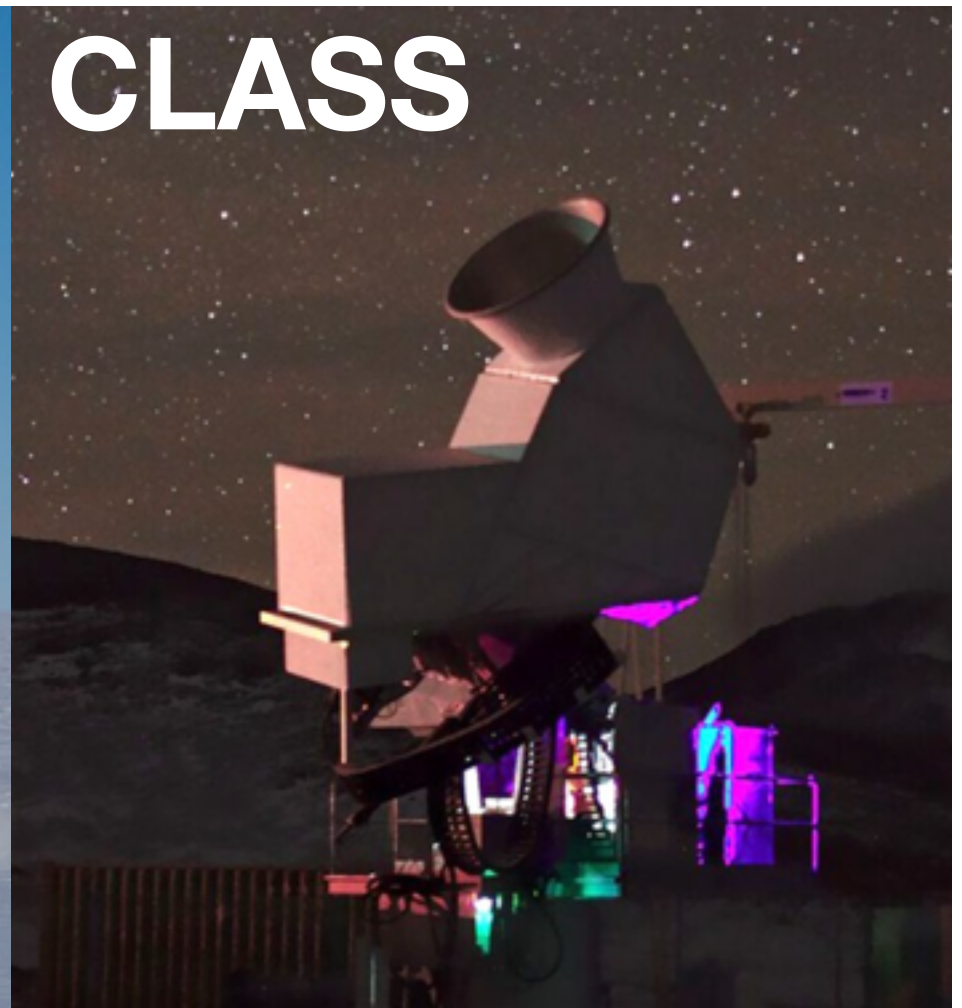
Appendix:
Experimental Landscape



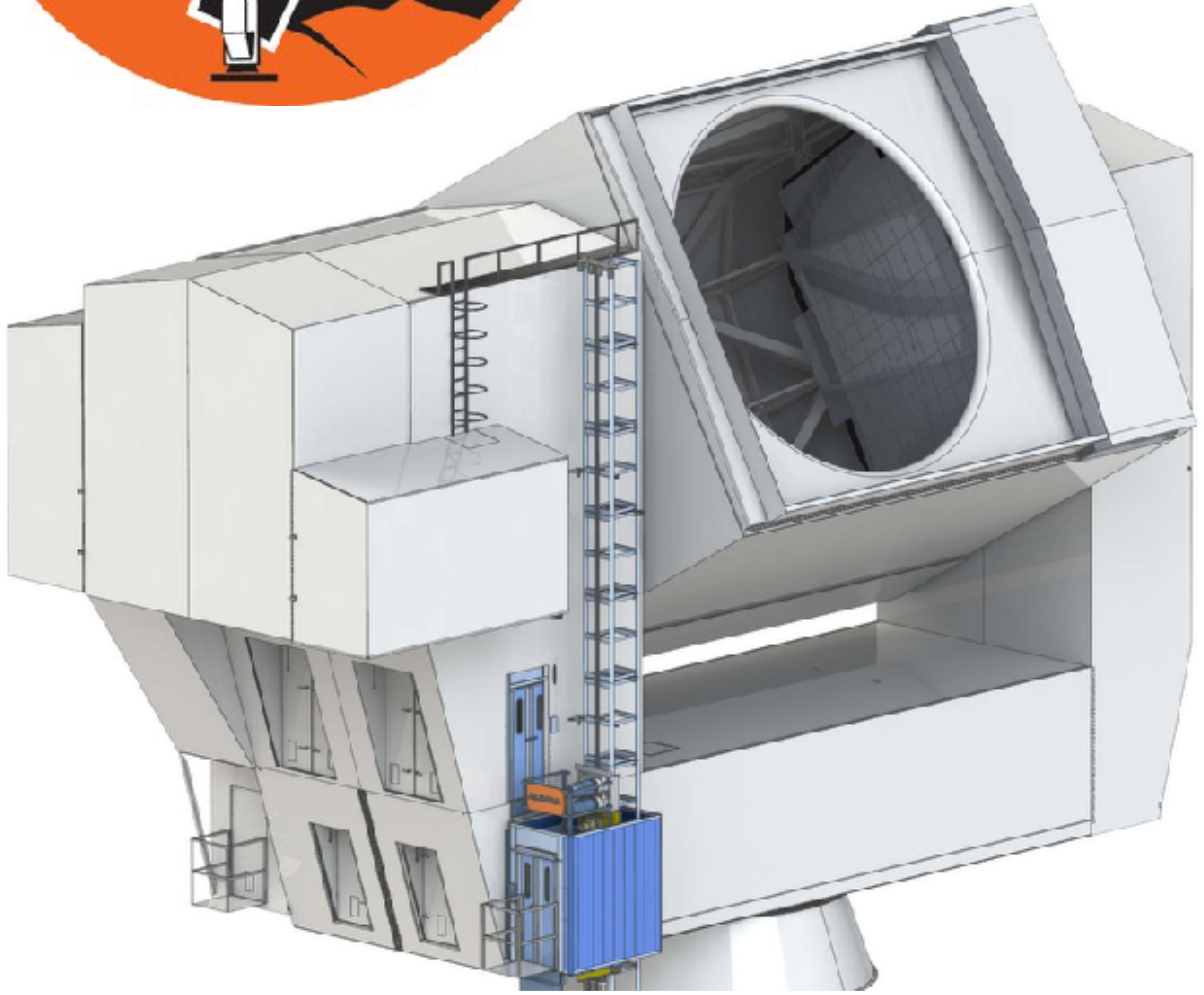
What comes next?

The Simons Array



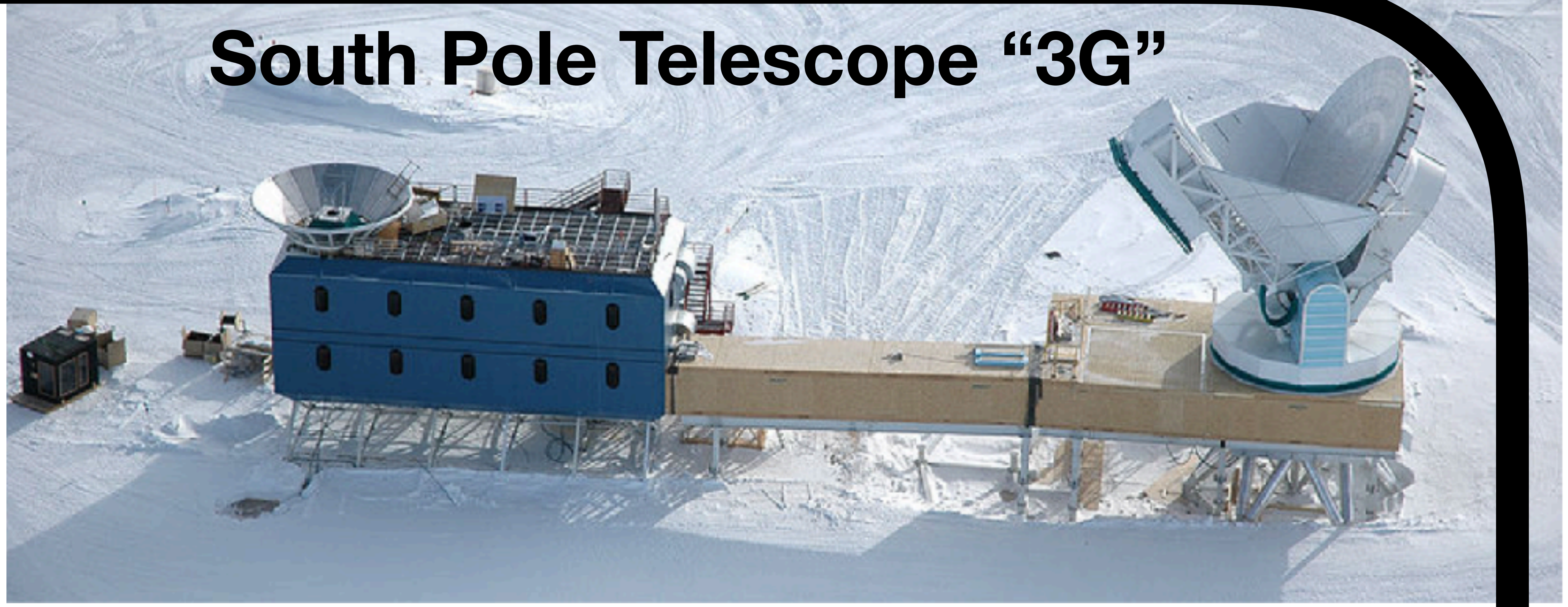


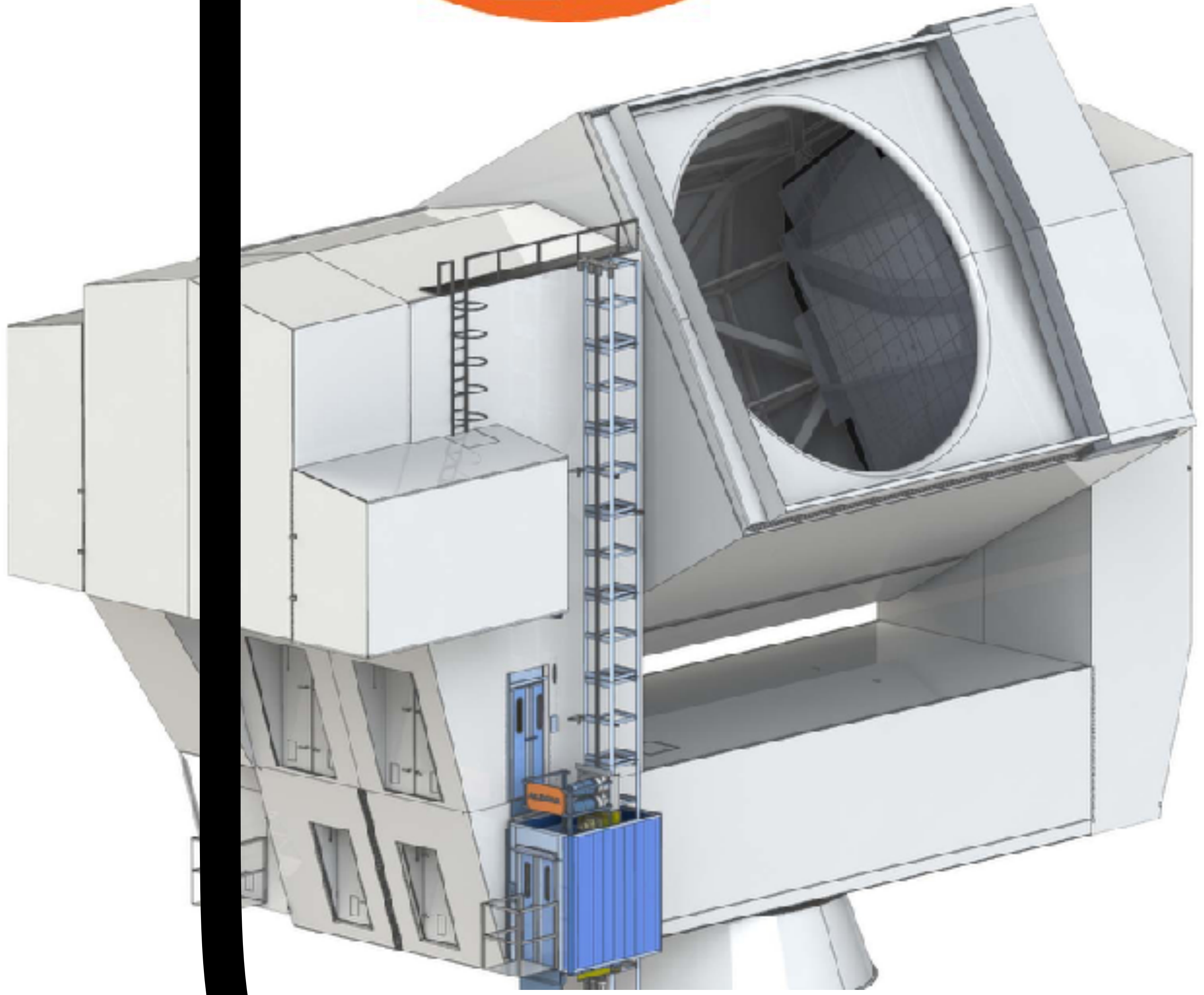
SIMONS OBSERVATORY



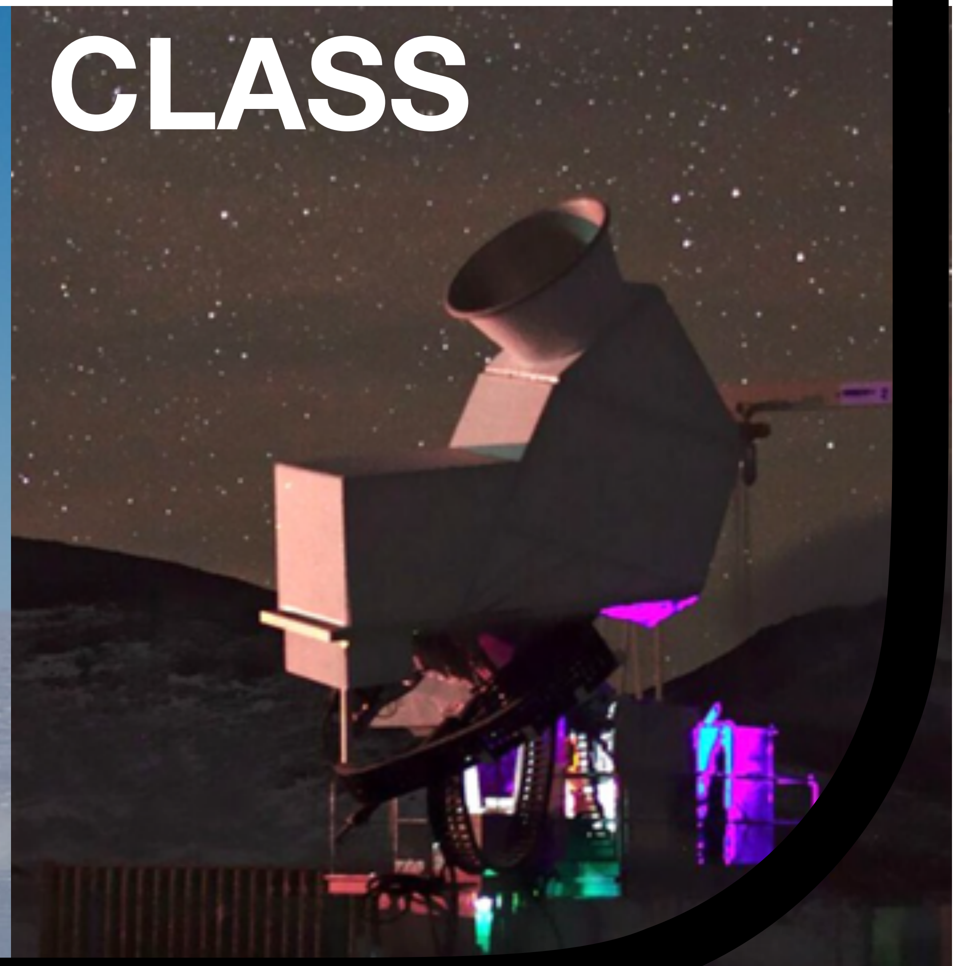
The Simons Array

SIMONS OBSERVATORY





CMB-S4(?)

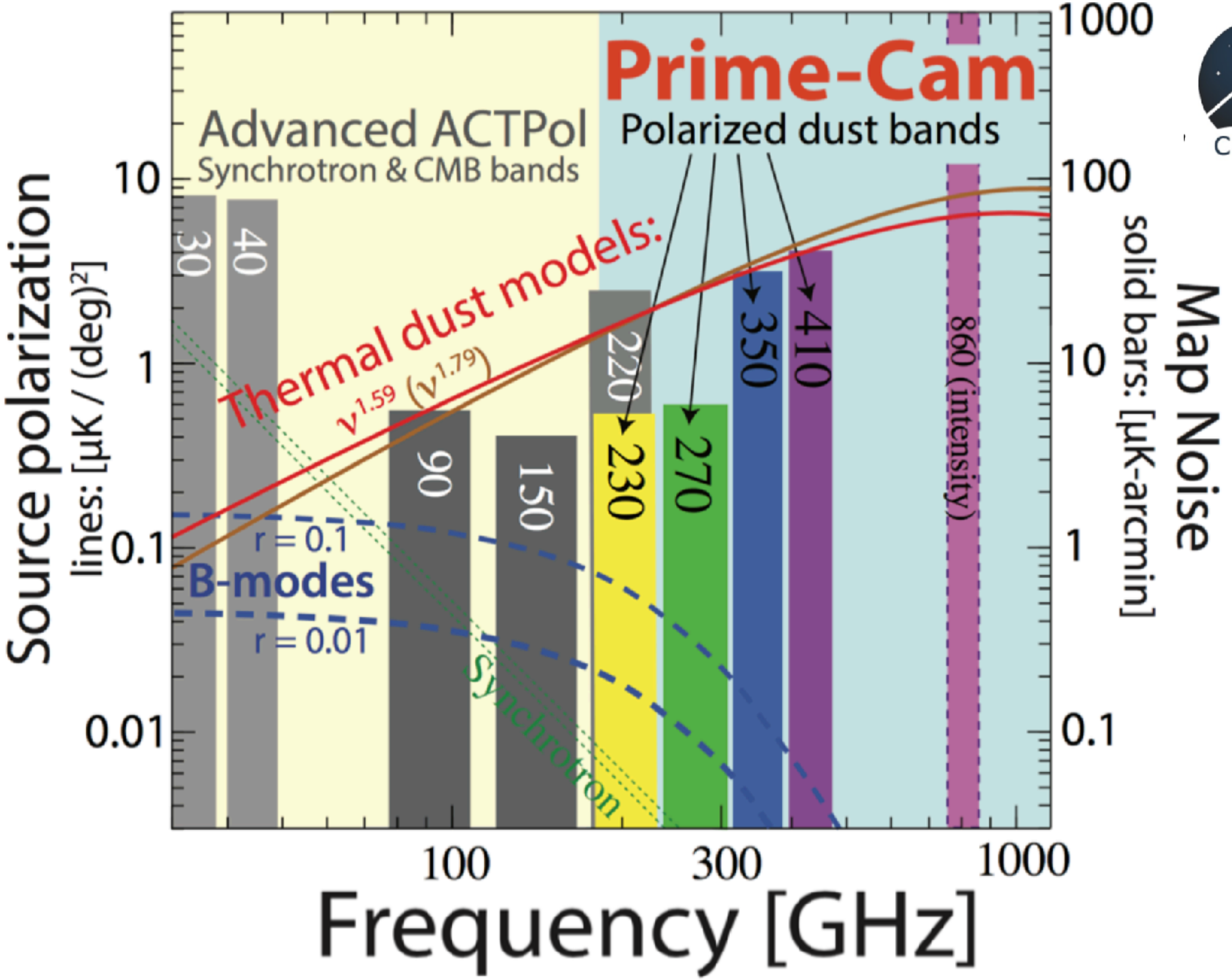


The Biggest Enemy: Polarised Dust Emission

- The upcoming data will **NOT** be limited by statistics, but by systematic effects such as the Galactic contamination
- **Solution**: Observe the sky at multiple frequencies, especially at high frequencies (>300 GHz)
- This is challenging, unless we have a superb, high-altitude site with low water vapour

- CCAT-p!

CCAT-p Collaboration



Frank Bertoldi's slide from the Florence meeting

Where is CCAT-p?

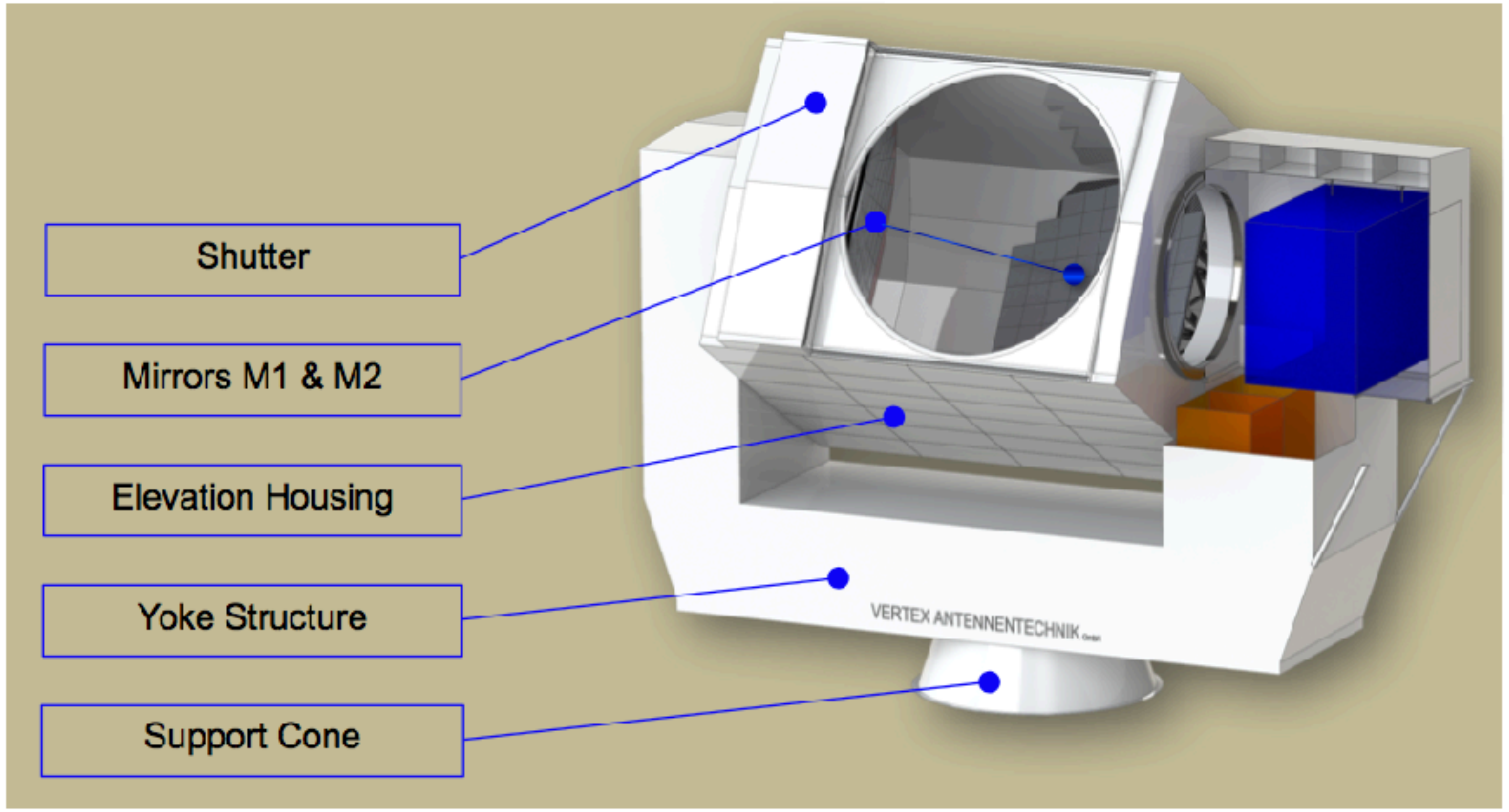
Cerro Chajnantor at 5600 m w/ TAO

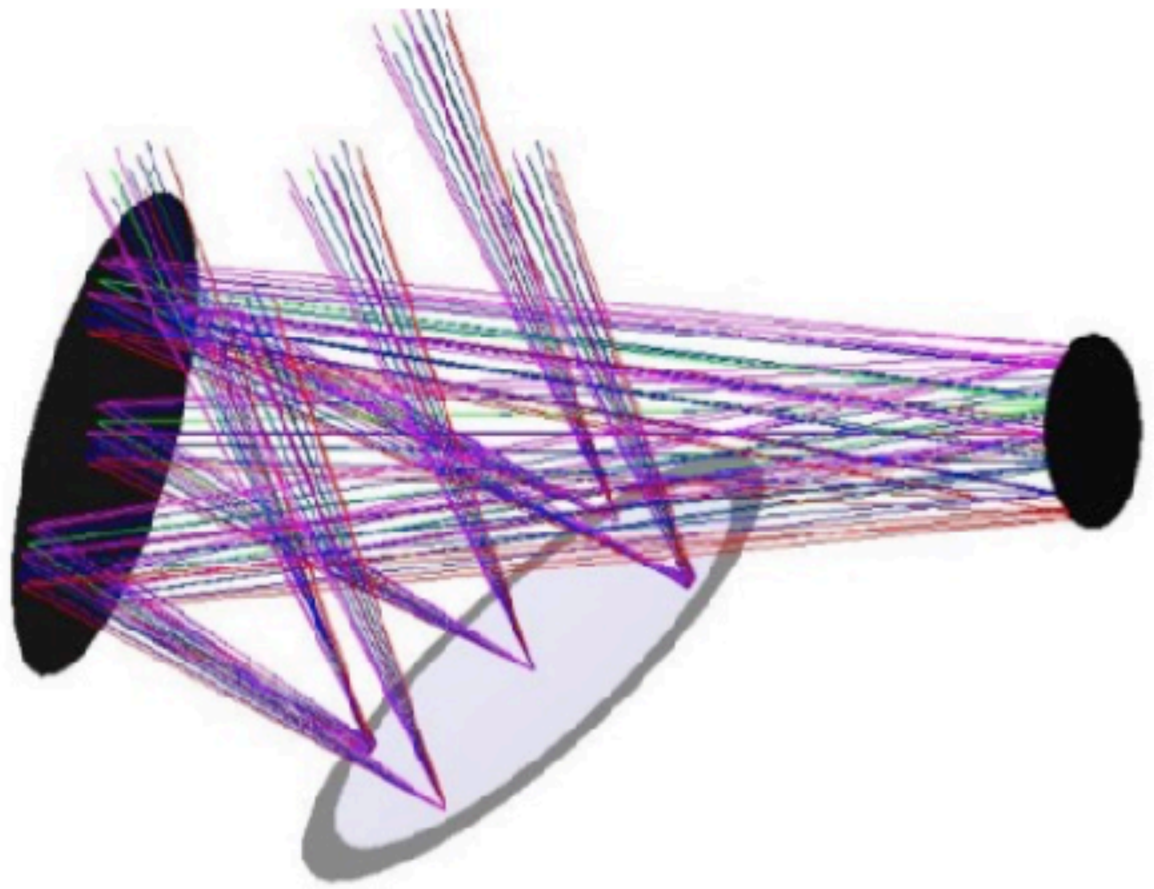
Frank Bertoldi's slide from the Florence meeting

What is CCAT-p?

CCAT-p

CCAT-prime is a high surface accuracy / throughput 6 m submm (0.3-3mm) telescope



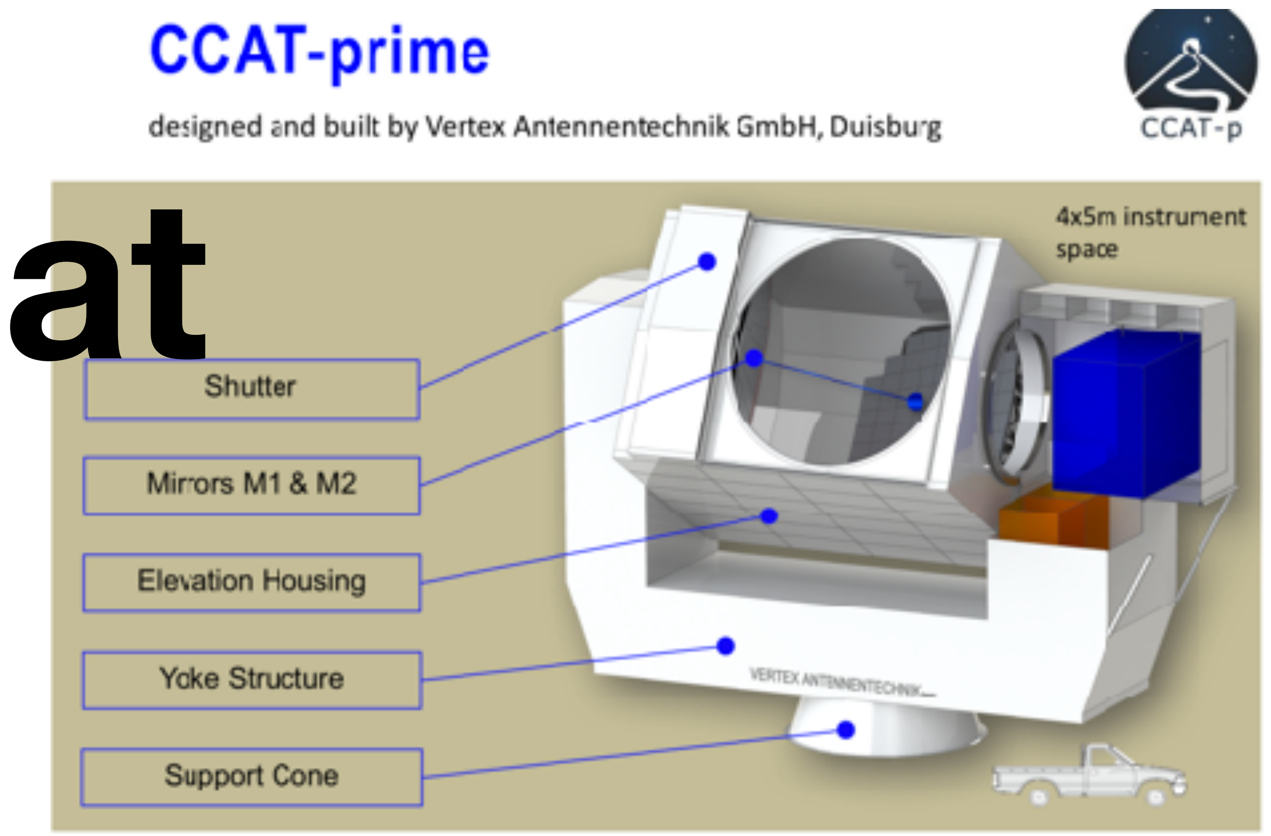


Cornell U. + German consortium + Canadian consortium + …

A Game Changer

- **CCAT-p**: 6-m, **Cross-dragone** design, on Cerro Chajnantor (5600 m)

- **Germany makes great telescopes!**



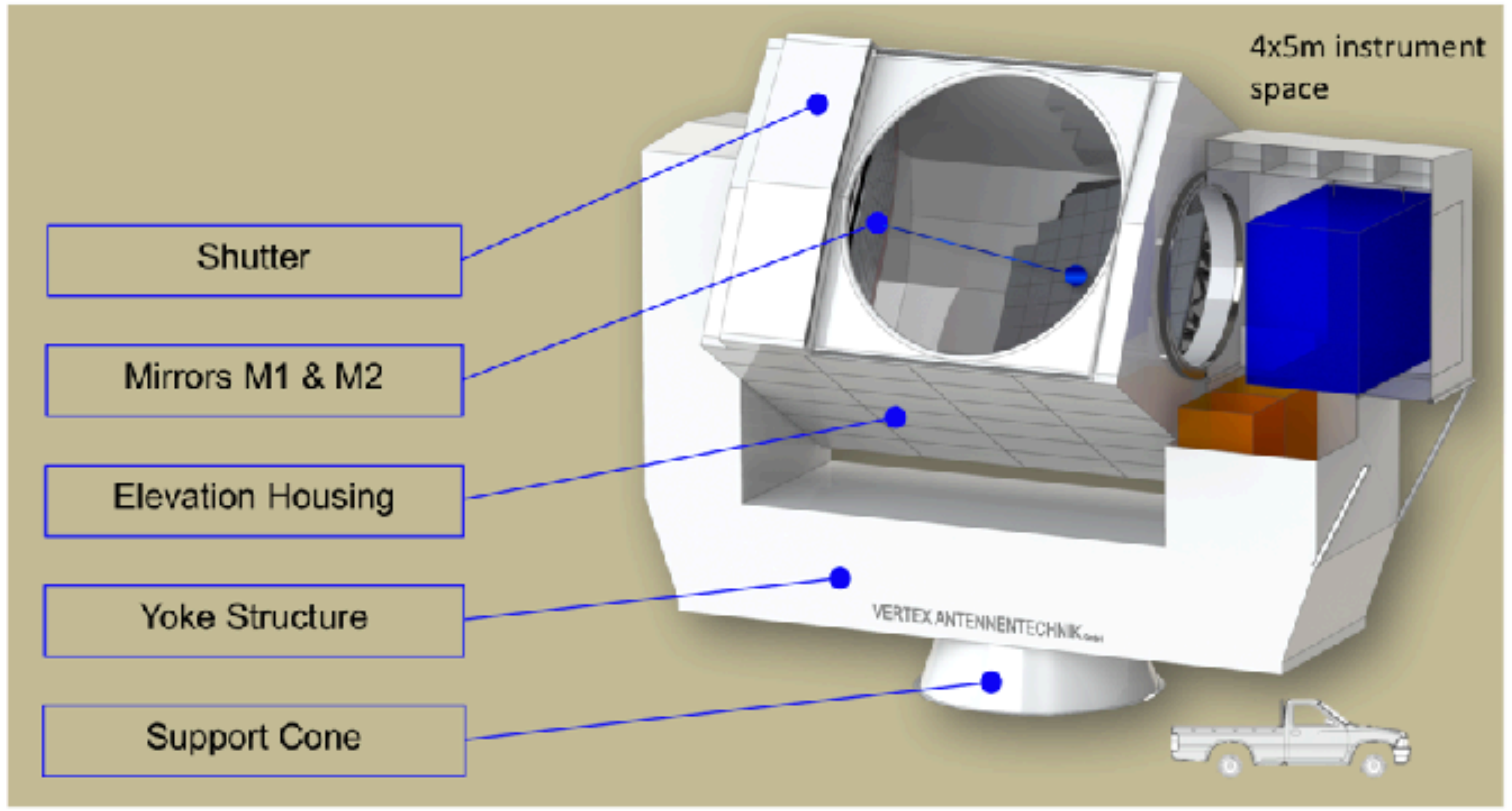
- Design study completed, and the contract has been signed by "VERTEX Antennentechnik GmbH"

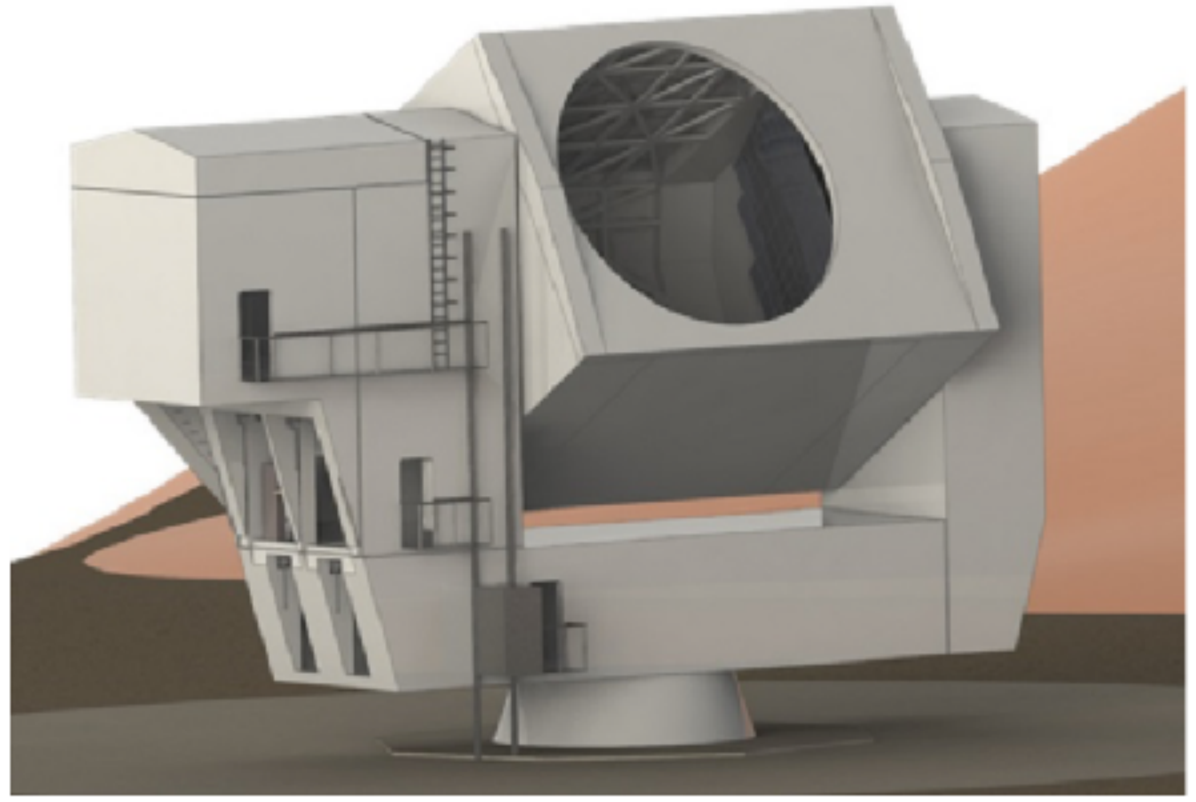
    - CCAT-p is a great opportunity for Germany to make significant contributions towards the CMB S-4 landscape (both US and Europe) by providing telescope designs and the "lessons learned" with prototypes.

CCAT-prime

designed and built by Vertex Antennentechnik GmbH, Duisburg

CCAT-p





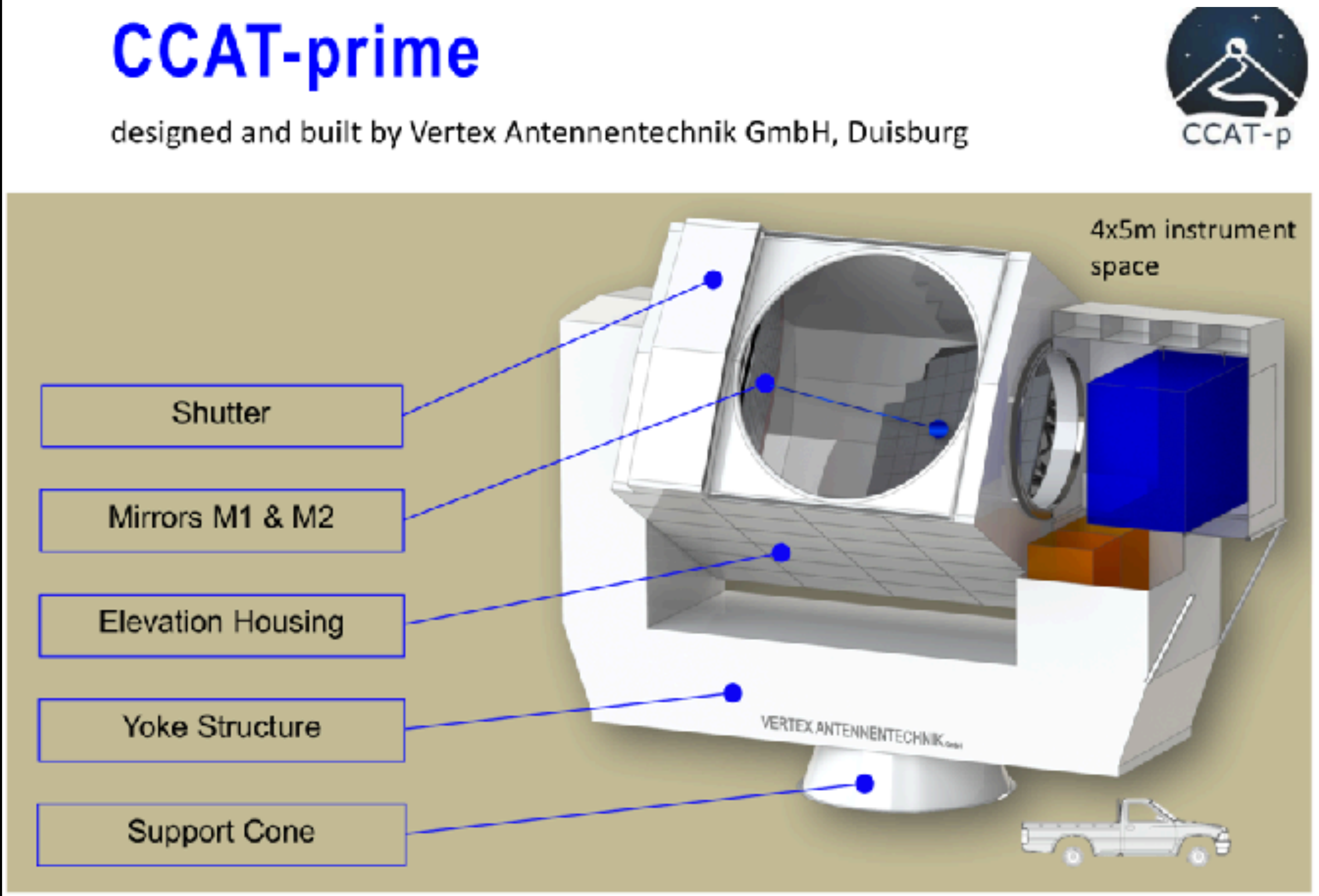
A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK.

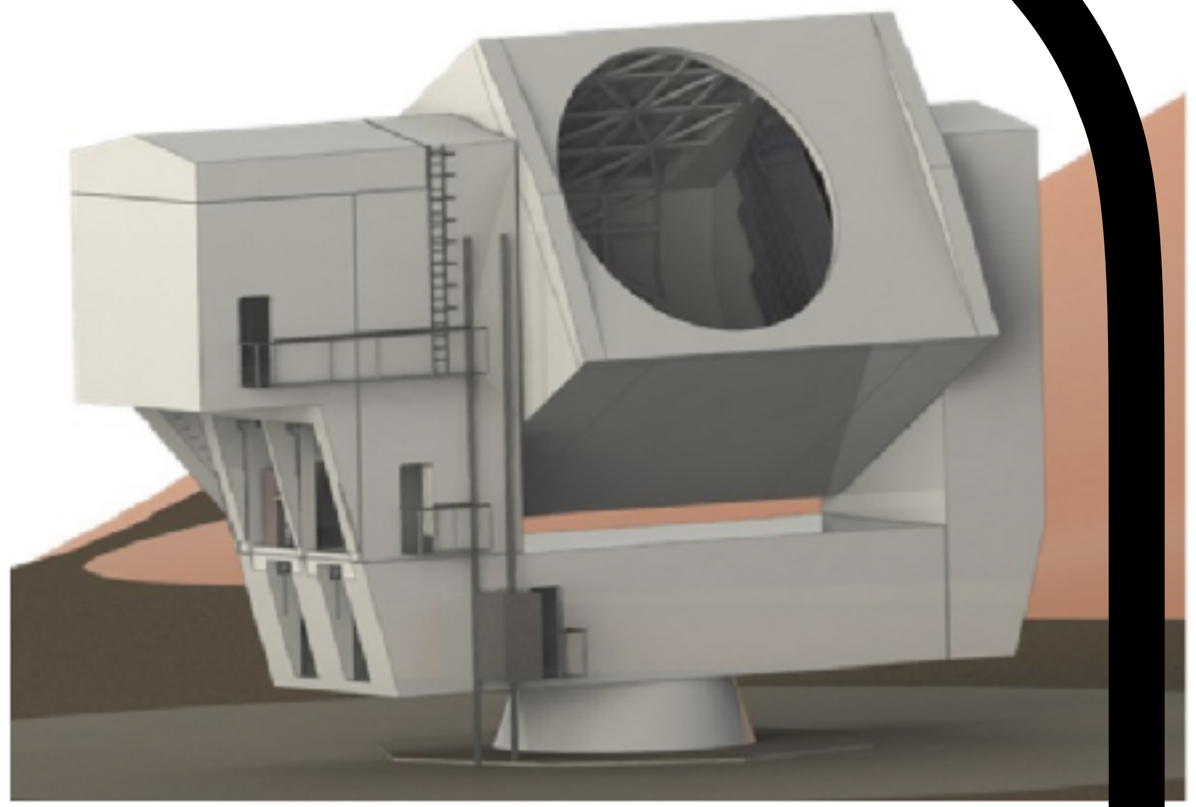
Simons Observatory (USA)

in collaboration

South Pole?

This could be “CMB-S4”





A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK.

Simons Observatory (USA)

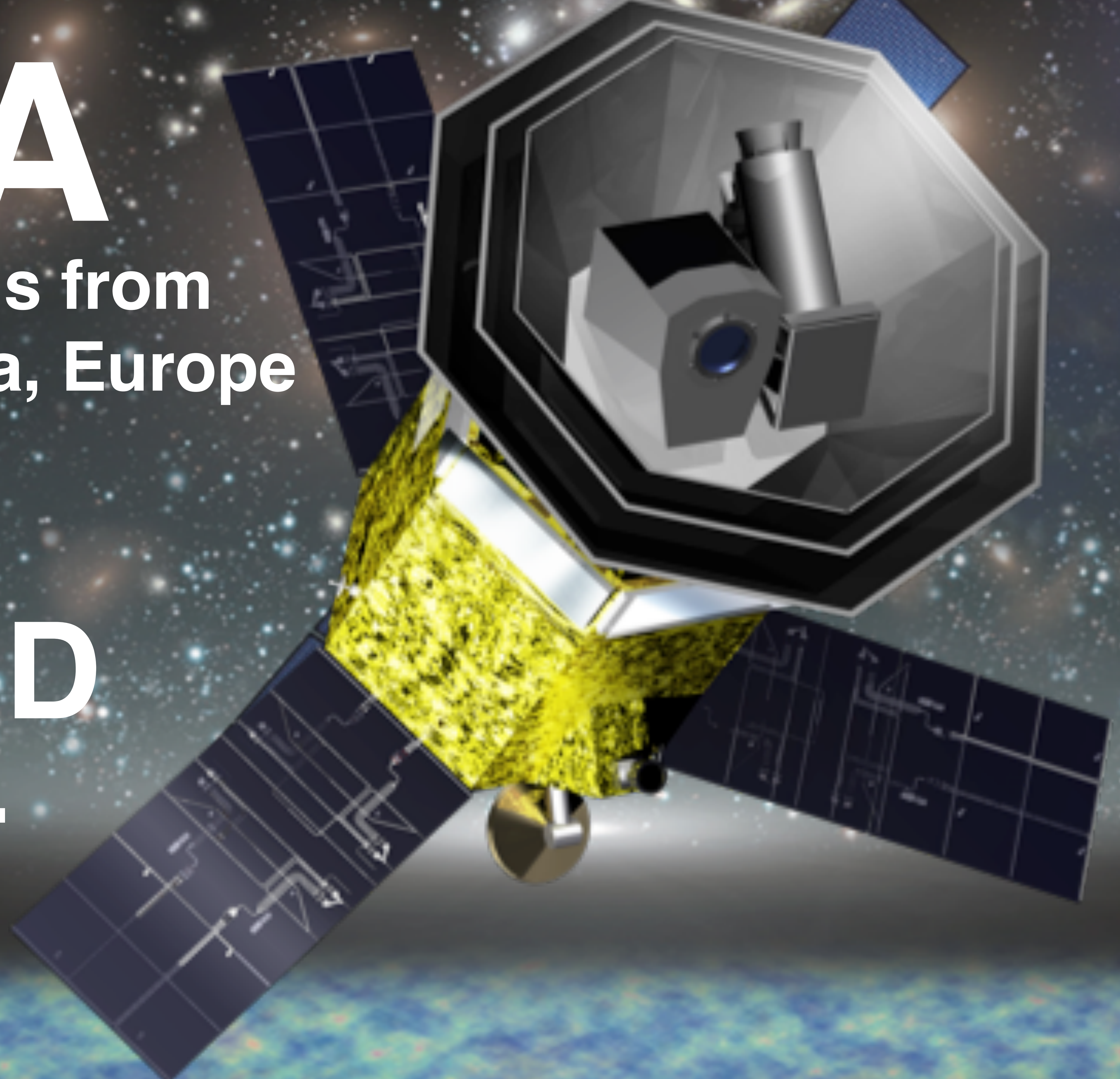
in collaboration

South Pole?

To have even more frequency coverage…

JAXA

+ participations from USA, Canada, Europe

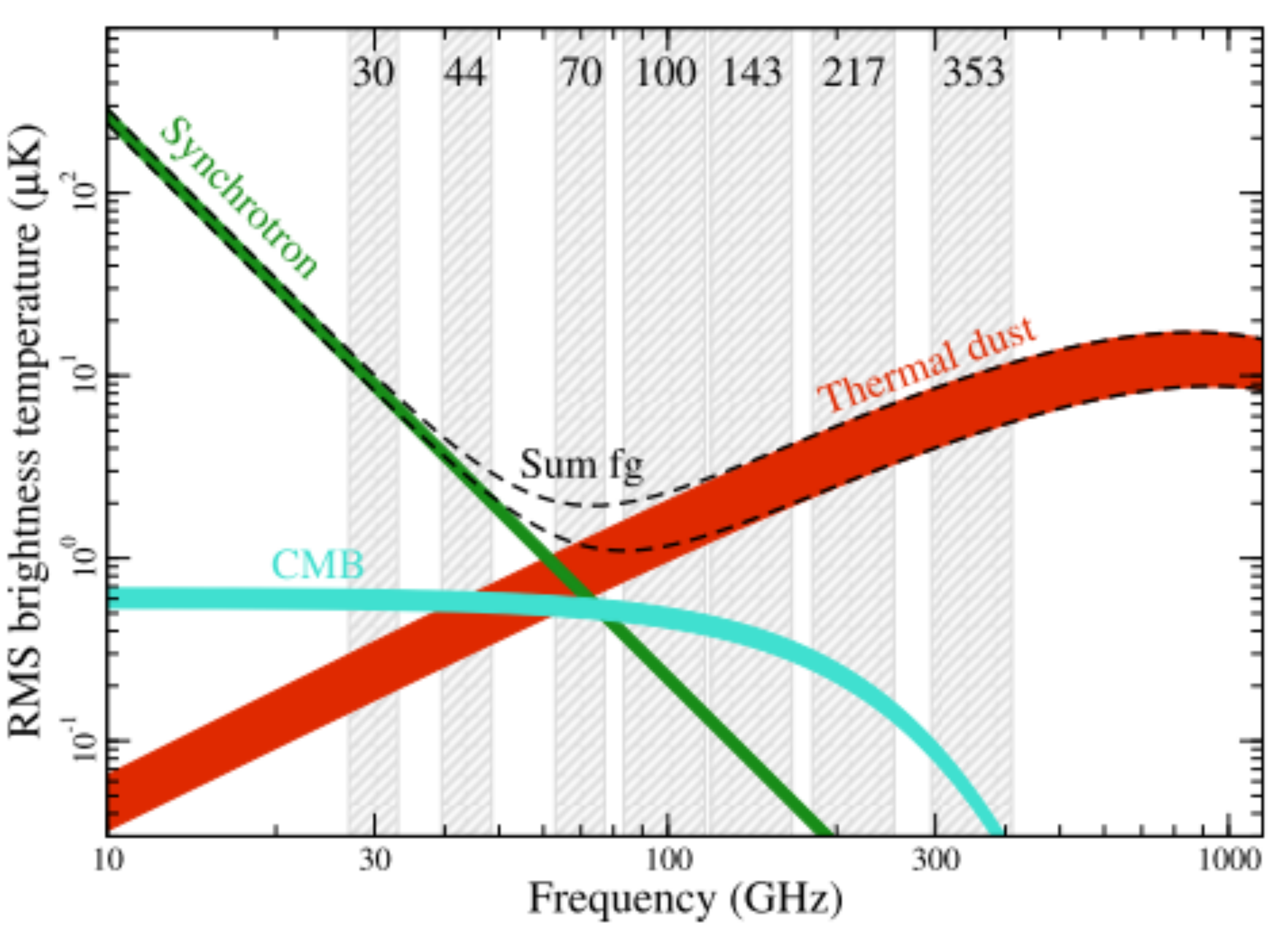


LiteBIRD 2028–

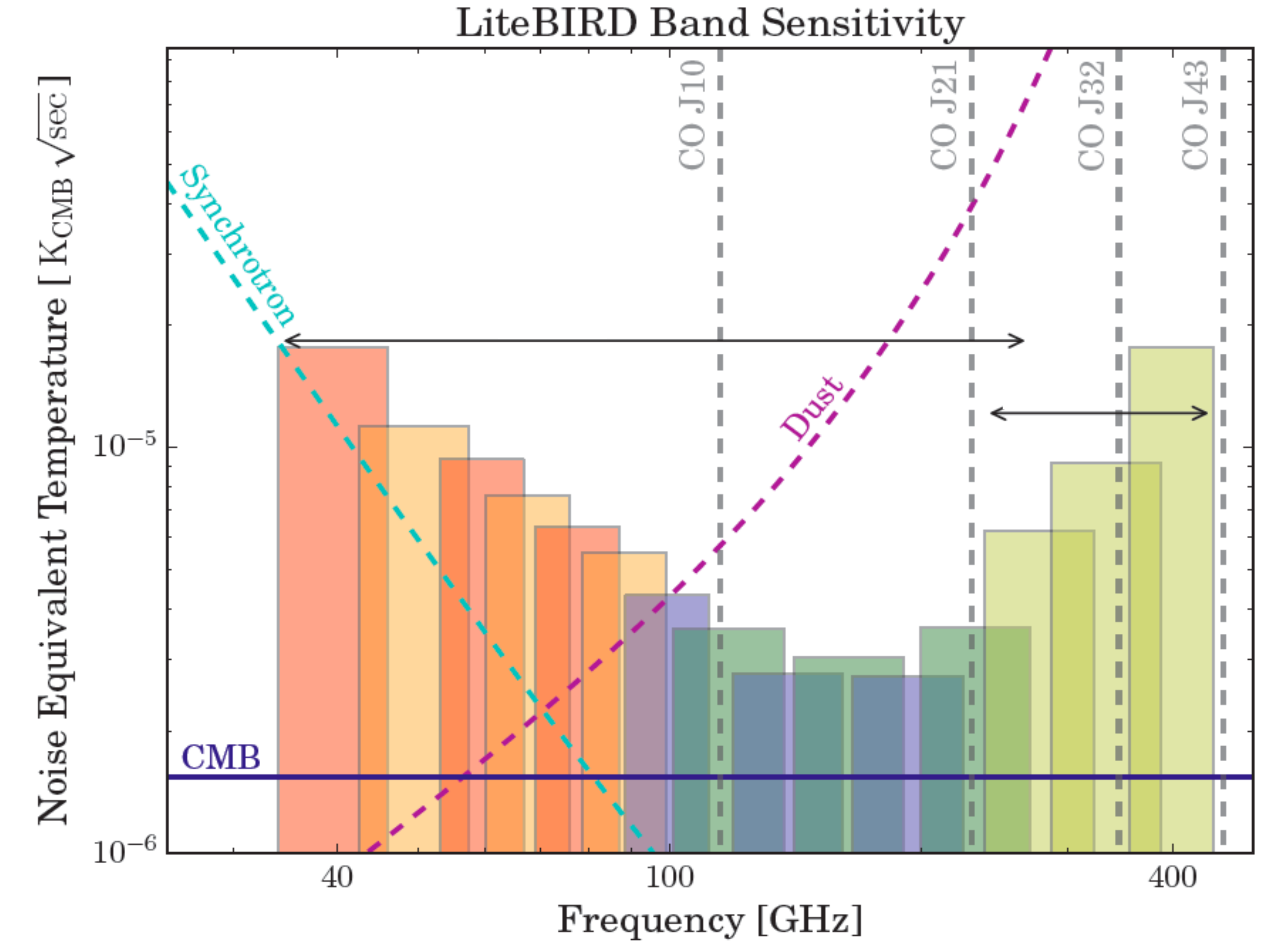
Polarisation satellite dedicated to measure CMB
arisation from primordial GW, with a few thous
TES bolometers in space


JAXA
+ participations from
USA, Canada, Europe
LiteBIRD
2028–
Selected!
May 21, 2019: JAXA has chosen LiteBIRD
as the strategic large-class mission.
We will go to L2!
LiteBIRD

Foreground Removal



Polarized galactic emission (Planck X)

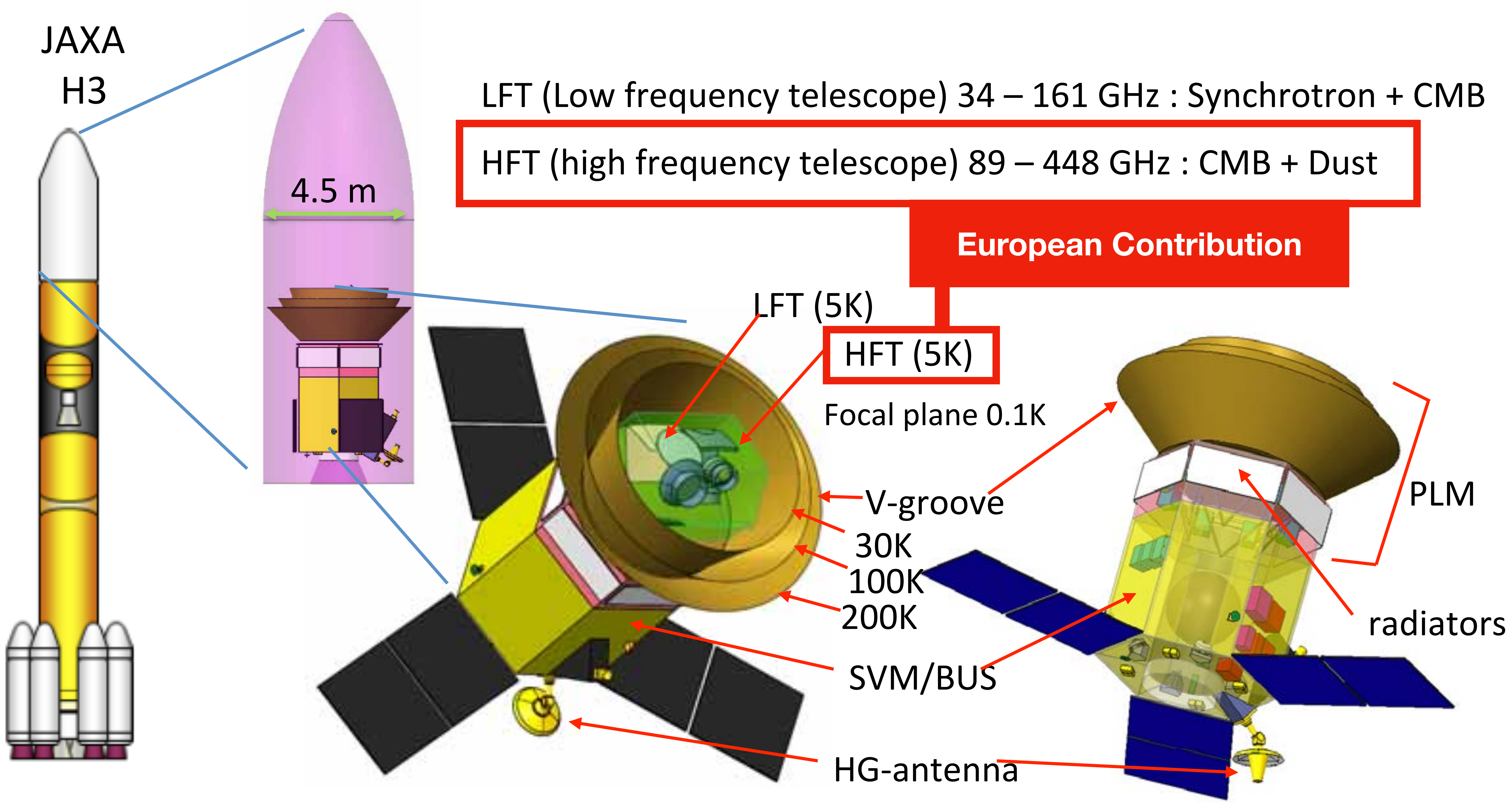


LiteBIRD: 15 frequency bands

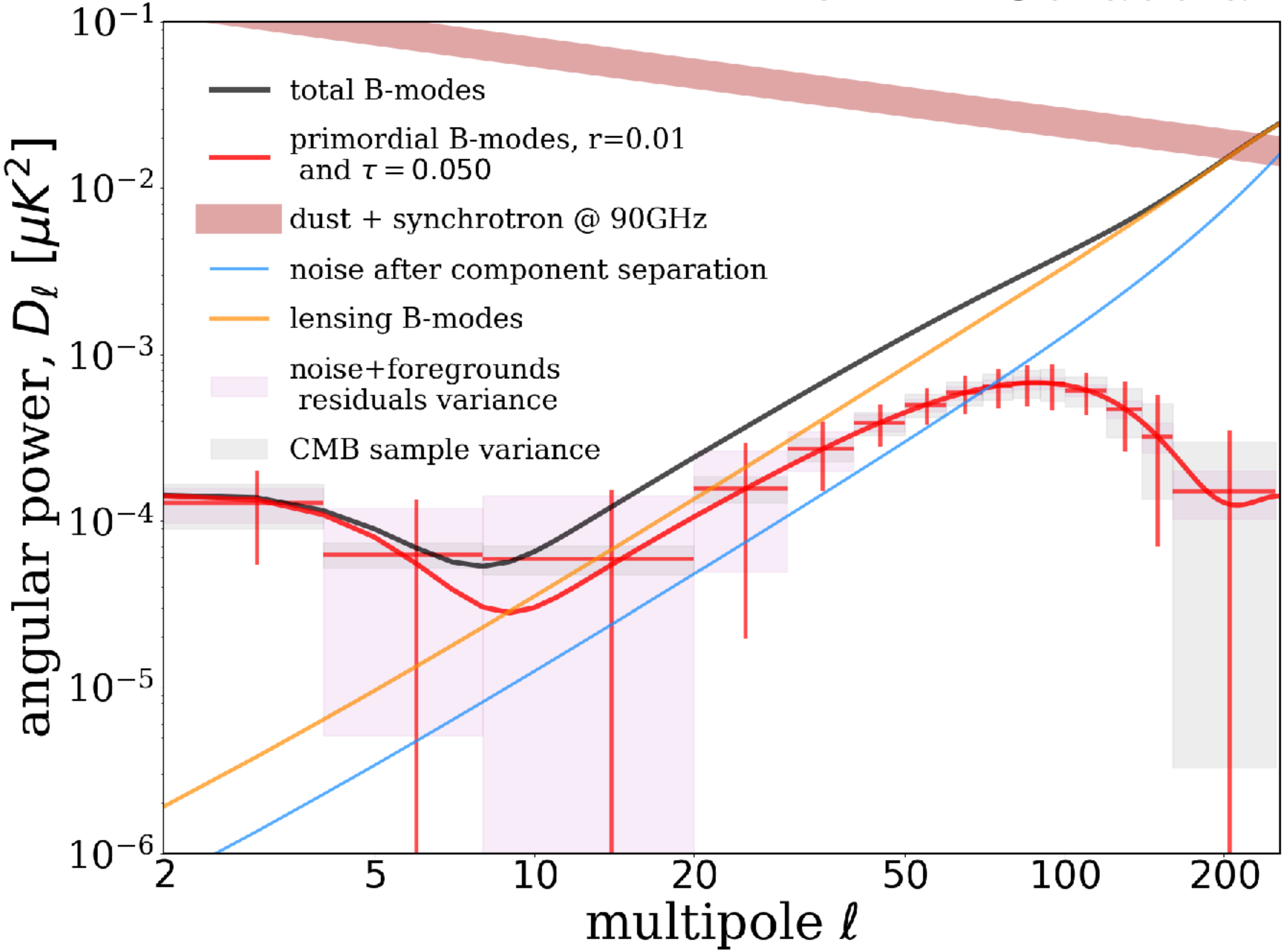
- Polarized foregrounds
  - Synchrotron radiation and thermal emission from inter-galactic dust
  - Characterize and remove foregrounds
- 15 frequency bands between 40 GHz - 400 GHz
  - Split between Low Frequency Telescope (LFT) and High Frequency Telescope (HFT)
  - LFT: 40 GHz – 235 GHz
  - HFT: 280 GHz – 400 GHz

Slide courtesy Toki Suzuki (Berkeley)

LiteBIRD Spacecraft



LiteBIRD Collaboration



LiteBIRD Collaboration