Testing Physics of the Early Universe **Observationally**: *Are Primordial Fluctuations Gaussian, or Non-Gaussian?*

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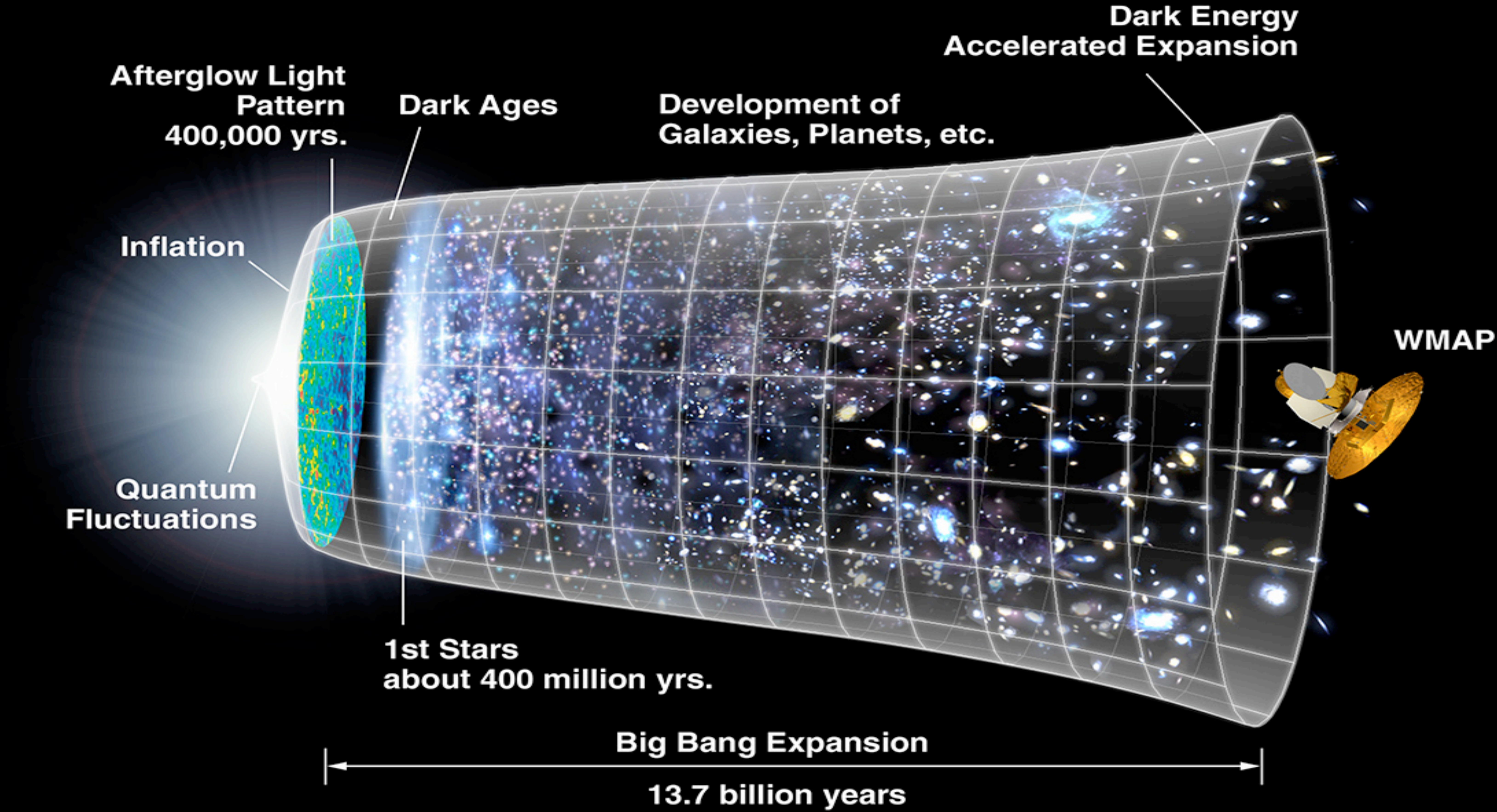
How?

- Einstein equations are differential equations. So...
- **Cosmology as a boundary condition problem**
  - We measure the physical condition of the universe today (or some other time for which we can make measurements, e.g., z=1090), and carry it backwards in time to a primordial universe.
- **Cosmology as an initial condition problem**
  - We use theoretical models of the primordial universe to make predictions for the observed properties of the universe.

- **Not surprisingly, we use both approaches.**

Messages From the Primordial Universe...



NASA/WMAP Science Team

WMAP5+BAO+SN

Observations I: Homogeneous Universe

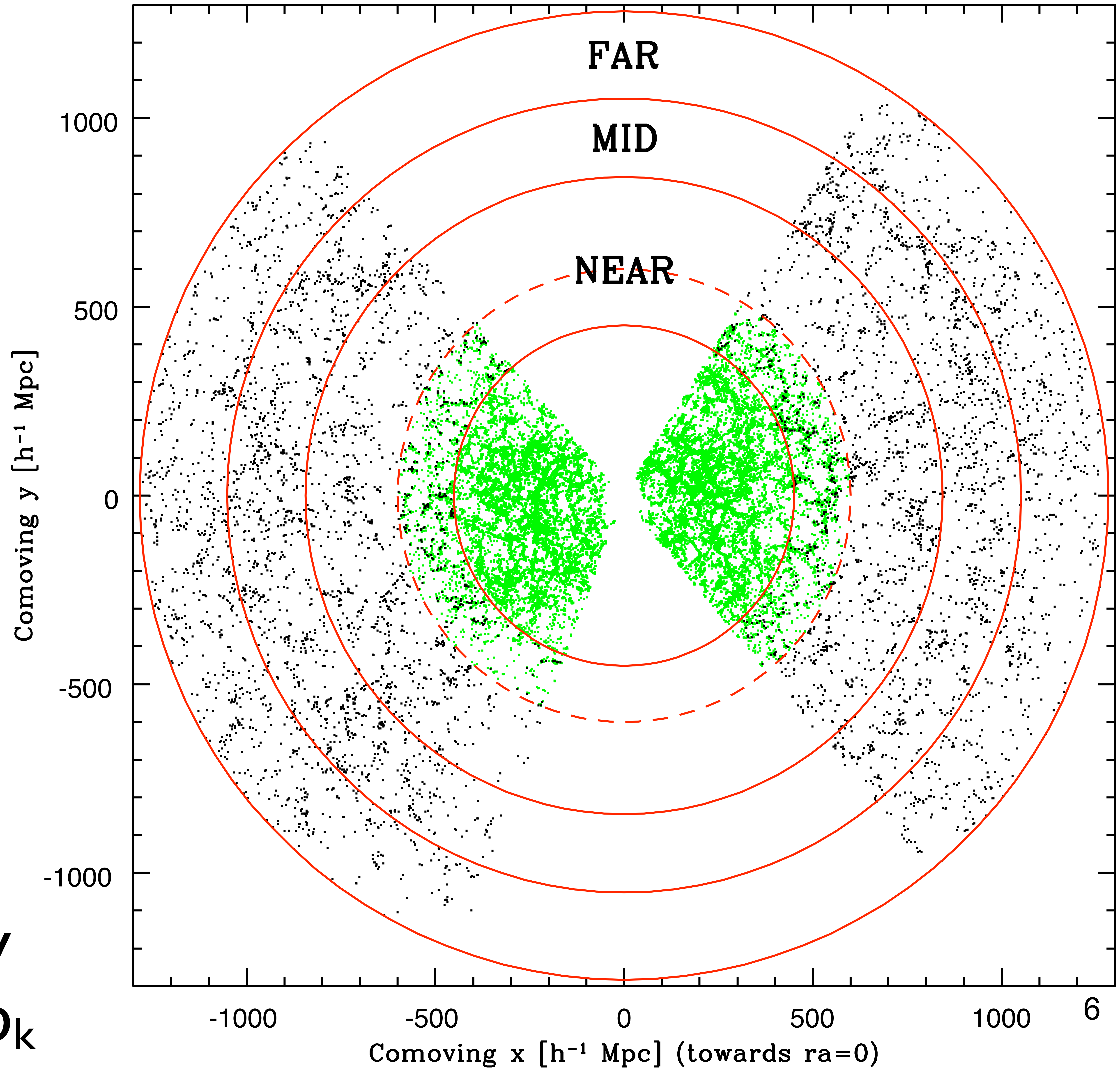
- $H^2(z) = H^2(0)[\Omega_r(1+z)^4+\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_{de}(1+z)^{3(1+w)}]$
  - (expansion rate) $H^2(0) = 70.5 \pm 1.3$ km/s/Mpc
  - (radiation) $\Omega_r = (8.4\pm0.3)\times10^{-5}$
  - (matter) $\Omega_m = 0.274\pm0.015$
  - **(curvature) $\Omega_k < 0.008$ (95%CL) -> Inflation**
  - (dark energy) $\Omega_{de} = 0.726\pm0.015$
  - (DE equation of state) $1+w = -0.006\pm0.068$

Observations II: Density Fluctuations, $\delta(x)$

- In Fourier space, $\delta(k) = A(k)\exp(i\varphi_k)$
  - **Power**: $P(k) = \langle|\delta(k)|^2\rangle = A^2(k)$
  - **Phase**: φ_k
- We can use the observed distribution of...
  - matter (e.g., galaxies, gas)
  - radiation (e.g., Cosmic Microwave Background)
- to learn about both $P(k)$ and φ_k.

Galaxy Distribution

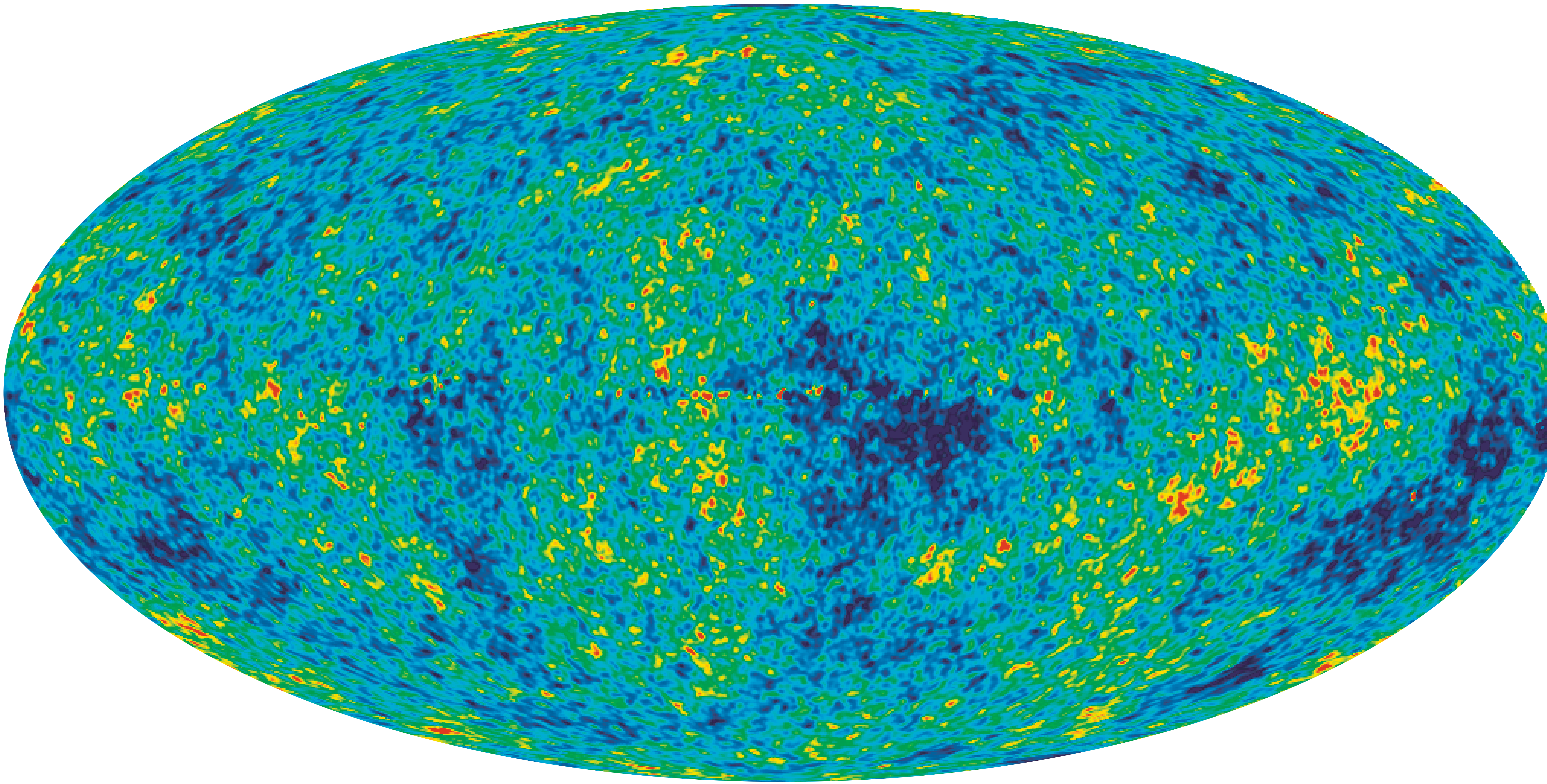
SDSS



- Matter distribution today (z=0~0.2): P(k), φ_k

Radiation Distribution

WMAP5

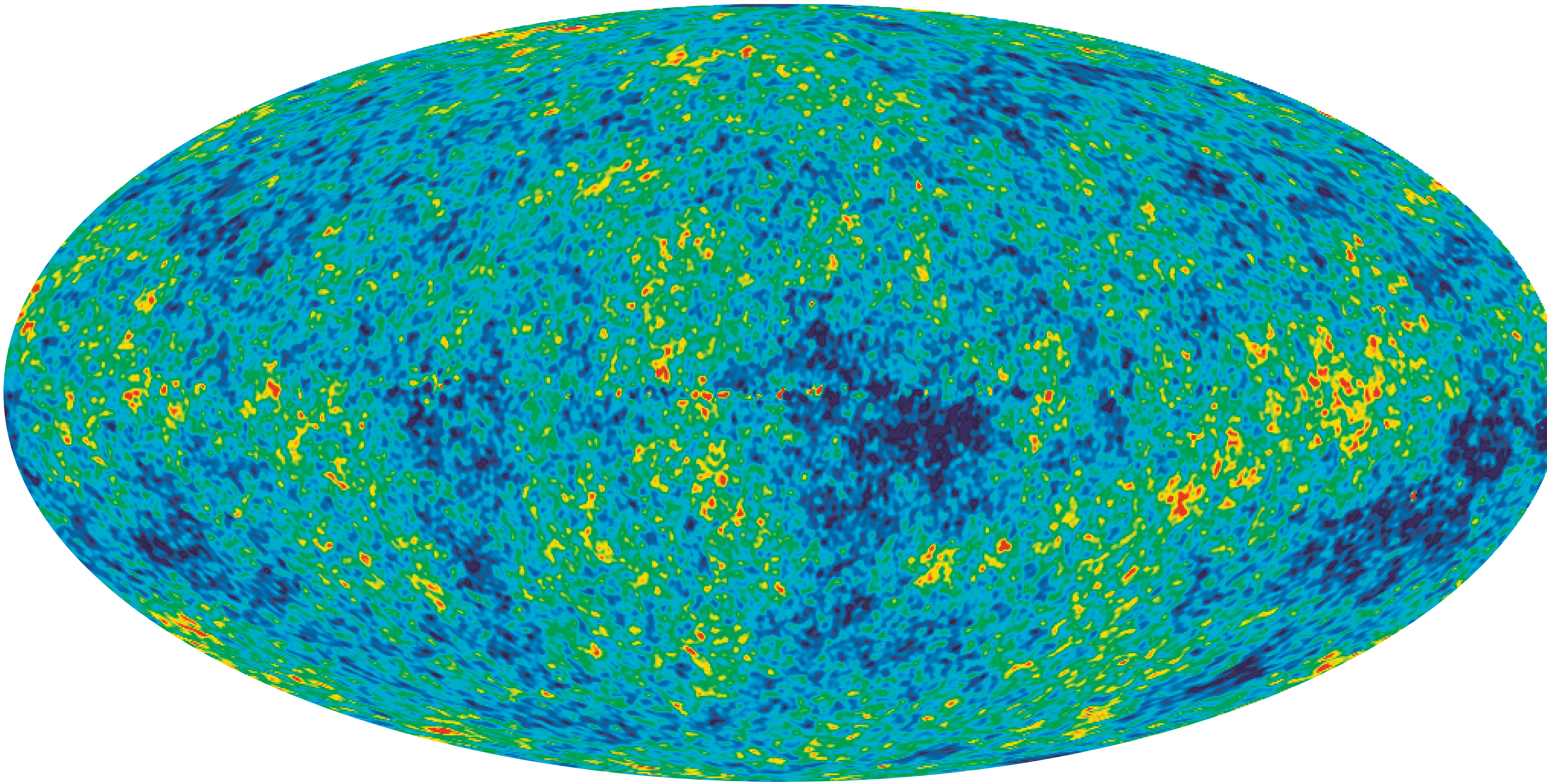


- Matter distribution at z=1090: P(k), φ_k

P(k): There were expectations

- Metric perturbations in g_{ij} (let's call that "curvature perturbations" Φ) is related to δ via
  - $k^2\Phi(k)=4\pi G\rho a^2\delta(k)$
- Variance of $\Phi(x)$ in position space is given by
  - $<\Phi^2(x)>=\int \ln k\ \mathbf{k^3|\Phi(k)|^2}$
  - In order to avoid the situation in which curvature (geometry) diverges on small or large scales, a "scale-invariant spectrum" was proposed: $\mathbf{k^3|\Phi(k)|^2 = const.}$
  - This leads to the expectation: $\mathbf{P(k)}=|\delta(k)|^2\mathbf{=k}$
    - *Harrison 1970; Zel'dovich 1972; Peebles&Yu 1970*

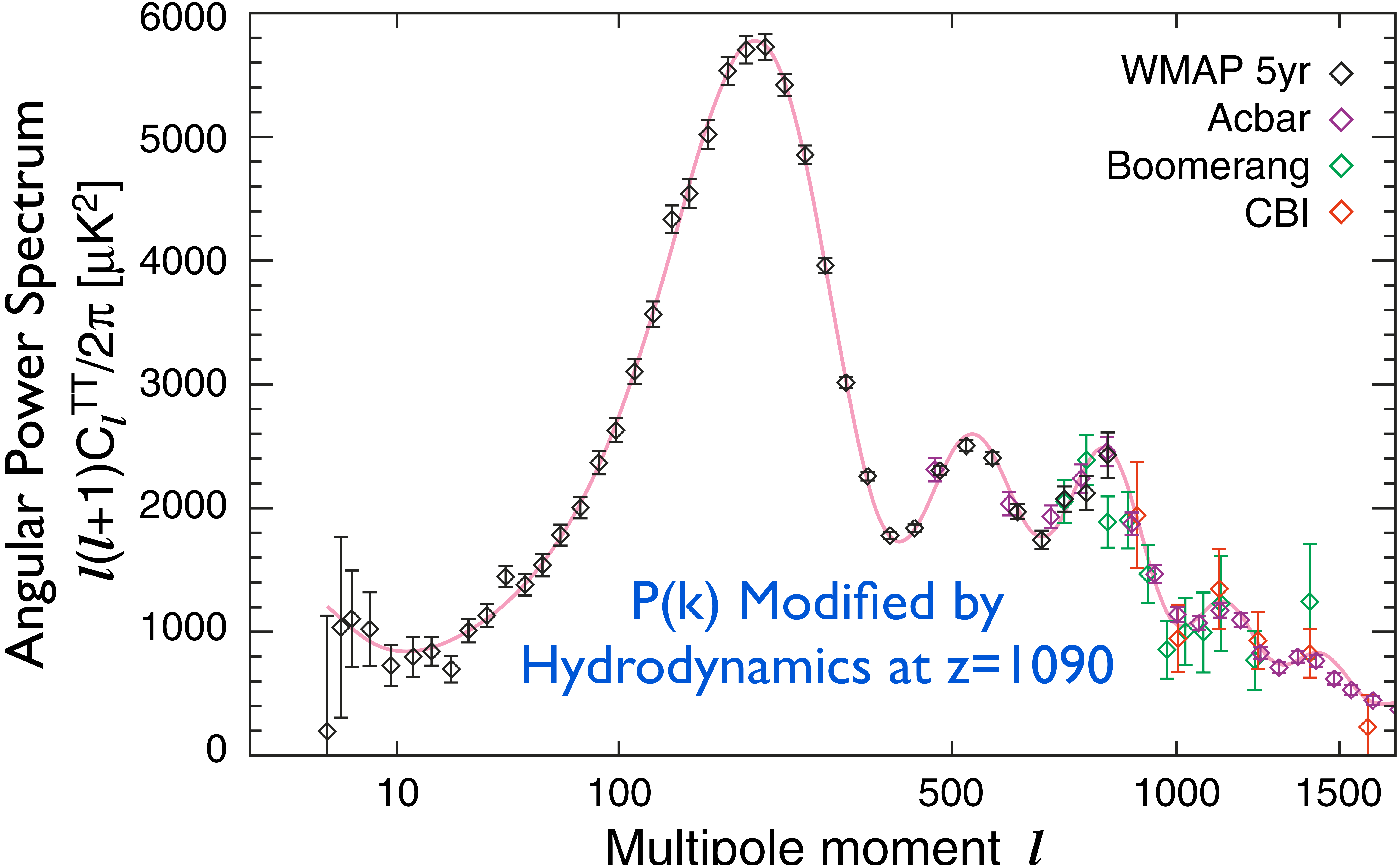
Take Fourier Transform of *WMAP5*



- ...and, square it in your head...

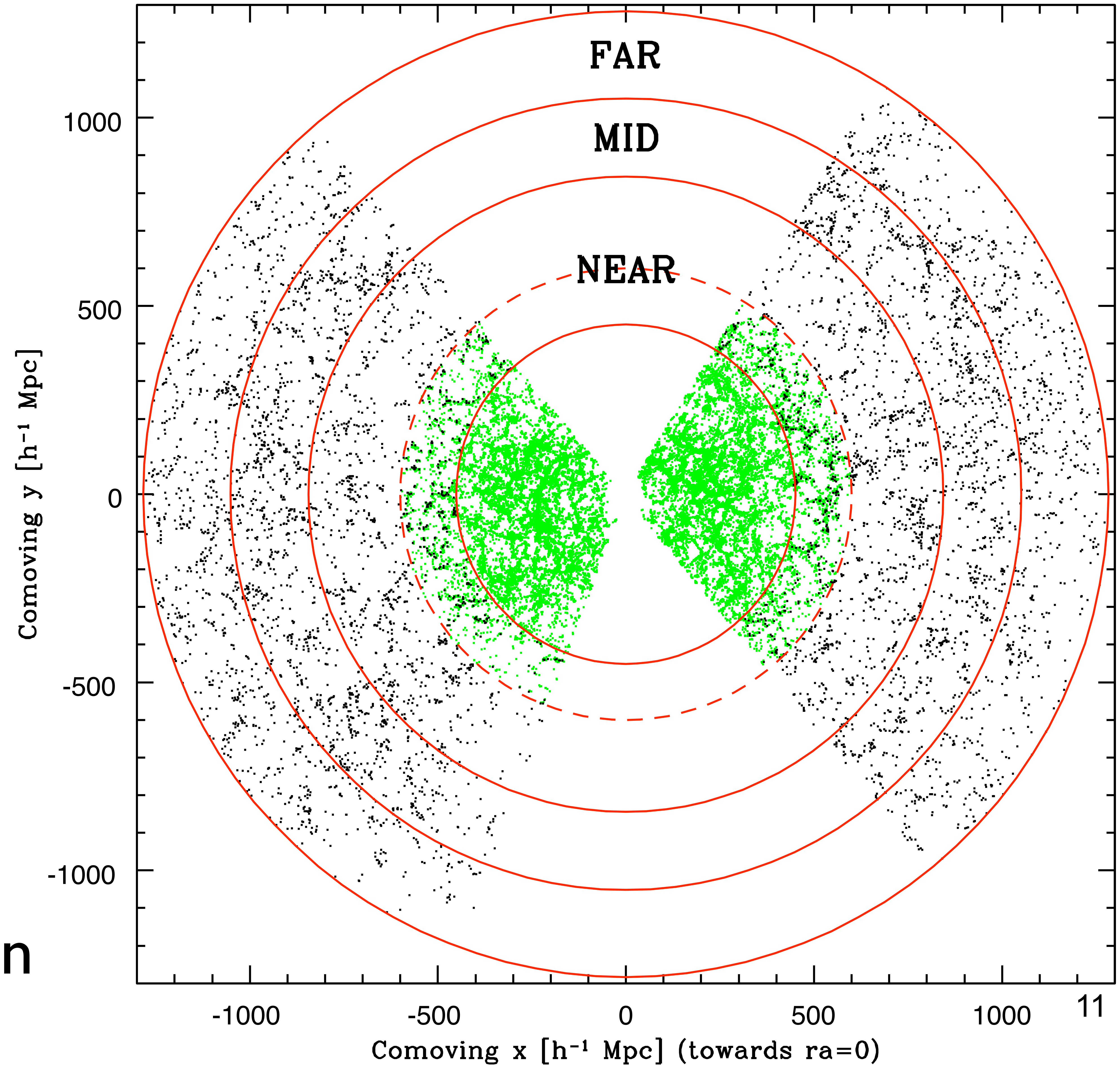
...and decode it.

Nolta et al. (2008)



Take Fourier Transform of

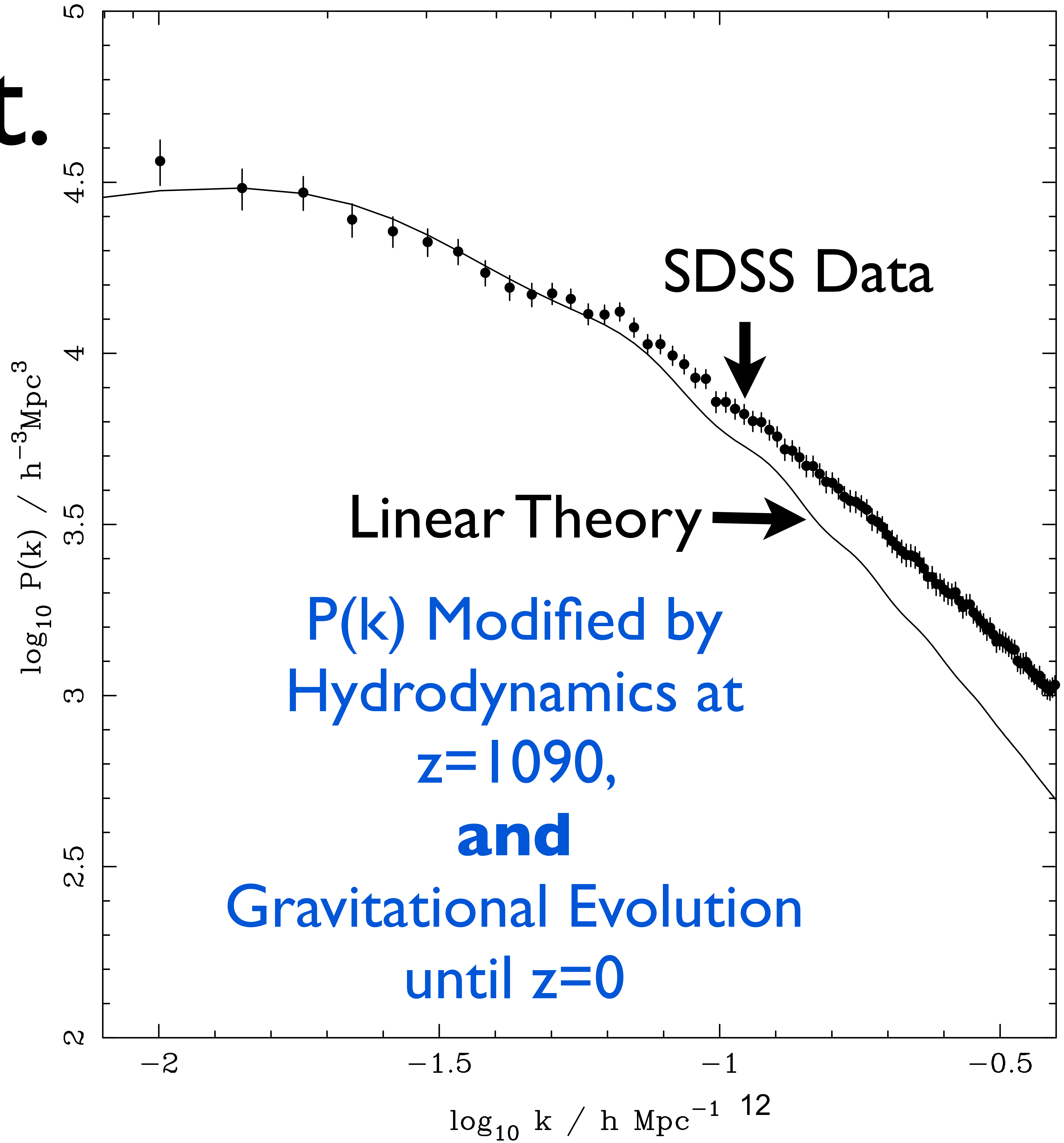
SDSS



- ...and square it in your head...

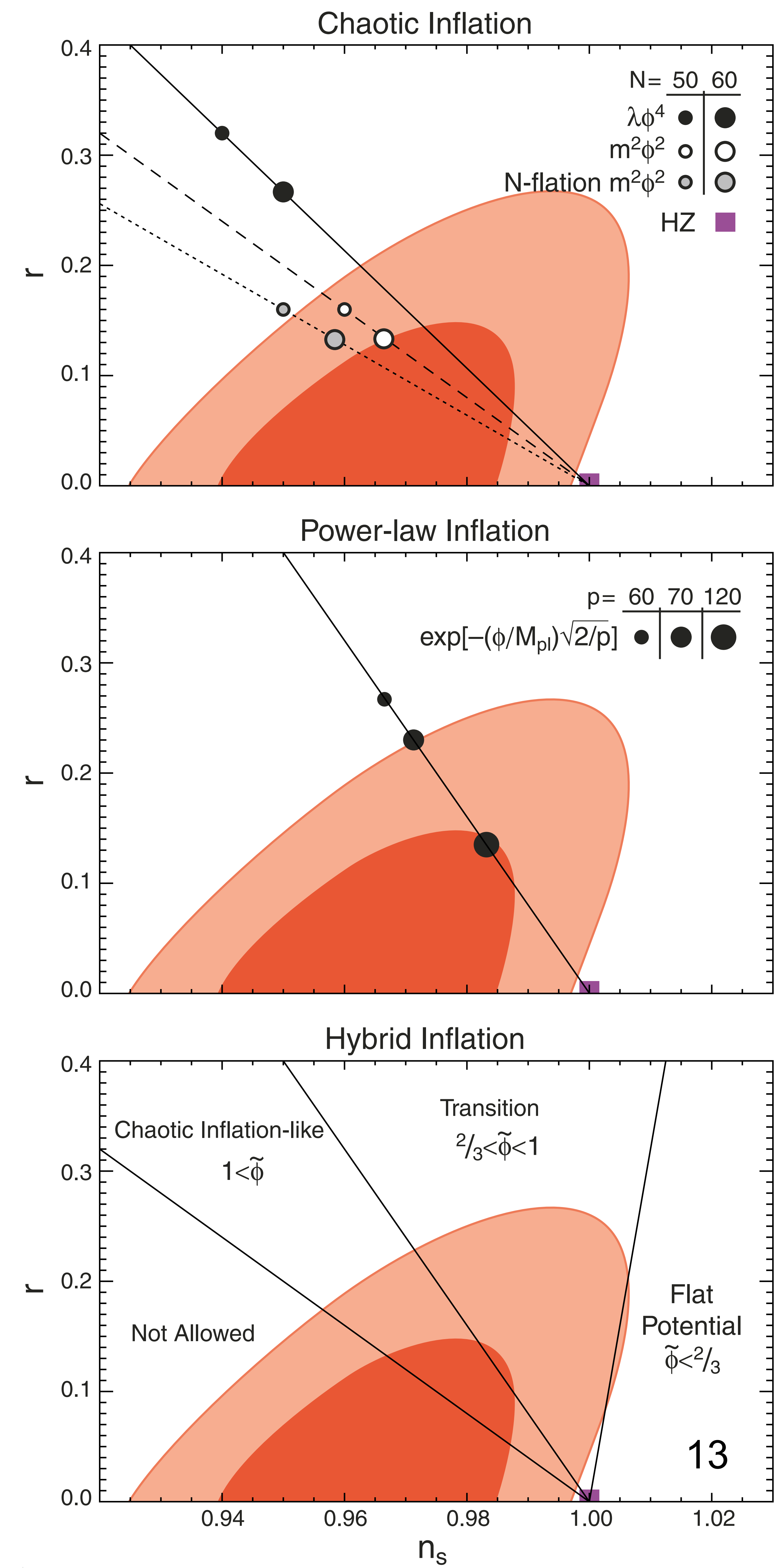
...and decode it.

- Decoding is complex, but you can do it.
- The latest result (from WMAP+: *Komatsu et al.*)
  - $P(k)=k^{ns}$
  - **$n_s=0.960\pm0.013$**
  - 3.1σ away from scale-invariance, $n_s=1$!



Deviation from $n_s=1$

- This was expected by many inflationary models
- In n_s–r plane (where r is called the "tensor-to-scalar ratio," which is P(k) of gravitational waves divided by P(k) of density fluctuations) **many inflationary models are compatible with the current data**
- Many models have been excluded also



Searching for Primordial Gravitational Waves in CMB

- Not only do inflation models produce density fluctuations, but also primordial gravitational waves
- Some predict the observable amount ($r>0.01$), some don't
  - Current limit: $r<0.22$ (95%CL) (*WMAP5+BAO+SN*)
- Alternative scenarios (e.g., New Ekpyrotic) don't
- A powerful probe for testing inflation and testing specific models: next "Holy Grail" for CMBist (Lyman, Suzanne)

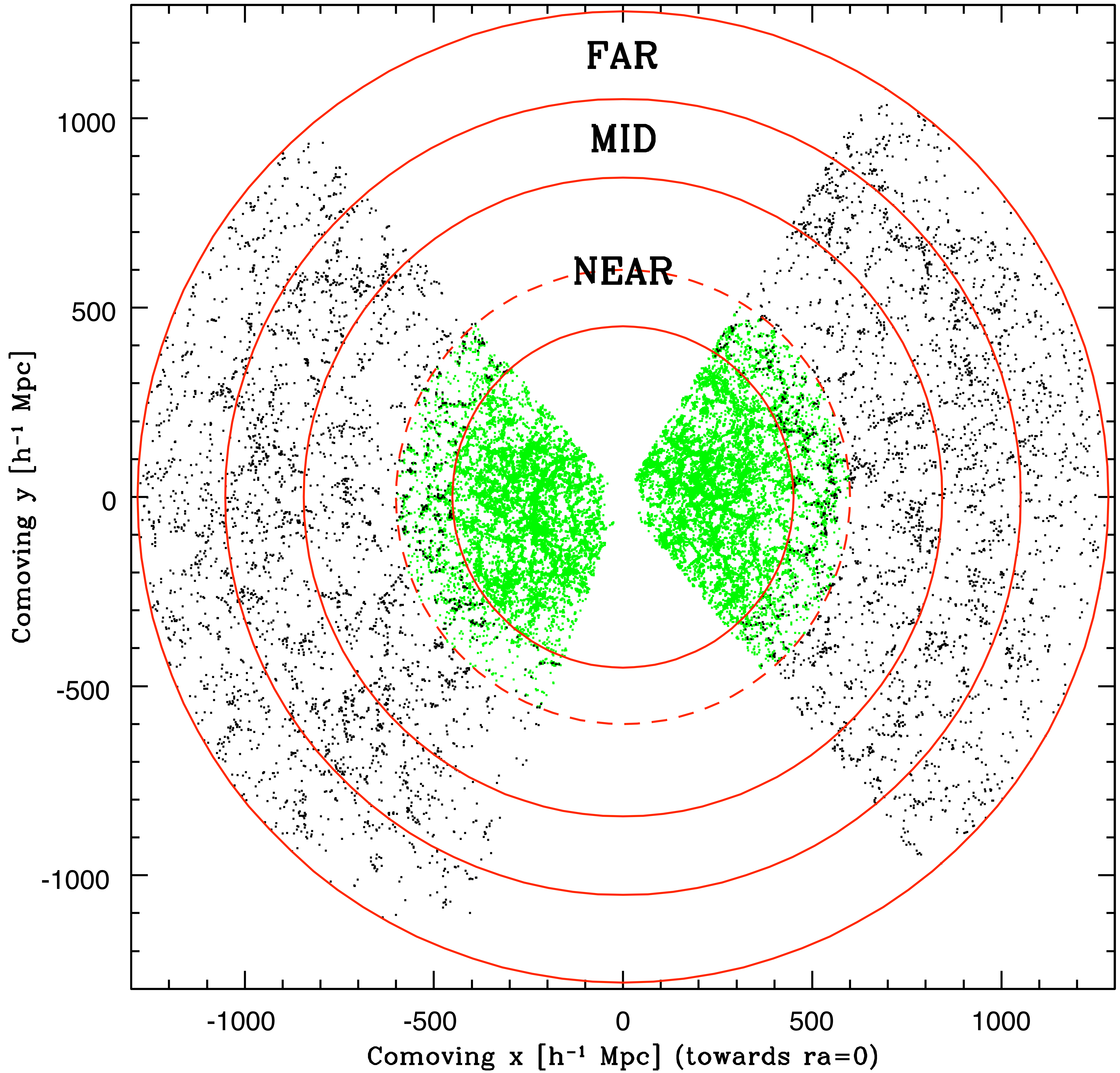
What About Phase, φ_k

- There were expectations also:
  - Random phases! (Peebles, ...)
- Collection of random, uncorrelated phases leads to the most famous probability distribution of δ:

Gaussian Distribution

Gaussian?

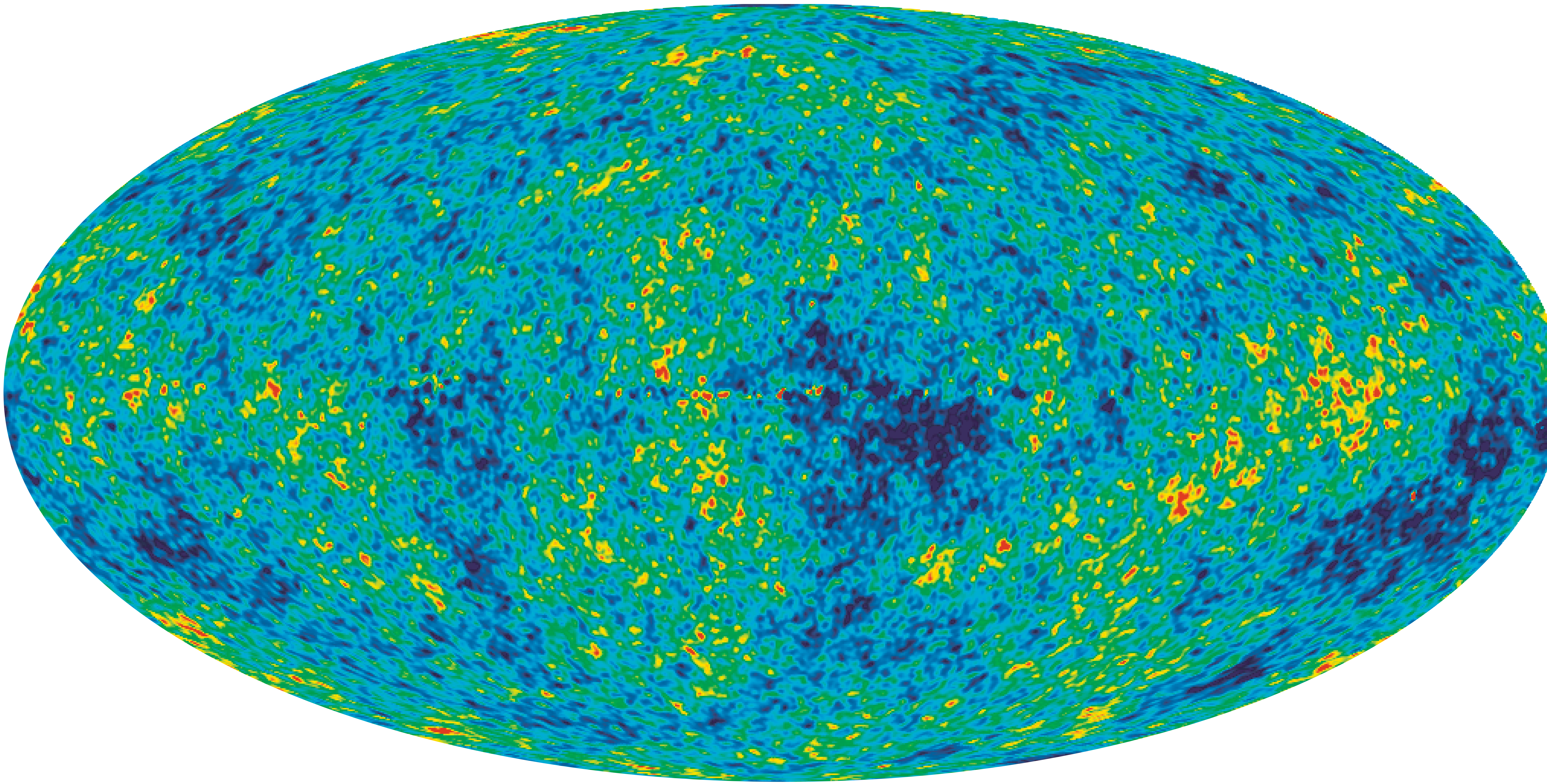
SDSS



- Phases are not random, due to non-linear gravitational evolution

Gaussian?

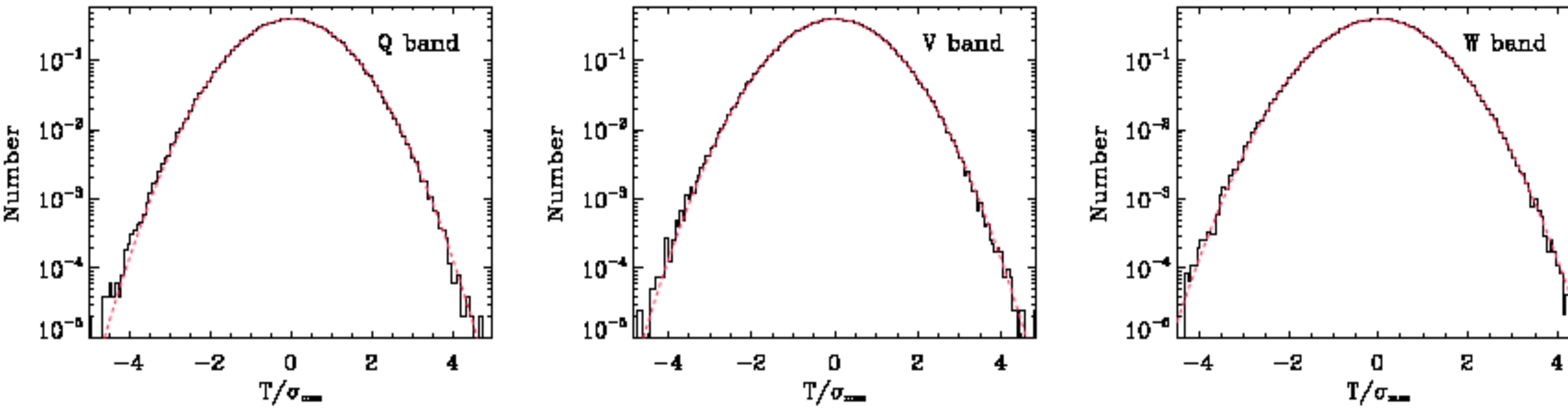
WMAP5



- Promising probe of Gaussianity – fluctuations still linear!

Spergel et al. (2008)

Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - –Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation Likes This Result

- According to inflation (Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Banch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

But, Not Exactly Gaussian

- Of course, there are always corrections to the simplest statement like this
- For one, inflaton field **does** have interactions. They are simply weak – of order the so-called slow-roll parameters, ε and η, which are O(0.01)

Non-Gaussianity from Inflation

- You need cubic interaction terms (or higher order) of fields.
  - $V(\phi) \sim \phi^3$: *Falk, Rangarajan & Srendnicki (1993)* [gravity not included yet]
  - Full expansion of the action, including gravity action, to cubic order was done a decade later by *Maldacena (2003)*

$$\phi = \phi(t) + \varphi(t,x)$$

$$\partial^2\chi = \frac{\dot\phi^2}{2\dot\rho^2}\frac{d}{dt}\left(-\frac{\dot\rho}{\dot\phi}\varphi\right)$$

$$h_{ij} = e^{2\rho}\hat h_{ij}$$

$$S_3 = \int e^{3\rho}\left(-\frac{\dot\phi}{4\dot\rho}\varphi\dot\varphi^2 - e^{-2\rho}\frac{\dot\phi}{4\dot\rho}\varphi(\partial\varphi)^2 - \dot\varphi\partial_i\chi\partial_i\varphi + \right.$$
$$+\frac{3\dot\phi^3}{8\dot\rho}\varphi^3 - \frac{\dot\phi^5}{16\dot\rho^3}\varphi^3 - \frac{\dot\phi V''}{4\dot\rho}\varphi^3 - \frac{V'''}{6}\varphi^3 + \frac{\dot\phi^3}{4\dot\rho^2}\varphi^2\dot\varphi + \frac{\dot\phi^2}{4\dot\rho}\varphi^2\partial^2\chi$$
$$\left. + \frac{\dot\phi}{4\dot\rho}(-\varphi\partial_i\partial_j\chi\partial_i\partial_j\chi + \varphi\partial^2\chi\partial^2\chi)\right)$$

Computing Primordial Bispectrum

- Three-point function, using in-in formalism (*Maldacena 2003; Weinberg 2005*)

$$\text{3-point function}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \left\langle \text{in} \left| \tilde{T} e^{i\int_{-\infty}^{t} H_I(t')dt'} \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\Phi(\mathbf{x}_3) T e^{-i\int_{-\infty}^{t} H_I(t')dt'} \right| \text{in} \right\rangle$$

- $H_I(t)$: Hamiltonian in interaction picture
  - Model-dependent: this determines which triangle shapes will dominate the signal
- $\Phi(x)$: operator representing curvature perturbations in interaction picture

Simplified Treatment

- Let's try to capture field interactions, or whatever non-linearities that might have been there during inflation, by the following simple, order-of-magnitude form (*Komatsu & Spergel 2001*):
  - $\mathbf{\Phi(x) = \Phi_{gaussian}(x) + f_{NL}[\Phi_{gaussian}(x)]^2}$
  - One finds f_{NL}=O(0.01) from inflation (*Maldacena 2003*; *Acquaviva et al. 2003*)
- **This is a powerful prediction of inflation**

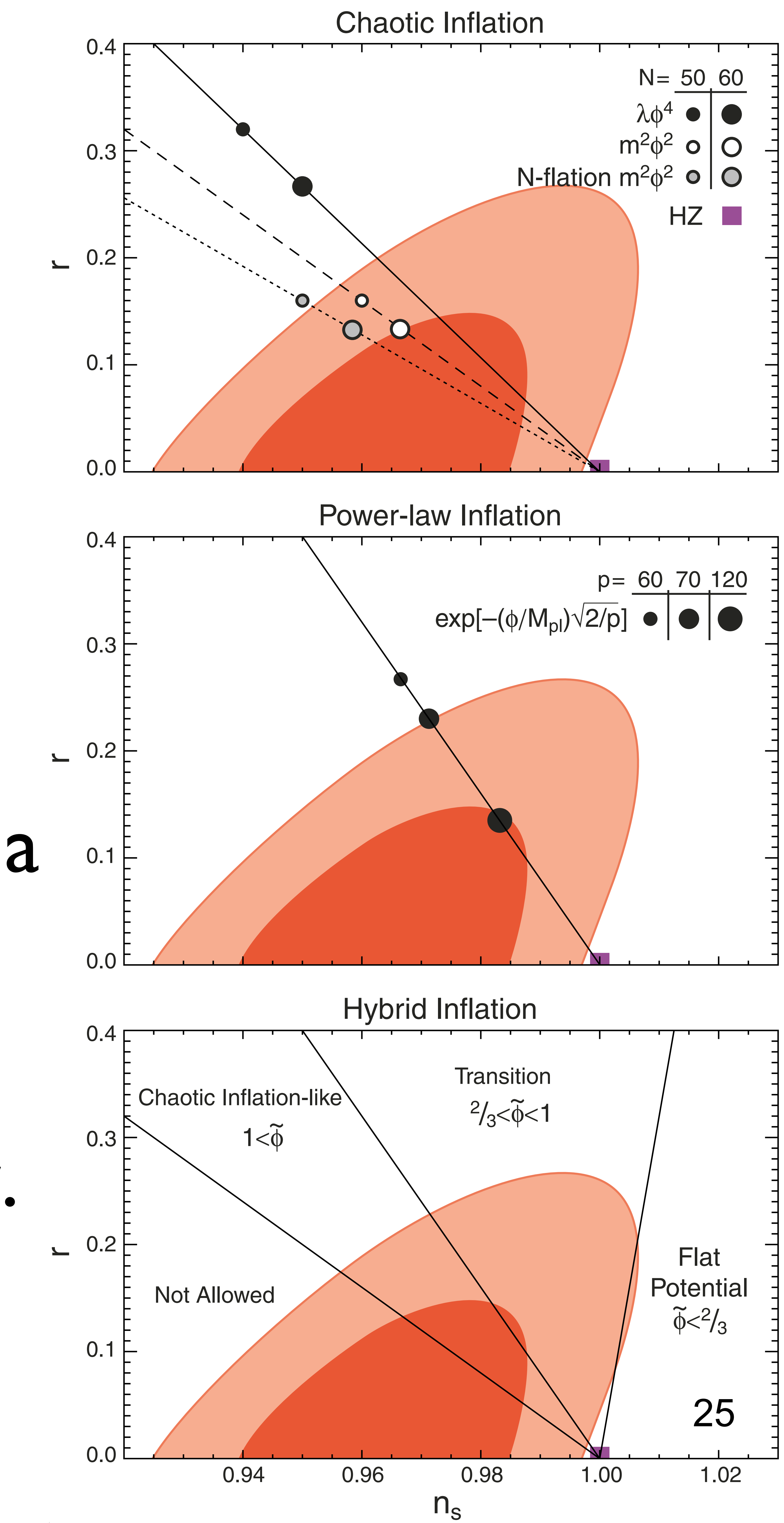
Earlier work on this form: Salopek&Bond (1990); Gangui et al. (1994); Verde et al. (2000); Wang&Kamionkowski (2000)

Why Study Non-Gaussianity?

- Because a detection of f_{NL} has a best chance of **ruling out the largest class of inflation models**.
- Namely, it will rule out inflation models based upon
  - a single scalar field with
  - the canonical kinetic term that
  - rolled down a smooth scalar potential slowly, and
  - was initially in the Banch-Davies vacuum.
- ***Detection of non-Gaussianity would be a major breakthrough in cosmology.***

We have *r* and *n*$_s$. Why Bother?

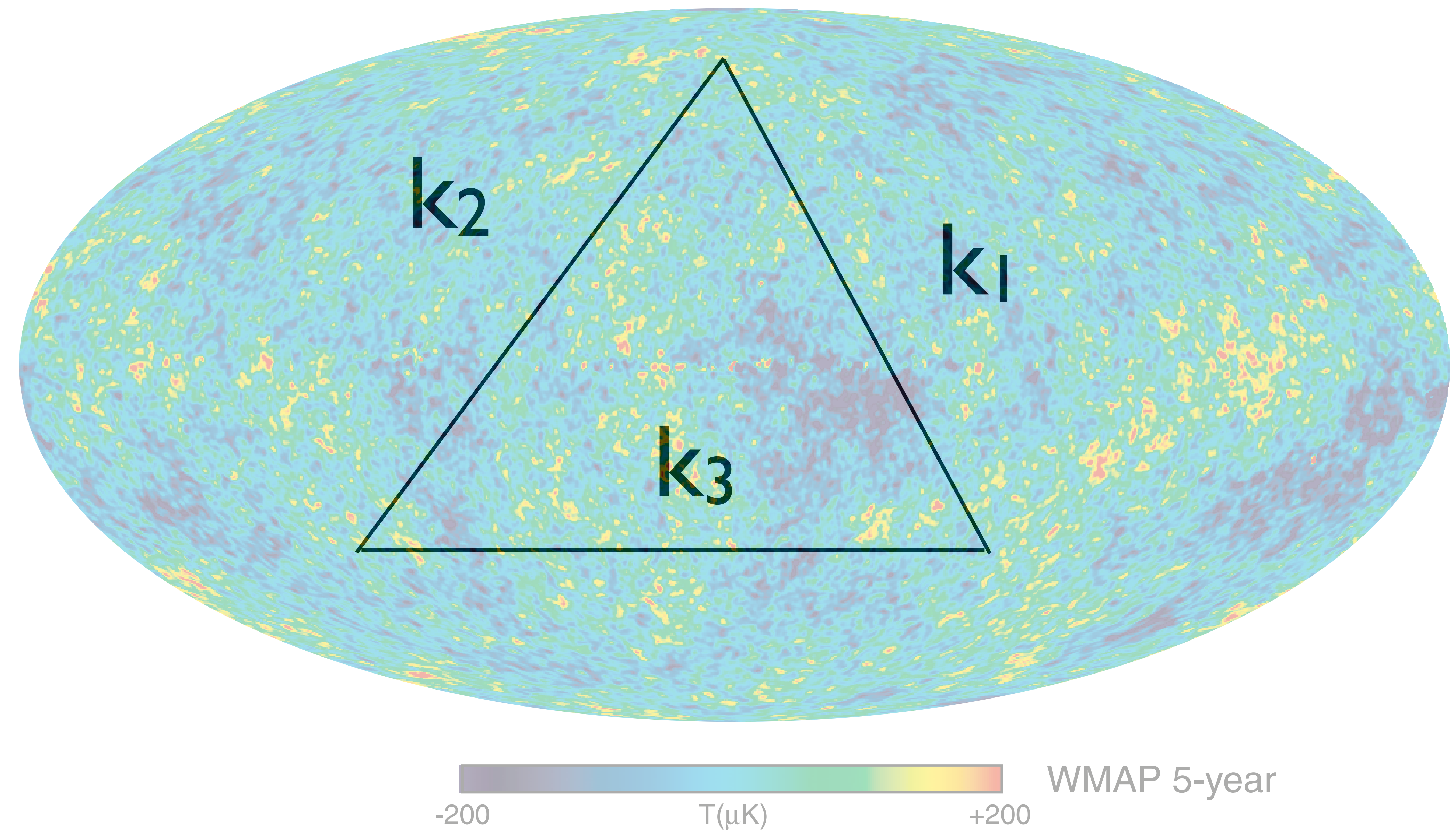
- While the current limit on the power-law index of the primordial power spectrum, **n$_s$**, and the amplitude of gravitational waves, **r**, have ruled out many inflation models already, many still survive (which is a good thing!)
- A convincing detection of f_{NL} would rule out most of them **regardless of n$_s$ or r**.
- f_{NL} offers more ways to test various early universe models!



Tool: Bispectrum

- **Bispectrum = Fourier Trans. of 3-pt Function**
- **The bispectrum vanishes** for Gaussian fluctuations with random phases.
- Any non-zero detection of the bispectrum indicates the presence of (some kind of) non-Gaussianity.
- A sensitive tool for finding non-Gaussianity.

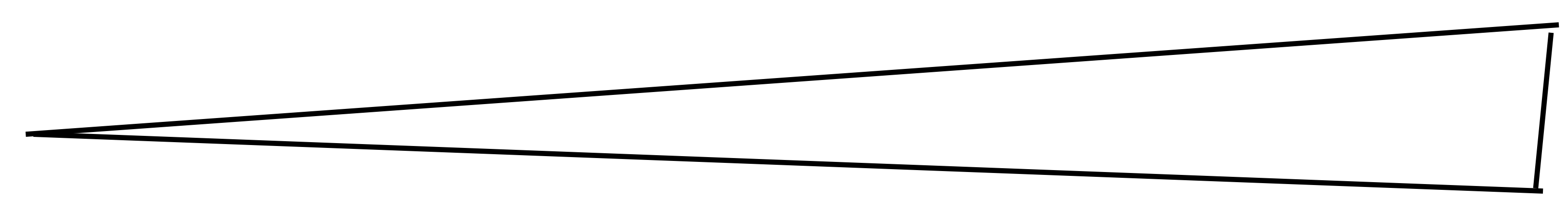
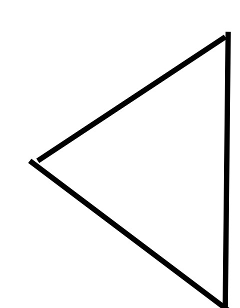
f_{NL} Generalized



- **f_{NL} = the amplitude of bispectrum**, which is
  - $=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\rangle = f_{NL}(2\pi)^3\delta^3(k_1+k_2+k_3)b(k_1,k_2,k_3)$
  - where $\Phi(k)$ is the Fourier transform of the curvature perturbation, and $b(k_1,k_2,k_3)$ is a model-dependent function that defines the shape of triangles predicted by various models.

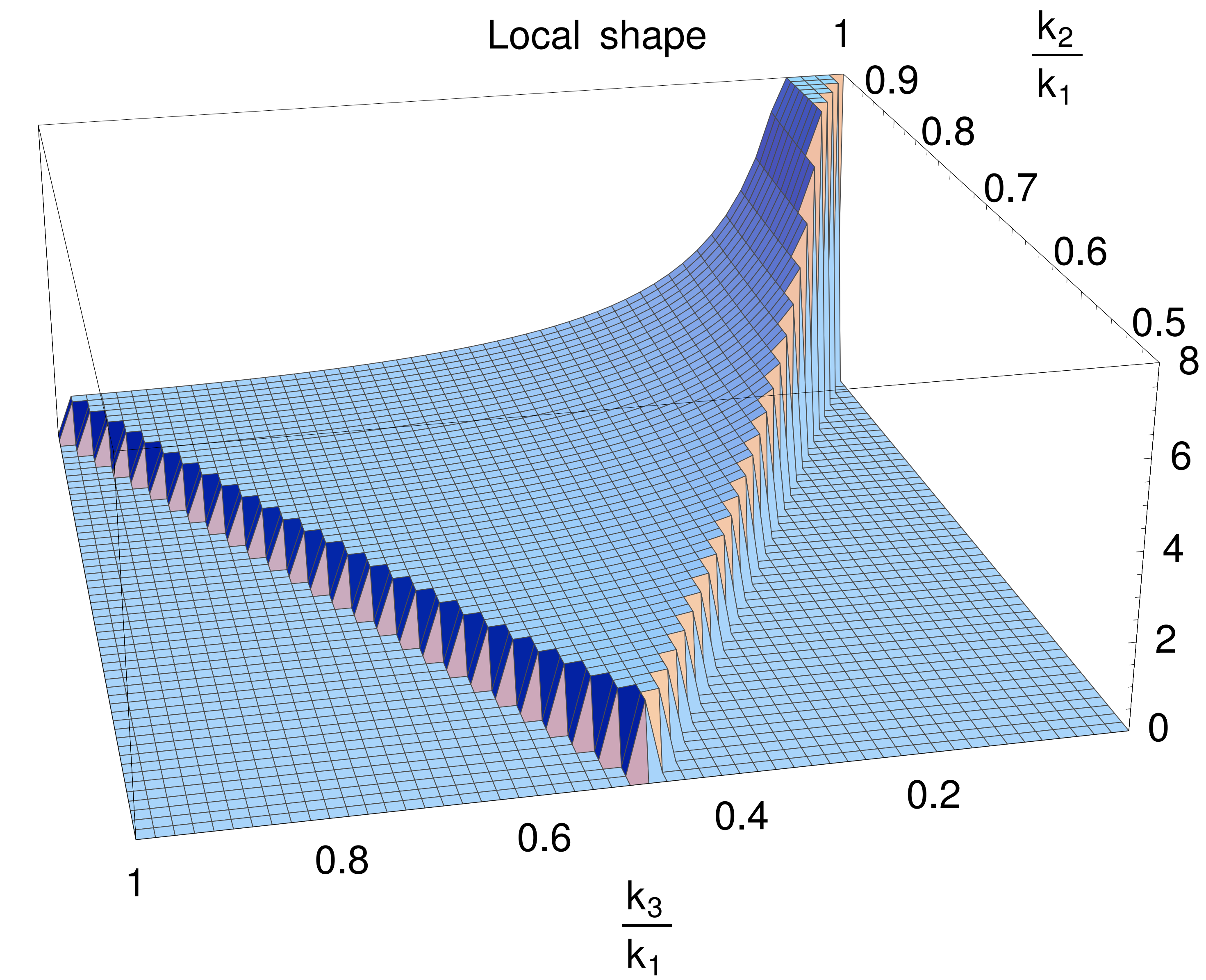
Two f_{NL}'s

There are more than two; I will come back to that later.

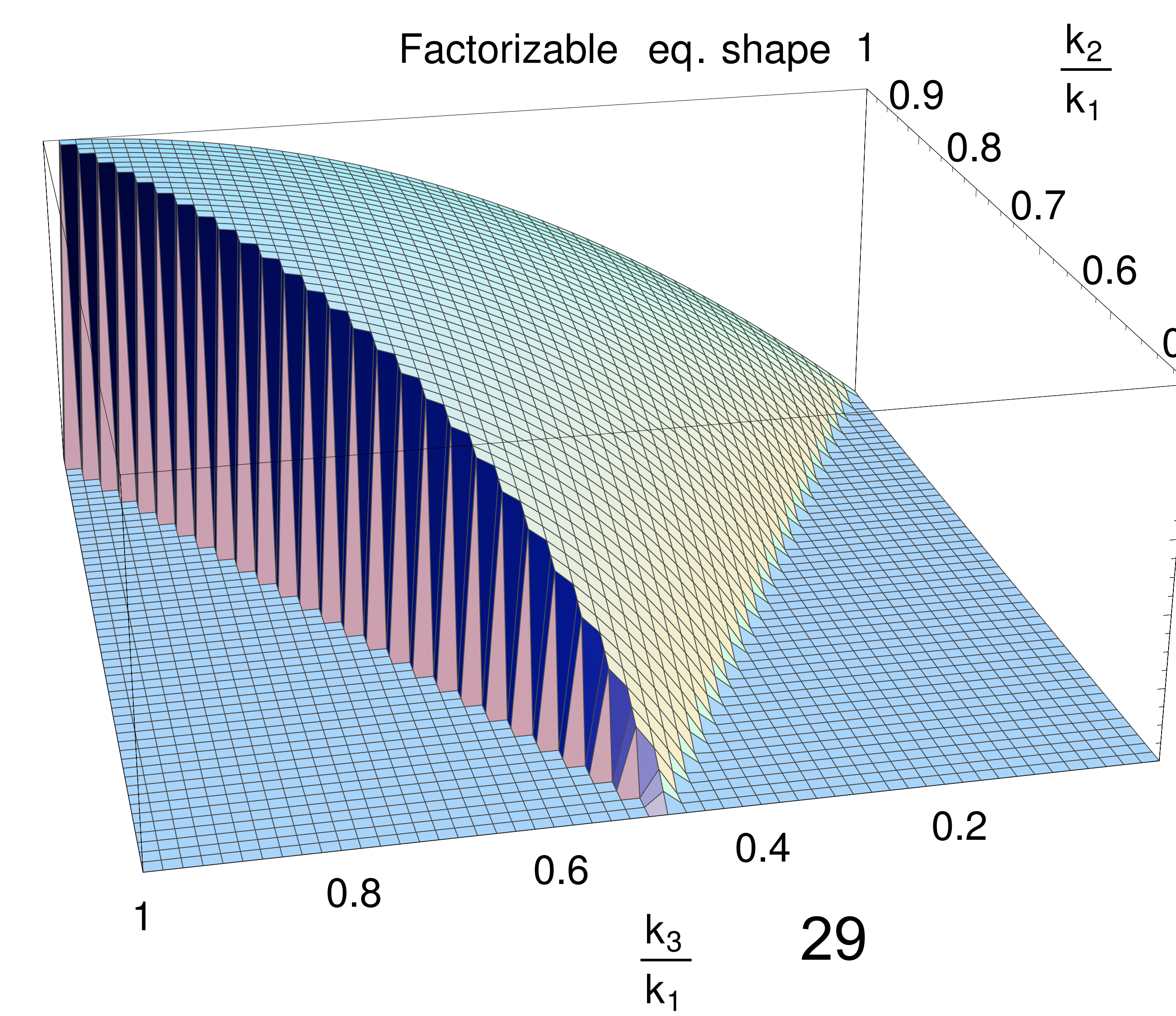
- Depending upon the shape of triangles, one can define various f_{NL}'s:
- "Local" form 
  - which generates non-Gaussianity locally in position space via $\Phi(x)=\Phi_{gaus}(x)+f_{NL}^{local}[\Phi_{gaus}(x)]^2$
- "Equilateral" form 
  - which generates non-Gaussianity locally in momentum space (e.g., k-inflation, DBI inflation)

Forms of $b(k_1,k_2,k_3)$

- Local form (*Komatsu & Spergel 2001*)
  - $b^{local}(k_1,k_2,k_3) = 2[P(k_1)P(k_2)+cyc.]$

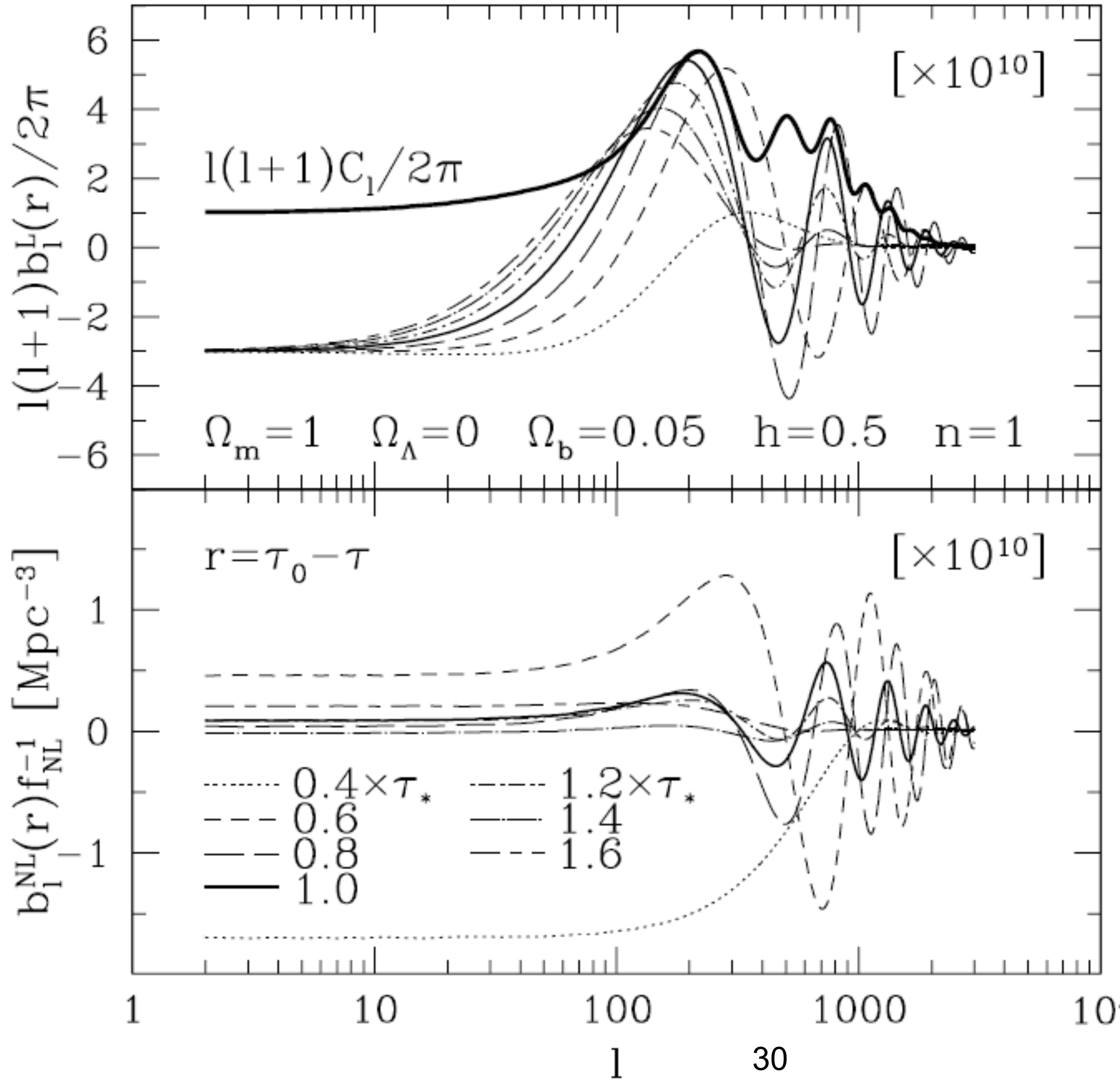


- Equilateral form (*Babich, Creminelli & Zaldarriaga 2004*)
  - $b^{equilateral}(k_1,k_2,k_3) = 6\{-[P(k_1)P(k_2)+cyc.] - 2[P(k_1)P(k_2)P(k_3)]^{2/3} + [P(k_1)^{1/3}P(k_2)^{2/3}P(k_3)+cyc.]\}$



Decoding Bispectrum

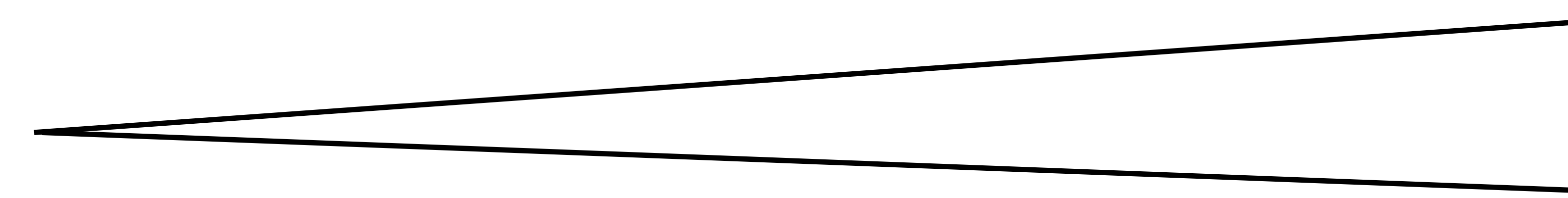
- Hydrodynamics at z=1090 generates acoustic oscillations in the bispectrum
- Well understood at the linear level (*Komatsu & Spergel 2001*)
- Non-linear extension?
  - *Nitta, Komatsu, Bartolo, Matarrese & Riotto* in prep.



What if f_{NL} is detected?

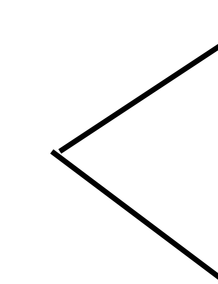
- A single field, canonical kinetic term, slow-roll, and/or Banch-Davies vacuum, must be modified.

Local • Multi-field (curvaton);



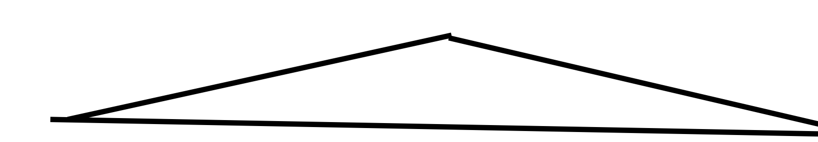
Preheating (e.g., *Chambers & Rajantie 2008*)

Equil. • Non-canonical kinetic term (k-inflation, DBI)



Bump +Osci. • Temporary fast roll (features in potential)

Folded • Departures from the Banch-Davies vacuum



- *It will give us a lot of clues as to what the correct early universe models should look like.*

...or, simply not inflation?

- It has been pointed out recently that New Ekpyrotic scenario generates f_{NL}^{local} ~100 generically
  - *Koyama et al.; Buchbinder et al.; Lehners & Steinhardt*

Measurement

- Use everybody's favorite: χ^2 minimization.
  - Minimize:

$$\chi^2 \equiv \sum_{2\le l_1\le l_2\le l_3} \frac{\left(B^{obs}_{l_1 l_2 l_3} - \sum_i A_i B^{(i)}_{l_1 l_2 l_3}\right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

  - with respect to $A_i=(f_{NL}^{local}, f_{NL}^{equilateral}, b_{src})$
  - B^{obs} is the observed bispectrum
  - $B^{(i)}$ is the theoretical template from various predictions

Journal on f_{NL}

- Local
  - $-3500 < f_{NL}^{local} < 2000$ [COBE 4yr, l_{max}=20] Komatsu et al. (2002)
  - $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, l_{max}=265] Komatsu et al. (2003)
  - $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, l_{max}=350] Spergel et al. (2007)
  - **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, l_{max}=500]** Komatsu et al. (2008)
- Equilateral
  - $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, l_{max}=405] Creminelli et al. (2006)
  - $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, l_{max}=475] Creminelli et al. (2007)
  - **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, l_{max}=700]** Komatsu et al. (2008)

What does f_{NL}~100 mean?

- Recall this form: $\Phi(x)=\Phi_{gaus}(x)+f_{NL}^{local}[\Phi_{gaus}(x)]^2$
  - Φ_{gaus} is small, of order 10^{-5}; thus, the second term is 10^{-3} times the first term, if f_{NL}~100
  - Precision test of inflation: **non-Gaussianity term is less than 0.1% of the Gaussian term**
    - cf: flatness tests inflation at 1% level

Non-Gaussianity Has Not Been Discovered Yet, but...

- At 68% CL, we have **f_{NL}=51±30** (positive 1.7σ)
  - Shift from Yadav & Wandelt's 2.8σ "hint" (f_{NL}~80) from the 3-year data can be explained largely by adding more years of data, i.e., statistical fluctuation, and a new 5-year Galaxy mask that is 10% larger than the 3-year mask
- There is a room for improvement
  - More years of data (WMAP 9-year survey funded!)
  - Better statistical analysis (*Smith & Zaldarriaga 2006*)
  - IF (big if) f_{NL}=50, we would see it at **3σ** in the 9-year data

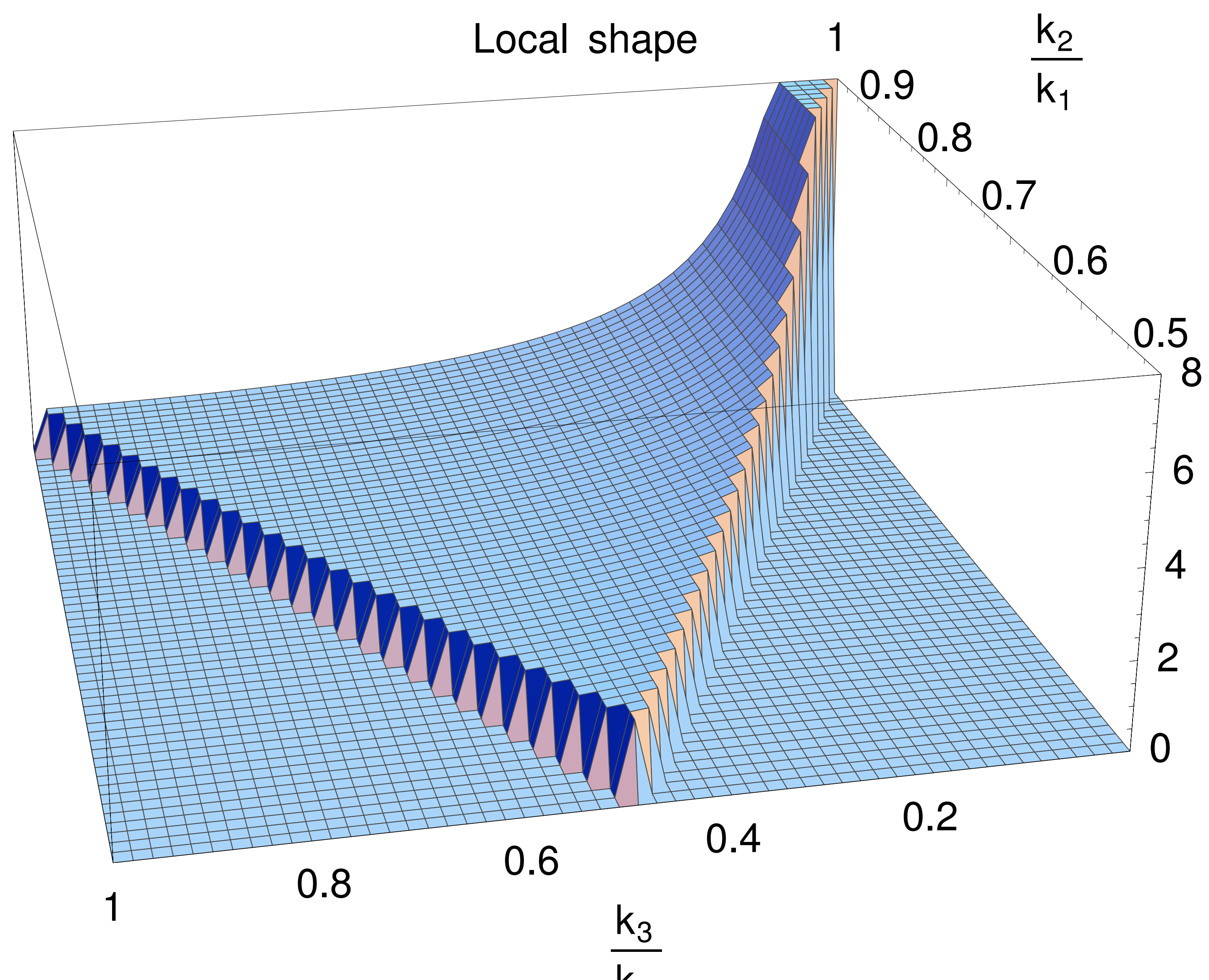
Exciting Future Prospects

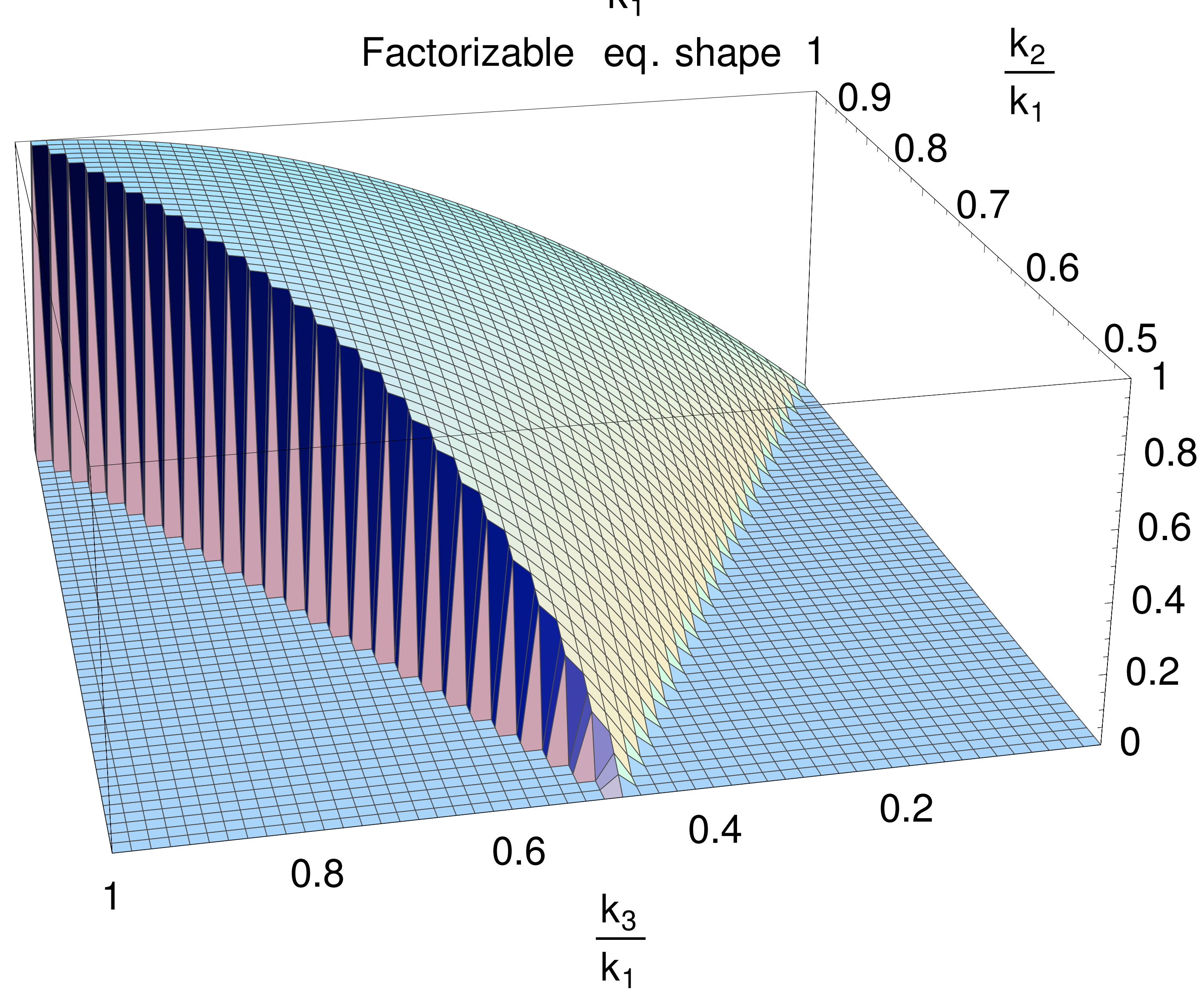
- Planck satellite (to be launched in March 2009)
  - will see f_{NL}^{local} at **17σ**, IF (big if) f_{NL}^{local}=50

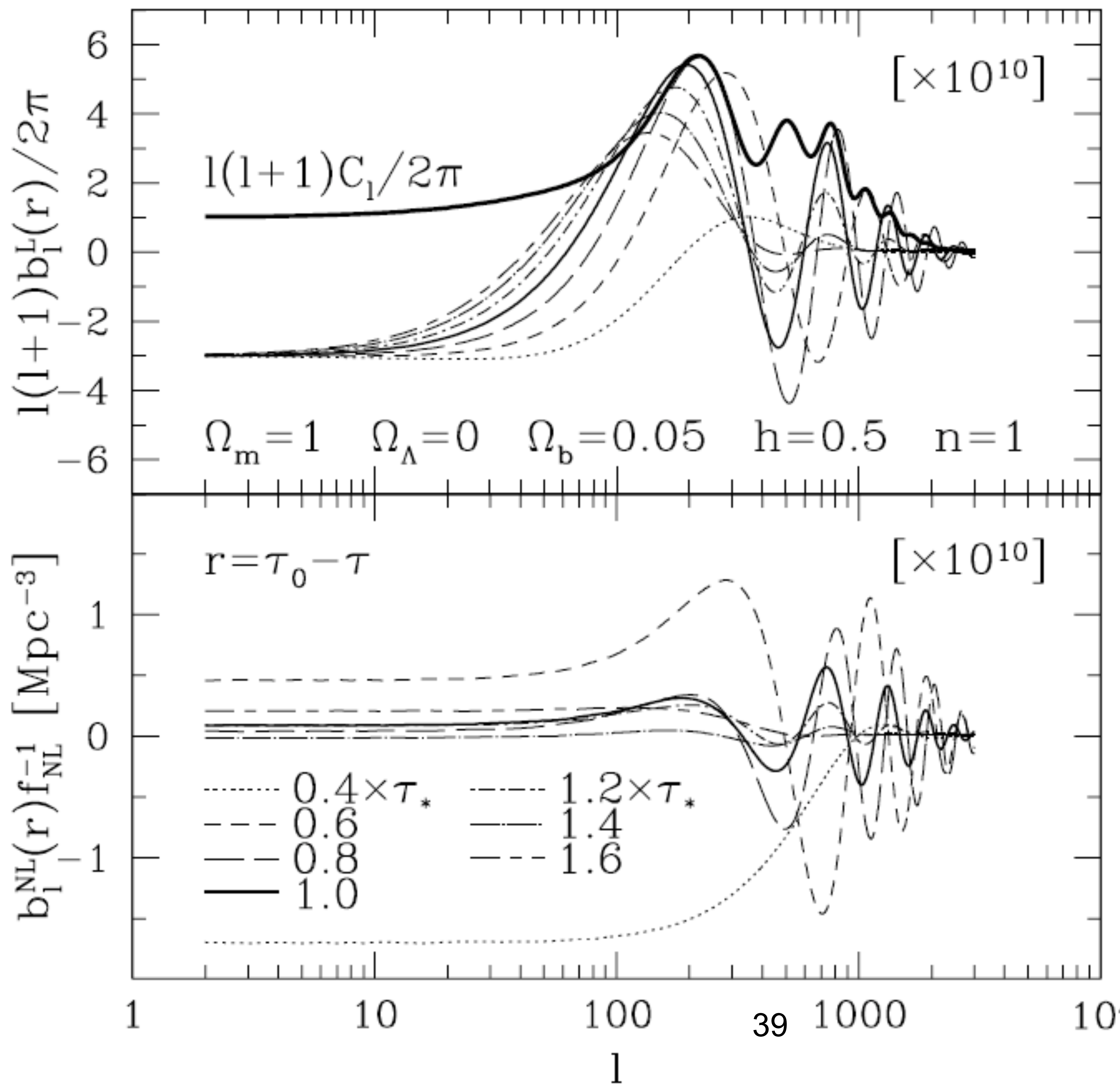
A Big Question

- Suppose that f_{NL} was found in, e.g., WMAP 9-year or Planck. That would be a profound discovery. ***However***:
  - **Q**: How can we convince ourselves and other people that primordial non-Gaussianity was found, rather than some junk?
  - **A**: (i) shape dependence of the signal, (ii) different statistical tools, and (iii) difference tracers

(i) Remember These Plots?







(ii) Different Tools

- How about 4-point function (trispectrum)?
- Beyong n-point function: How about morphological characterization (Minkowski Functionals)?

Beyond Bispectrum: Trispectrum of Primordial Perturbations

- Trispectrum is the Fourier transform of four-point correlation function.
- Trispectrum(k_1,k_2,k_3,k_4)

$$=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\Phi(k_4)\rangle$$

which can be sensitive to the higher-order terms:

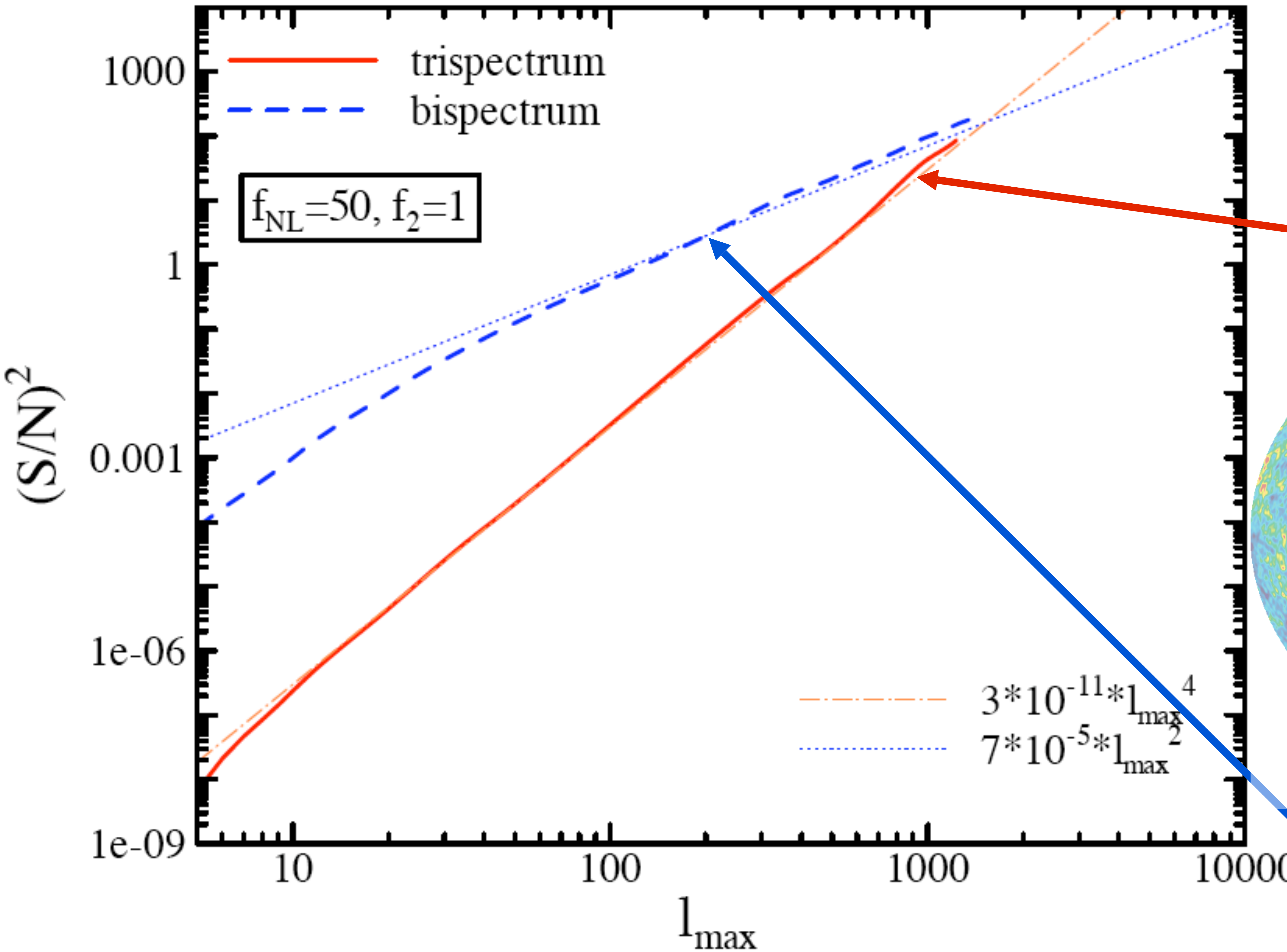
$$\Phi(x) = \Phi_{\rm L}(x) + f_{\rm NL}\left[\Phi_{\rm L}^2(x) - \langle\Phi_{\rm L}^2(x)\rangle\right] + f_2\Phi_{\rm L}^3(x)$$

Measuring Trispectrum

- It's pretty painful to measure all the quadrilateral configurations.
  - Measurements from the COBE 4-year data were possible and done (*Komatsu 2001; Kunz et al. 2001*)
- Only limited configurations measured from the WMAP 3-year data
  - *Spergel et al. (2007)*
- No evidence for non-Gaussianity, but f_{NL} or f_2 has not been constrained by the trispectrum yet. (Work in progress: *Smith, Komatsu, et al*)

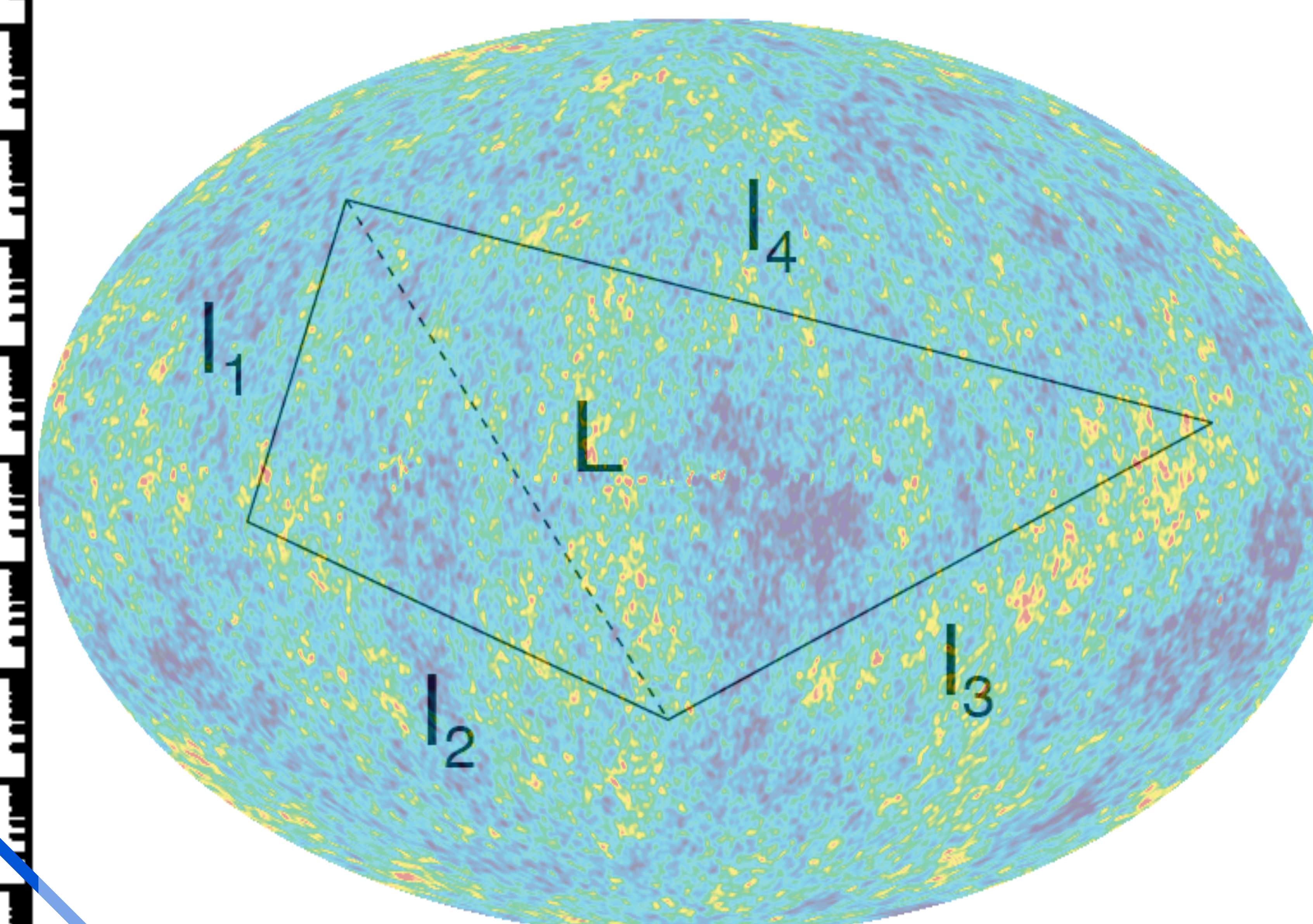
Kogo & Komatsu (2006)

Trispectrum: if f_{NL} is greater than ~50, excellent cross-check for Planck

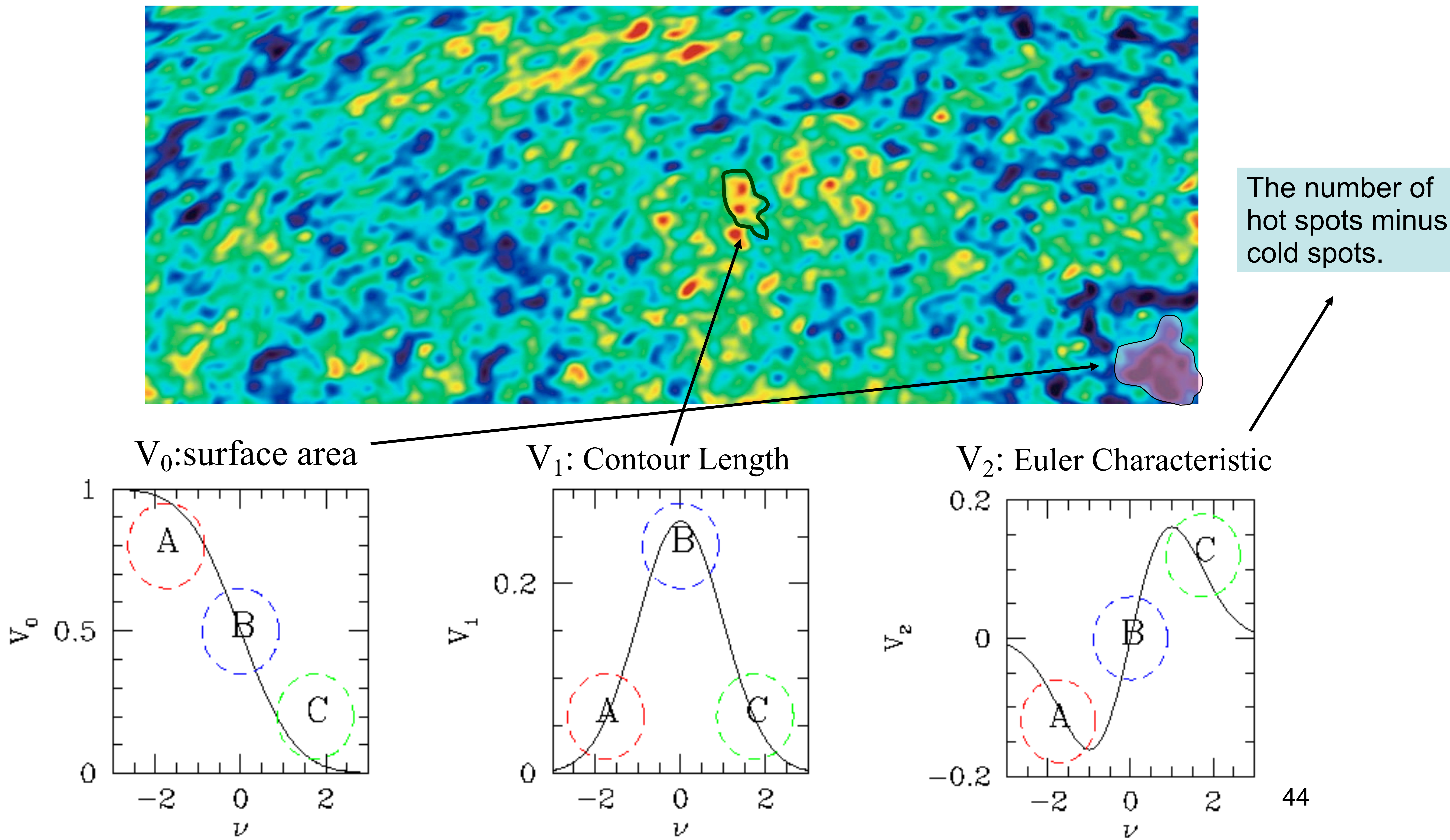


Trispectrum ($\sim f_{NL}^2$)

Bispectrum ($\sim f_{NL}$)



Minkowski Functionals (MFs)



Hikage, Komatsu & Matsubara (2006)

Analytical formulae of MFs

Perturbative formulae of MFs (Matsubara 2003)

$$V_k(\nu) = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_2}{\omega_{2-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{2}\sigma_0}\right)^k e^{-\nu^2/2} \{H_{k-1}(\nu)$$

Gaussian term

$(k = 0,1,2)$

$$+ \left[\frac{1}{6} S^{(0)} H_{k+2}(\nu) + \frac{k}{3} S^{(1)} H_k(\nu) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(\nu)\right]\sigma_0 + O(\sigma_0^{\ 2})\}$$

leading order of Non-Gaussian term

$$\sigma_j^2 = \frac{1}{4}\sum_l (2l+1)[l(l+1)]^j C_l W_l^2$$

W_l: smoothing kernel

$$\omega_0 = 1, \omega_1 = 1, \omega_2 = \pi, \omega_3 = 4\pi/3$$

H_k : k- th Hermite polynomial

$S^{(a)}$: skewness parameters (a = 0,1,2)

In weakly non-Gaussian fields (σ_0<<1) , the non-Gaussianity in MFs is characterized by three skewness parameters $S^{(a)}$.

Matsubara (2003)

3 “Skewness Parameters”

- Ordinary skewness

$$S^{(0)} \equiv \frac{\langle f^3 \rangle}{\sigma_0^4},$$

- Second derivative

$$S^{(1)} \equiv -\frac{3}{4}\frac{\langle f^2(\nabla^2 f)\rangle}{\sigma_0^2\sigma_1^2},$$

- (First derivative)^2 x Second derivative

$$S^{(2)} \equiv -\frac{3d}{2(d-1)}\frac{\langle(\nabla f)\cdot(\nabla f)(\nabla^2 f)\rangle}{\sigma_1^4},$$

$$S^{(0)} = \frac{3}{2\pi\sigma_0^4} \sum_{2\le l_1 \le l_2 \le l_3} I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3},$$

$$S^{(1)} = \frac{3}{8\pi\sigma_0^2\sigma_1^2} \sum_{2\le l_1 \le l_2 \le l_3} [l_1(l_1+1) + l_2(l_2+1) + l_3(l_3+1)] \times I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3},$$

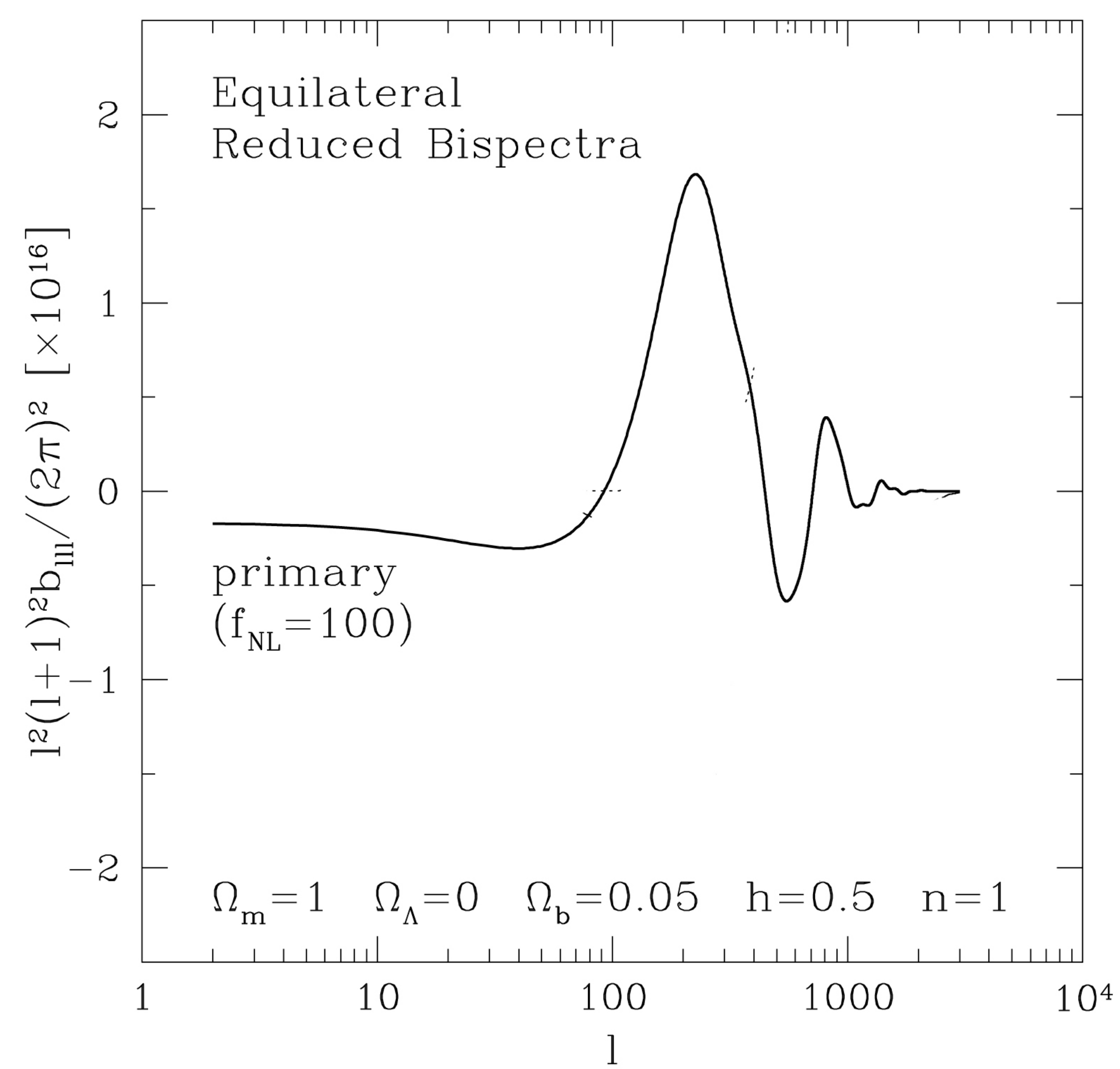
$$S^{(2)} = \frac{3}{4\pi\sigma_1^4} \sum_{2\le l_1 \le l_2 \le l_3} \{[l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1)] \times l_3(l_3+1) + (\text{cyc.})\} I^2_{l_1 l_2 l_3} b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3},$$

$S^{(0)}$: Simple average of b_{l1l2l3}

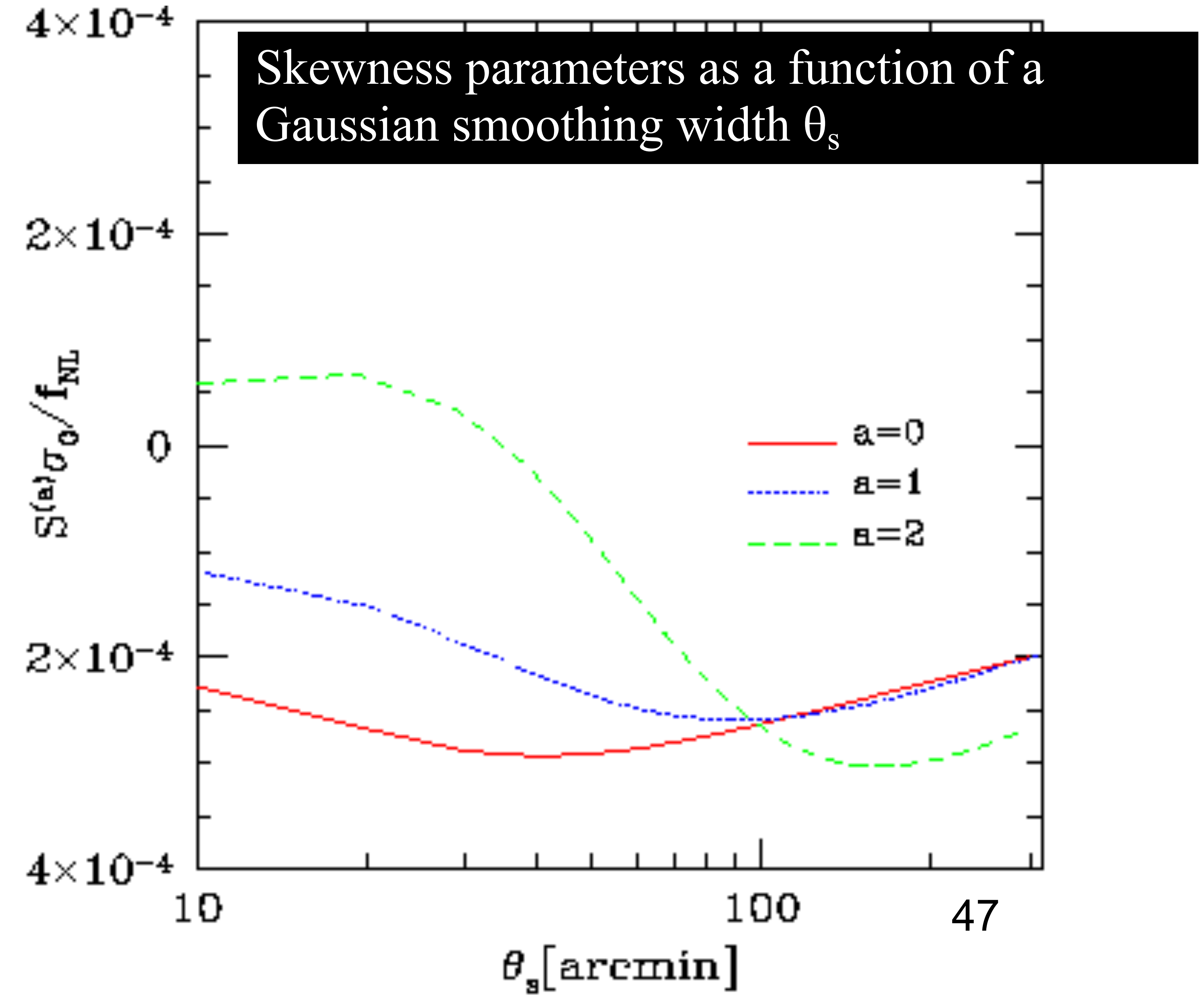
$S^{(1)}$: l^2 weighted average

$S^{(2)}$: l^4 weighted average

Analytical predictions of bispectrum at $f_{NL}=100$ (Komatsu & Spergel 2001)



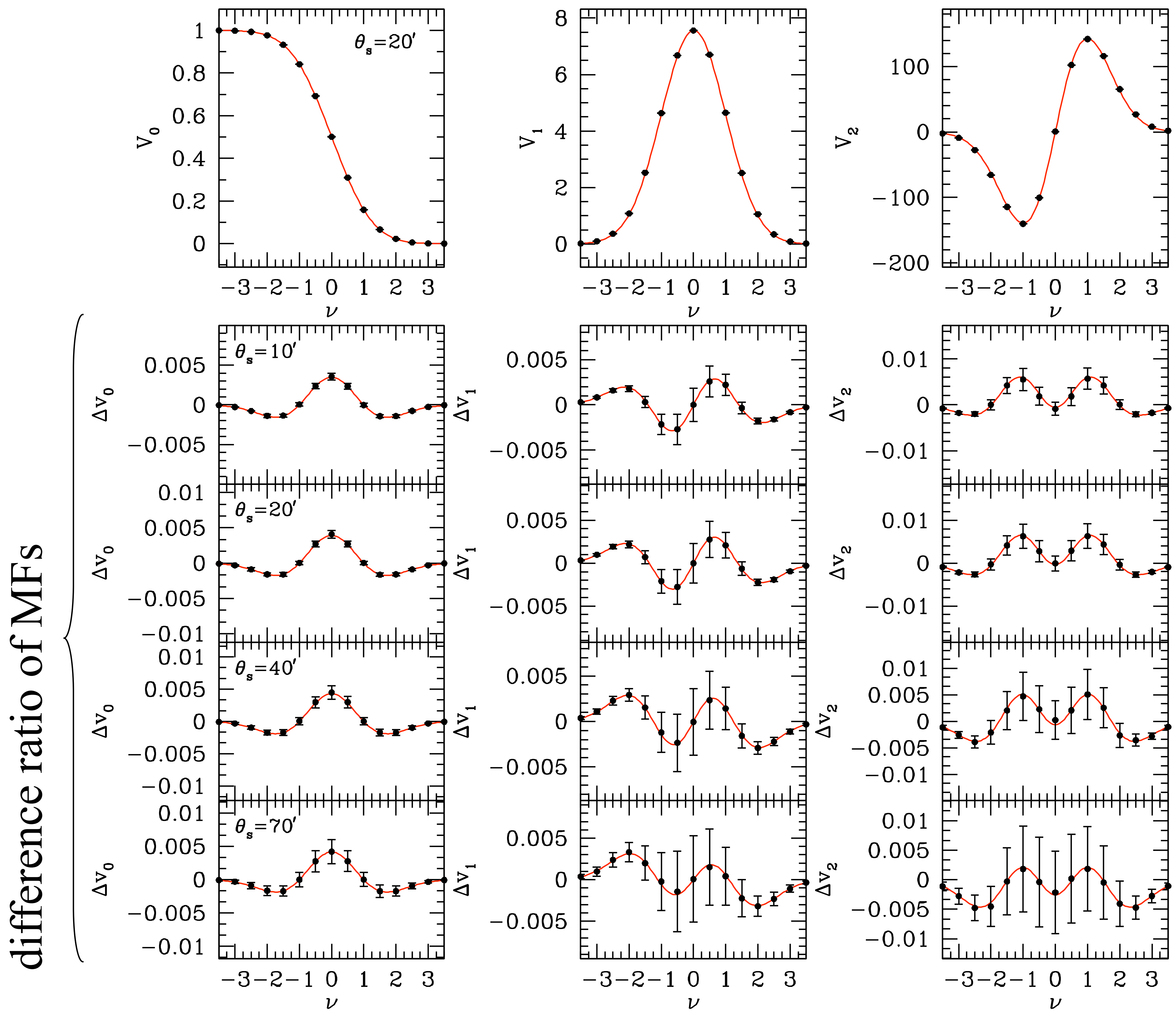
Skewness parameters as a function of a Gaussian smoothing width θ_s



Hikage et al. (2008)

Comparison of analytical formulae with Non-Gaussian simulations

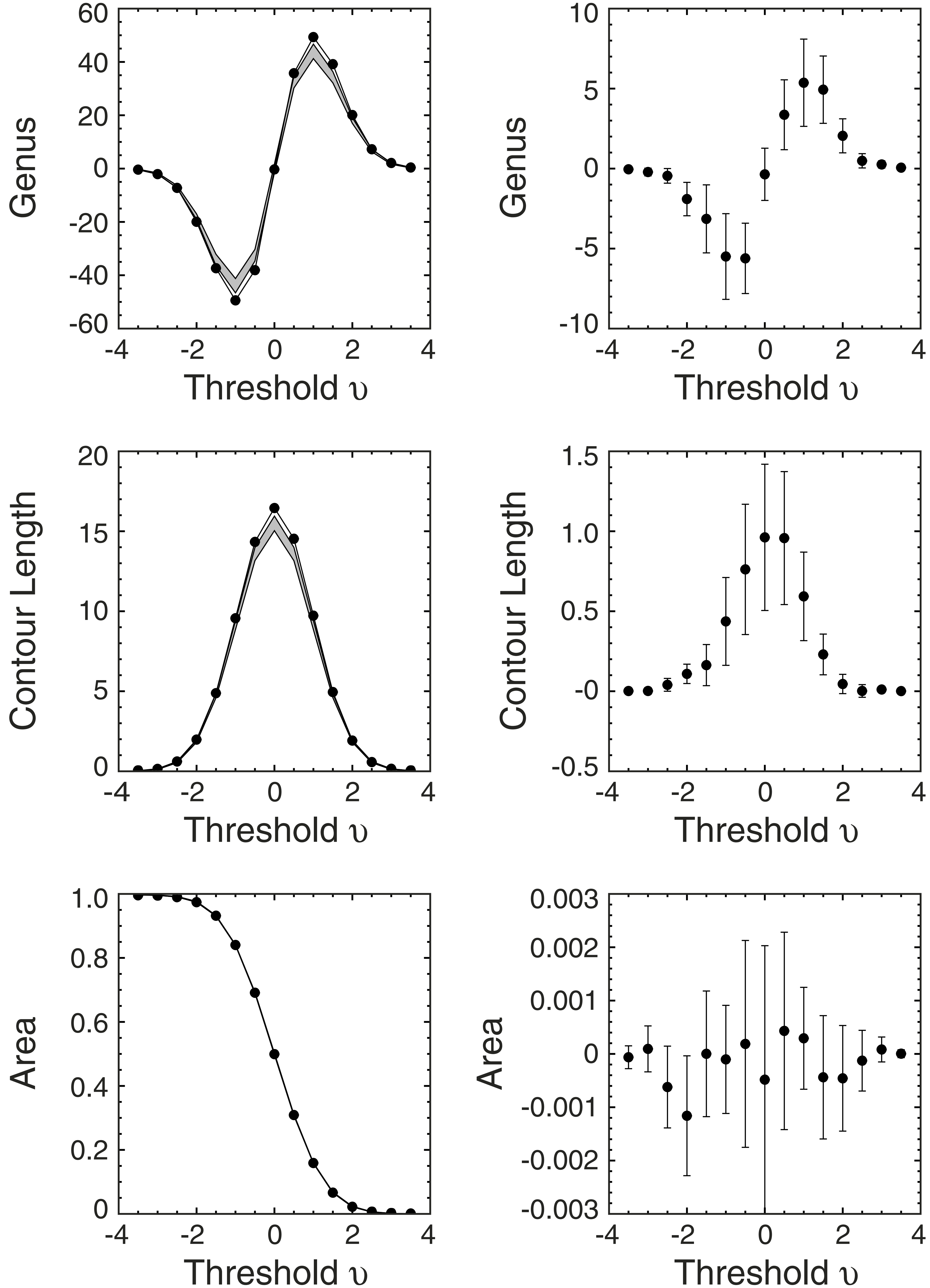
Surface area   Contour Length   Euler Characteristic



Comparison of MFs between analytical predictions and non-Gaussian simulations with f_{NL}=100 at different Gaussian smoothing scales, $\boldsymbol{\theta_s}$

Simulations are done for WMAP.

Analytical formulae agree with non-Gaussian simulations very well.



MFs from *WMAP* *5-Year Data (V+W)*

Result from a single resolution
(N_{side}=128; 28 arcmin pixel)
[*analysis done by Al Kogut*]

f_{NL}^{local} = –57 +/- 60 (68% CL)

–178 < f_{NL}^{local} < 64 (95% CL)

See *Hikage et al.* for an extended analysis of MFs from the 5-year data.

(ii) Different Tracers

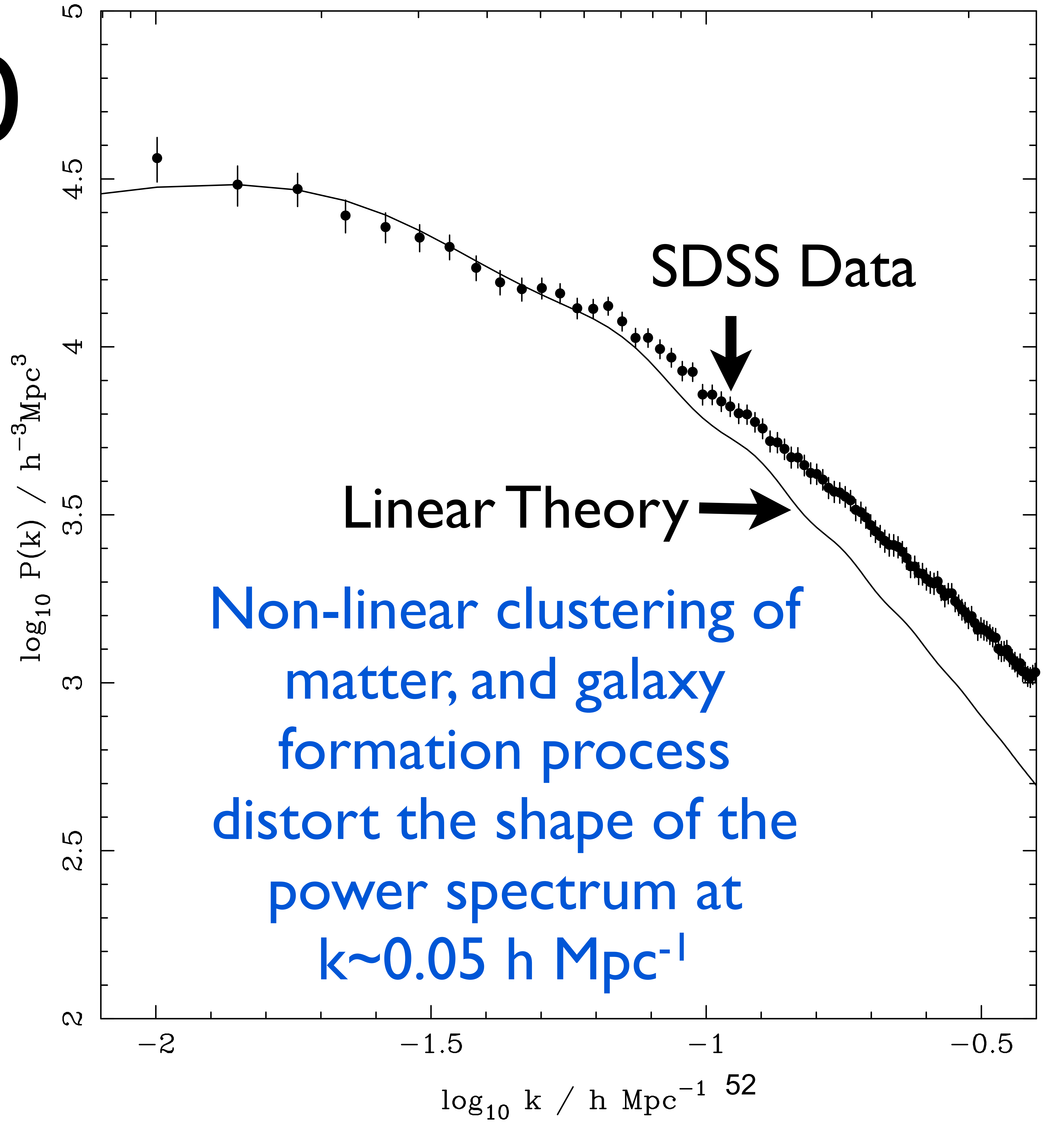
- CMB is a powerful probe of non-Gaussianity; however, there is a fundamental limitation
- The number of Fourier modes is limited because it is a 2-dimensional field: $N_{mode} \sim l^2$
- ***3-dimensional tracers*** of primordial fluctuations will provide far better constraints as the number of modes grows faster: $N_{mode} \sim k^3$
    - Are there any?

Believe it or not:

- Galaxy redshift surveys can yield competitive constraints.

But, not at z~0

- The number of modes available at z~0 is limited because of non-linearity
- We can use modes up to k_{max}~0.05hMpc^{-1}, for which we know how to model the power spectrum
- Beyond that, non-linearity is too strong to understand

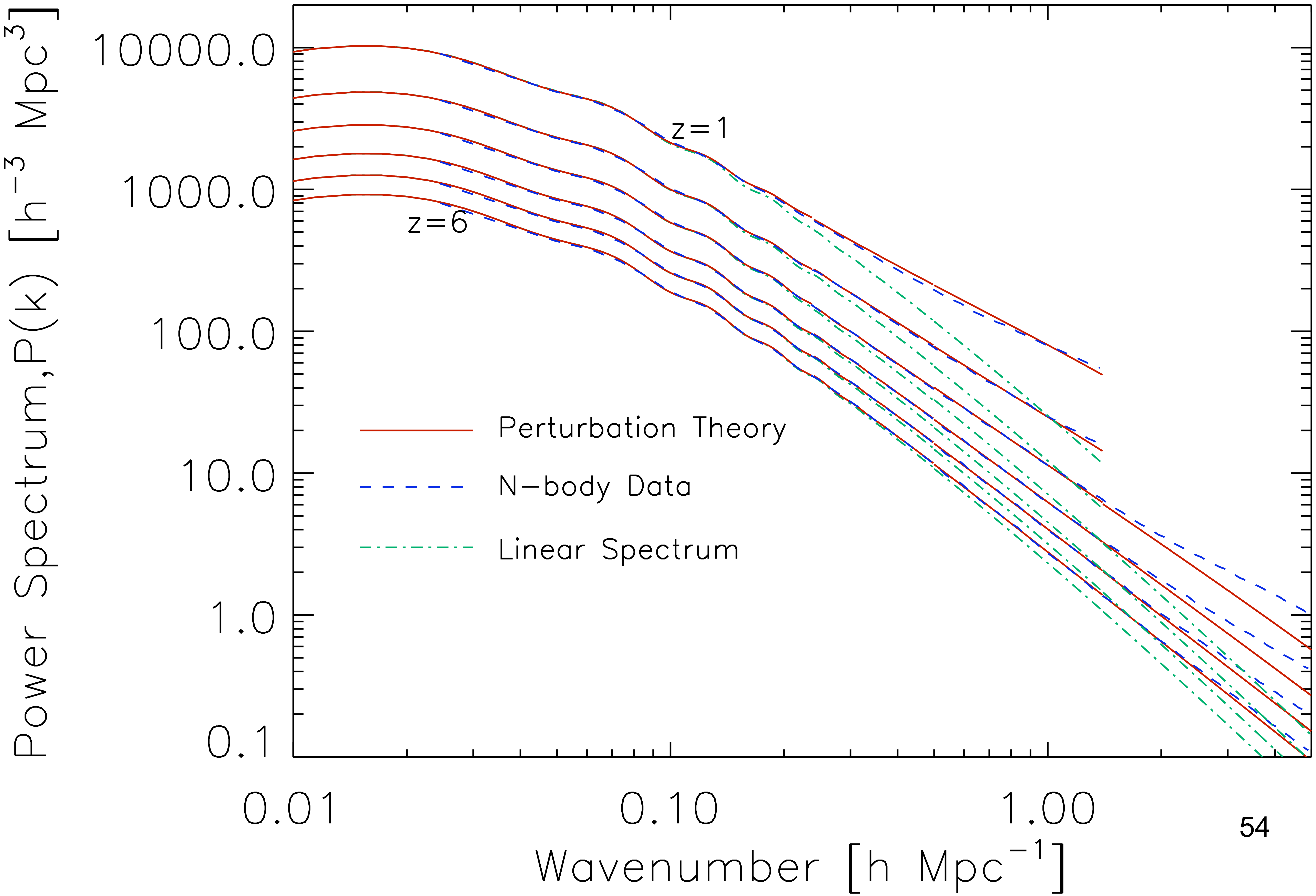


High-z Galaxy Surveys! (SDSS@z>1)

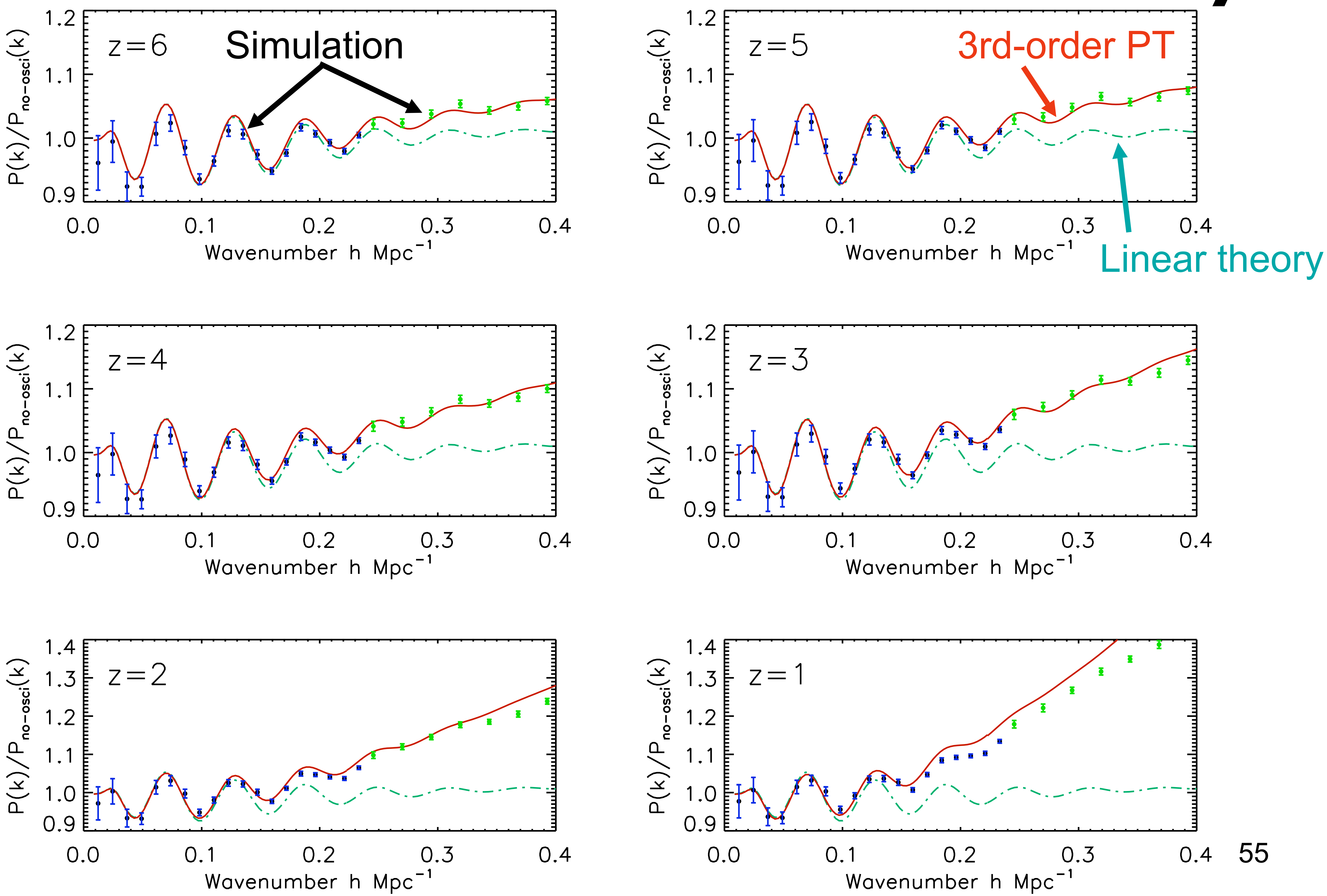
- Thanks to advances in technology...
- **High-redshift (z>1) galaxy redshift surveys are now possible.**
- And now, such surveys are needed for different reasons: Dark Energy studies
- **Non-linearities are weaker at z>1, making it possible to use the cosmological perturbation theory to calculate P(k) and $B(k_1,k_2,k_3)$!**

Jeong & Komatsu (2006)

"Perturbation Theory Reloaded"



BAO: Matter Non-linearity



Sefusatti & Komatsu (2006)

f_{NL} from Galaxy Bispectrum

- Planned future large-scale structure surveys such as
  - **HETDEX** (Hobby-Eberly Dark Energy Experiment)
    - UT Austin (PI: G.Hill) 0.8M galaxies, $1.9<z<3.5$, 8 Gpc^3
    - 3-year survey begins in 2011; Comparable to WMAP for f_{NL}^{local}
  - **ADEPT** (Advanced Dark Energy Physics Telescope)
    - NASA/GSFC (PI: C.L.Bennett), 100M galaxies, $1<z<2$, 290 Gpc^3
    - Comparable to Planck for f_{NL}^{local}
  - **CIP** (Cosmic Inflation Probe)
    - Harvard+UT (PI: G.Melnich), 10 M galaxies, $2<z<6$, 50 Gpc^3
    - Comparable to Planck for f_{NL}^{local}

Summary

- **Non-Gaussianity is a *new, powerful* probe of physics of the early universe**
    - It has a best chance of ruling out the largest class of inflation models — could even rule out the inflationary paradigm, and support alternatives
- Various forms of f_{NL} available today — 1.7σ at the moment, wait for WMAP 9-year (2011) and Planck (2012) for >3σ
- To convince ourselves of detection, we need to see the acoustic oscillations, and the same signal in bispectrum, trispectrum, Minkowski functionals, of both CMB and large-scale structure of the universe
- New "industry" — active field! (unlike stock market today)