Lecture notes:
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 3: Sourced Contribution

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March 19, 2020

We continue to use D_{ij} for the gravitation wave

$$ds_4^2 = -\exp(2\Phi)dt^2 + a^2\exp(-2\Psi)\sum_{i=1}^{3}\sum_{j=1}^{3}[\exp(D)]_{ij}dx^i dx^j$$

Φ **: Newton's gravitational potential**

Ψ **: Spatial scalar curvature perturbation**

D_{ij} **: Tensor metric perturbation [=gravitational waves]**

$$[\exp(D)]_{ij} \equiv \delta_{ij} + D_{ij} + \frac{1}{2}\sum_{k=1}^{3} D_{ik}D_{kj} + \frac{1}{6}\sum_{km} D_{ik}D_{km}D_{mj} + \cdots$$

Are GWs from vacuum fluctuation in spacetime, or from sources?

$$\Box D_{ij} = -16\pi G \pi_{ij}^{GW}$$

$$T_{ij} = a^2 \pi_{ij}$$

- **Homogeneous solution**: “GWs from the vacuum fluctuation”
  - We covered this on Day 1
- **Inhomogeneous solution**: “GWs from sources”
  - Topic of today’s lecture

Which sources?

- Scalar, vector, tensor decomposition
  - When the unperturbed space is homogeneous and isotropic, we can classify perturbations based on **how they transform under spatial rotation:**
  - Spin 0: Scalar
  - Spin 1: Vector
  - Spin 2: Tensor

$$x^i \to x^{i\prime} = \sum_{j=1}^{3} R^i_j x^j$$

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$$x^i \to x^{i\prime} = \sum_{j=1}^{3} R^i_j x^j$$

  - **Spin 0: Scalar**

$$f(\mathbf{x}) \to \tilde{f}(\mathbf{x}') = f(\mathbf{x})$$

  - Spin 1: Vector
  - Spin 2: Tensor

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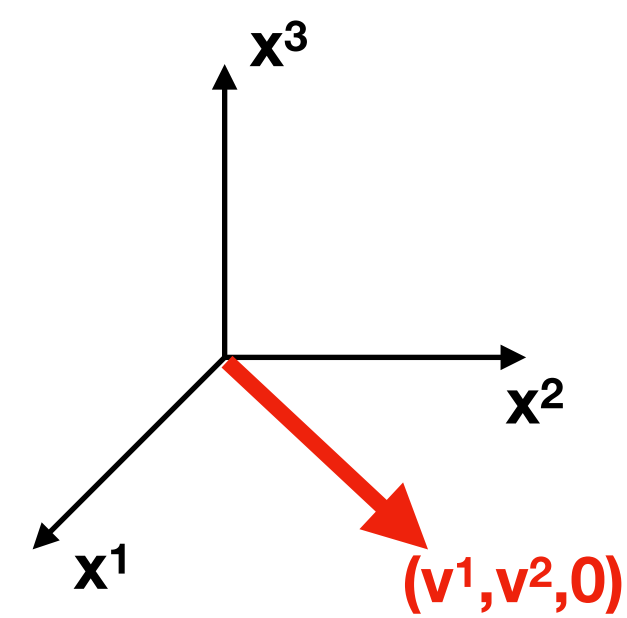

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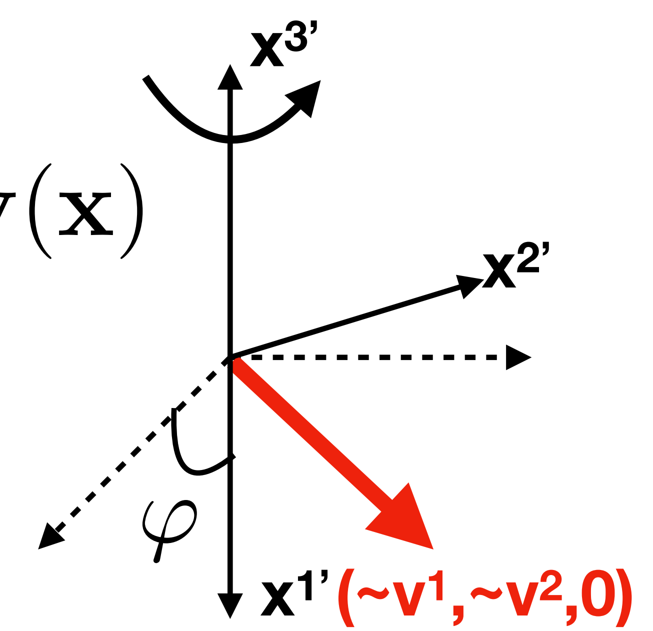
  - Spin 0: Scalar
  - **Spin 1: Vector**

$$\mathbf{v}(\mathbf{x}) \to \tilde{\mathbf{v}}(\mathbf{x}') = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{v}(\mathbf{x})$$

  - Spin 2: Tensor



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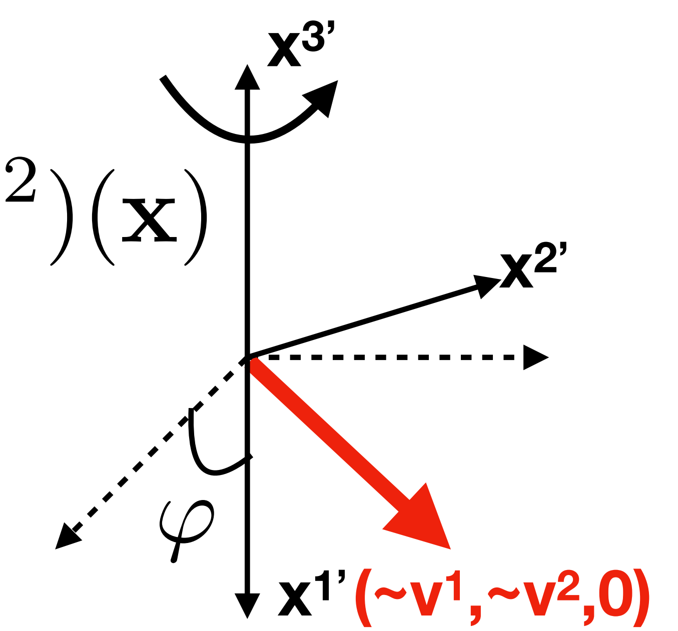
$$x^i \to x^{i\prime} = \sum_{j=1}^{3} R^i_j x^j$$

  - Spin 0: Scalar
  - **Spin 1: Vector**

spin 1

$$(v^1 \pm i v^2)(\mathbf{x}) \to (\tilde{v}^1 \pm i\tilde{v}^2)(\mathbf{x}') = e^{\mp i\varphi}(v^1 \pm i v^2)(\mathbf{x})$$

  - Spin 2: Tensor



Which sources?

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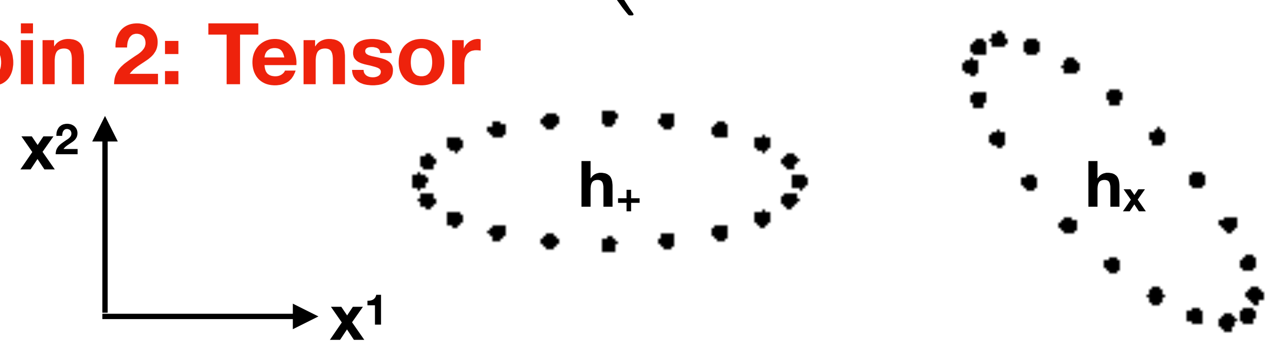
  - When the unperturbed space is homogeneous and isotropic, we can classify perturbations based on **how they transform under spatial rotation:**

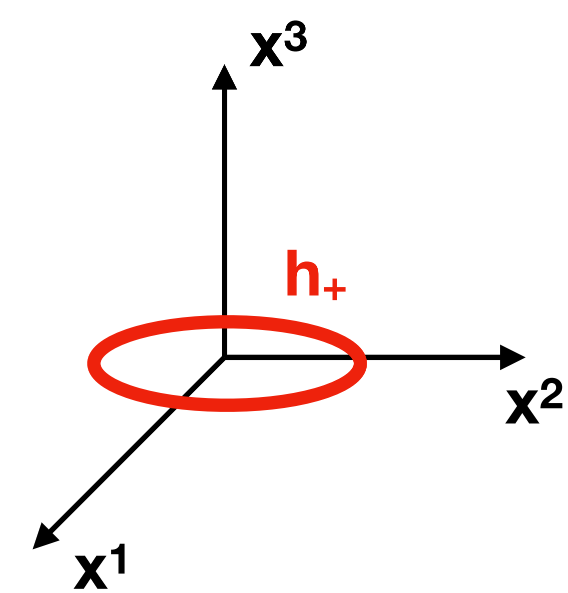
$$x^i \to x^{i\prime} = \sum_{j=1}^{3} R^i_j x^j$$

  - Spin 0: Scalar

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

  - **Spin 2: Tensor**





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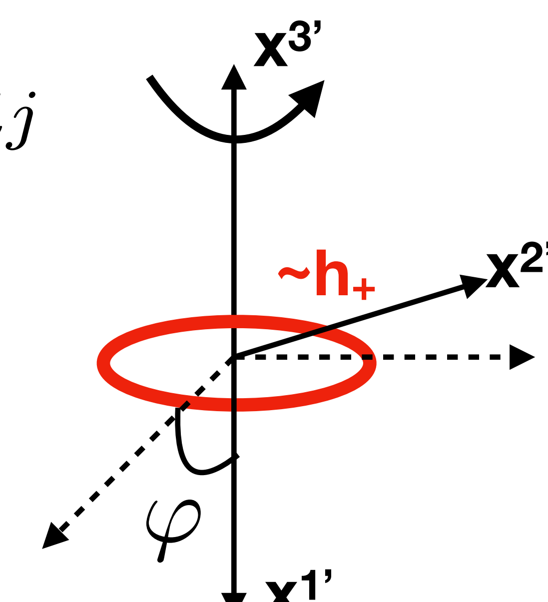
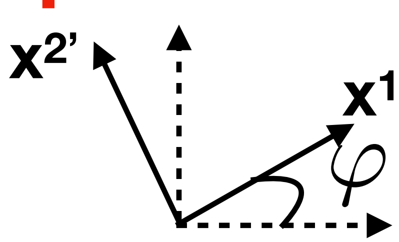
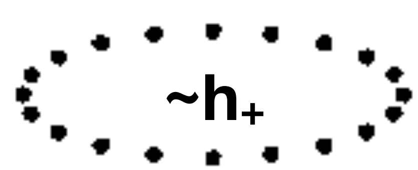
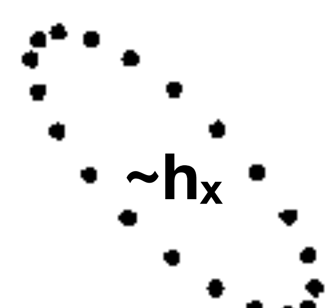
$$D_{ij} \to \tilde{D}_{ij} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi & 0 \\ -\sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} D_{ij}$$



  - **Spin 2: Tensor**







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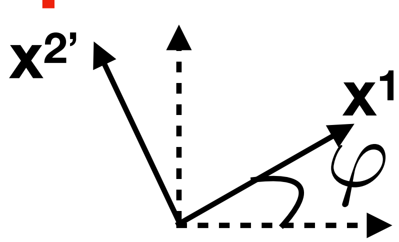
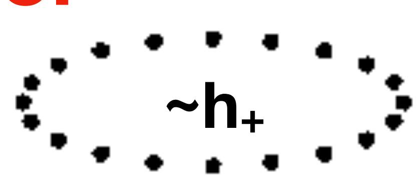
$$(h_+ \pm i h_\times)(\mathbf{x}) \to (\tilde{h}_+ \pm i\tilde{h}_\times)(\mathbf{x}')$$

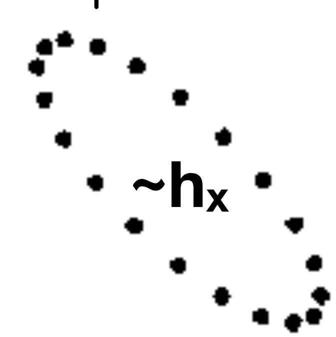
$$= e^{\mp 2i\varphi}(h_+ \pm i h_\times)(\mathbf{x})$$

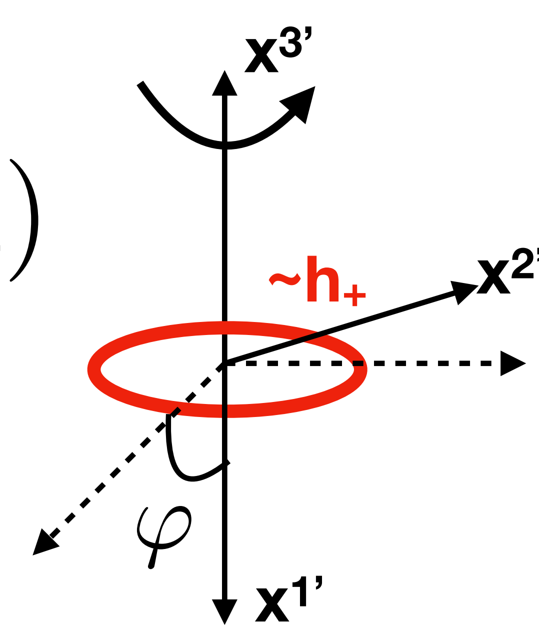
spin 2

  - **Spin 2: Tensor**









Vector and Tensor Modes

2 degrees of freedom

- <u>Recap:</u>
  - **Vector**: Transverse

$$\sum_{i=1}^{3} \partial_i v^i = 0 \rightarrow \sum_{i=1}^{3} k^i v^i = 0$$

  - **Tensor**: Transverse and traceless

$$\sum_{i=1}^{3} \partial_i D_{ij} = 0 \rightarrow \sum_{i=1}^{3} k^i D_{ij} = 0,$$

$$\sum_{i=1}^{3} D_{ii} = 0$$

2 degrees of freedom

Lifshitz (1946); Bardeen (1980); Kodama & Sasaki (1984)

Scalar-Vector-Tensor Decomposition Theorem

- At linear order, scalar, vector, and tensor components are **decoupled** (different spins do not mix at linear order)
- That is to say, **tensor modes cannot be sourced by scalar or vector modes at linear order** (and vice versa)
    - Scalars and vectors can source tensor modes at non-linear order (e.g., second order)

EoM of GW with source

By this, we mean transverse and traceless

$$a^2\Box D_{ij} = -16\pi G T_{ij}^{GW}$$

$$\Box \equiv \frac{1}{\sqrt{-g}}\sum_{\mu=0}^{3}\sum_{\nu=0}^{3}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right)$$

$$g^{00} = -1, \quad g^{0i} = 0, \quad g^{ij} = a^{-2}(t)(\delta^{ij} - D^{ij}),$$

$$g_{ij} = a^2(t)(\delta_{ij} + D_{ij}), \quad \sqrt{-g} = a^3(t)$$

EoM of GW with source

Using $M_{\rm pl} = (8\pi G)^{-1/2}$

$$a^2 \Box D_{ij} = -(2/M_{\rm pl}^2) T_{ij}^{GW}$$

- This can be derived from variation of the action:

$$I = \int \sqrt{-g} d^4x \left(\frac{1}{2} M_{\rm pl}^2 R + \mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm vector} + \mathcal{L}_{\rm tensor} \right)$$

$$\frac{\delta I}{\delta g^{ij}} = -\frac{1}{4} M_{\rm pl}^2 \sqrt{-g} a^2 \Box D_{ij} + (\text{2nd and higher order terms})$$

$$+ \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{ij}} = 0$$

Stress-energy Tensor

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$$\frac{\delta I}{\delta g^{ij}} = -\frac{1}{4} M_{\rm pl}^2 \sqrt{-g} a^2 \Box D_{ij} + (\text{2nd and higher order terms})$$

$$+ \boxed{\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{ij}}} = 0 \quad , \quad \rightarrow \quad \boxed{T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{ij}}}$$

Scalar Source

Real Scalar Field

$$\mathcal{L}_\phi = -\frac{1}{2}\sum_{\mu\nu} g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} - V(\phi)$$

$$T_{ij}^\phi = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_\phi}{\delta g^{ij}}$$

$$= \frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^j} - g_{ij}\left[\frac{1}{2}\sum_{\mu\nu} g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} + V(\phi)\right]$$

- The second term (proportional to g_{ij}) disappears when taking the traceless component, $T_{ij} - g_{ij}T/3$ **[T is the trace of T_{ij}]**

Real Scalar Field

$$\mathcal{L}_\phi = -\frac{1}{2}\sum_{\mu\nu} g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} - V(\phi)$$

$$T^\phi_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_\phi}{\delta g^{ij}}$$

This is second order! Because:
$\phi(t,\mathbf{x}) = \bar\phi(t) + \delta\phi(t,\mathbf{x})$

$$= \frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^j} - g_{ij}\left[\frac{1}{2}\sum_{\mu\nu} g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} + V(\phi)\right]$$

- The second term (proportional to g_{ij}) disappears when taking the traceless component, $T_{ij} - g_{ij}T/3$ **[T is the trace of T_{ij}]**

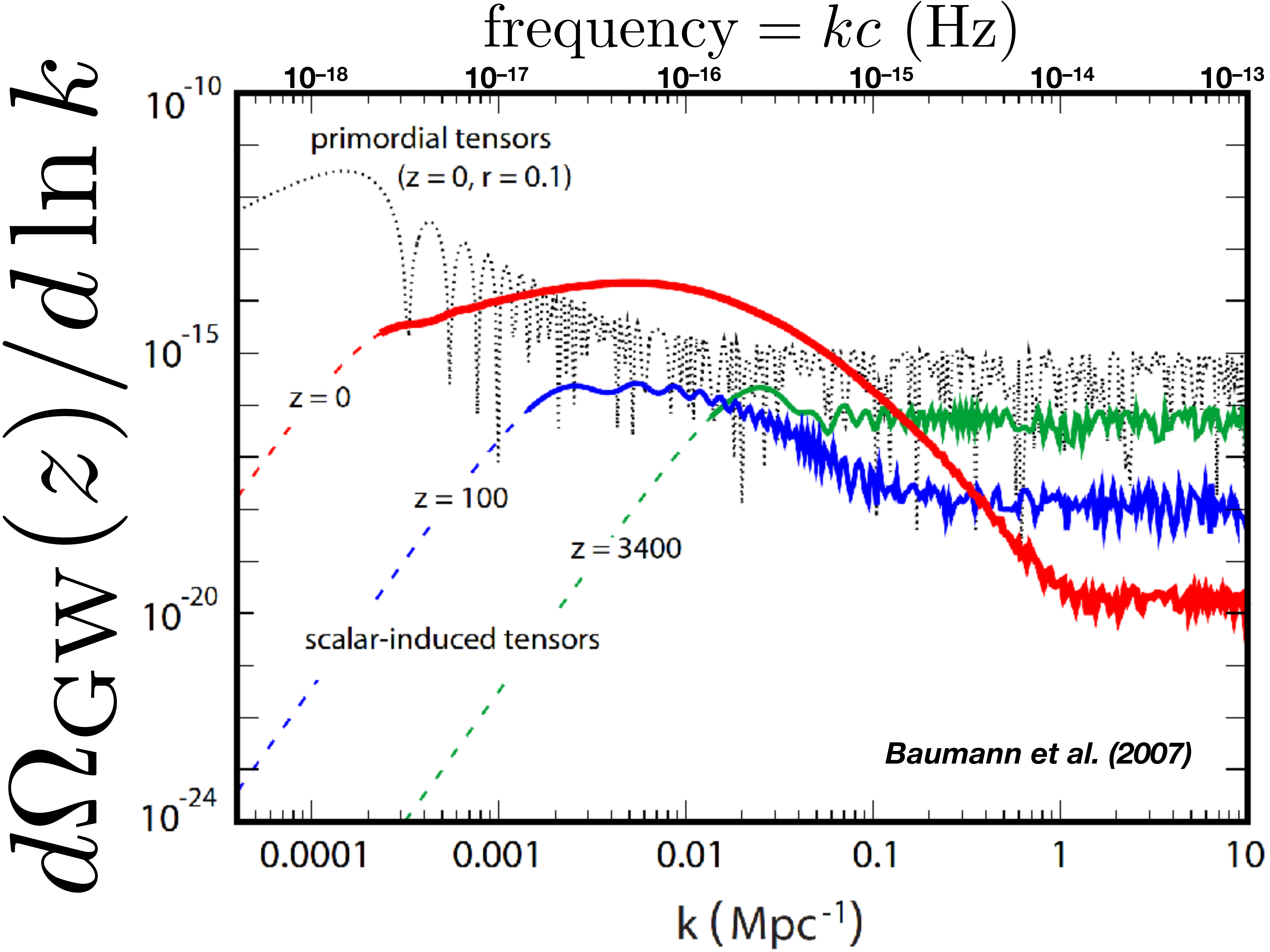
Acquaviva et al. (2003); Baumann et al. (2007)

GW from second-order scalar perturbations

$$\frac{\delta I}{\delta g^{ij}} = -\frac{1}{4}M_{\rm pl}^2\sqrt{-g}\,a^2\Box D_{ij} + (\text{2nd and higher order terms}) + \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{ij}} = 0$$

$$\frac{1}{2}\Big[2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi\Big]$$

- *Not necessarily inflationary source*; the structure formation in the Universe gives the ***guaranteed*** amount of GW from second-order scalar perturbation



frequency = kc (Hz)
$d\Omega_{\rm GW}(z)/d\ln k$
primordial tensors
(z = 0, r = 0.1)
z = 0
z = 100
z = 3400
scalar-induced tensors
Baumann et al. (2007)
k (Mpc^{-1})

Vector Source

Electro-magnetic Field

$$\mathcal{L}_A = -\frac{1}{4}\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu}$$

with $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$

$$F_{i0} = E_i,$$

$$F_{12} = B_3,$$

$$F_{23} = B_1,$$

$$F_{31} = B_2$$

[up to a^2 factors]

Electro-magnetic Field

$$\mathcal{L}_A = -\frac{1}{4}\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu}$$

with $F_{\mu\nu} = \dfrac{\partial A_\nu}{\partial x^\mu} - \dfrac{\partial A_\mu}{\partial x^\nu}$

for $x^0 = \eta$

and x^i =com. coord.

Turner & Widrow (1988)

$$F_{\mu\nu} = a^2 \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

$F_{i0} = E_i,$

$F_{12} = B_3,$

$F_{23} = B_1,$

$F_{31} = B_2$

[up to a² factors]

Then, $$-\frac{1}{4}\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B})$$

I.e., the form remains the same as in non-expanding space

Electro-magnetic Field

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with $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$

$$F_{i0} = E_i,$$
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[up to a² factors]

Stress-energy Tensor

$$T^A_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_A}{\delta g^{ij}}$$

$$= \sum_{\mu\nu} g^{\mu\nu}F_{i\mu}F_{j\nu} - \frac{1}{4}g_{ij}\sum_{\mu\nu}F_{\mu\nu}F^{\mu\nu}$$

EM Stress-Energy Tensor

$$T^A_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta \sqrt{-g} \mathcal{L}_A}{\delta g^{ij}}$$

$$= \sum_{\mu\nu} g^{\mu\nu} F_{i\mu} F_{j\nu} - \frac{1}{4} g_{ij} \sum_{\mu\nu} F_{\mu\nu} F^{\mu\nu}$$

$$F_{i0} = E_i, \quad F_{12} = B_3, \quad F_{23} = B_1, \quad F_{31} = B_2$$

[up to a² factors]

Check: Isotropic Pressure

$$P_A = \frac{1}{3} T^A \equiv \frac{1}{3} \sum_{ij} g^{ij} T^A_{ij} = \frac{1}{6} (\mathbf{E} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{B}) = \frac{1}{3} \rho_A$$

OK!

EM Stress-Energy Tensor

$$T^A_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta \sqrt{-g} \mathcal{L}_A}{\delta g^{ij}}$$

$$= \sum_{\mu\nu} g^{\mu\nu} F_{i\mu} F_{j\nu} - \frac{1}{4} g_{ij} \sum_{\mu\nu} F_{\mu\nu} F^{\mu\nu}$$

$$F_{i0} = E_i,$$
$$F_{12} = B_3,$$
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[up to a^2 factors]

Traceless Component

$$T^A_{ij} - \frac{1}{3} g_{ij} T^A = -a^2 (E_i E_j + B_i B_j) + \frac{1}{3} g_{ij} (\mathbf{E} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{B})$$

EM Stress-Energy Tensor

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$$= \sum_{\mu\nu} g^{\mu\nu} F_{i\mu} F_{j\nu}$$

This is second order because E_i and B_i cannot have the mean values; otherwise the background space wouldn't be isotropic

Traceless Component

$$T^A_{ij} - \frac{1}{3} g_{ij} T^A = -a^2 (E_i E_j + B_i B_j) + \frac{1}{3} g_{ij} (\mathbf{E}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{B})$$

"Magnetogenesis" by quantum fluctuation during inflation?

- On Day 1, we learned that the **equation of motion of gravitational waves** during inflation had **a constant (conserved) solution in the super-horizon limit**

- Can we do the same for electromagnetic fields? Then perhaps we can generate the intergalactic magnetic fields naturally also from inflation?

Recap: Tensor Mode

- On Day 1, we learned that the **equation of motion of gravitational waves** during inflation had **a constant (conserved) solution in the super-horizon limit**

    - This was due to the time-dependent mass:

$$u_{ij}'' + \left[k^2 + m^2(\eta)\right] u_{ij} = 0$$

$$\begin{cases} u_{ij}(\eta, \mathbf{k}) = a(\eta) D_{ij}(\eta, \mathbf{k}), \quad dt = a(\eta) d\eta \\ m^2(\eta) = -\dfrac{a''}{a} = -a^2(2H^2 + \dot{H}) \end{cases}$$

conformal time

Recap: Tensor Mode

- On Day 1, we learned that the **equation of motion of gravitational waves** during inflation had **a constant (conserved) solution in the super-horizon limit**
    - This was due to the time-dependent mass:

$$u''_{ij} + \left[k^2 + m^2(\eta)\right] u_{ij} = 0$$

- For k << m,

$$u_{ij} \propto a(\eta) \to D_{ij} = \text{constant}$$

How about Vector Mode?

- What happens to electromagnetic (EM) fields? Can we generate the super-horizon EM field during inflation?

- **The answer is no** in the Standard Model of elementary particles and fields, and **no for the fundamental reason**

(Massless) Vector Mode

- The equation of motion for $A_i(\eta,k)$:

$$A_i'' + k^2 A_i = 0$$

  - EM fields decay as a^{-2}:

$$\mathbf{E} = -a^{-2}\mathbf{A}' \propto a^{-2},$$

$$\mathbf{B} = a^{-2}\nabla \times \mathbf{A} \propto a^{-2}$$

- The EoM of A_i has no time-dependent mass term due to the expansion of the Universe!!

- **The massless vector field does not feel the expansion of the Universe. How come?**

Conformal Invariance

- It turns out that the electromagnetic action

$$I = -\frac{1}{4}\int \sqrt{-g}d^4x \sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu}$$

is "conformally invariant", in the sense that it remains unchanged under the so-called "conformal transformation" of the metric

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

Conformal Invariance

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$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$\sqrt{-g} \to \sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g}$$

Conformal Invariance

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$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$F^{\mu\nu} = \sum_{\alpha\beta} g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta} \to \sum_{\alpha\beta} \tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\alpha\beta} = \Omega^{-4}\sum_{\alpha\beta} g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}$$

Conformal Invariance

- It turns out that the electromagnetic action

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is "conformally invariant", in the sense that it remains unchanged under the so-called "conformal transformation" of the metric

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

Thus, $\sqrt{-g}\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu}$ **remains unchanged!**

Conformal Invariance

- It turns out that the electromagnetic action

$$I = -\frac{1}{4}\int \sqrt{-g}d^4x \sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu}$$

- This means that we can “undo” the expansion of the Universe and yet the EM field does not feel it!

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = a^{-2}g_{\mu\nu} = \eta_{\mu\nu}$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$$

$$ds^2 = a^2(-d\eta^2 + d\mathbf{x}^2) \rightarrow -d\eta^2 + d\mathbf{x}^2$$

Therefore:

- **Scalar field**: Super-horizon modes are amplified during inflation and yield seeds for the cosmic structure (colloquium last week)
- **Tensor field**: Super-horizon modes are amplified during inflation and yield a background of stochastic gravitational waves (Day1) and B-mode polarisation of the CMB (Day 2)
- **Electromagnetic field**: Nothing happens during inflation!

More general result

- One can show that the action is conformally invariant when the derived stress-energy tensor is traceless:

$$\sum_{\mu\nu} g^{\mu\nu} T_{\mu\nu} = 0$$

- This is certainly the case for the electromagnetic field:

$$T_{\mu\nu} = \sum_{\alpha\beta} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} - \frac{1}{4} g_{\mu\nu} \sum_{\alpha\beta} F_{\alpha\beta} F^{\alpha\beta}, \quad \sum_{\mu\nu} g^{\mu\nu} g_{\mu\nu} = 4$$

More general result

- More generally, the stress energy tensor of a perfect fluid is

$$T_{\mu\nu} = P g_{\mu\nu} + (P+\rho) u_\mu u_\nu, \quad \sum_{\mu\nu} g^{\mu\nu} u_\mu u_\nu = -1$$

- The trace is $$\sum_{\mu\nu} g^{\mu\nu} T_{\mu\nu} = 3P - \rho$$

- **Thus, the trace vanishes for any relativistic perfect fluids satisfying P=ρ/3!**

Side Note: Vanishing time-dependent mass during the radiation era

- The time-dependent mass for the equation of motion of gravitational waves vanishes during the radiation era: **a(η) ~ η**

$$u''_{ij} + \left[k^2 + m^2(\eta)\right] u_{ij} = 0$$

The GW mode function does not "feel" the expansion of the Universe (except redshifts) during the radiation era

$$\begin{cases} u_{ij}(\eta, \mathbf{k}) = a(\eta) D_{ij}(\eta, \mathbf{k}), \quad dt = a(\eta) d\eta \\ m^2(\eta) = -\frac{a''}{a} = 0, \quad \text{for } a(\eta) \propto \eta \end{cases}$$

conformal time ($d\eta$)

Breaking of Conformal Invariance

PHYSICAL REVIEW D VOLUME 37, NUMBER 10 15 MAY 1988

Inflation-produced, large-scale magnetic fields

Michael S. Turner and Lawrence M. Widrow

- **Add terms to break conformal invariance:**

B. RA^2 terms

Consider the Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{b}{2}RA^2 - \frac{c}{2}R_{\mu\nu}A^\mu A^\nu ,$$

Both can generate super-horizon scale vector fields. Though they are no longer considered as a mechanism to produce sufficient magnetic fields, the basic idea is there. **What do they do to the gravitational waves?**

C. RF^2 terms

We now consider the coupling of gravitational and electromagnetic fields through terms in the Lagrangian of the form RF^2. The most general Lagrangian containing such terms can be written

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}g , \tag{2.20}$$

$$\mathcal{L}g = -\frac{1}{4m_e^2}(bRF_{\mu\nu}F^{\mu\nu} + cR_{\mu\nu}F^{\mu\kappa}F^\nu{}_\kappa + dR_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa}) , \tag{2.21}$$

Turner & Widrow (1988)

Breaking of Conformal Invariance

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- **Add te**

Consider the La

$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu}$

gravitational and

the Lagrangian of

rangian containing

(2.20)

$+dR_{\mu\nu\lambda\kappa}F^{\mu\nu}F^{\lambda\kappa})$,

(2.21)

Next consider axion electrodynamics. For energies well below the Peccei-Quinn symmetry-breaking scale f_a, the effective Lagrangian for axion electrodynamics is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (3.7)$$

where g_a is a coupling constant of the order α, and the vacuum angle $\theta = \phi_a/f_a$ (ϕ_a = axion field). The equations of motion are

$$-\frac{1}{a^2}\frac{\partial}{\partial\eta}a^2\mathbf{E} + \nabla\times\mathbf{B} = g_a(\dot{\theta}\mathbf{B} + \nabla\theta\times\mathbf{E}) , \quad (3.8)$$

$$\frac{1}{a^2}\frac{\partial}{\partial\eta}a^2\mathbf{B} + \nabla\times\mathbf{E} = 0 , \quad (3.9)$$

$$\ddot{\theta} + 2\frac{\dot{a}}{a}\dot{\theta} + k^2\theta + g_a a^2\mathbf{E}\cdot\mathbf{B} = 0 . \quad (3.10)$$

Both can ge
scale vector
no longer cons
to produce sufficient magnetic fields, the basic idea is there. **What do they do to the gravitational waves?**

Turner & Widrow (1988)

Chern-Simons Term

the effective Lagrangian for axion electrodynamics is

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\theta\partial^\mu\theta - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (3.7)$$

$$\tilde{F}^{\mu\nu} = \sum_{\alpha\beta} \frac{\epsilon^{\mu\nu\alpha\beta}}{2\sqrt{-g}} F_{\alpha\beta}$$

Chern-Simons term

where g_a is a coupling constant of the order α, and the vacuum angle $\theta = \phi_a / f_a$ (ϕ_a = axion field).

$$\sum_{\mu\nu} F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$ **Parity Even**

$$\sum_{\mu\nu} F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B}\cdot\mathbf{E}$$ **Parity Odd**

- The axion field, θ, is a “pseudo scalar”, which is parity odd; thus, the last term in Eq.3.7 is parity even as a whole.

Anber & Sorbo (2010)

New Equation of Motion for the Vector Mode

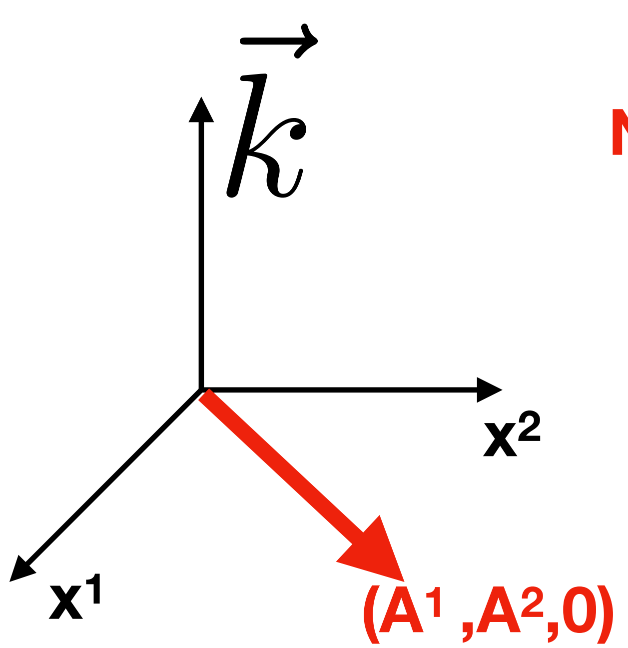
the effective Lagrangian for axion electrodynamics is

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\theta\partial^\mu\theta - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_a\theta F_{\mu\nu}\tilde{F}^{\mu\nu}, \qquad (3.7)$$

Chern-Simons term

$$A''_\pm + \left(k^2 \pm 2k\frac{\xi}{\eta}\right) A_\pm = 0$$

New, helicity-dependent term, with $\xi = \frac{2g_a\dot{\theta}}{H}$, during inflation $-\infty < \eta < 0$



$$A_\pm = \frac{A_1 \mp iA_2}{\sqrt{2}}$$

- A± is the mode function of each helicity state

Comparison to EoM of GW

Gravitational Wave (From Day 1)

$$u_\lambda'' + \left[k^2 + m^2(\eta)\right] u_\lambda = 0, \qquad m^2 = -\frac{a''}{a} = -\frac{2}{\eta^2}$$

with λ = -2, +2 (spin 2)

This minus sign was the key

Vector Field

$$A_\lambda'' + \left[k^2 + m_A^2(k,\eta,\lambda)\right] A_\lambda = 0, \qquad m_A^2 = \lambda \frac{4kg_a\dot{\theta}}{H\eta}$$

$$(-\infty < \eta < 0)$$

with λ = -1, +1 (spin 1)

- Therefore, for k << $|m_A|$, one of the helicities, for which **λ(dθ/dt) > 0,** is amplified relative to the other! The vector field becomes **"chiral"**

(*) The exact solution can be given in the form of a "Whittaker function"

Anber & Sorbo (2010)

Large-scale Solution

$$A''_{\pm} + \left(k^2 \pm 2k\frac{\xi}{\eta}\right) A_{\pm} = 0$$

For $\xi > 0, \quad \frac{1}{8\xi} \ll -k\eta \ll 2\xi^{(*)}$**,**

$$A_+ \approx \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} \exp\left(\boxed{\pi\xi} - 2\sqrt{2\xi k/aH}\right)$$

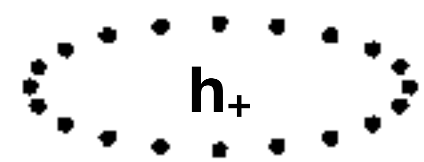
- **Exponential dependence on ξ!**

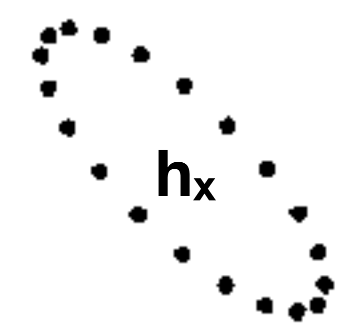
Helicity decomposition of GW

$$\Box D_{ij} = 16\pi G(E_i E_j + B_i B_j)^{\mathrm{TT}}$$

To extract the transverse and traceless component

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





$$D_L = \frac{h_+ + ih_\times}{\sqrt{2}}, \quad D_R = \frac{h_+ - ih_\times}{\sqrt{2}}$$

Left-handed: Helicity –2 **Right-handed: Helicity +2**

Sorbo (2011); Barnaby, Namba & Peloso (2011)

Power Spectrum of GW

$$D_L = \frac{h_+ + ih_\times}{\sqrt{2}}, \quad D_R = \frac{h_+ - ih_\times}{\sqrt{2}}$$

Left-handed: Helicity –2 **Right-handed: Helicity +2**

$$\frac{k^3\langle|D_R|^2\rangle}{2\pi^2} = \frac{4}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\left[1 + 8.6\times10^{-7}\frac{H^2}{M_{\rm pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\frac{k^3\langle|D_L|^2\rangle}{2\pi^2} = \frac{4}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\left[1 + 1.8\times10^{-9}\frac{H^2}{M_{\rm pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

Vacuum contribution (From Day 1)

- The above is for dθ/dt > 0 (hence ξ>0). **Chiral gravitational waves!**

Sorbo (2011); Barnaby, Namba & Peloso (2011)

Power Spectrum of GW

$$D_L = \frac{h_+ + ih_\times}{\sqrt{2}}, \qquad D_R = \frac{h_+ - ih_\times}{\sqrt{2}}$$

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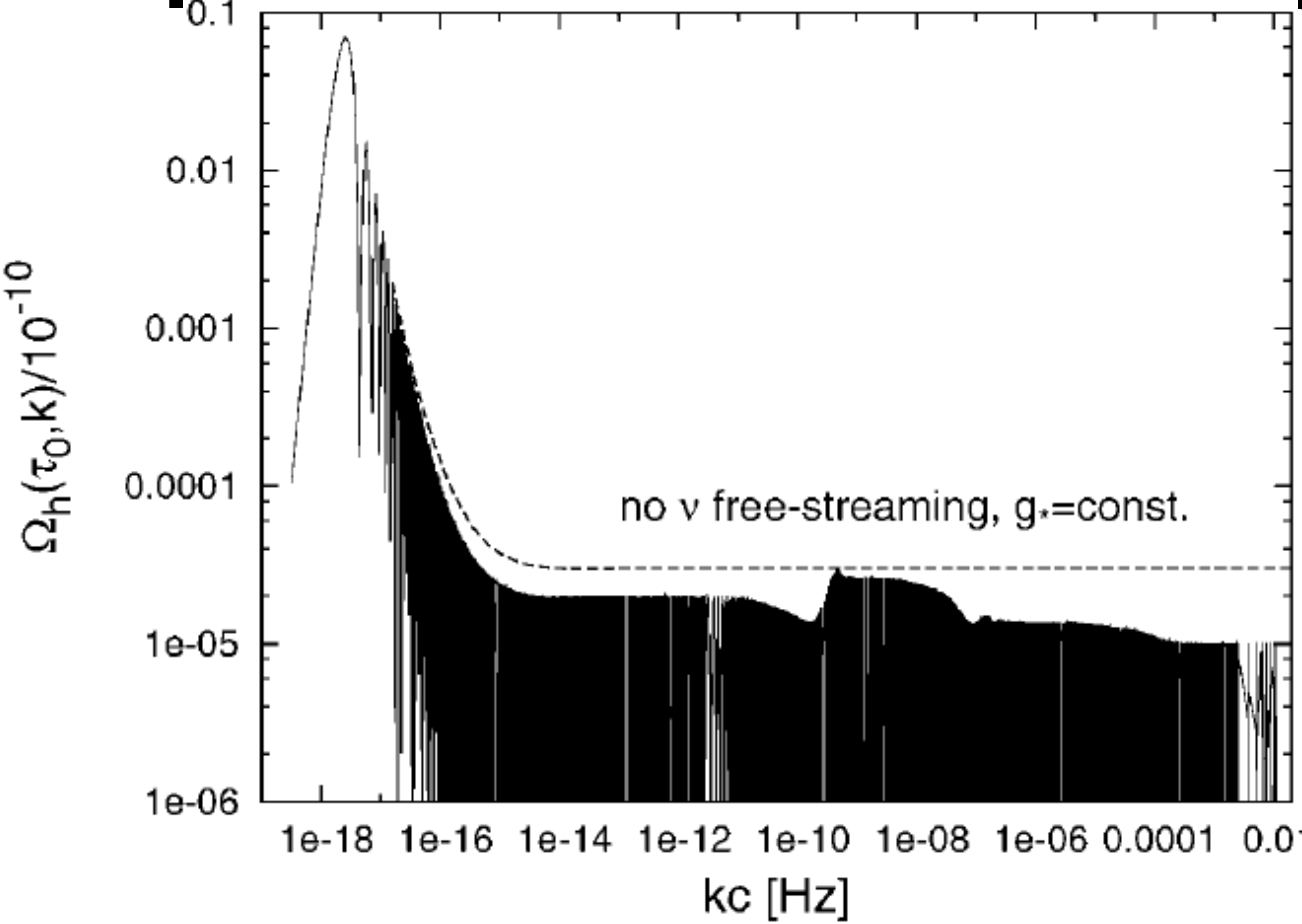
- If $\dot\theta$ (hence ξ) increases in time (axion speeds up), we will have a **rising** spectrum of GW; **completely new phenomenology!**

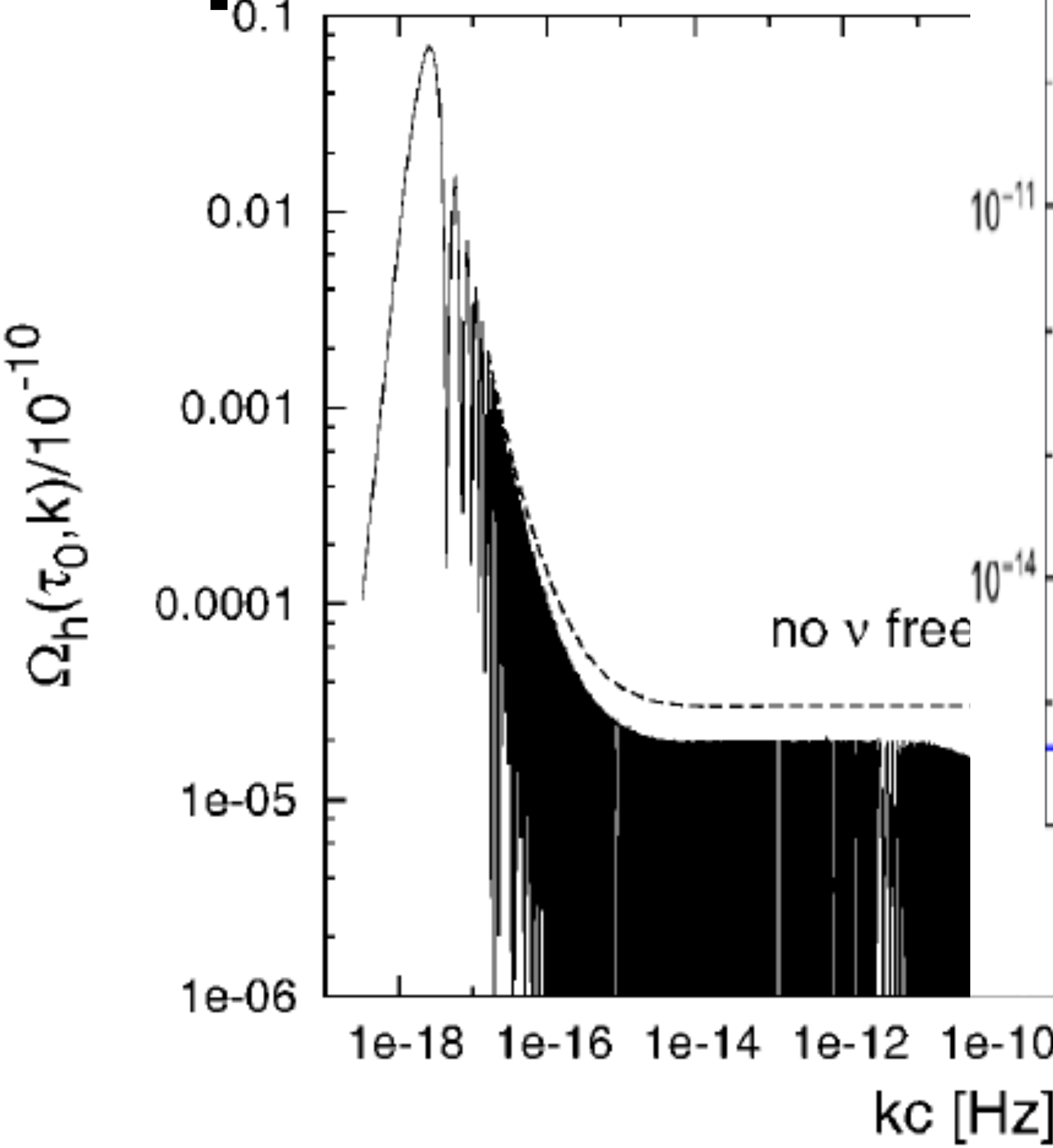
Theoretical energy density

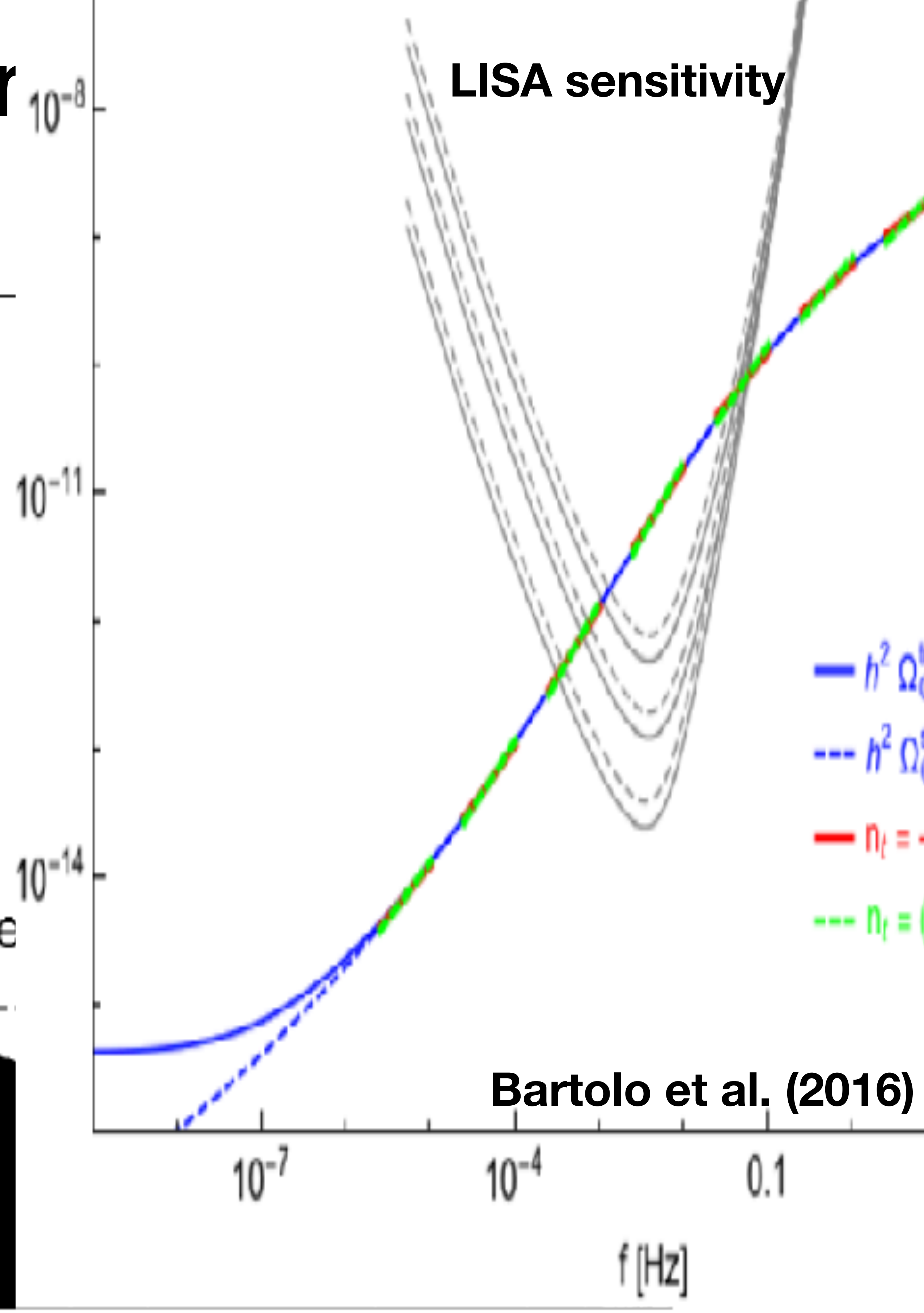
Spectrum of GW today



Theoretical energy der

Spectrum of





New Phenomenology

- **Vacuum Contribution**
  - Scale-invariant
  - Gaussian
  - No chirality
    - No circular polarisation in GW
    - No TB/EB correlation in CMB

- **Axion-U(1) gauge field Sourced Contribution**
  - **Non**-scale-invariant
  - **Non**-Gaussian
  - **Chiral**
    - GW is circularly polarised
    - TB/EB correlations do not vanish

Concluding Message

$$a^2\Box D_{ij} = -16\pi G T_{ij}^{GW}$$

- **Do not take it for granted if someone told you that detection of the primordial gravitational waves would be a signature of "quantum gravity"!**

  - Only the homogeneous solution corresponds to the vacuum tensor metric perturbation. **There is no *a priori* reason to neglect an inhomogeneous solution!**

  - Contrary, we have several examples in which detectable GWs are generated by **sources** [e.g., U(1) and SU(2) gauge fields]

Appendix:
Linearly sourcing GW by SU(2) Gauge Field

Challenge for vector-sourced GW on CMB scales

- Can we generate GW on CMB scales (~10^{-18} Hz) by the vector field and a Chern-Simons coupling?
    - The answer is "***not easy***", because it also creates the scalar perturbation that is too non-Gaussian
- Not only does the second-order vector perturbation generate non-Gaussian GW, but it also generates the non-Gaussian scalar perturbation, which is not seen on the CMB scale

Anber & Sorbo (2010); Barnaby & Peloso (2010)

Scalar perturbation from the second-order vector field

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}$$

The equation of motion (Euler-Lagrange equation) for ϕ is

$$\Box\phi - \frac{\partial V}{\partial\phi} = \frac{\alpha}{4f}\sum_{\mu\nu} F^{\mu\nu}\tilde{F}_{\mu\nu} = -\frac{\alpha}{f}\underline{\mathbf{E}\cdot\mathbf{B}}$$

- The scalar field perturbation is sourced **non-linearly** by the vector field -> Highly non-Gaussian contribution!

What went wrong?

$$\Box D_{ij} = 16\pi G(E_i E_j + B_i B_j)^{\mathrm{TT}}$$

To extract the transverse and traceless component

- The vector mode could not source the tensor mode at linear order in homogeneous and isotropic background, as E_i and B_i cannot take the mean values
    - Isotropy is broken otherwise
    - The same non-linear source generates the scalar perturbation that is too non-Gaussian to be consistent with CMB data
- Can we find a field which can source the tensor mode **linearly**?

Maleknejad & Sheikh-Jabbari (2011, 2013); Adshead & Wyman (2012)

A Solution: U(1) -> SU(2)

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a - \frac{\alpha}{4f}\phi F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a$$

$$F_{\mu\nu}^a = \frac{\partial A_\nu^a}{\partial x^\mu} - \frac{\partial A_\mu^a}{\partial x^\nu} + g_A \sum_{b,c=1}^{3} \epsilon^{abc} A_\mu^b A_\nu^c$$

[a=1,2,3; μ=0,1,2,3]

self-interaction term

SU(2) gauge field: $\mathbf{A}_\mu = \sum_{a=1}^{3} A_\mu^a \frac{\sigma^a}{2}$

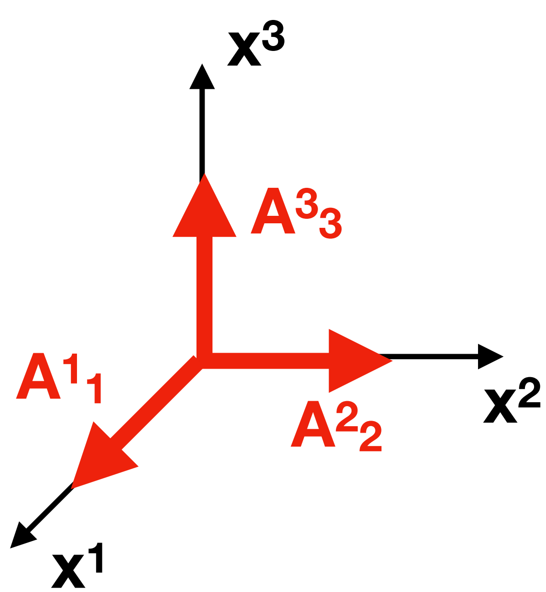
Pauli matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Remarkable Discovery

- The SU(2) gauge field has a solution, in which A^a_μ establishes α homogeneous and isotropic mean value Q(t):

$$A_i^a = a(t)Q(t)\delta_i^a$$

- You can picture this configuration by aligning a=1 with the x-axis, a=2 with the y-axis, and a=3 with the z-axis:



- This configuration is stable against a perturbation, and it is in fact the attractor solution for fairly generic initial conditions.

Maleknejad & Erfani (2014); Wolfson, Maleknejad & Komatsu (2020)

Remarkable Discovery

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U(1) [EM]

$$F_{i0} = a^2 E_i,$$
$$F_{12} = a^2 B_3,$$
$$F_{23} = a^2 B_1,$$
$$F_{31} = a^2 B_2$$

SU(2)

$$F_{i0}^a = -(aQ)'\delta_i^a,$$
$$F_{12}^a = g_A a^2 Q^2 \delta_3^a,$$
$$F_{23}^a = g_A a^2 Q^2 \delta_1^a,$$
$$F_{31}^a = g_A a^2 Q^2 \delta_2^a$$

Stress-energy Tensor

$$T_{ij}^{\mathrm{SU}(2)} = \sum_{\mu\nu} g^{\mu\nu} \sum_{a=1}^{3} F_{i\mu}^{a} F_{j\nu}^{a} - \frac{1}{4} g_{ij} \sum_{\mu\nu} \sum_{a=1}^{3} F_{\mu\nu}^{a} F_{a}^{\mu\nu}$$

This term disappears in the traceless component

Perturbation:

$$\delta T_{ij}^{\mathrm{SU}(2)} = \sum_{\mu\nu} g^{\mu\nu} \sum_{a=1}^{3} \left[\bar{F}_{i\mu}^{a} (\delta F_{j\nu}^{a}) + (\delta F_{i\mu}^{a}) \bar{F}_{j\nu}^{a} \right] + g_{ij}(\dots)$$

- The perturbed stress energy tensor is **linear** in the vector perturbation!

Tensor Mode in the SU(2) Gauge Field

- When expanded around the homogeneous and isotropic solution, the perturbation of the SU(2) gauge field contains scalar, vector, and **tensor** modes:

$$A_i^a = (aQ)\delta_i^a + \text{scalar} + \text{vector} + t_{ai}$$

- **symmetric**
- **transverse**
- **traceless**

Maleknejad & Sheikh-Jabbari (2011, 2013)

Tensor Mode in the SU(2) Gauge Field

- When expanded around the homogeneous and isotropic solution, the perturbation of the SU(2) gauge field contains scalar, vector, and **tensor** modes:

$$A_i^a = (aQ)\delta_i^a + \mathrm{scalar} + \mathrm{vector} + \underline{t_{ai}}$$

- symmetric
- transverse
- traceless

$$\delta T_{ij}^{\mathrm{SU}(2)} = -\frac{2}{a}\frac{d(aQ)}{dt}t'_{ij} + 2g_A Q^2\left\{g_A Q a t_{ij} - \frac{1}{2}\left[\sum_{ab}\epsilon^{iba}\frac{\partial t_{aj}}{\partial x^b} + (i \leftrightarrow j)\right]\right\}$$

Maleknejad & Sheikh-Jabbari (2011, 2013)

Helicity Decomposition

For tensor modes going in k_3 direction:

$$t_{ij} = \begin{pmatrix} t_+ & t_\times & 0 \\ t_\times & -t_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \begin{cases} t_L = \dfrac{t_+ + i t_\times}{\sqrt{2}} & \textbf{helicity –2} \\ t_R = \dfrac{t_+ - i t_\times}{\sqrt{2}} & \textbf{helicity +2} \end{cases}$$

$$\delta T_L^{\mathrm{SU}(2)} = -\frac{2}{a}\frac{d(aQ)}{dt} t'_L + 2 g_A Q^2 \left(g_A Q a t_L + k_3 t_L \right)$$

$$\delta T_R^{\mathrm{SU}(2)} = -\frac{2}{a}\frac{d(aQ)}{dt} t'_R + 2 g_A Q^2 \left(g_A Q a t_R - k_3 t_R \right)$$

- The perturbed stress energy tensor is **linear** in $t_{L,R}$!

Helicity Decomposition

For tensor modes going in k_3 direction:

$$t_{ij} = \begin{pmatrix} t_+ & t_\times & 0 \\ t_\times & -t_+ & 0 \\ 0 & 0 & 0 \end{pmatrix},\qquad \begin{cases} t_L = \dfrac{t_+ + i t_\times}{\sqrt{2}} & \textbf{helicity −2} \\ t_R = \dfrac{t_+ - i t_\times}{\sqrt{2}} & \textbf{helicity +2} \end{cases}$$

$$\delta T_L^{\mathrm{SU}(2)} = -\frac{2}{a}\frac{d(aQ)}{dt} t'_L + 2 g_A Q^2 \left(g_A Q a t_L + k_3 t_L \right)$$

$$\delta T_R^{\mathrm{SU}(2)} = -\frac{2}{a}\frac{d(aQ)}{dt} t'_R + 2 g_A Q^2 \left(g_A Q a t_R - k_3 t_R \right)$$

Using symmetry, this result is valid for all $k_i = k$ ($k_3 \to k$)

- The perturbed stress energy tensor is **linear** in $t_{L,R}$!

Adshead, Martinec & Wyman (2013); Dimastrogiovanni & Peloso (2013)
Maleknejad, Sheikh-Jabbari & Soda (2013)

$t_{L,R}$: Equations of Motion

$$t_L'' + \left[k^2 + \frac{2}{\eta^2}\left(m_Q\xi + (-k\eta)(m_Q+\xi)\right)\right] t_L = \mathcal{O}(D_L)$$

$$t_R'' + \left[k^2 + \frac{2}{\eta^2}\left(m_Q\xi - (-k\eta)(m_Q+\xi)\right)\right] t_R = \mathcal{O}(D_R)$$

$$-\infty < \eta < 0 \qquad \begin{cases} m_Q = gQ/H \\ \xi = \lambda\dot{\phi}/(2fH) \end{cases}$$

- During inflation, $\xi \approx m_Q + m_Q^{-1}$

[m_Q ~ a few, for successful phenomenology of this model]

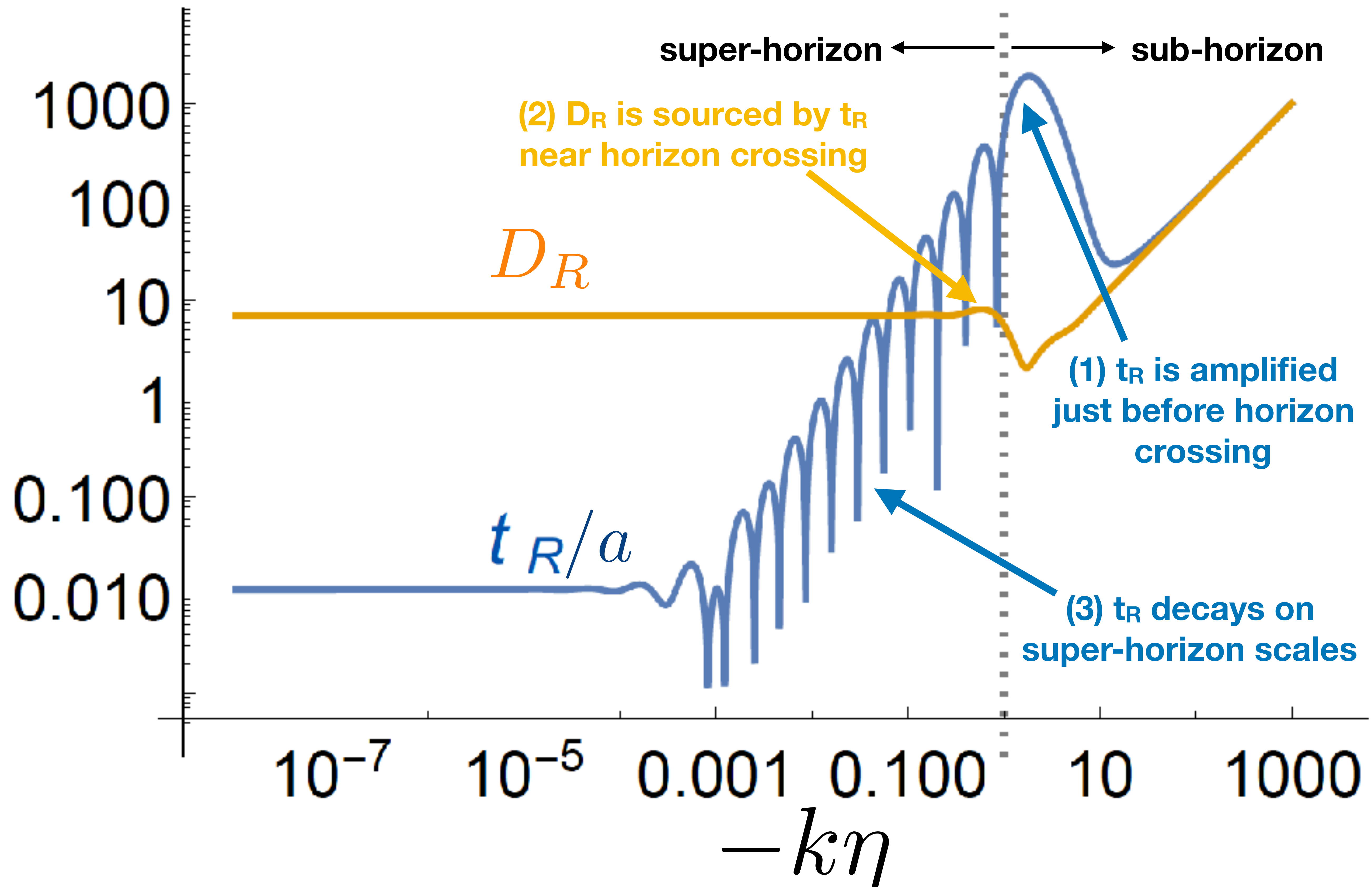
- For ξ>0, the right-handed mode is amplified for

$$\sqrt{2}(-1+\sqrt{2})m_Q < -k\eta < \sqrt{2}(1+\sqrt{2})m_Q$$

$$\Rightarrow \quad 0.6 m_Q < -k\eta < 3.6 m_Q$$

Dimastrogiovanni, Fasiello & Fujita (2016)

Sourced GW



Maleknejad (2016); Dimastrogiovanni, Fasiello & Fujita (2016); Maleknejad & Komatsu (2019)

Power Spectrum of GW

$$D_L = \frac{h_+ + ih_\times}{\sqrt{2}}, \quad D_R = \frac{h_+ - ih_\times}{\sqrt{2}}$$

Left-handed: Helicity –2 **Right-handed: Helicity +2**

$$\frac{k^3\langle|D_R|^2\rangle}{2\pi^2} = \frac{4}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\left[1 + \frac{Q^2}{2M_{\rm pl}^2}|\mathcal{G}_R(m_Q)|^2 e^{\pi(m_Q+\xi)}\right]$$

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Vacuum contribution (From Day 1)

$$|\mathcal{G}_R|^2 \gg |\mathcal{G}_L|^2$$

- The above is for dϕ/dt > 0 (hence ξ>0). **Chiral gravitational waves!**

Maleknejad (2016); Dimastrogiovanni, Fasiello & Fujita (2016); Maleknejad & Komatsu (2019)

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- Time dependence of $\xi \sim m_Q + m_Q^{-1}$ results in various **non-scale-invariant** power spectrum shapes

How about scalar modes?

- The scalar mode is not amplified for $m_Q > \sqrt{2}$

Dimastrogiovanni & Peloso (2013)

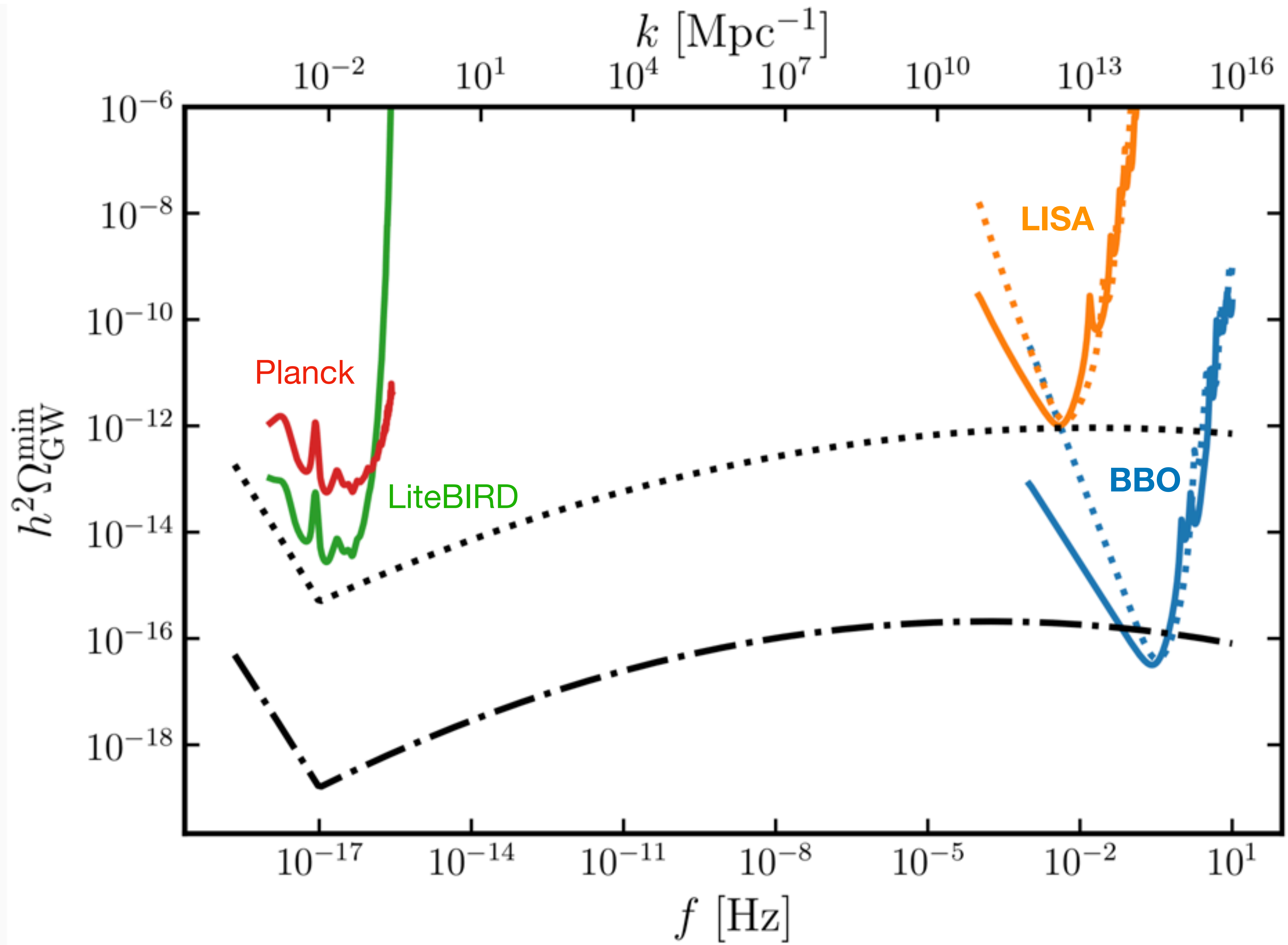
- Therefore, the picture is:
  - The scalar (curvature) perturbation is given by the vacuum fluctuation (nearly scale invariant and Gaussian), consistent with the CMB data (colloquium last week) unlike the U(1) case!
  - The tensor perturbation (GW) is given by the sourced contribution

Dimastrogiovanni & Peloso (2013); Adshead, Martinec & Wyman (2013); Maleknejad, Sheikh-Jabbari & Soda (2013)

Phenomenology, and more reading

- **Non**-scale invariant spectrum
  - See **Fujita, Sfakianakis & Shiraishi (2019)** for various power spectrum shapes
- **Non**-Gaussian
  - It is linearly sourced by t_R, but t_R itself is highly non-Gaussian because of self-interaction. See **Agrawal, Fujita & Komatsu (2018a,b)**
- **Chiral**
  - Circular polarisation of GW and TB/EB correlation in CMB as observable signatures. See **Thorne et al. (2018)**

Thorne, Fujita, Hazumi, Katayama, Komatsu & Shiraishi, PRD, 97, 043506 (2018)



CMB Experimental Strategy Commonly Assumed So Far

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)

2. Check for scale invariance: Consistent with a scale invariant spectrum?

   - Yes => Announce discovery of the vacuum fluctuation in spacetime

   - No => WTF?

New CMB Experimental Strategy: New Standard!

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)

2. Consistent with a scale invariant spectrum?

3. Consistent with Gaussianity?

4. TB/EB correlations consistent with zero?

- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

If not, you may have just discovered new physics during inflation!

2. Consistent with a scale invariant spectrum?

3. Consistent with Gaussianity?

4. TB/EB correlations consistent with zero?

- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime