Physics of CMB Anisotropies

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"*The CMB from A to Z*", November 13–15, 2017

Planning: Day 1 (today)

- **Lecture 1 [8:30–9:15]**
  - Brief introduction of the CMB research
  - Temperature anisotropy from gravitational effects
- **Lecture 2 [14:00–14:45]**
  - Power spectrum basics
  - Temperature anisotropy from hydrodynamical effects (sound waves)

Planning: Day 2

- **Lecture 3 [8:30–9:15]**
  - Temperature anisotropy from sound waves [continued]
  - Cosmological parameter dependence of the temperature power spectrum
- **Lecture 4 [14:00–14:45]**
  - Cosmological parameter dependence of the temperature power spectrum [continued]
  - Polarisation

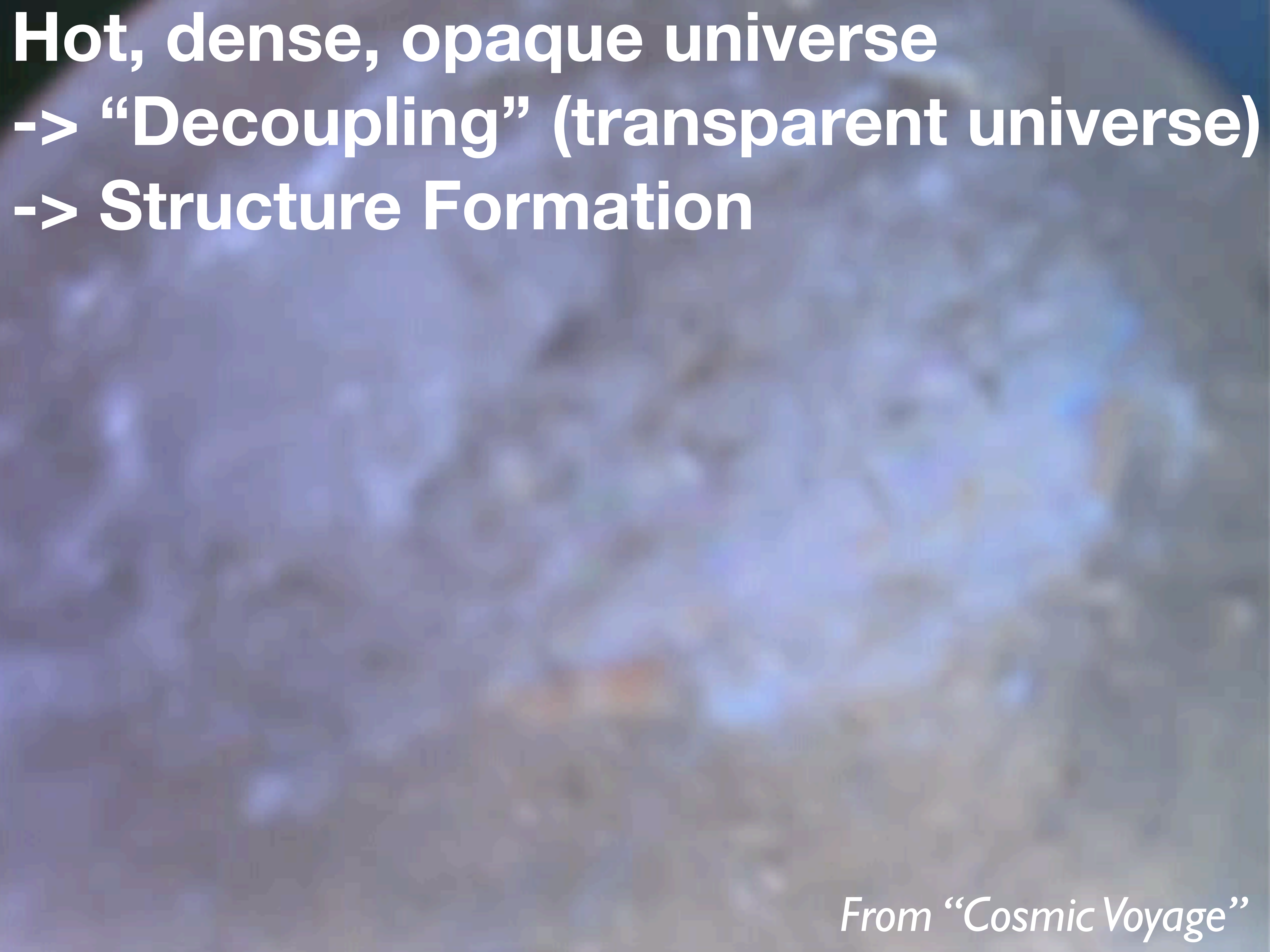
Planning: Day 3

- **Lecture 5 [8:30-9:15]**
    - Polarisation [continued]
    - Gravitational waves and their imprints on the CMB

Hot, dense, opaque universe

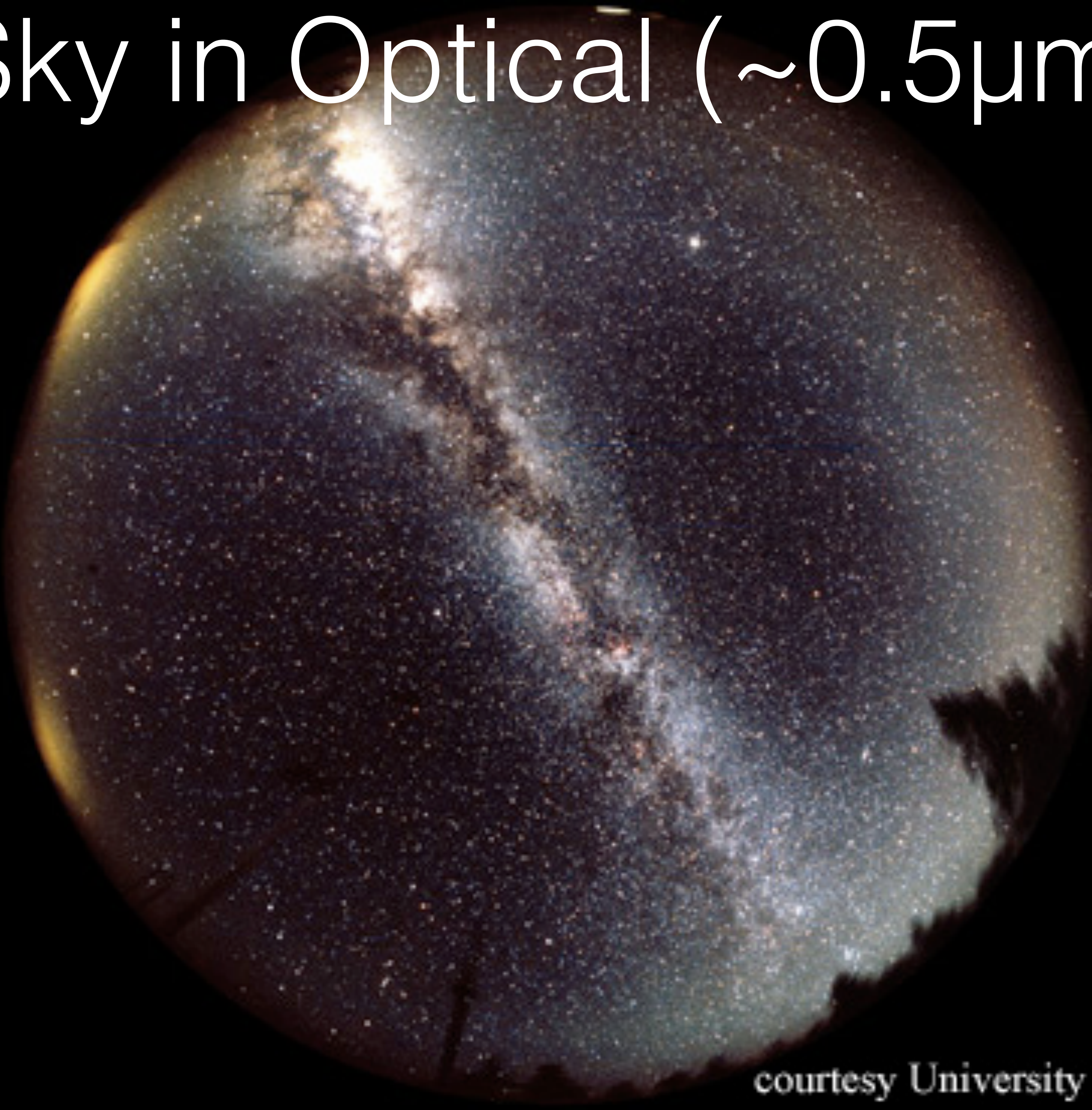
-> “Decoupling” (transparent universe)

-> Structure Formation



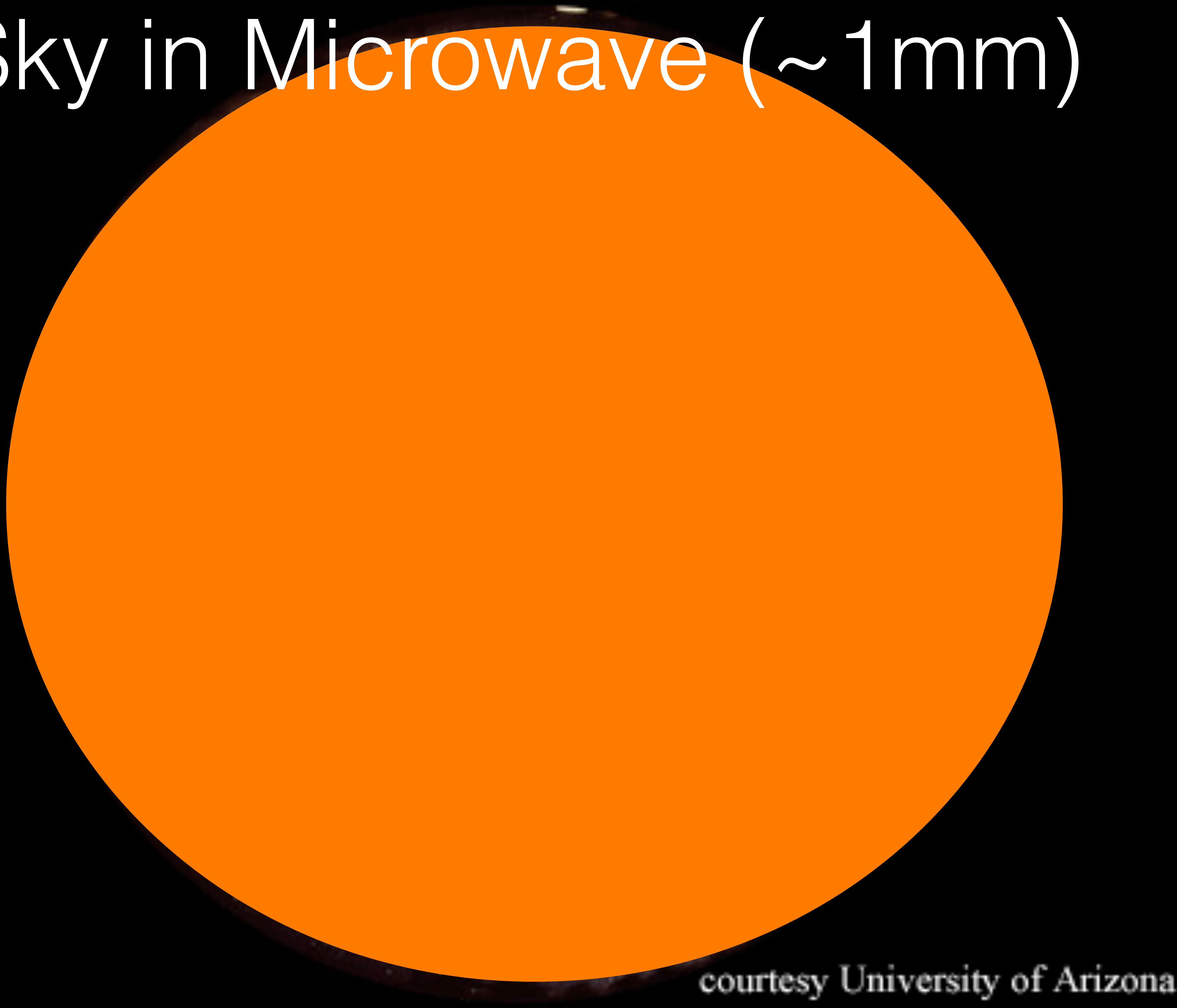
From “Cosmic Voyage”

Sky in Optical (~0.5μm)



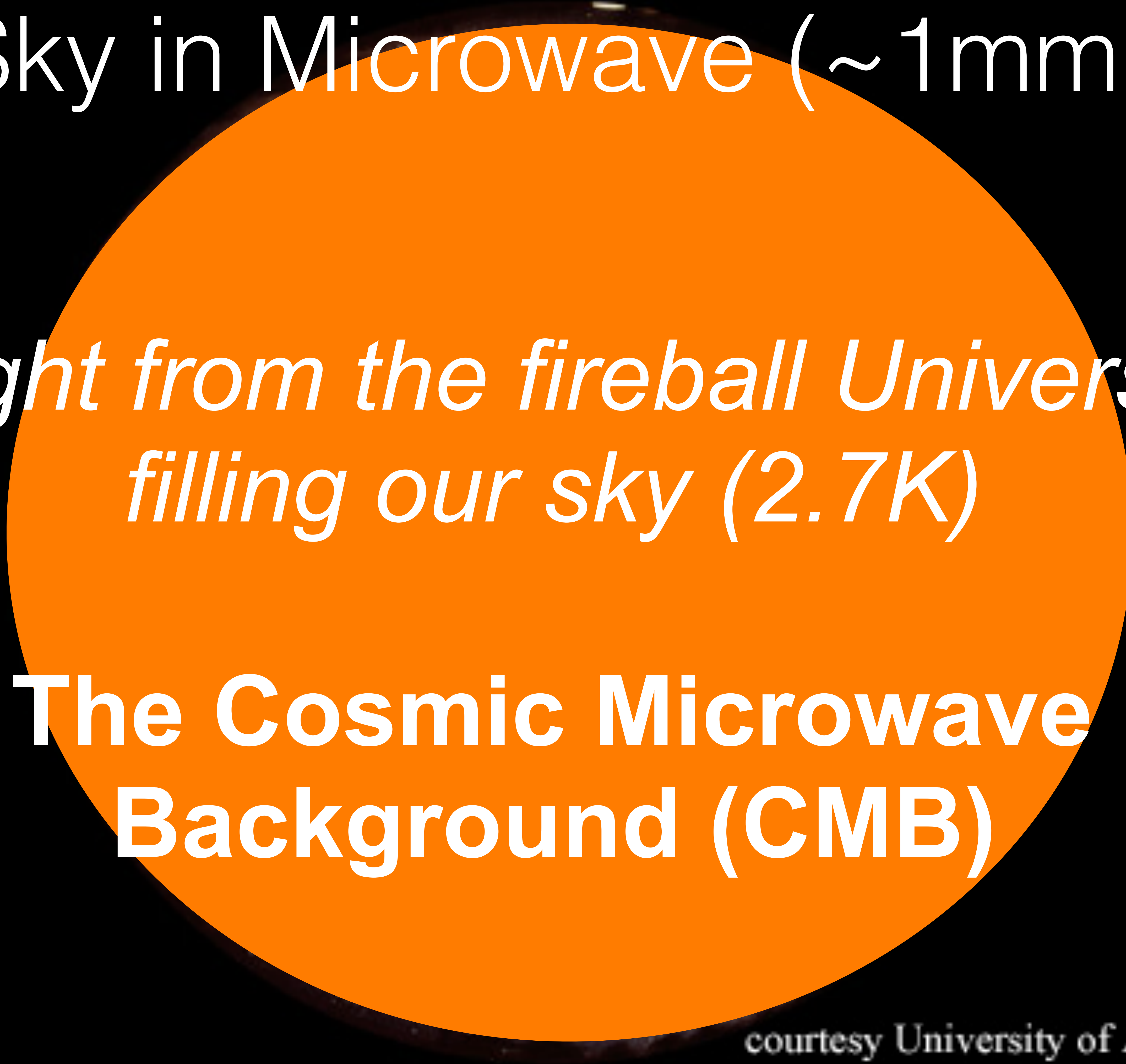
courtesy University of Arizona

Sky in Microwave (~1mm)



courtesy University of Arizona

Sky in Microwave (~1mm)



Light from the fireball Universe filling our sky (2.7K)

The Cosmic Microwave Background (CMB)

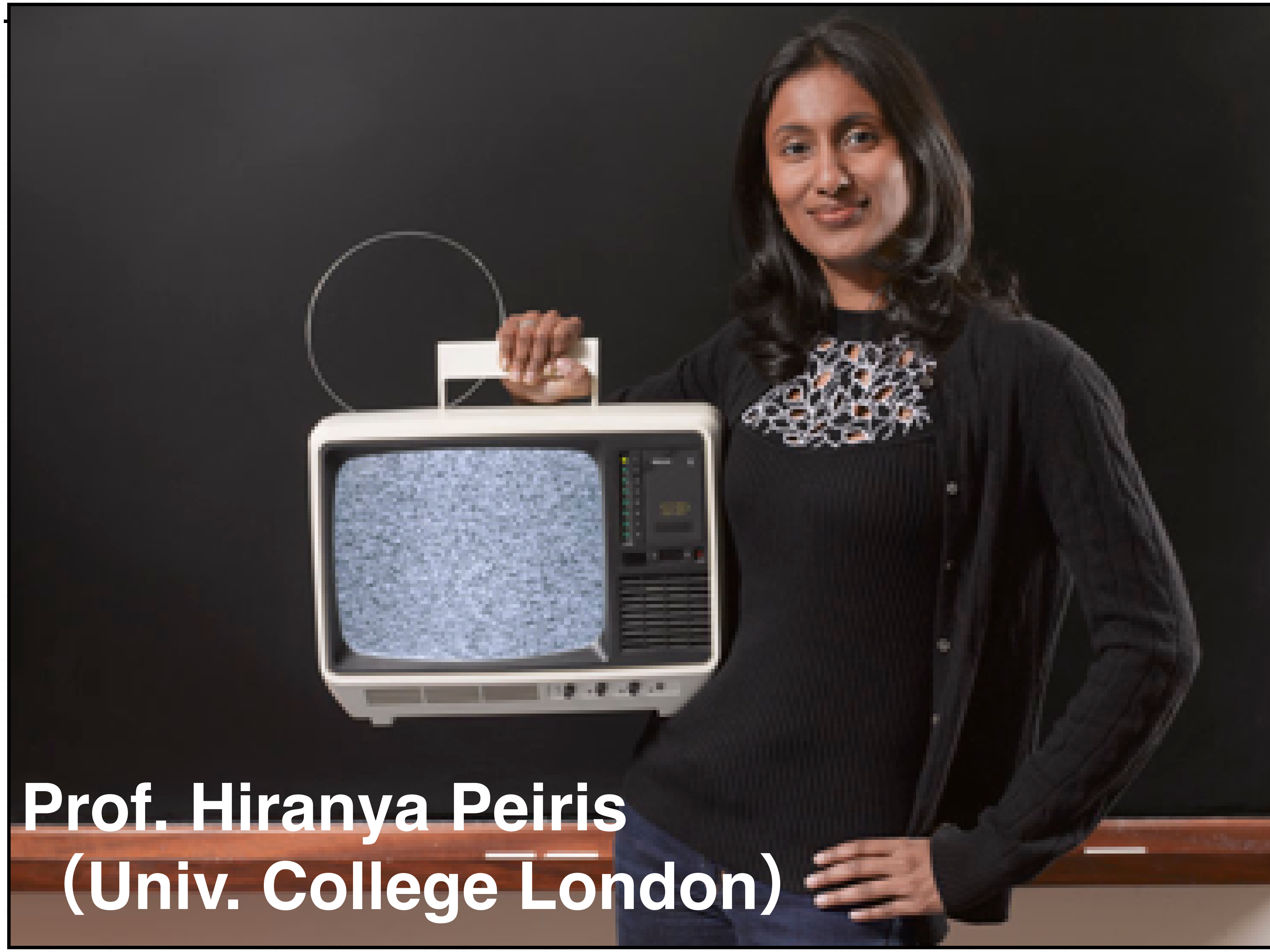
courtesy University of Arizona

410 photons

per

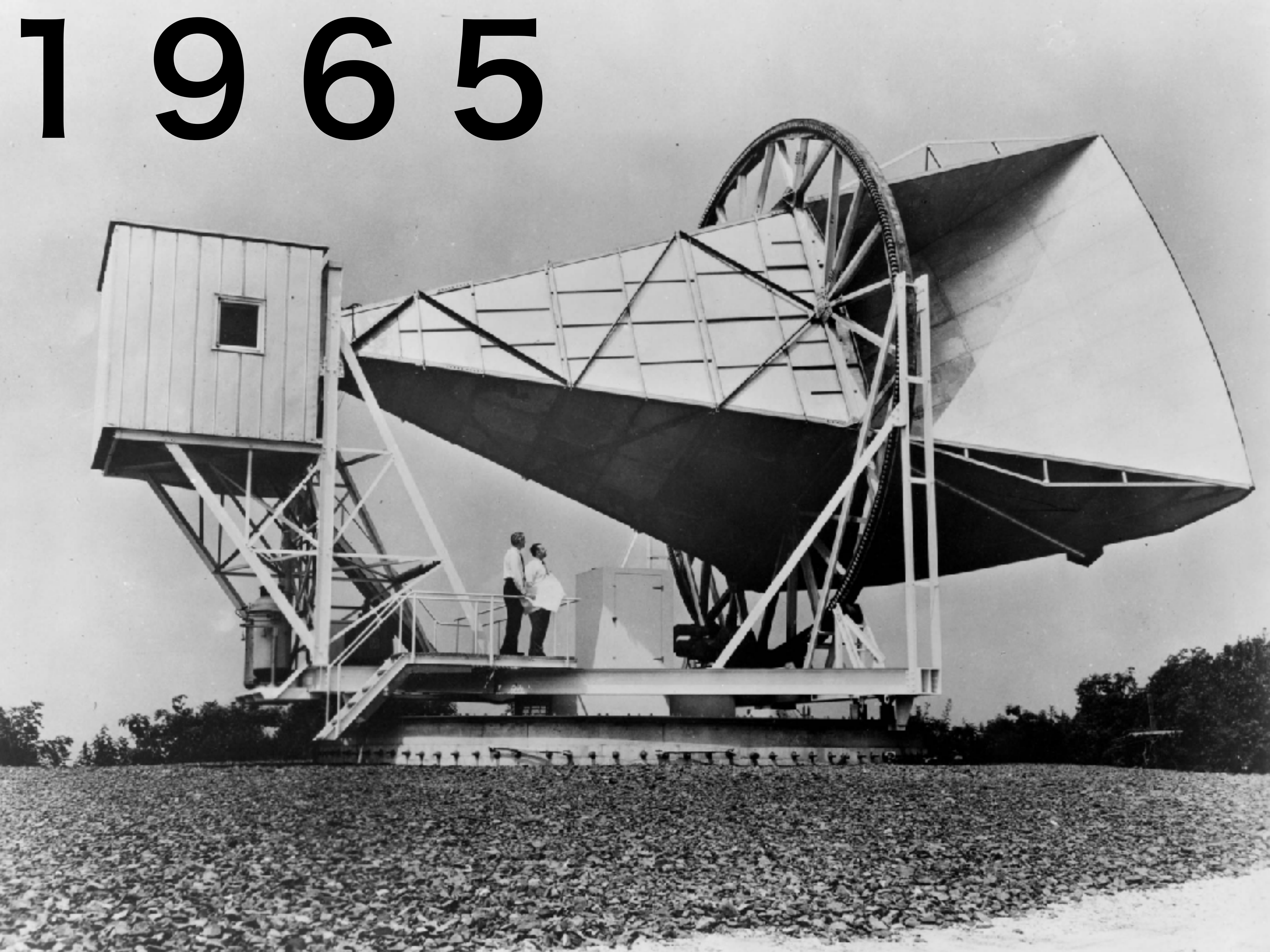
cubic centimeter!!

courtesy University of Arizona

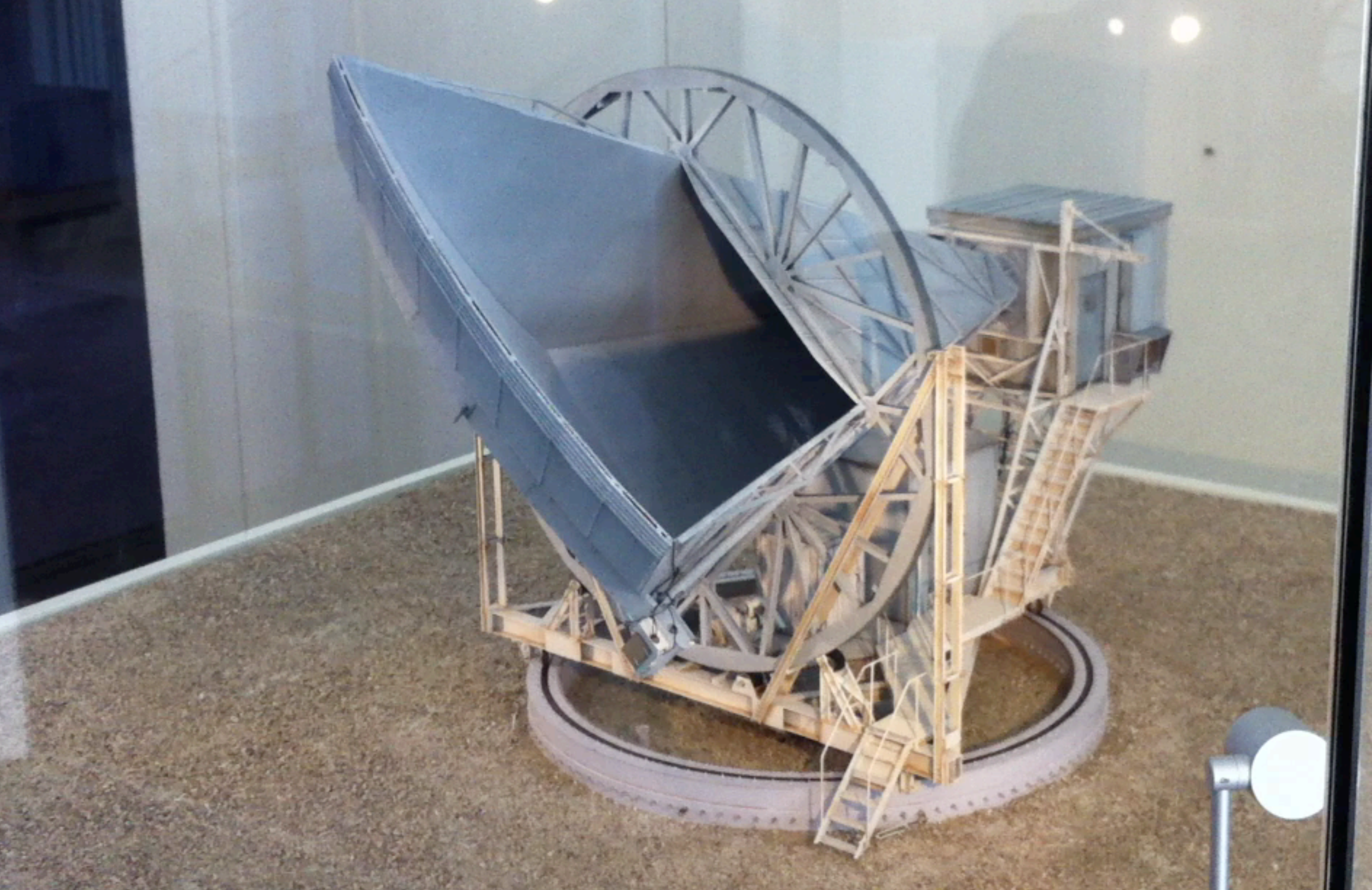


All you need to do is to detect radio waves. For example, 1% of noise on the TV is from the fireball Universe

1965

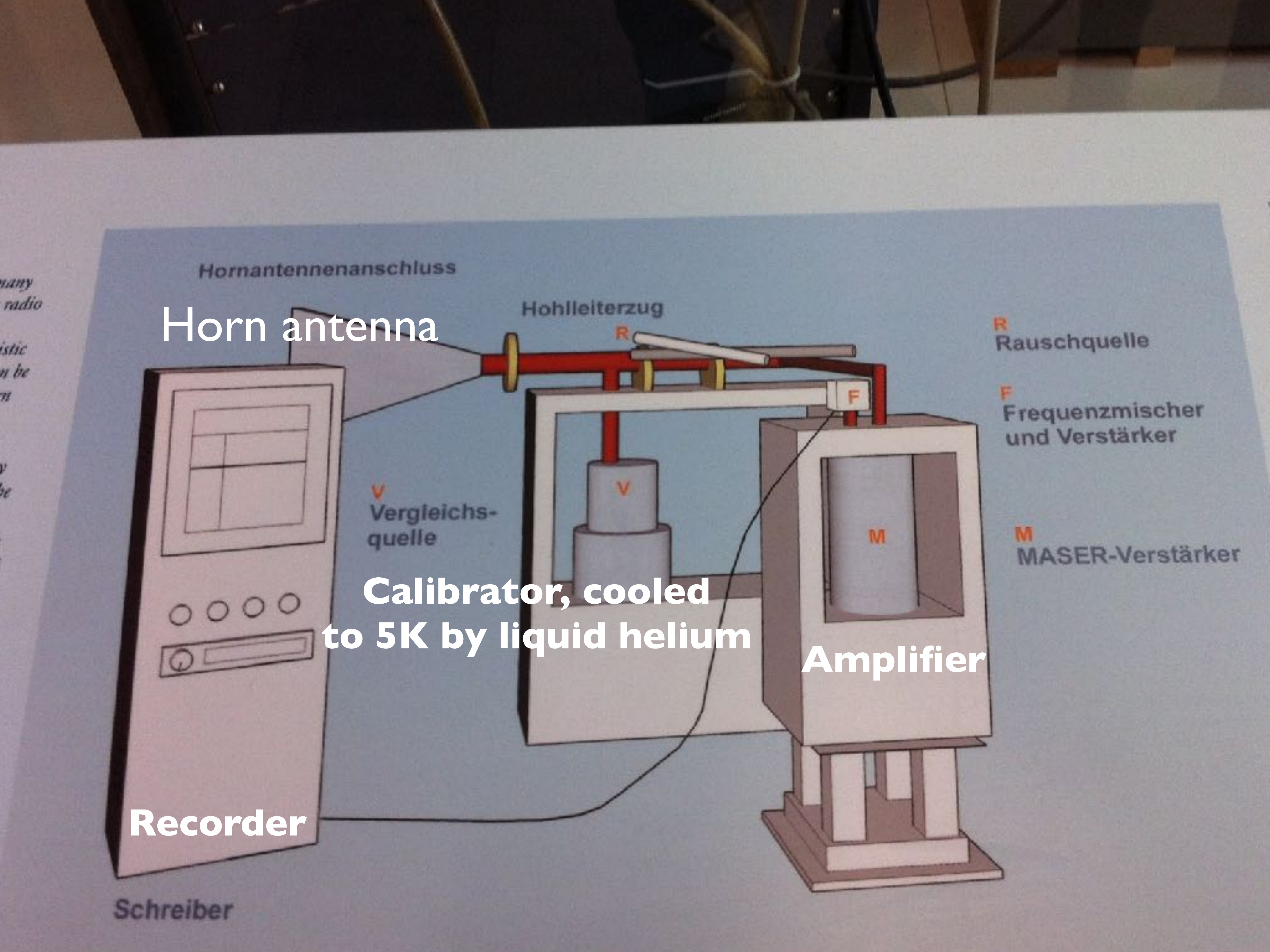


1:25 model of the antenna at Bell Lab
The 3rd floor of Deutsches Museum



The real detector system used by Penzias & Wilson
The 3rd floor of Deutsches Museum

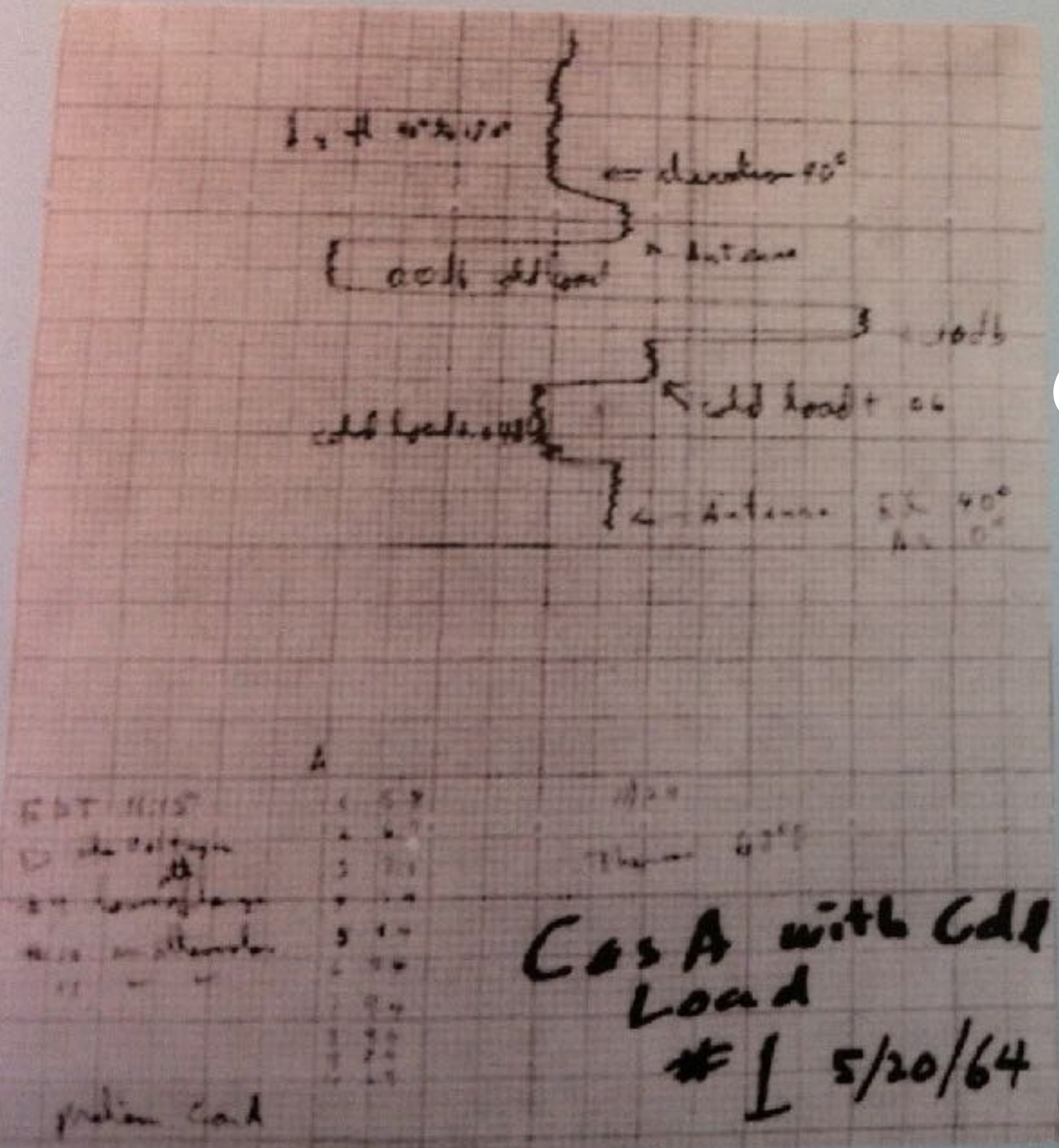
Arno Penzias



Hornantennenanschluss
Horn antenna
Hohlleiterzug
R
R
Rauschquelle
F
F
Frequenzmischer und Verstärker
V
Vergleichs-quelle
V
M
M
MASER-Verstärker
Calibrator, cooled to 5K by liquid helium
Amplifier
Recorder
Schreiber

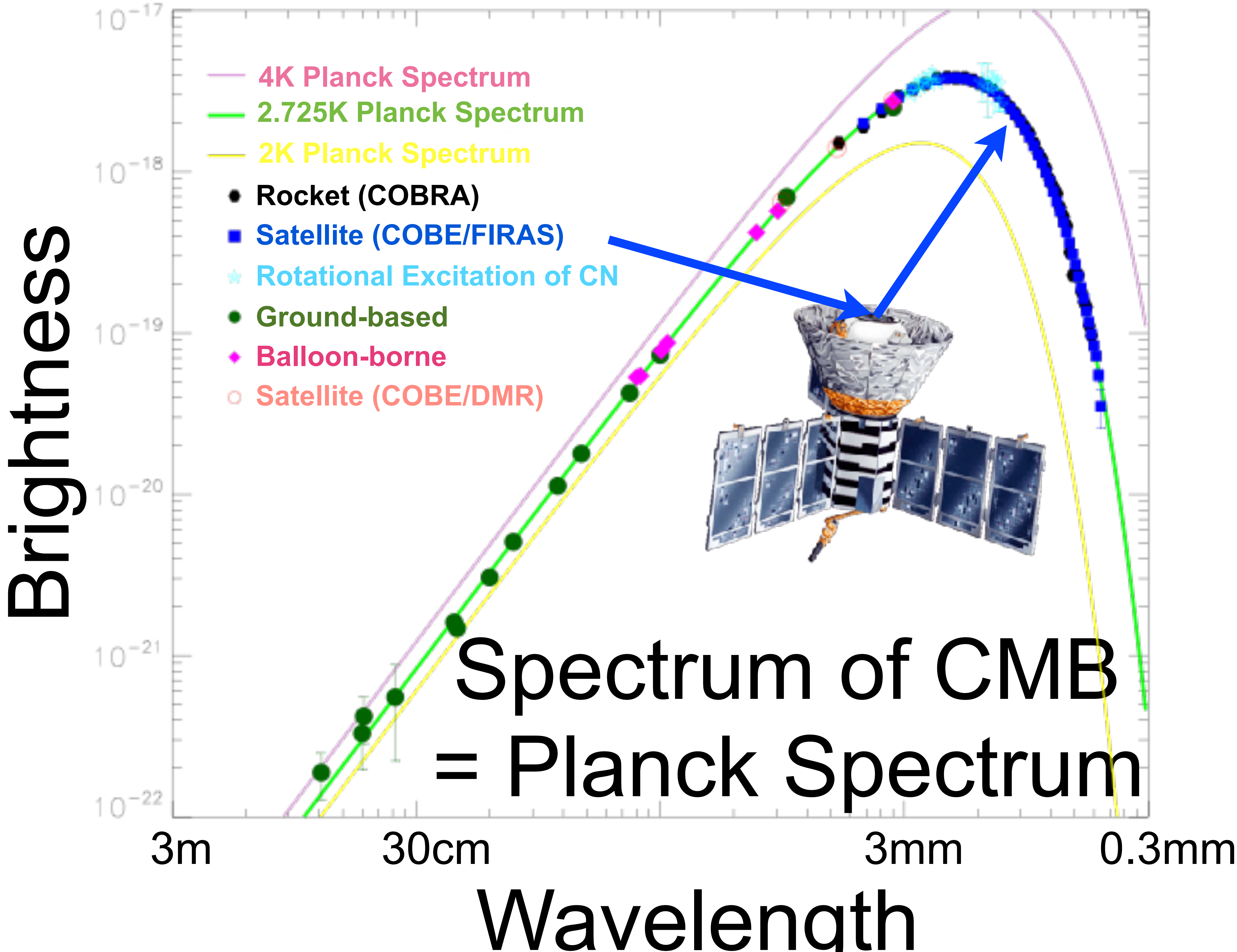
May 20, 1964
CMB
Discovered

6.7–2.3–0.8–0.1
= 3.5±1.0 K



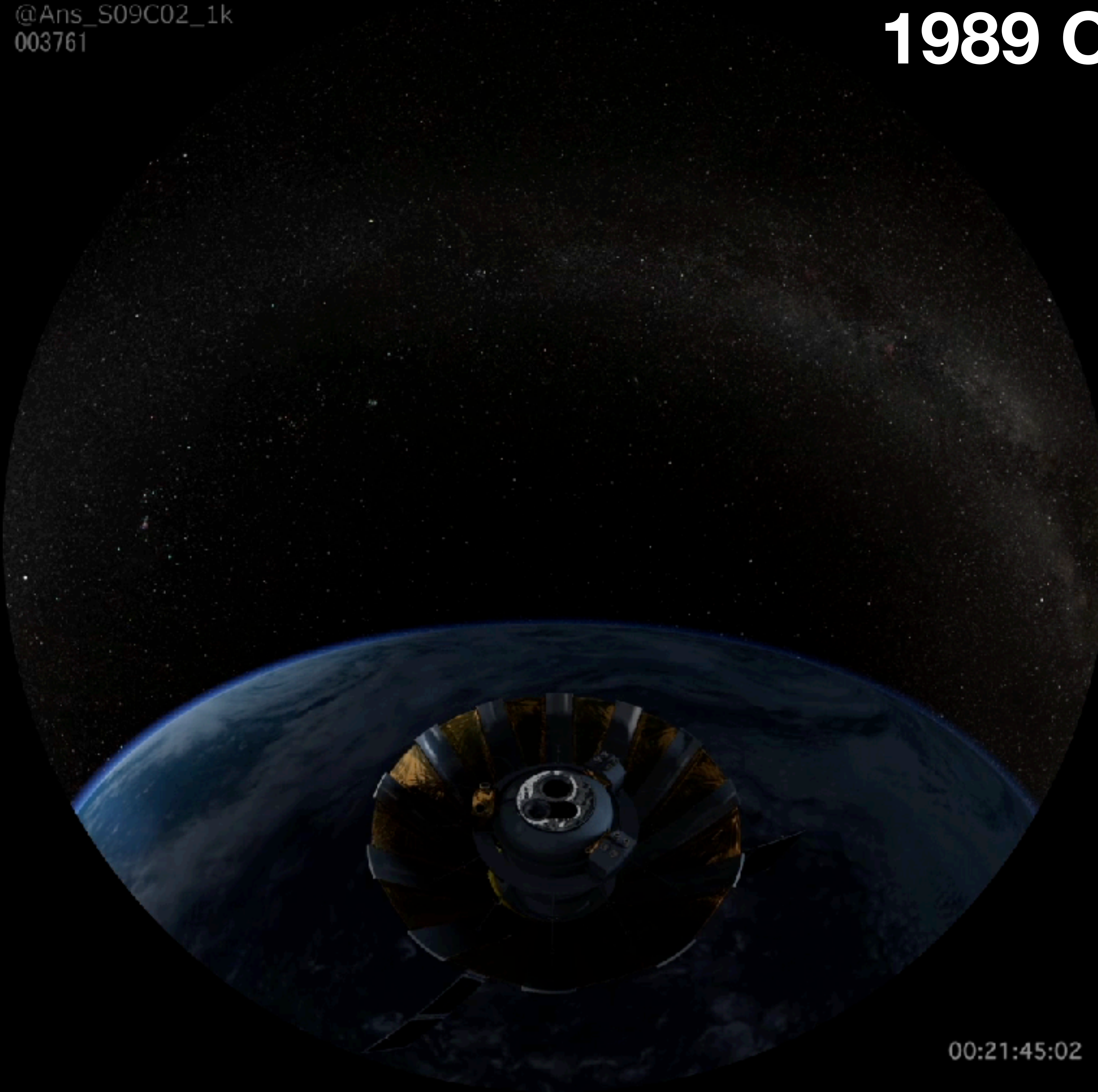
Schreiberaufzeichnung der ersten Messung des Mikrowellenhintergrundes am 20.5.1964

Recording of the first measurement of cosmic microwave background radiation taken on 5/20/1964.

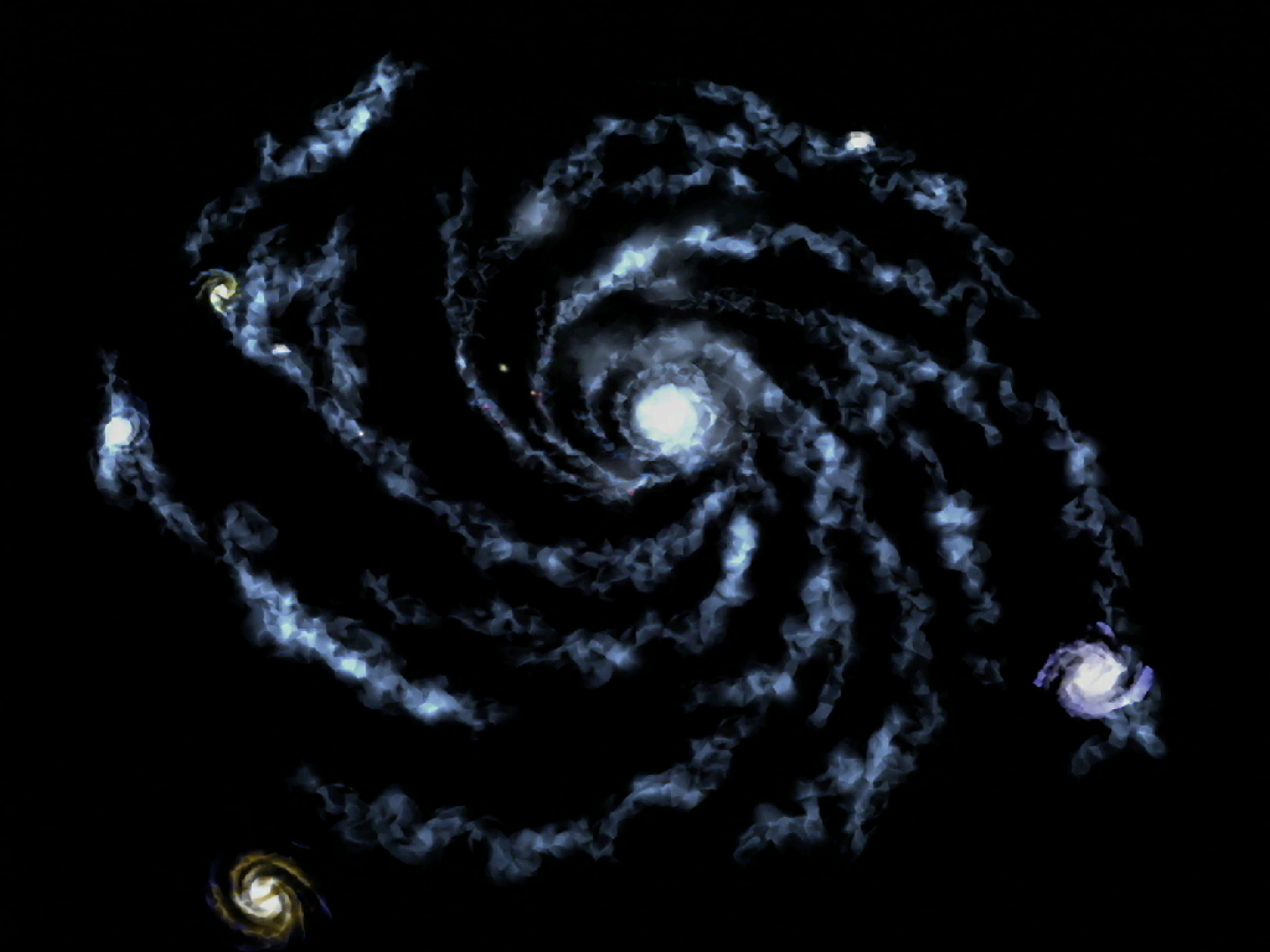


Brightness
10⁻¹⁷
10⁻¹⁸
10⁻¹⁹
10⁻²⁰
10⁻²¹
10⁻²²
4K Planck Spectrum
2.725K Planck Spectrum
2K Planck Spectrum
Rocket (COBRA)
Satellite (COBE/FIRAS)
Rotational Excitation of CN
Ground-based
Balloon-borne
Satellite (COBE/DMR)
Spectrum of CMB
= Planck Spectrum
3m
30cm
3mm
0.3mm
Wavelength

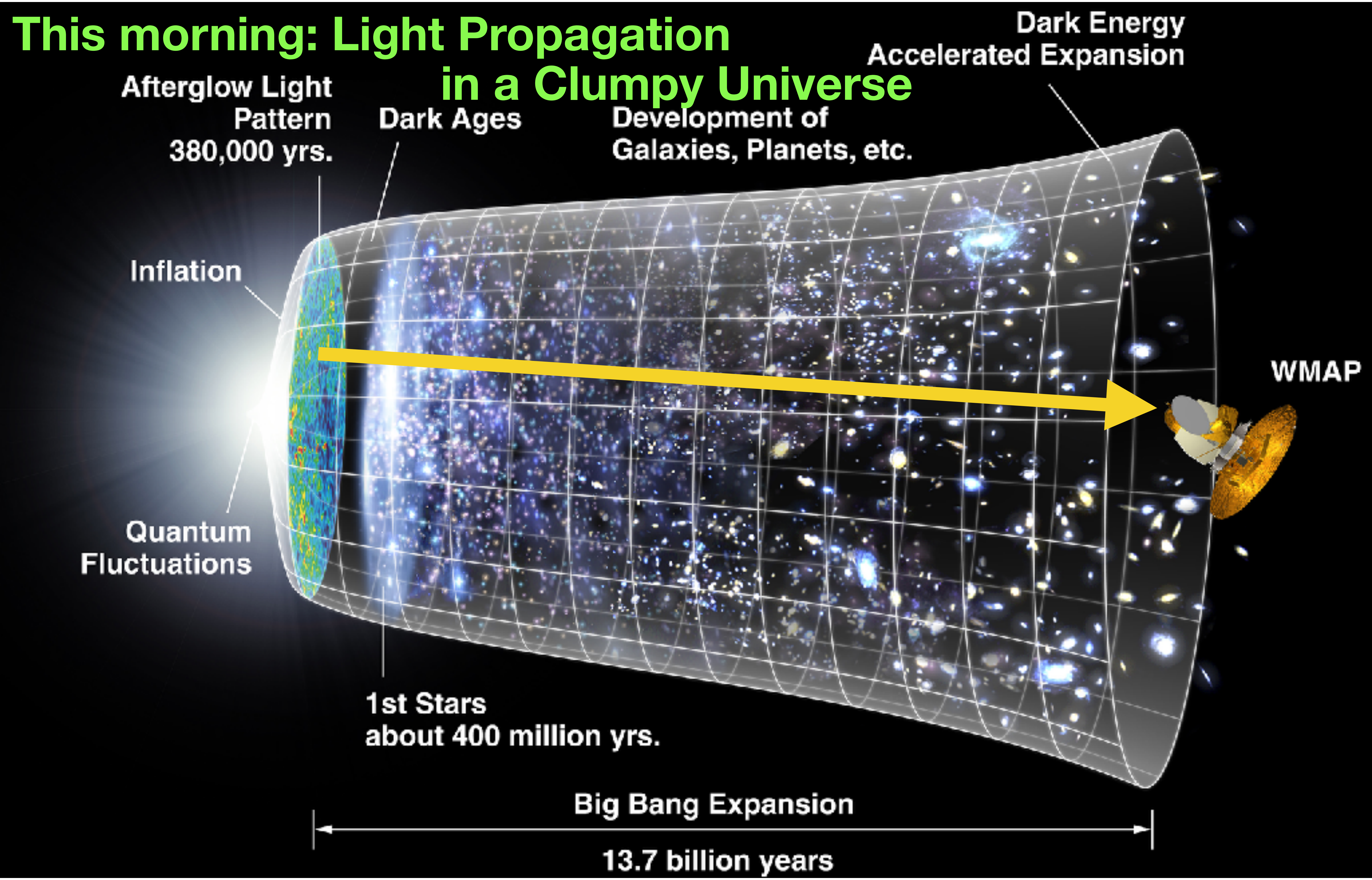
1989 COBE



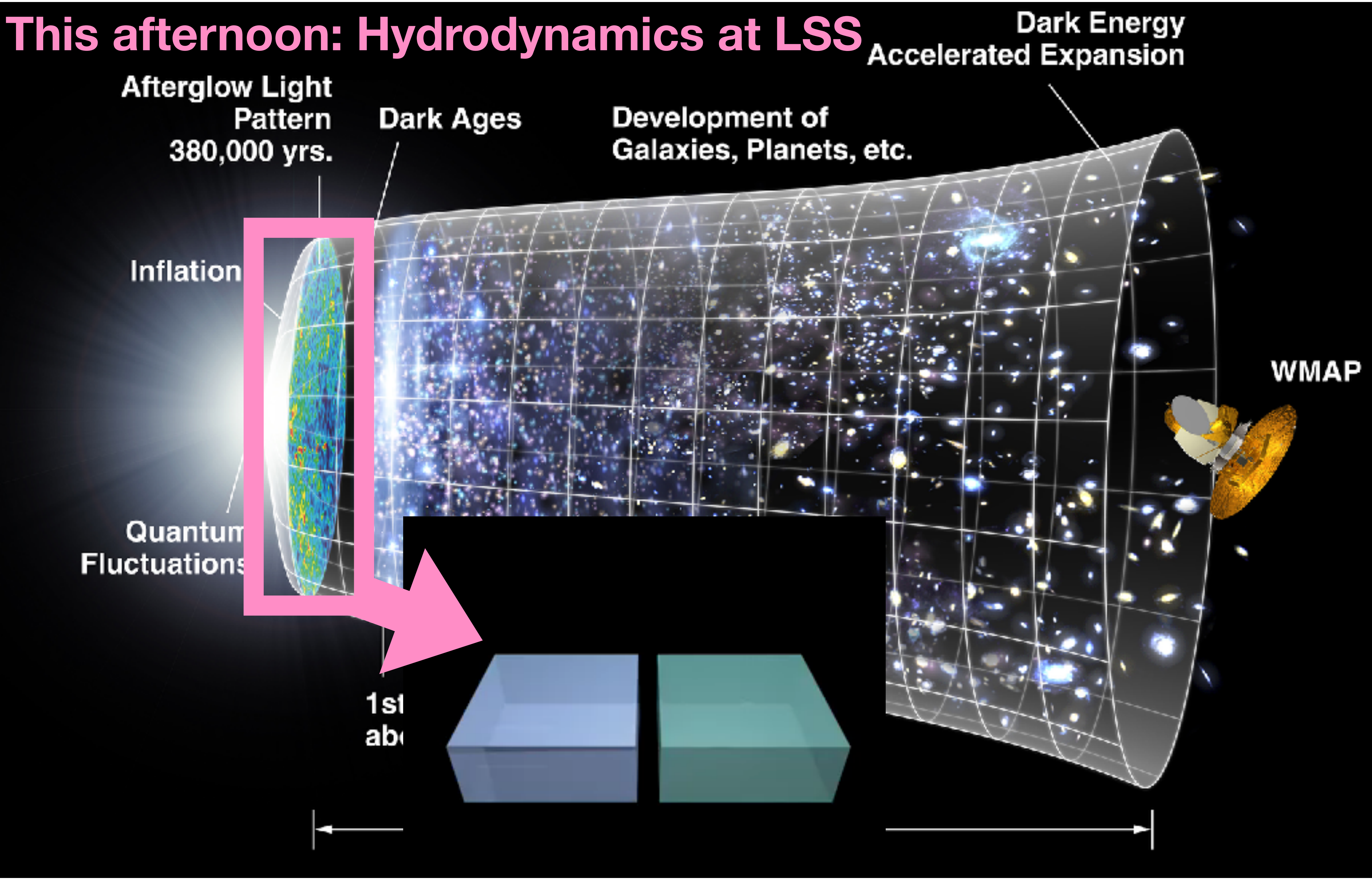
2001 WMAP



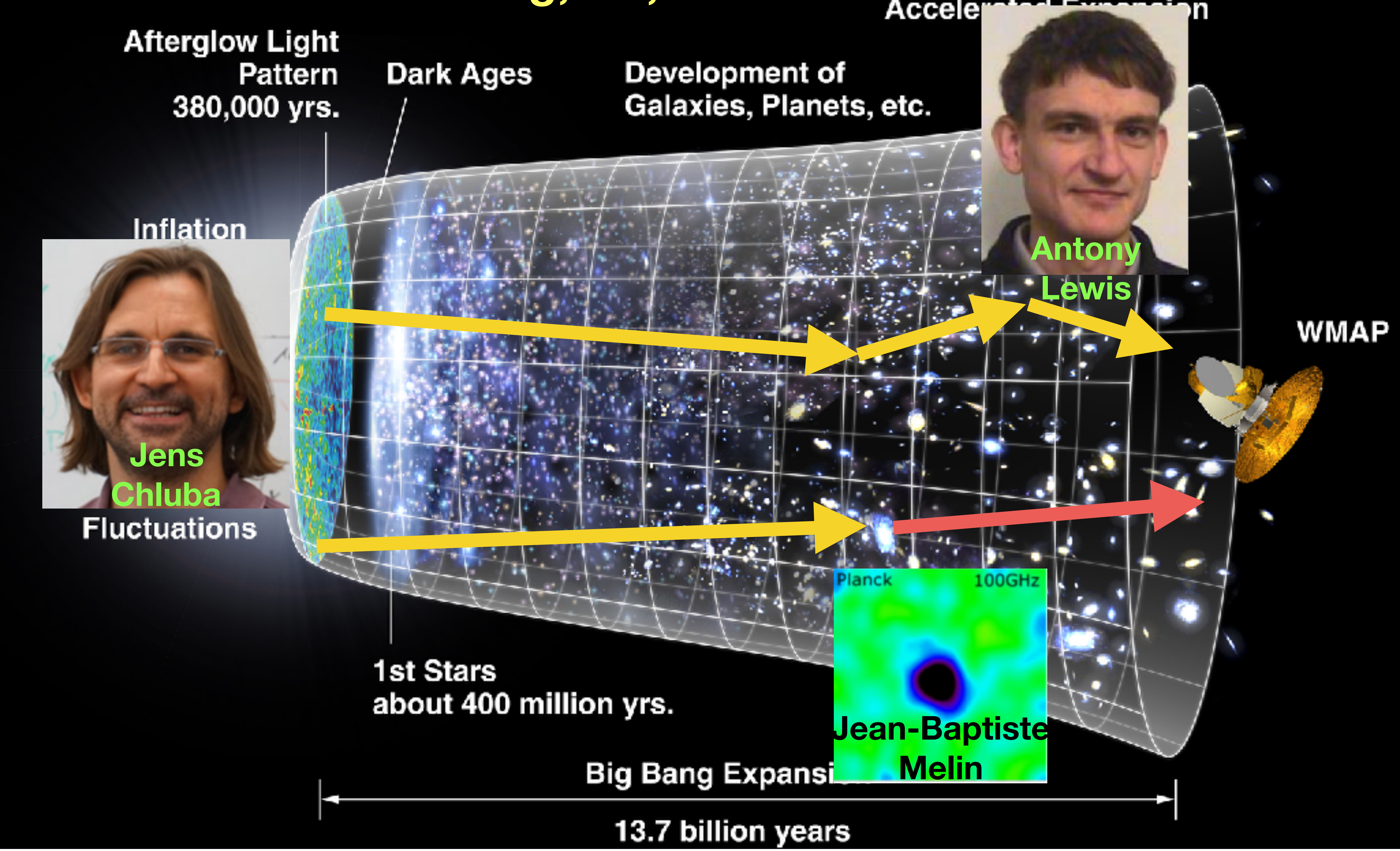
Concept of “Last Scattering Surface”


This morning: Light Propagation
in a Clumpy Universe
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Dark Energy
Accelerated Expansion
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

This afternoon: Hydrodynamics at LSS

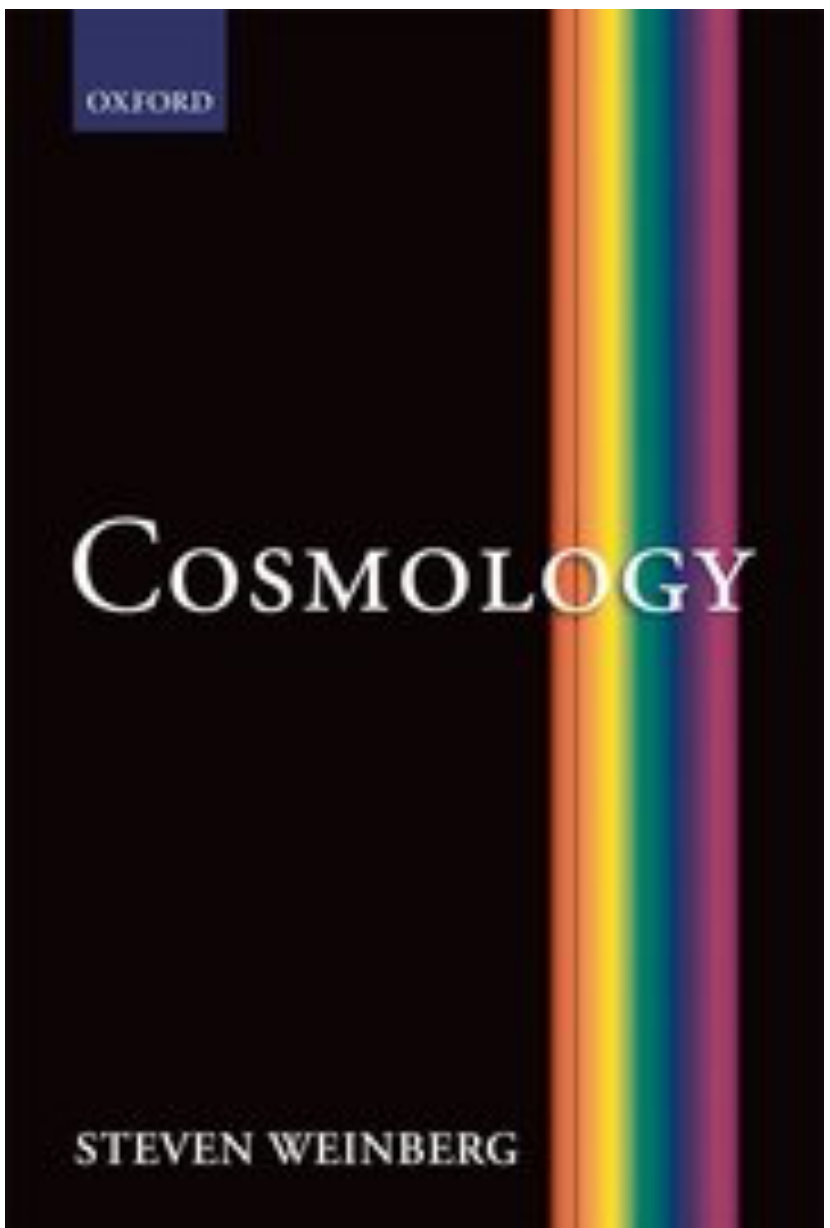


Other lecturers: Lensing, SZ, Recombination



Notation

- Notation in my lectures follows that of the text book "Cosmology" by Steven Weinberg



Cosmological Parameters

- Unless stated otherwise, we shall assume a **spatially-flat Λ Cold Dark Matter** (ΛCDM) model with

$$\Omega_B h^2 = 0.022$$ **[baryon density]**

$$\Omega_M h^2 = 0.14$$ **[total mass density]**

$$\Omega_M = 0.3$$

which implies:

$$\Omega_\Lambda = 0.7, \quad \Omega_D h^2 = 0.118, \quad \Omega_B = 0.04714$$

$$H_0 = 100\ h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}; \quad H_0 = 68.31\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

How light propagates in a clumpy universe?

- Photons gain/lose energy by **gravitational blue/redshifts**

this lecture

- Photons change their directions via **gravitational lensing**

Distance between two points in space

- Static (i.e., non-expanding) Euclidean space
  - In Cartesian coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = dx^2 + dy^2 + dz^2$$

Distance between two points in space

- Homogeneously expanding Euclidean space
  - In Cartesian **comoving** coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = \boxed{a^2(t)}(dx^2 + dy^2 + dz^2)$$

"scale factor"

Distance between two points in space

- Homogeneously expanding Euclidean space
  - In Cartesian **comoving** coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = \boxed{a^2(t)} \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij}\, dx^i dx^j$$

"scale factor"

δ_{ij} **=1 for i=j**

=0 otherwise

Distance between two points in space

- Inhomogeneous curved space
  - In Cartesian **comoving** coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + \boxed{h_{ij}}) dx^i dx^j$$

"**metric perturbation**"

-> CURVED SPACE!

Not just space…

- Einstein told us that a clock ticks slowly when gravity is strong…
- Space-time distance, ds_4, is modified by the presence of gravitational fields

$$ds_4^2 = -\exp(2\Phi)dt^2 + a^2 \exp(-2\Psi) \sum_{i=1}^{3} \sum_{j=1}^{3} [\exp(D)]_{ij} dx^i dx^j$$

Φ : Newton's gravitational potential

Ψ : Spatial scalar curvature perturbation

D_{ij} : Tensor metric perturbation [=gravitational waves]

Tensor perturbation D_{ij}: Area-conserving deformation

- Determinant of a matrix

$$[\exp(D)]_{ij} \equiv \delta_{ij} + D_{ij} + \frac{1}{2}\sum_{k=1}^{3} D_{ik}D_{kj} + \frac{1}{6}\sum_{km} D_{ik}D_{km}D_{mj} + \cdots$$

is given by $\exp(\sum_i D_{ii})$

- **Thus, D_{ij} must be trace-less** $\sum_i D_{ii} = 0$

if it is area-conserving deformation of two points in space

Not just space…

- Einstein told us that a clock ticks slowly when gravity is strong…
- Space-time distance, ds_4, is modified by the presence of gravitational fields

$$ds_4^2 = -\exp(2\Phi)dt^2 + a^2\exp(-2\Psi)\sum_{i=1}^{3}\sum_{j=1}^{3}[\exp(D)]_{ij}dx^i dx^j$$

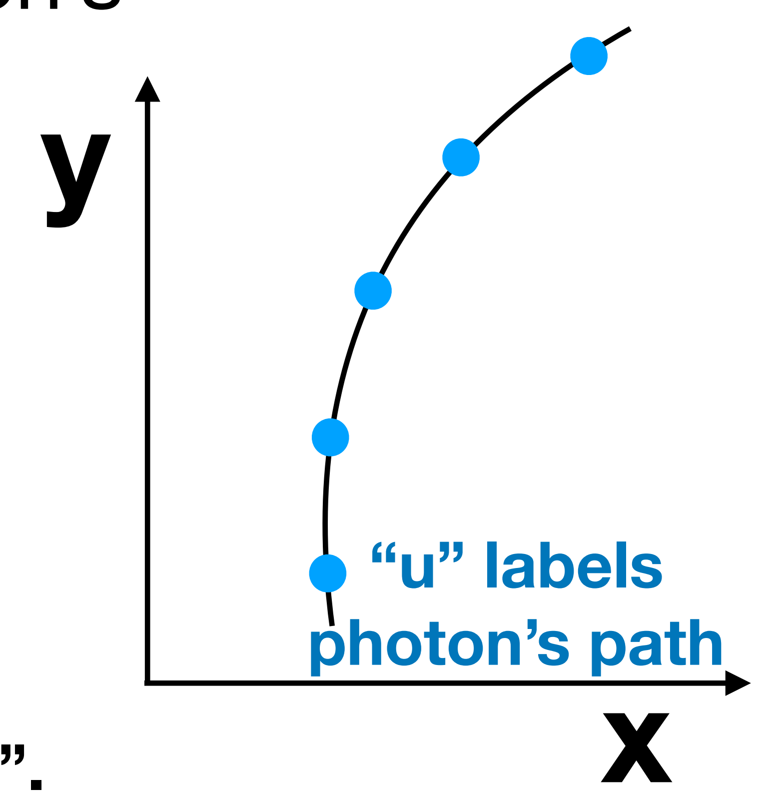
Φ **: Newton's gravitational potential**

Ψ **: Spatial scalar curvature perturbation**
is a perturbation to the determinant of spatial metric

Evolution of photon's coordinates

- Photon's path is determined such that the distance traveled by a photon between two points is minimised. This yields the equation of motion for photon's coordinates $x^\mu = (t, x^i)$

$$\frac{d^2 x^\lambda}{du^2} + \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{du} \frac{dx^\nu}{du} = 0$$

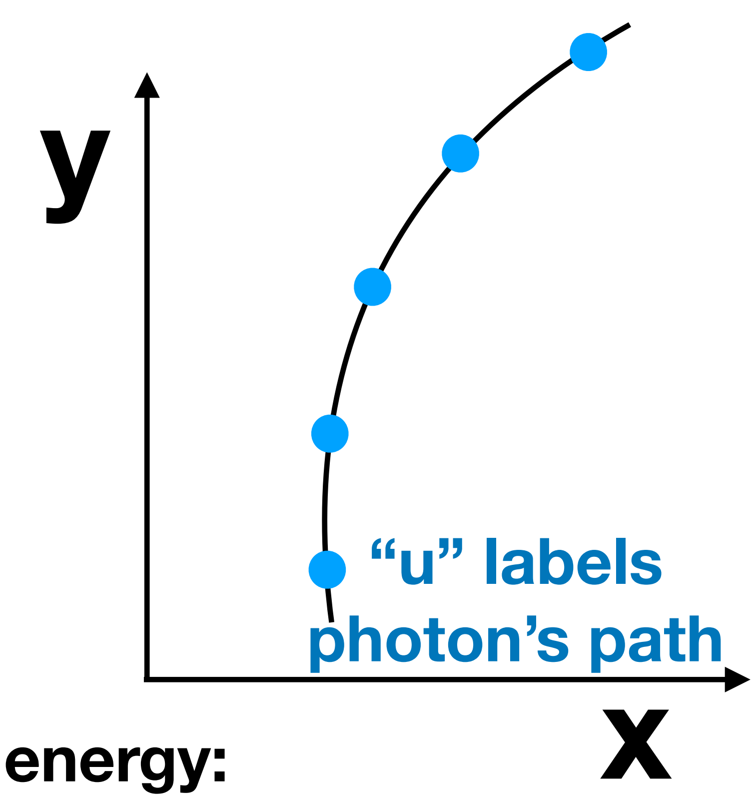


This equation is known as the "geodesic equation".
The second term is needed to keep the form of the equation unchanged under general coordinate transformation => GRAVITATIONAL EFFECTS!

Evolution of photon's momentum

- It is more convenient to write down the geodesic equation in terms of the **photon momentum**:

$$p^\mu \equiv \frac{dx^\mu}{du}$$



then

$$\frac{dp^\lambda}{dt} + \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \Gamma^\lambda_{\mu\nu}\frac{p^\mu p^\nu}{p^0} = 0$$

Magnitude of the photon momentum is equal to the photon energy:

$$p^2 \equiv \sum_{i=1}^{3}\sum_{j=1}^{3} g_{ij}p^i p^j$$

Some calculations…

$$\frac{dp^\lambda}{dt} + \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \Gamma^\lambda_{\mu\nu} \frac{p^\mu p^\nu}{p^0} = 0$$

With $ds_4^2 = \sum_{\mu\nu} g_{\mu\nu} dx^\mu dx^\nu$ $\left(\begin{array}{l} g_{00} = -\exp(2\Phi), \quad g_{0i} = 0, \\ g_{ij} = a^2 \exp(-2\Psi)[\exp(D)]_{ij} \end{array} \right)$

$$\Gamma^\lambda_{\mu\nu} \equiv \frac{1}{2} \sum_{\rho=0}^{3} g^{\lambda\rho} \left(\frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right)$$

Scalar perturbation [valid to all orders]

$$\Gamma^0_{00} = \dot\Phi, \quad \Gamma^0_{0i} = \frac{\partial\Phi}{\partial x^i}, \quad \Gamma^i_{00} = \exp(2\Phi) \sum_j g^{ij} \frac{\partial\Phi}{\partial x^j},$$

$$\Gamma^i_{0j} = \left(\frac{\dot a}{a} - \dot\Psi\right)\delta^i_j, \quad \Gamma^0_{ij} = \exp(-2\Phi)\left(\frac{\dot a}{a} - \dot\Psi\right) g_{ij},$$

$$\Gamma^k_{ij} = \delta_{ij} \sum_\ell \delta^{k\ell} \frac{\partial\Psi}{\partial x^\ell} - \delta^k_i \frac{\partial\Psi}{\partial x^j} - \delta^k_j \frac{\partial\Psi}{\partial x^i},$$

Tensor perturbation [valid to 1st order in D]

$$\Gamma^i_{0j} = \frac{\dot a}{a}\delta^i_j + \frac{1}{2}\sum_k \delta^{ik}\dot D_{kj}, \quad \Gamma^0_{ij} = \frac{\dot a}{a} g_{ij} + \frac{a^2}{2}\dot D_{ij},$$

$$\Gamma^k_{ij} = \frac{1}{2}\sum_\ell \delta^{k\ell}\left(\frac{D_{i\ell}}{\partial x^j} + \frac{D_{\ell j}}{\partial x^i} - \frac{D_{ij}}{\partial x^\ell}\right),$$

Recap

Math may be messy but the concept is transparent!

- Requiring **photons to travel between two points in space-time with the minimum path length**, we obtained the geodesic equation

- The geodesic equation contains $\Gamma^{\lambda}_{\mu\nu}$ that is required to make **the form of the equation unchanged under general coordinate transformation**

- Expressing $\Gamma^{\lambda}_{\mu\nu}$ in terms of the metric perturbations, we obtain the desired result - **the equation that describes the rate of change of the photon energy!**

$$p^2 \equiv \sum_{i=1}^{3}\sum_{j=1}^{3} g_{ij} p^i p^j$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

γ^i is a unit vector of the direction of photon's momentum:

$$\sum_i (\gamma^i)^2 = 1$$

- Let's interpret this equation *physically*

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = \boxed{-\frac{\dot{a}}{a}} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

γ^i is a unit vector of the direction of photon's momentum:

$$\sum_i (\gamma^i)^2 = 1$$

- **Cosmological redshift**
  - Photon's wavelength is stretched in proportion to the scale factor, and thus the photon energy decreases as

$$p \propto a^{-1}$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} \boxed{+\dot{\Psi}} - \frac{1}{a}\sum_i \frac{\partial\Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Cosmological redshift - part II**
  - The spatial metric is given by $ds^2 = a^2(t)\exp(-2\Psi)d\mathbf{x}^2$
  - Thus, locally we can define a new scale factor:
    $\tilde{a}(t,\mathbf{x}) = a(t)\exp(-\Psi)$
  - Then the photon momentum decreases as

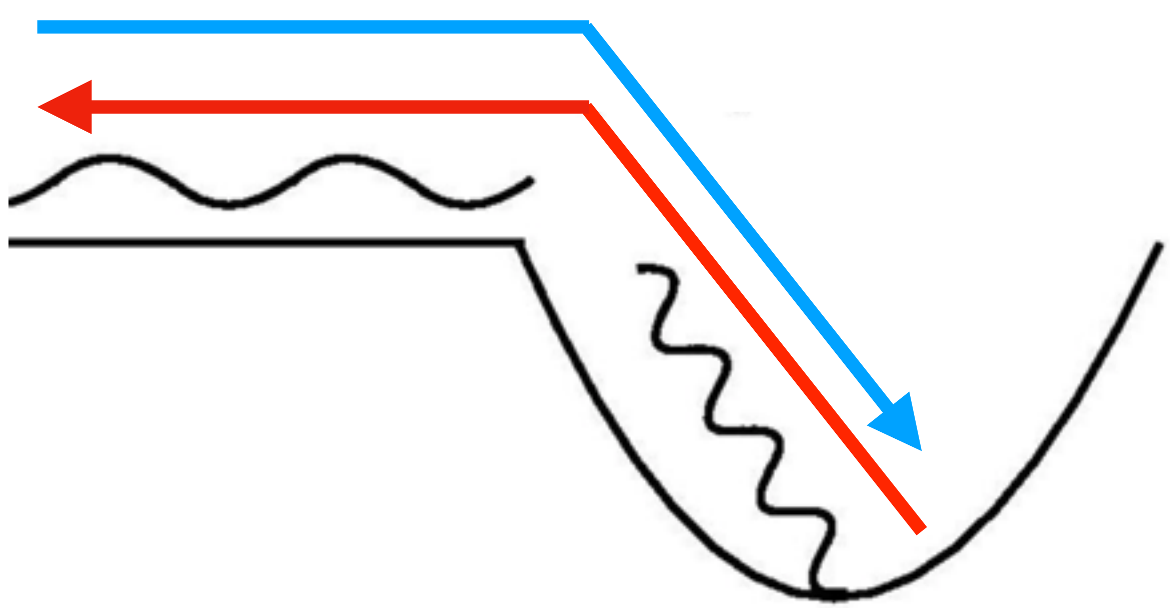
$$p \propto \tilde{a}^{-1}$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Gravitational blue/redshift (Scalar)**



Potential well (ϕ < 0)

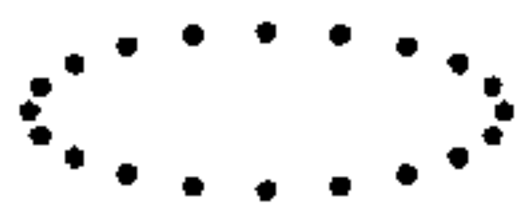
Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i \boxed{-\frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j}$$

- **Gravitational blue/redshift (Tensor)**

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Sachs & Wolfe (1967)

The Result

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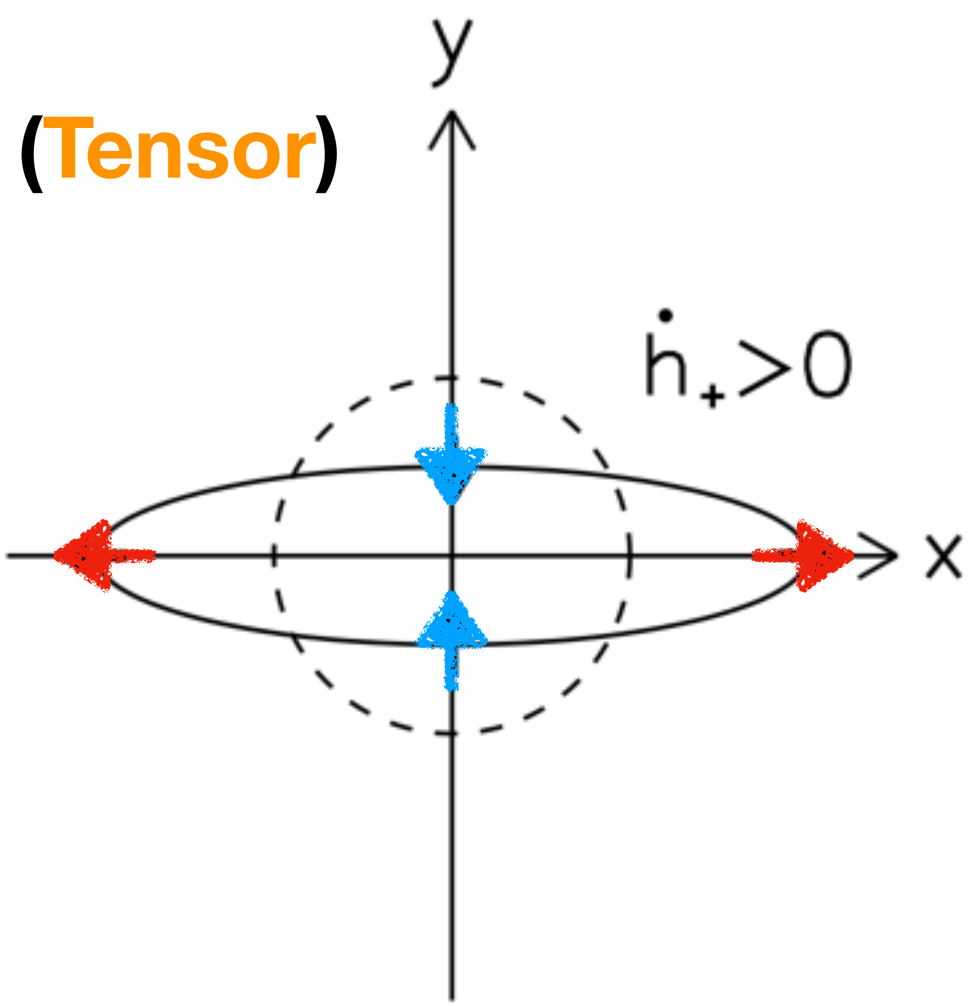


Sachs & Wolfe (1967)

Formal Solution (Scalar)

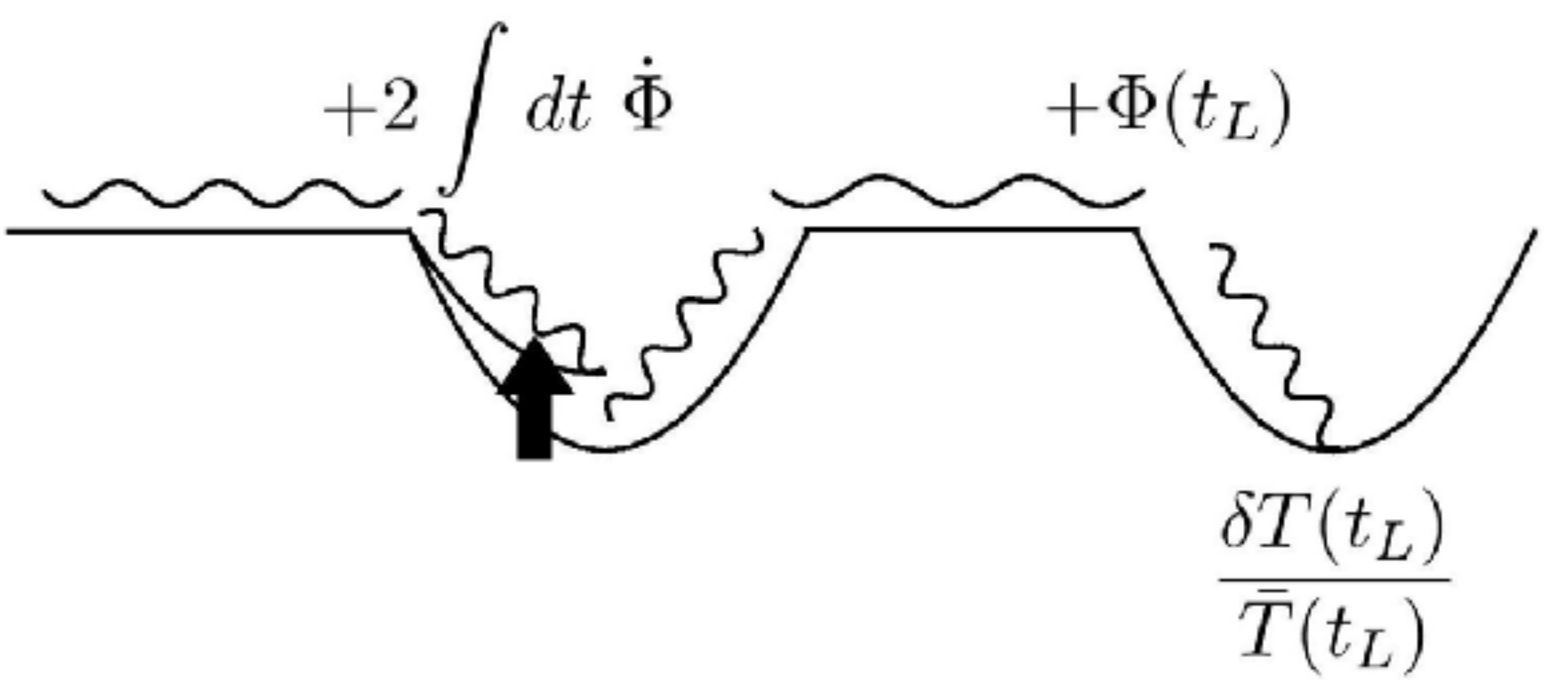
"L" for "Last scattering surface"

$$\ln(ap)(t_0) = \ln(ap)(t_L) + \Phi(t_L) - \Phi(t_0) + \int_{t_L}^{t_0} dt\ (\dot{\Phi} + \dot{\Psi})$$

or

$$\frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i = \frac{d\Phi}{dt} - \dot{\Phi}$$

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0) + \int_{t_L}^{t_0} dt\ (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Coming distance (r)

$$x^i = \hat{n}^i r$$

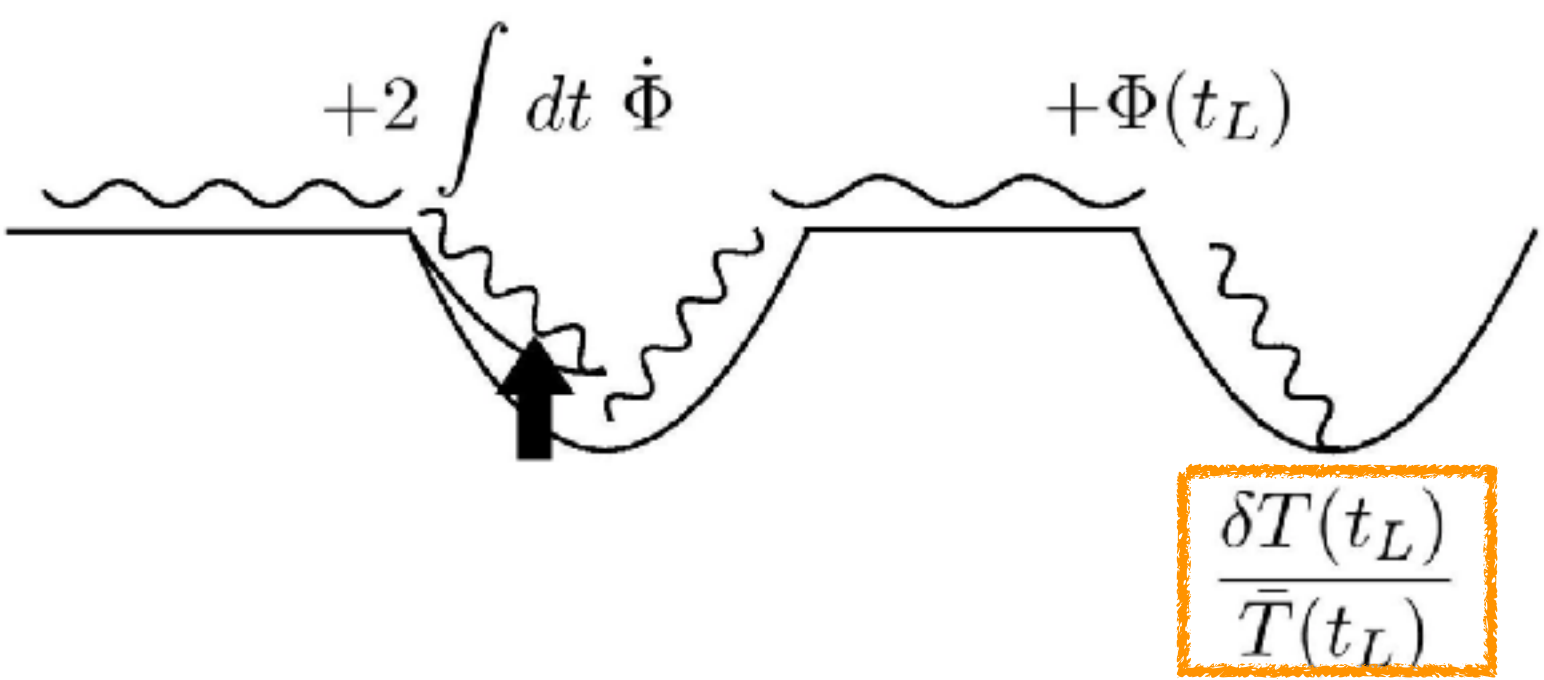
$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

Sachs & Wolfe (1967)

Formal Solution (Scalar)

Initial Condition

$$\frac{\Delta T(\hat{n})}{T_0} = \boxed{\frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)}} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0) + \int_{t_L}^{t_0} dt \, (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



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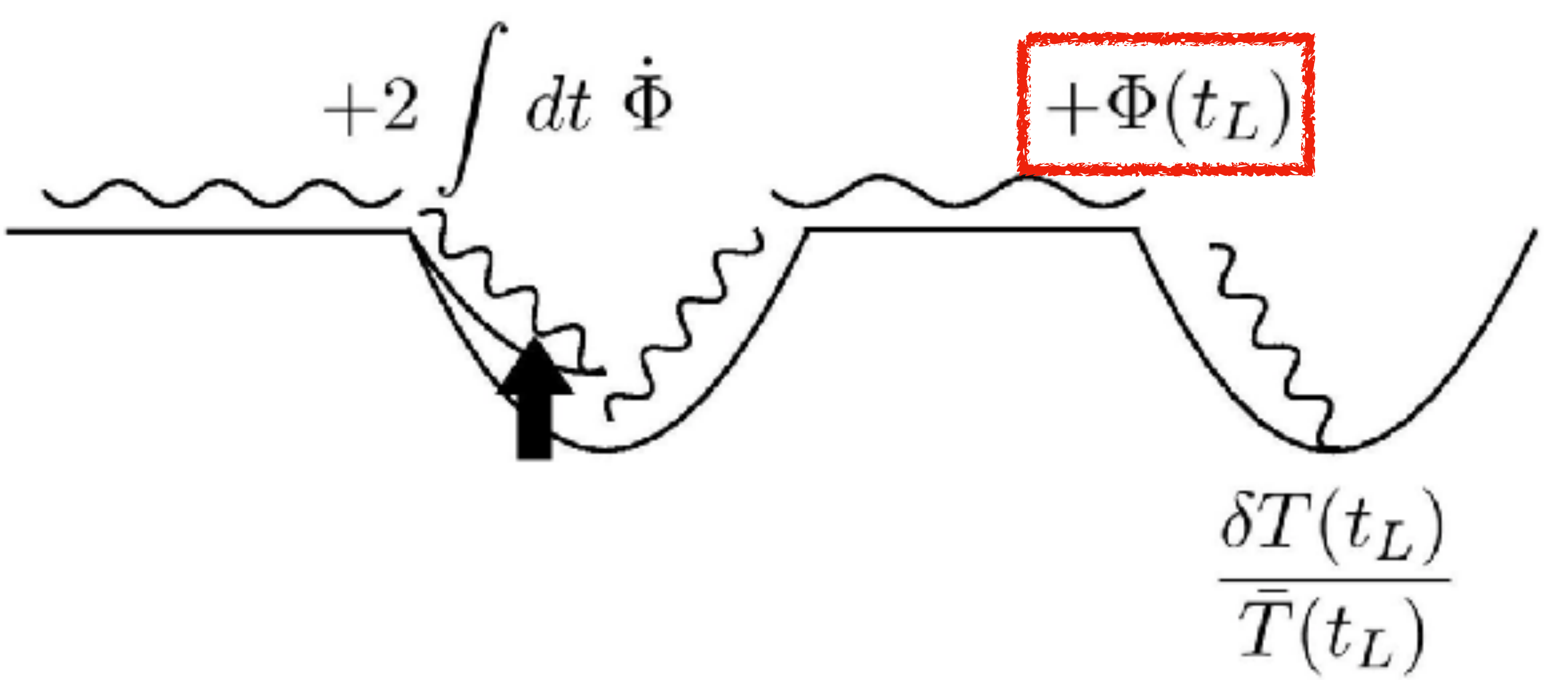
Sachs & Wolfe (1967)

Formal Solution (Scalar)

Gravitational Redshit

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0)$$

$$+ \int_{t_L}^{t_0} dt \; (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Comoving distance (r)

$$x^i = \hat{n}^i r$$

$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

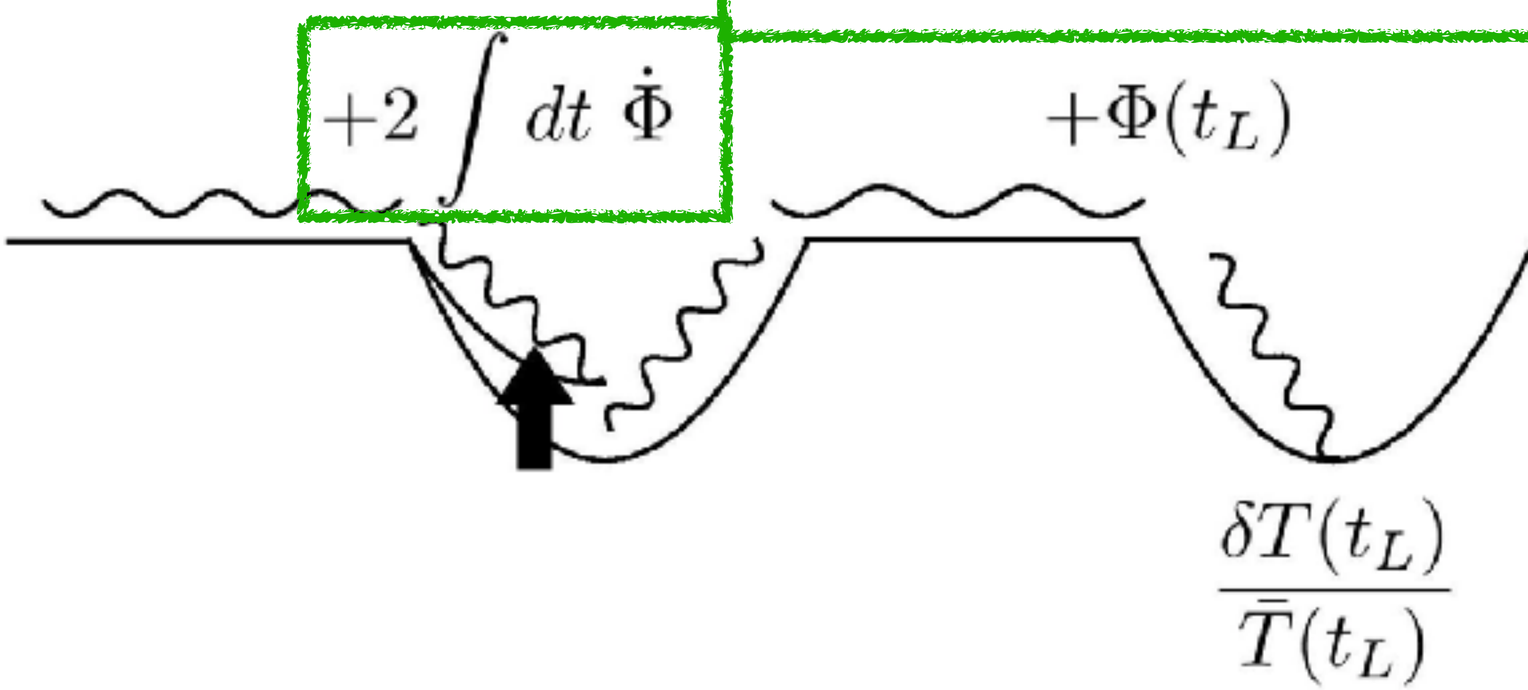
Sachs & Wolfe (1967)

Formal Solution (Scalar)

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0)$$

“integrated Sachs-Wolfe” (ISW) effect

$$+ \int_{t_L}^{t_0} dt \; (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

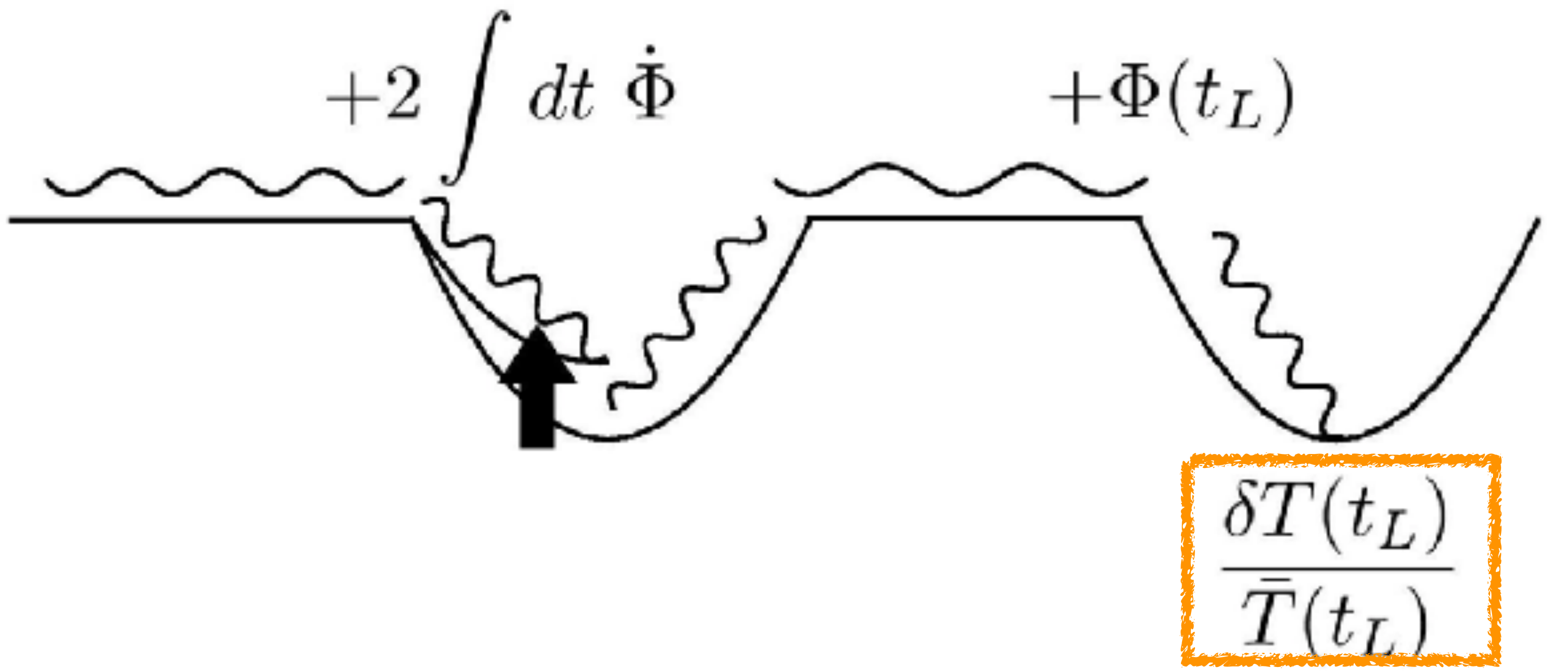
$$\hat{n}^i = -\gamma^i$$

Coming distance (r)

$$x^i = \hat{n}^i r$$

$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

Initial Condition



- "*Were photons hot or cold at the bottom of the potential well at the last scattering surface?"*
- This must be assumed a priori - **only the data can tell us!**

“Adiabatic” Initial Condition

- <u>Definition</u>: “*Ratios of the number densities of all species are equal everywhere initially*”
  - For i^{th} and j^{th} species, **$n_i(x)/n_j(x)$ = constant**
- For a quantity X(t,x), let us define the **fluctuation, *δX*,** as

$$\delta X(t,\boldsymbol{x}) \equiv X(t,\boldsymbol{x}) - \bar{X}(t)$$

- Then, the adiabatic initial condition is

$$\frac{\delta n_i(t_{\rm initial},\mathbf{x})}{\bar{n}_i(t_{\rm initial})} = \frac{\delta n_j(t_{\rm initial},\mathbf{x})}{\bar{n}_j(t_{\rm initial})}$$

Example: Thermal Equilibrium

- When photons and baryons were in thermal equilibrium in the past, then
  - $n_{photon} \sim T^3$ and $n_{baryon} \sim T^3$
  - That is to say, **thermal equilibrium naturally gives the adiabatic initial condition**
  - This gives

$$3\frac{\delta T(t_i, \boldsymbol{x})}{\bar{T}(t_i)} = \frac{\delta\rho_B(t_i, \boldsymbol{x})}{\bar{\rho}_B(t_i)}$$

- **“B” for “Baryons”**
- **ρ is the mass density**

Big Question

- ***How about dark matter?***

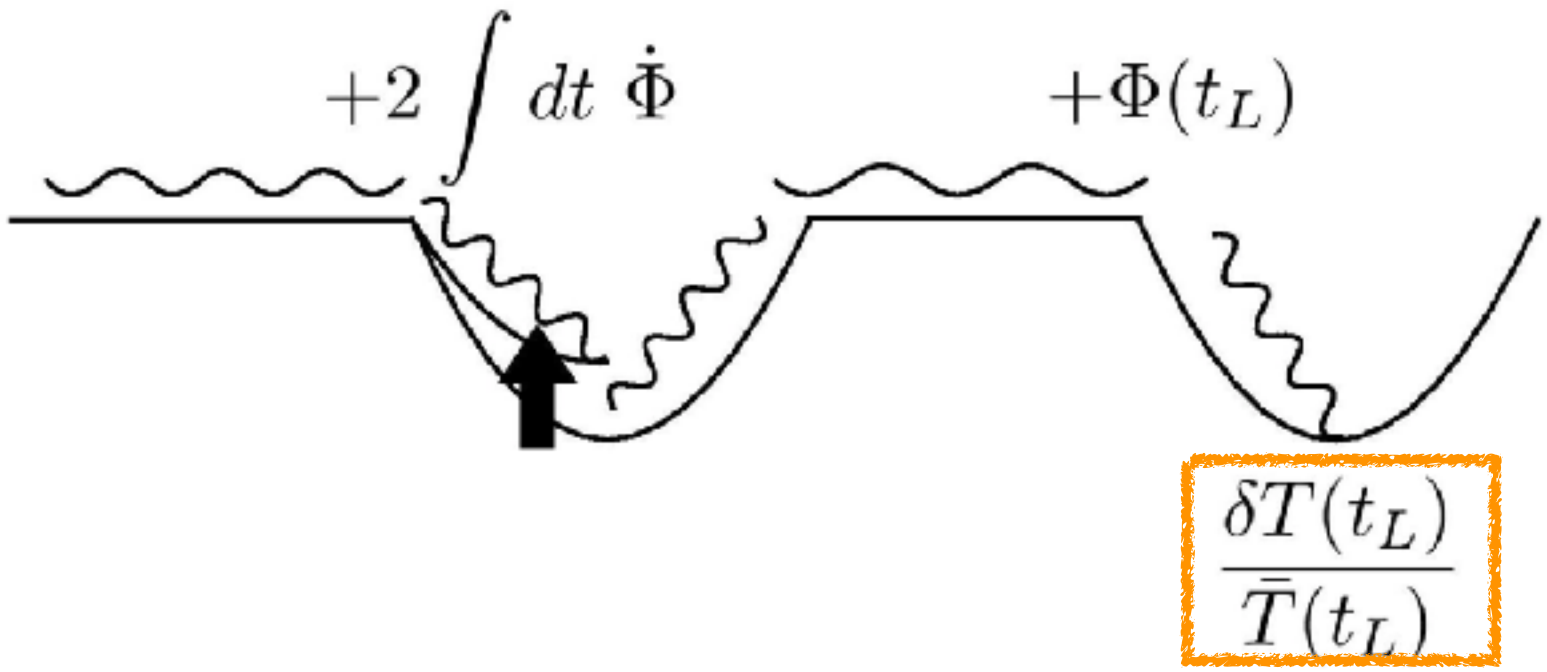
- If dark matter and photons were in thermal equilibrium in the past, then they should also obey the adiabatic initial condition

  - If not, *there is no a priori reason to expect the adiabatic initial condition!*

- The current data are consistent with the adiabatic initial condition. This means something important for the nature of dark matter!

We shall assume the adiabatic initial condition throughout the lectures

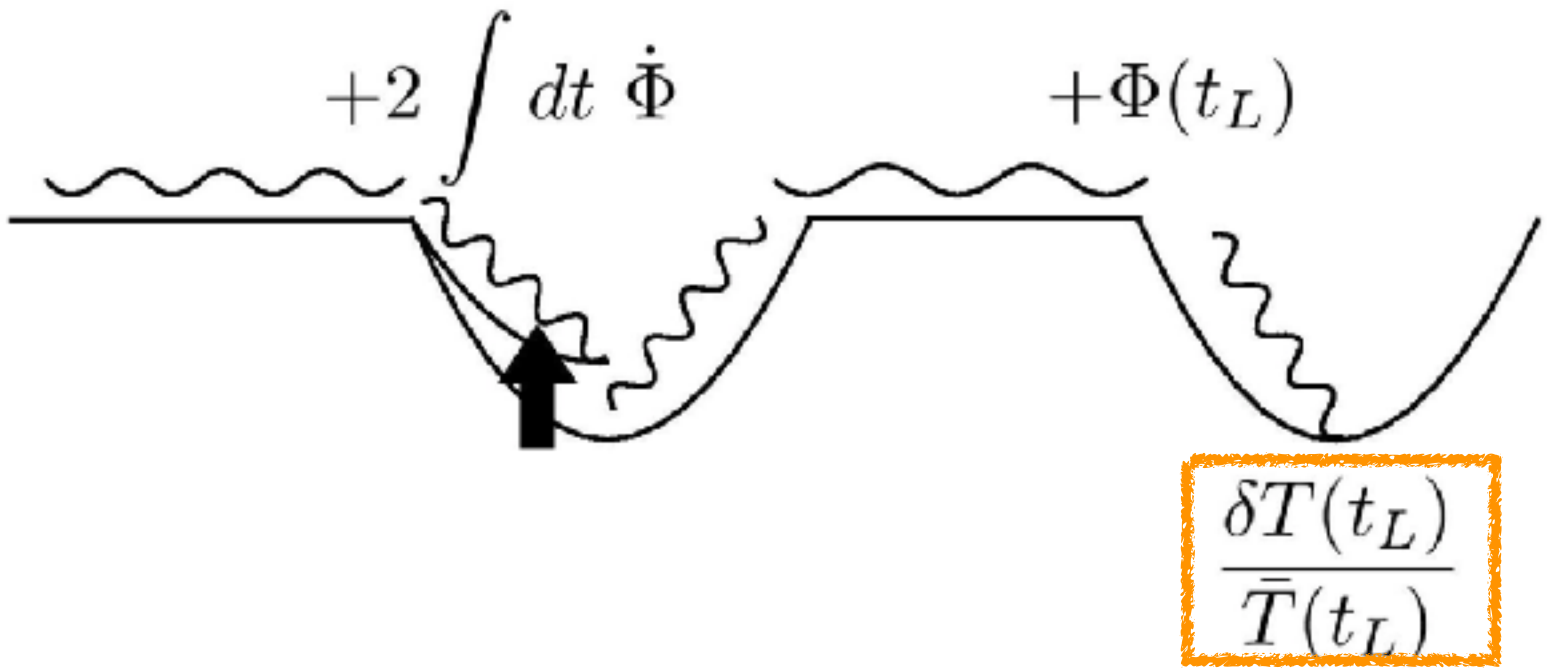
Adiabatic Solution



- At the last scattering surface, the temperature fluctuation is given by the matter density fluctuation as

$$\frac{\delta T(t_L,\mathbf{x})}{\bar{T}(t_L)} = \frac{1}{3}\frac{\delta\rho_M(t_L,\mathbf{x})}{\bar{\rho}_M(t_L)}$$

Adiabatic Solution

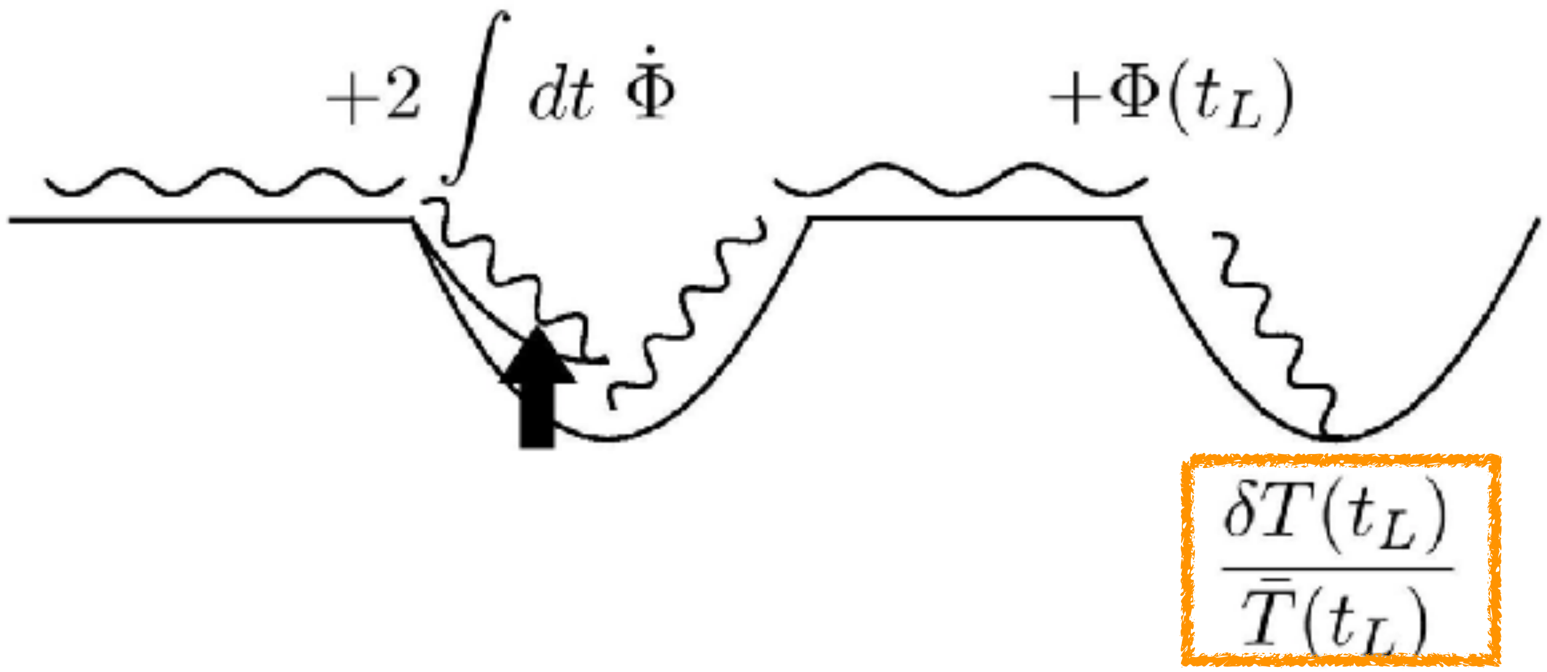


- On large scales, the matter density fluctuation during the matter-dominated era is given by $\delta\rho_M/\bar{\rho}_M = -2\Phi$; thus,

$$\frac{\delta T(t_L, \mathbf{x})}{\bar{T}(t_L)} = \frac{1}{3}\frac{\delta\rho_M(t_L, \mathbf{x})}{\bar{\rho}_M(t_L)} = -\frac{2}{3}\Phi(t_L, \mathbf{x})$$

***Hot* at the bottom of the potential well, but…**

Over-density = Cold spot



- Therefore: $$\frac{\Delta T(\hat{n})}{T_0} = \frac{1}{3}\Phi(t_L, \hat{r}_L)$$

This is negative in an over-density region!