Lecture notes:
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 2: Polarisation of the CMB

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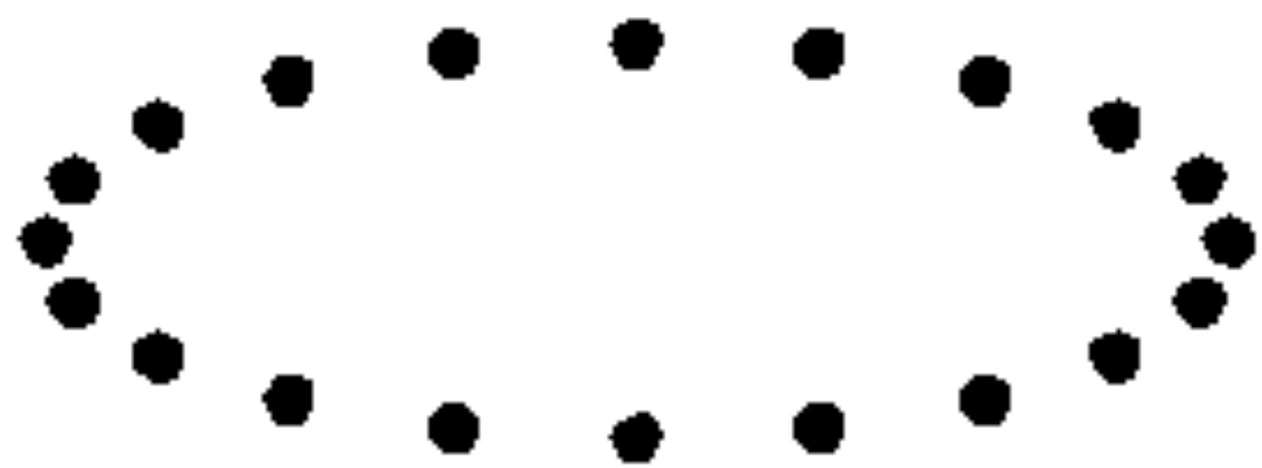
How do we measure gravitational waves?

Measuring GW

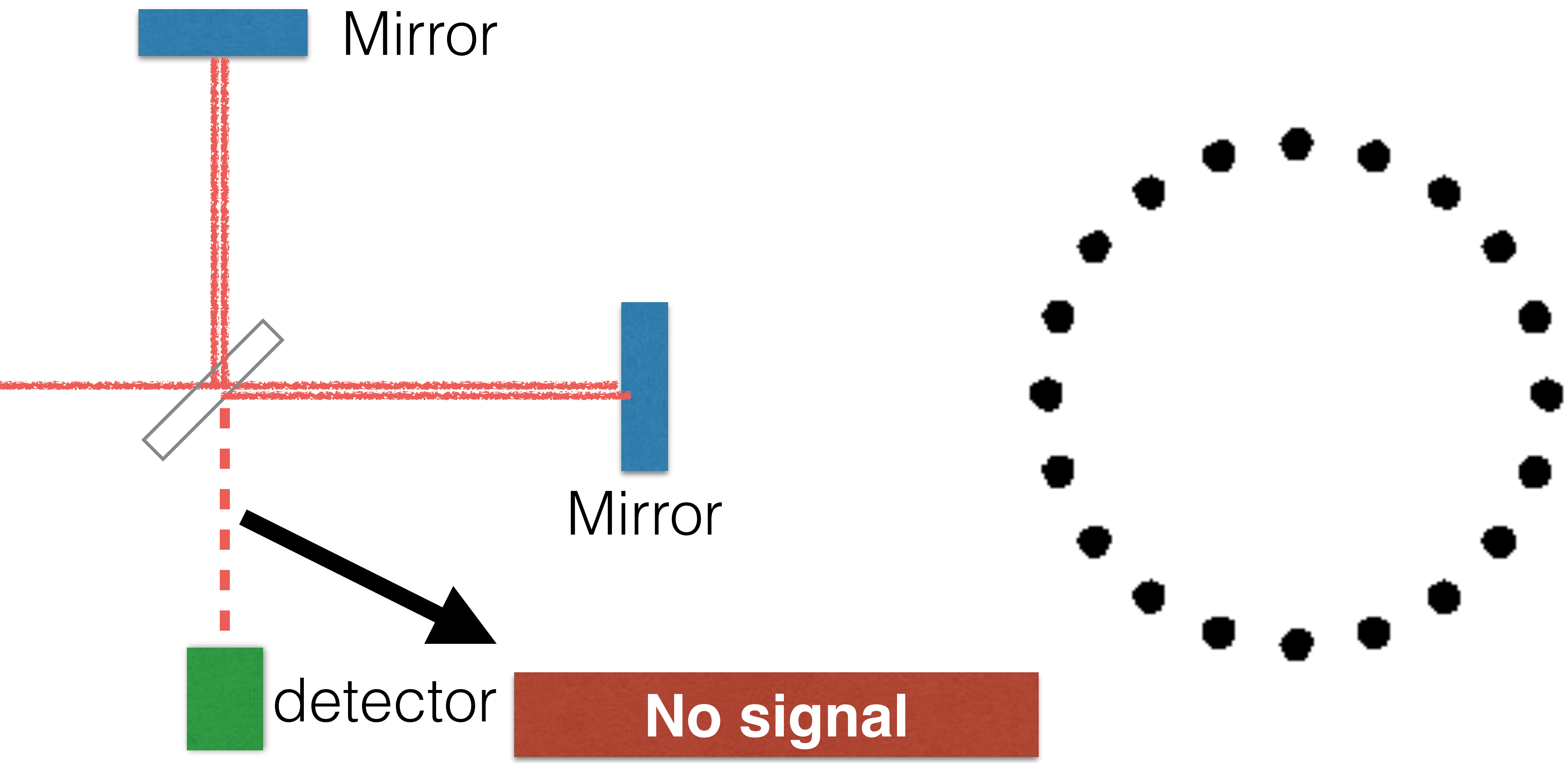
- GW changes distances between two points

$$d\ell^2 = d\mathbf{x}^2 = \sum_{ij} \delta_{ij} dx^i dx^j$$

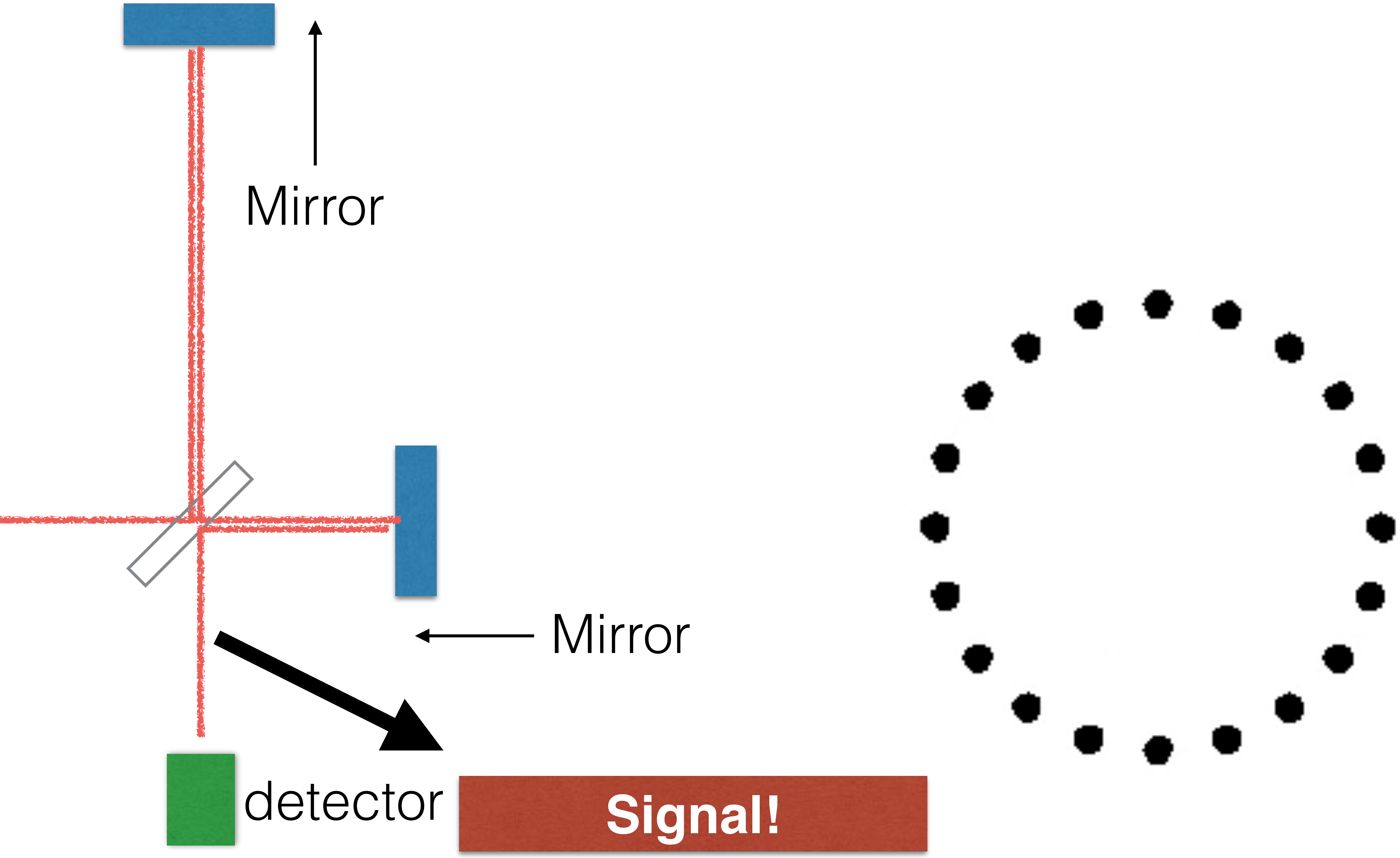
$$d\ell^2 = \sum_{ij} (\delta_{ij} + \underline{h_{ij}}) dx^i dx^j$$



Laser Interferometer



Laser Interferometer



LIGO detected GW from a binary blackholes, with a frequency of **~100 Hz** = the wavelength of thousands of kilometres

But, the primordial GW affecting the **CMB** has a frequency of **10^{-18} Hz** = the wavelength of **billions of light-years**!!

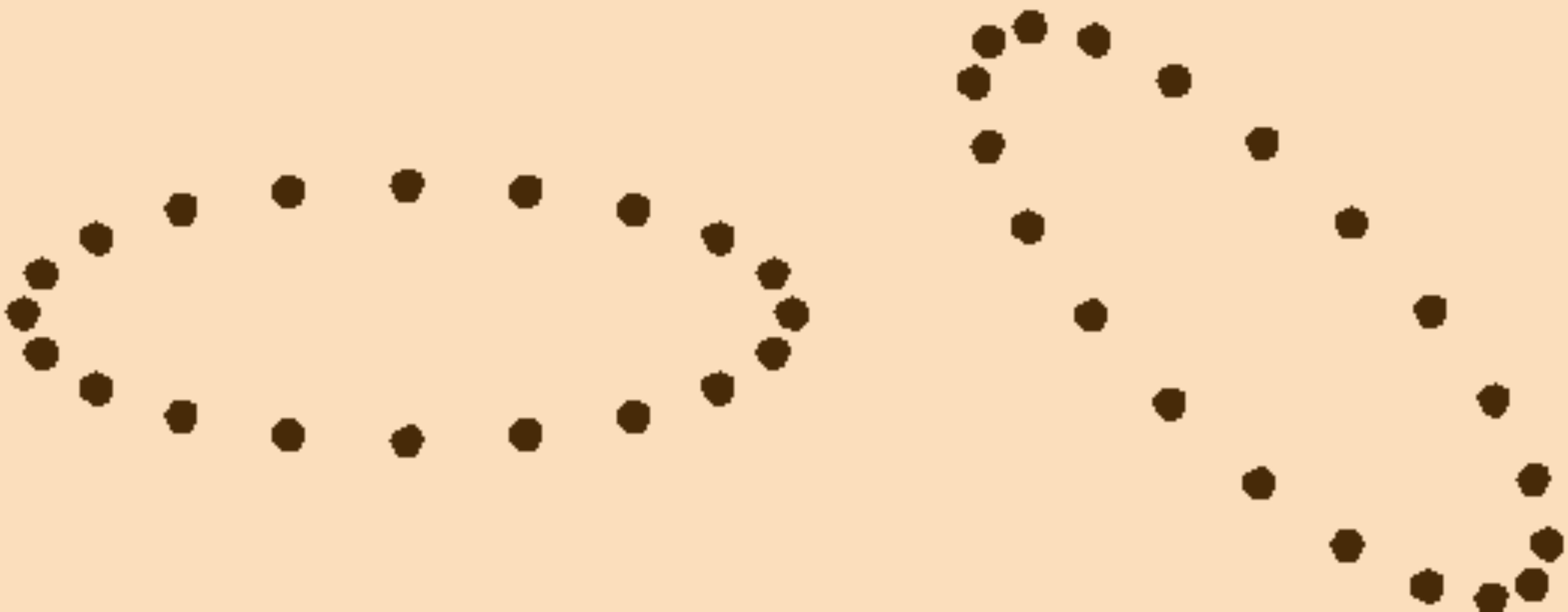
How do we find it?

Detecting GW by CMB

Isotropic electro-magnetic fields

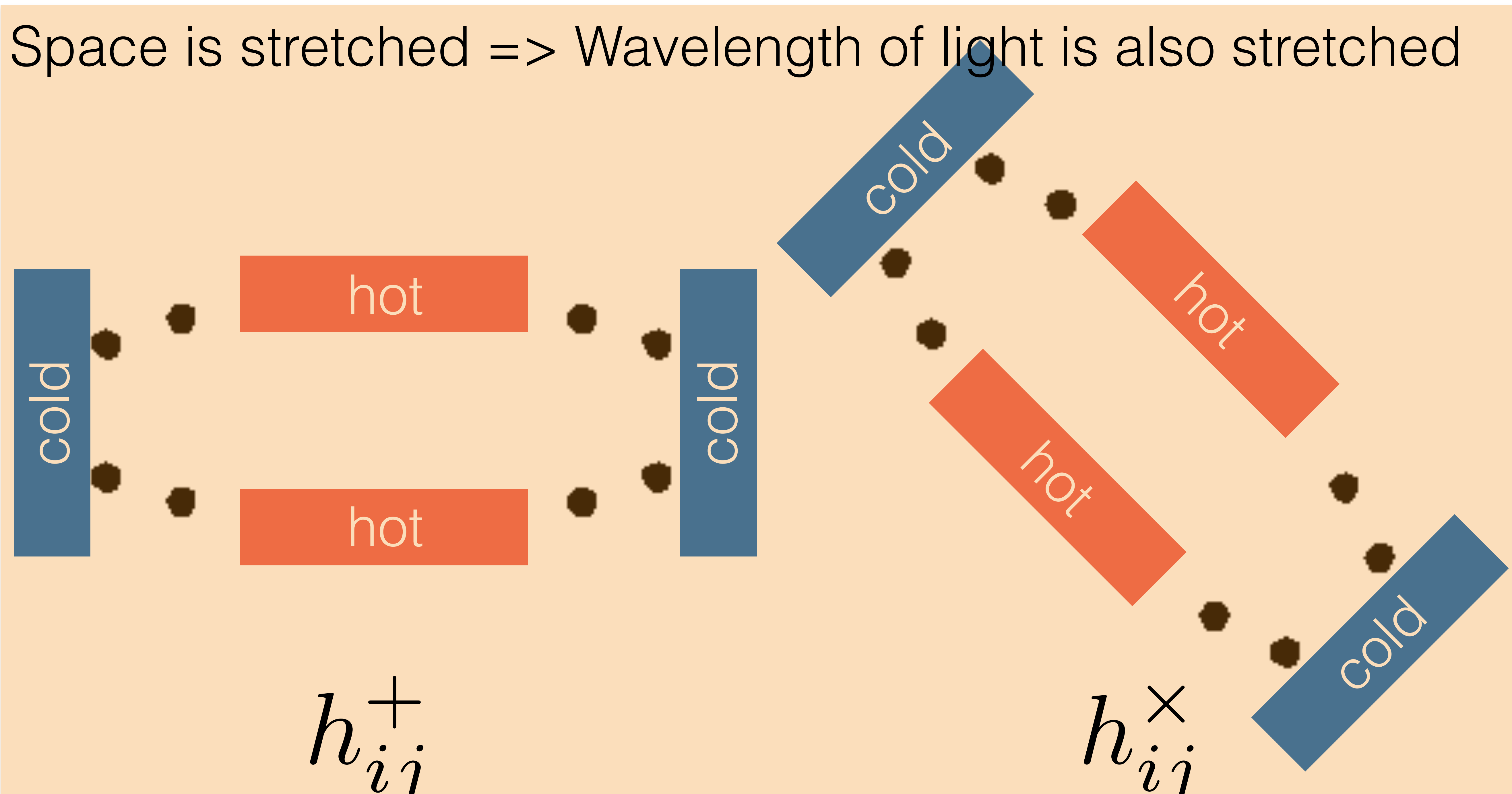
Detecting GW by CMB

GW propagating in isotropic electro-magnetic fields



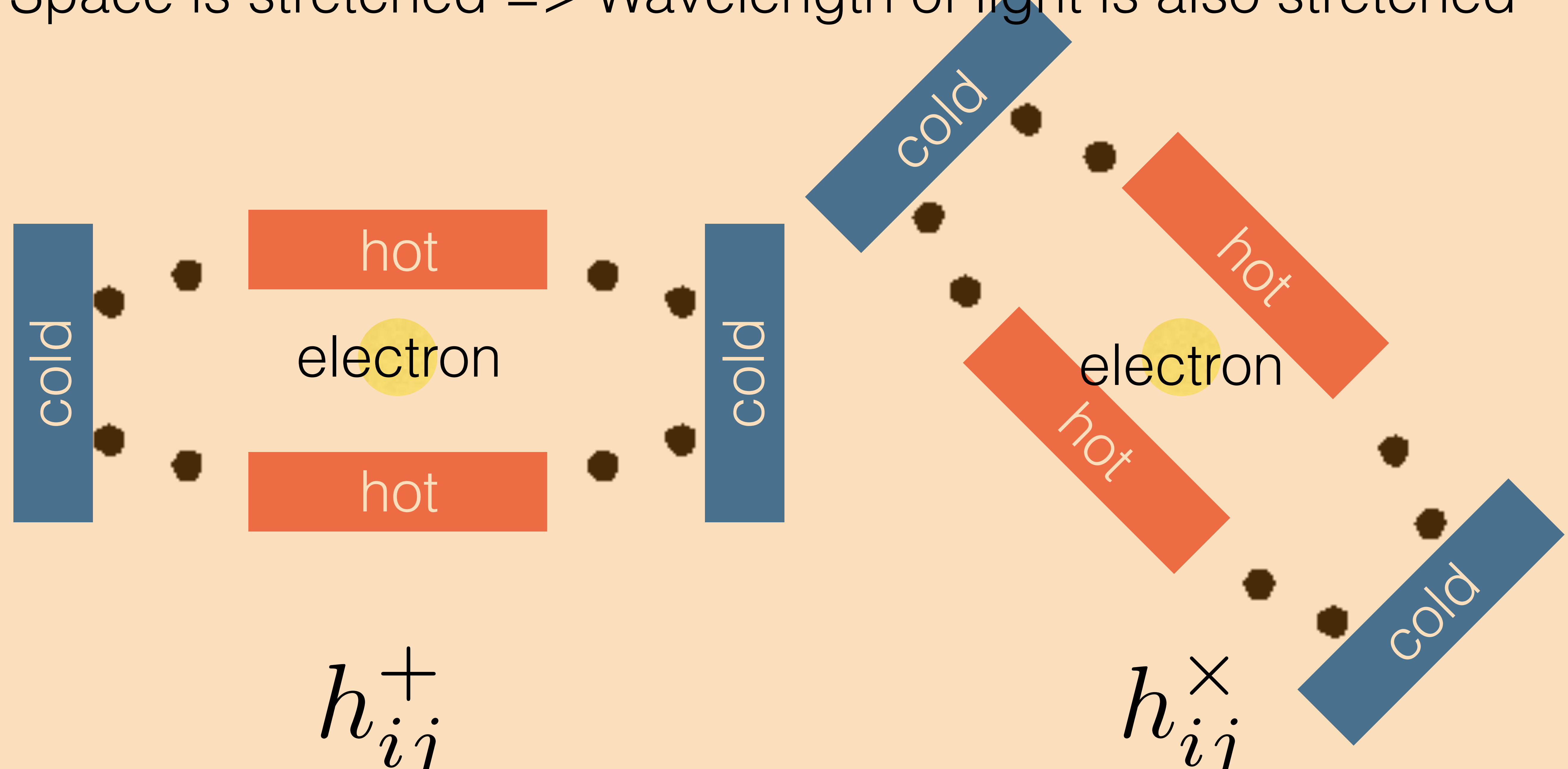
h_{ij}^{+} $h_{ij}^{\times}$

Detecting GW by CMB



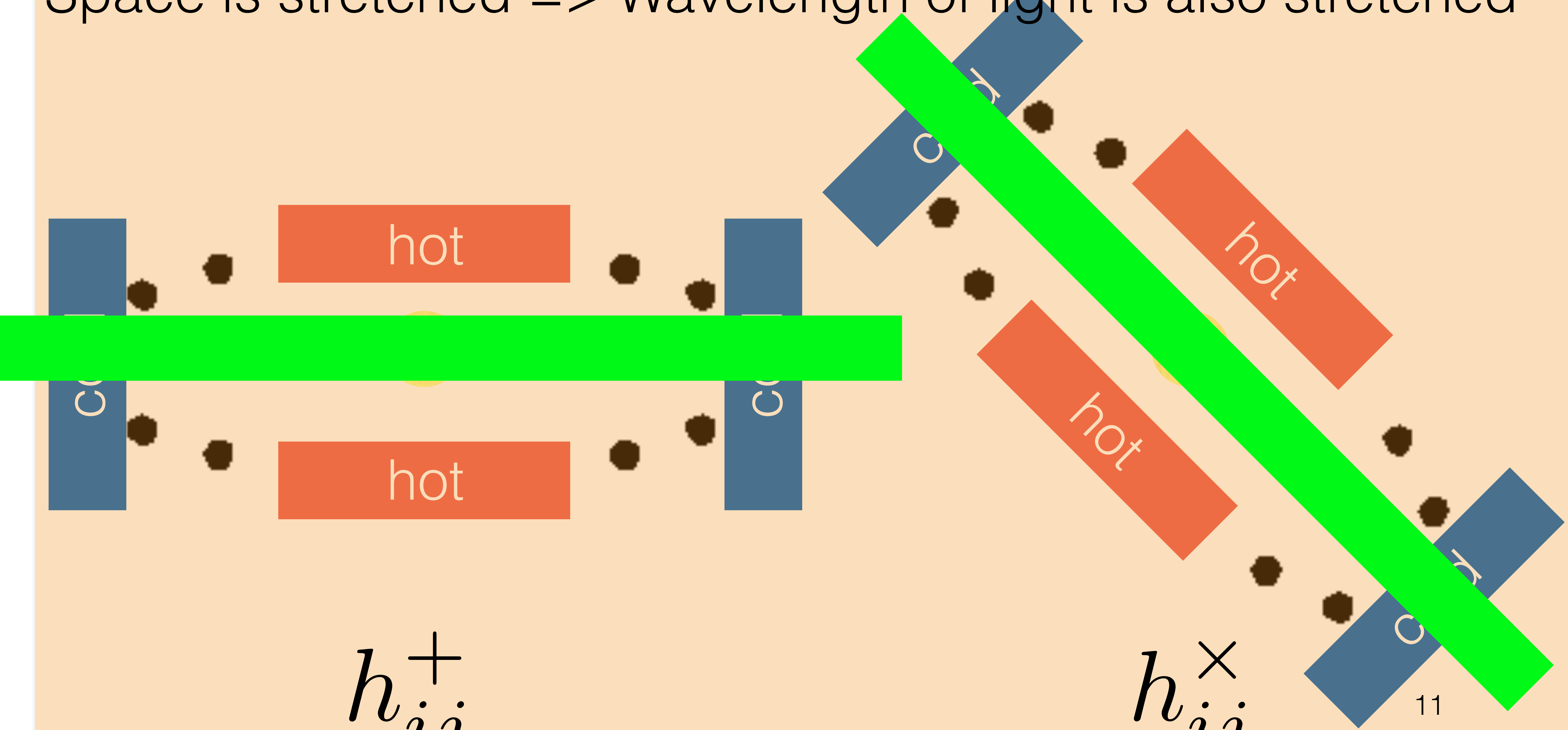
Detecting GW by CMB
Polarisation

Space is stretched => Wavelength of light is also stretched



Detecting GW by CMB
Polarisation

Space is stretched => Wavelength of light is also stretched

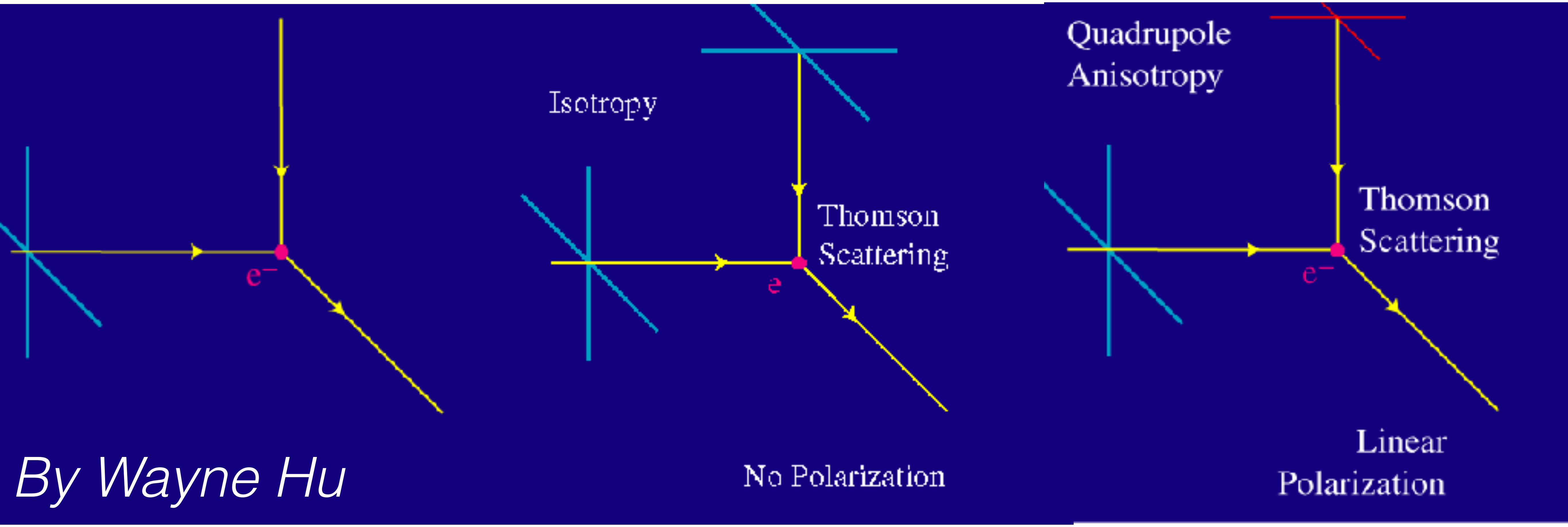


h_{ij}^{+} $h_{ij}^{\times}$

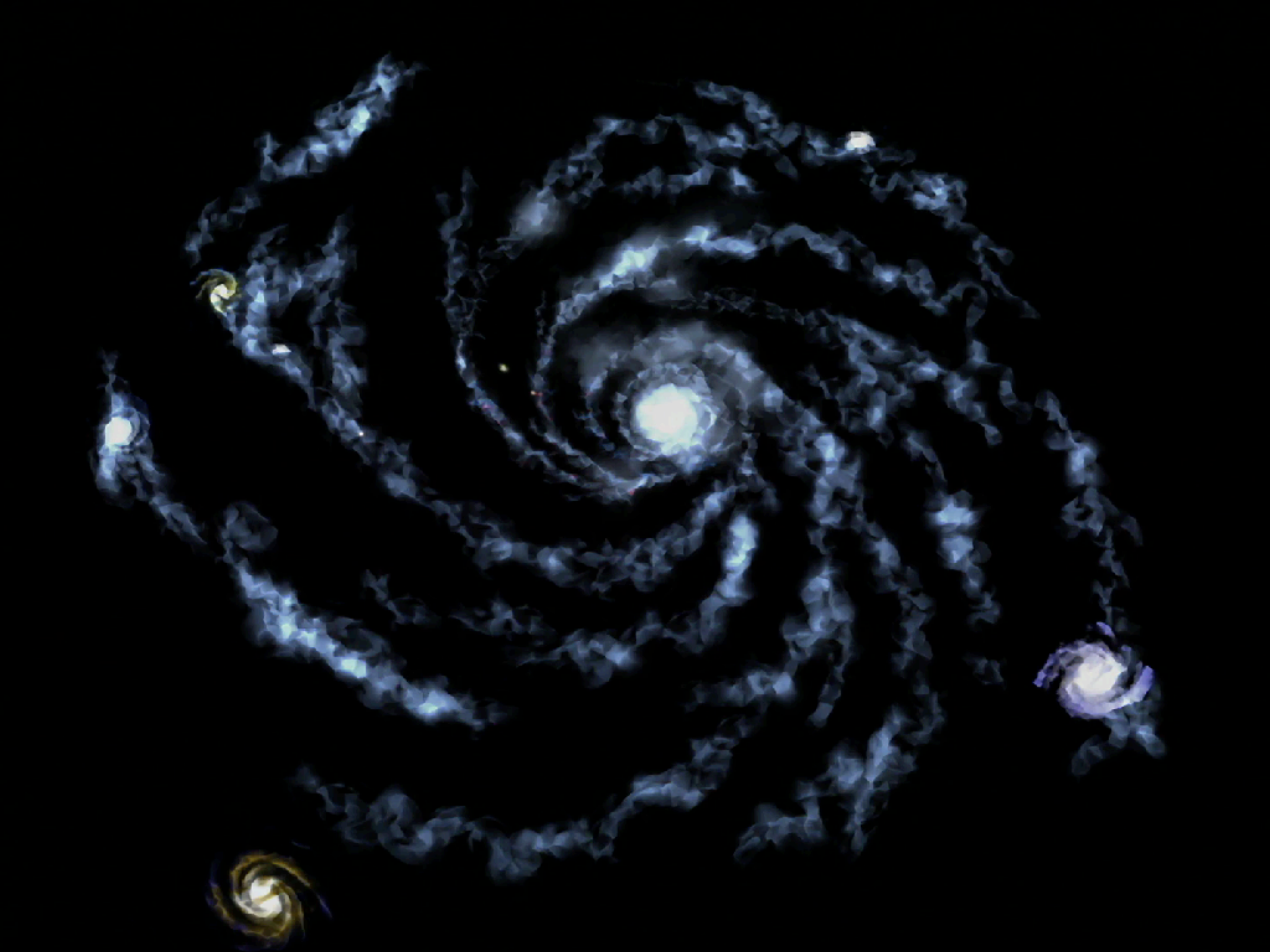

Photo Credit: TALEX
horizontally polarised

Photo Credit: TALEX

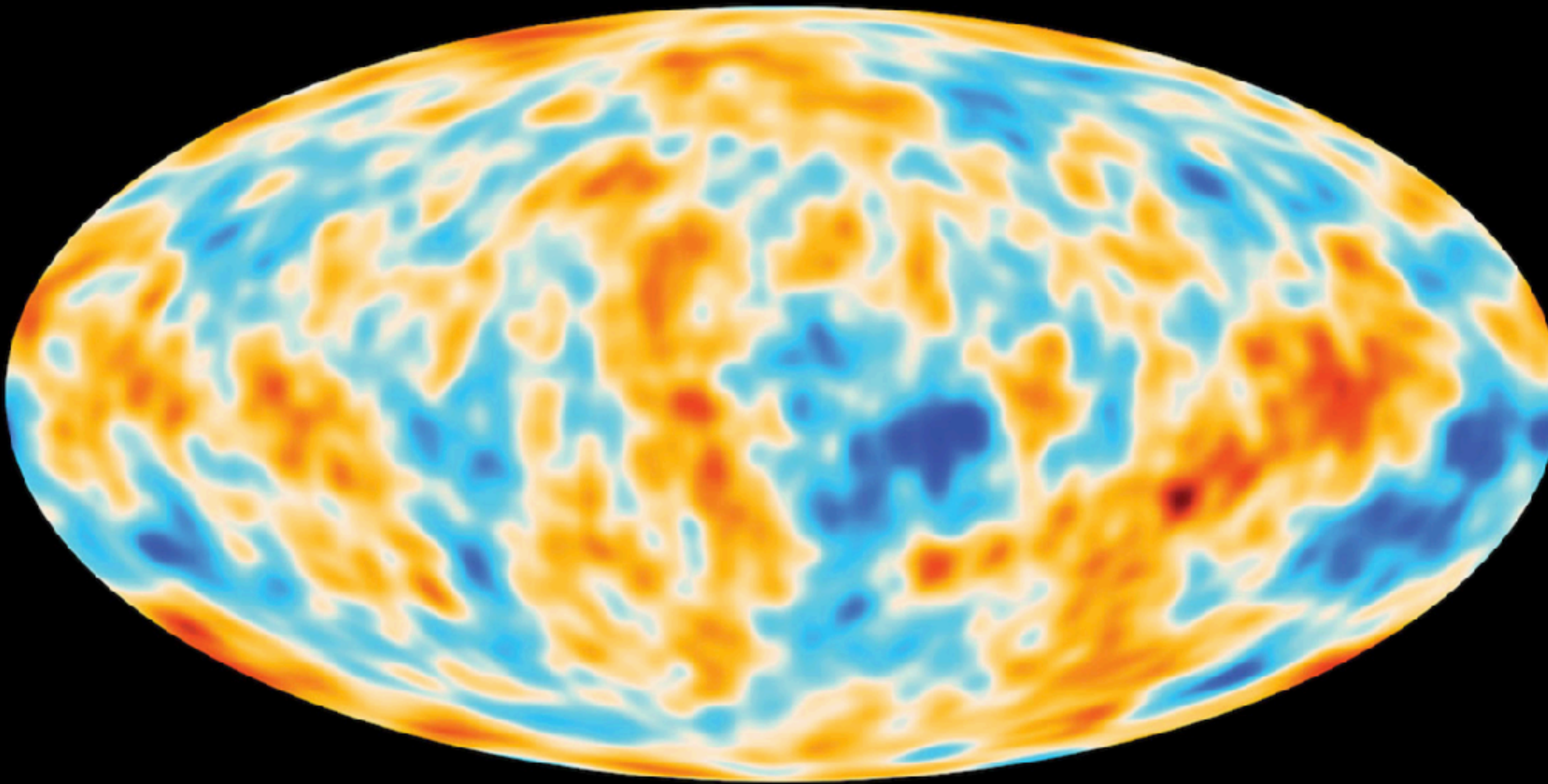
Physics of CMB Polarisation



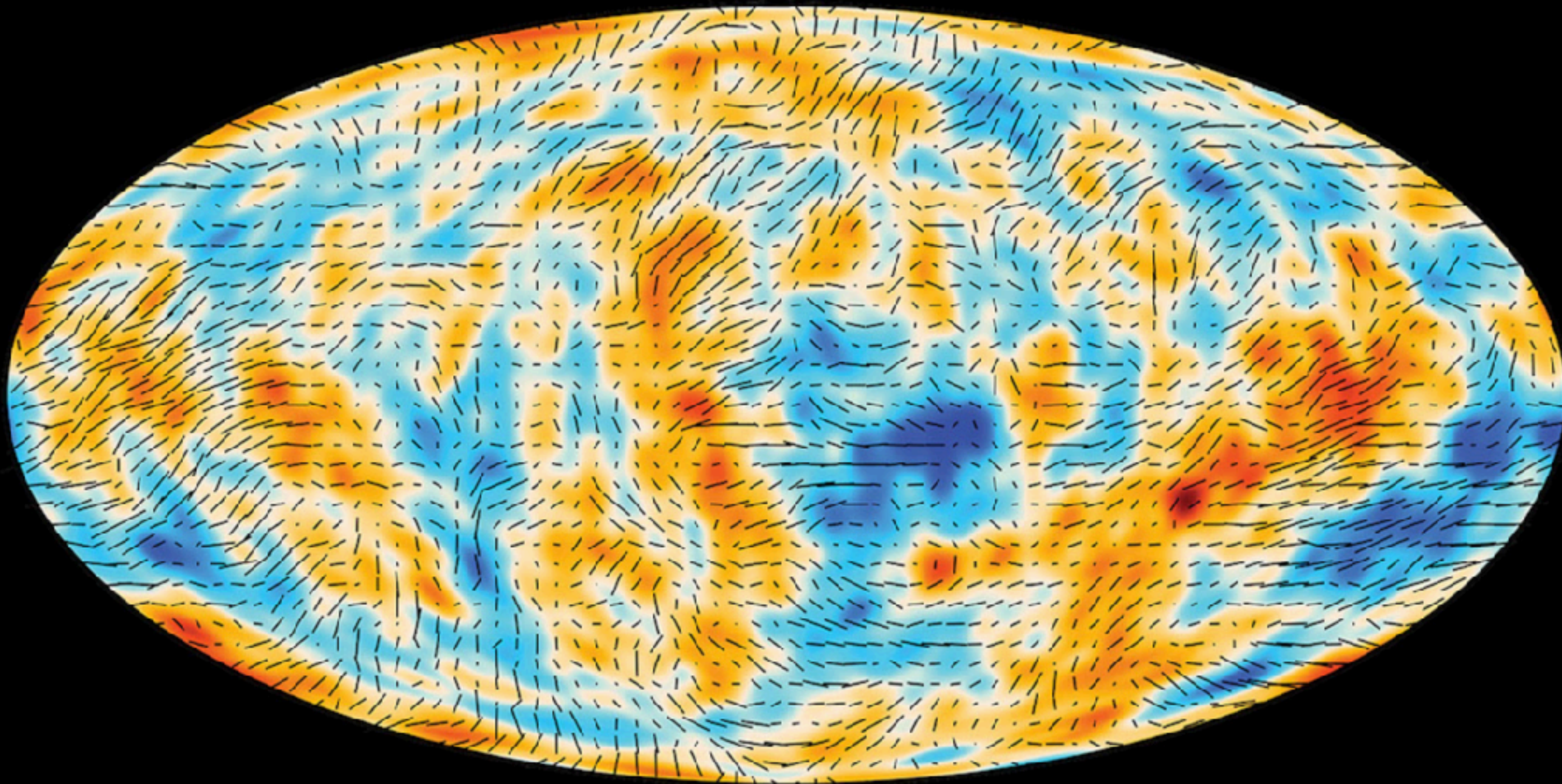
- Necessary and sufficient conditions for generating polarisation in CMB:
  - Thomson scattering
  - **Quadrupolar** temperature anisotropy around an electron



Credit: ESA

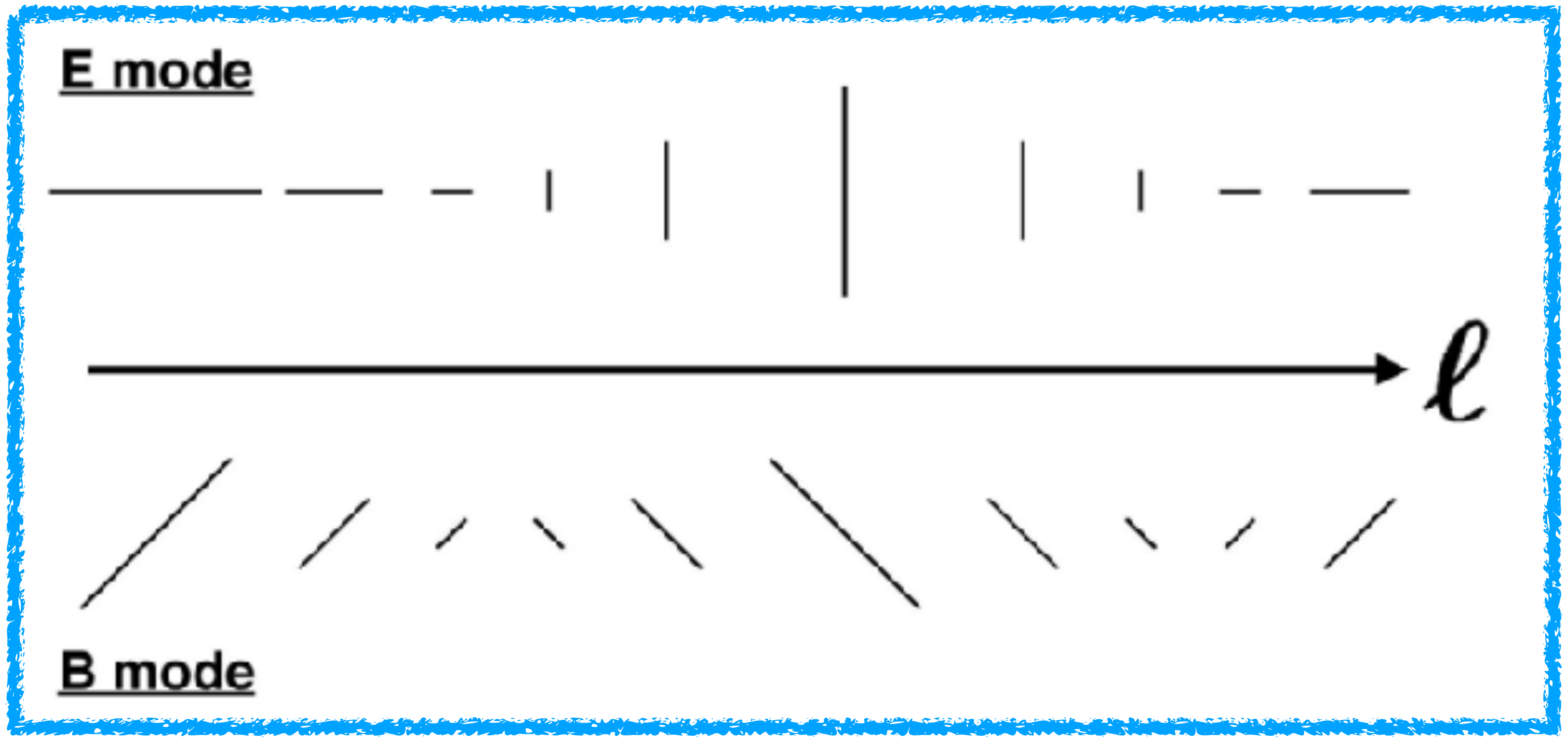


Temperature (smoothed)



Temperature (smoothed) + Polarisation

E and B mode

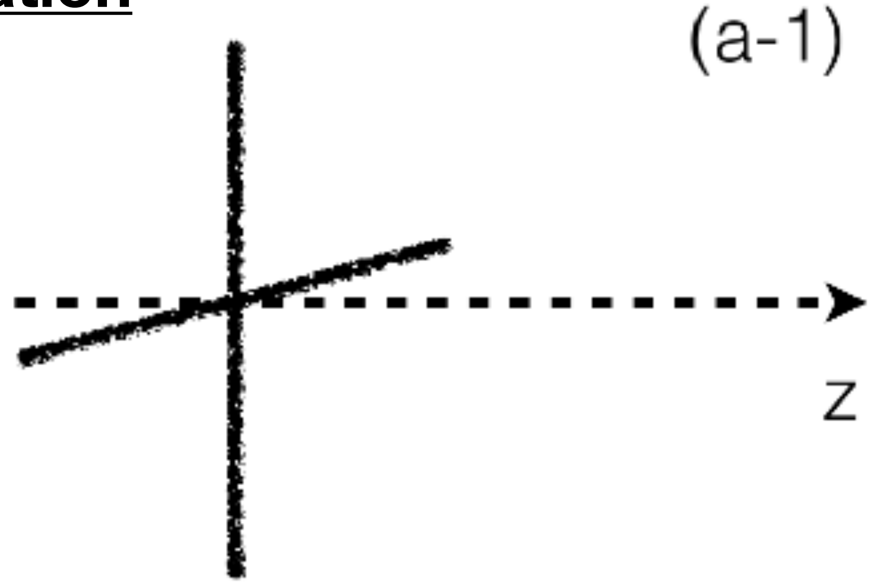


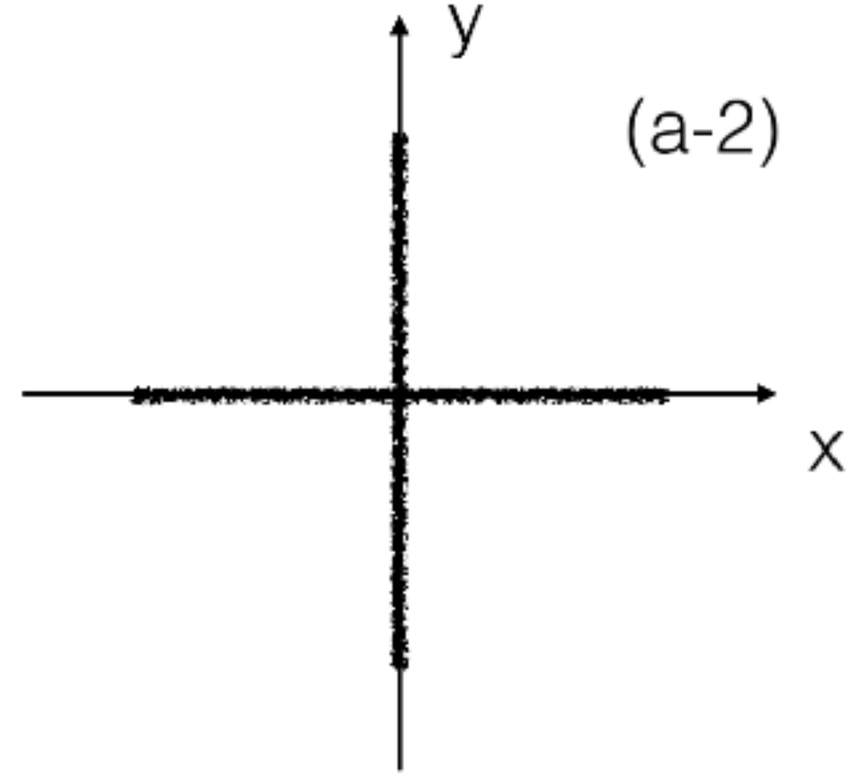
- **E mode**: Polarisation directions parallel or perpendicular to the wavevector
- **B mode**: Polarisation directions 45 degree tilted with respect to the wavevector

E&B decomposition:
A closer look

Polarisation

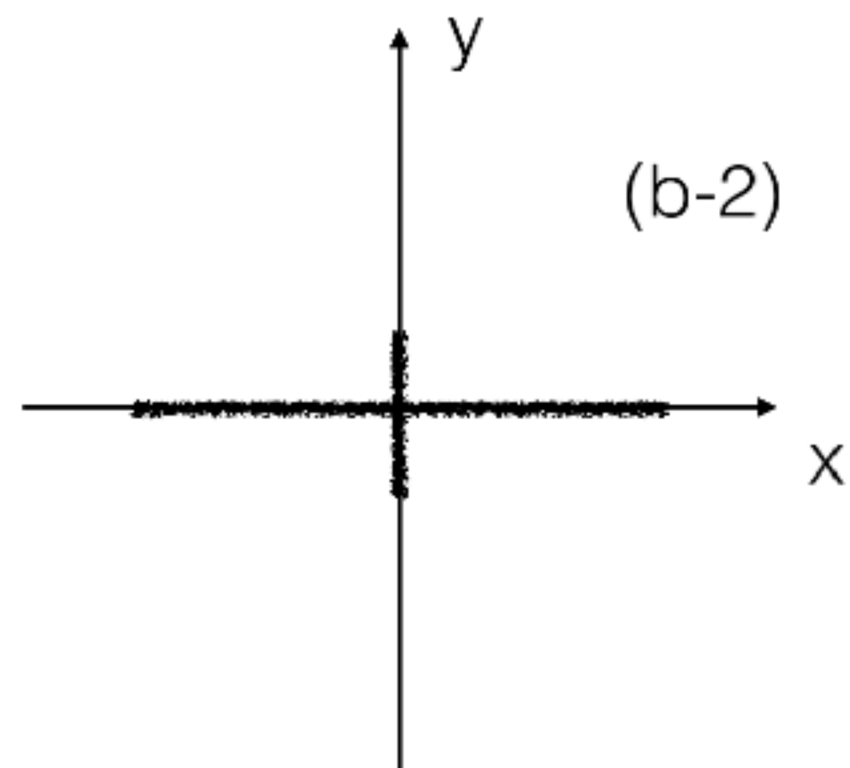
No polarisation





(b-1)

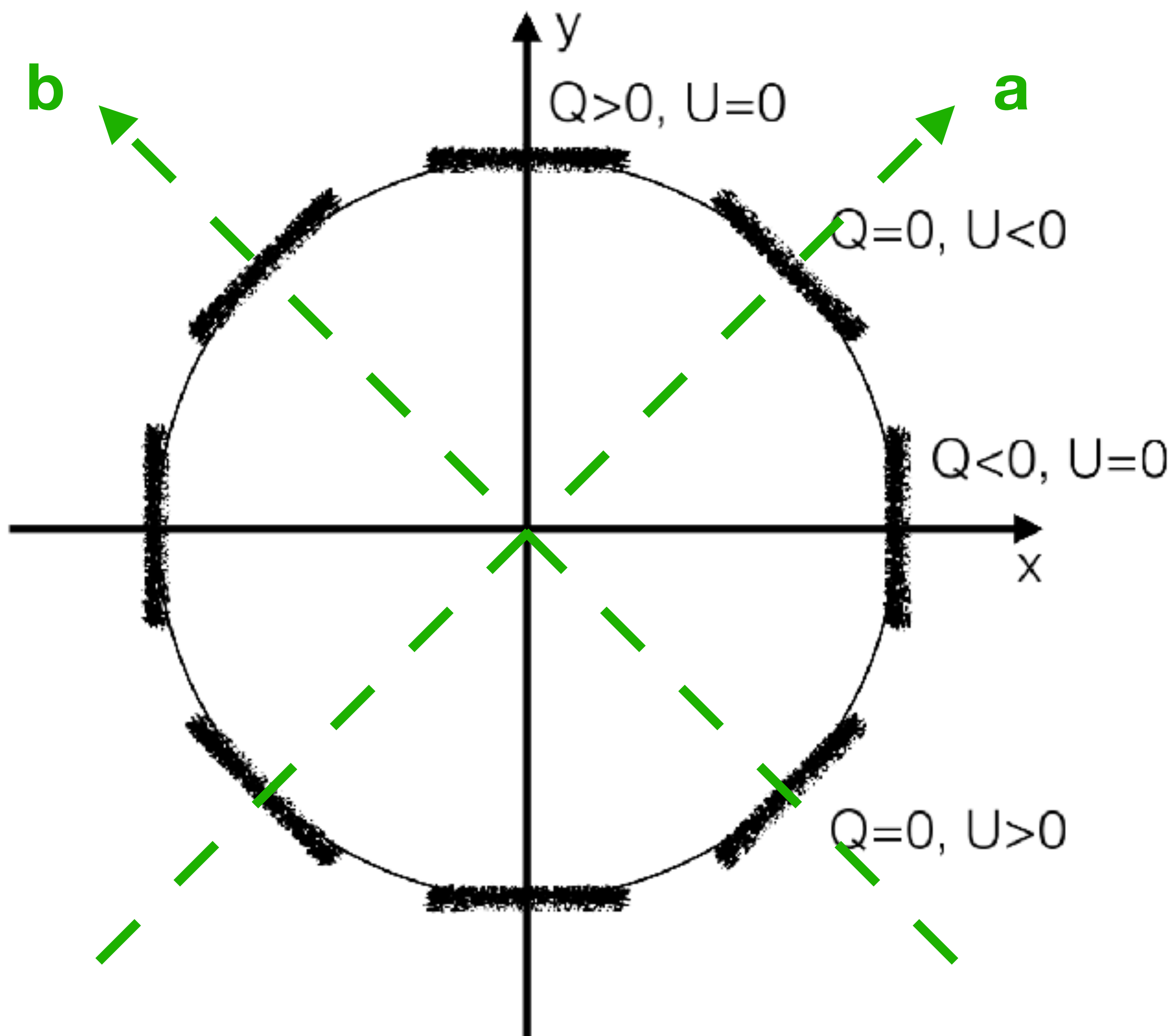
Polarised in x-direction



Stokes Parameters
[Flat Sky, Cartesian coordinates]

$$Q \propto E_x^2 - E_y^2$$

$$U \propto E_a^2 - E_b^2$$

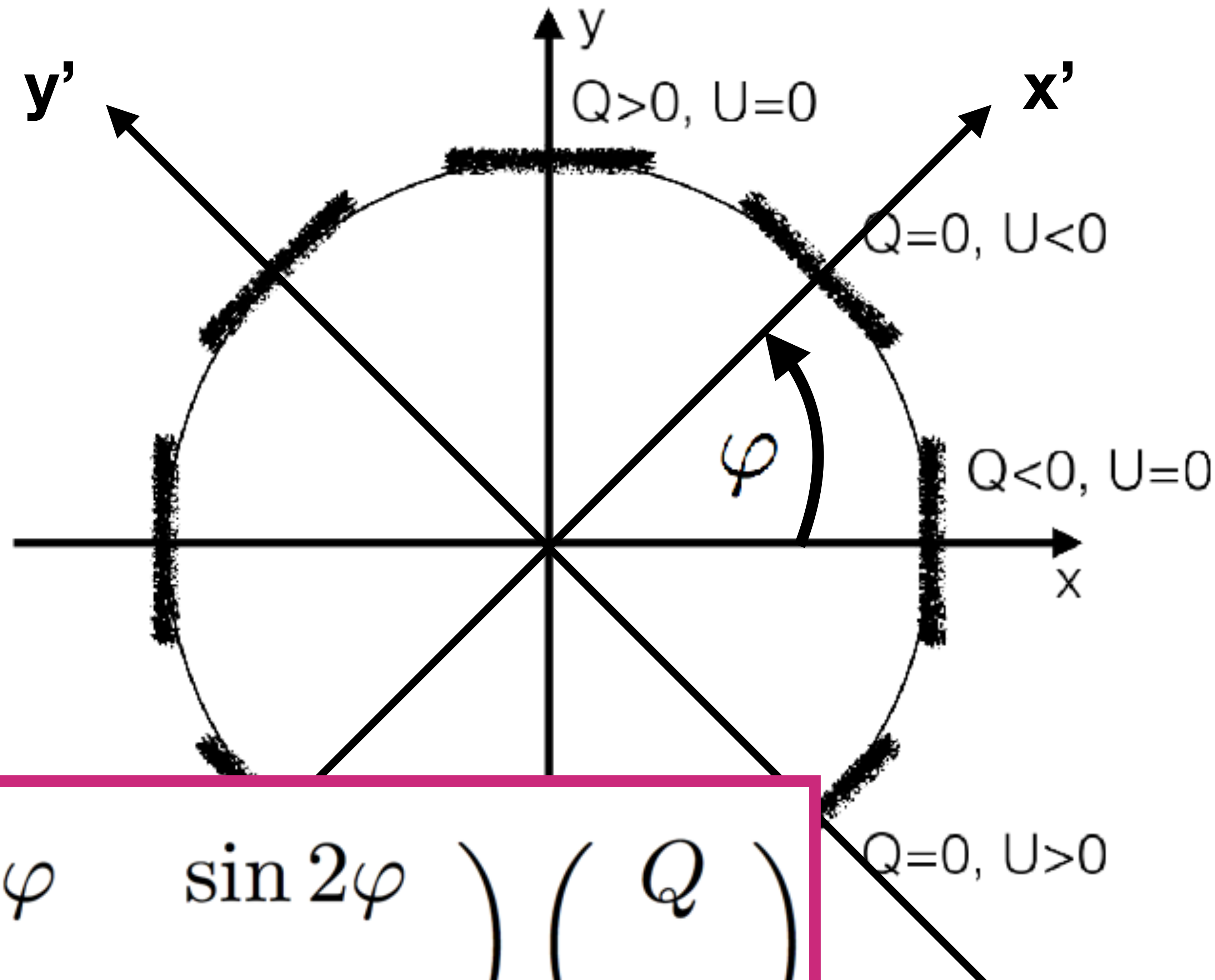


Stokes Parameters change under coordinate rotation

Under (x,y) -> (x',y'):

$$Q \longrightarrow \tilde{Q}$$

$$U \longrightarrow \tilde{U}$$



$$\begin{pmatrix} \tilde{Q} \\ \tilde{U} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$

Compact Expression

- Using an imaginary number, write $Q + iU$

Then, under coordinate rotation we have

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

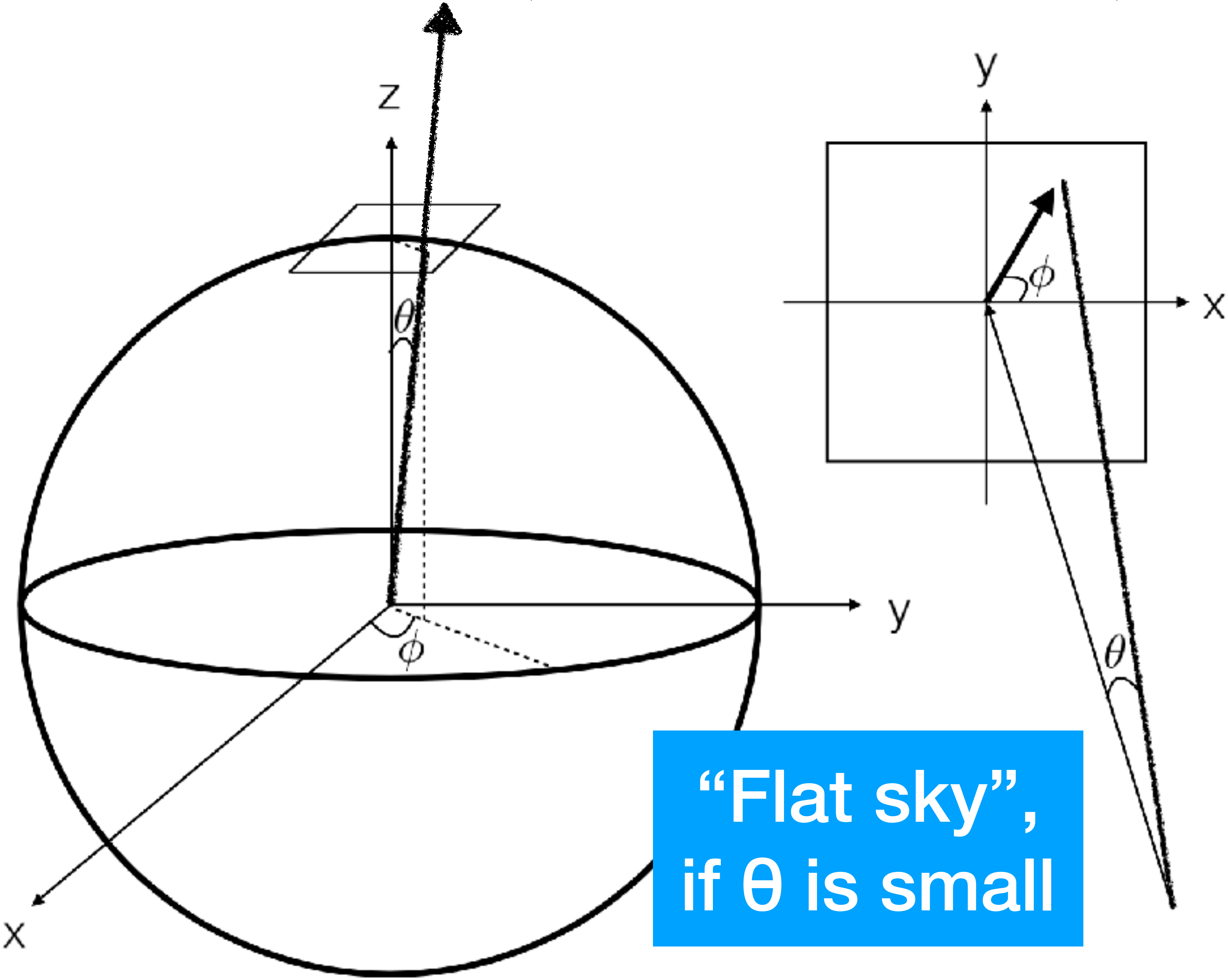
$$\tilde{Q} - i\tilde{U} = \exp(2i\varphi)(Q - iU)$$

E and B decomposition

- That Q and U depend on coordinates is not very convenient…
  - Someone said, “I measured Q!” but then someone else may say, “No, it’s U!”. They flight to death, only to realise that their coordinates are 45 degrees rotated from one another…
- The best way to avoid this unfortunate fight is to define a coordinate-independent quantity for the distribution of polarisation patterns in the sky

To achieve this, we need to go to Fourier space

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$



"Flat sky", if θ is small

Fourier-transforming Stokes Parameters?

$$Q(\boldsymbol{\theta}) + iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2}\; a_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

where

$$\boldsymbol{\ell} = (\ell\cos\phi_\ell, \ell\sin\phi_\ell)$$

- As Q+iU changes under rotation, the Fourier coefficients $a_{\boldsymbol{\ell}}$ change as well
- So…

(*) Nevermind the overall minus sign. This is just for convention

Tweaking Fourier Transform

$$Q(\boldsymbol{\theta}) + iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2}\; a_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

where we write the coefficients as(*)

$$a_{\boldsymbol{\ell}} = -{}_2a_{\boldsymbol{\ell}} \exp(2i\phi_\ell)$$

- Under rotation, the azimuthal angle of a Fourier wavevector, ϕ_l, changes as $\phi_\ell \to \tilde{\phi}_\ell = \phi_\ell - \varphi$
- This cancels the factor in the left hand side:

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

Seljak (1997); Zaldarriaga & Seljak (1997); Kamionkowski, Kosowky, Stebbins (1997)

Tweaking Fourier Transform

- We thus write

$$Q(\boldsymbol{\theta}) \pm iU(\boldsymbol{\theta}) = -\int \frac{d^2\ell}{(2\pi)^2}\, {}_{\pm 2}a_{\boldsymbol{\ell}} \exp(\pm 2i\phi_\ell + i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

- And, defining ${}_{\pm 2}a_{\boldsymbol{\ell}} \equiv -(E_{\boldsymbol{\ell}} \pm iB_{\boldsymbol{\ell}})$

$$Q(\boldsymbol{\theta}) \pm iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2}\, (E_{\boldsymbol{\ell}} \pm iB_{\boldsymbol{\ell}}) \exp(\pm 2i\phi_\ell + i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

By construction E_l and B_l do not pick up a factor of exp(2iϕ) under coordinate rotation. **That's great!** What kind of polarisation patterns do these quantities represent?

Pure E, B Modes

- Q and U produced by E and B modes are given by

$$Q(\boldsymbol{\theta}) \quad = \quad \int \frac{d^2\ell}{(2\pi)^2}(E_{\boldsymbol{\ell}}\cos 2\phi_\ell - B_{\boldsymbol{\ell}}\sin 2\phi_\ell)\exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

$$U(\boldsymbol{\theta}) \quad = \quad \int \frac{d^2\ell}{(2\pi)^2}(E_{\boldsymbol{\ell}}\sin 2\phi_\ell + B_{\boldsymbol{\ell}}\cos 2\phi_\ell)\exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

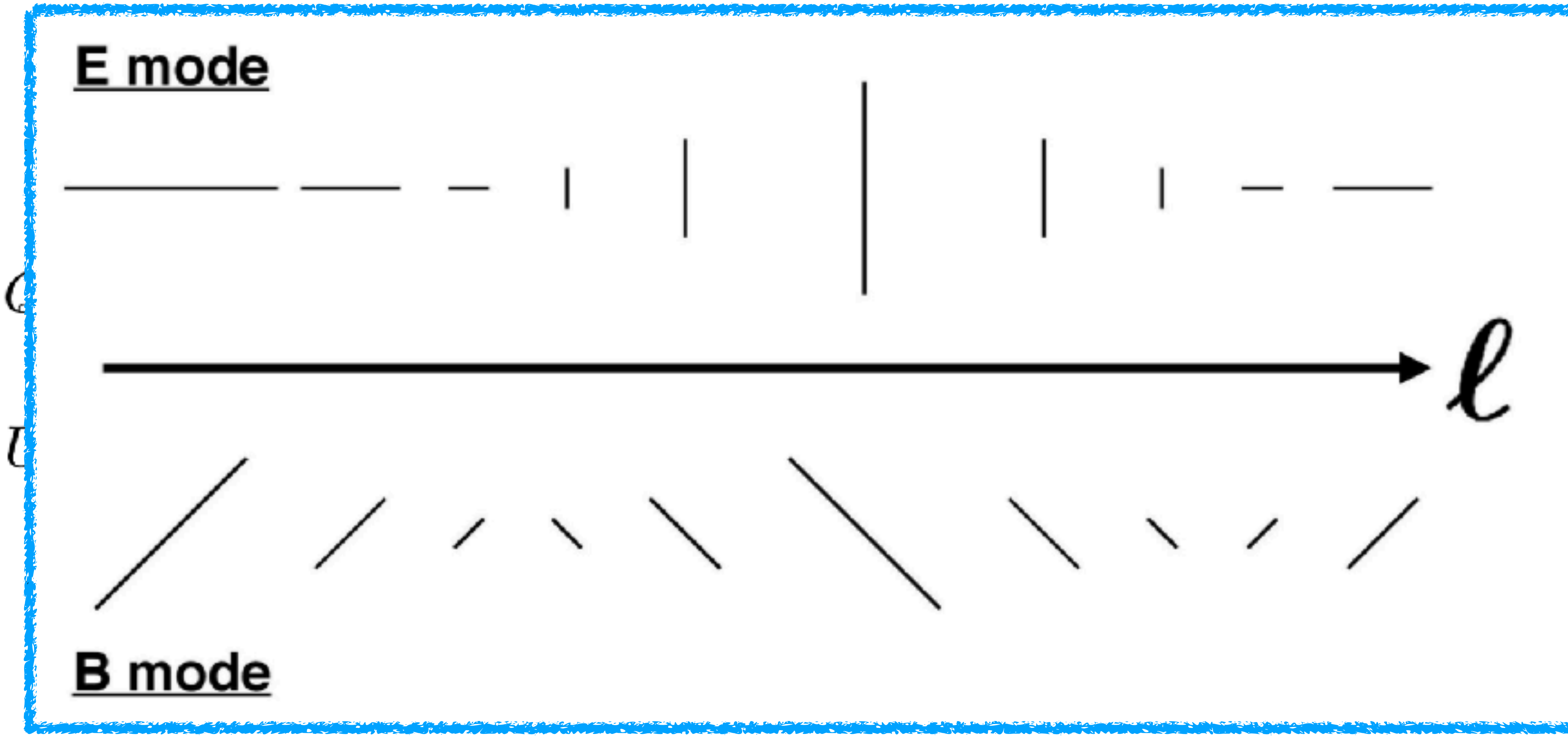
- Let’s consider Q and U that are produced by a single Fourier mode

- Taking the x-axis to be the direction of a wavevector, we obtain

$$Q(\theta) = \Re\left[E_\ell \exp(i\ell\theta)\right]$$

$$U(\theta) = \Re\left[B_\ell \exp(i\ell\theta)\right]$$

Pure E, B Modes

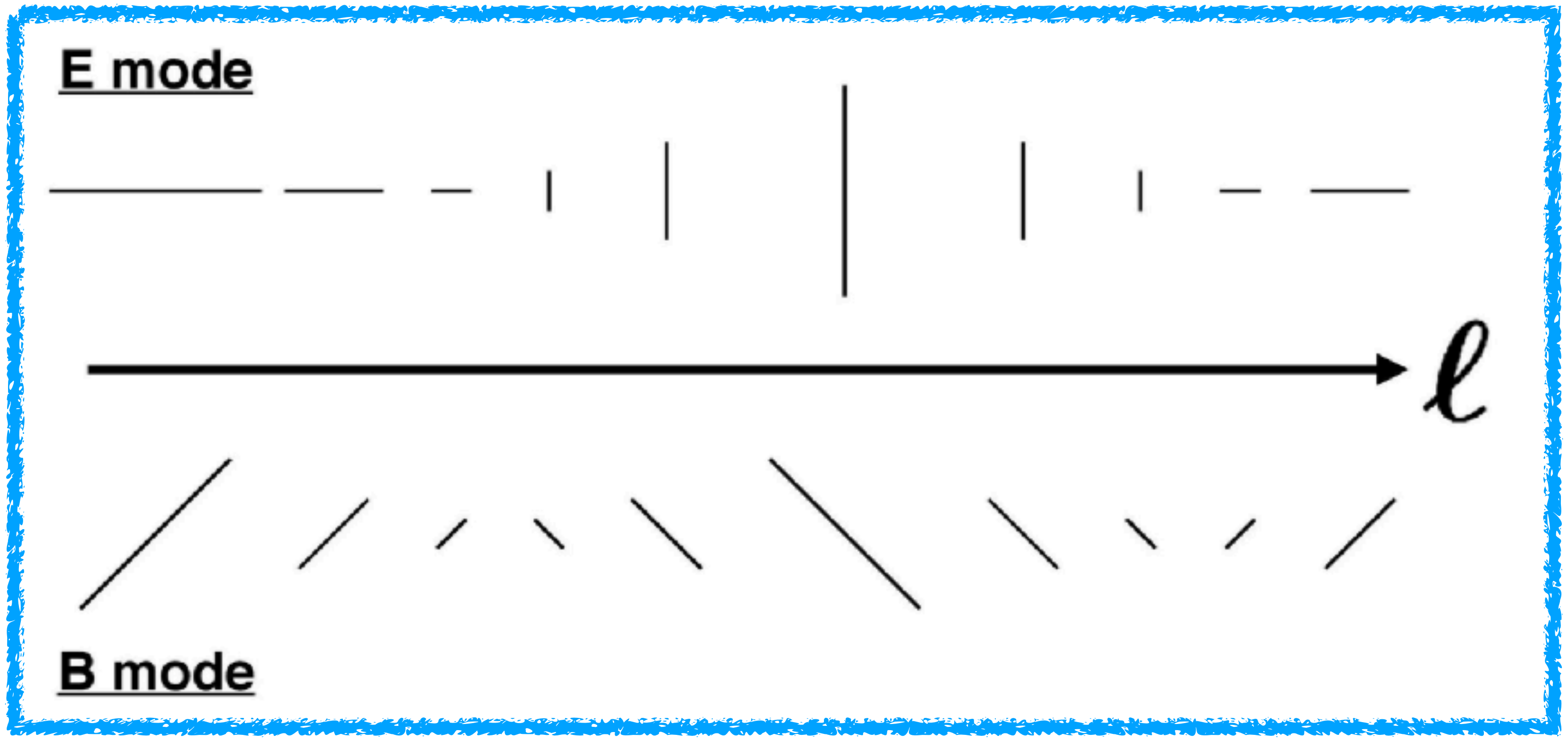


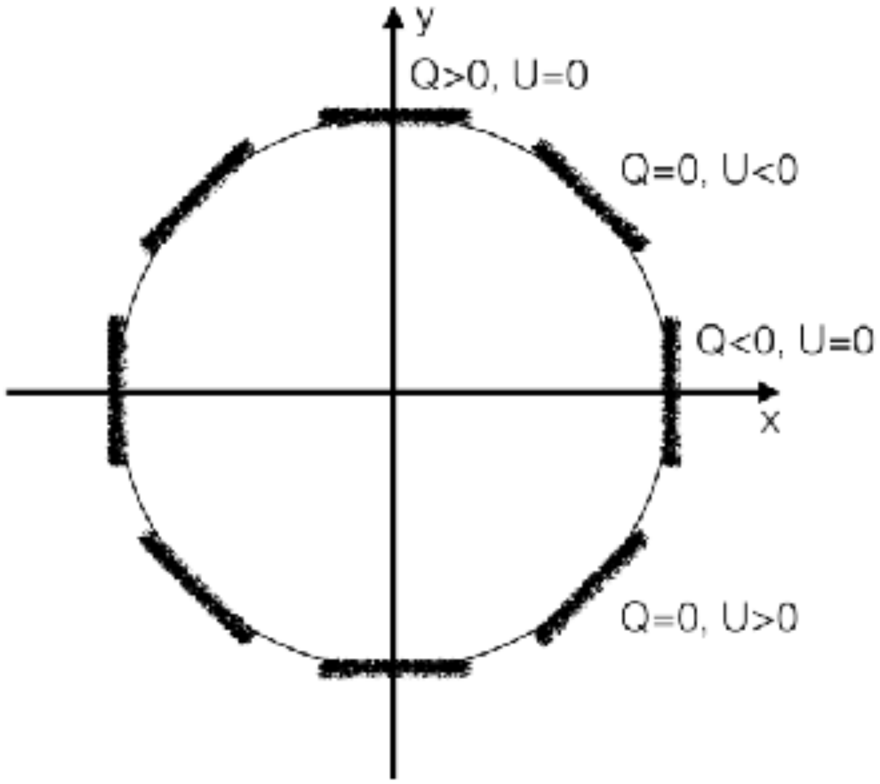
- Taking the x-axis to be the direction of a wavevector, we obtain

$$Q(\theta) = \Re\left[E_\ell \exp(i\ell\theta)\right]$$

$$U(\theta) = \Re\left[B_\ell \exp(i\ell\theta)\right]$$

Geometric Meaning



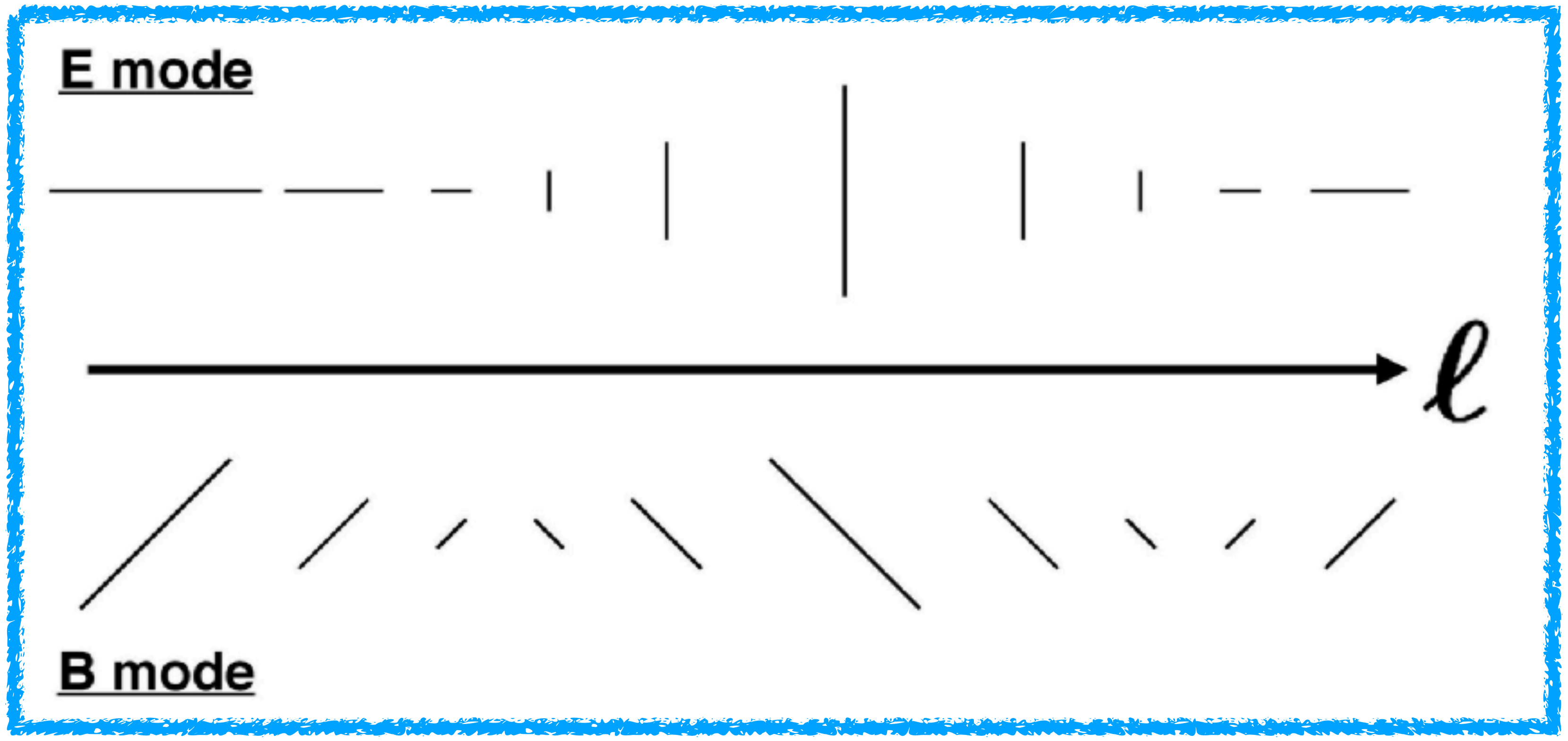


- **E mode**: Stokes Q, defined with respect to ℓ as the x-axis

- **B mode**: Stokes U, defined with respect to ℓ as the y-axis

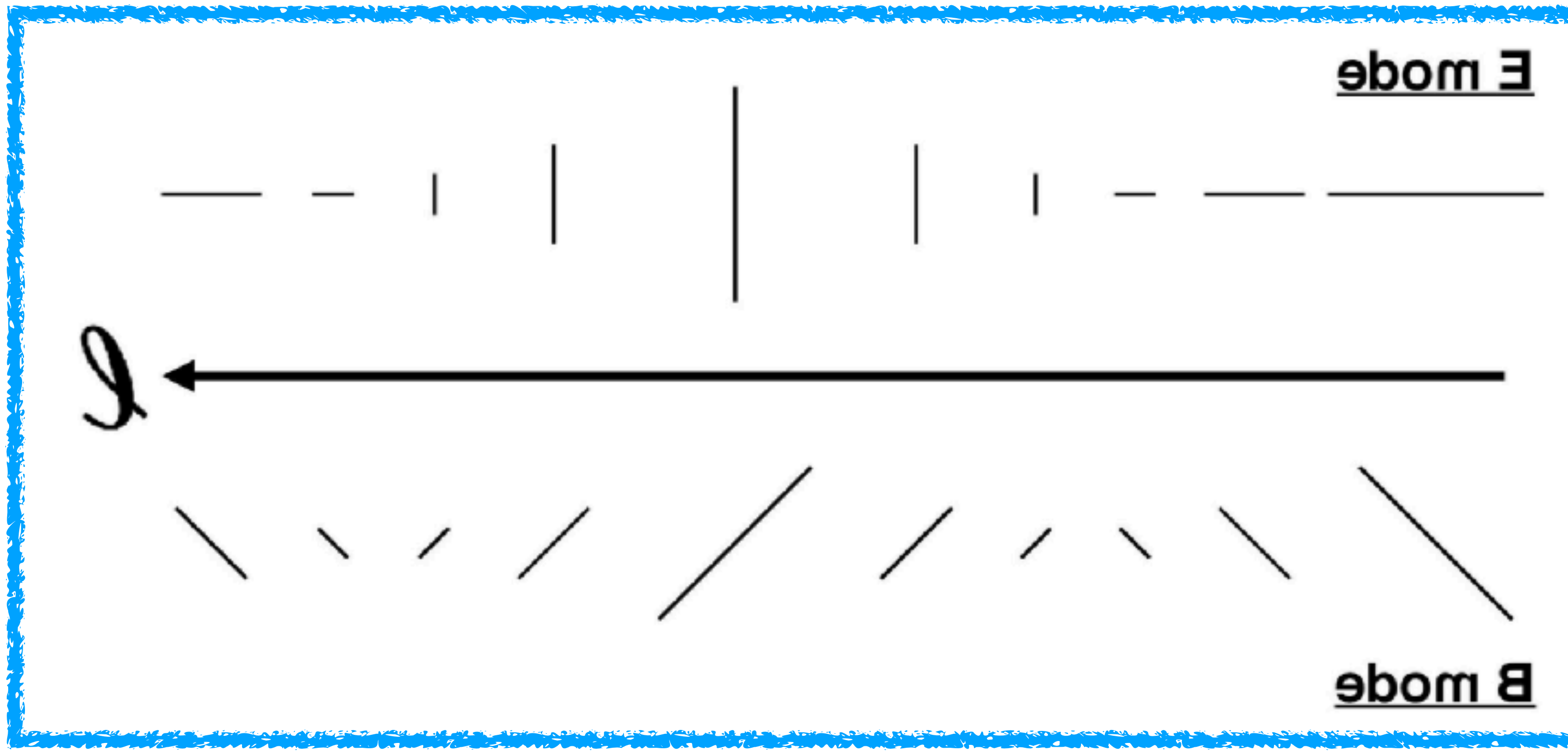
IMPORTANT: These are all coordinate-independent statements

Parity



- **<u>E mode</u>**: Parity even
- **<u>B mode</u>**: Parity odd

Parity



- **E mode**: Parity even
- **B mode**: Parity odd

Power Spectra

$$\langle E_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{EE}$$

$$\langle B_{\boldsymbol{\ell}} B^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{BB}$$

$$\langle T_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = \langle T^*_{\boldsymbol{\ell}} E_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{TE}$$

- However, <EB> and <TB> vanish for parity-preserving fluctuations because <EB> and <TB> change sign under parity flip

Power Spectra

$$\langle E_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{EE}$$

$$\langle B_{\boldsymbol{\ell}} B^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{BB}$$

$$\langle T_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = \langle T^*_{\boldsymbol{\ell}} E_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{TE}$$

<EB> and <TB> can be non-zero if physics that produced gravitational waves during information broke parity! This will be the topic on March 19

How do gravitational waves generate E&B polarisation?

Distance between two points in space

- Inhomogeneous curved space
  - In Cartesian **comoving** coordinates $\boldsymbol{x} = (x, y, z)$

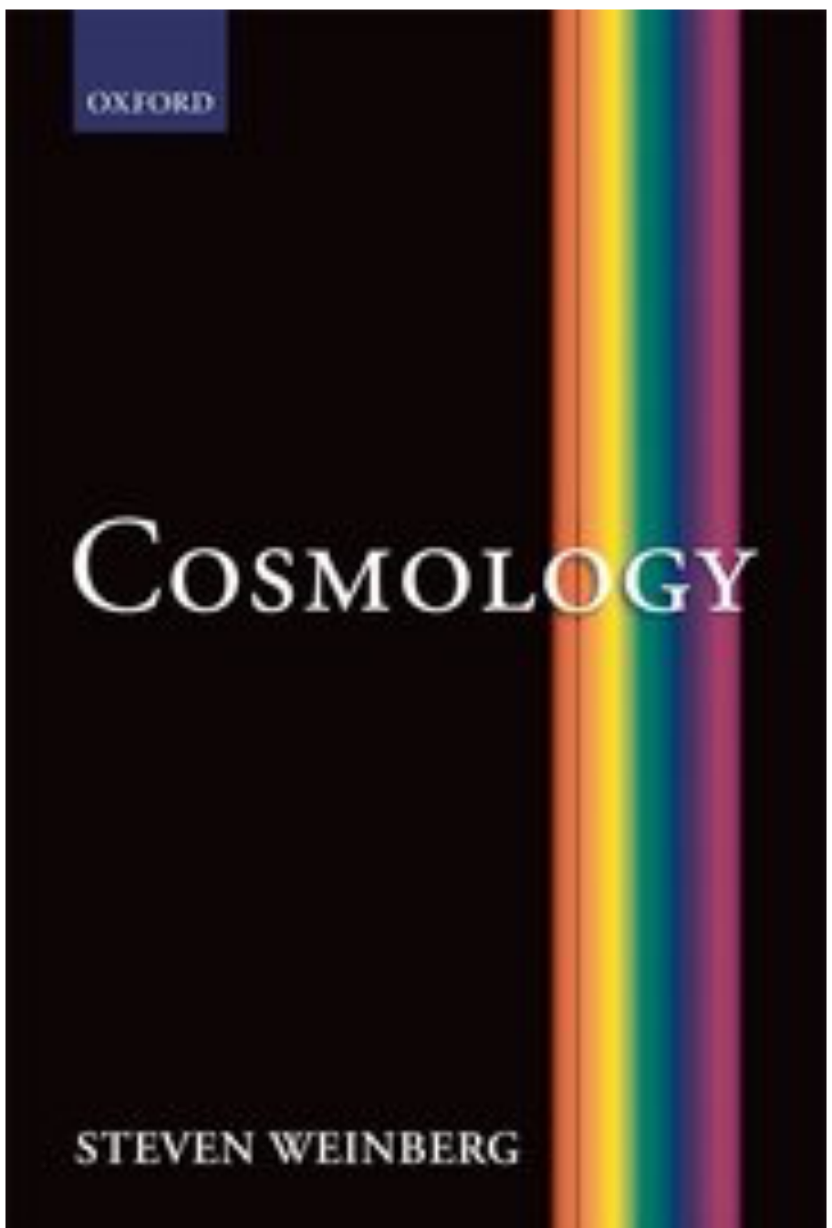
$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + \boxed{h_{ij}}) dx^i dx^j$$

"**metric perturbation**"

-> CURVED SPACE!

Now I am going to change notation (sorry!): h_{ij} -> D_{ij}

- Notation in today's lecture follows that of the text book "Cosmology" by Steven Weinberg



Space-time Distance

- Einstein told us that a clock ticks slowly when gravity is strong…
- Space-time distance, ds_4, is modified by the presence of gravitational fields

$$ds_4^2 = -\exp(2\Phi)dt^2 + a^2\exp(-2\Psi)\sum_{i=1}^{3}\sum_{j=1}^{3}[\exp(D)]_{ij}dx^i dx^j$$

Φ **: Newton's gravitational potential**

Ψ **: Spatial scalar curvature perturbation**

D_{ij} **: Tensor metric perturbation [=gravitational waves]**

Tensor perturbation D_{ij}: Area-conserving deformation

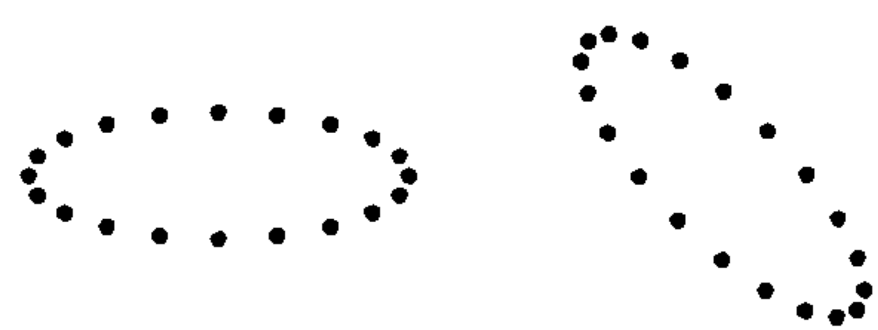
- Determinant of a matrix

$$[\exp(D)]_{ij} \equiv \delta_{ij} + D_{ij} + \frac{1}{2}\sum_{k=1}^{3} D_{ik}D_{kj} + \frac{1}{6}\sum_{km} D_{ik}D_{km}D_{mj} + \cdots$$

is given by $\exp(\sum_i D_{ii})$

- **Thus, D_{ij} must be trace-less** $\sum_i D_{ii} = 0$

if it is area-conserving deformation of two points in space



Curvature Perturbation

- Einstein told us that a clock ticks slowly when gravity is strong…
- Space-time distance, ds_4, is modified by the presence of gravitational fields

$$ds_4^2 = -\exp(2\Phi)dt^2 + a^2 \exp(-2\Psi) \sum_{i=1}^{3} \sum_{j=1}^{3} [\exp(D)]_{ij} dx^i dx^j$$

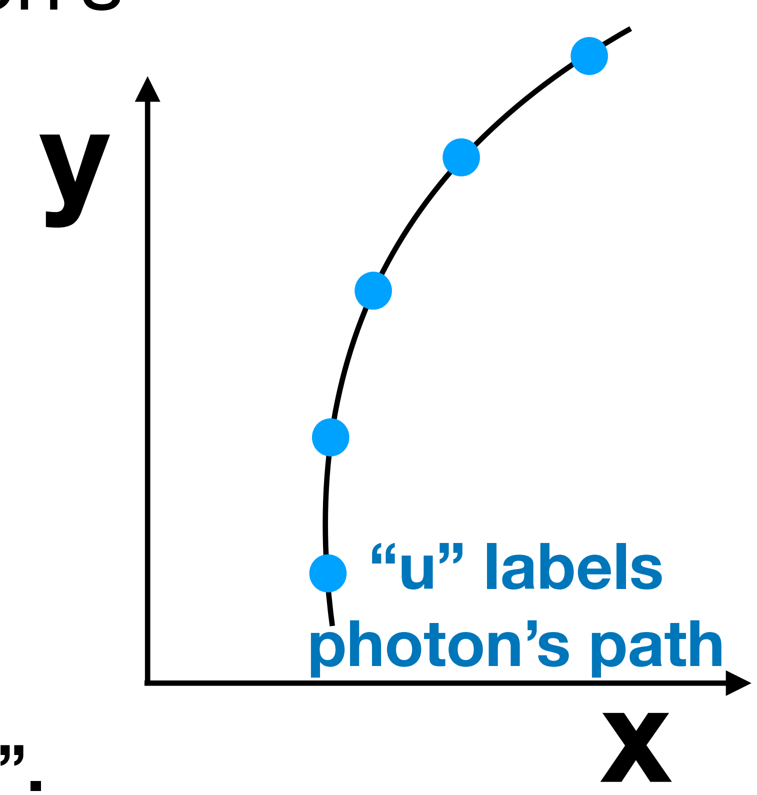
Φ **: Newton's gravitational potential**

Ψ **: Spatial scalar curvature perturbation**
is a perturbation to the determinant of spatial metric

Evolution of photon's coordinates

- Photon's path is determined such that the distance traveled by a photon between two points is minimised. This yields the equation of motion for photon's coordinates $x^\mu = (t, x^i)$

$$\frac{d^2x^\lambda}{du^2} + \sum_{\mu=0}^{3}\sum_{\nu=0}^{3}\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{du}\frac{dx^\nu}{du} = 0$$



This equation is known as the "geodesic equation".

The second term is needed to keep the form of the equation unchanged under general coordinate transformation => GRAVITATIONAL EFFECTS!

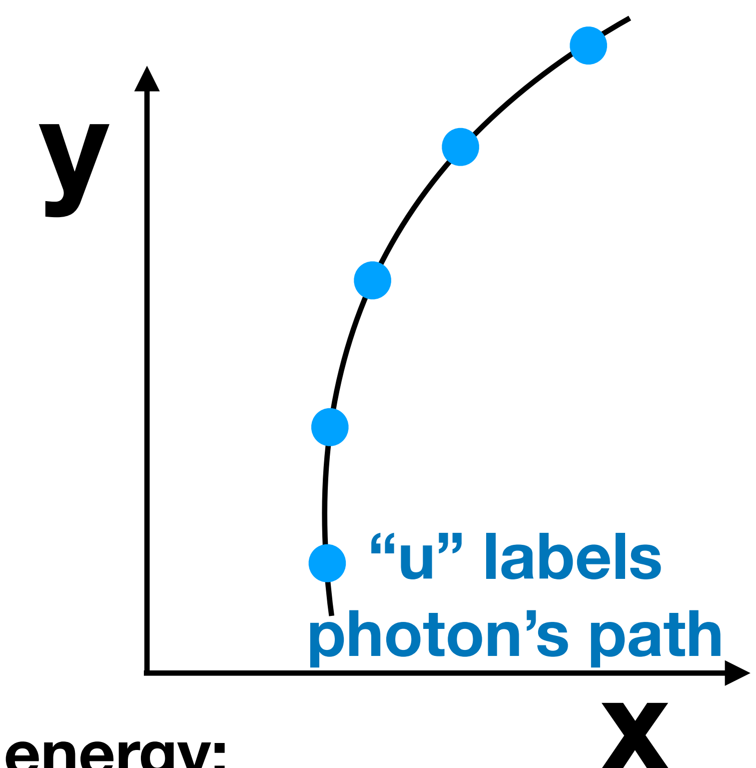
Evolution of photon's momentum

- It is more convenient to write down the geodesic equation in terms of the **photon momentum**:

$$p^\mu \equiv \frac{dx^\mu}{du}$$

then

$$\frac{dp^\lambda}{dt} + \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \Gamma^\lambda_{\mu\nu}\frac{p^\mu p^\nu}{p^0} = 0$$



Magnitude of the photon momentum is equal to the photon energy:

$$p^2 \equiv \sum_{i=1}^{3}\sum_{j=1}^{3} g_{ij}p^i p^j$$

Some calculations...

$$\frac{dp^\lambda}{dt} + \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \Gamma^\lambda_{\mu\nu} \frac{p^\mu p^\nu}{p^0} = 0$$

With $ds_4^2 = \sum_{\mu\nu} g_{\mu\nu} dx^\mu dx^\nu$ $\left(\begin{array}{l} g_{00} = -\exp(2\Phi), \quad g_{0i} = 0, \\ g_{ij} = a^2 \exp(-2\Psi)[\exp(D)]_{ij} \end{array} \right)$

$$\Gamma^\lambda_{\mu\nu} \equiv \frac{1}{2} \sum_{\rho=0}^{3} g^{\lambda\rho} \left(\frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right)$$

Scalar perturbation [valid to all orders]

$$\Gamma^0_{00} = \dot{\Phi}, \quad \Gamma^0_{0i} = \frac{\partial \Phi}{\partial x^i}, \quad \Gamma^i_{00} = \exp(2\Phi) \sum_j g^{ij} \frac{\partial \Phi}{\partial x^j},$$

$$\Gamma^i_{0j} = \left(\frac{\dot{a}}{a} - \dot{\Psi} \right) \delta^i_j, \quad \Gamma^0_{ij} = \exp(-2\Phi) \left(\frac{\dot{a}}{a} - \dot{\Psi} \right) g_{ij},$$

$$\Gamma^k_{ij} = \delta_{ij} \sum_\ell \delta^{k\ell} \frac{\partial \Psi}{\partial x^\ell} - \delta^k_i \frac{\partial \Psi}{\partial x^j} - \delta^k_j \frac{\partial \Psi}{\partial x^i},$$

Tensor perturbation [valid to 1st order in D]

$$\Gamma^i_{0j} = \frac{\dot{a}}{a} \delta^i_j + \frac{1}{2} \sum_k \delta^{ik} \dot{D}_{kj}, \quad \Gamma^0_{ij} = \frac{\dot{a}}{a} g_{ij} + \frac{a^2}{2} \dot{D}_{ij},$$

$$\Gamma^k_{ij} = \frac{1}{2} \sum_\ell \delta^{k\ell} \left(\frac{D_{i\ell}}{\partial x^j} + \frac{D_{\ell j}}{\partial x^i} - \frac{D_{ij}}{\partial x^\ell} \right),$$

Recap

Math may be messy but the concept is transparent!

- Requiring **photons to travel between two points in space-time with the minimum path length**, we obtained the geodesic equation

- The geodesic equation contains $\Gamma^{\lambda}_{\mu\nu}$ that is required to make **the form of the equation unchanged under general coordinate transformation**

- Expressing $\Gamma^{\lambda}_{\mu\nu}$ in terms of the metric perturbations, we obtain the desired result - **the equation that describes the rate of change of the photon energy!**

$$p^2 \equiv \sum_{i=1}^{3} \sum_{j=1}^{3} g_{ij} p^i p^j$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

γ^i is a unit vector of the direction of photon's momentum:

$$\sum_i (\gamma^i)^2 = 1$$

- Let's interpret this equation *physically*

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

γ^i is a unit vector of the direction of photon's momentum:

$$\sum_i (\gamma^i)^2 = 1$$

- **Cosmological redshift**
  - Photon's wavelength is stretched in proportion to the scale factor, and thus the photon energy decreases as

$$p \propto a^{-1}$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Cosmological redshift - part II**
  - The spatial metric is given by $ds^2 = a^2(t)\exp(-2\Psi)d\mathbf{x}^2$
  - Thus, locally we can define a new scale factor: $\tilde{a}(t,\mathbf{x}) = a(t)\exp(-\Psi)$
  - Then the photon momentum decreases as

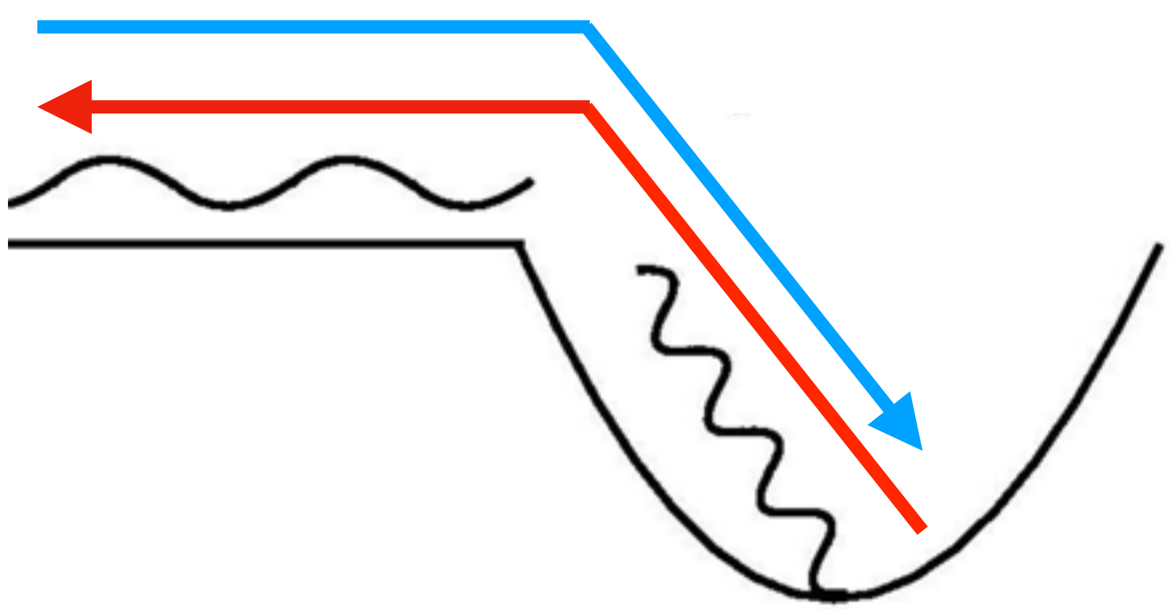
$$p \propto \tilde{a}^{-1}$$

Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Gravitational blue/redshift (Scalar)**



Potential well (ϕ < 0)

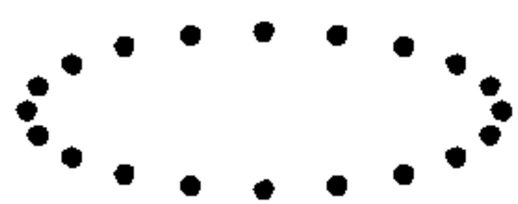
Sachs & Wolfe (1967)

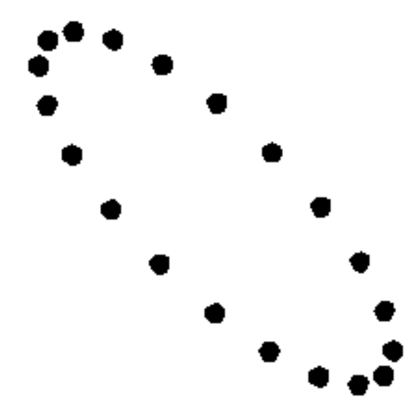
The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i \boxed{- \frac{1}{2}\sum_{ij} \dot{D}_{ij}\gamma^i\gamma^j}$$

- **Gravitational blue/redshift (Tensor)**

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





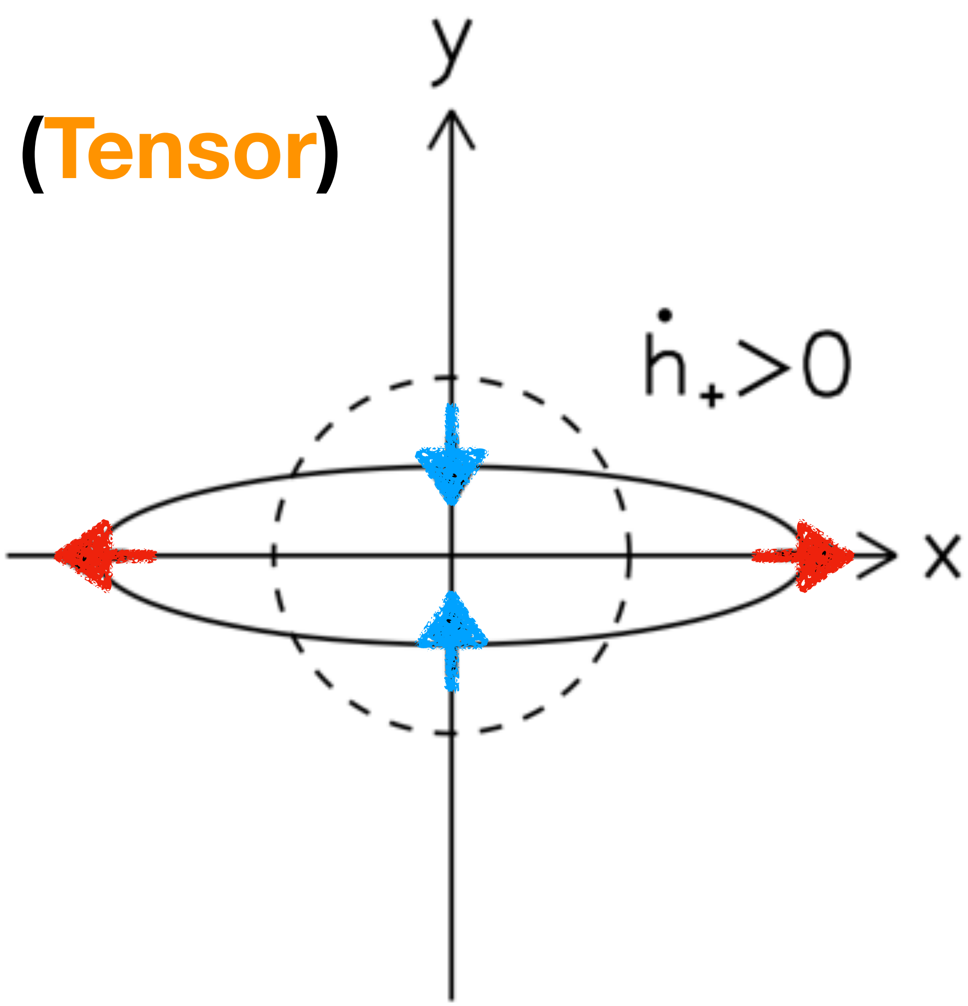
Sachs & Wolfe (1967)

The Result

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Gravitational blue/redshift (Tensor)**

$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Sachs & Wolfe (1967)

Formal Solution (Scalar)

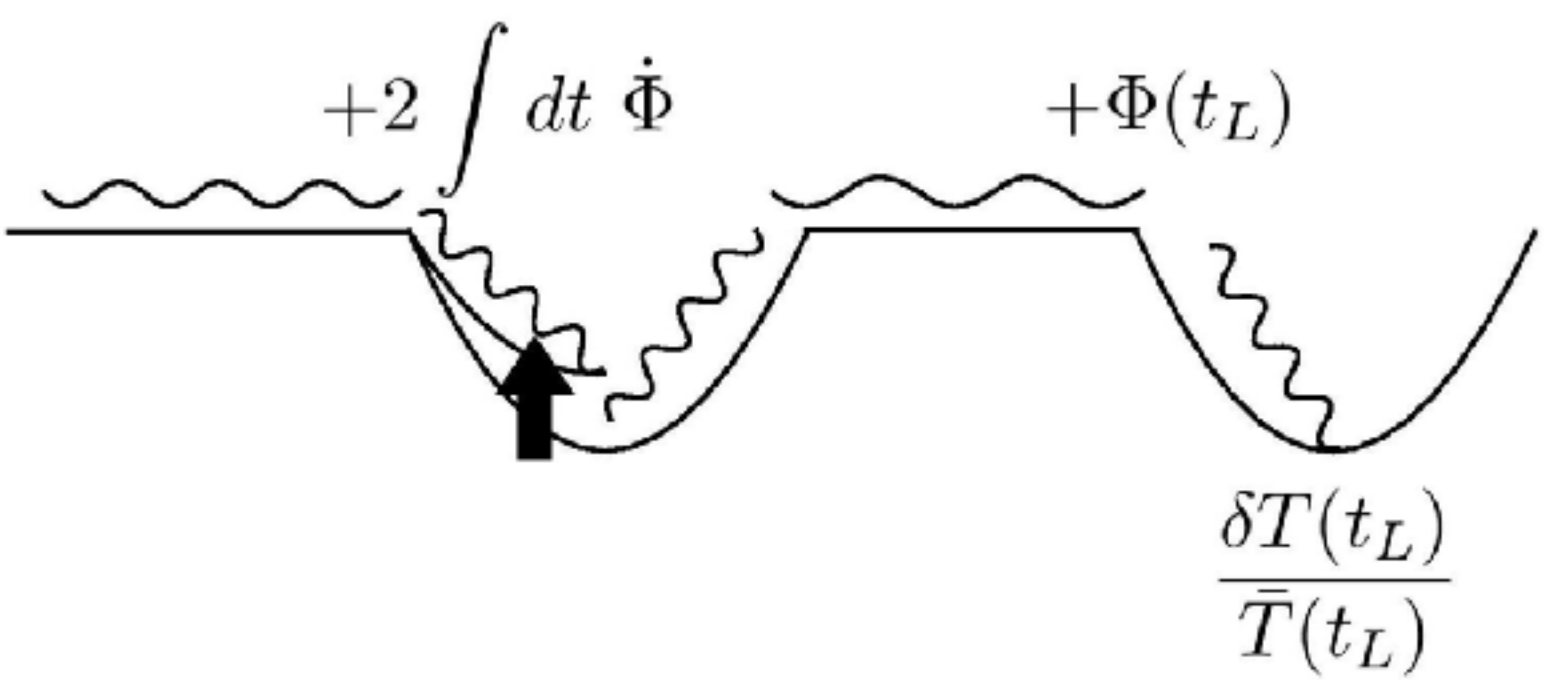
"L" for "Last scattering surface"

$$\ln(ap)(t_0) = \ln(ap)(t_L) + \Phi(t_L) - \Phi(t_0) + \int_{t_L}^{t_0} dt\ (\dot{\Phi} + \dot{\Psi})$$

or

$$\frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i = \frac{d\Phi}{dt} - \dot{\Phi}$$

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0) + \int_{t_L}^{t_0} dt\ (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Coming distance (r)

$$x^i = \hat{n}^i r$$

$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

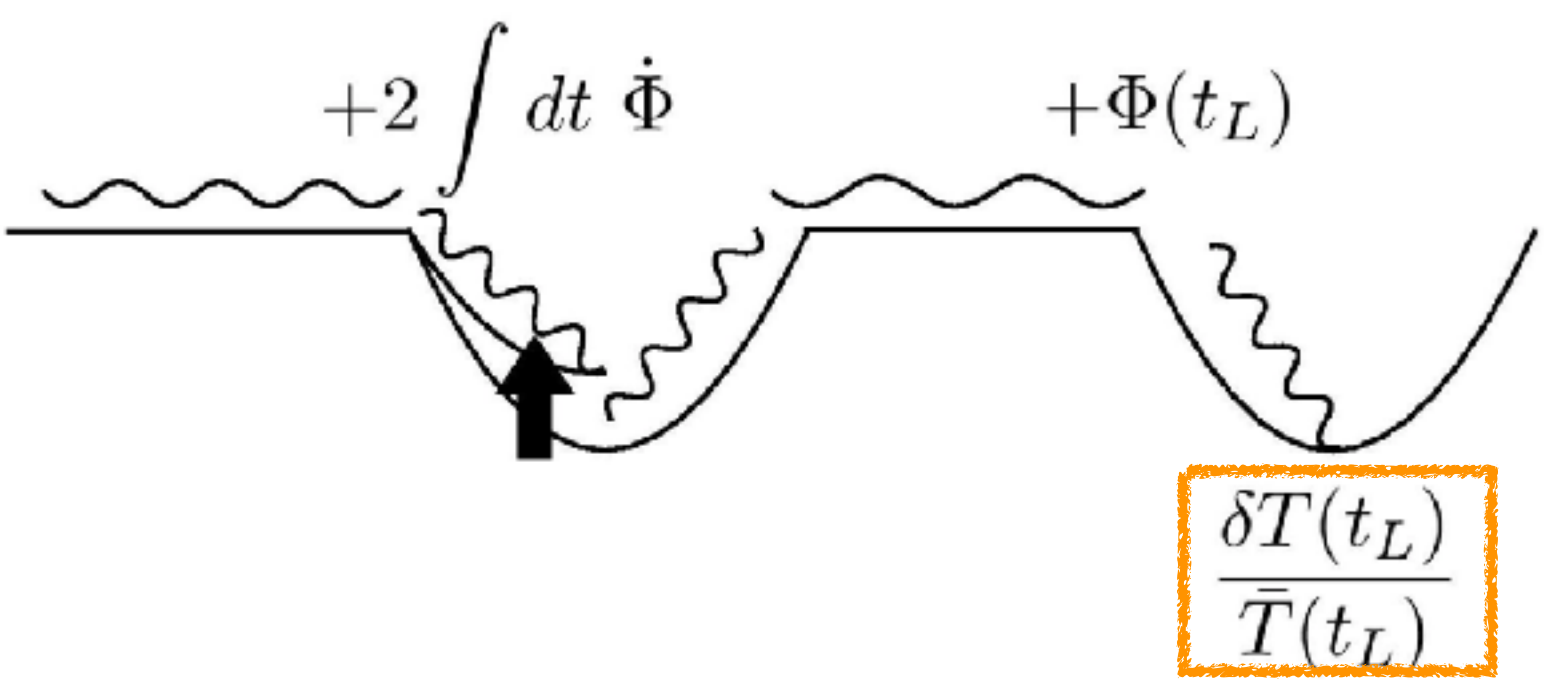
Sachs & Wolfe (1967)

Formal Solution (Scalar)

Initial Condition

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0)$$

$$+ \int_{t_L}^{t_0} dt \, (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Coming distance (r)

$$x^i = \hat{n}^i r$$

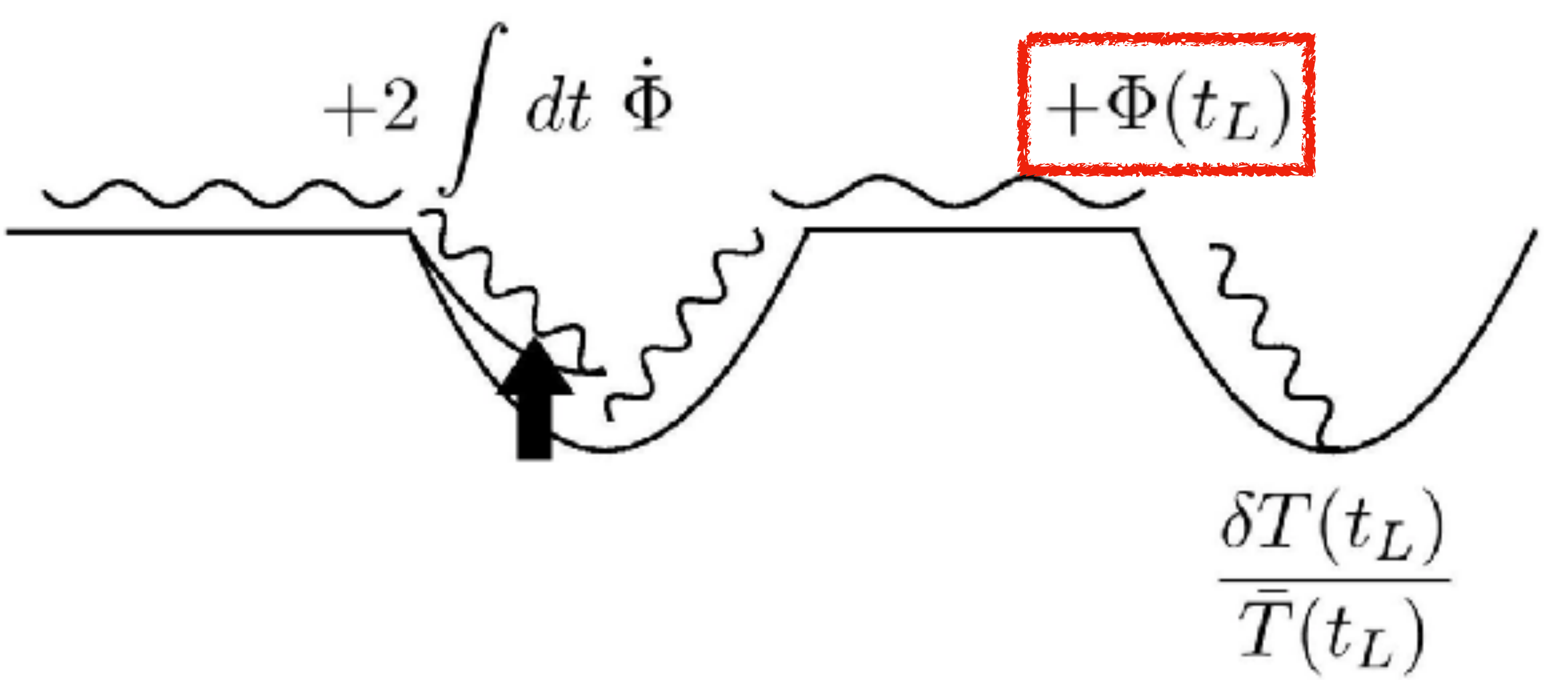
$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

Sachs & Wolfe (1967)

Formal Solution (Scalar)

Gravitational Redshit

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0) + \int_{t_L}^{t_0} dt \; (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Comoving distance (r)

$$x^i = \hat{n}^i r$$

$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

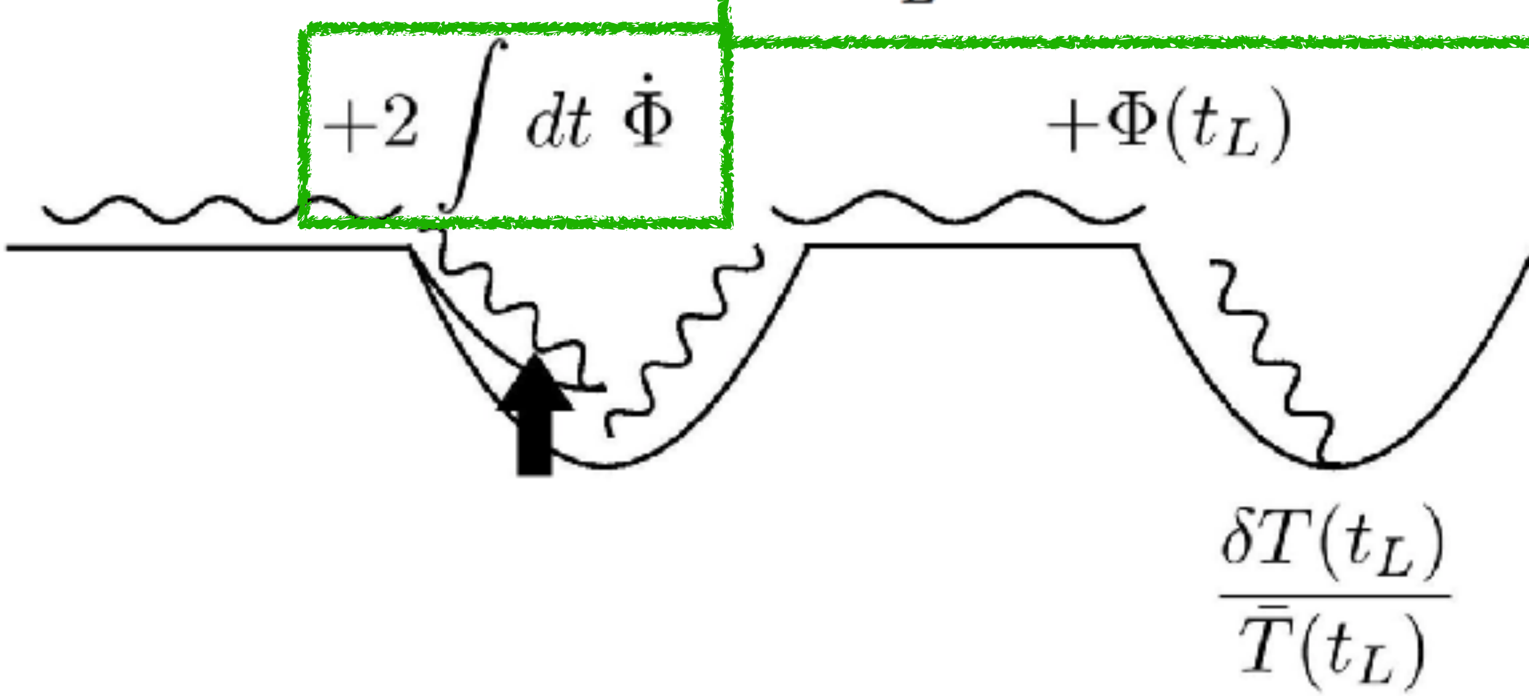
Sachs & Wolfe (1967)

Formal Solution (Scalar)

$$\frac{\Delta T(\hat{n})}{T_0} = \frac{\delta T(t_L, \hat{n} r_L)}{\bar{T}(t_L)} + \Phi(t_L, \hat{n} r_L) - \Phi(t_0, 0)$$

"integrated Sachs-Wolfe" (ISW) effect

$$+ \int_{t_L}^{t_0} dt \; (\dot{\Phi} + \dot{\Psi})(t, \hat{n} r)$$



Line-of-sight direction

$$\hat{n}^i = -\gamma^i$$

Coming distance (r)

$$x^i = \hat{n}^i r$$

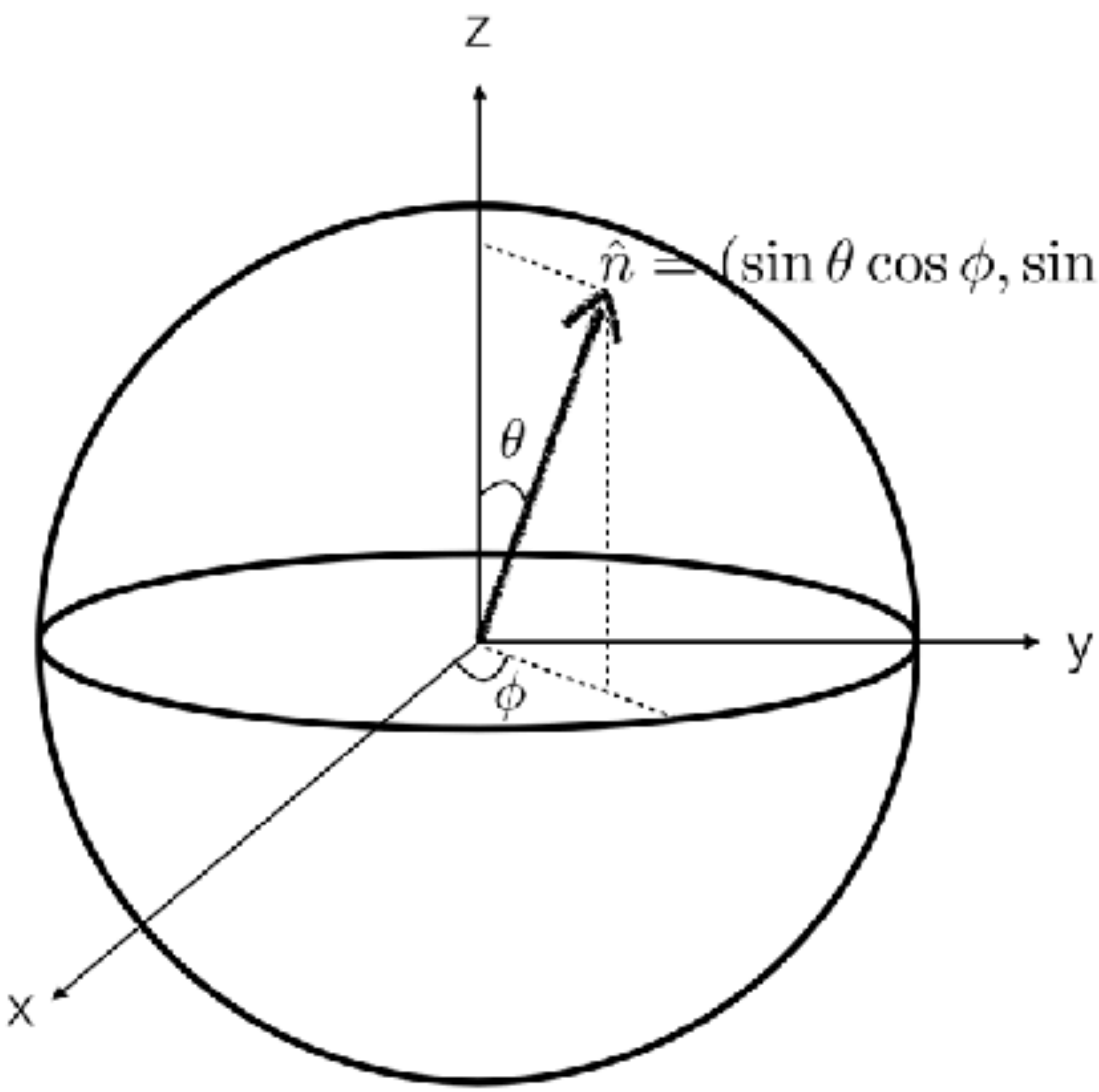
$$r(t) = \int_t^{t_0} \frac{dt'}{a(t')}$$

Sachs & Wolfe (1967)

Formal Solution (Tensor)

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\rm ISW} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

negligible contribution before the last scattering



$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

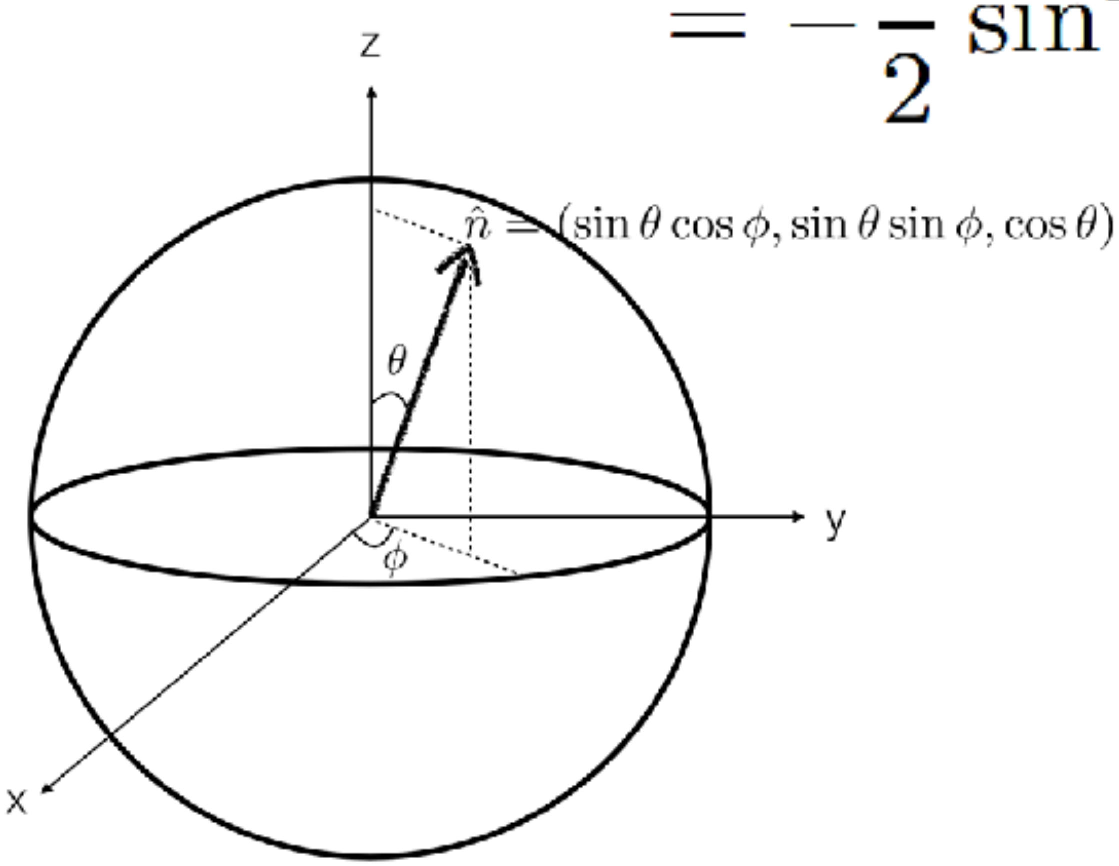
Sachs & Wolfe (1967)

Formal Solution (Tensor)

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\rm ISW} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

negligible contribution before the last scattering

$$= -\frac{1}{2}\sin^2\theta\int_{t_L}^{t_0} dt\left(\dot{h}_+\cos 2\phi + \dot{h}_\times \sin 2\phi\right)$$



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Sachs & Wolfe (1967)

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negligible contribution before the last scattering

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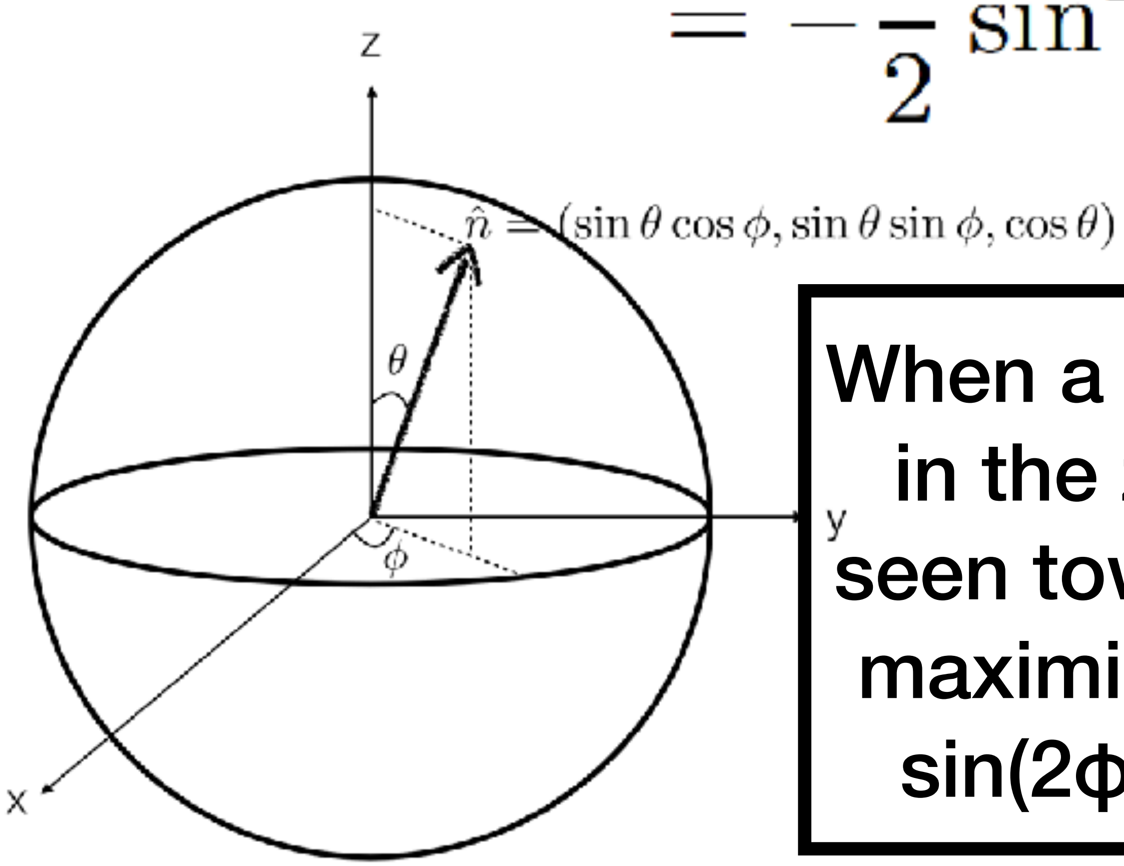


When a plane wave gravitational wave propagates in the z direction, no temperature anisotropy is seen towards the poles (θ=0, π). The anisotropy is maximised on the horizon (θ=π/2) with cos(2ϕ) & sin(2ϕ) modulation in the azimuthal directions.

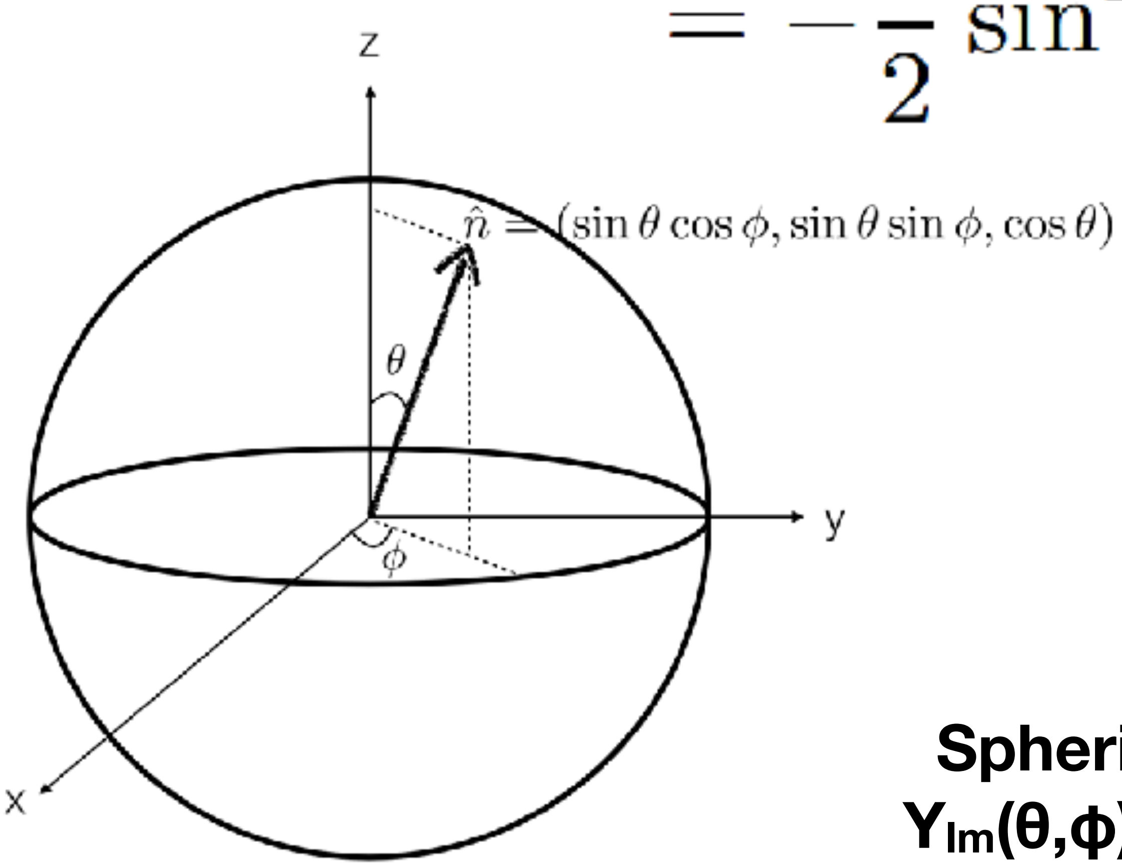
Sachs & Wolfe (1967)

Formal Solution (Tensor)

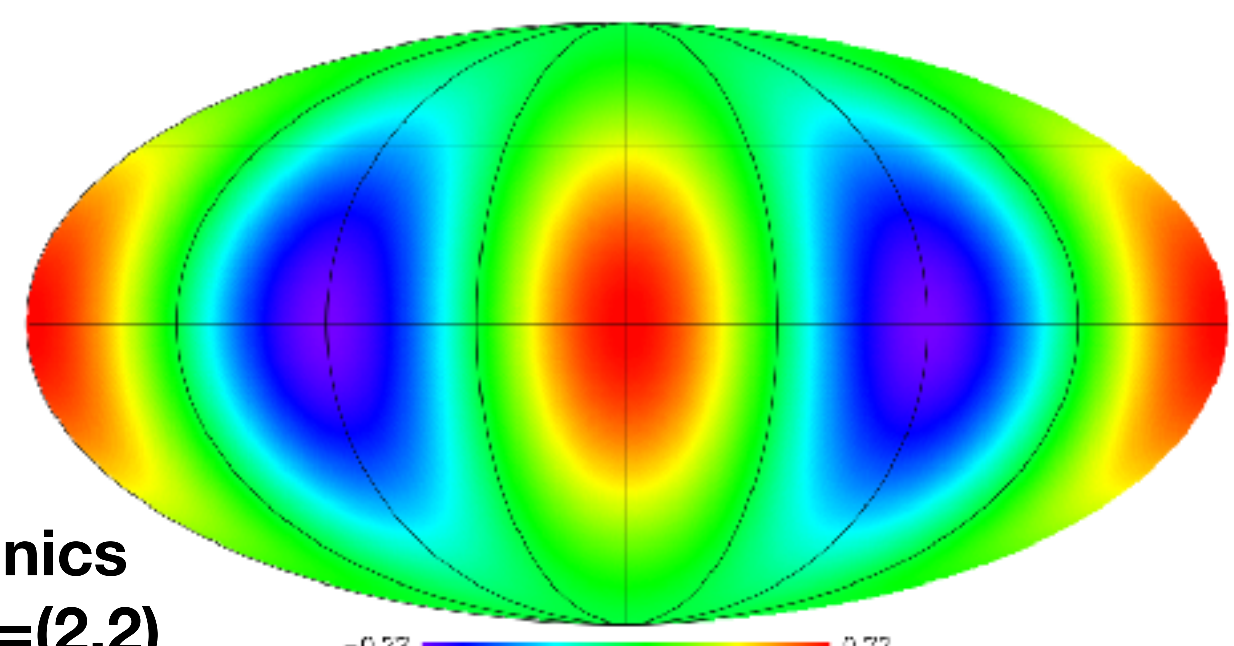
$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\rm ISW} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

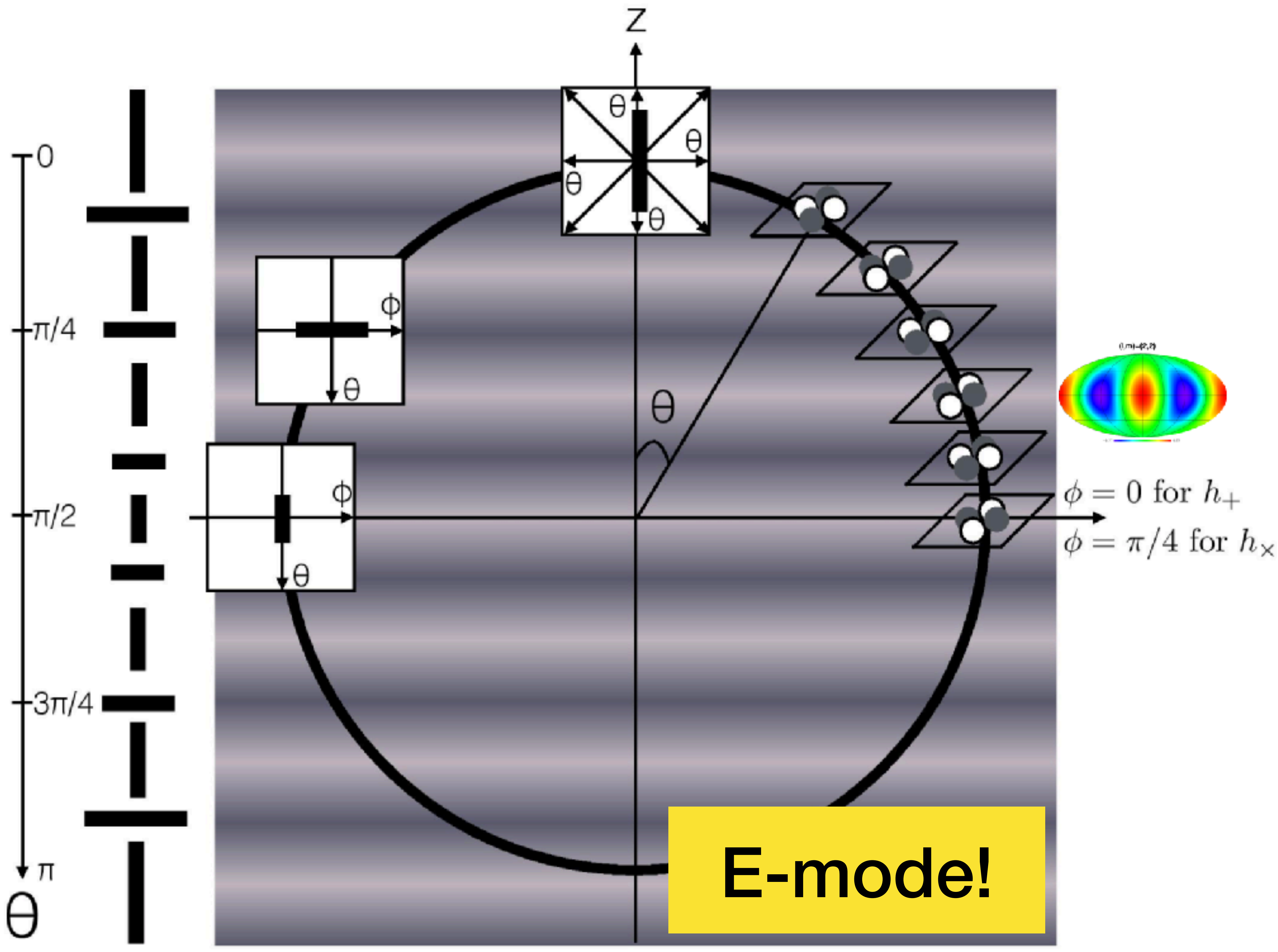
negligible contribution before the last scattering

$$= -\frac{1}{2}\sin^2\theta\int_{t_L}^{t_0} dt\left(\dot{h}_+\cos 2\phi + \dot{h}_\times \sin 2\phi\right)$$

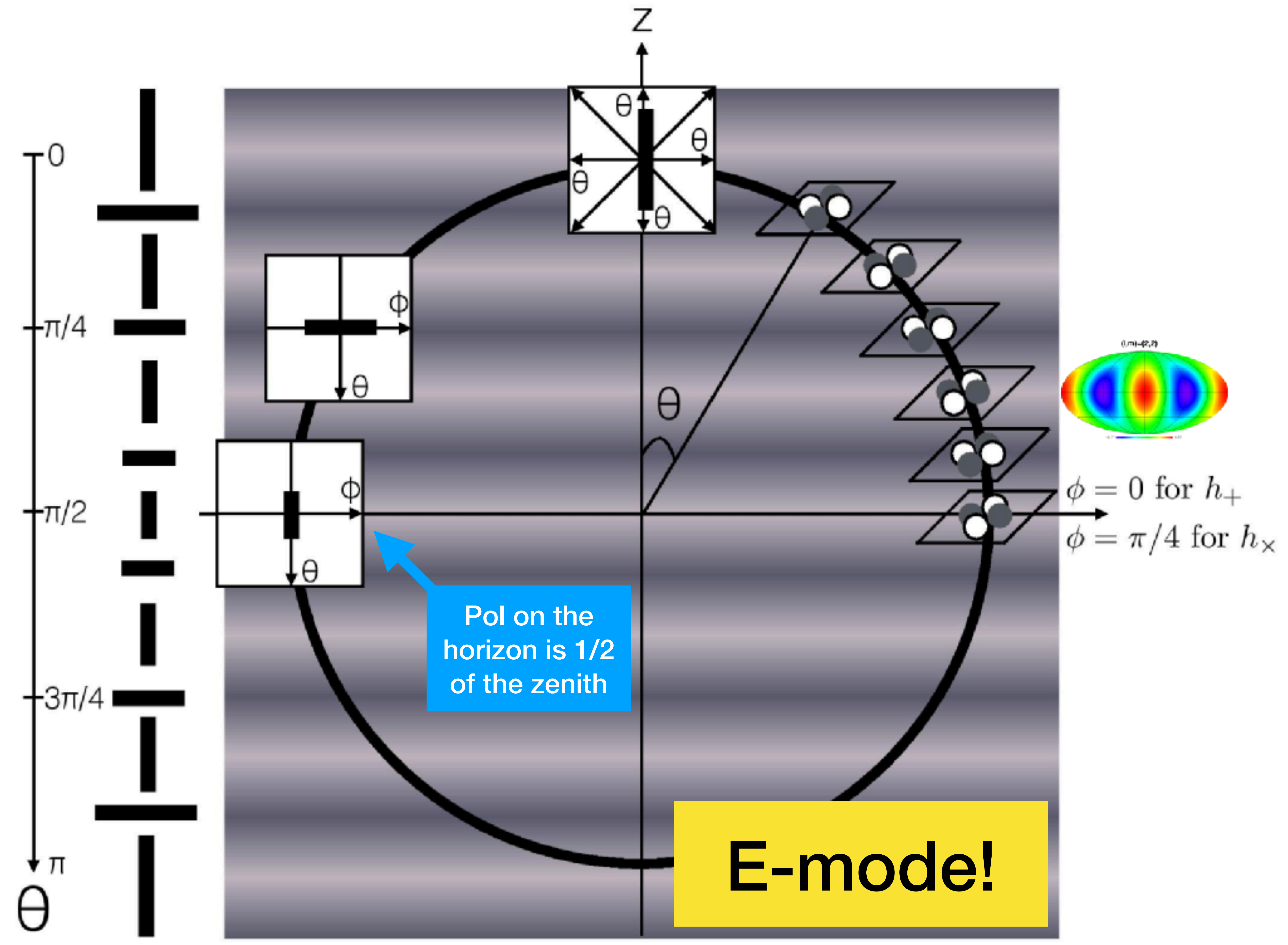


Spherical harmonics
$Y_{lm}(\theta,\phi)$ with (l,m)=(2,2)

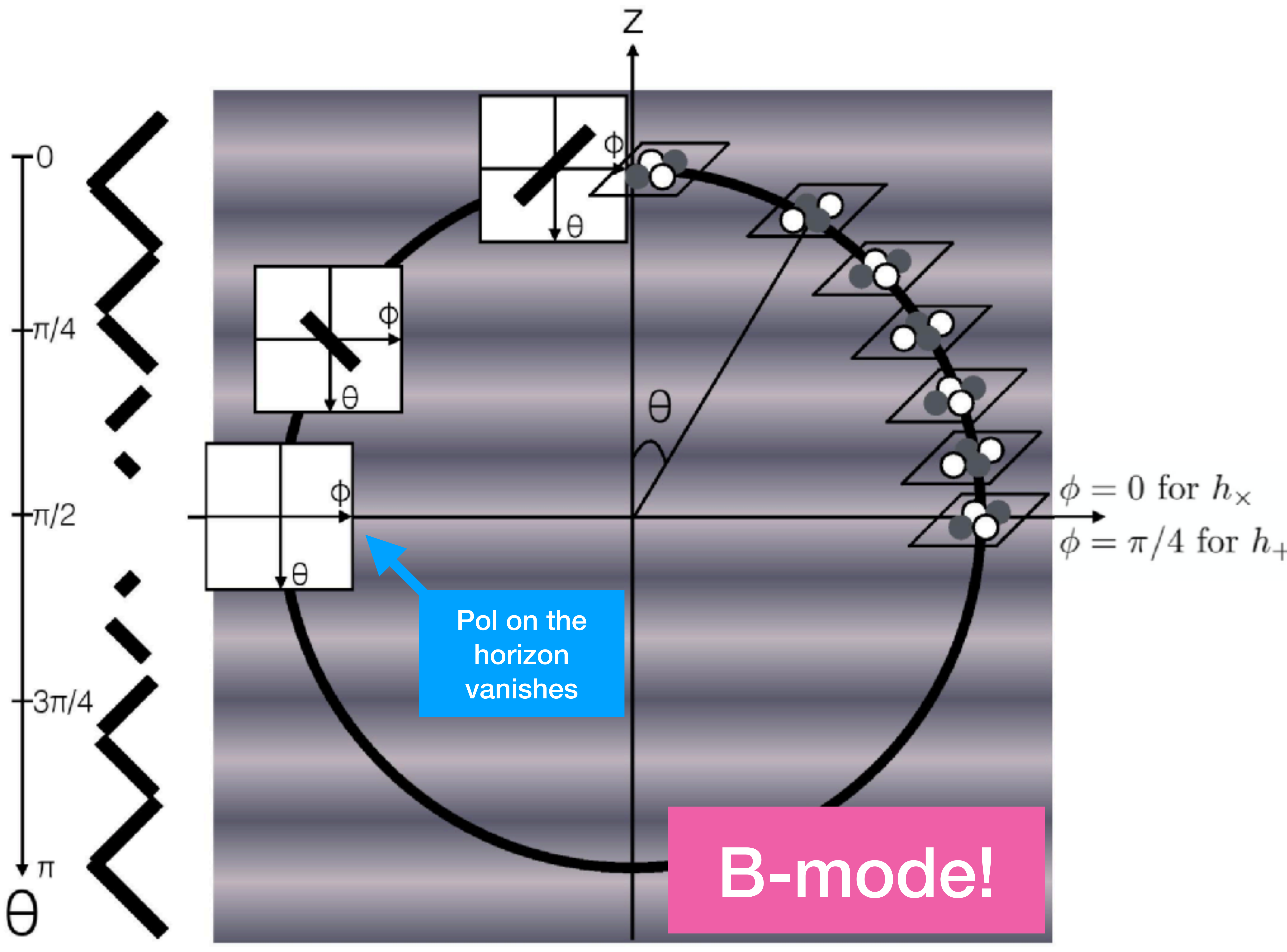




Z
θ
φ
0
π/4
π/2
3π/4
π
$\phi = 0$ for h_+
$\phi = \pi/4$ for $h_\times$
E-mode!

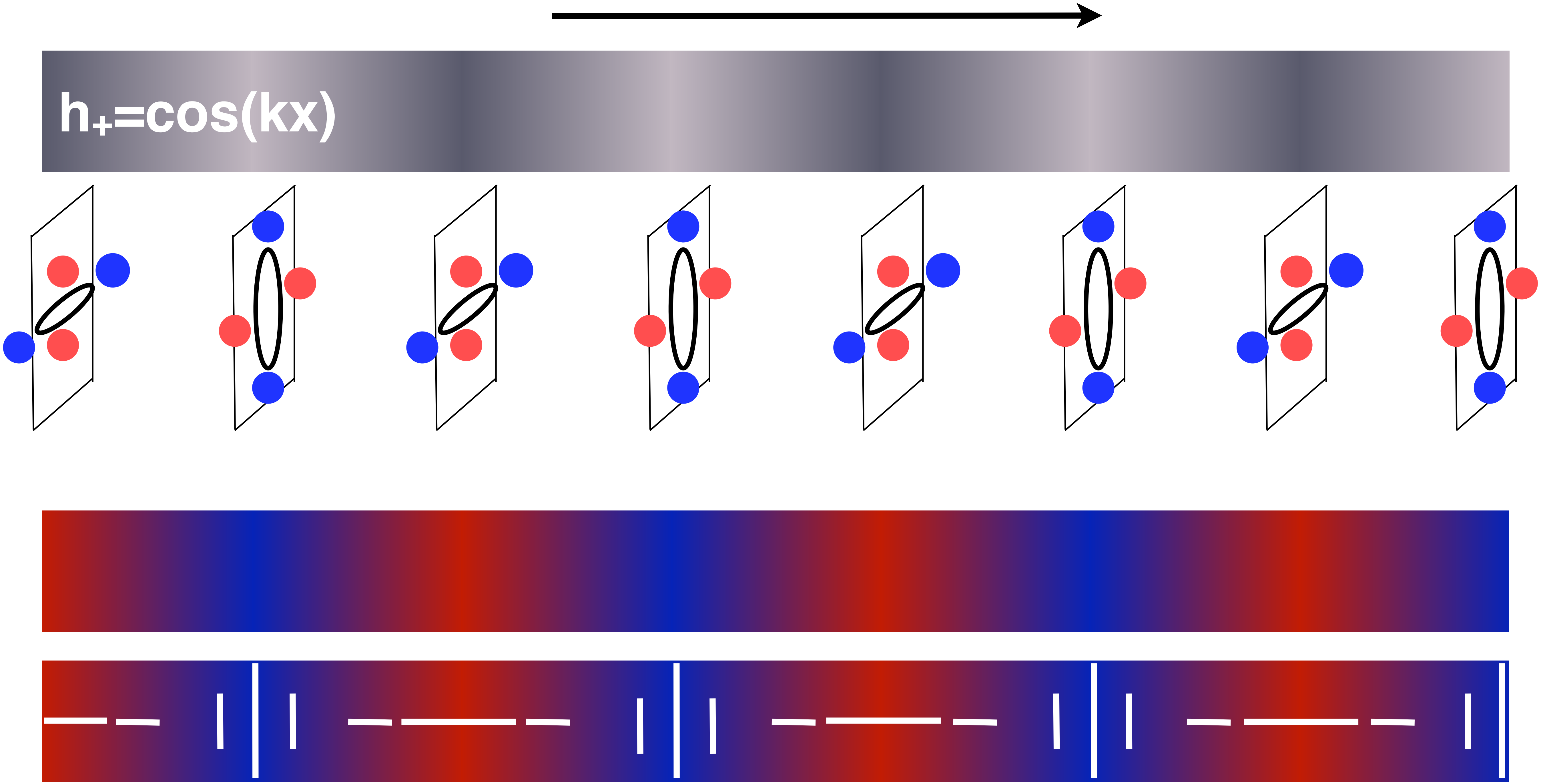


Z
θ
θ
θ
θ
φ
θ
φ
θ
θ
0
π/4
π/2
3π/4
π
θ
$\phi = 0$ for h_+
$\phi = \pi/4$ for $h_\times$
Pol on the horizon is 1/2 of the zenith
E-mode!

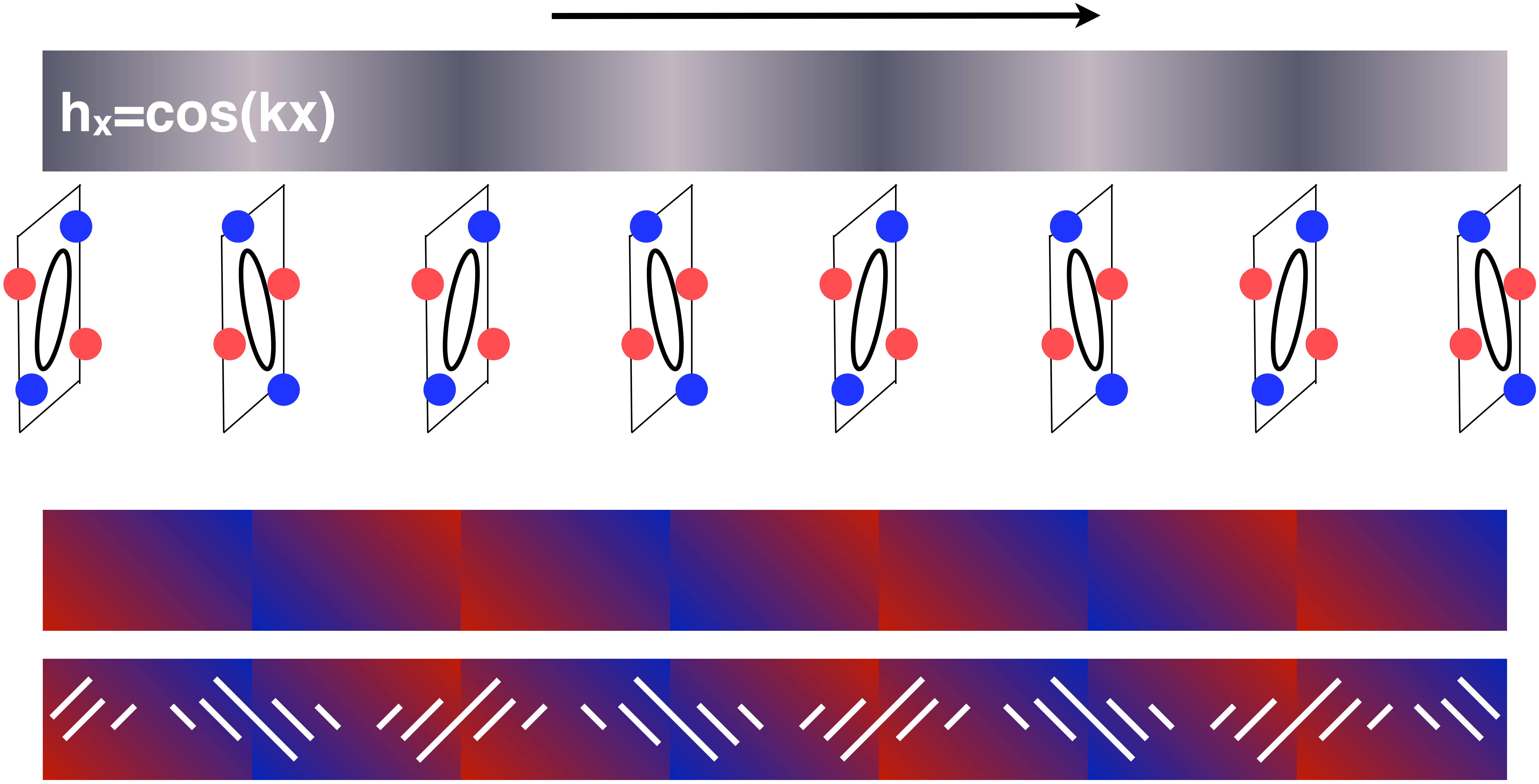


z
0
π/4
π/2
3π/4
π
θ
ϕ
θ
$\phi = 0$ for $h_\times$
$\phi = \pi/4$ for h_+
Pol on the horizon vanishes
B-mode!

propagation direction of GW

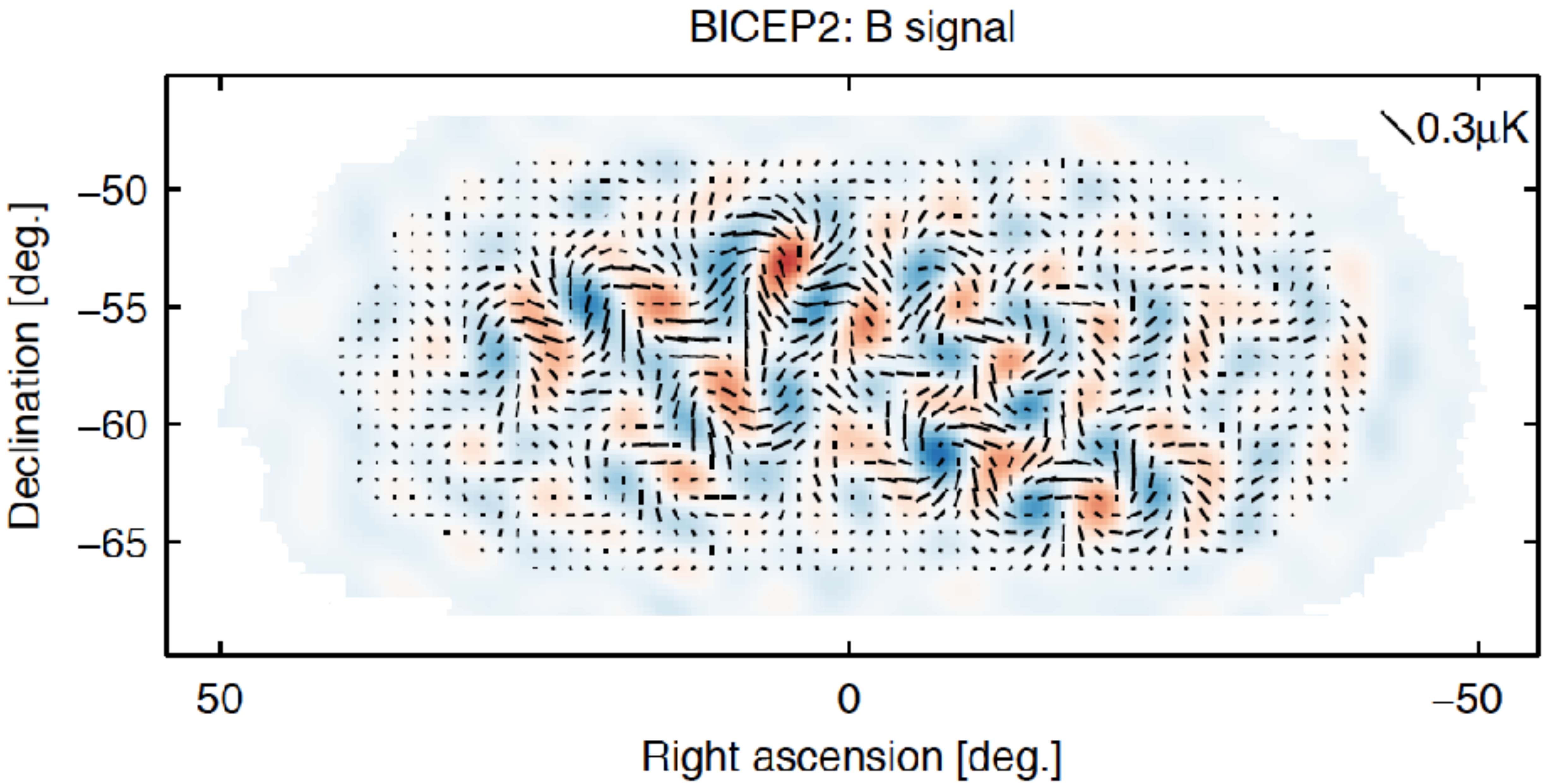


Polarisation directions perpendicular/parallel to the wavenumber vector -> **E mode polarisation** propagation direction of GW



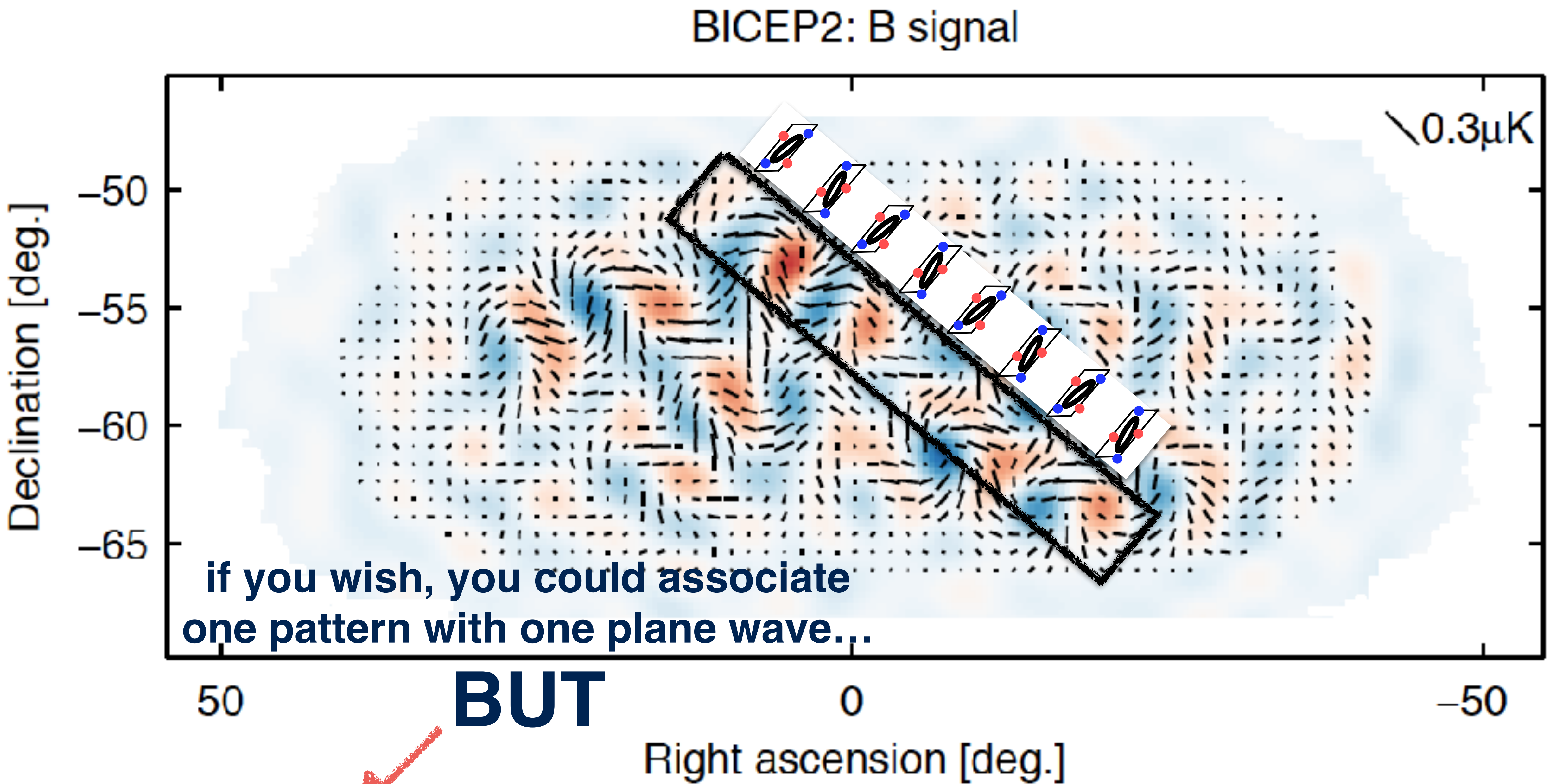
Polarisation directions 45 degrees tilted from to the wavenumber vector -> **B mode polarisation**

Signature of gravitational waves in the sky [?]



<u>CAUTION</u>: we are NOT seeing a single plane wave propagating perpendicular to our line of sight

Signature of gravitational waves in the sky [?]



BUT

<u>CAUTION</u>: we are NOT seeing a single plane wave propagating perpendicular to our line of sight

Propagation of cosmological gravitational waves

$$\ddot{D}_{ij} + \frac{3\dot{a}}{a}\dot{D}_{ij} - \frac{1}{a^2}\nabla^2 D_{ij} = 16\pi G\pi_{ij}^{\text{tensor}}$$

- Tensor anisotropic stress can do two things:
  - It can ***generate*** gravitational waves
  - It can ***damp*** gravitational waves (neutrino anisotropic stress)

But we ignore the tensor anisotropic stress today

Super-horizon Solution

$$\ddot{D}_{ij} + \frac{3\dot{a}}{a}\dot{D}_{ij} = 0$$

$$\Rightarrow \quad D_{ij} = \text{constant} + \text{decaying term}$$

- Super-horizon tensor perturbation is conserved
- Thus, **no ISW temperature anisotropy on super-horizon scales**
- It does not look like “gravitational waves”, but it will start oscillating and behaving like waves once it enters the horizon

η: *“conformal time”, or the distance traveled by photons*

Matter-dominated Solution

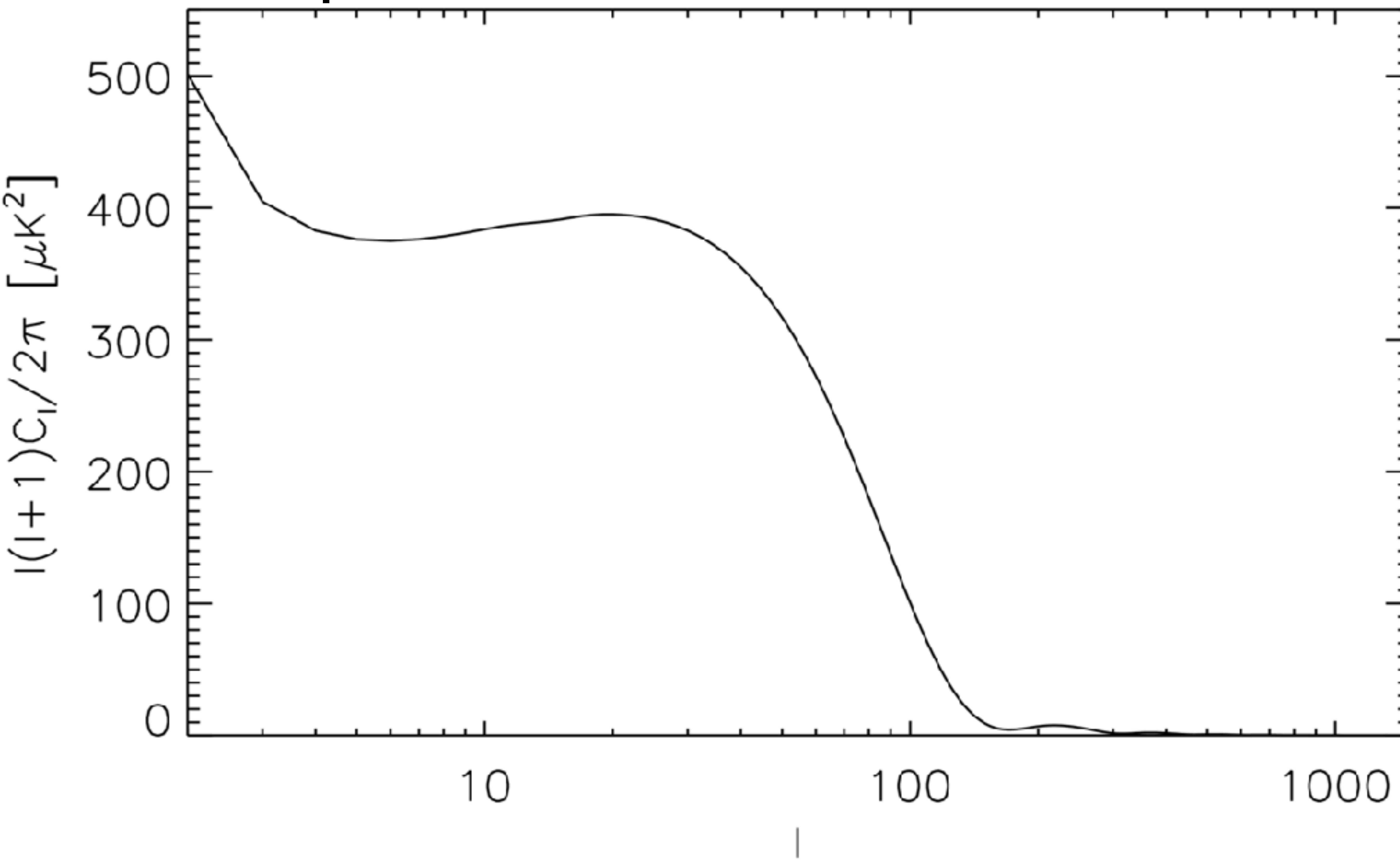
$$D_{ij,\boldsymbol{q}}(t) = C_{ij,\boldsymbol{q}} \frac{3j_1(q\eta)}{q\eta} \propto \frac{1}{a(t)}$$

$$\dot{D}_{ij,\boldsymbol{q}}(t) = -C_{ij,\boldsymbol{q}} \frac{q}{a(t)} \frac{3j_2(q\eta)}{q\eta} \propto \frac{1}{a^2(t)}$$

- $\partial D_{ij}/\partial t$ gives the ISW. It peaks at the horizon crossing, qη~2
- The energy density is given by $(\partial D_{ij}/\partial t)^2$, which indeed decays like radiation, a^{-4}

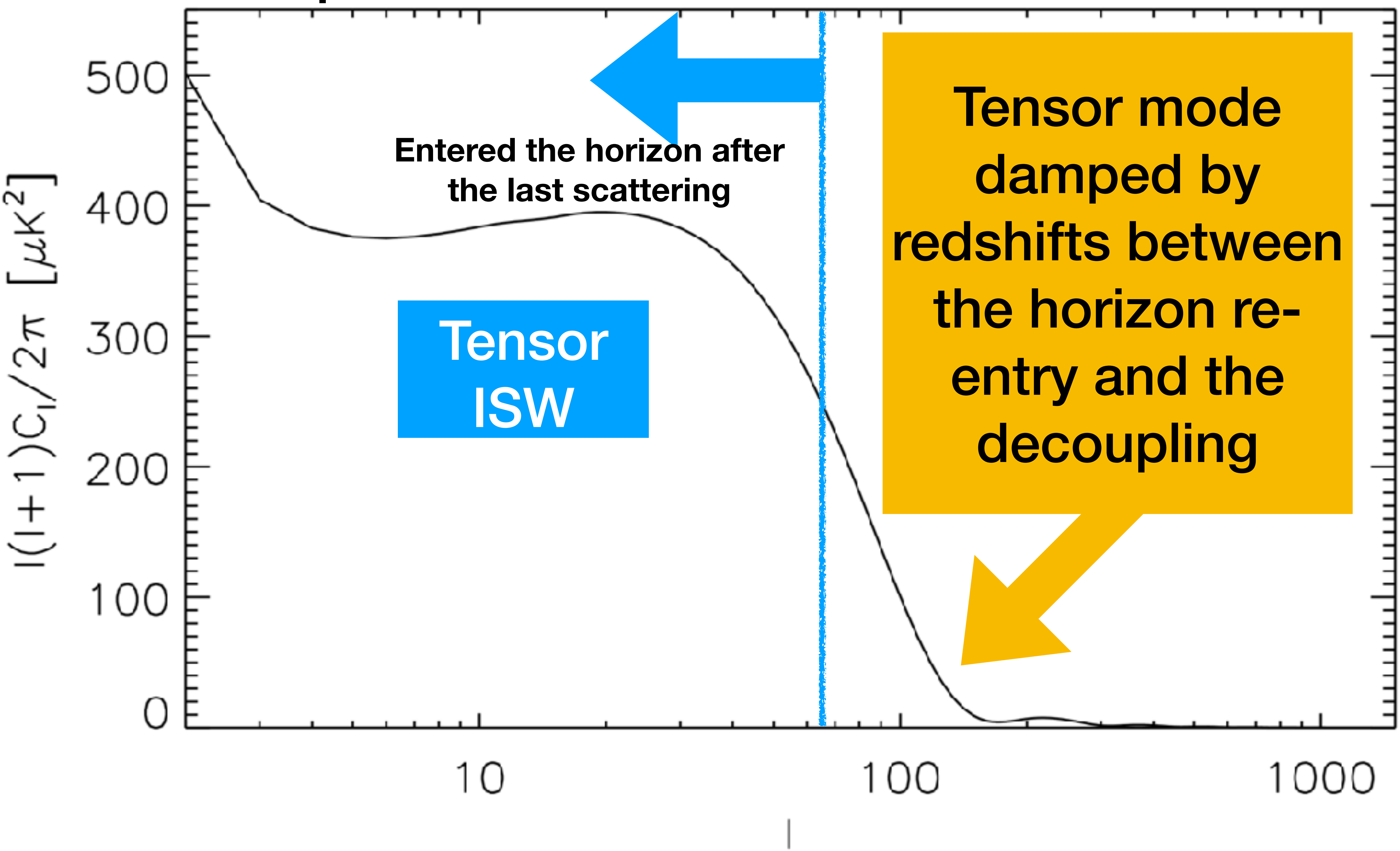
Scale-invariant

Temperature C_l from GW



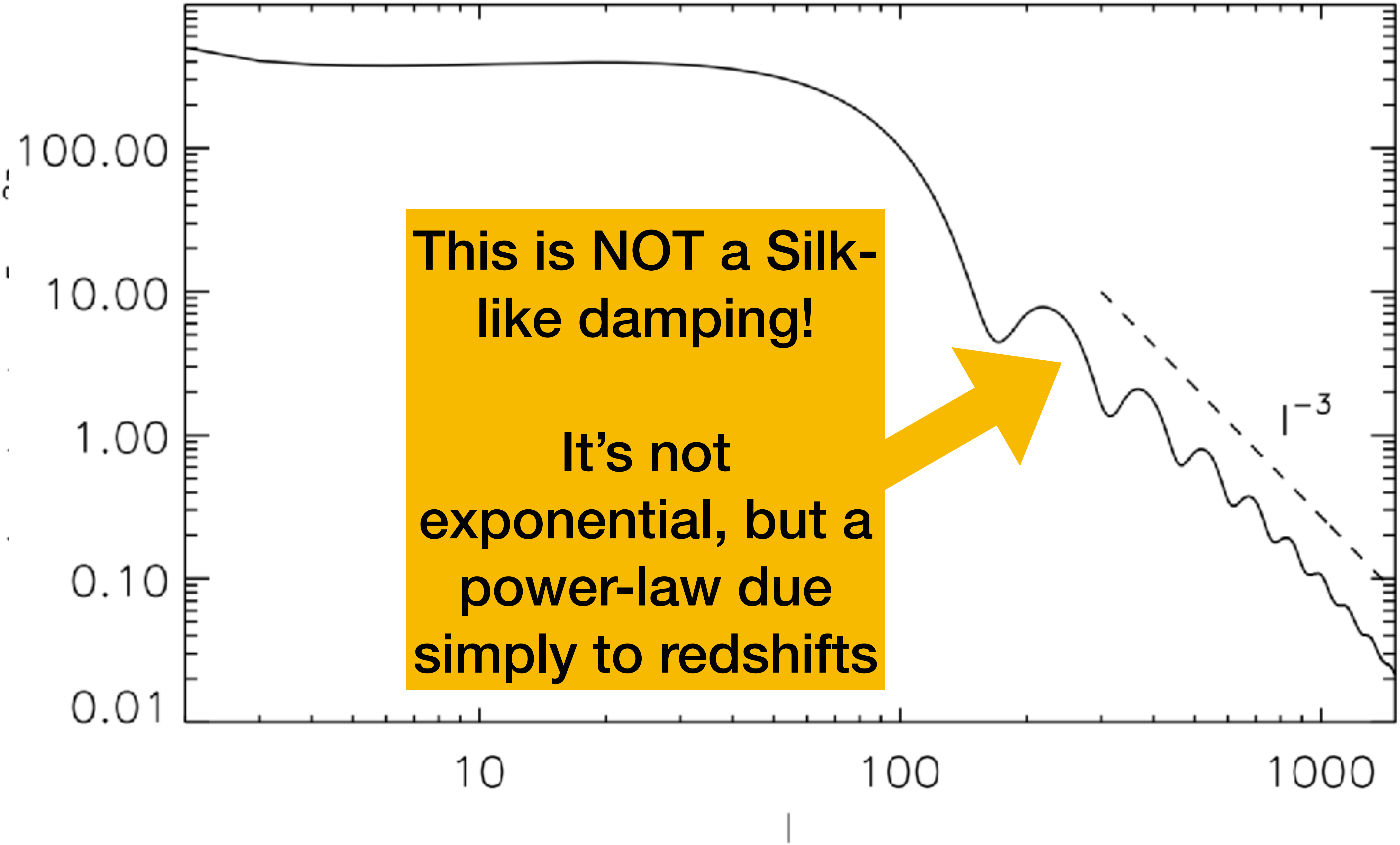
Scale-invariant

Temperature C_l from GW



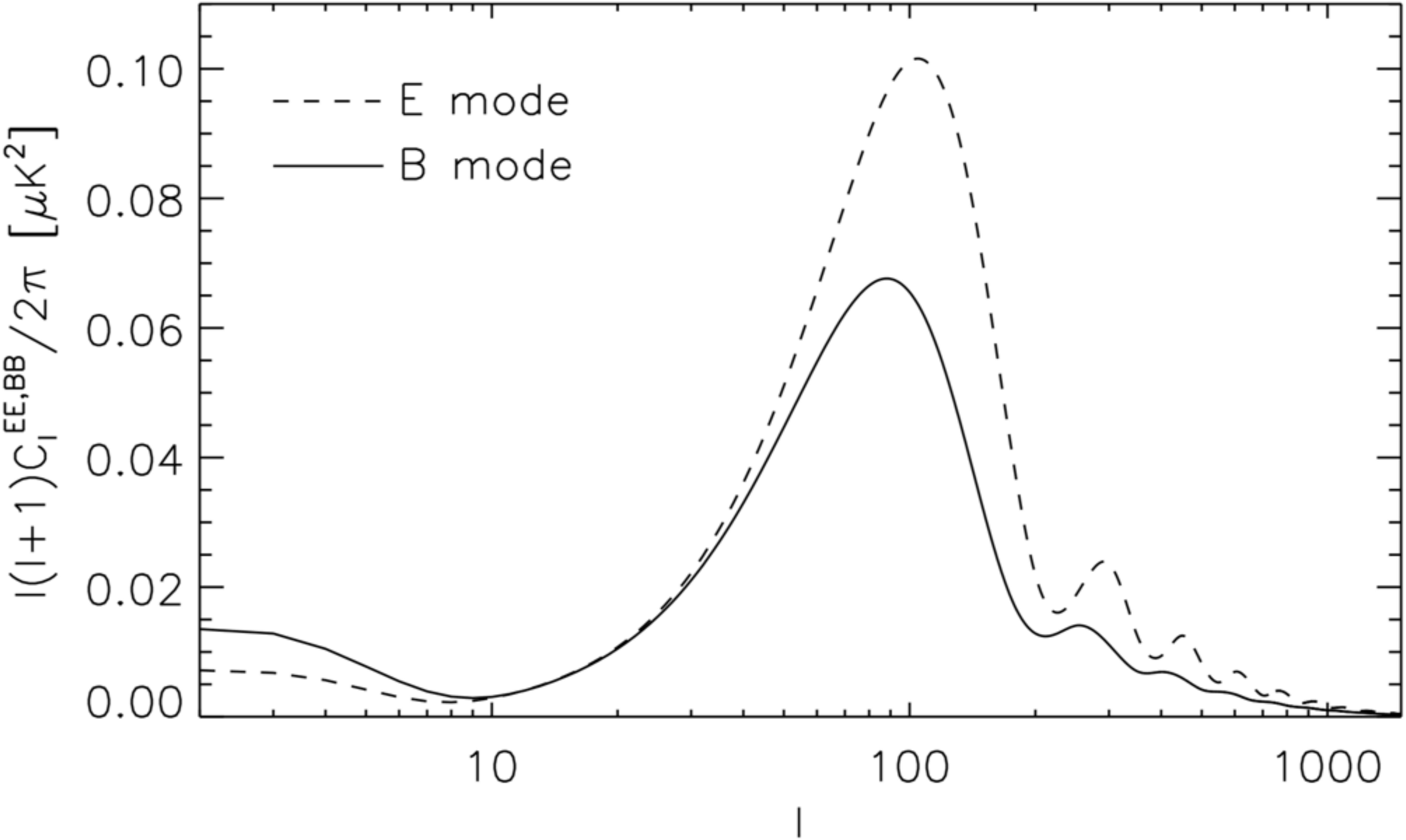
Scale-invariant

Temperature C_l from GW

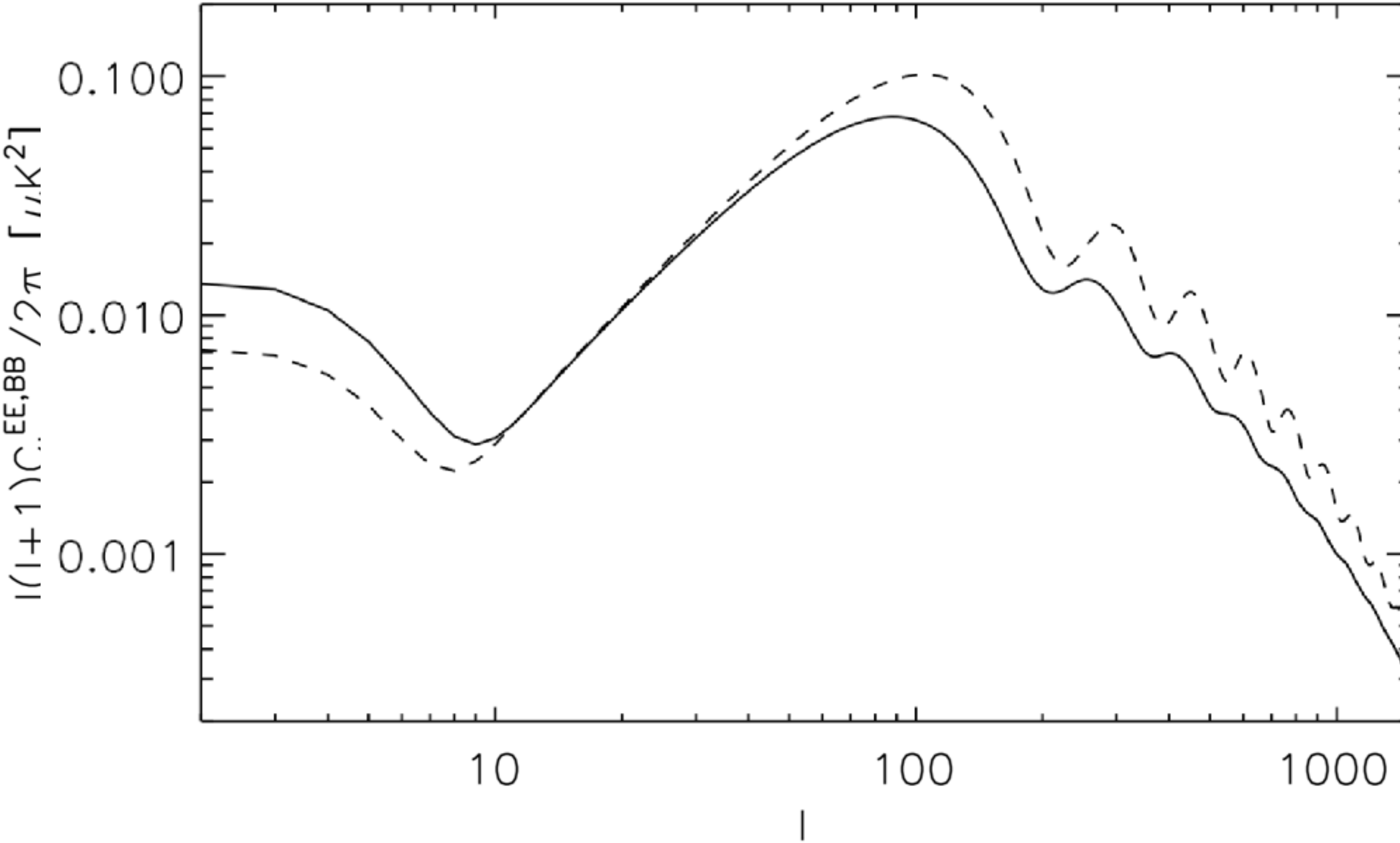


This is NOT a Silk-like damping!

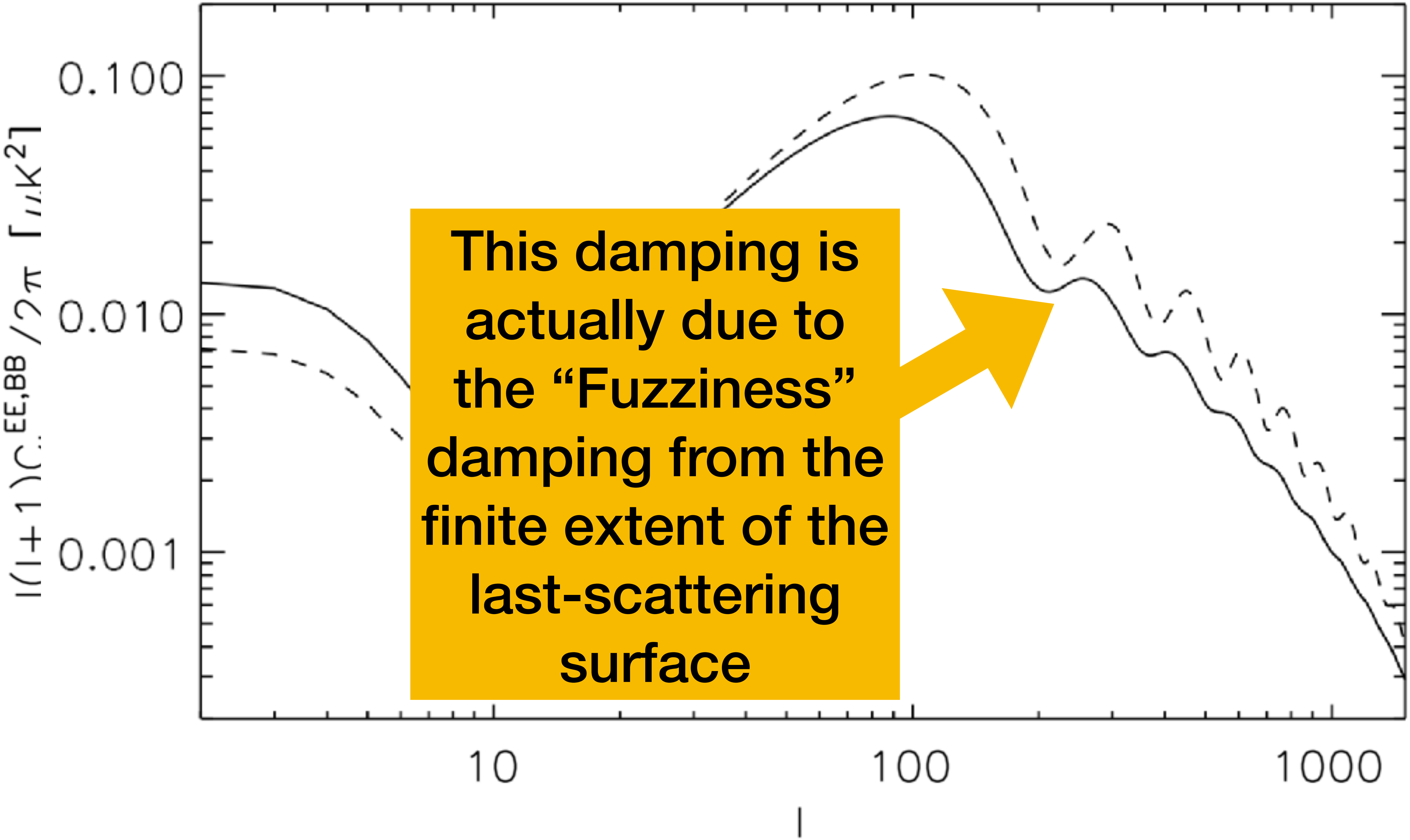
It's not exponential, but a power-law due simply to redshifts



- E and B modes are produced nearly equally, but on small scales B is smaller than E because B vanishes on the horizon



- E and B modes are produced nearly equally, but on small scales B is smaller than E because B vanishes on the horizon



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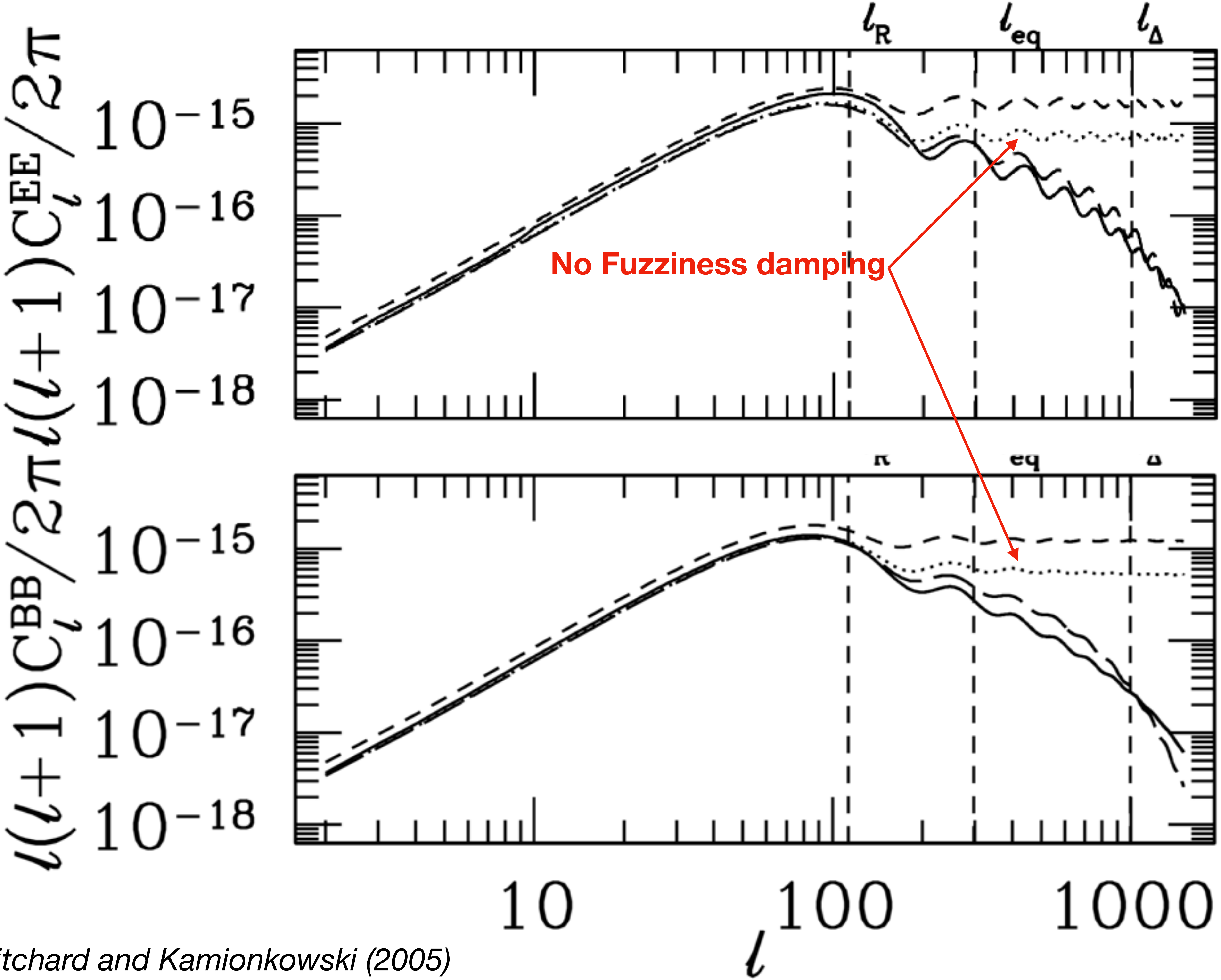
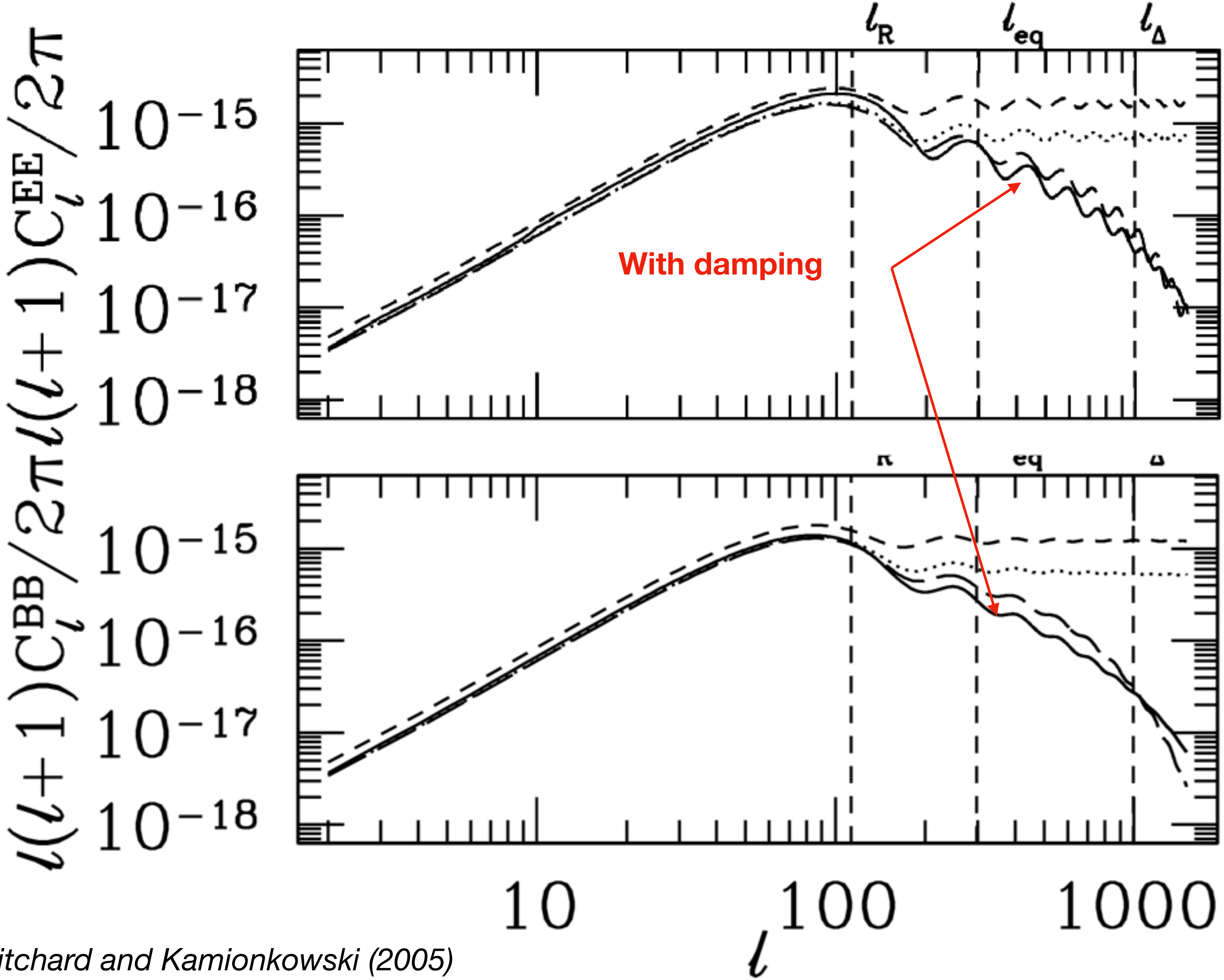


Pritchard and Kamionkowski (2005)



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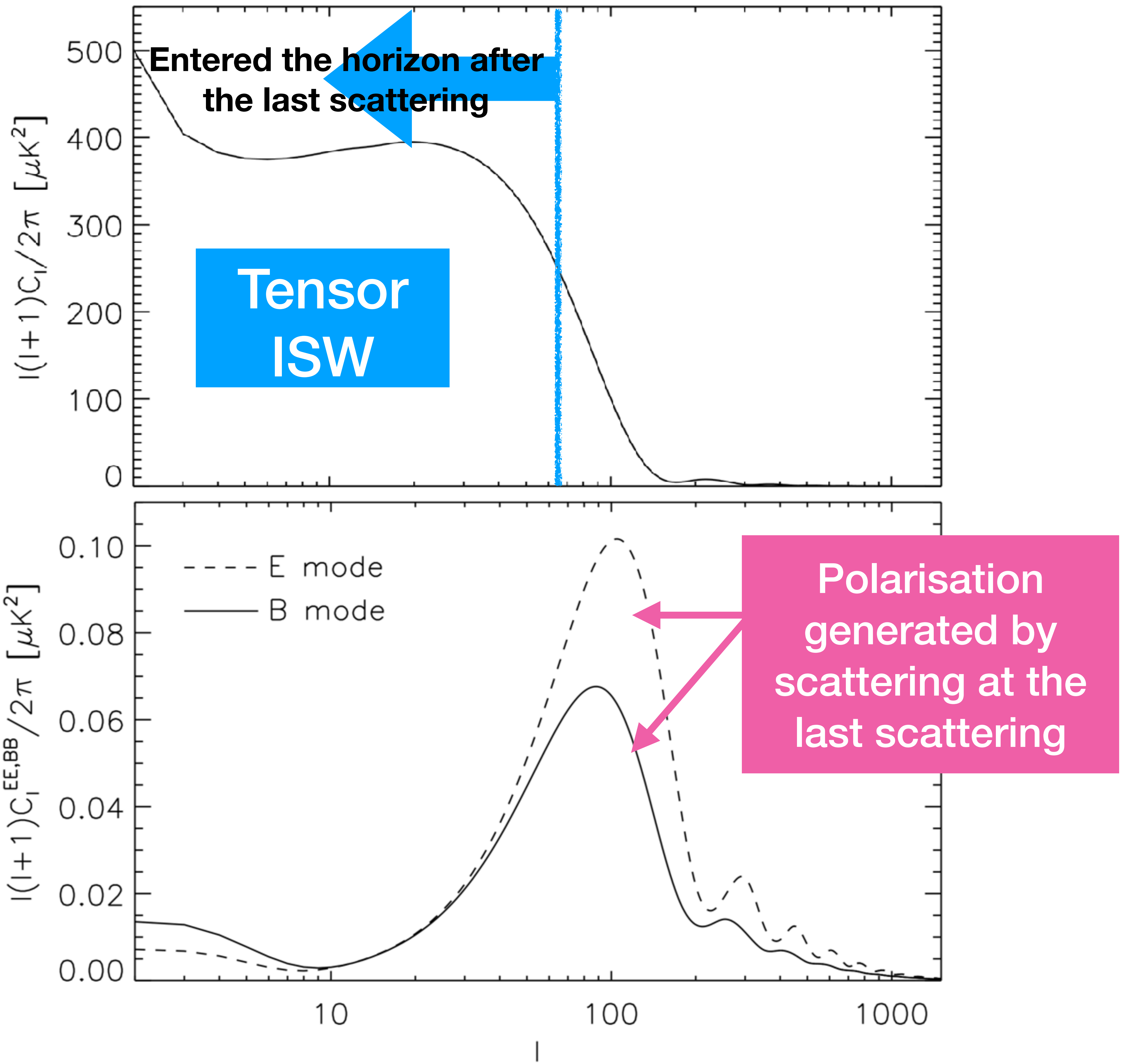
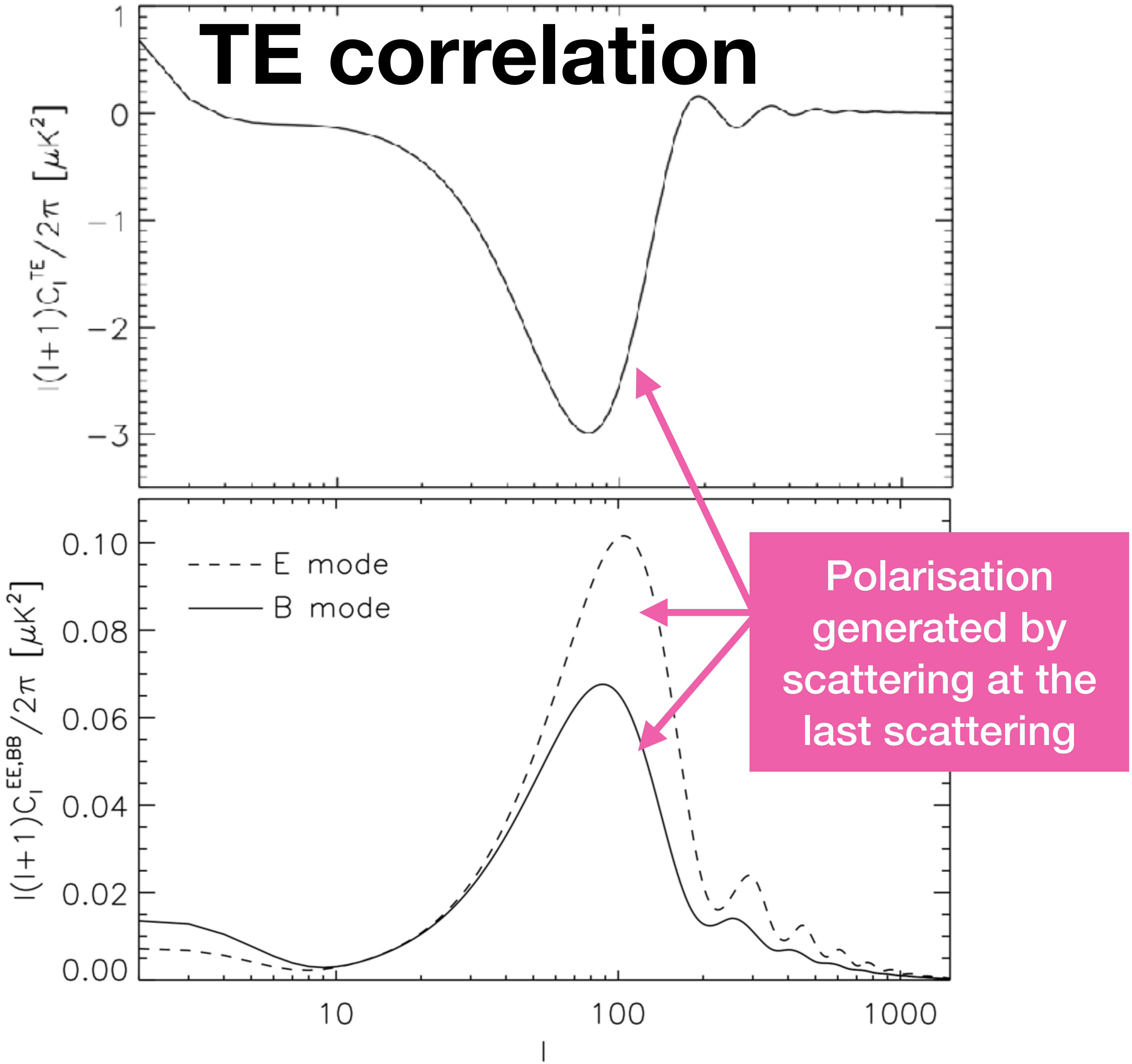
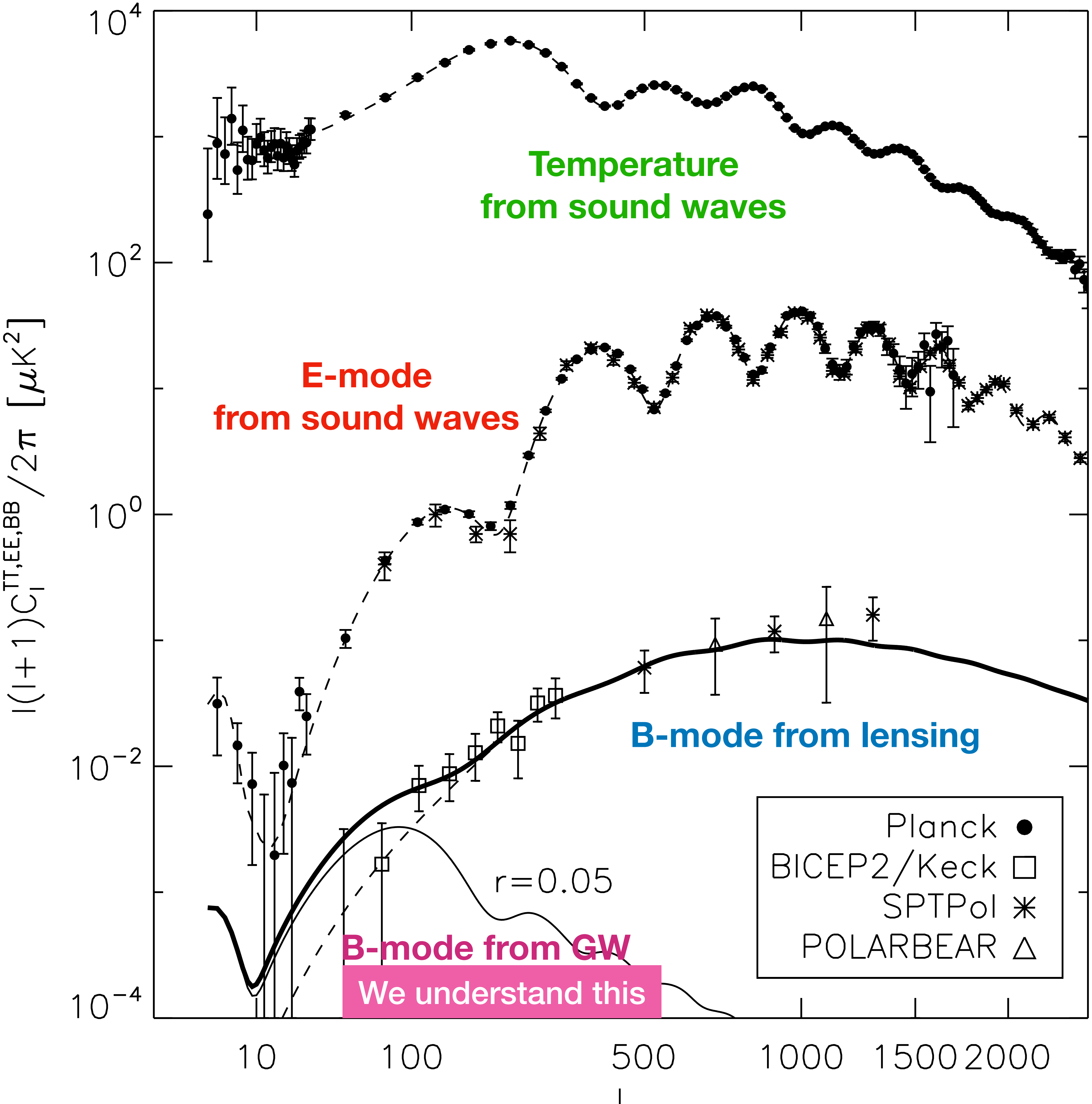


Entered the horizon after the last scattering
Tensor ISW
Polarisation generated by scattering at the last scattering
E mode
B mode
l(l+1)C_l/2π [μK²]
l(l+1)C_l^EE,BB/2π [μK²]
l

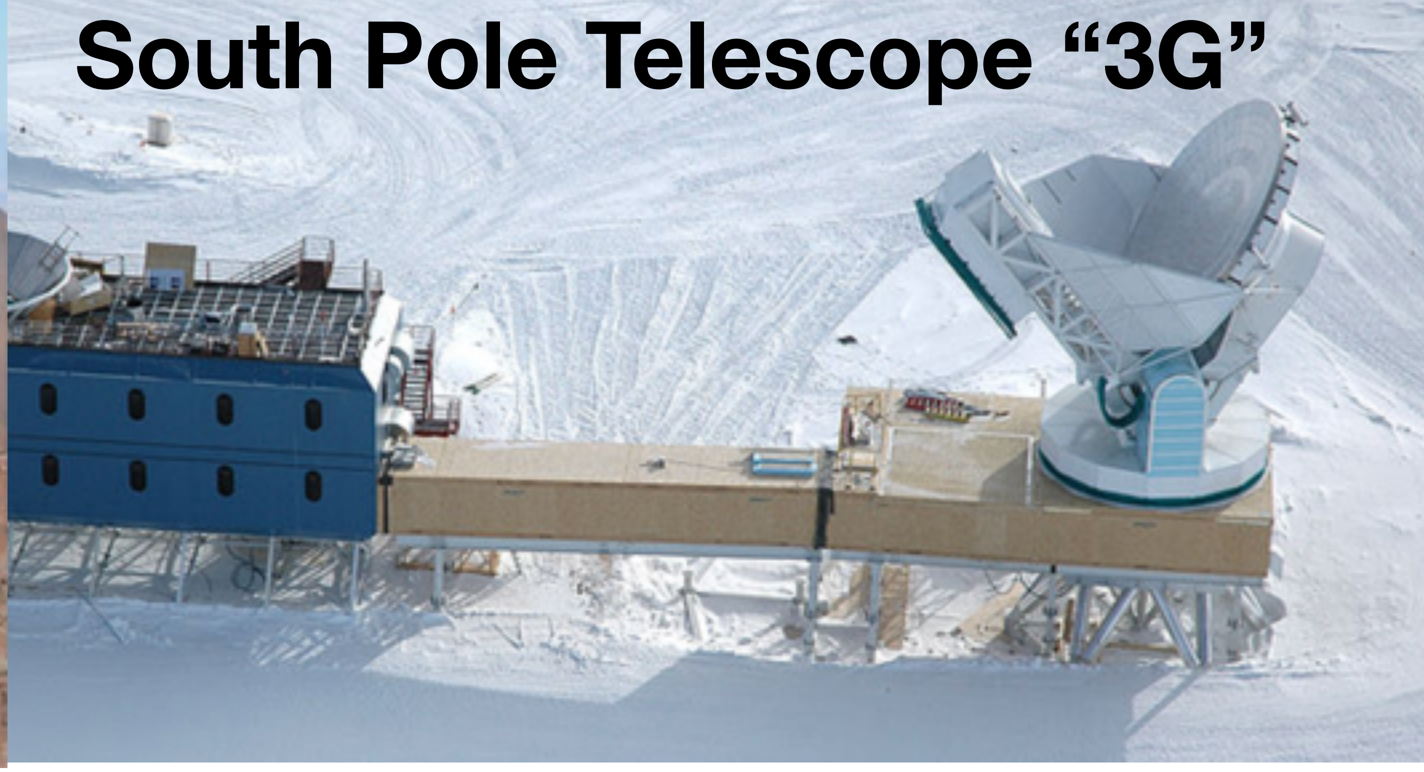


TE correlation
$l(l+1)C_l^{TE}/2\pi\ [\mu K^2]$
1
0
−1
−2
−3
$l(l+1)C_l^{EE,BB}/2\pi\ [\mu K^2]$
0.10
0.08
0.06
0.04
0.02
0.00
E mode
B mode
10
100
1000
l
Polarisation generated by scattering at the last scattering



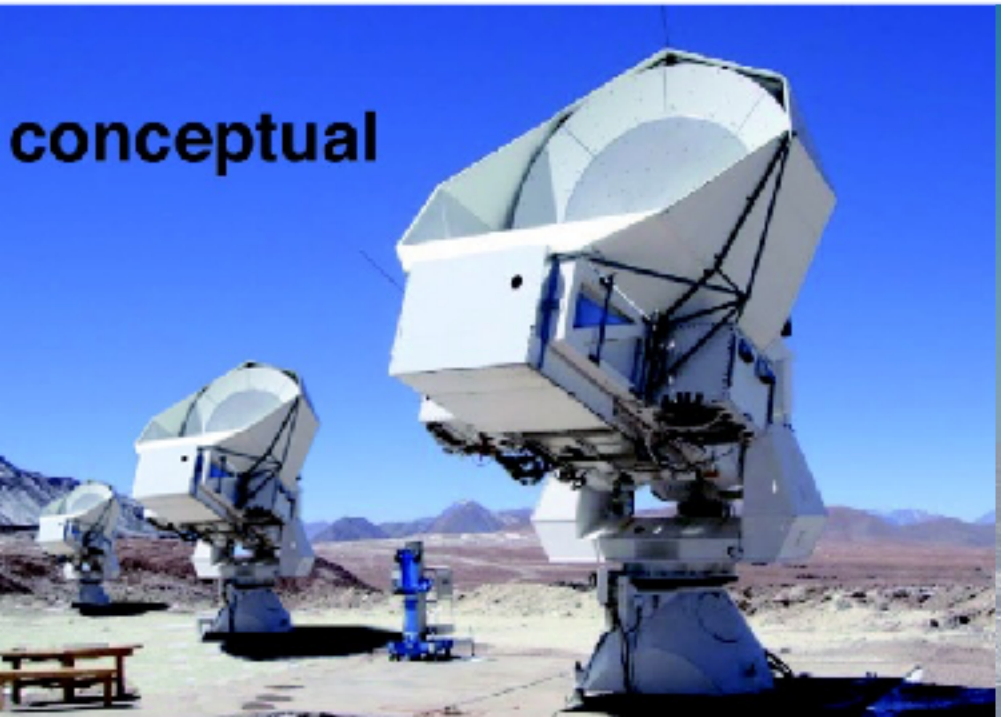
Temperature
from sound waves
E-mode
from sound waves
B-mode from lensing
B-mode from GW
We understand this
r=0.05
Planck
BICEP2/Keck
SPTPol
POLARBEAR
10^4
10^2
10^0
10^{-2}
10^{-4}
10
100
500
1000
1500
2000
l
$l(l+1)C_l^{TT,EE,BB}/2\pi\ [\mu K^2]$

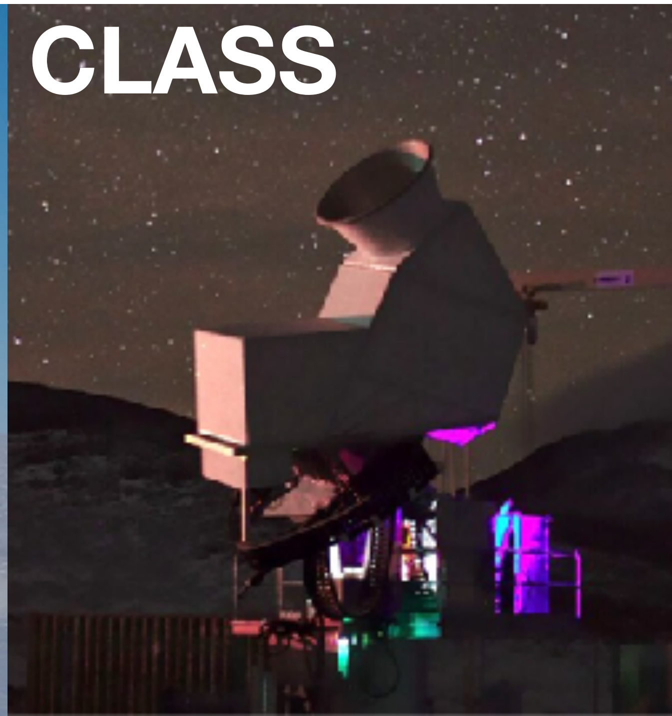
Appendix: Experimental Landscape



What comes next?

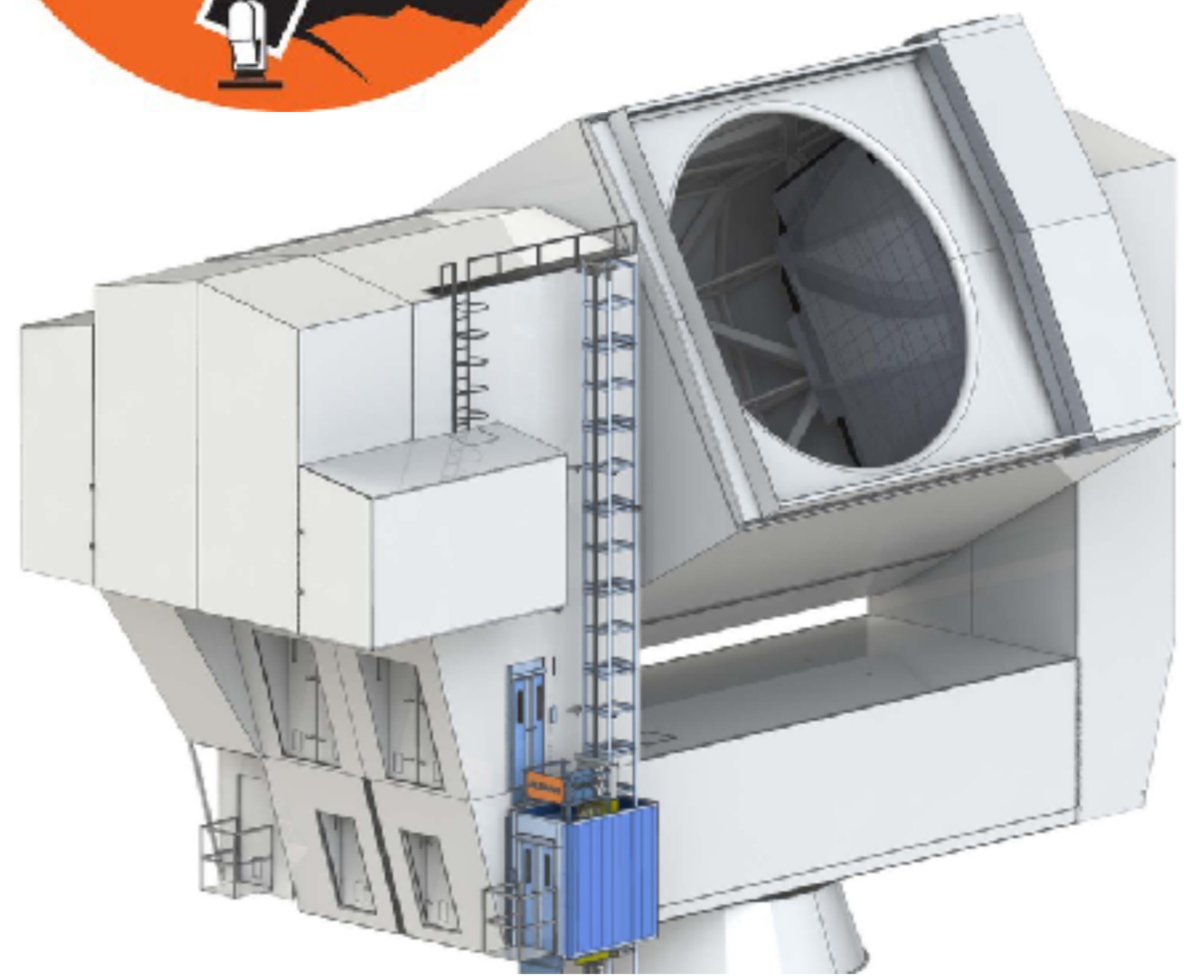
The Simons Array



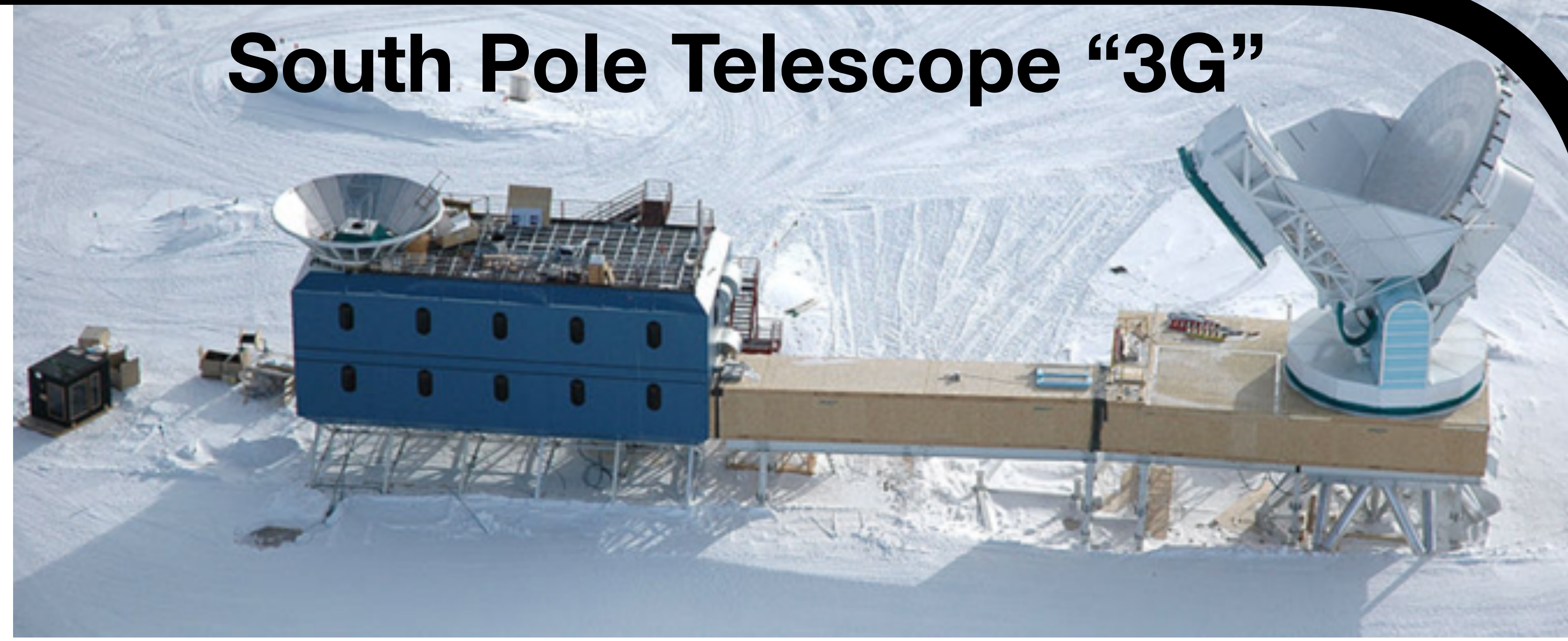


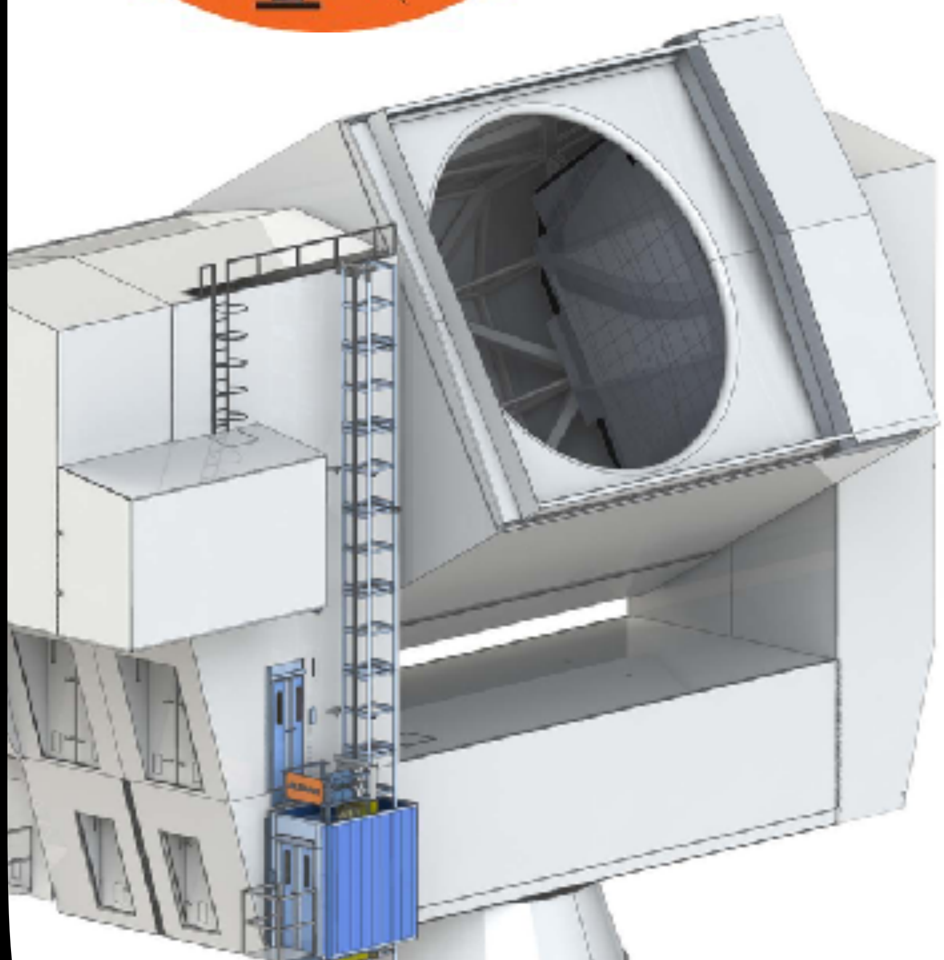
SIMONS OBSERVATORY

The Simons Array



SIMONS OBSERVATORY





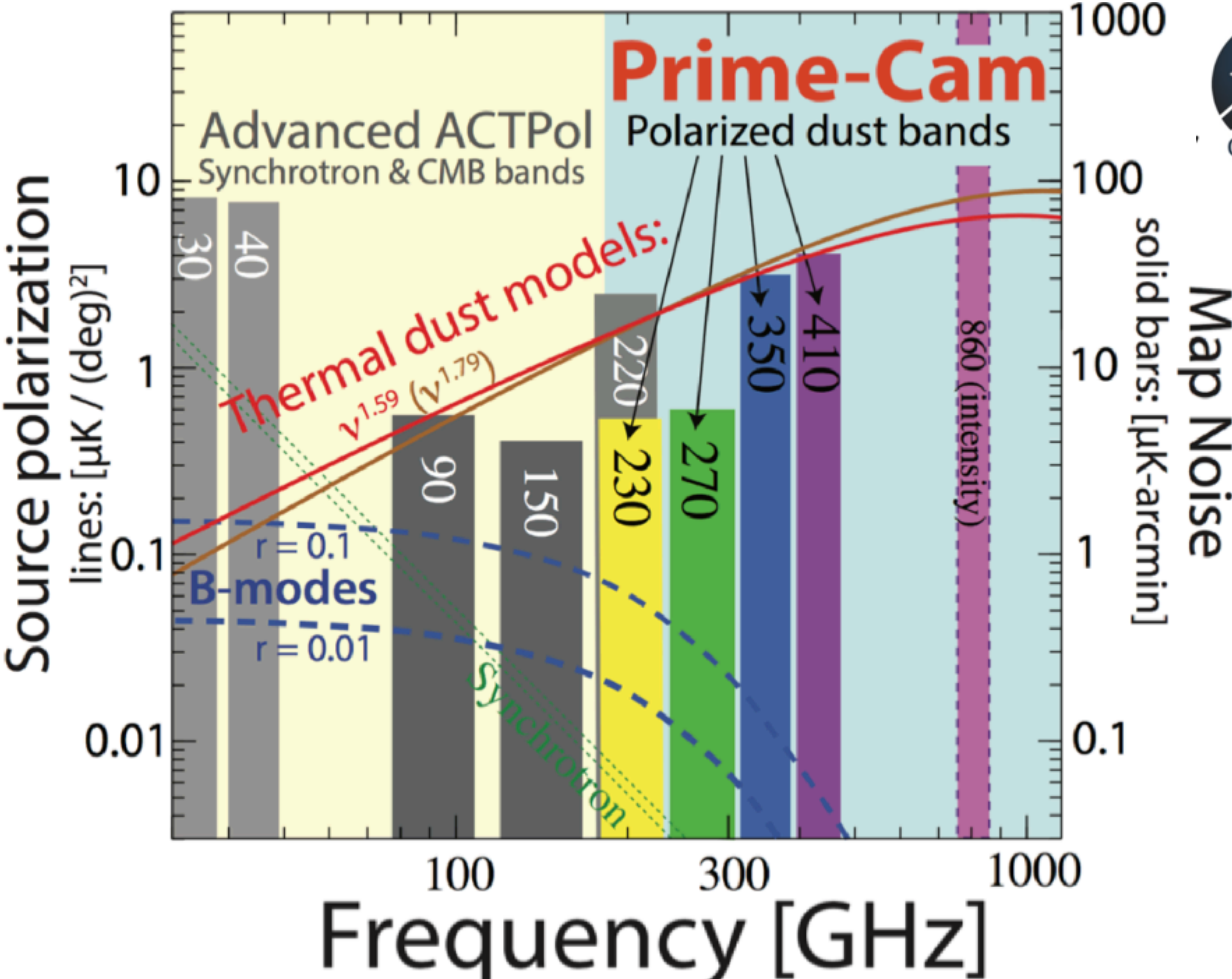
CMB-S4(?)

The Biggest Enemy: Polarised Dust Emission

- The upcoming data will **NOT** be limited by statistics, but by systematic effects such as the Galactic contamination
- **Solution**: Observe the sky at multiple frequencies, especially at high frequencies (>300 GHz)
- This is challenging, unless we have a superb, high-altitude site with low water vapour

- CCAT-p!

CCAT-p Collaboration



Frank Bertoldi’s slide from the Florence meeting

Where is CCAT-p?

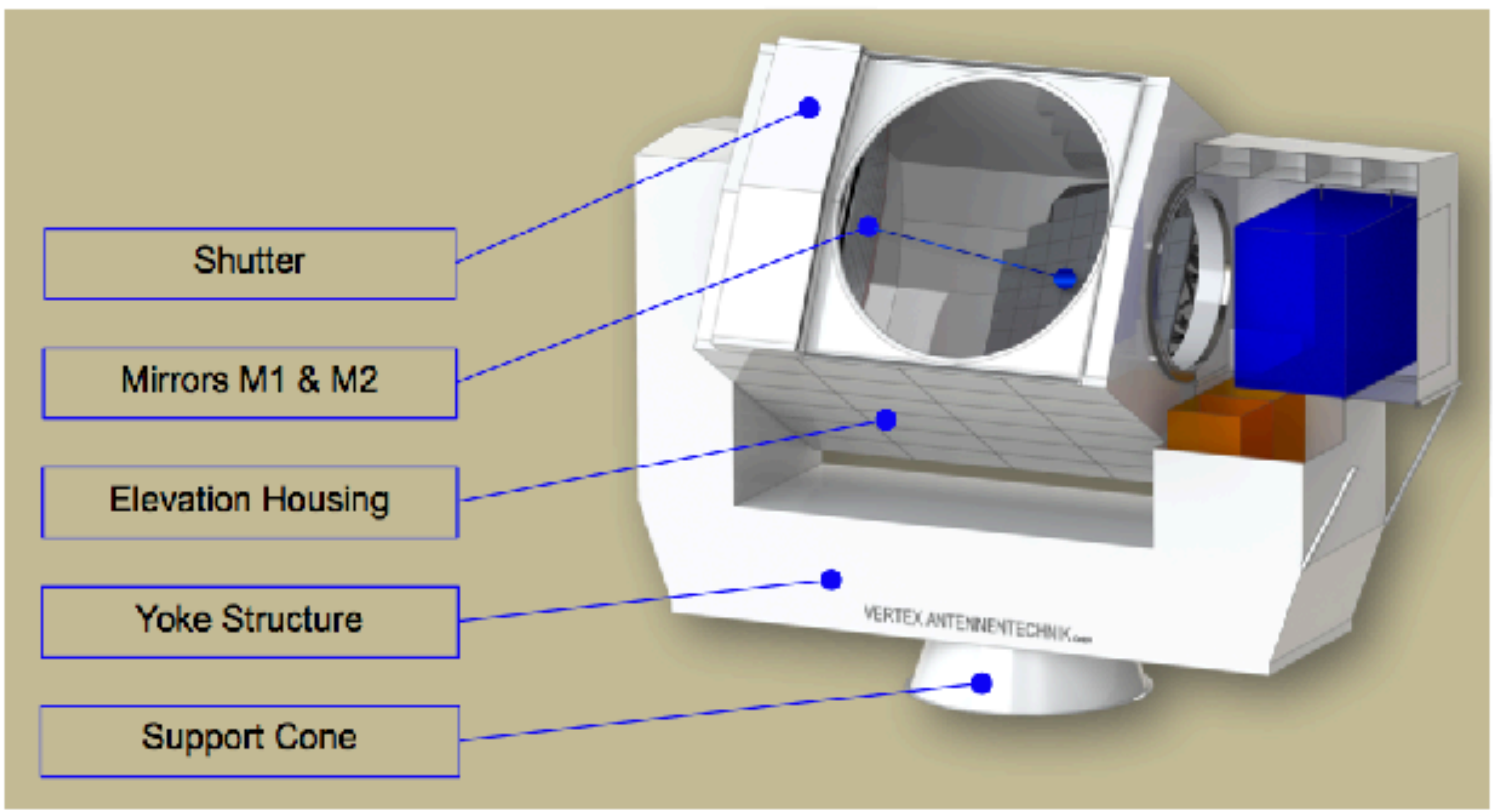
Cerro Chajnantor at 5600 m w/ TAO

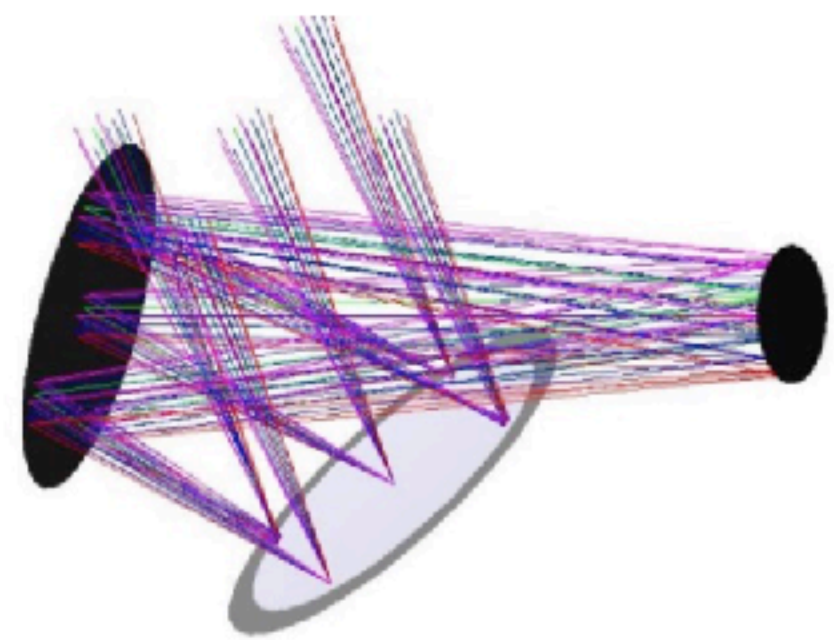
Frank Bertoldi's slide from the Florence meeting

What is CCAT-p?

CCAT-p

CCAT-prime is a high surface accuracy / throughput 6 m submm (0.3-3mm) telescope

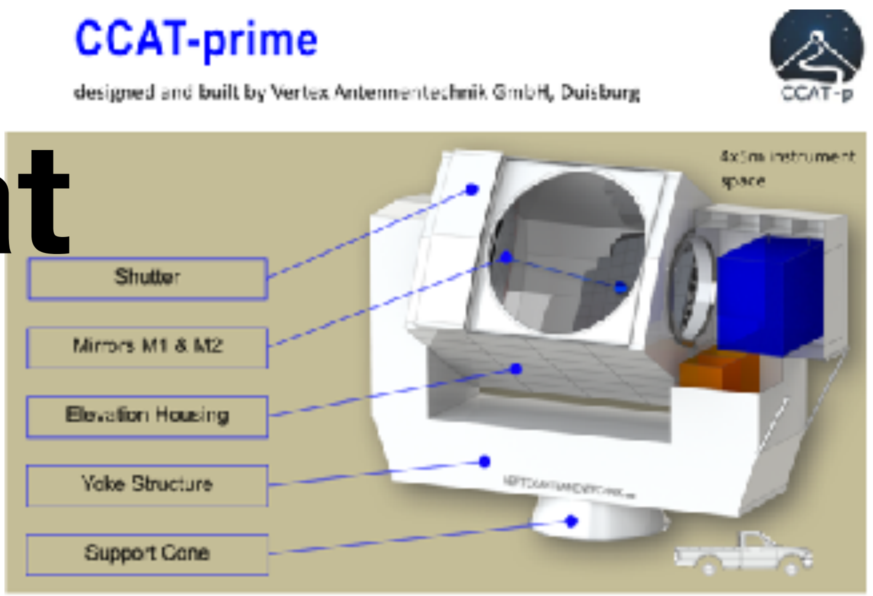




Cornell U. + German consortium + Canadian consortium + …

A Game Changer

- **CCAT-p**: 6-m, **Cross-dragone** design, on Cerro Chajnantor (5600 m)



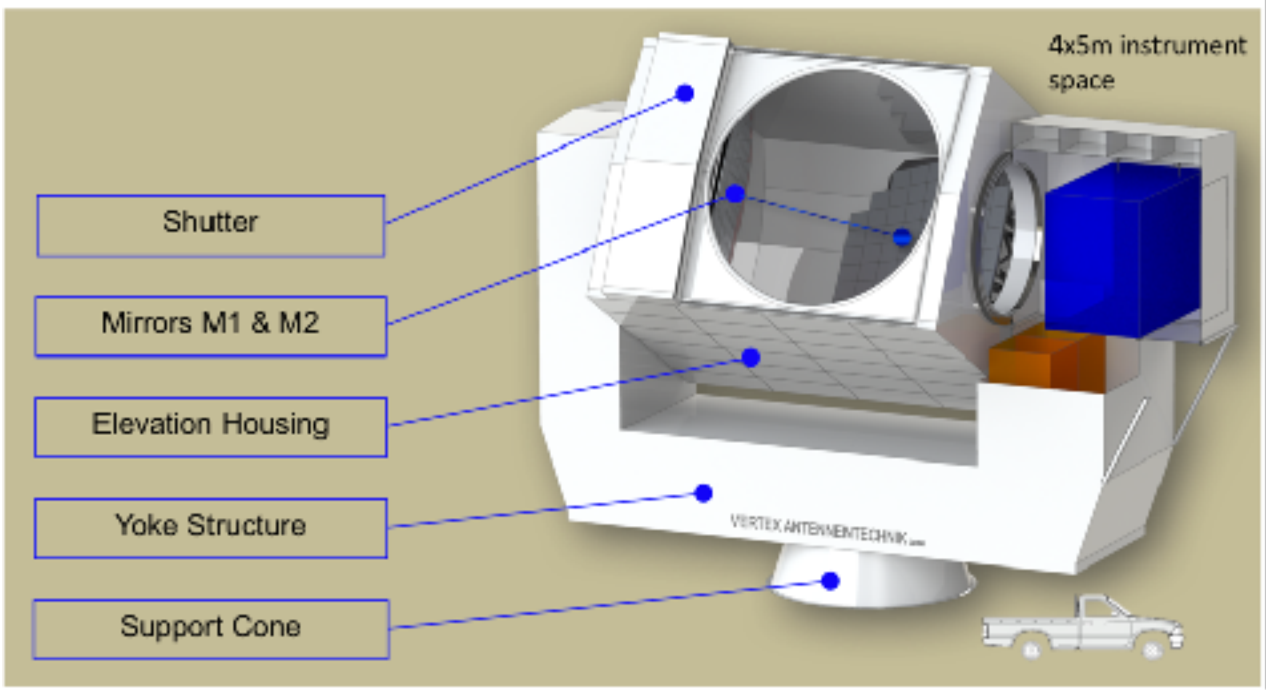
- **Germany makes great telescopes!**

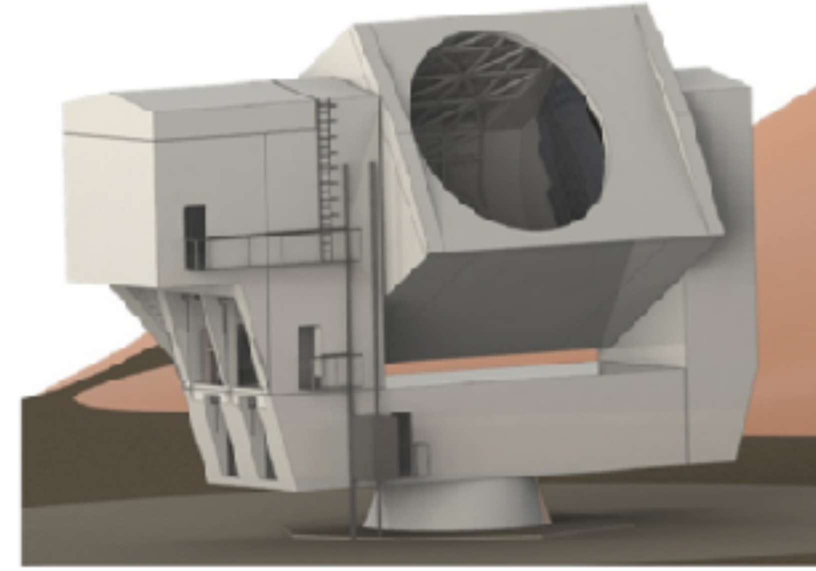
- Design study completed, and the contract has been signed by "VERTEX Antennentechnik GmbH"
    - CCAT-p is a great opportunity for Germany to make <u>significant contributions</u> towards the CMB S-4 landscape (both US and Europe) by providing telescope designs and the "lessons learned" with prototypes.

CCAT-prime

designed and built by Vertex Antennentechnik GmbH, Duisburg

CCAT-p





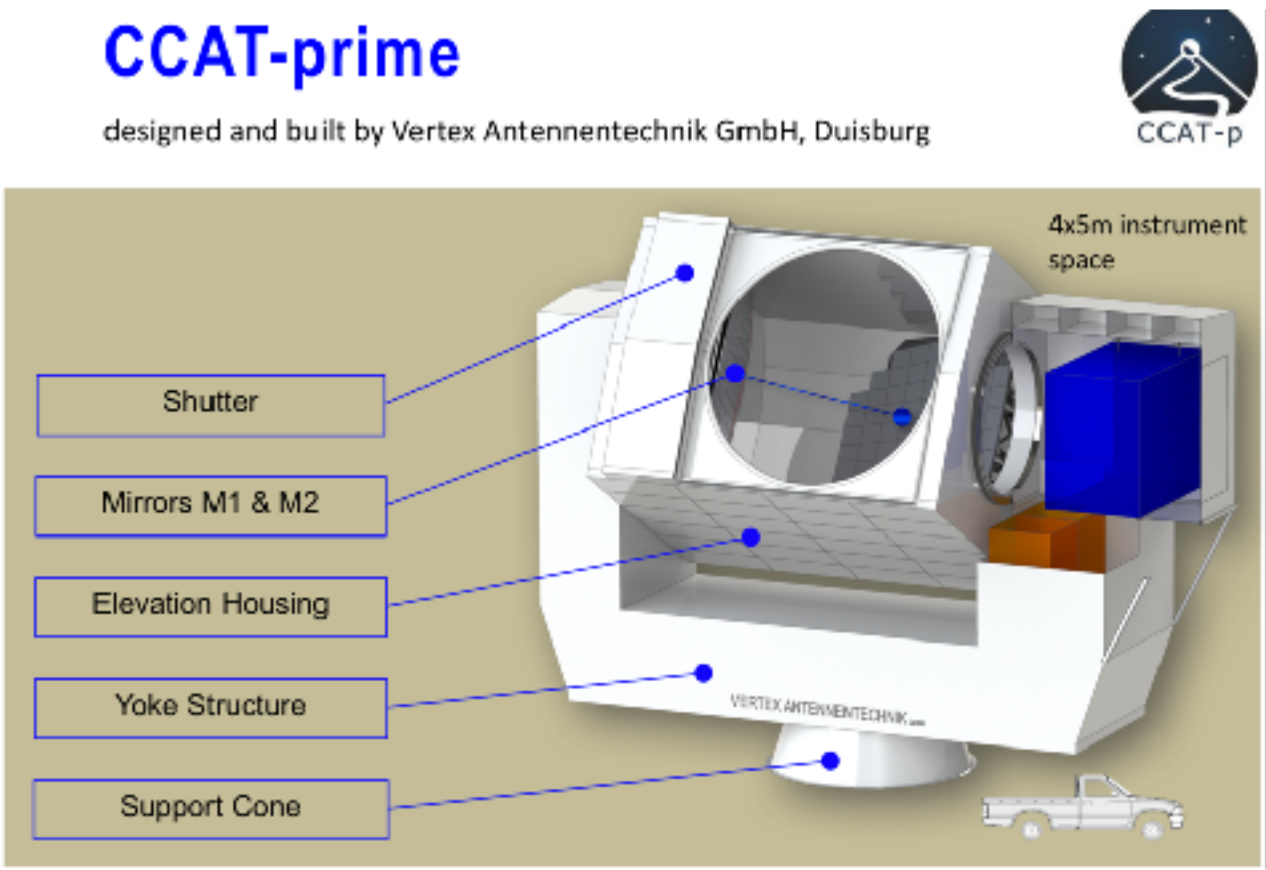
A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK.

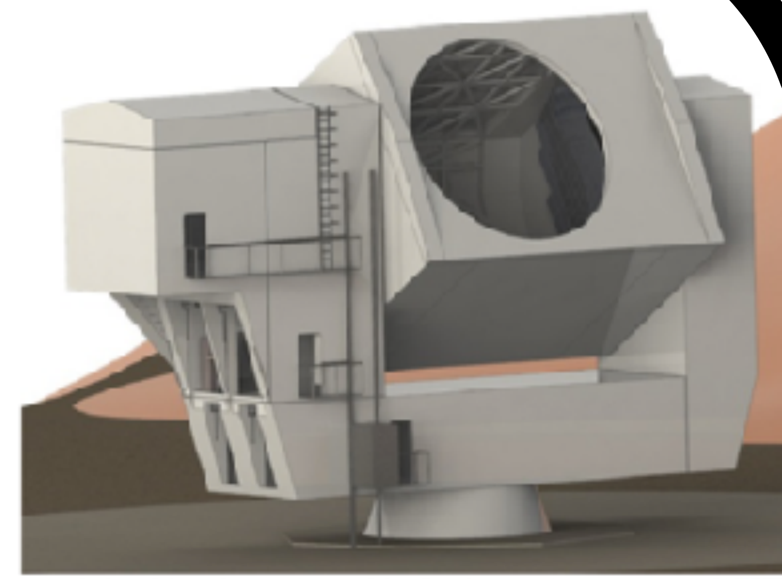
Simons Observatory (USA)

in collaboration

South Pole?

This could be "CMB-S4"





A rendering of the unique and powerful radio telescope. Image courtesy of VERTEX ANTENNENTECHNIK

Simons Observatory (USA)

in collaboration

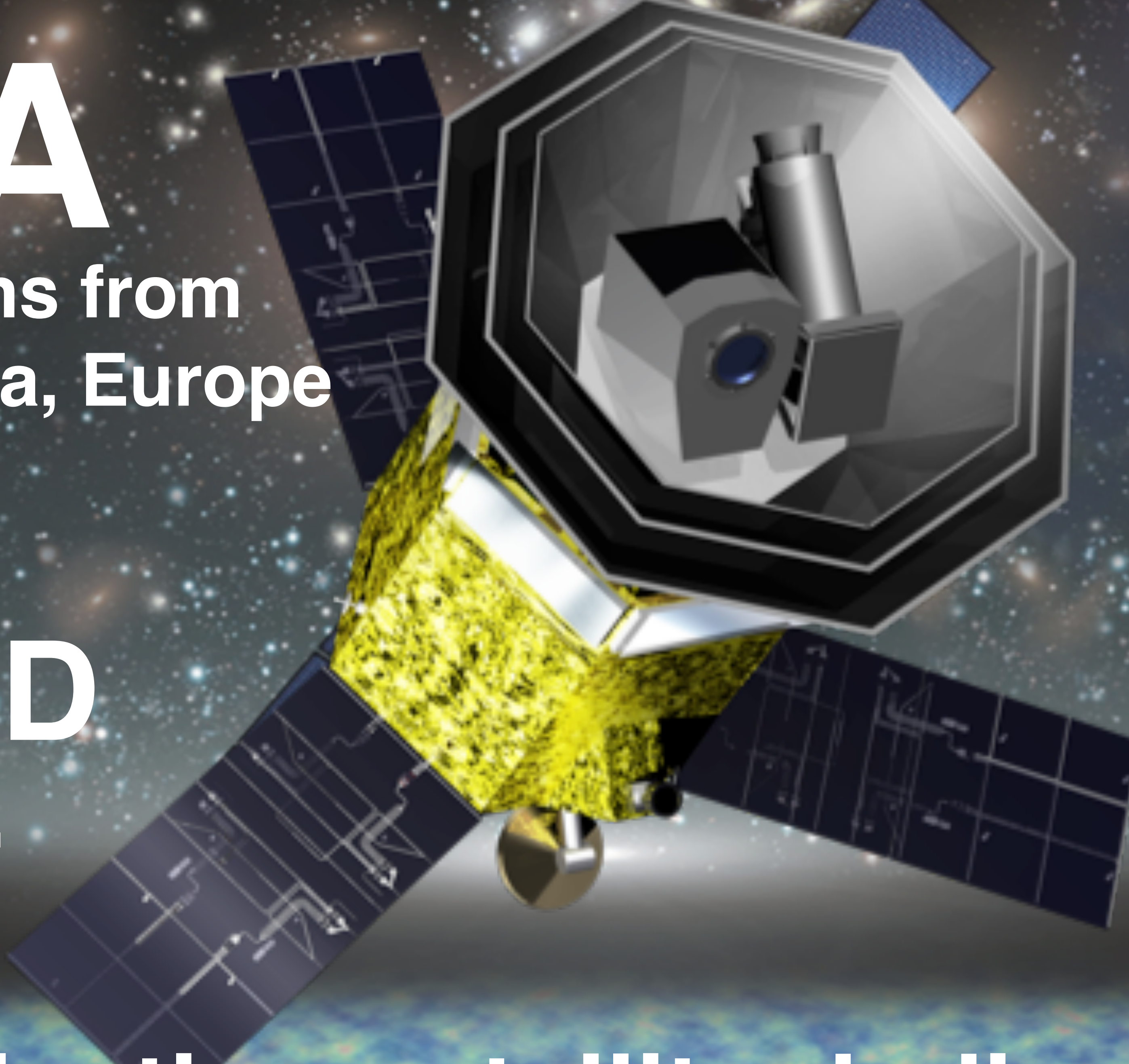
South Pole?

To have even more frequency coverage…

JAXA

+ participations from USA, Canada, Europe

LiteBIRD 2028–



Polarisation satellite dedicated to measure CMB polarisation from primordial GW, with a few thousand TES bolometers in space

JAXA

+ participations from USA, Canada, Europe

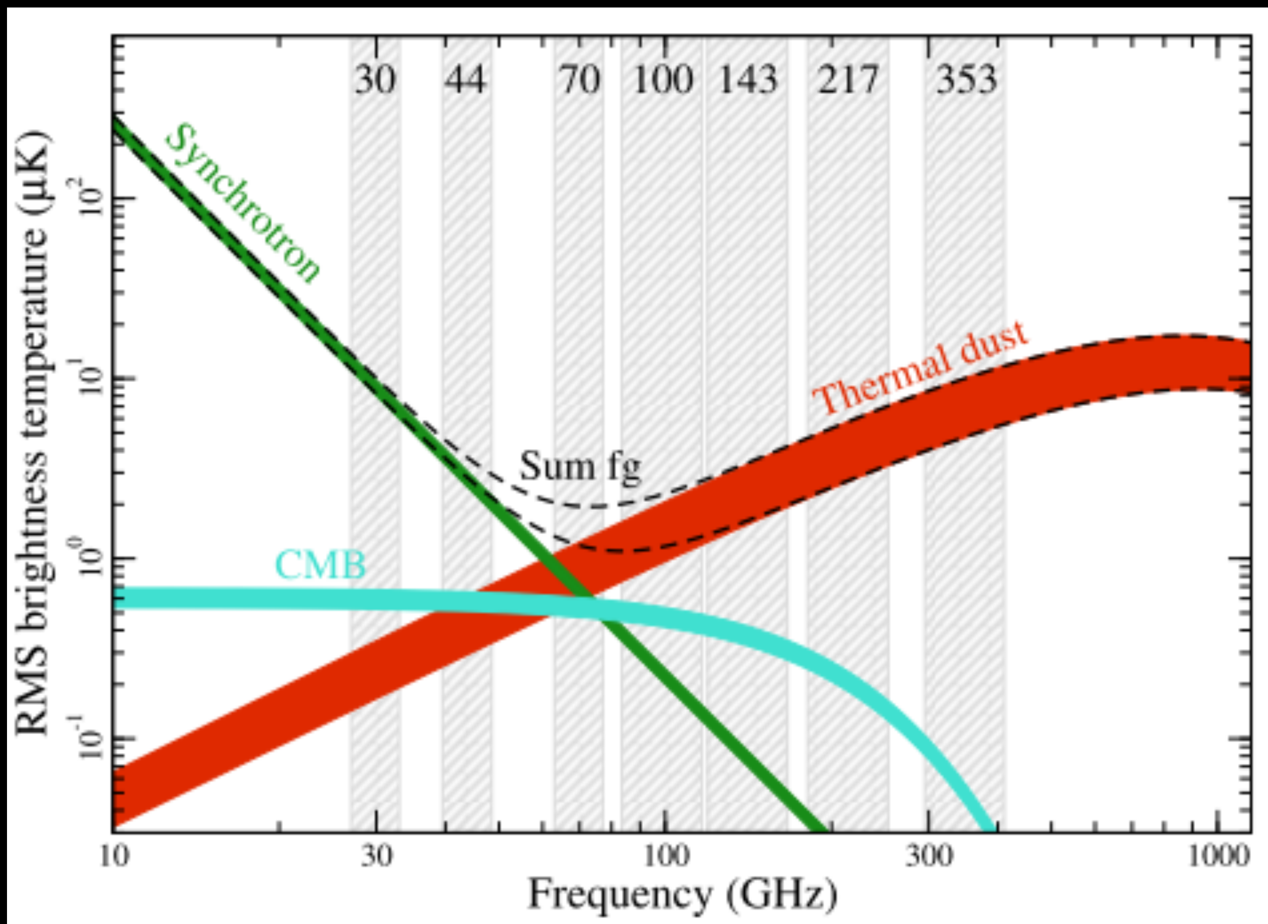
LiteBIRD 2028–

Selected!

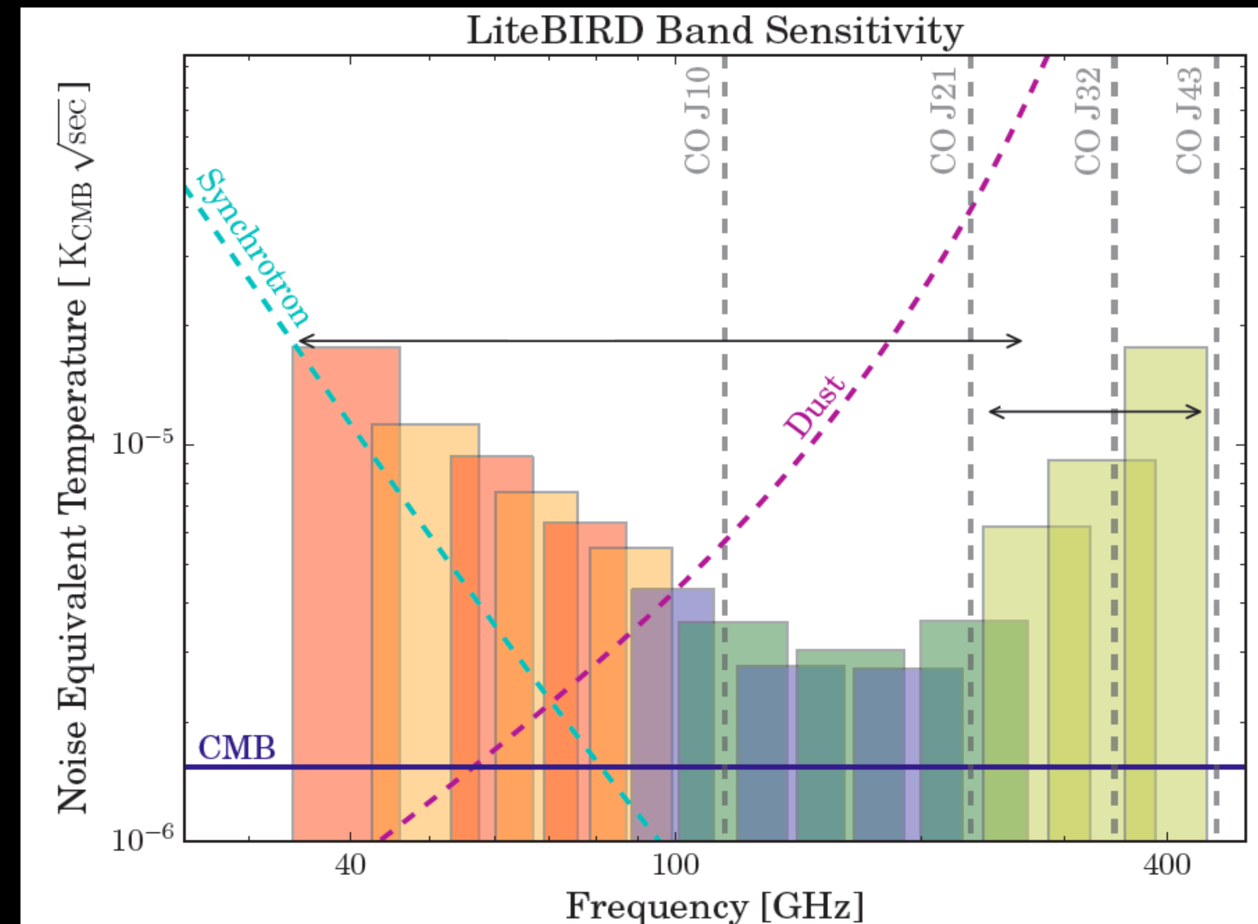
May 21, 2019: JAXA has chosen LiteBIRD as the strategic large-class mission.

We will go to L2!

Foreground Removal



Polarized galactic emission (Planck X)



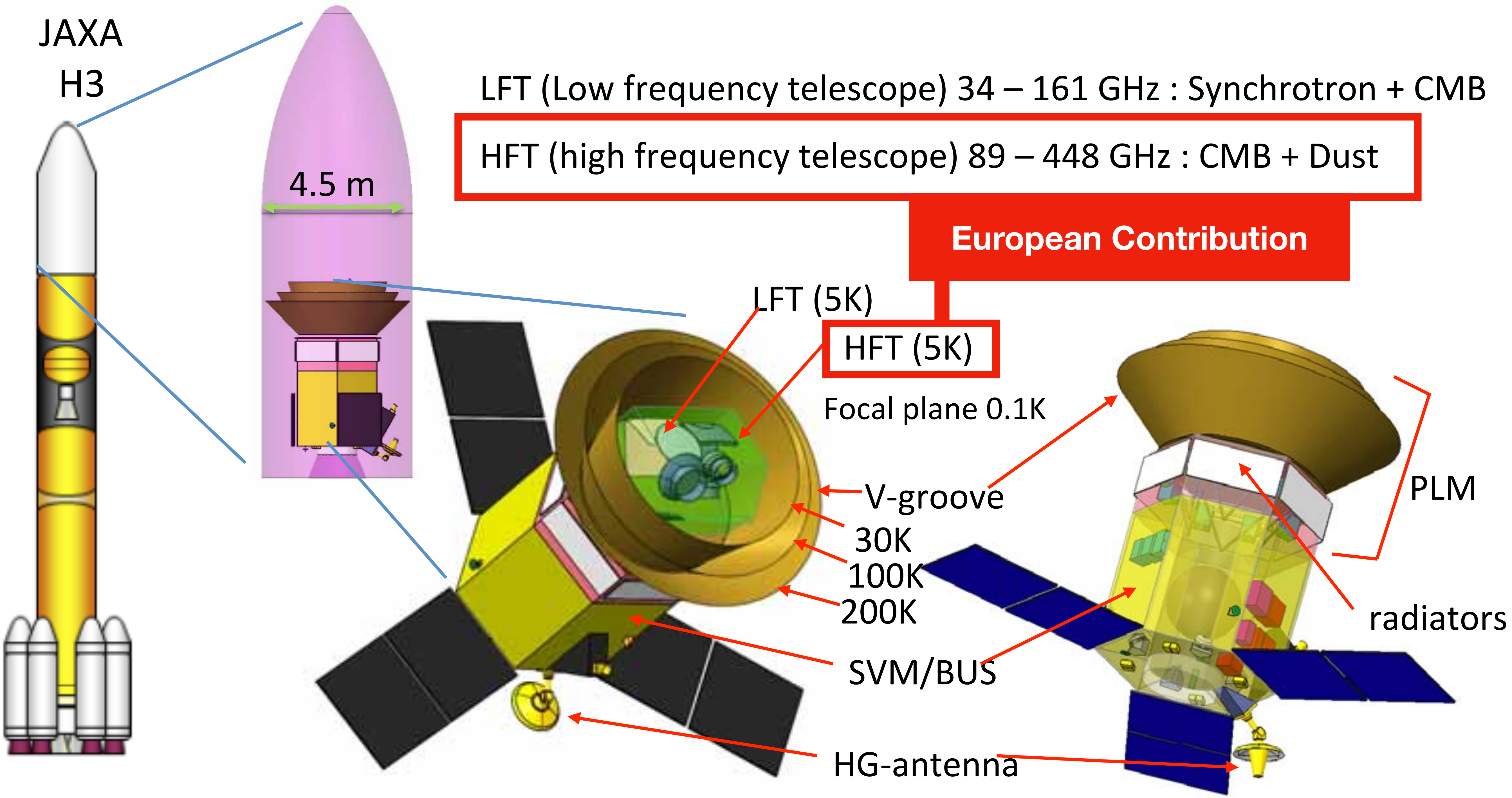
LiteBIRD: 15 frequency bands

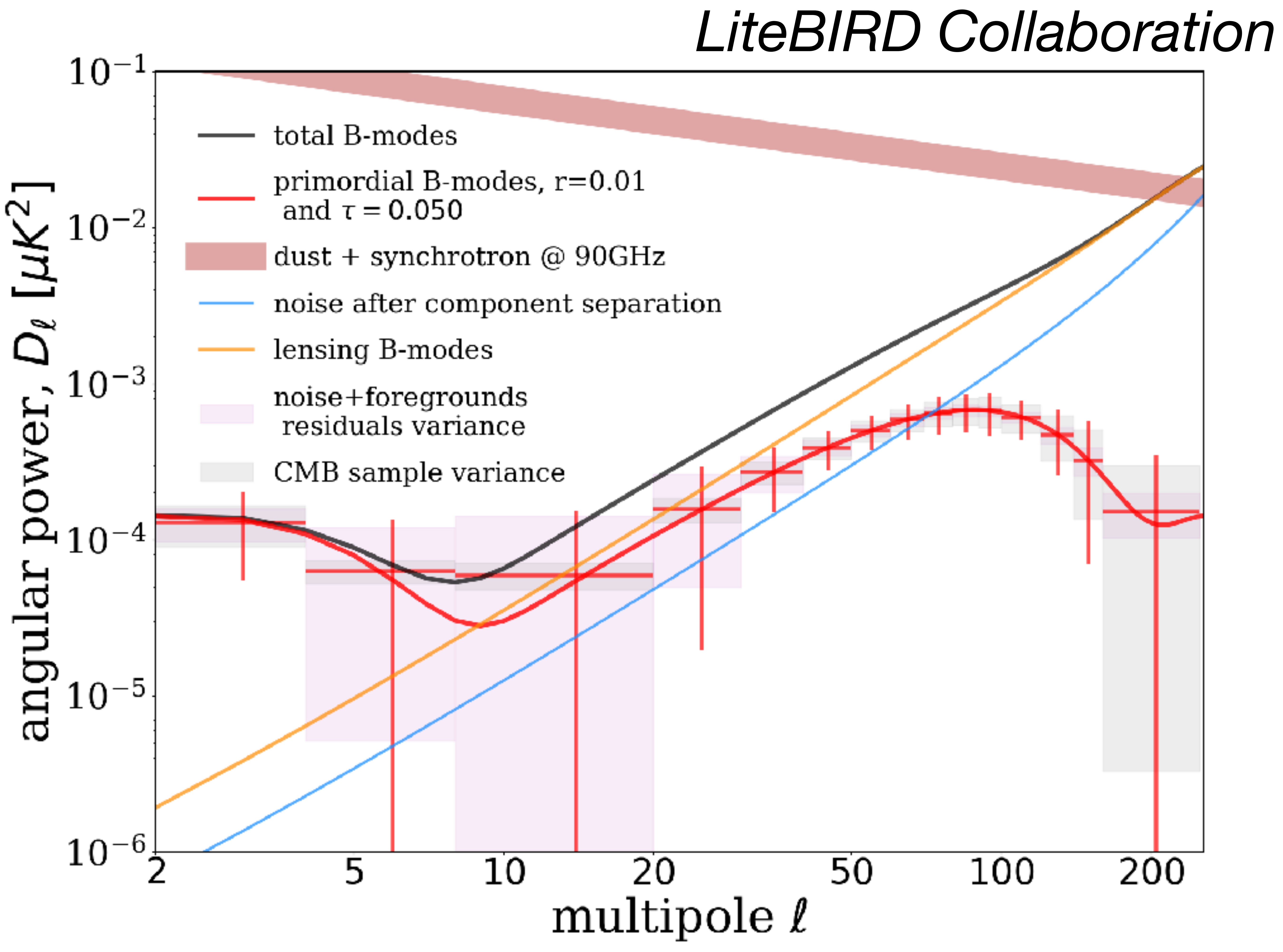
- Polarized foregrounds
  - Synchrotron radiation and thermal emission from inter-galactic dust
  - Characterize and remove foregrounds
- 15 frequency bands between 40 GHz - 400 GHz
  - Split between Low Frequency Telescope (LFT) and High Frequency Telescope (HFT)
  - LFT: 40 GHz – 235 GHz
  - HFT: 280 GHz – 400 GHz

Slide courtesy Toki Suzuki (Berkeley)

Slide courtesy Yutaro Sekimoto (ISAS/JAXA)

LiteBIRD Spacecraft





LiteBIRD Collaboration
total B-modes
primordial B-modes, r=0.01
and τ = 0.050
dust + synchrotron @ 90GHz
noise after component separation
lensing B-modes
noise+foregrounds
residuals variance
CMB sample variance
angular power, D_ℓ [μK^2]
multipole ℓ
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
2
5
10
20
50
100
200

LiteBIRD Collaboration