Lectures on Dark Energy Probes

Eiichiro Komatsu (Max-Planck-Institut für Astrophysik)
Challenges in Modern Cosmology, Natal, Brazil
May 8 and 9, 2014

The lecture slides are available at
http://www.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Topics

- In this lecture, we will cover
  - Cosmic microwave background
  - Galaxy redshift surveys
  - Galaxy clusters
- as “dark energy probes.” However, we do not have time to cover
  - Type Ia supernovae
  - Weak gravitational lensing

- Simple routines for computing various cosmological quantities [many of which are shown in this lecture] are available at

- Cosmology Routine Library (CRL):

    - **http://www.mpa-garching.mpg.de/~komatsu/crl/**

Defining “Dark Energy”

- It is often said that there are two approaches to explain the observed acceleration of the universe.
  - One is “*dark energy*,” and
  - The other is a “*modification to General Relativity*.”
- However, **there is no clear distinction between them**, unless we impose some constraints on what we mean by “dark energy.”

DE vs MG: Example #1

- Consider an action given by [with 8πG=1]

$$\int d^4x\sqrt{-g}\,\left(\frac{R+\alpha R^2}{2}+\mathcal{L}_{\mathrm{matter}}\right)$$

Matter is minimally coupled to gravity via √-*g*

- Perform a conformal transformation

$$g_{\mu\nu}\to\hat{g}_{\mu\nu}=(1+2\alpha R)g_{\mu\nu}$$

- Define a scalar field

$$\phi=\sqrt{\frac{3}{2}}\ln(1+2\alpha R)$$

- Then…

DE vs MG: Example #1

- The action becomes

$$\int d^4x\sqrt{-\hat{g}}\left(\frac{\hat{R}}{2}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)+e^{-2\sqrt{\frac{2}{3}}\phi}\mathcal{L}_{\mathrm{matter}}\right)$$

- with a potential

$$V(\phi)=\frac{1}{8\alpha}\left(1-e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$$

- Therefore, a modified GR model with R^2 is equivalent to a model with a dark energy field, ϕ, **coupled to matter**
  - **This is generic to models with** $\alpha R^2 \to f(R)$

DE vs MG: Example #2

- Consider an action given by [with 8πG=1]

$$\int d^4x\sqrt{-g}\left(\frac{R+f(R)}{2}+\mathcal{L}_{\rm matter}\right)$$

- And a FLRW metric with scalar perturbations

$$ds^2=-(1+2\Psi)dt^2+a^2(t)(1+2\Phi)d\mathbf{x}^2$$

- Then the relation between Φ and Ψ is given by

$$\nabla^2(\Psi+\Phi)=-\frac{\frac{d^2f}{dR^2}}{1+\frac{df}{dR}}\nabla^2(\delta R)\neq 0$$

- (Here, “matter” does not have anisotropic stress)

DE vs MG: Example #2

- Consider an action given by [with 8πG=1]

$$\int d^4x\sqrt{-g}\left(\frac{R}{2}+\mathcal{L}_{\rm dark\ energy}+\mathcal{L}_{\rm matter}\right)$$

- And anisotropic stress of dark energy

$$T^i_j = P_{\rm de}\delta^i_j + P_{\rm de}(\nabla^i\nabla_j - \frac{1}{3}\delta^i_j\nabla^2)\pi_{\rm de}$$

- Then the relation between Φ and Ψ is given by

$$\nabla^2(\Psi+\Phi) = a^2 P_{\rm de}\pi_{\rm de} \neq 0$$

- **DE anisotropic stress can mimic f(R) gravity**

Defining “Dark Energy”

- Therefore, we shall use the following terminology:
- By “dark energy”, we mean a fluid which
  - has an equation of state of $P_{de} < -\rho_{de}/3$,
  - does not couple to matter, and
  - does not have anisotropic stress
- This “dark energy” fluid can be distinguished from modifications to General Relativity

Goals of Dark Energy Research

- We wish to determine the nature of dark energy. But, *where should we start*?

- A breakthrough in science is often made when the standard model is ruled out.

    - "Standard model" in cosmology is the ΛCDM model. We wish to rule out dark energy being Λ, a cosmological constant

- The most important goal of dark energy research is to find that **the dark energy density, ρ_{de}, depends on time**

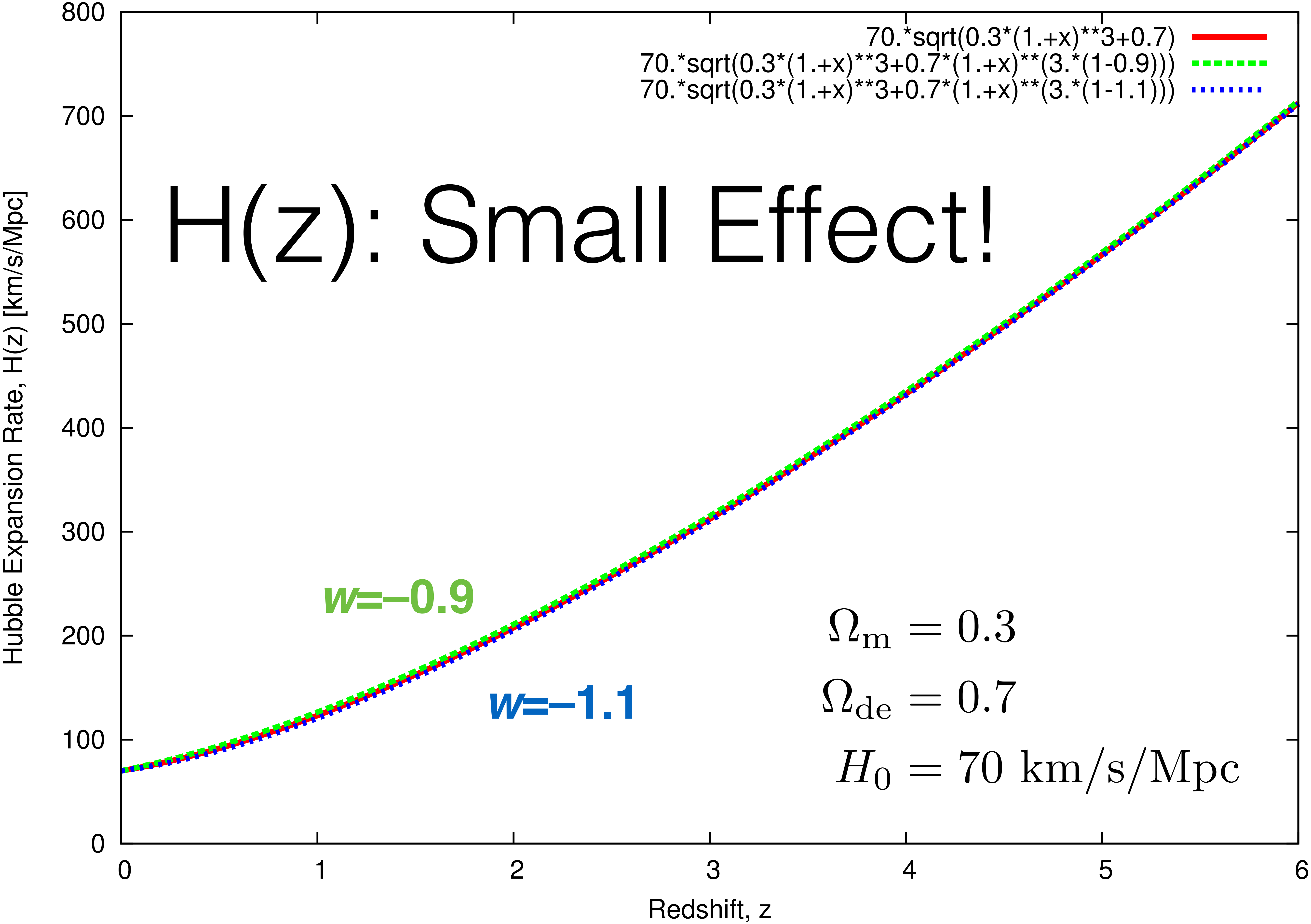
Measuring Dark Energy

- We can measure the dark energy density only via its effect on the expansion of the universe. Namely, we wish to **measure the Hubble expansion rate**, H(z), as a function of redshifts

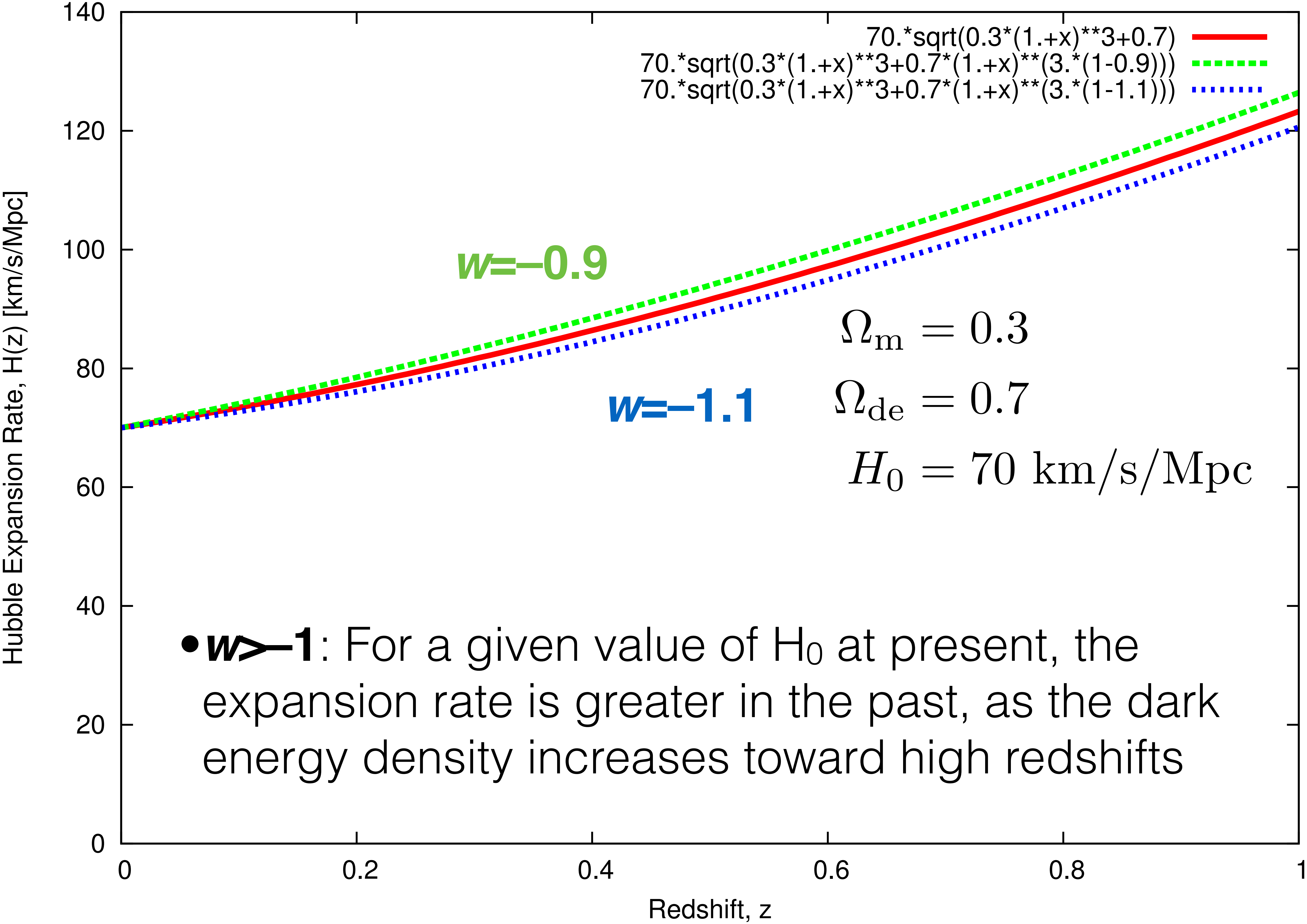
$$H^2(z) = \frac{8\pi G}{3}\left[\rho_{\rm matter}(0)(1+z)^3 + \rho_{\rm de}(z)\right]$$

- Energy conservation gives [with $w(z)=P_{\rm de}(z)/\rho_{\rm de}(z)$]

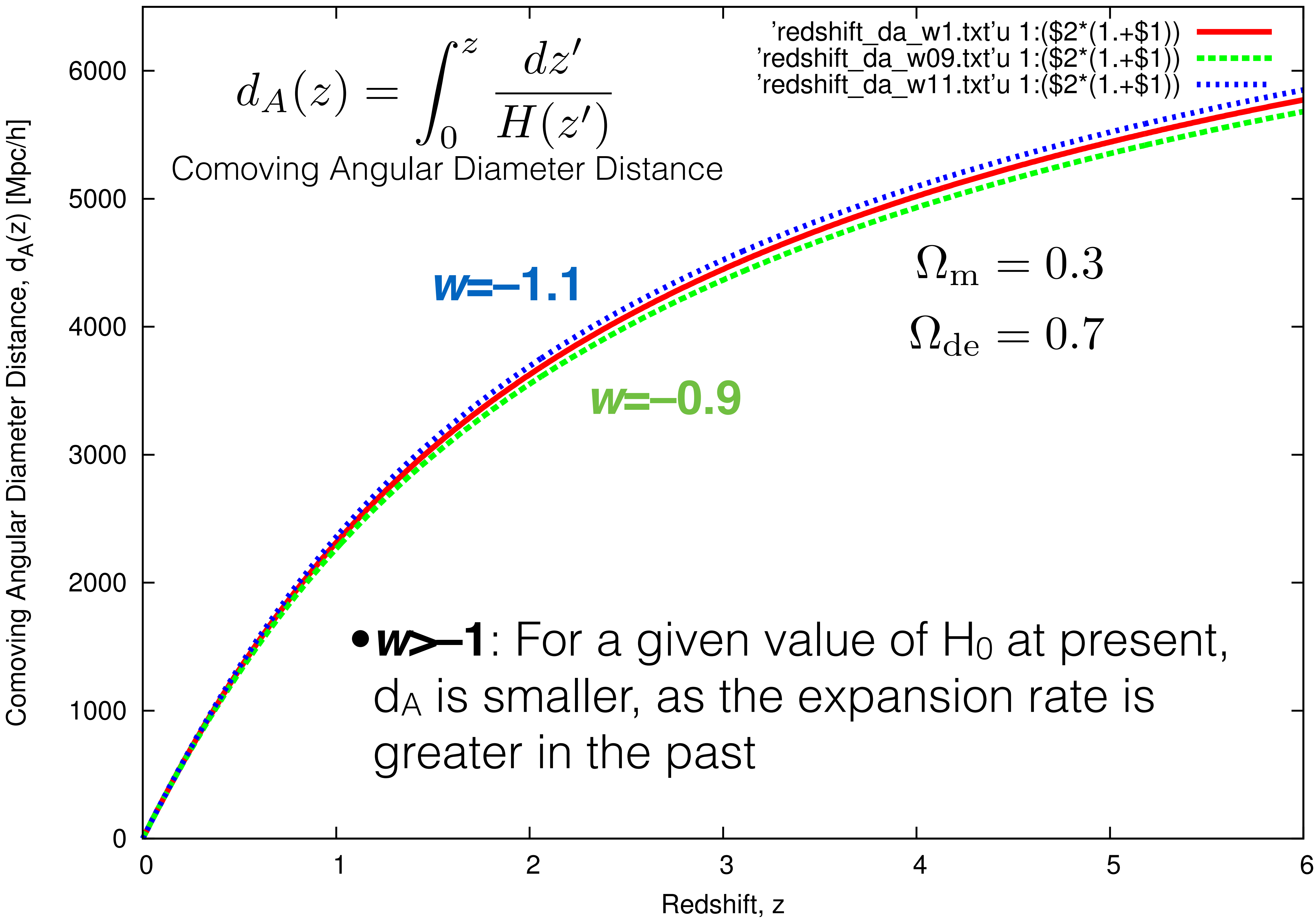
$$\ln\frac{\rho_{\rm de}(z)}{\rho_{\rm de}(0)} = 3\int_0^z \frac{dz'}{1+z'}[1+w(z')]$$



H(z): Small Effect!
70.*sqrt(0.3*(1.+x)**3+0.7)
70.*sqrt(0.3*(1.+x)**3+0.7*(1.+x)**(3.*(1-0.9)))
70.*sqrt(0.3*(1.+x)**3+0.7*(1.+x)**(3.*(1-1.1)))
w=–0.9
w=–1.1
$\Omega_{\rm m} = 0.3$
$\Omega_{\rm de} = 0.7$
$H_0 = 70$ km/s/Mpc
Hubble Expansion Rate, H(z) [km/s/Mpc]
Redshift, z
0
100
200
300
400
500
600
700
800
0
1
2
3
4
5
6



70.*sqrt(0.3*(1.+x)**3+0.7)
70.*sqrt(0.3*(1.+x)**3+0.7*(1.+x)**(3.*(1-0.9)))
70.*sqrt(0.3*(1.+x)**3+0.7*(1.+x)**(3.*(1-1.1)))
w=–0.9
w=–1.1
$\Omega_{\rm m} = 0.3$
$\Omega_{\rm de} = 0.7$
$H_0 = 70$ km/s/Mpc
• w>–1: For a given value of H_0 at present, the expansion rate is greater in the past, as the dark energy density increases toward high redshifts
Hubble Expansion Rate, H(z) [km/s/Mpc]
Redshift, z
0
20
40
60
80
100
120
140
0
0.2
0.4
0.6
0.8
1



$d_A(z) = \int_0^z \frac{dz'}{H(z')}$
Comoving Angular Diameter Distance
'redshift_da_w1.txt'u 1:($2*(1.+$1))
'redshift_da_w09.txt'u 1:($2*(1.+$1))
'redshift_da_w11.txt'u 1:($2*(1.+$1))
w=–1.1
w=–0.9
$\Omega_{\rm m} = 0.3$
$\Omega_{\rm de} = 0.7$
• ***w*>–1**: For a given value of H_0 at present, d_A is smaller, as the expansion rate is greater in the past
Comoving Angular Diameter Distance, $d_A(z)$ [Mpc/h]
Redshift, z
0
1000
2000
3000
4000
5000
6000
0
1
2
3
4
5
6

Growth of Perturbation

- The expansion of the universe also determines how fast perturbations grow. An intuitive argument is as follows.

- The growth time scale of matter perturbations [free-fall time, t_{ff}] is given by

$$\frac{d^2r}{dt^2} = -\frac{4\pi G\rho_{\mathrm{matter}}}{3}r \quad \Rightarrow \quad t_{\mathrm{ff}} \approx \frac{1}{\sqrt{G\rho_{\mathrm{matter}}}}$$

- The matter perturbation growth is determined by competition between the free-fall time and the expansion time scale, t_{exp},

$$t_{\mathrm{exp}} \equiv \frac{1}{H} \approx \frac{1}{\sqrt{G(\rho_{\mathrm{matter}} + \rho_{\mathrm{de}})}}$$

Growth of Perturbation

- The matter perturbation cannot grow during the dark-energy-dominated era, $\rho_{de} >> \rho_{matter}$, because the expansion is too fast

$$t_{\rm exp} \approx \frac{1}{\sqrt{G(\rho_{\rm matter} + \rho_{\rm de})}} \ll \frac{1}{\sqrt{G\rho_{\rm matter}}} \approx t_{\rm ff}$$

- Therefore, **measuring the [suppression of] growth rate of matter perturbations** can also be used to measure the effect of dark energy on the expansion rate of the universe

Growth Equation

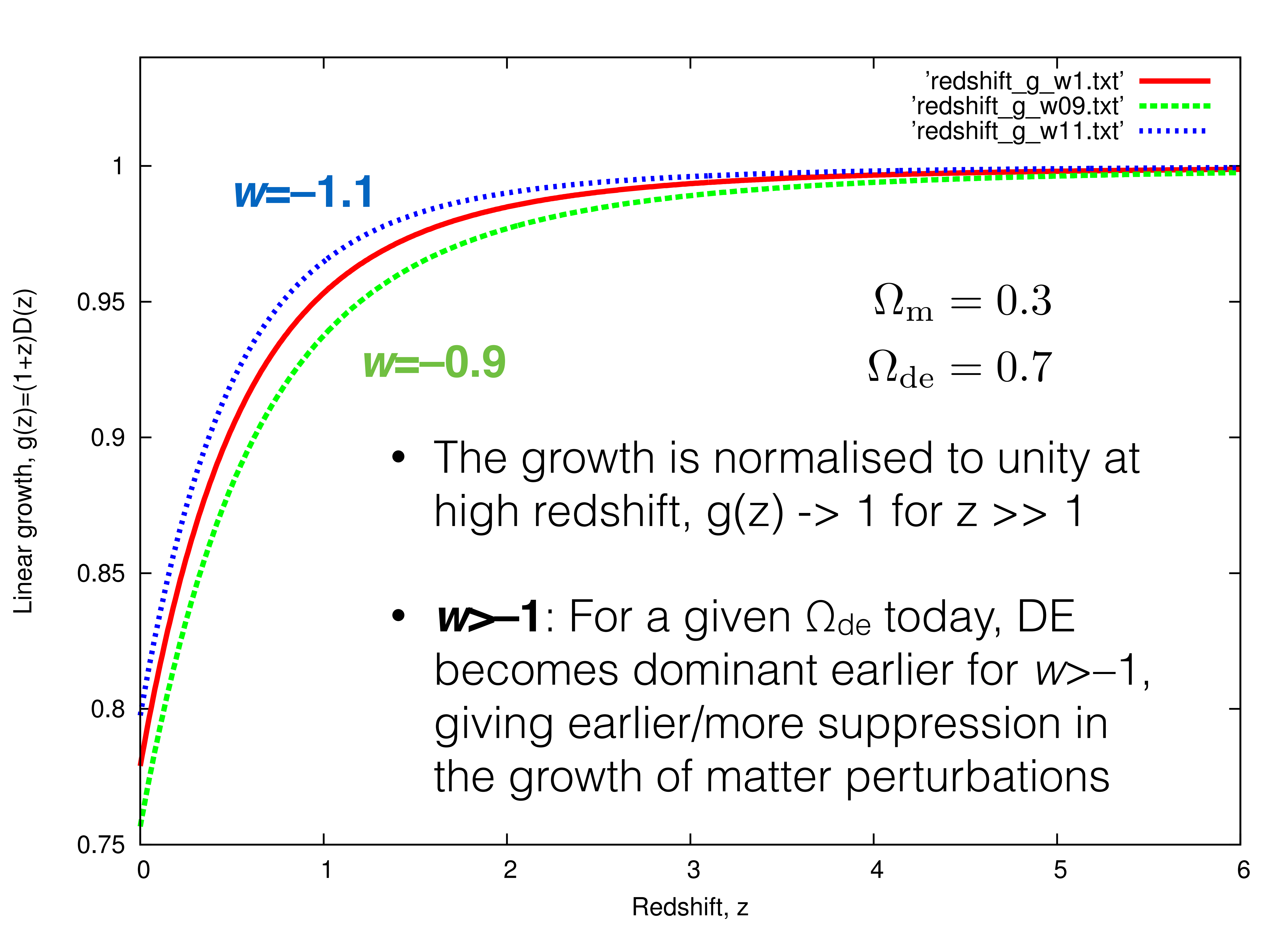
- Writing the redshift dependence of matter density perturbations as

$$\delta_{\mathrm{matter}}(z) \propto \frac{g(z)}{1+z}$$

- The evolution equation of g(z) is given by

$$\frac{d^2 g}{d\ln(1+z)^2} - \left[\frac{5}{2} + \frac{1}{2}(\Omega_k(z) - 3w(z)\Omega_{\mathrm{de}}(z))\right]\frac{dg}{d\ln(1+z)} + \left[2\Omega_k(z) + \frac{3}{2}(1-w(z))\Omega_{\mathrm{de}}(z)\right] g(z) = 0$$

**Strictly speaking, this formula is valid when the contribution of DE fluctuations to the gravitational potential is negligible compared to matter*


'redshift_g_w1.txt'
'redshift_g_w09.txt'
'redshift_g_w11.txt'
w=–1.1
w=–0.9
$\Omega_{\rm m} = 0.3$
$\Omega_{\rm de} = 0.7$
• The growth is normalised to unity at high redshift, g(z) -> 1 for z >> 1
• w>–1: For a given Ω_{de} today, DE becomes dominant earlier for w>–1, giving earlier/more suppression in the growth of matter perturbations
Linear growth, g(z)=(1+z)D(z)
Redshift, z
0.75
0.8
0.85
0.9
0.95
1
0
1
2
3
4
5
6

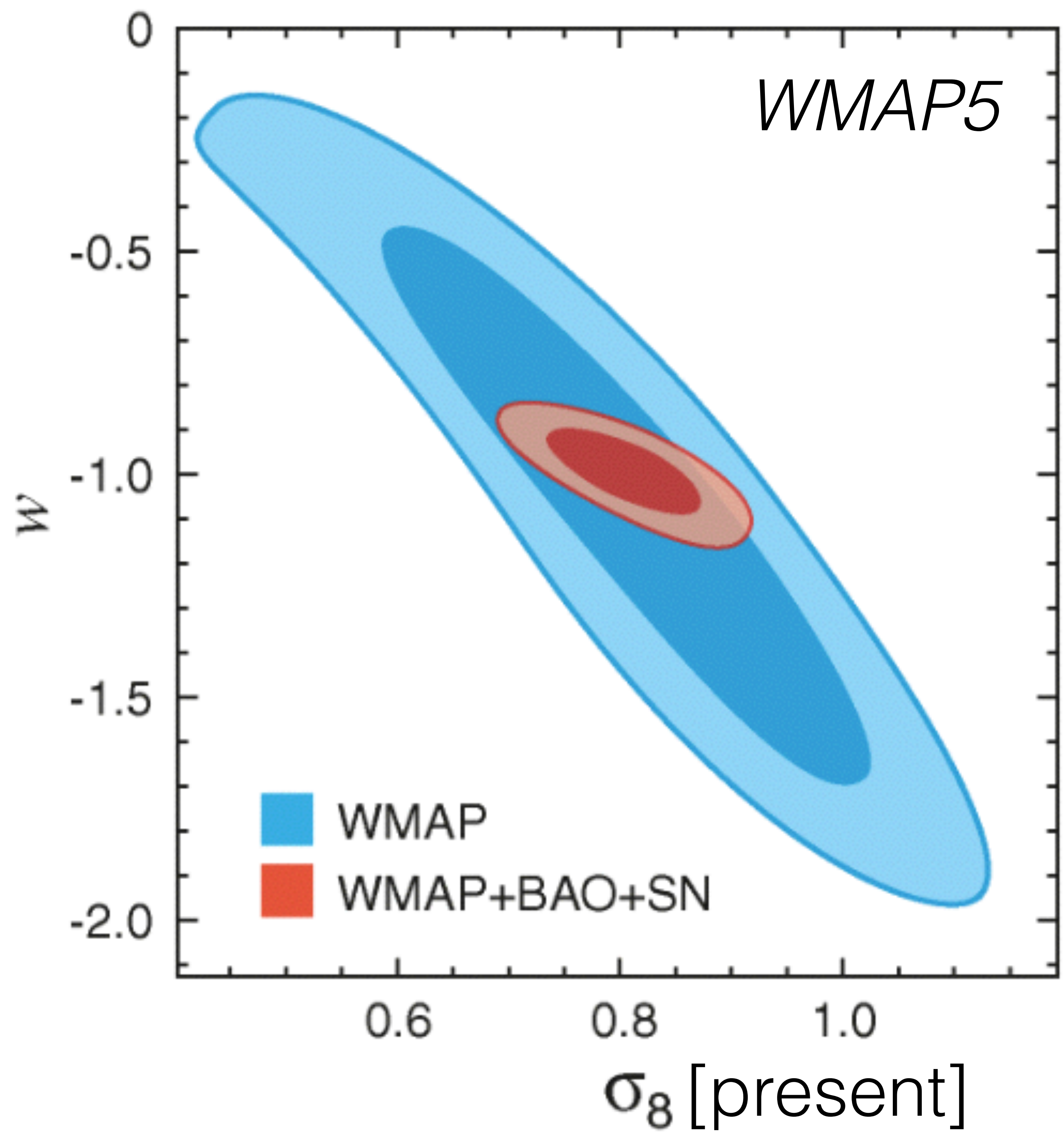
Cosmic Microwave Background

DE vs CMB

- Temperature anisotropy of the cosmic microwave background provides information on dark energy by
  - Providing the amplitude of fluctuations at $z=1090$
  - Providing the angular diameter distance to $z=1090$
  - Integrated Sachs-Wolfe (ISW) effect

Growth: Application #1

- Use the CMB data to fix the amplitude of fluctuations at z=1090
- Varying *w* then gives various values of the present-day matter fluctuation amplitude, σ_8
- Data on σ_8 [i.e., large-scale structure data at lower redshifts] can then determine the value of *w*



Growth: Application #2

- **Integrated Sachs-Wolfe effect** [Sachs&Wolfe 1967]

  - As CMB photons travel from z=1090 to the present epoch, their energies change due to time-dependent gravitational potentials

$$\frac{dp^\mu}{dt} + \Gamma^\mu_{\alpha\beta}\frac{p^\alpha p^\beta}{p^0} = 0 \qquad \text{[geodesic equation]}$$

with $ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\mathbf{x}^2$

$$\Rightarrow \quad \frac{d[\ln(ap)+\Psi]}{dt} = \dot{\Psi} - \dot{\Phi} \qquad [p^2 \equiv g_{ij}p^i p^j]$$

Growth: Application #2

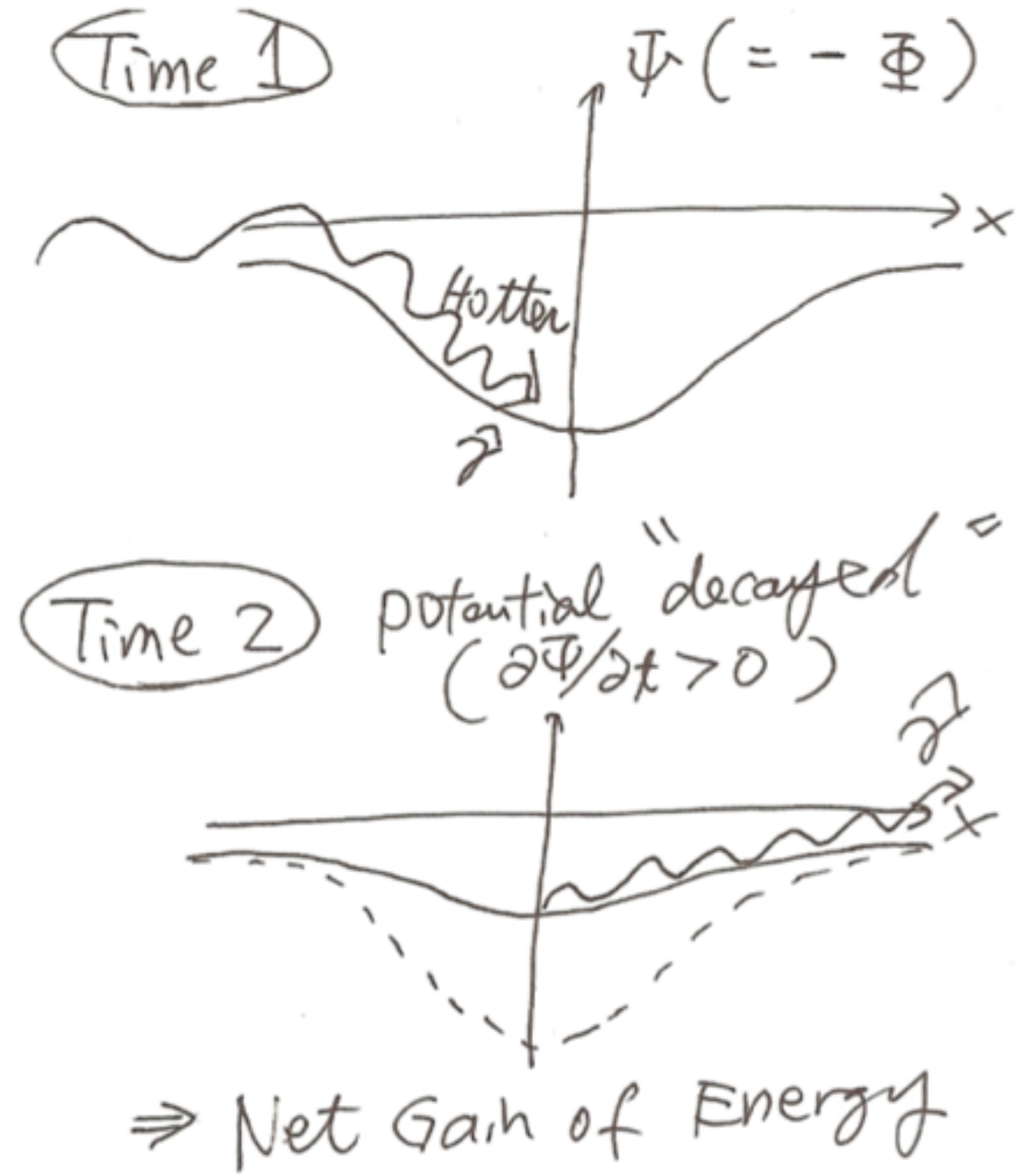
- **Integrated Sachs-Wolfe effect**

$$\frac{\delta T_{\rm ISW}}{T} = \int_{t_*}^{t_0} dt\ (\dot\Psi - \dot\Phi)$$

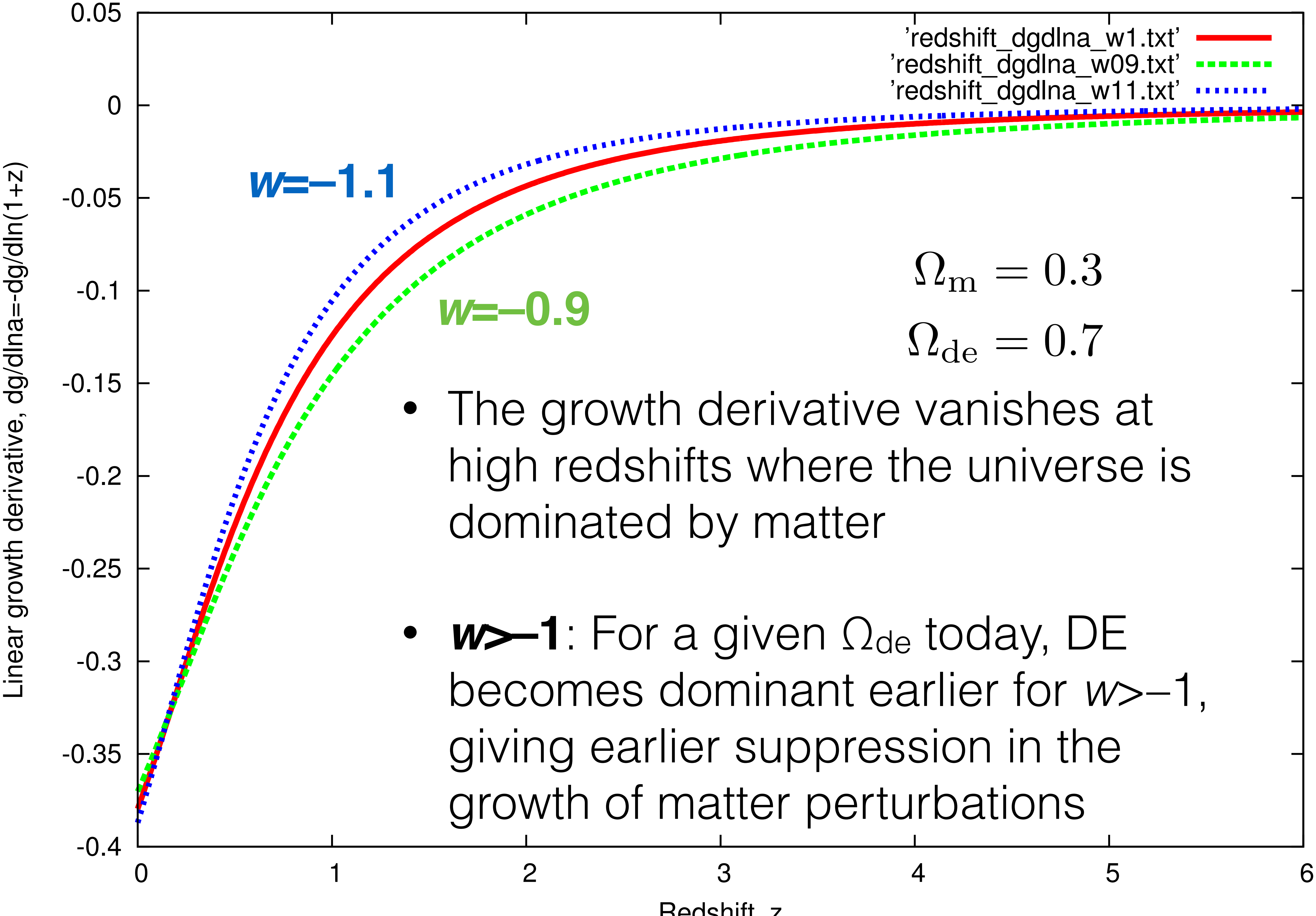
$$= 2\Psi(t_{\rm MD})\int_{t_{\rm MD}}^{t_0} dt\ \dot g$$

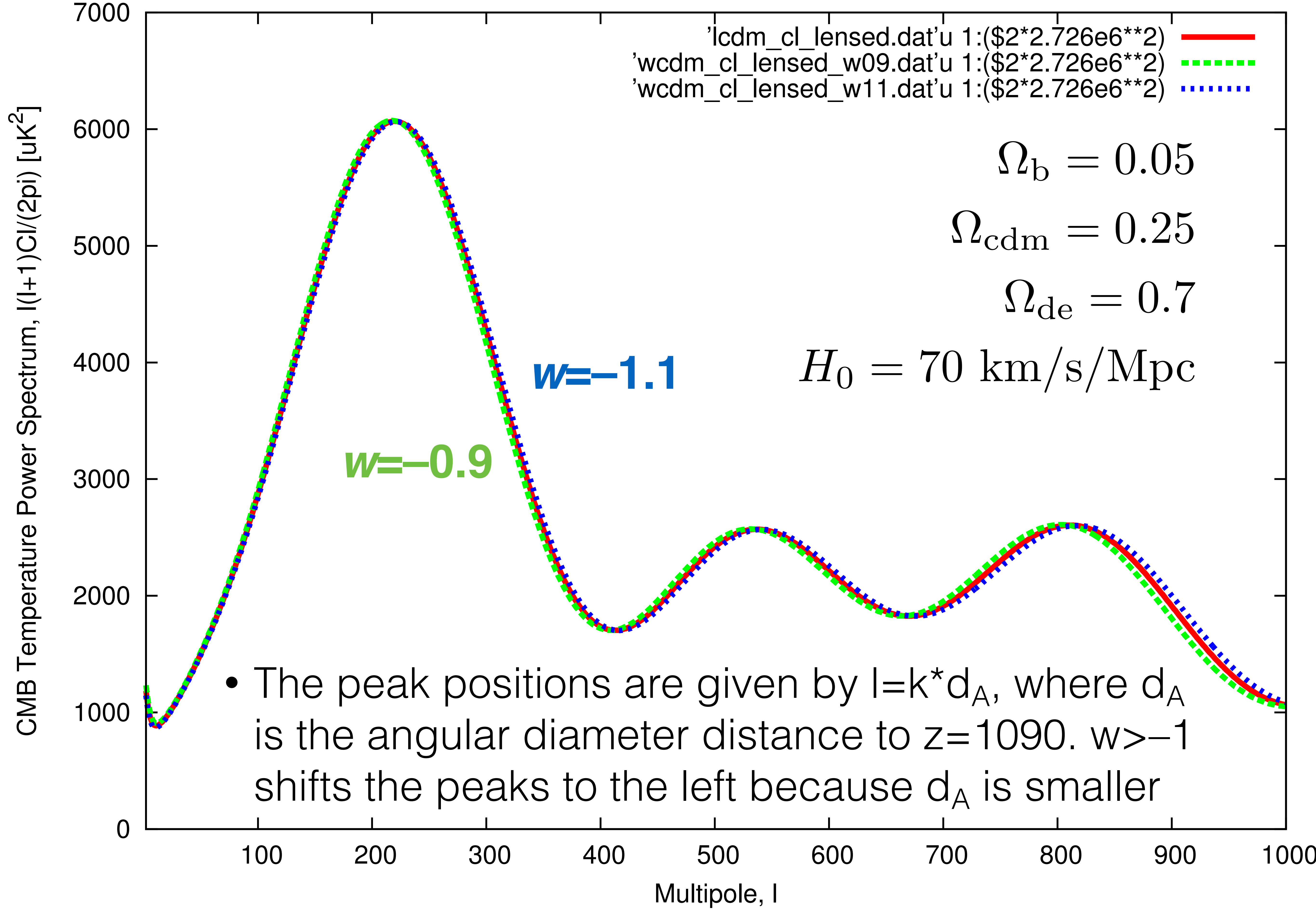
- The right hand side vanishes during the matter-dominated (MD) era, while Ψ and Φ **decay** during the DE-dominated era

- **ISW is a direct probe of d*g*/d*t***

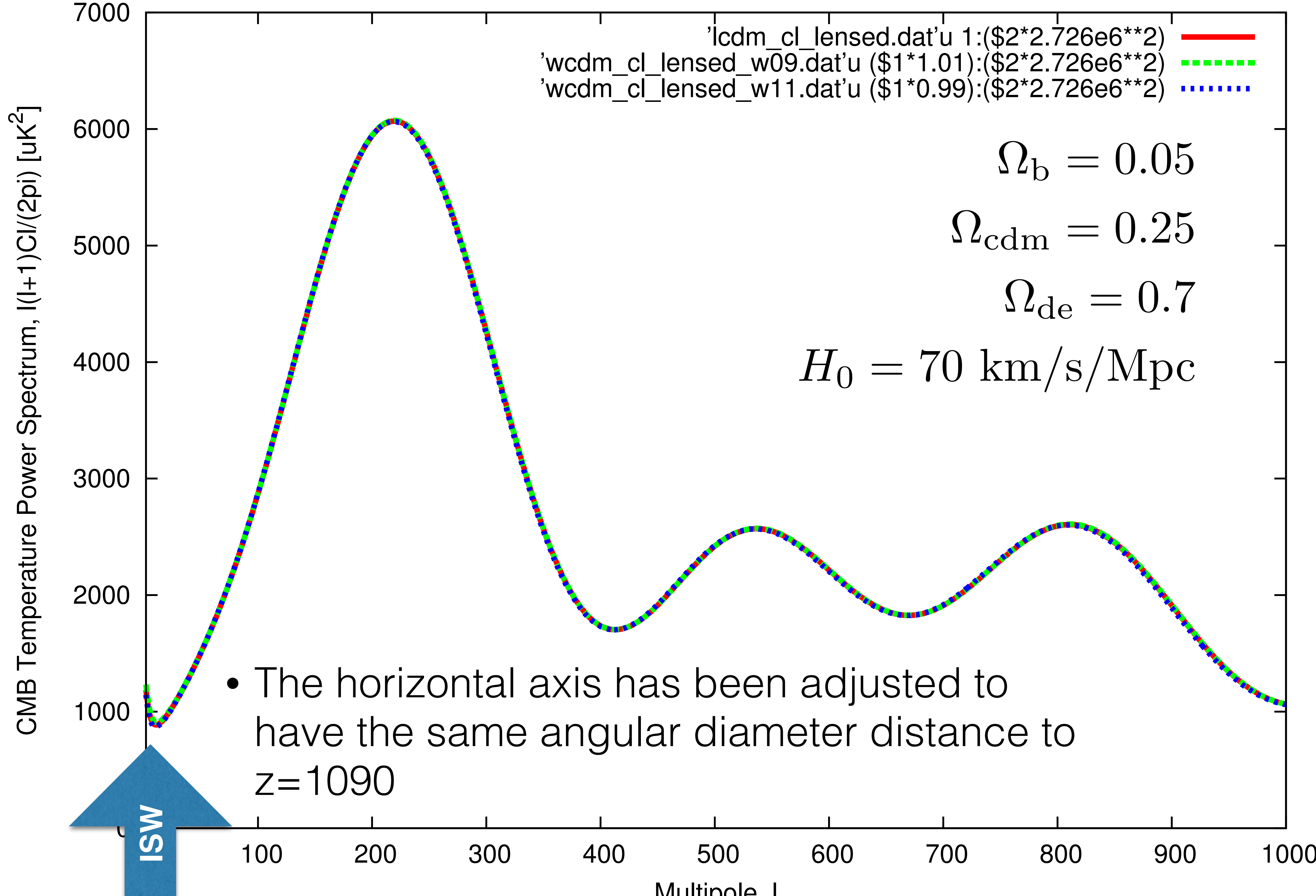


$$\frac{1}{p}\frac{dp}{dt} = -\frac{\partial\Phi}{\partial t} = \frac{\partial\Psi}{\partial t}$$

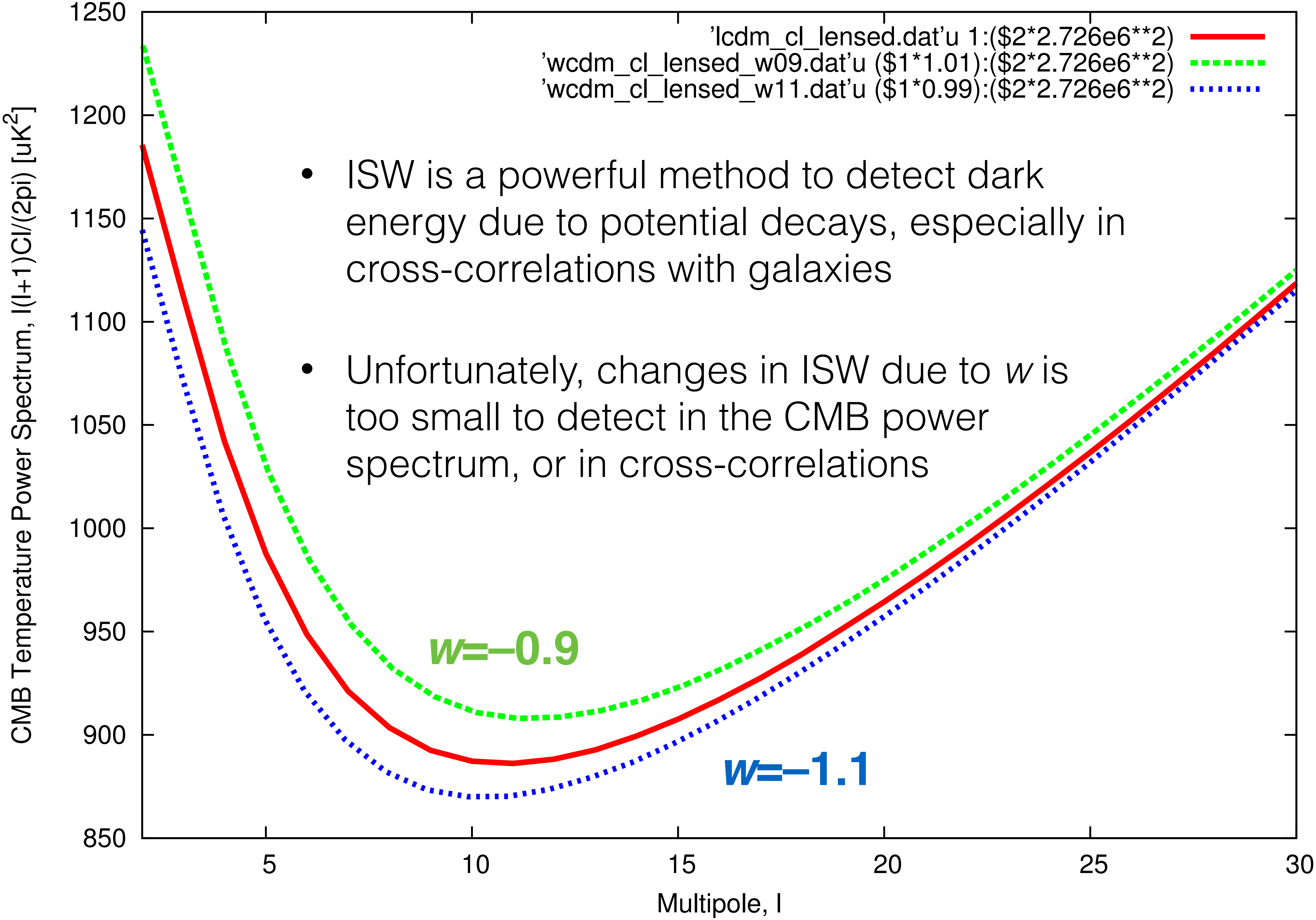

0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
0
1
2
3
4
5
6
Redshift, z
Linear growth derivative, dg/dlna=-dg/dln(1+z)
'redshift_dgdlna_w1.txt'
'redshift_dgdlna_w09.txt'
'redshift_dgdlna_w11.txt'
w=–1.1
w=–0.9
$\Omega_{\rm m} = 0.3$
$\Omega_{\rm de} = 0.7$
• The growth derivative vanishes at high redshifts where the universe is dominated by matter
• ***w*>–1**: For a given Ω_{de} today, DE becomes dominant earlier for *w*>–1, giving earlier suppression in the growth of matter perturbations



- The peak positions are given by $l=k^*d_A$, where d_A is the angular diameter distance to $z=1090$. $w>-1$ shifts the peaks to the left because d_A is smaller



7000
6000
5000
4000
3000
2000
1000
0
CMB Temperature Power Spectrum, l(l+1)Cl/(2pi) [uK^2]
'lcdm_cl_lensed.dat'u 1:($2*2.726e6**2)
'wcdm_cl_lensed_w09.dat'u ($1*1.01):($2*2.726e6**2)
'wcdm_cl_lensed_w11.dat'u ($1*0.99):($2*2.726e6**2)
$\Omega_b = 0.05$
$\Omega_{cdm} = 0.25$
$\Omega_{de} = 0.7$
$H_0 = 70$ km/s/Mpc
• The horizontal axis has been adjusted to have the same angular diameter distance to z=1090
ISW
100
200
300
400
500
600
700
800
900
1000
Multipole, l



- ISW is a powerful method to detect dark energy due to potential decays, especially in cross-correlations with galaxies
- Unfortunately, changes in ISW due to *w* is too small to detect in the CMB power spectrum, or in cross-correlations

Galaxy Redshift Survey

DE vs Galaxy Survey

- Galaxy redshift surveys provide information on dark energy by
  - Measuring $d_A(z)$ and $H(z)$ from the standard ruler and Alcock-Paczynski methods
  - Measuring the linear growth of matter perturbations from the redshift space distortion

Measuring H(z)

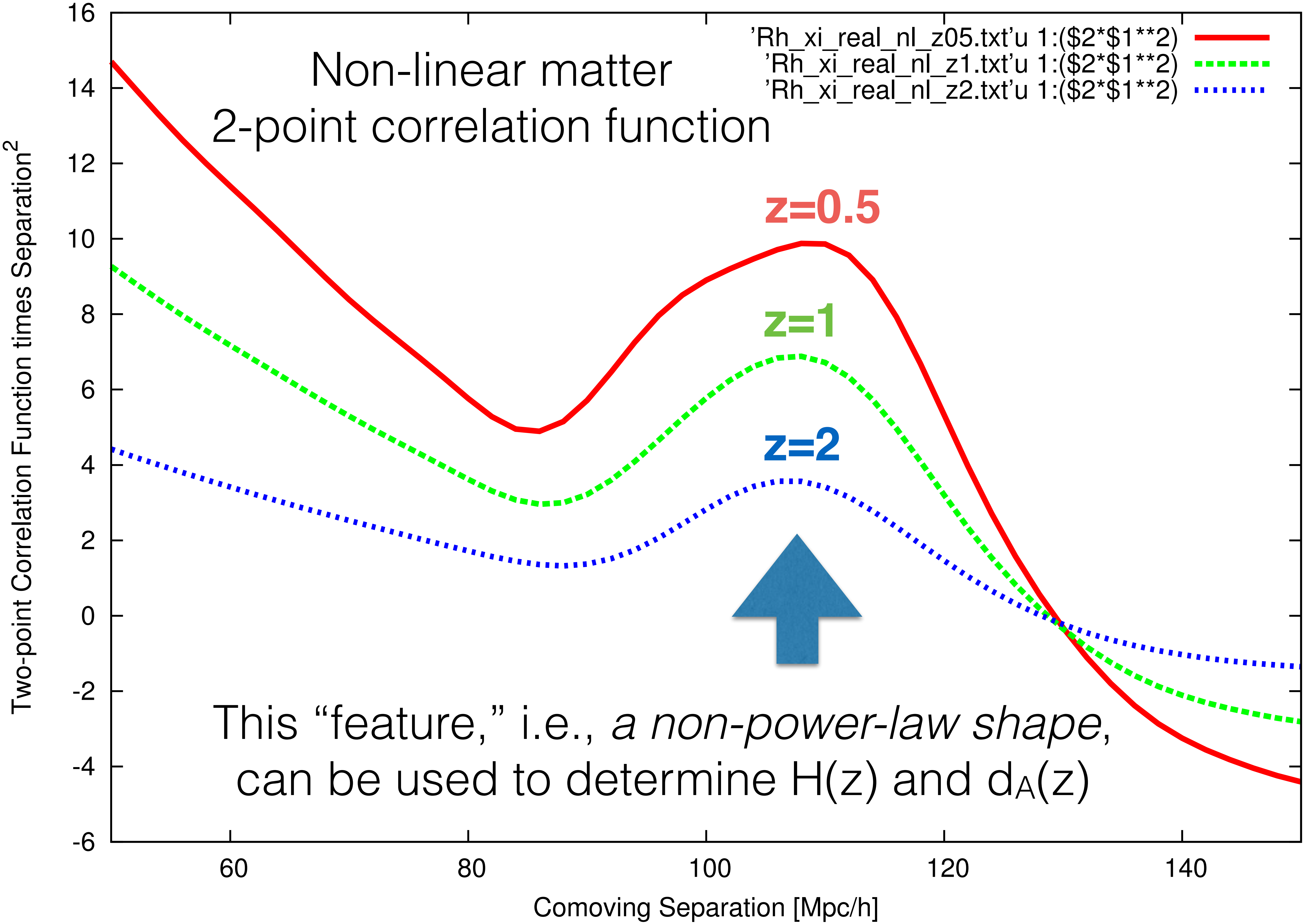
- **Standard ruler method applied to correlation functions of galaxies**

  - Use known, well-calibrated, specific features in N-point correlation functions [usually N=2] of matter in angular and redshift directions

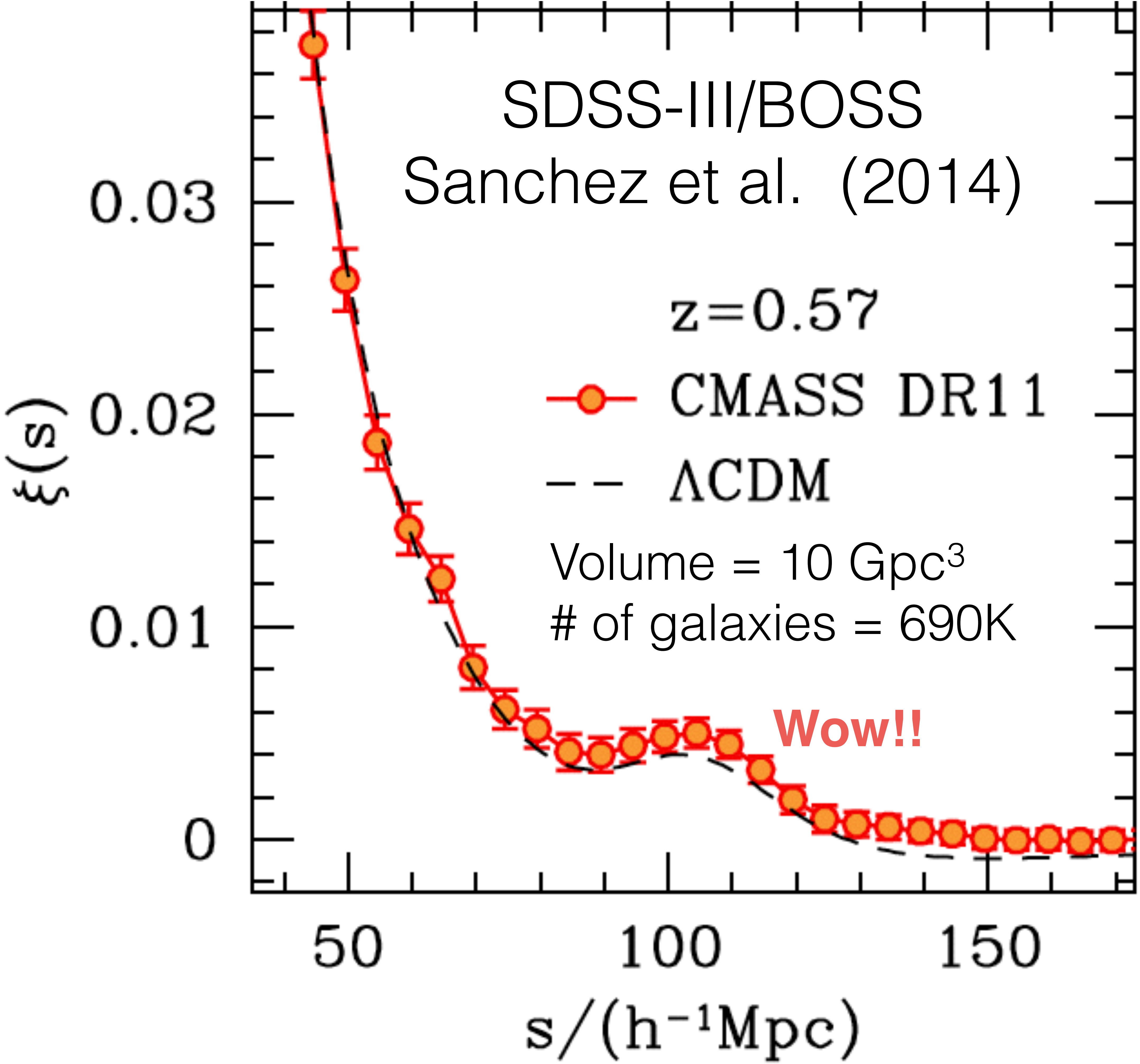
  - Mapping the observed separations of galaxies to the comoving separations:

$$\Delta z = H(z)\Delta r_{\|} \quad \text{[Line-of-sight direction]}$$

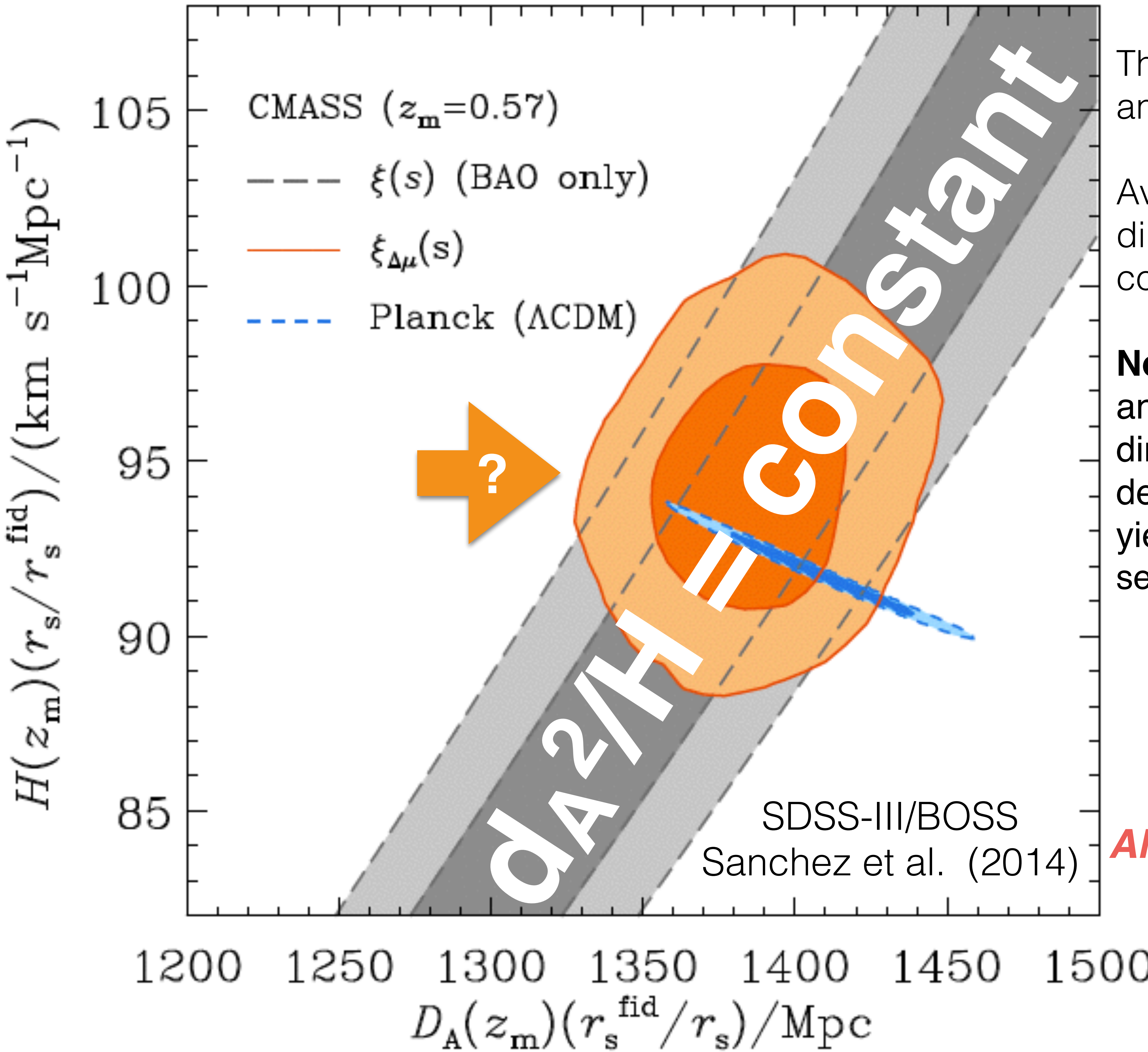
$$\Delta\theta = \frac{\Delta r_{\perp}}{d_A(z)} \quad \text{[Angular directions]} \quad d_A = \int_0^z \frac{dz'}{H(z')}$$



Non-linear matter
2-point correlation function
'Rh_xi_real_nl_z05.txt'u 1:($2*$1**2)
'Rh_xi_real_nl_z1.txt'u 1:($2*$1**2)
'Rh_xi_real_nl_z2.txt'u 1:($2*$1**2)
z=0.5
z=1
z=2
This "feature," i.e., *a non-power-law shape*,
can be used to determine H(z) and d_A(z)
Two-point Correlation Function times Separation^2
Comoving Separation [Mpc/h]
16
14
12
10
8
6
4
2
0
-2
-4
-6
60
80
100
120
140



SDSS-III/BOSS
Sanchez et al. (2014)
z=0.57
CMASS DR11
ΛCDM
Volume = 10 Gpc³
# of galaxies = 690K
Wow!!
0.03
0.02
0.01
0
ξ(s)
50
100
150
s/(h⁻¹Mpc)



There are 2 angular and 1 LOS directions.

Averaging all three directions yields a constraint on $\mathbf{d_A{}^2/H}$

Not averaging angular and LOS directions breaks degeneracy and yields d_A and H separately; **but how?**

Alcock-Paczynski Test

Alcock&Paczynski (1979)

Alcock-Paczynski [AP] Test

- **The key idea**: homogeneity and isotropy of the universe demands that the two-point correlation be **isotropic** in all three directions
  - (in the absence of redshift space distortion [RSD] - we shall come back to this shortly; but let us ignore RSD here for simplicity)

Alcock&Paczynski (1979)

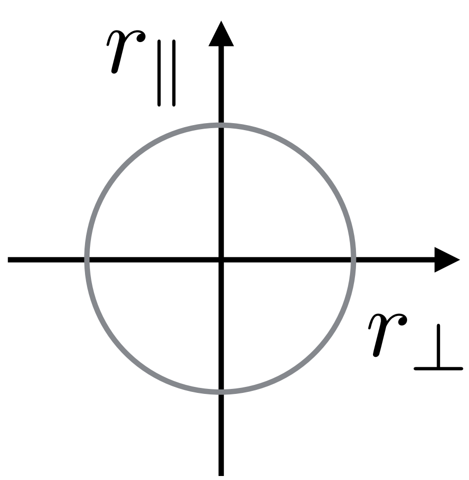
How the AP test works

- We convert the observed angular and redshift separations into the comoving separations, assuming $d_A(z)$ and $H(z)$.

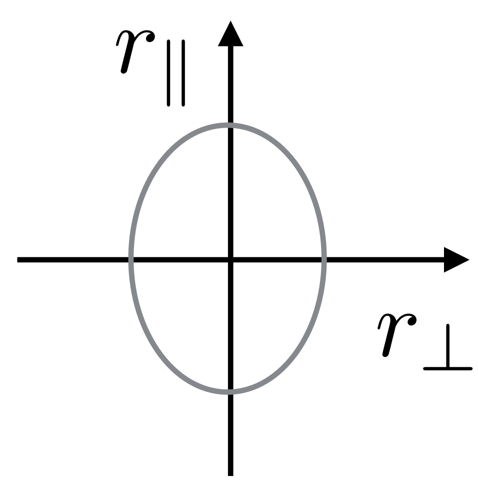
$$\Delta z = H(z)\Delta r_{\|} \quad \text{[Line-of-sight direction]}$$

$$\Delta\theta = \frac{\Delta r_{\perp}}{d_A(z)} \quad \text{[Angular directions]}$$

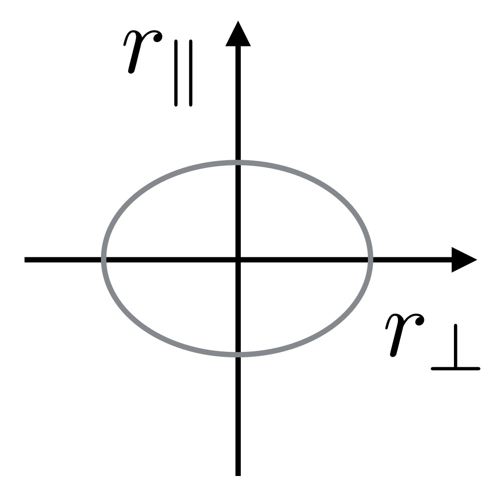
Both d_A and H are correct



If d_A is wrong



If H is wrong



Alcock&Paczynski (1979)

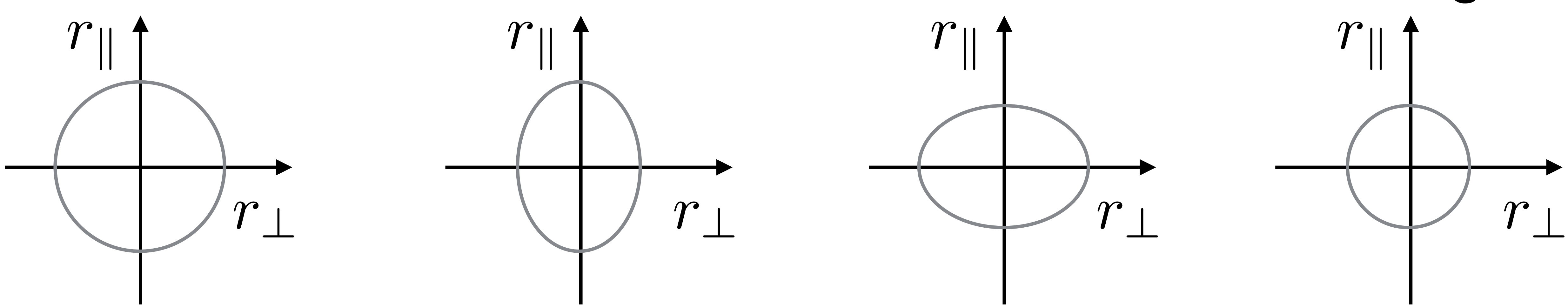
How the AP test works

- We convert the observed angular and redshift separations into the comoving separations, assuming $d_A(z)$ and $H(z)$.

$$\Delta z = H(z)\Delta r_{||} \quad \text{[Line-of-sight direction]}$$

$$\Delta \theta = \frac{\Delta r_\perp}{d_A(z)} \quad \text{[Angular directions]}$$

Both d_A and H are correct | If d_A is wrong | If H is wrong | If both are wrong

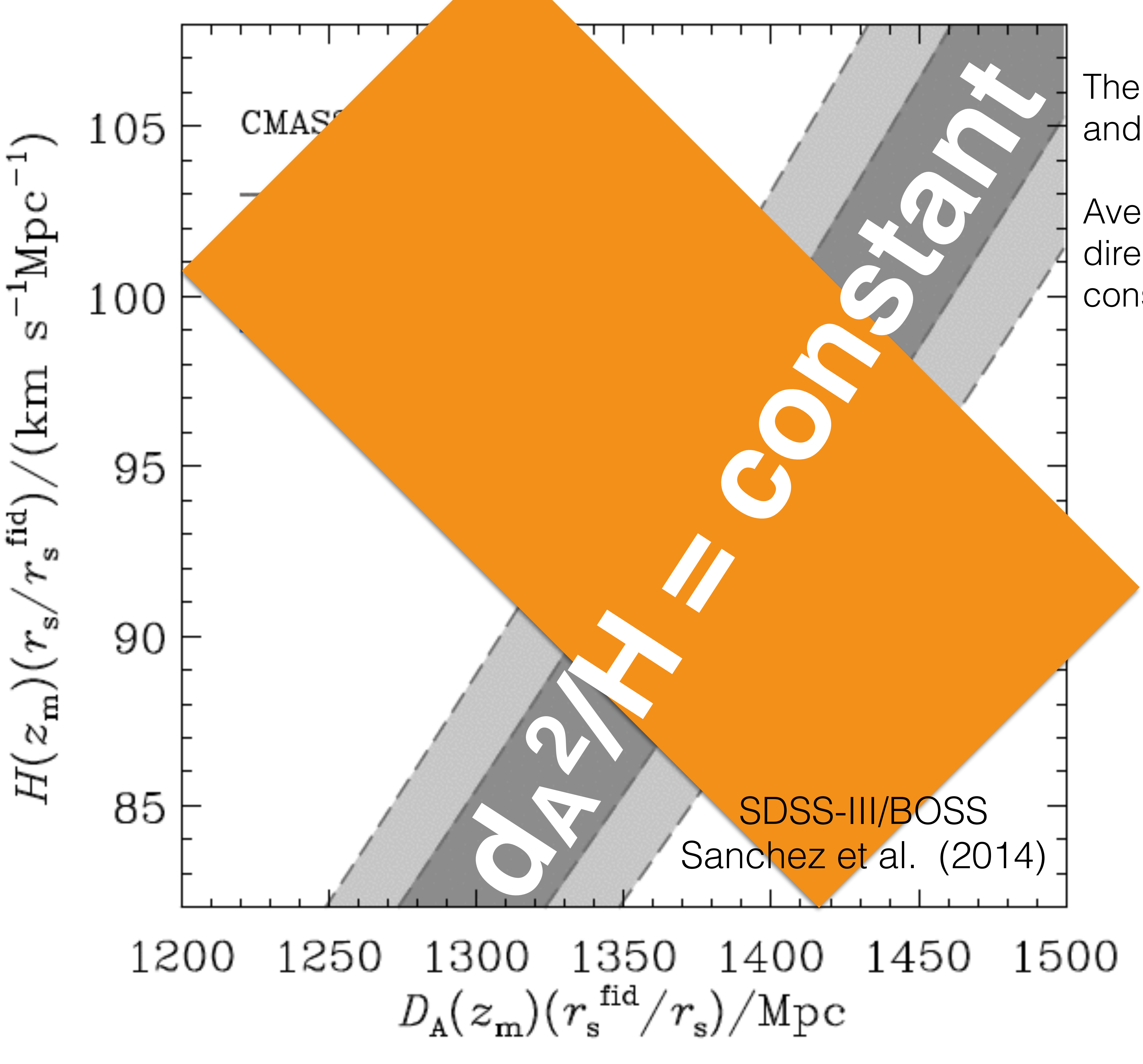


$d_A H$ from the AP test

- We tune d_A and H until the correlation function in comoving coordinates becomes isotropic [modulo RSD].

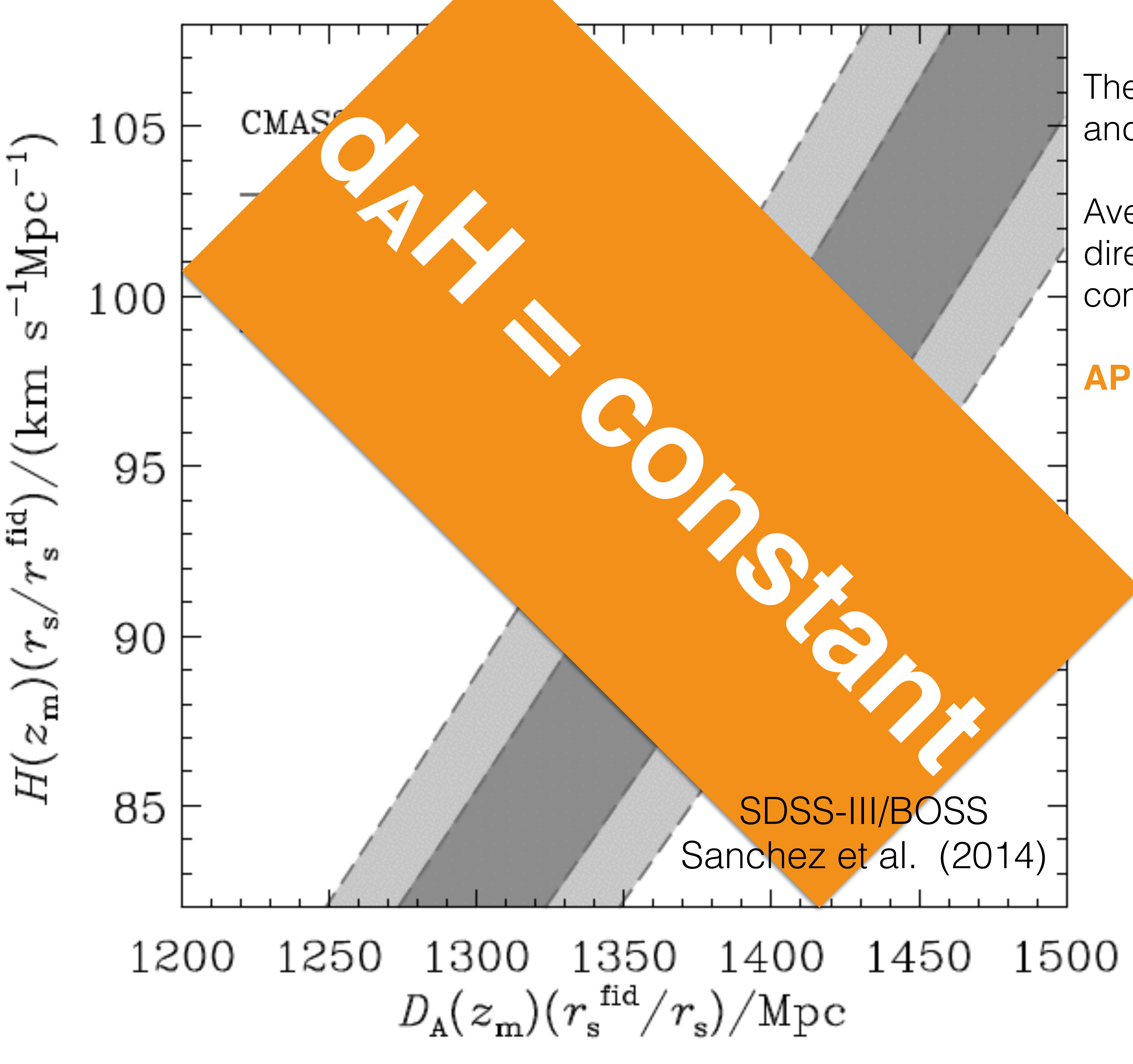
- However, the AP test cannot give d_A and H separately; it can only give $d_A H$.

- **Combining the AP test with the standard ruler method** giving d_A^2/H gives tight constraints on d_A and H separately! [Shoji, Jeong & Komatsu 2009]



There are 2 angular and 1 LOS directions.

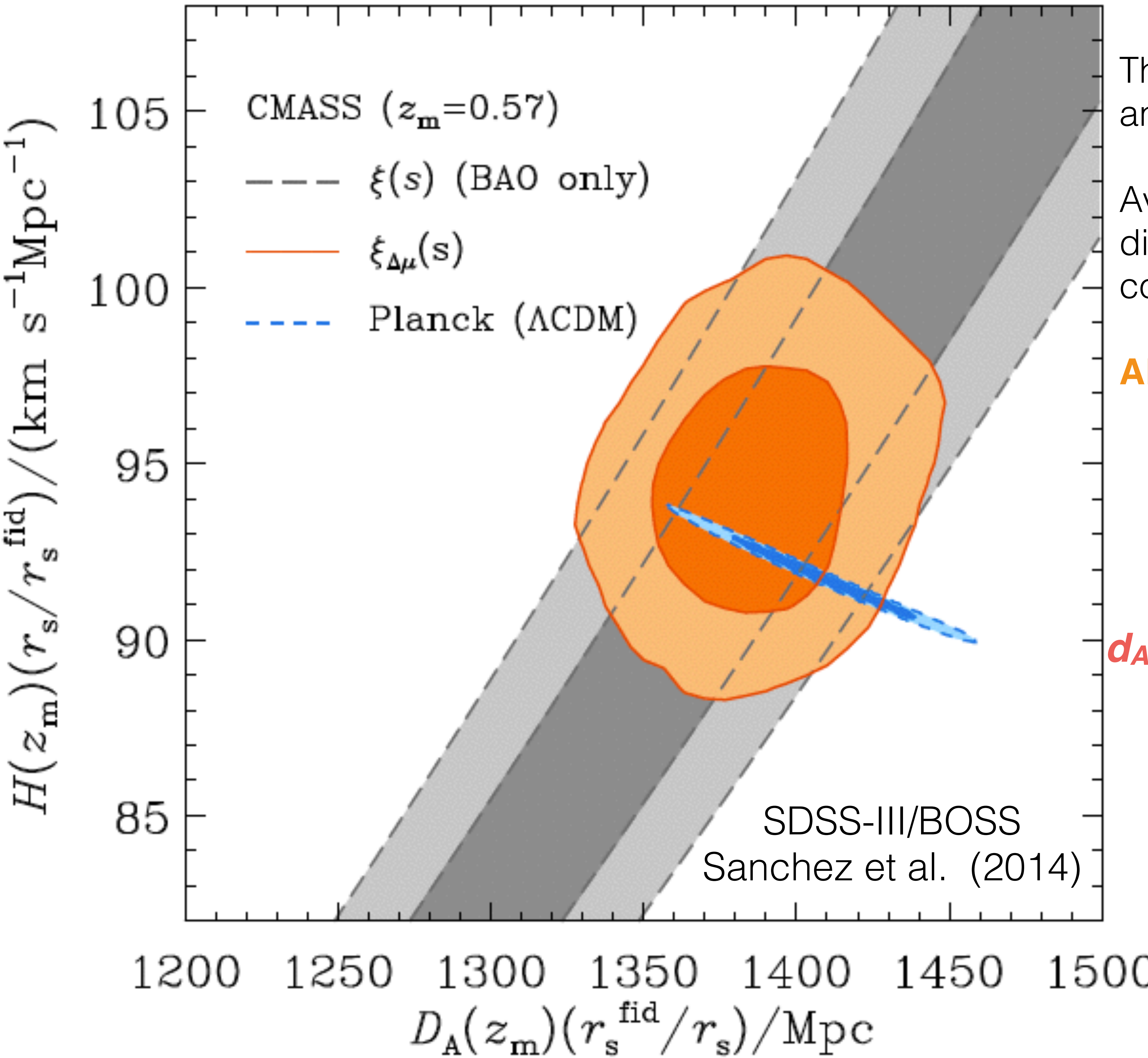
Averaging all three directions yields a constraint on **d_A^2/H**



There are 2 angular and 1 LOS directions.

Averaging all three directions yields a constraint on **d_A^2/H**

AP test gives $d_A H$



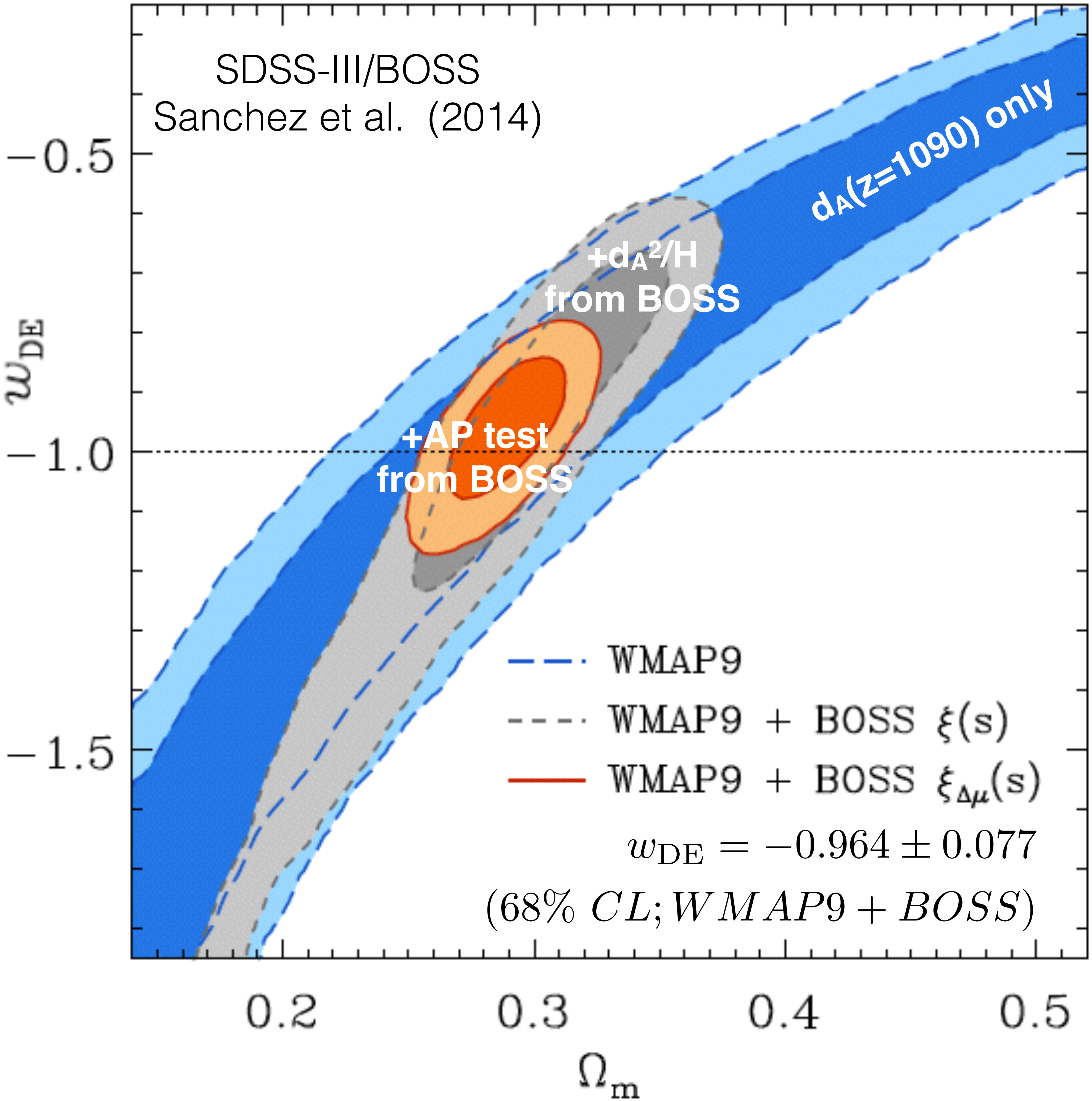
There are 2 angular and 1 LOS directions.

Averaging all three directions yields a constraint on $\mathbf{d_A^2/H}$

AP test gives $\mathbf{d_A H}$

d_A & H determined separately!

Limits on DE

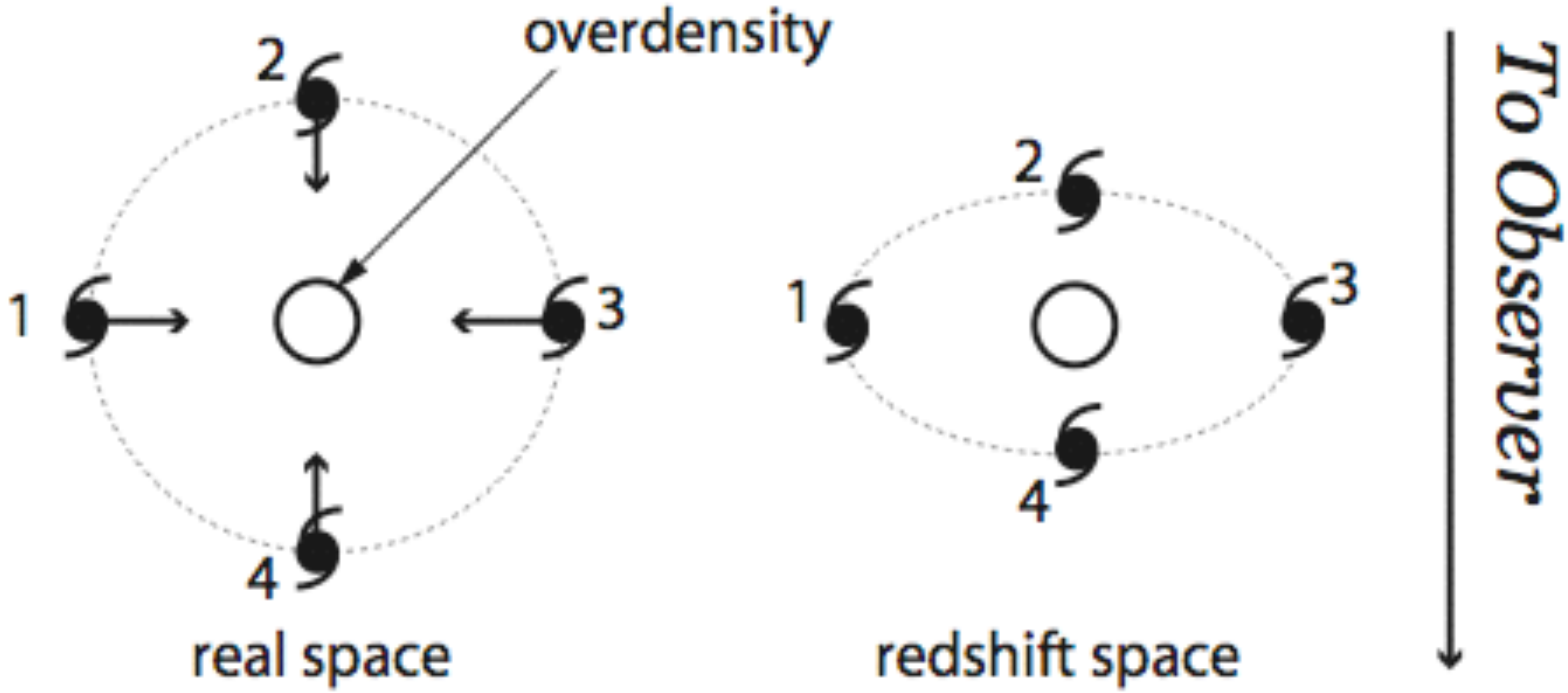


For a long time, we had to use Type Ia supernova data to put a competitive limit on the equation of state of DE, w_{DE}.

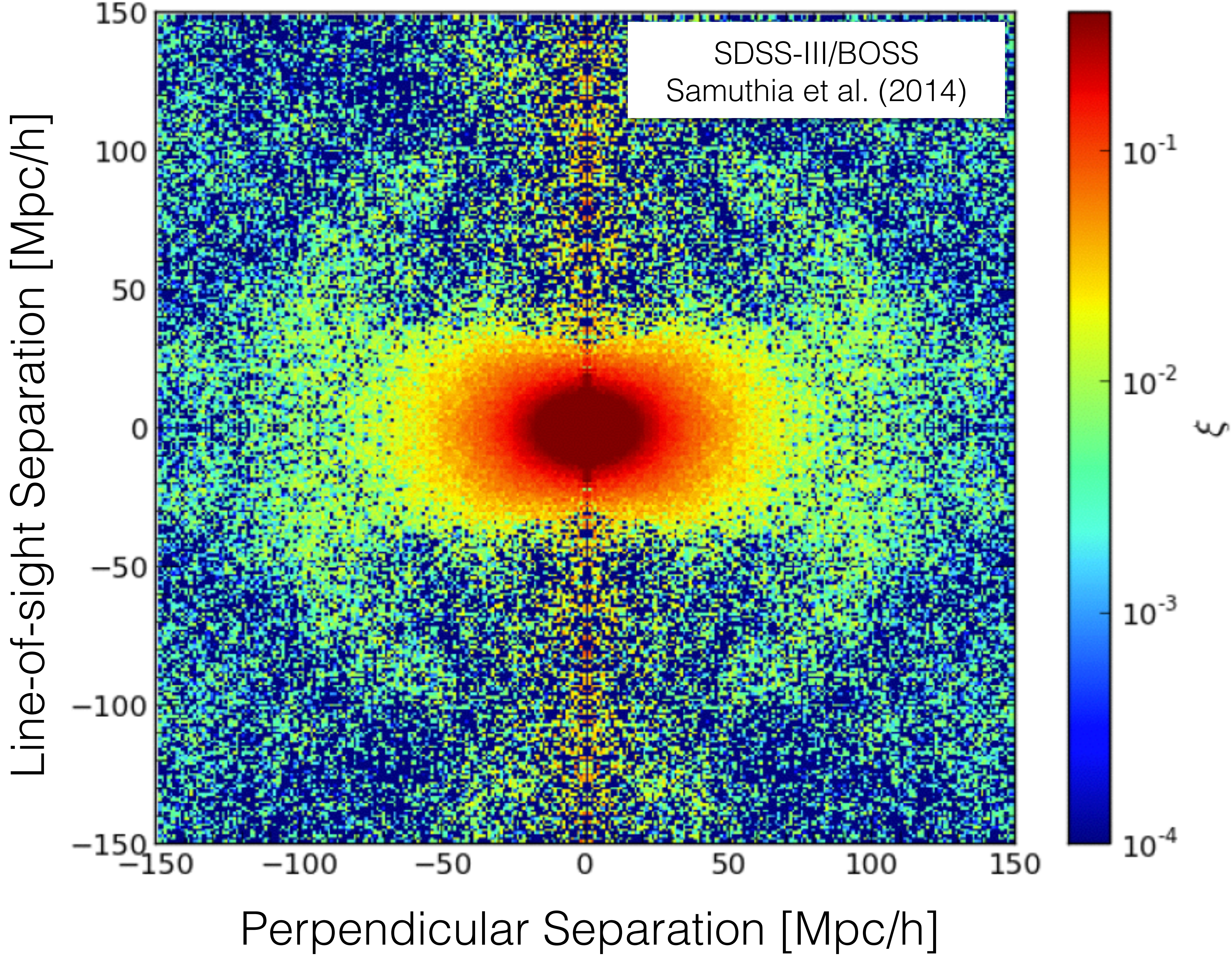
With the AP test, we can finally constrain w_{DE} **without using supernovae**!

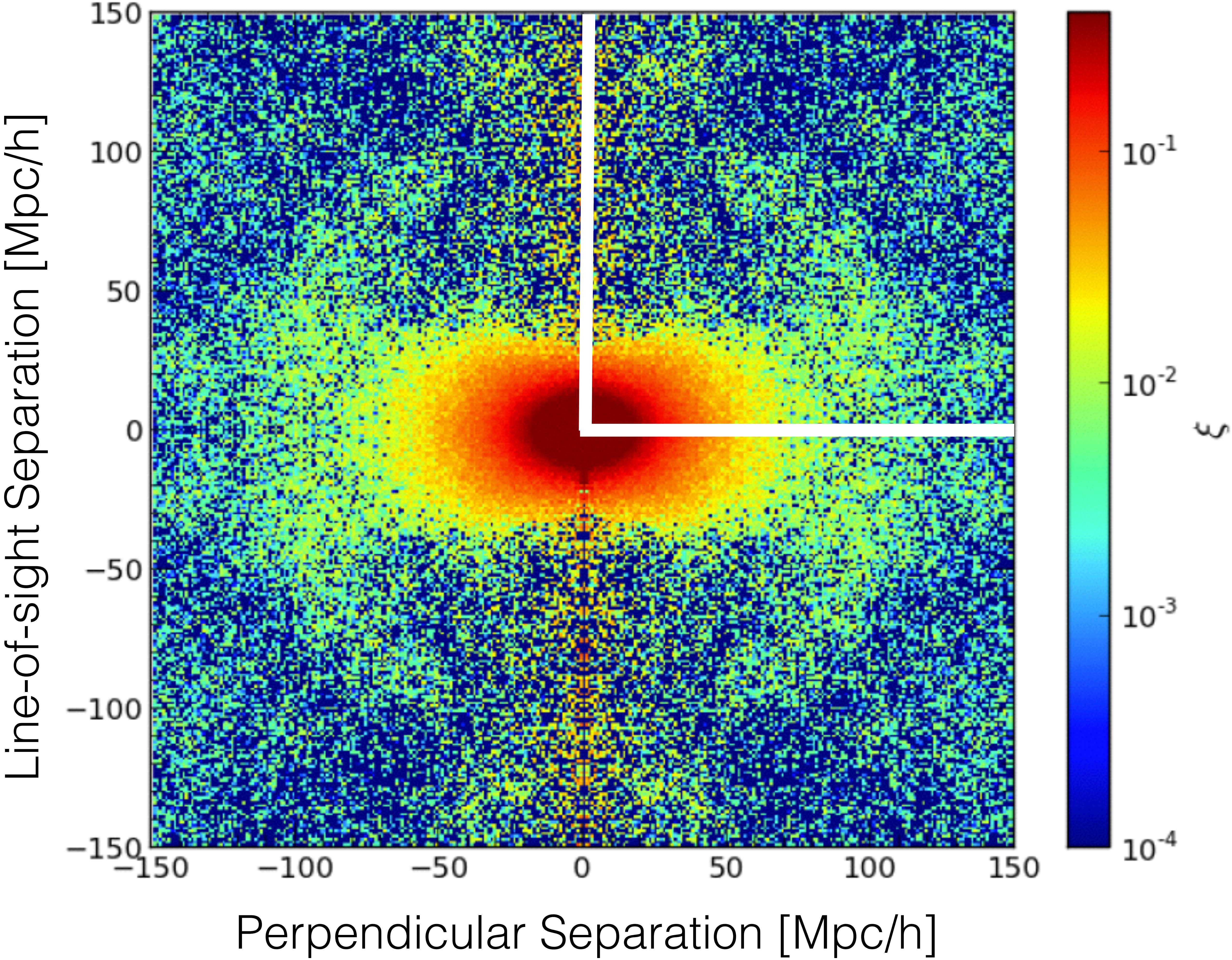
-> Powerful check of systematics

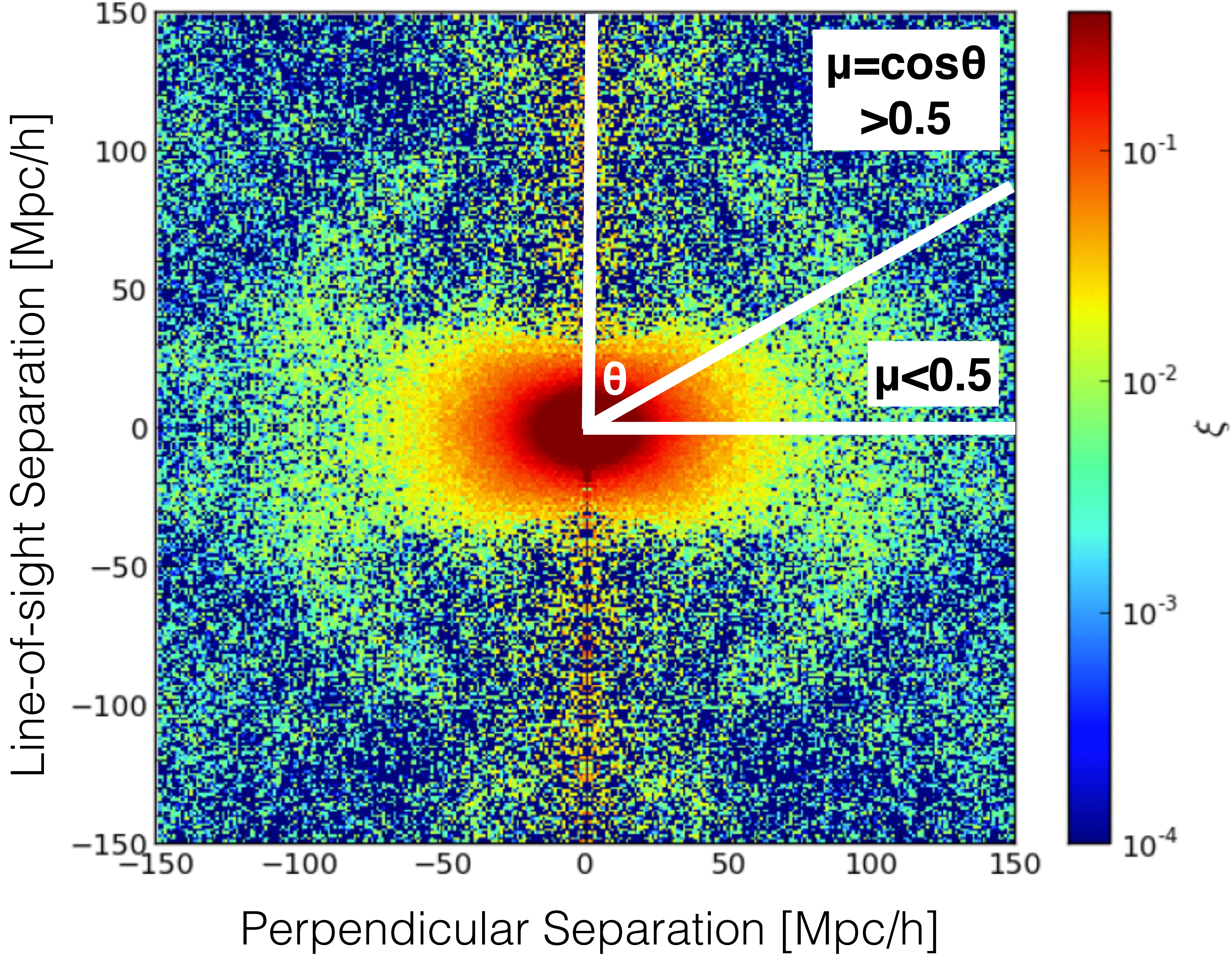
Redshift Space Distortion



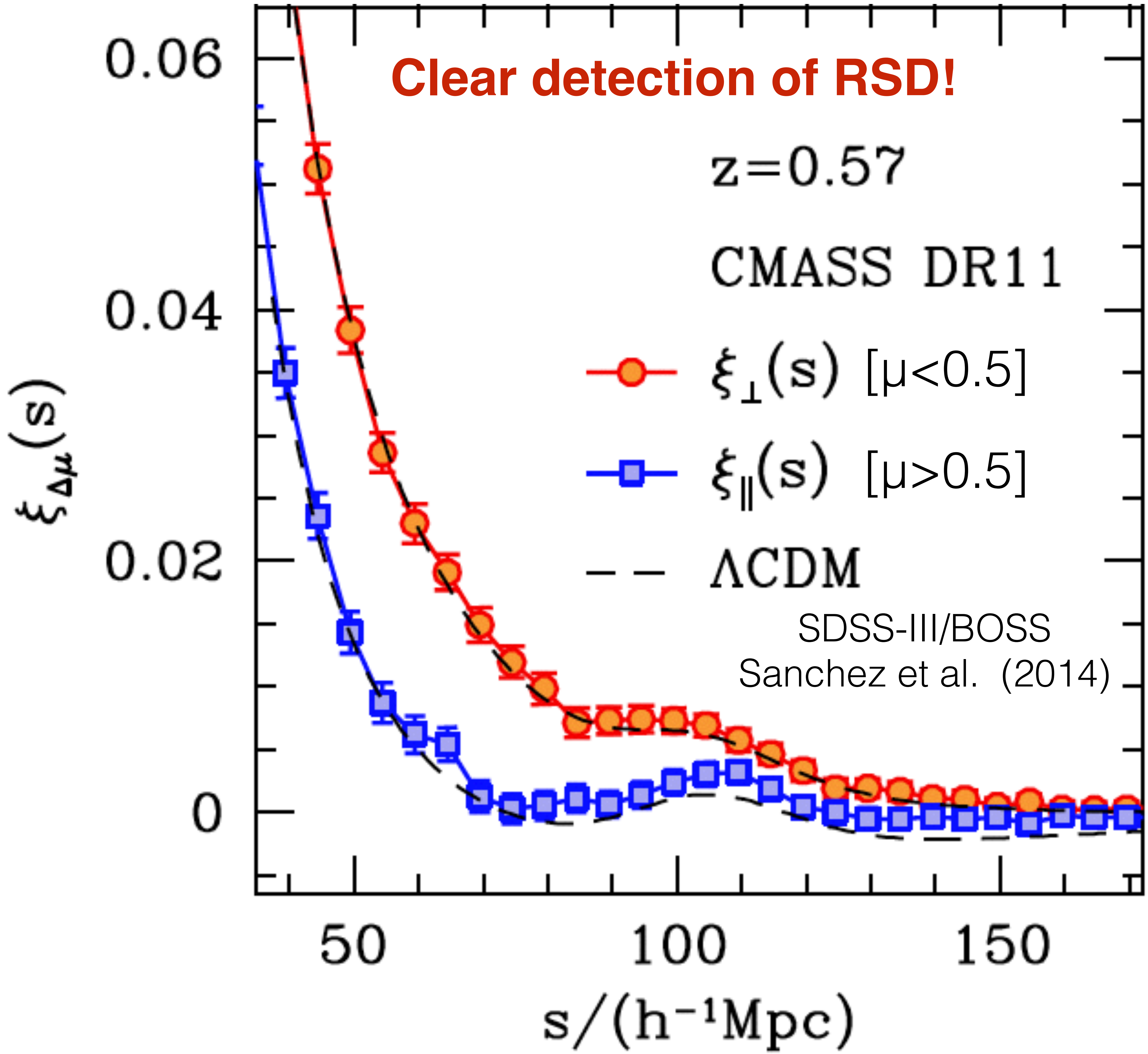
- Large-scale flow of galaxies into an over-density region **enhances** clustering along the line of sight


SDSS-III/BOSS
Samuthia et al. (2014)
Line-of-sight Separation [Mpc/h]
Perpendicular Separation [Mpc/h]
150
100
50
0
−50
−100
−150
−150
−100
−50
0
50
100
150
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ξ


150
100
50
0
−50
−100
−150
−150
−100
−50
0
50
100
150
Line-of-sight Separation [Mpc/h]
Perpendicular Separation [Mpc/h]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ξ



μ=cosθ
>0.5
θ
μ<0.5
150
100
50
0
−50
−100
−150
−150
−100
−50
0
50
100
150
Line-of-sight Separation [Mpc/h]
Perpendicular Separation [Mpc/h]
10^-1
10^-2
10^-3
10^-4
ξ



Clear detection of RSD!
z=0.57
CMASS DR11
$\xi_\perp(s)$ [μ<0.5]
$\xi_\parallel(s)$ [μ>0.5]
ΛCDM
SDSS-III/BOSS
Sanchez et al. (2014)
0.06
0.04
0.02
0
$\xi_{\Delta\mu}(s)$
50
100
150
$s/(h^{-1}\mathrm{Mpc})$

Kaiser Effect: Derivation

- Conservation of the number of galaxies

$$\bar{n}(1+\delta_s)d^3s = \bar{n}(1+\delta_r)d^3r$$

redshift space **real space**

$$\delta_s = \frac{1}{|J|}(1+\delta_r) - 1$$

- Jacobian matrix for real to redshift space trans. is

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{aH}\frac{\partial v_\parallel}{\partial x^1} & \frac{1}{aH}\frac{\partial v_\parallel}{\partial x^2} & 1 + \frac{1}{aH}\frac{\partial v_\parallel}{\partial x^3} \end{pmatrix}$$

$$\Rightarrow \quad |J| = 1 + \frac{1}{aH}\frac{\partial v_\parallel}{\partial x^3}$$

Kaiser Effect: Derivation

- Expanding to first order in perturbations

$$\delta_s = \frac{1}{|J|}(1+\delta_r) - 1 \quad \text{with} \quad |J| = 1 + \frac{1}{aH}\frac{\partial v_{\parallel}}{\partial x^3}$$

$$\Rightarrow \qquad \delta_s = \delta_r - \frac{1}{aH}\frac{\partial v_{\parallel}}{\partial x^3}$$

- To determine the 2nd term, use continuity equation

$$\dot{\delta}_r + \frac{1}{a}\nabla \cdot \mathbf{v} = 0$$

- The linear growth rate gives

$$\dot{\delta}_r = fH\delta_r \qquad \text{with} \qquad f \equiv 1 + \frac{d\ln g}{d\ln a}$$

Kaiser Effect: Derivation

- Going to Fourier space

$$\dot{\delta}_r + \frac{1}{a}\nabla \cdot \mathbf{v} = 0 \qquad \text{with} \qquad \dot{\delta}_r = fH\delta_r$$

$$\Rightarrow \qquad v_{\parallel,k} = iafH\frac{k_\parallel}{k^2}\delta_{r,k}$$

- Therefore

$$\delta_{s,k} = \left(1 + f\frac{k_\parallel^2}{k^2}\right)\delta_{r,k} = \left(1 + f\mu^2\right)\delta_{r,k}$$

where μ=cosθ, and θ is the angle between k and the line of sight

The Kaiser effect gives quadrupole dependence on μ

Constraining Growth from the Kaiser Effect

- The Kaiser effect gives a specific angular dependence of the correlation function, with the coefficient given by $f=1+d\ln g/d\ln a$
    - **It can be used to constrain $d\ln g/d\ln a$**
- However, the Kaiser formula is valid only in the linear regime. We must extend it to include non-linear effects. This calculation has not been completed yet, and **it is the most pressing issue in the large-scale structure community**

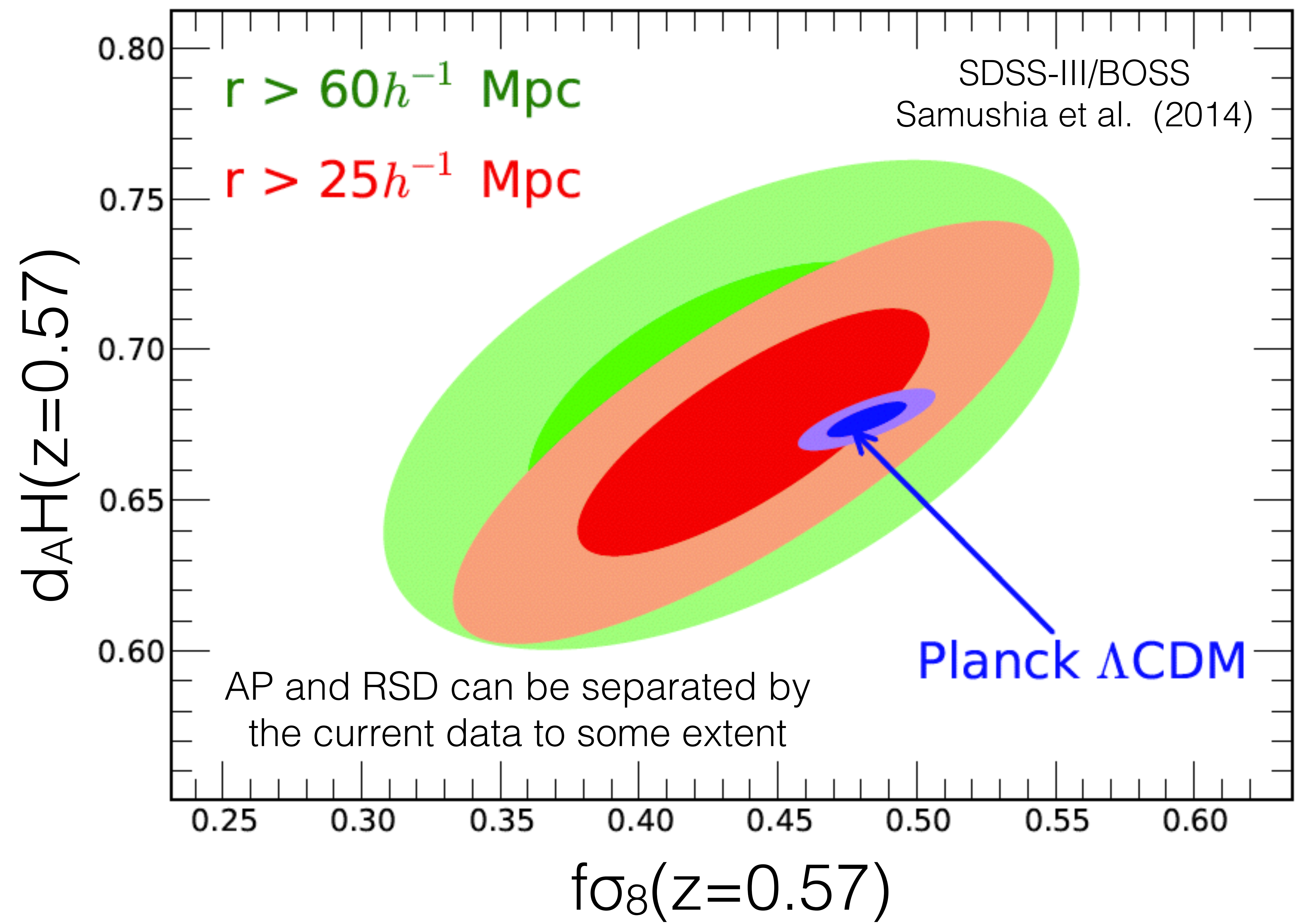
Galaxy Bias

- Another complication is that galaxies are biased tracers of the underlying mass distribution. In the linear regime, $\delta_{galaxy}=b\delta_{matter} \sim b\sigma_8$, in real space

- In redshift space, schematically

$$\delta_{\mathrm{g}}(\mu = 0) \propto b\sigma_8$$
$$\delta_{\mathrm{g}}(\mu = 1) \propto (b + f)\sigma_8$$

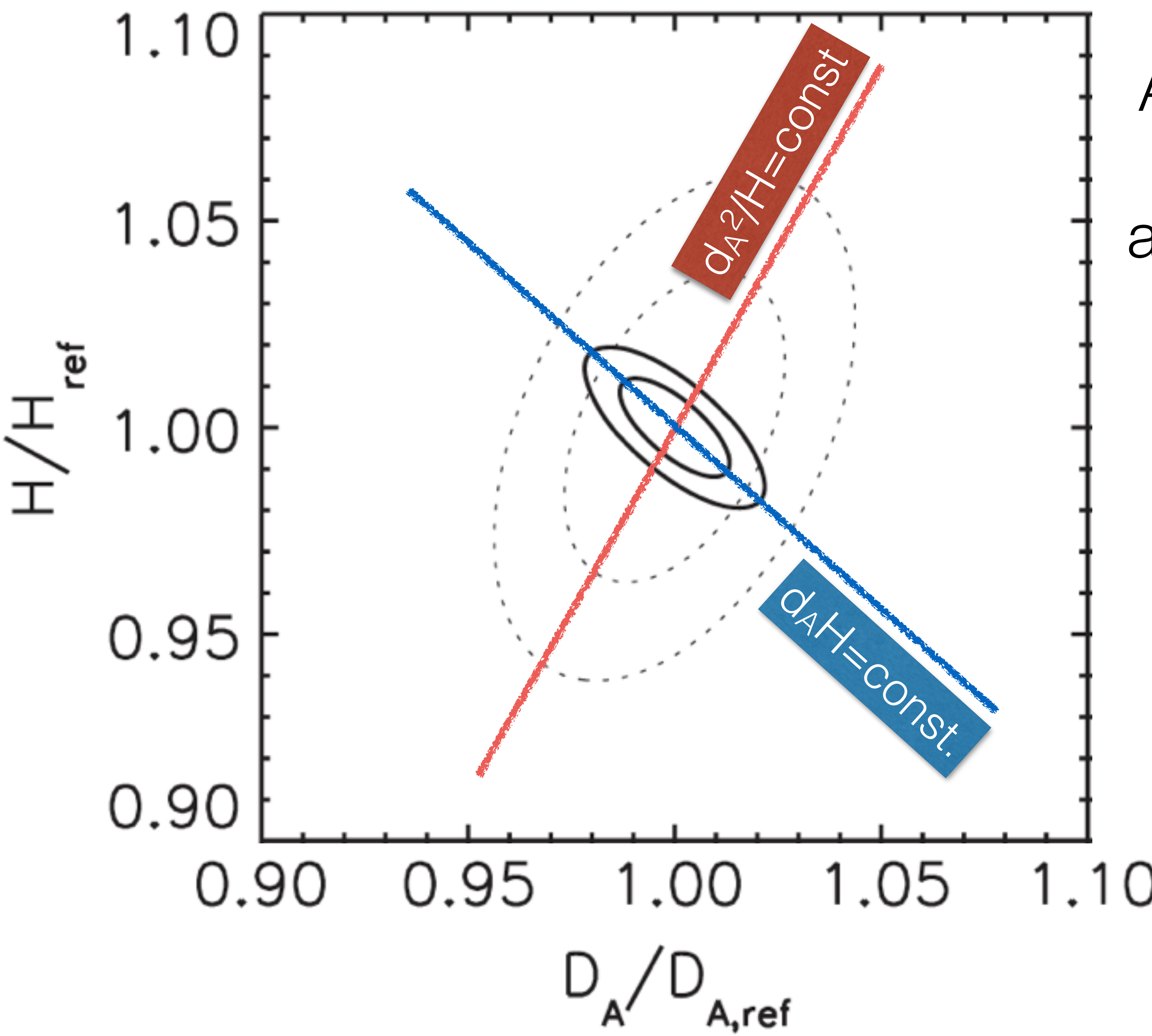
- Therefore, **the Kaiser effect yields $f\sigma_8$**, rather than f itself, unless we know the value of the bias factor, b. [This information can be obtained from weak lensing data, if available]



SDSS-III/BOSS
Samushia et al. (2014)
r > $60h^{-1}$ Mpc
r > $25h^{-1}$ Mpc
Planck ΛCDM
AP and RSD can be separated by
the current data to some extent
0.80
0.75
0.70
0.65
0.60
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
$d_A H(z=0.57)$
$f\sigma_8(z=0.57)$

Shoji, Jeong & Komatsu (2009)

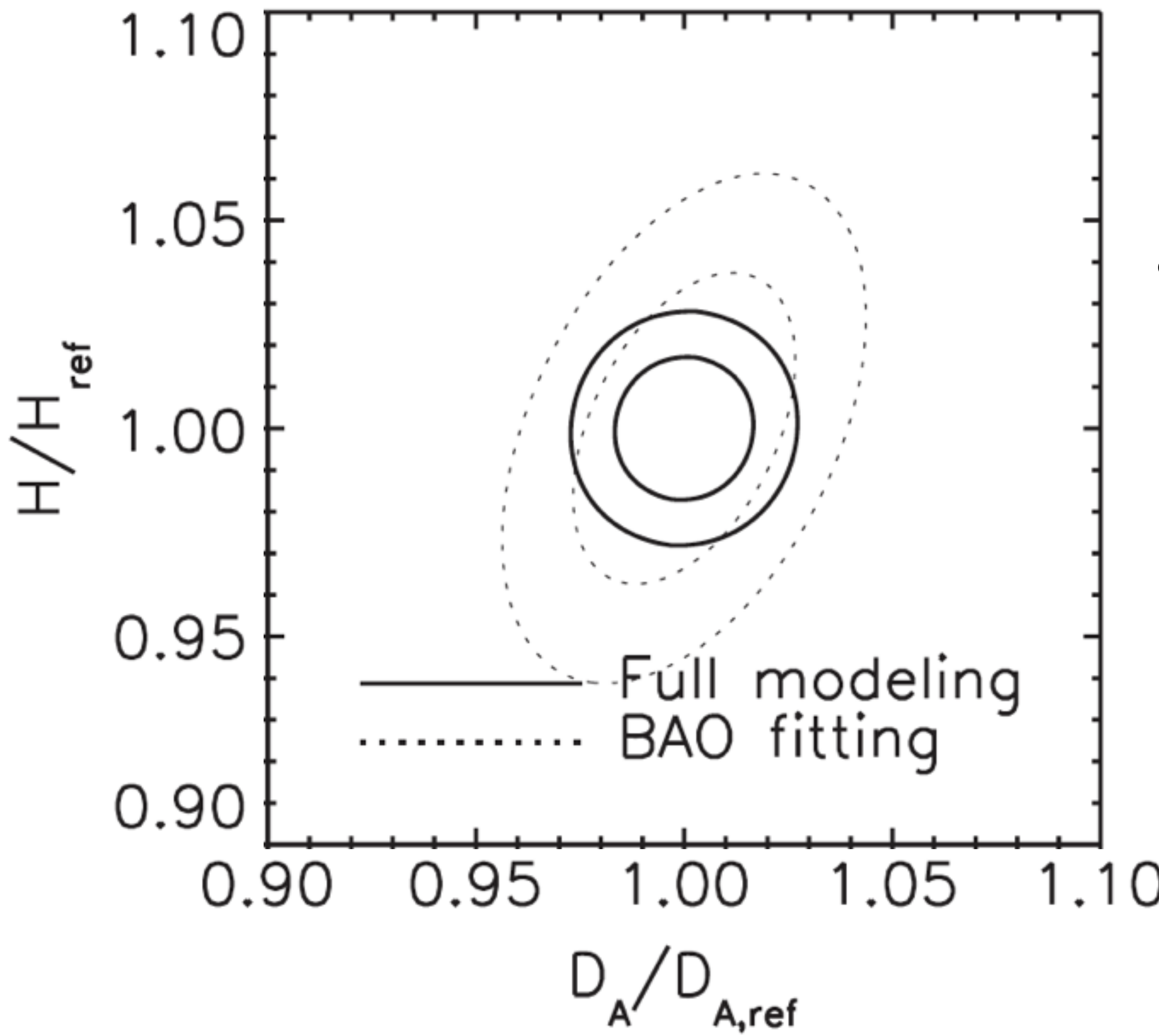
AP & RSD: Summary



All parameters but the over-all amplitude are fixed

Shoji, Jeong & Komatsu (2009)

AP & RSD: Summary

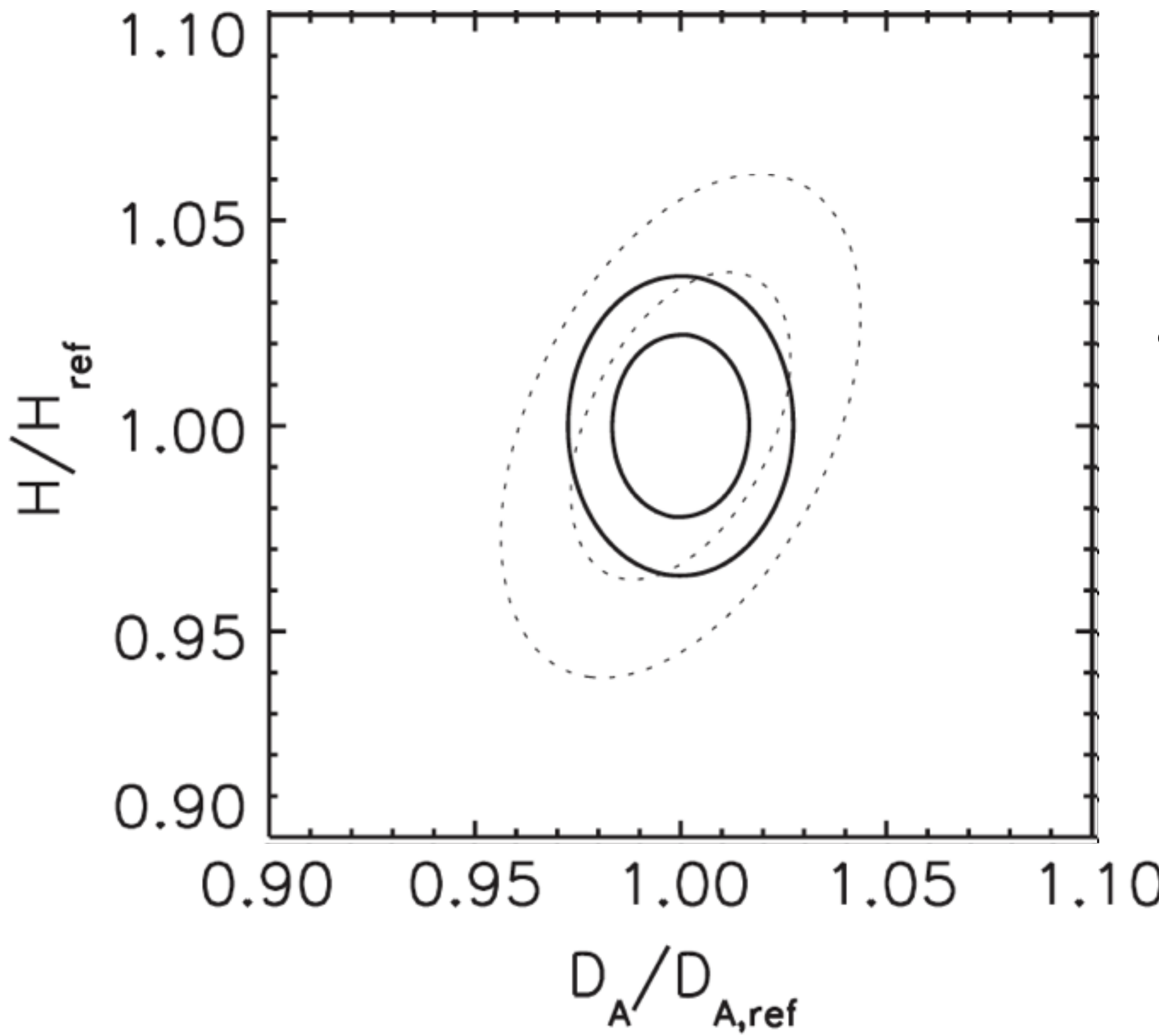


- Amplitude
- Linear RSD

are marginalised

Shoji, Jeong & Komatsu (2009)

AP & RSD: Summary

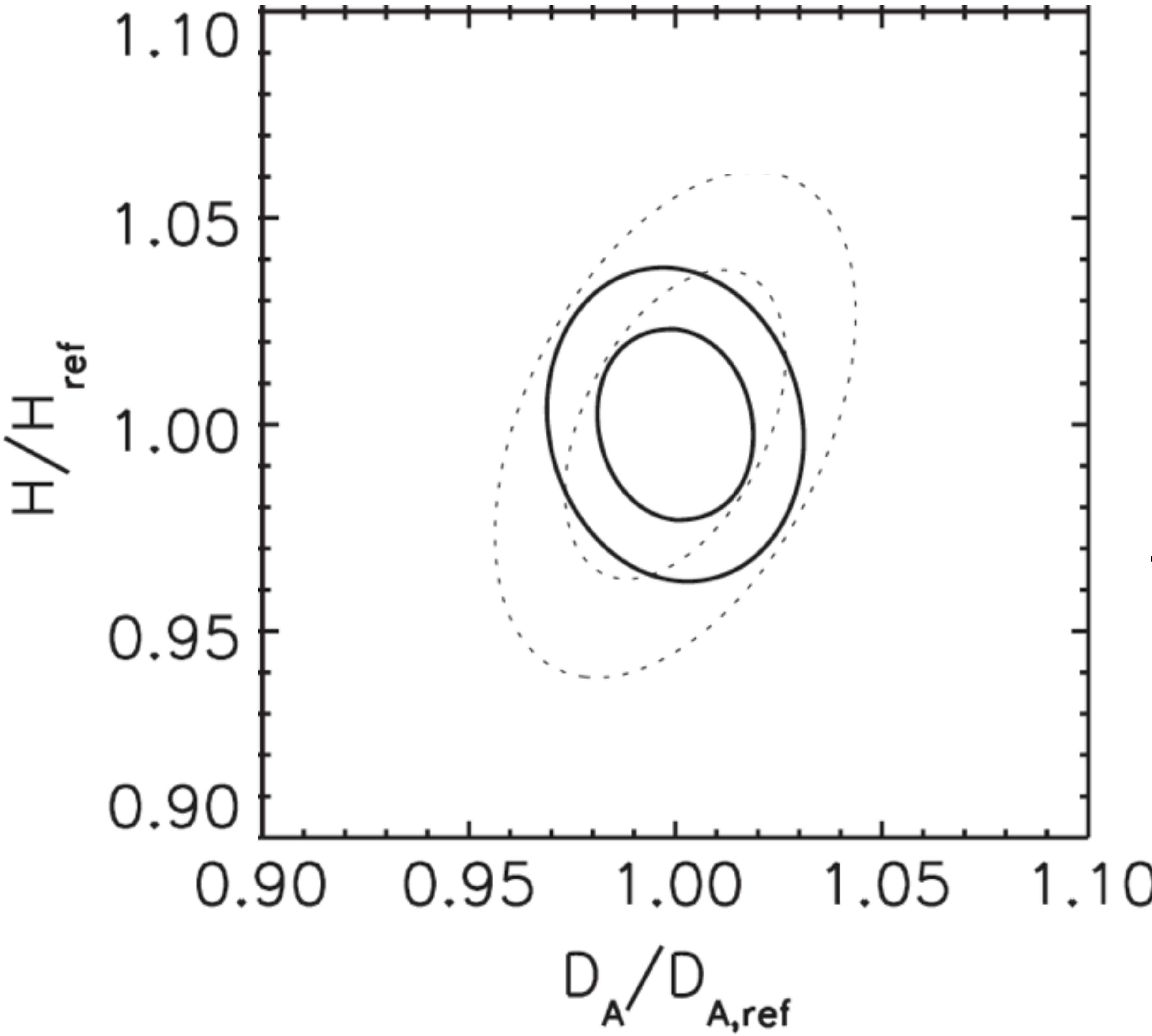


- Amplitude
- Linear RSD
- Non-linear RSD

are marginalised

Shoji, Jeong & Komatsu (2009)

AP & RSD: Summary



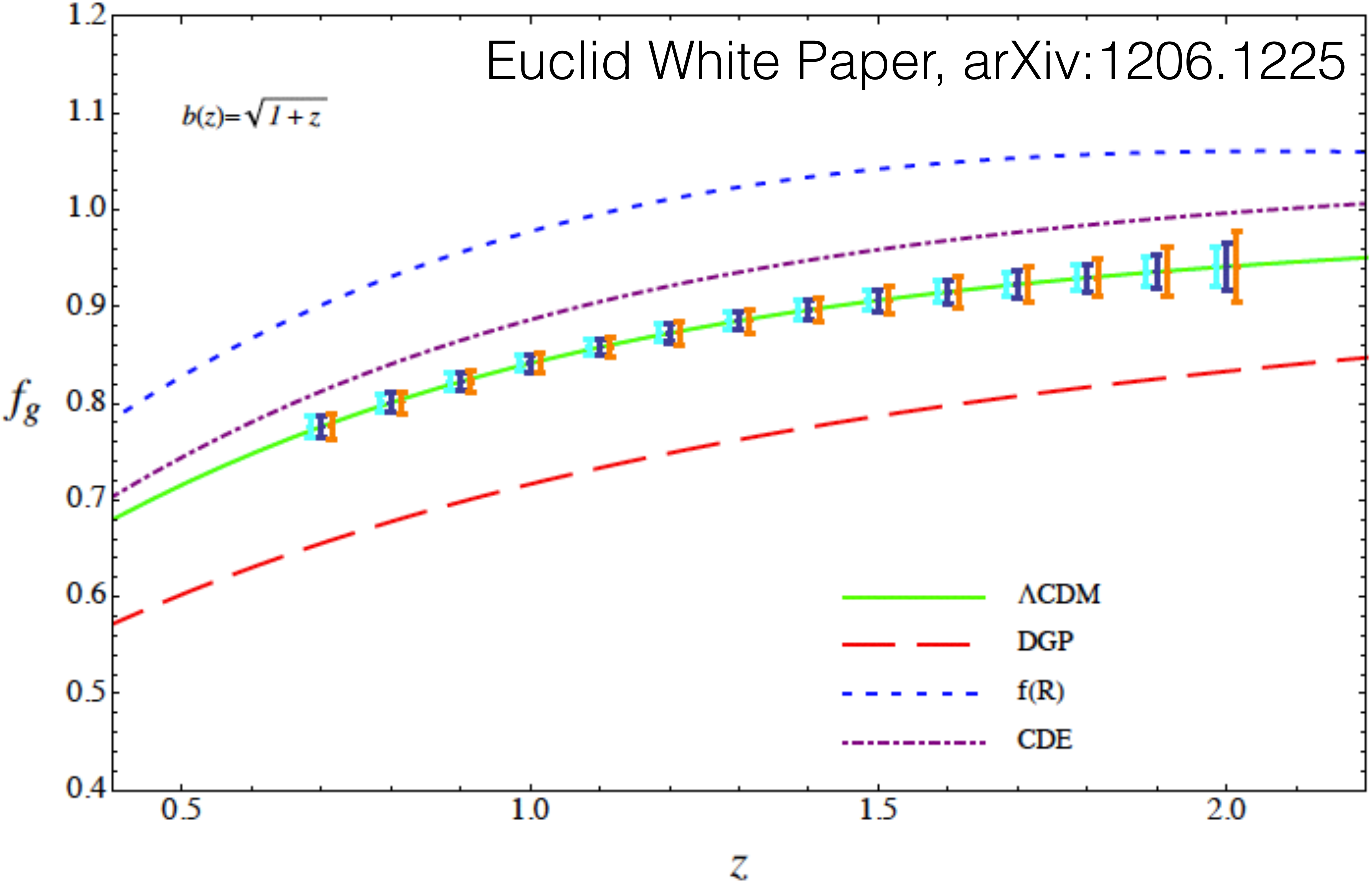
- Amplitude
- Linear RSD
- Non-linear RSD
- Primordial spectrum shape

are marginalised

DE and Galaxy Survey

- In summary, galaxy surveys can constrain DE via:
  - d_A^2/H from the standard ruler method,
  - d_AH from the AP test, and
  - $f=1+dg/d\ln a$ from [linear] RSD
- The first two constraints give the dark energy density, ρ_{DE}. Does it vary with time?
- GR+dark energy relates $dg/d\ln a$ with H. Does GR fit?

This may be possible in 10 years from now…



Galaxy Clusters

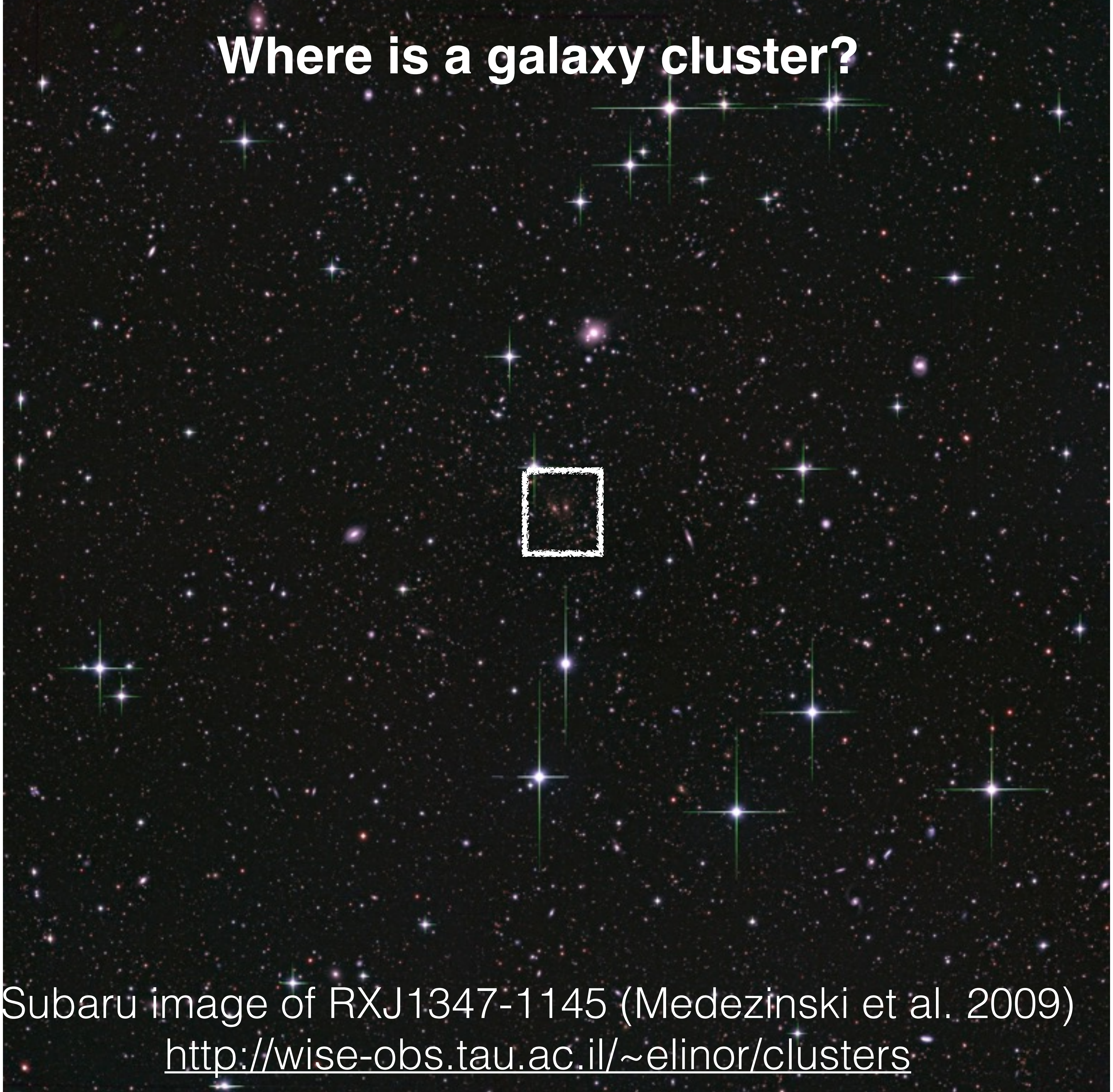
DE vs Galaxy Clusters

- **Counting** galaxy clusters provides information on dark energy by
  - Providing the comoving volume element which depends on $d_A(z)$ and $H(z)$
  - Providing the amplitude of matter fluctuations as a function of redshifts, $\sigma_8(z)$

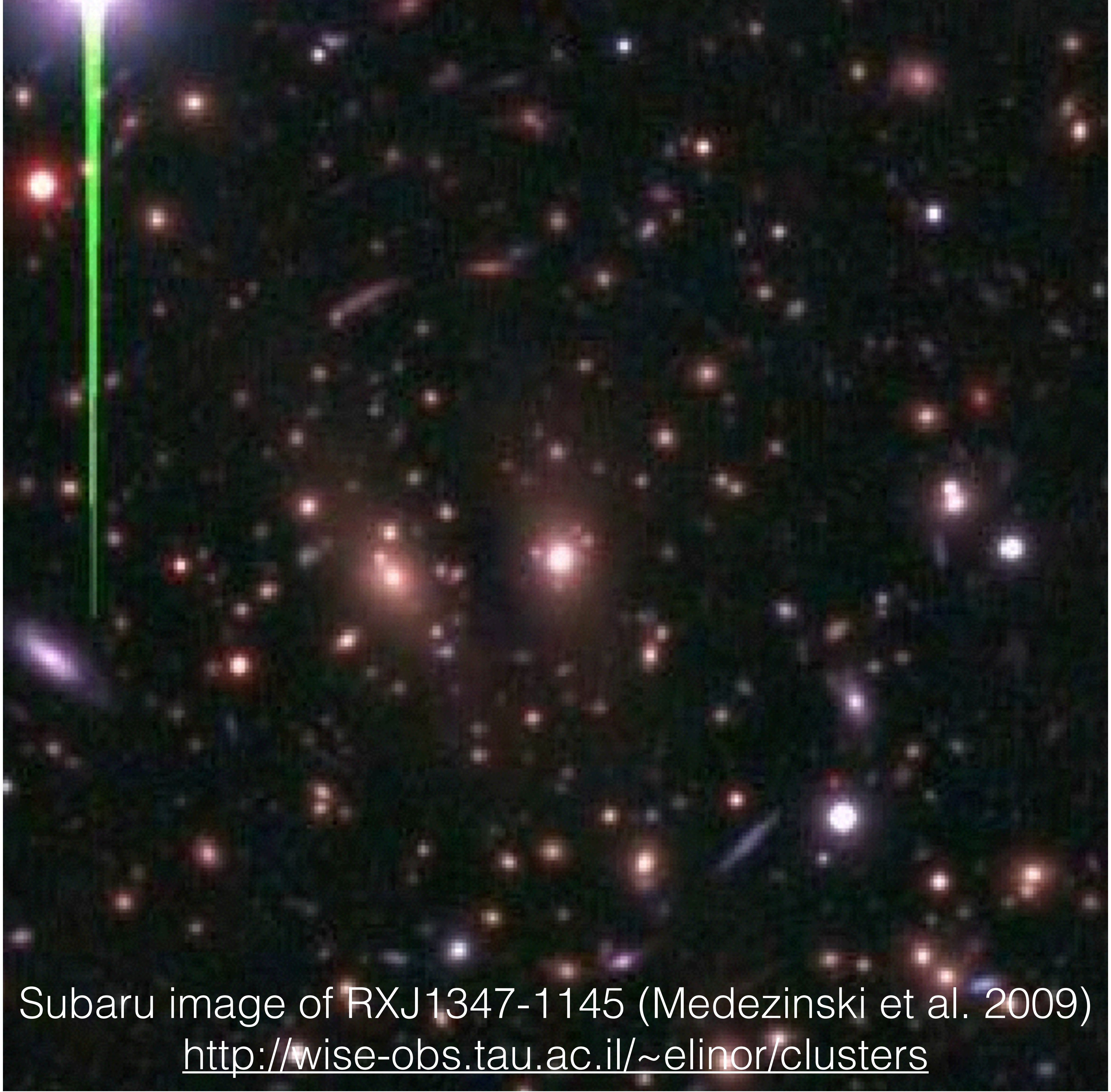
Where is a galaxy cluster?

Subaru image of RXJ1347-1145 (Medezinski et al. 2009)
http://wise-obs.tau.ac.il/~elinor/clusters

Where is a galaxy cluster?

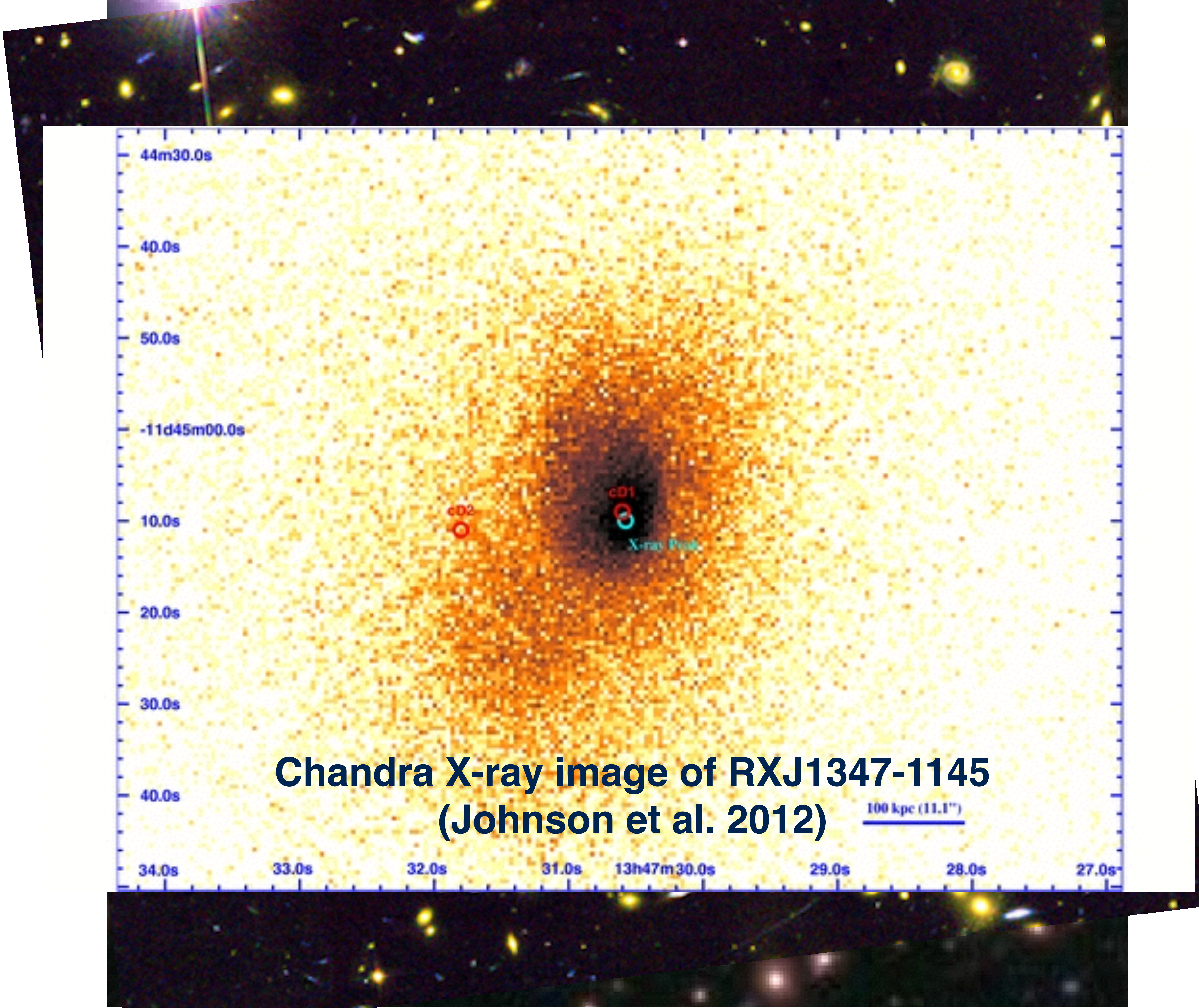


Subaru image of RXJ1347-1145 (Medezinski et al. 2009)
http://wise-obs.tau.ac.il/~elinor/clusters



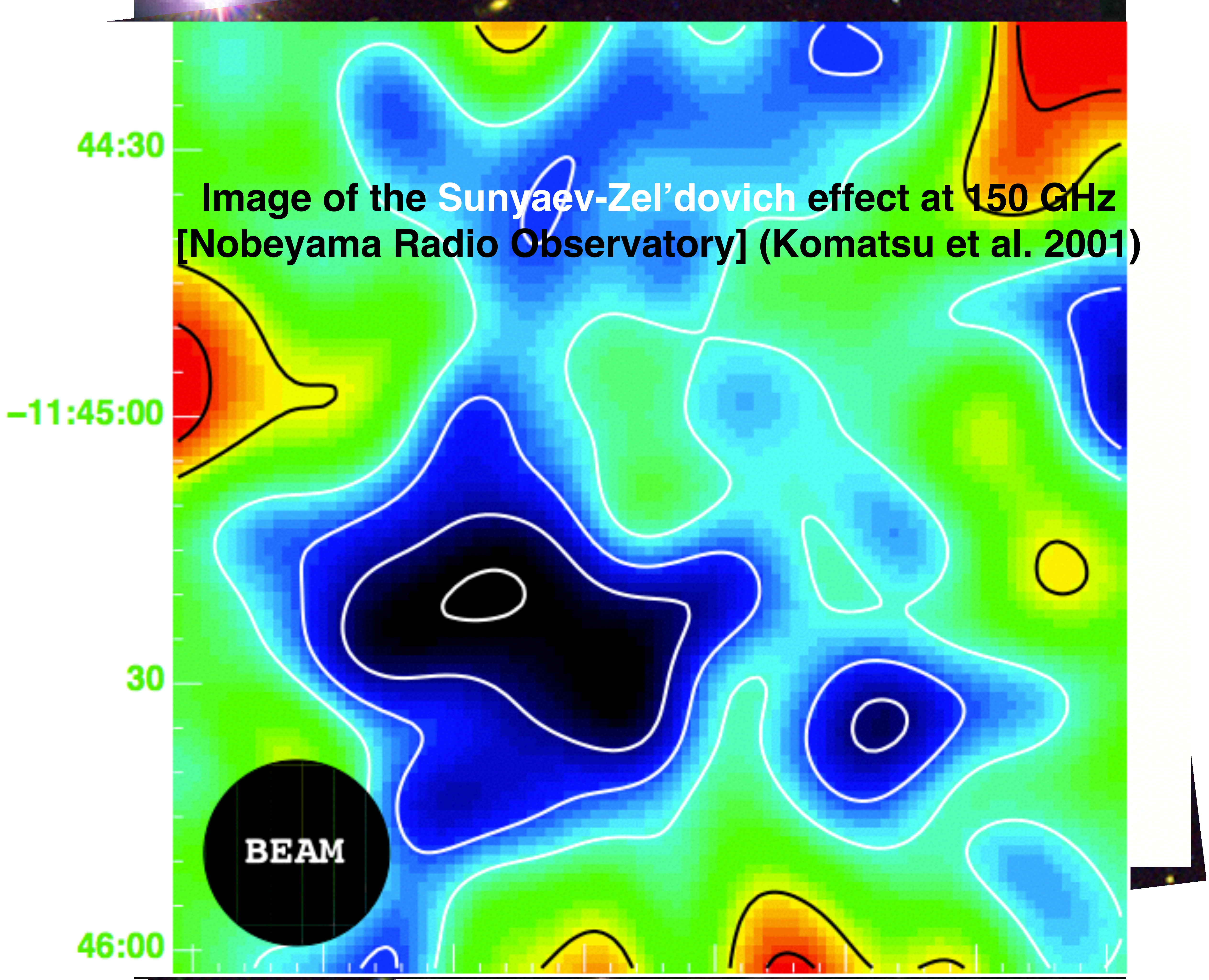
Subaru image of RXJ1347-1145 (Medezinski et al. 2009)
http://wise-obs.tau.ac.il/~elinor/clusters

Hubble image of RXJ1347-1145 (Bradac et al. 2008)



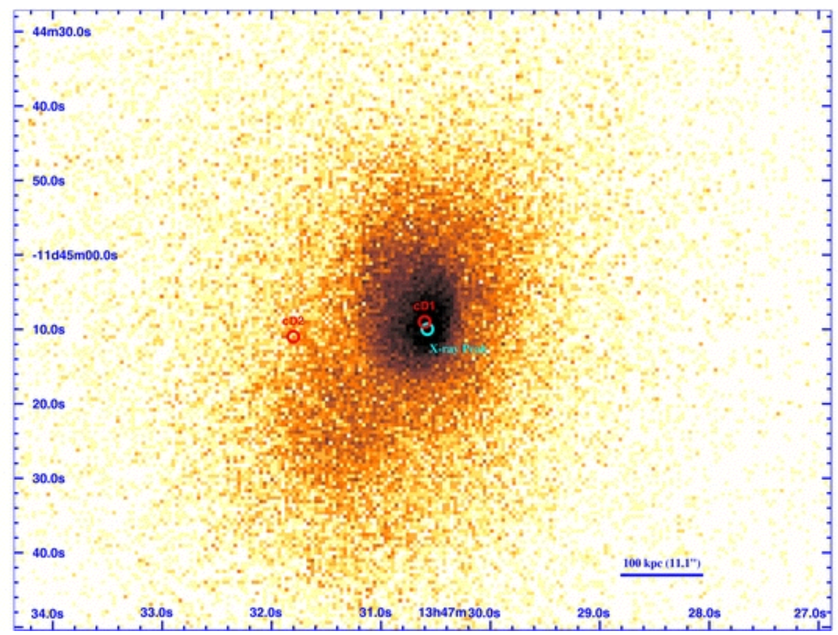
Chandra X-ray image of RXJ1347-1145 (Johnson et al. 2012)

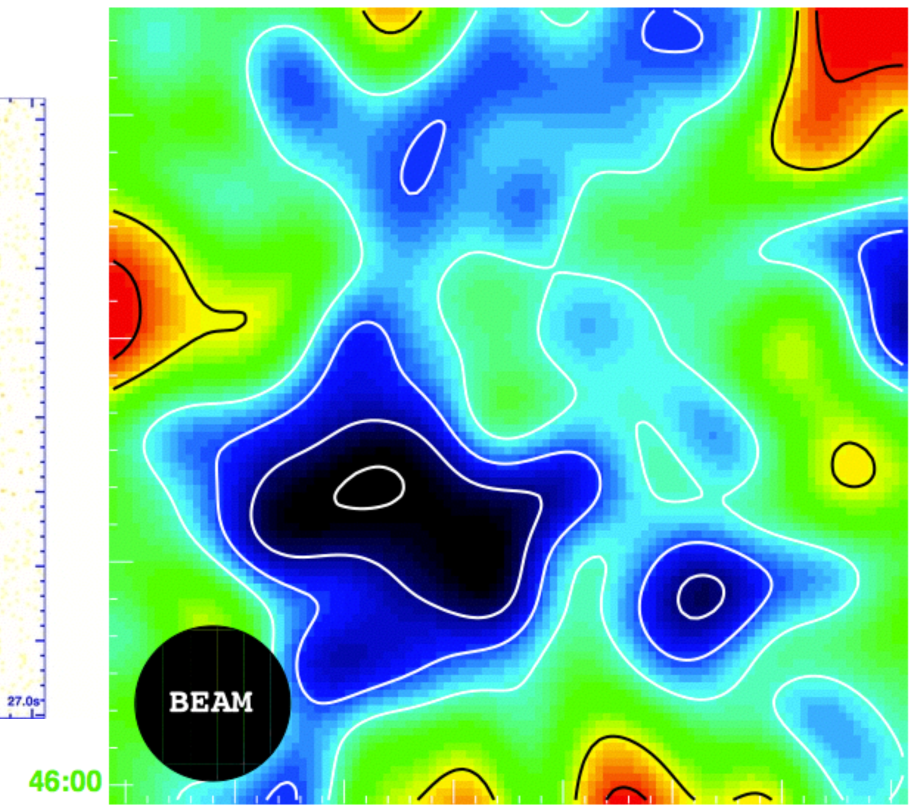
Image of the Sunyaev-Zel’dovich effect at 150 GHz [Nobeyama Radio Observatory] (Komatsu et al. 2001)



Multi-wavelength Data

$$I_X = \int dl\ n_e^2 \Lambda(T_X) \qquad I_{SZ} = g_\nu \frac{\sigma_T k_B}{m_e c^2} \int dl\ n_e T_e$$





Optical:
- $10^{2–3}$ galaxies
- velocity dispersion
- gravitational lensing

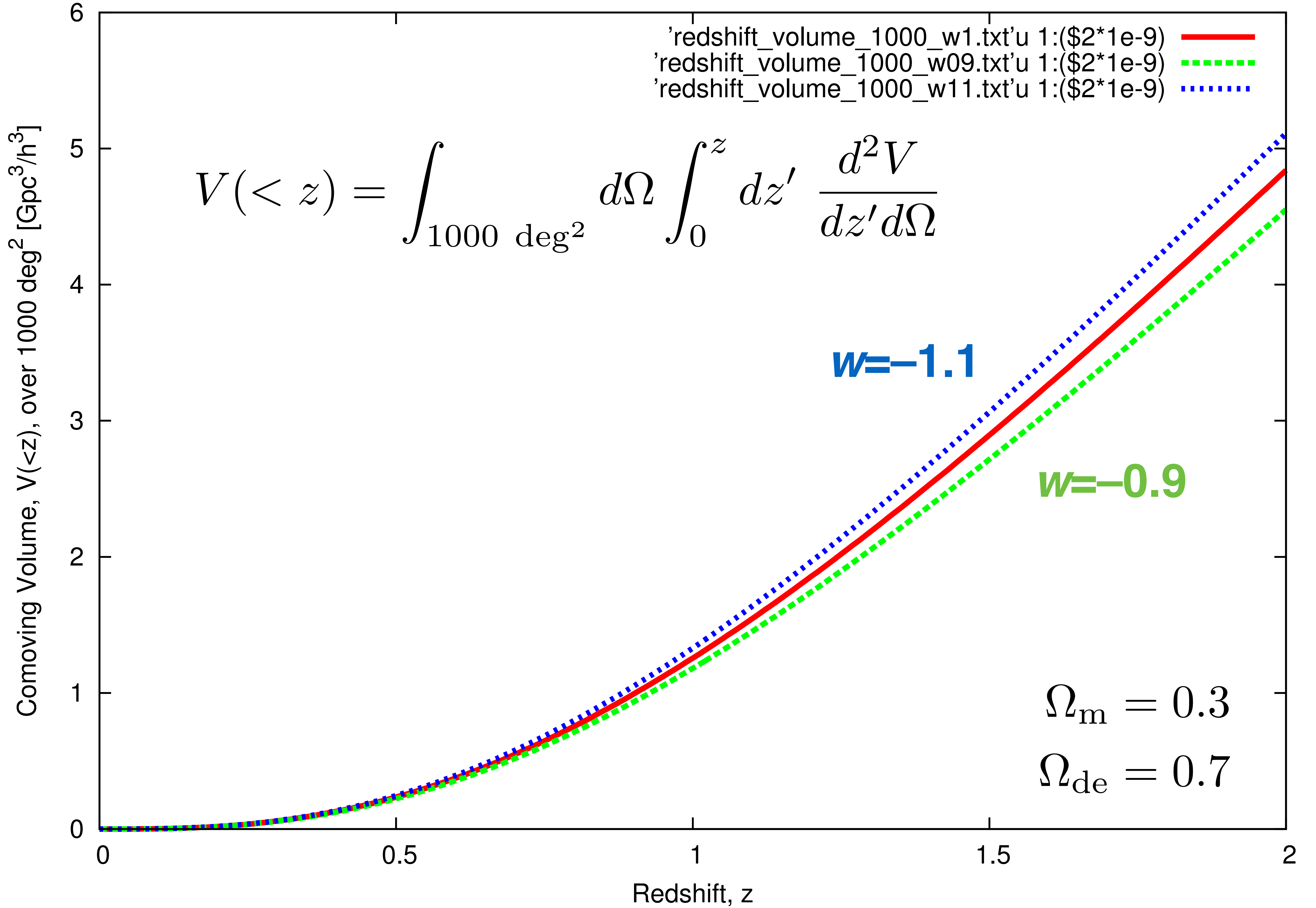
X-ray:
- hot gas ($10^{7–8}$ K)
- spectroscopic T_X
- Intensity ~ $n_e^2 L$

SZ [microwave]:
- hot gas (10^{7-8} K)
- electron pressure
- Intensity ~ $n_e T_e L$

Galaxy Cluster Counts

- We count galaxy clusters over a certain region in the sky [with the solid angle Ω_{obs}]
- Our ability to detect clusters is limited by noise [limiting flux, F_{lim}]
- For a comoving number density of clusters per unit mass, dn/dM, the observed number count is

$$N = \Omega_{\rm obs} \int_0^\infty dz \; \frac{d^2V}{dz d\Omega} \int_{F_{\rm lim}(z)}^\infty dF \; \frac{dn}{dM} \frac{dM}{dF}$$



'redshift_volume_1000_w1.txt'u 1:($2*1e-9)
'redshift_volume_1000_w09.txt'u 1:($2*1e-9)
'redshift_volume_1000_w11.txt'u 1:($2*1e-9)
$V(< z) = \int_{1000\ \mathrm{deg}^2} d\Omega \int_0^z dz' \ \frac{d^2V}{dz' d\Omega}$
w=−1.1
w=−0.9
$\Omega_\mathrm{m} = 0.3$
$\Omega_\mathrm{de} = 0.7$
Comoving Volume, V(<z), over 1000 deg^2 [Gpc^3/h^3]
Redshift, z
0
0.5
1
1.5
2
0
1
2
3
4
5
6

Mass Function, dn/dM

- The comoving number density per unit mass range, dn/dM, is **exponentially** sensitive to the amplitude of matter fluctuations, σ_8, for high-mass, rare objects
  - By "high-mass objects", we mean "high peaks," satisfying $1.68/\sigma(M) > 1$

δ is a Random Gaussian field:

$$\boxed{\underbrace{P(\delta)}_{\text{probability}} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\delta^2/2\sigma^2}}$$

$\int_{-\infty}^{\infty} d\delta \, P(\delta) = 1$

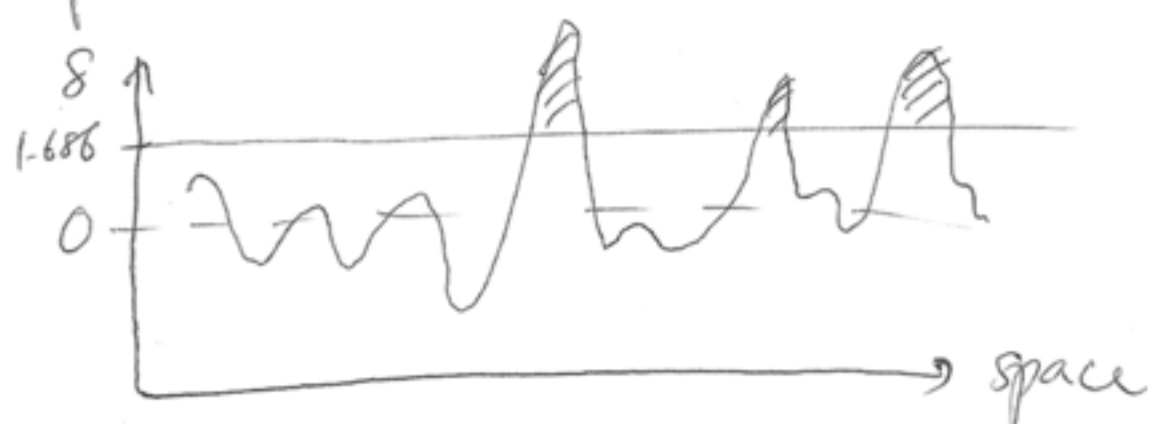
σ^2: variance of δ

Mass Function, dn/dM

- The comoving number density per unit mass range, dn/dM, is **exponentially** sensitive to the amplitude of matter fluctuations, σ_8, for high-mass, rare objects

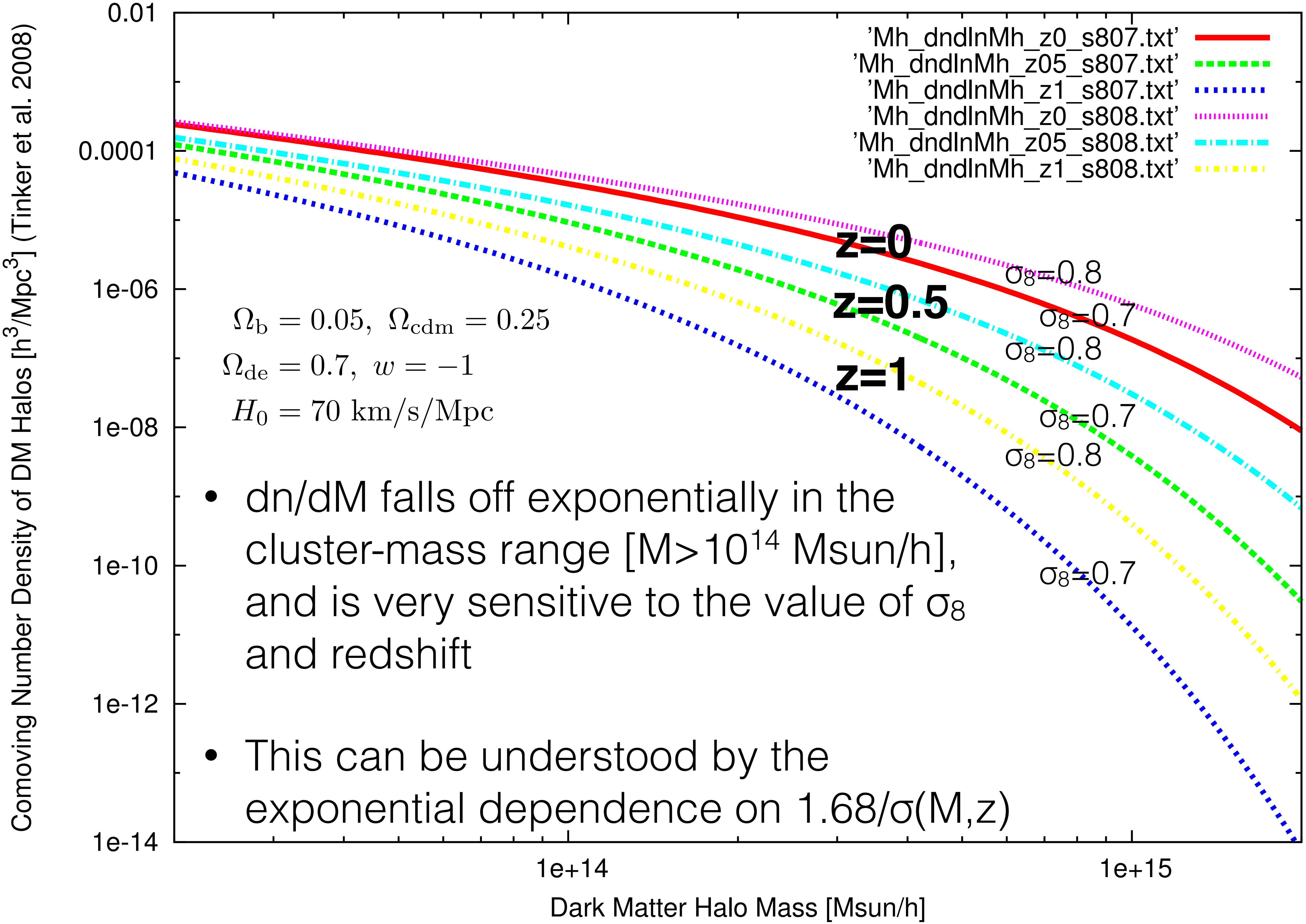
  - By “high-mass objects”, we mean “high peaks,” satisfying $1.68/\sigma(M) > 1$

Collapsed Regions have linearly-extrapolated $\delta \geq 1.686$



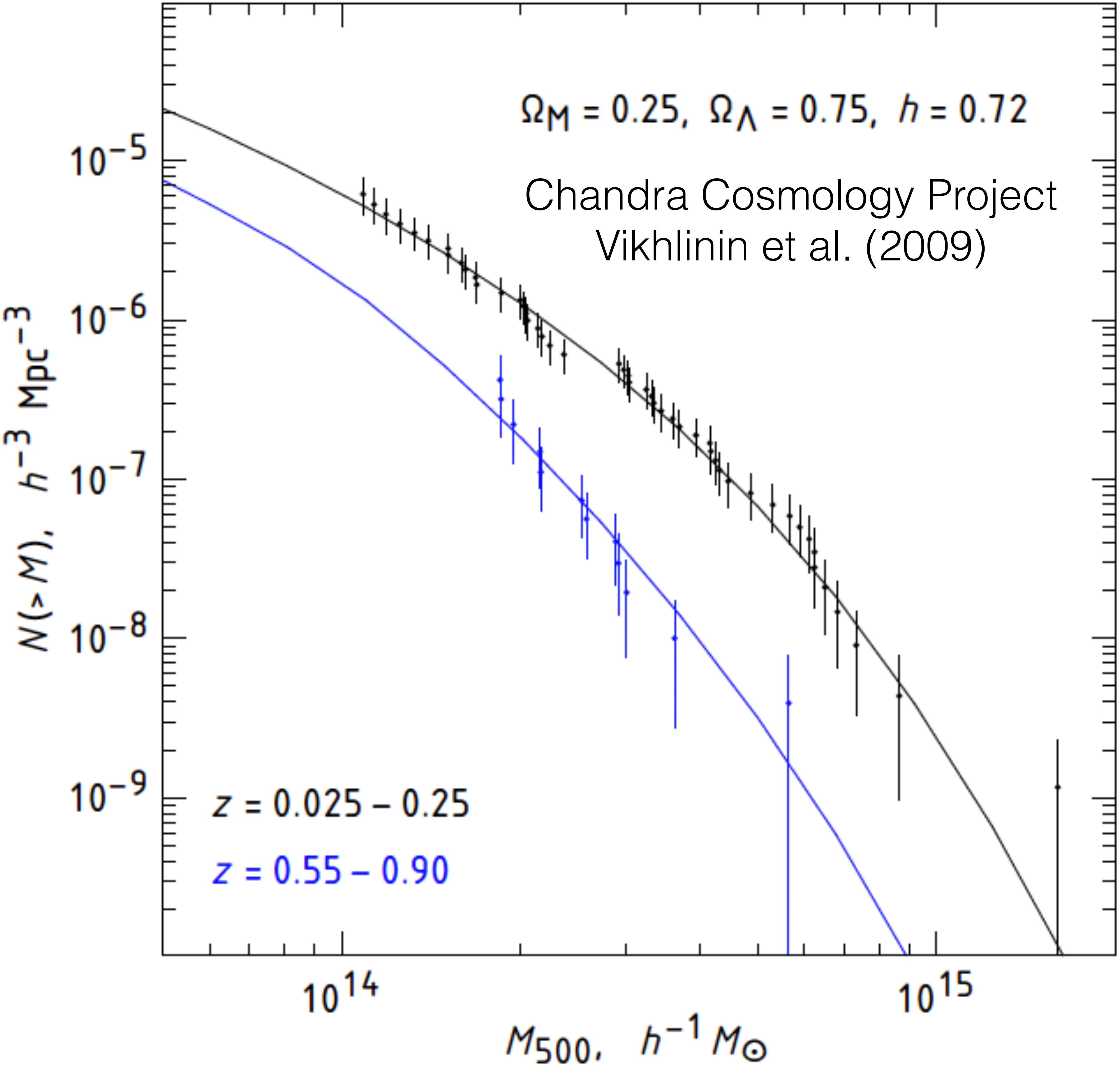
Areas of those collapsed regions

$$= \int_{\delta_c = 1.686}^{\infty} P(\delta)\, d\delta .$$

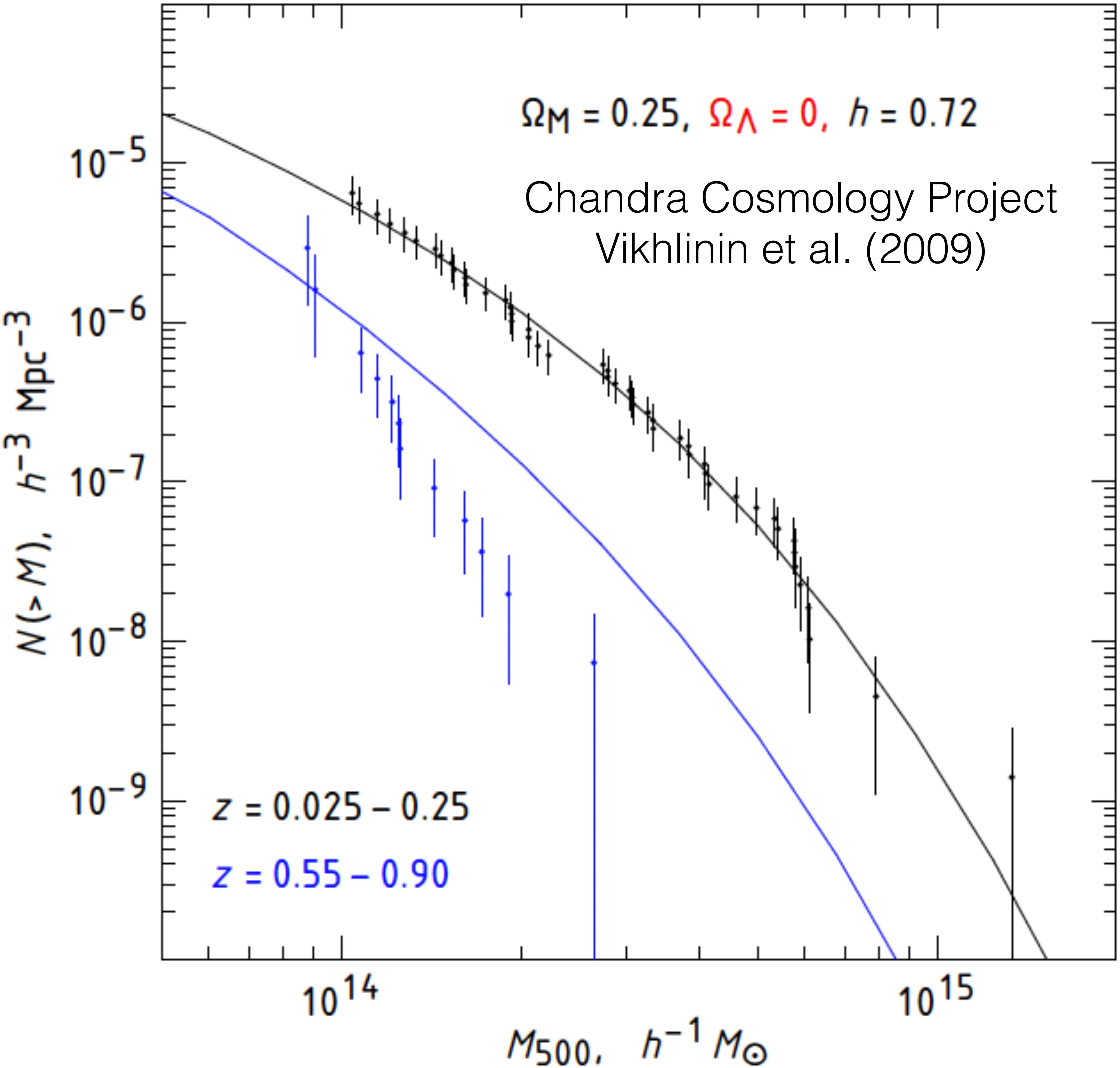


- dn/dM falls off exponentially in the cluster-mass range [M>10^{14} Msun/h], and is very sensitive to the value of σ_8 and redshift
- This can be understood by the exponential dependence on $1.68/\sigma(M,z)$

Cumulative mass function
from X-ray cluster samples



Cumulative mass function from X-ray cluster samples



The Challenge

$$N = \Omega_{\rm obs} \int_0^\infty dz \, \frac{d^2V}{dz d\Omega} \int_{F_{\rm lim}(z)}^\infty dF \, \frac{dn}{dM} \boxed{\frac{dM}{dF}}$$

- **Cluster masses are not directly observable**

Miss estimation of the masses from the observables severely compromises the statistical power of galaxy clusters as a DE probe

- The observables "F" include
  - Number of cluster member galaxies [optical]
  - Velocity dispersion [optical]
  - Strong- and weak-lensing masses [optical]
  - X-ray intensity [X-ray]
  - X-ray spectroscopic temperature [X-ray]
  - SZ intensity [microwave]

HSE: the leading method

- Currently, most of the mass cluster estimations rely on the X-ray data and the assumption of hydrostatic equilibrium [HSE]

  - The measured X-ray intensity is proportional to $\int n_e^2\, dl$, which can be converted into a radial profile of electron density, $\mathbf{n_e(r)}$, assuming spherical symmetry

  - The spectroscopic data give a radial electron temperature profile, $\mathbf{T_e(r)}$

These measurements give an estimate of the **electron pressure profile**, $P_e(r)=n_e(r)k_BT_e(r)$

HSE: the leading method

- Recently, more SZ measurements, which are proportional to $\int n_e k_B T_e \, dl$, are used to directly obtain an estimate of the **electron pressure profile**

HSE: the leading method

- In the usual HSE assumption, the total gas pressure [including contributions from ions and electrons] gradient balances against gravity

[X=0.75 is the hydrogen mass abundance]

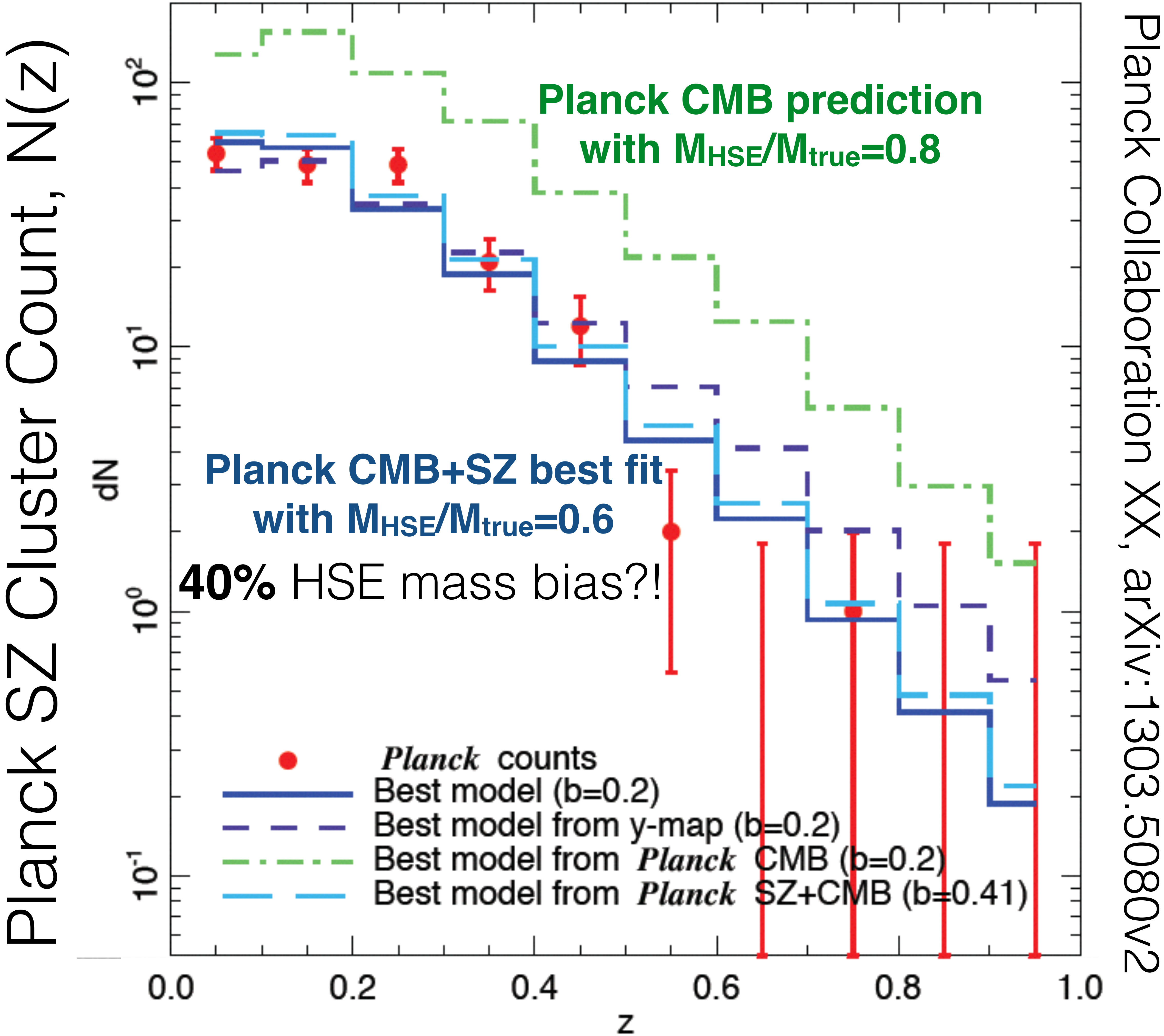
- $n_{gas} = n_{ion} + n_e = [(3+5X)/(2+2X)]n_e = 1.93n_e$
- Assuming $T_{ion} = T_e$ [which is not always satisfied!]
  - $P_{gas}(r) = 1.93P_e(r)$
- Then, HSE $$\frac{1}{\rho_{\rm gas}(r)}\frac{\partial P_{\rm gas}(r)}{\partial r} = -\frac{GM(<r)}{r^2}$$
  - gives an estimate of the total mass of a cluster, M

Limitation of HSE

- The HSE equation $$\frac{1}{\rho_{\rm gas}(r)}\frac{\partial P_{\rm gas}(r)}{\partial r} = -\frac{GM(<r)}{r^2}$$
  - only includes **thermal** pressure; however, not all kinetic energy of in-falling gas is thermalized
  - There is evidence that there is significant **non-thermal pressure support** coming from bulk motion of gas (e.g., turbulence)
- Therefore, the correct equation to use would be

$$\frac{1}{\rho_{\rm gas}(r)}\frac{\partial[P_{\rm th}(r) + P_{\rm non-th}(r)]}{\partial r} = -\frac{GM(<r)}{r^2}$$

Not including $P_{\rm non\text{-}th}$ leads to underestimation of the cluster mass!


Planck SZ Cluster Count, N(z)
Planck CMB prediction
with M_{HSE}/M_{true}=0.8
Planck CMB+SZ best fit
with M_{HSE}/M_{true}=0.6
40% HSE mass bias?!
Planck counts
Best model (b=0.2)
Best model from y-map (b=0.2)
Best model from Planck CMB (b=0.2)
Best model from Planck SZ+CMB (b=0.41)
dN
10^{2}
10^{1}
10^{0}
10^{-1}
0.0
0.2
0.4
0.6
0.8
1.0
z
Planck Collaboration XX, arXiv:1303.5080v2

Analytical Model for Non-Thermal Pressure

- **Basic idea 1**: non-thermal motion of gas in clusters is sourced by the mass growth of clusters [via mergers and mass accretion] with efficiency η

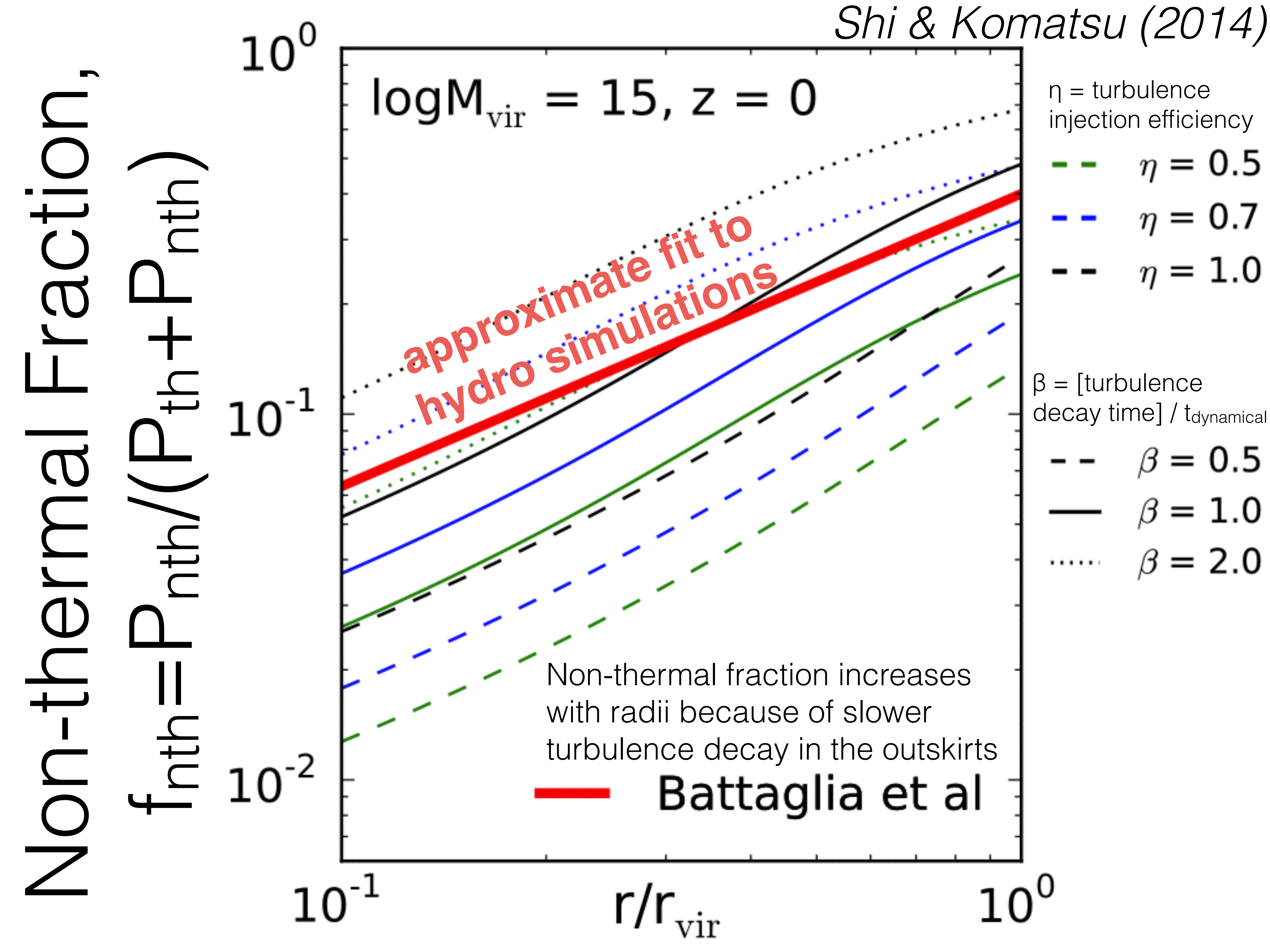
- **Basic idea 2**: induced non-thermal motion decays and thermalizes in a dynamical time scale

- Putting these ideas into a differential equation:

$$\frac{d\sigma^2_{nth}}{dt} = -\frac{\sigma^2_{nth}}{t_d} + \eta \frac{d\sigma^2_{tot}}{dt}$$

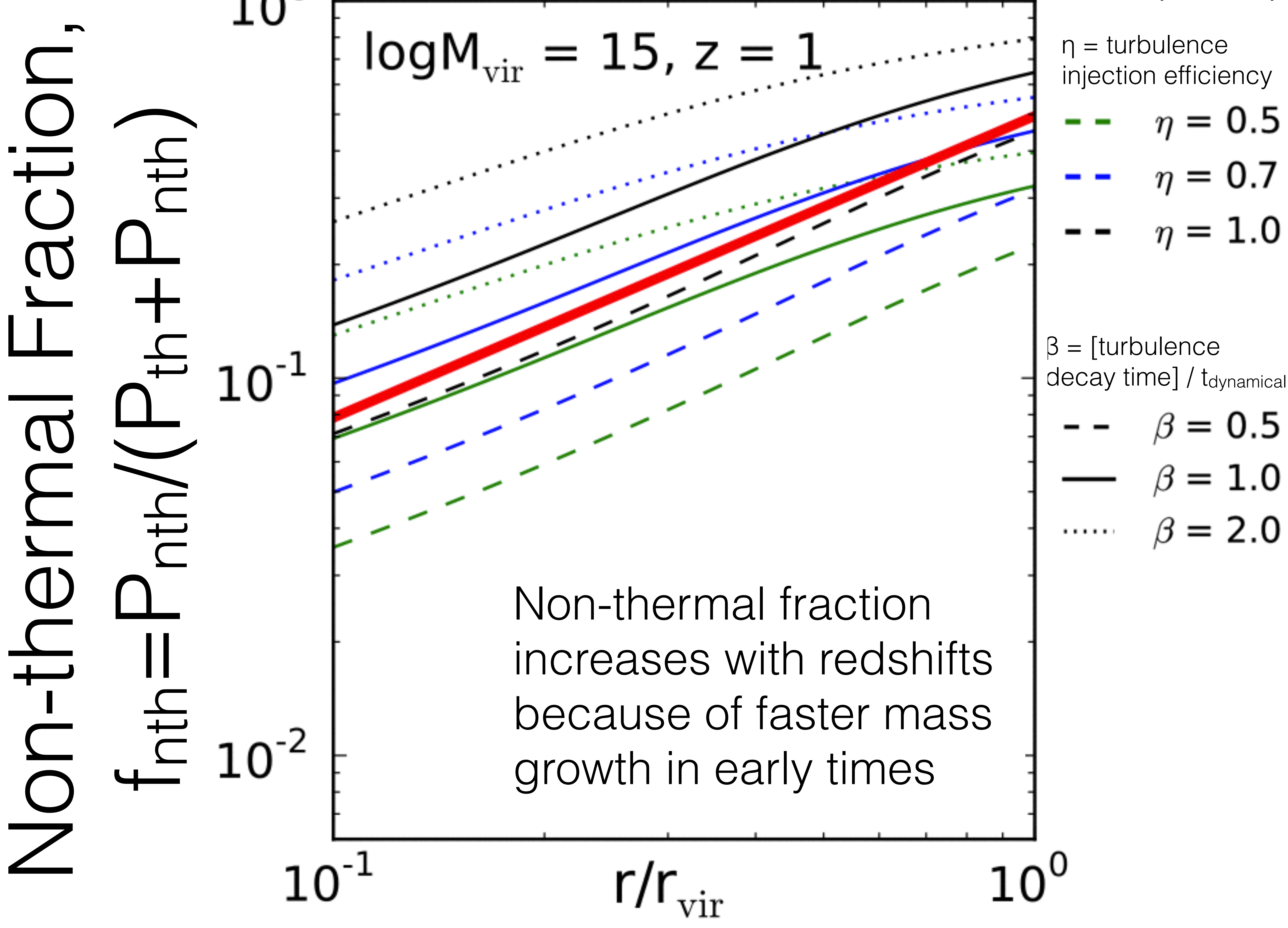
Shi & Komatsu (2014)

$[\sigma^2 = P/\rho_{gas}]$



Shi & Komatsu (2014)
Non-thermal Fraction,
$f_{nth}=P_{nth}/(P_{th}+P_{nth})$
$\log M_{vir} = 15, z = 0$
approximate fit to hydro simulations
η = turbulence injection efficiency
$\eta = 0.5$
$\eta = 0.7$
$\eta = 1.0$
β = [turbulence decay time] / $t_{dynamical}$
$\beta = 0.5$
$\beta = 1.0$
$\beta = 2.0$
Non-thermal fraction increases with radii because of slower turbulence decay in the outskirts
Battaglia et al
10^0
10^{-1}
10^{-2}
10^{-1}
10^0
r/r_{vir}

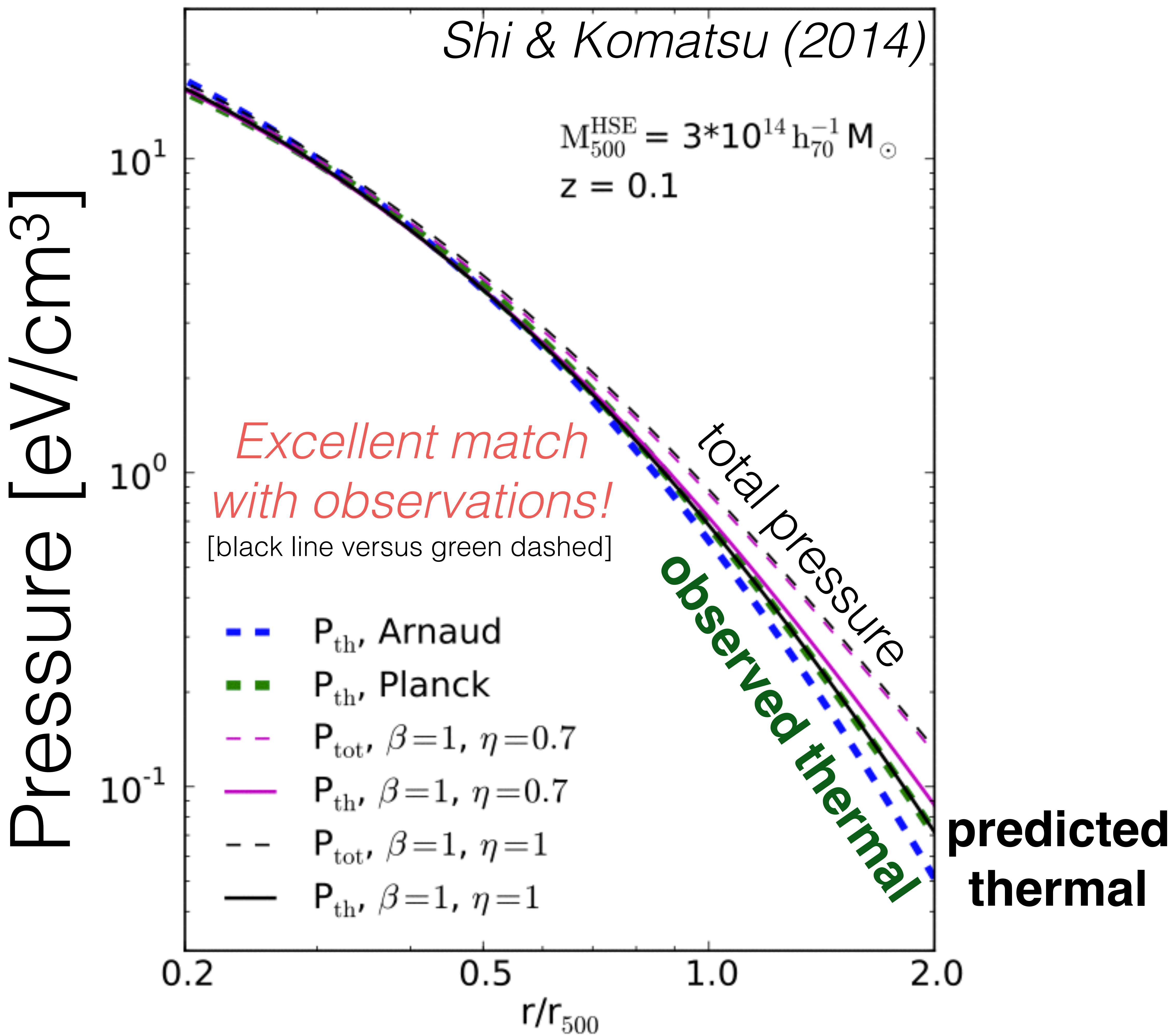
Shi & Komatsu (2014)



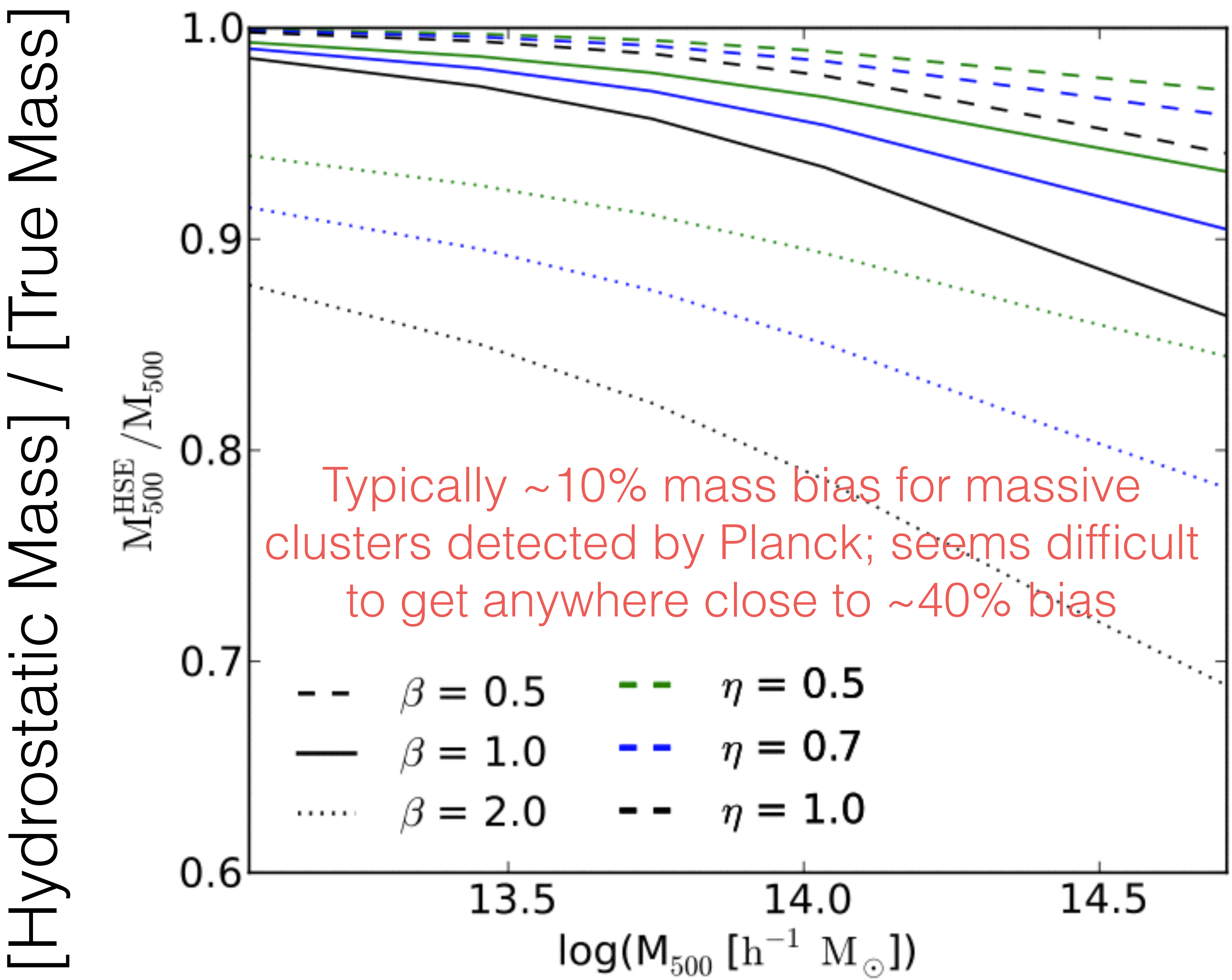
With $P_{non\text{-}thermal}$ computed

- We can now predict the X-ray and SZ observables, by subtracting $P_{non\text{-}thermal}$ from P_{total}, which is fixed by the total mass

- We can then predict what the bias in the mass estimation if hydrostatic equilibrium with thermal pressure is used



Shi & Komatsu (2014)
$M_{500}^{HSE} = 3*10^{14} h_{70}^{-1} M_\odot$
z = 0.1
Excellent match with observations!
[black line versus green dashed]
P_{th}, Arnaud
P_{th}, Planck
P_{tot}, $\beta=1$, $\eta=0.7$
P_{th}, $\beta=1$, $\eta=0.7$
P_{tot}, $\beta=1$, $\eta=1$
P_{th}, $\beta=1$, $\eta=1$
total pressure
observed thermal
predicted thermal
Pressure [eV/cm^3]
r/r_{500}
10^1
10^0
10^{-1}
0.2
0.5
1.0
2.0



[Hydrostatic Mass] / [True Mass]
M_{500}^{HSE}/M_{500}
1.0
0.9
0.8
0.7
0.6
13.5
14.0
14.5
log(M_{500} [h^{-1} $M_\odot$])
Typically ~10% mass bias for massive clusters detected by Planck; seems difficult to get anywhere close to ~40% bias
β = 0.5
β = 1.0
β = 2.0
η = 0.5
η = 0.7
η = 1.0

Remarks on Modifications to GR

Testing Modified GR #1

- Modifications to GR generically predict that two potentials in the metric are different: **$\Phi \neq -\Psi$**
- This, in principle, modifies gravitational lensing, which is proportional to **$\Psi-\Phi$**. This is equal to 2Ψ in GR, but not in modified GR
    - However, in scalar-tensor theories [i.e., modified gravity theories in which modifications are equivalent to introducing a new scalar degree of freedom], null geodesics is not modified
    - This happens because, schematically, two potentials are modified such that Φ -> $\Phi+\beta$, Ψ -> $\Psi+\beta$ [where β is some function], hence $\Psi-\Phi$ is unmodified
    - No effect on gravitational lensing in scalar-tensor theories

Testing Modified GR #1

- On the other hand, **only Ψ** enters in Euler's equation and determines velocities of motion of non-relativistic objects [such as galaxies]
    - Ψ is modified from GR even in scalar-tensor theories; thus, velocities of galaxies are also modified

Testing Modified GR #1

- **Implication**:
  - the “dynamical mass” of galaxy clusters estimated from velocity dispersion of the member galaxies, and
  - the “lensing mass” estimated from gravitational lensing
- are **different** in modified GR.
  - E.g., the lensing mass is equal to the true mass in scalar-tensor theories of gravity, but the dynamical mass is different from the true mass

Testing Modified GR #2

- In GR, knowing the expansion history of the universe yields the growth history of linear perturbations as well

$$\frac{d^2g}{d\ln(1+z)^2} - \left[\frac{5}{2} + \frac{1}{2}(\Omega_k(z) - 3w(z)\Omega_{\mathrm{de}}(z))\right]\frac{dg}{d\ln(1+z)} + \left[2\Omega_k(z) + \frac{3}{2}(1-w(z))\Omega_{\mathrm{de}}(z)\right]g(z) = 0$$

**Strictly speaking, this formula is valid when the contribution of DE fluctuations to the gravitational potential is negligible compared to matter*

- In modified GR, there is no such correspondence; thus, the data on both the expansion history [i.e., H(z)] and the data on the growth history [i.e., g] test modifications to GR

Summary

- CMB, galaxy surveys, and galaxy clusters can be used to measure two crucial quantities: the expansion rate, H(z), and the growth history, g(z), which in turn test the most important hypothesis: ***does the dark energy density vary with time?***
  - We did not cover Type Ia supernovae or weak/strong gravitational lensing in this lecture, but they also provide information on H(z) and g(z)
- CMB has limited sensitivity to *w* but provides an important anchor [the sound horizon and d_A to z=1090]
- Non-linearity in redshift space distortion must be understood before using galaxy surveys to learn about g(z)
- Understanding the hydrostatic mass bias is the most important challenge to using galaxy clusters as a cosmological probe