# On the analytical method of inflationary particle production with form fields

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科研費
KAKENHI

ダークマターの正体は何か？
広大なディスカバリースペースの網羅的研究
What is dark matter? - Comprehensive study of the huge discovery space in dark matter
文部科学省
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MAX-PLANCK-INSTITUT
FÜR ASTROPHYSIK

19.7.2022 mini-workshop

# Form fields with kinetic couplings

- In higher dimensional theories, scalar sectors are naturally coupled to gauge sectors (form fields):

$$\mathcal{L} \supset I(\varphi)^2 F_{\mu\nu} F^{\mu\nu}, \; I(\varphi)^2 H_{\mu\nu\rho} H^{\mu\nu\rho}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

: vector field (1-form field)

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

: antisymmetric tensor field (2-form field)
→ axions

- Time variation of the kinetic function could trigger the particle production of form fields during inflation

# Rich cosmological phenomena

## $I^2 FF$ model

- Generation of primordial magnetic fields ***Ratra (1992); Martin, Yokoyama (2008); Fujita, Mukohyama (2012);… (a lot)***
- Anisotropic inflation models ***Watanabe, Kanno, Soda (2009)…; Ito, Soda (2015);***
- Statistically-anisotropic primordial GWs ***Fujita, IO, Tanaka, Yokoyama (2018); Hiramatsu, Murai, IO, Yokoyama (2020);***
- Generation of primordial BHs & GWs ***Kawasaki, Nakatsuka, IO (2019)***

## $I^2 HH$ model

- Anisotropic inflation models ***Ohashi, Tsujikawa, Soda (2013)…; Ito, Soda (2015);***
- Statistically-anisotropic primordial GWs ***IO, Fujita (2018);***
- Generation of Primordial BHs & GWs ***Fujita, Nakatsuka, IO, Young (2022);***

# Inflation with form field (setup)

- Consider the vector field model:

$$\mathcal{L} = \frac{1}{2}M_p^2 R - \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4}I(\varphi)^2 F_{\mu\nu}F^{\mu\nu}$$

FLRW Metric: $ds^2 = -dt^2 + a(t)^2 d\boldsymbol{x}^2 = a(\tau)^2(-d\tau^2 + d\boldsymbol{x}^2)$

Gauge conditions: $A_0 = 0,\ \partial_i A_i = 0$

- Consider the operator expansion in Fourier space:

$$A_i(\tau, \boldsymbol{x}) = \sum_{s=X,Y} \int \frac{d\boldsymbol{k}}{(2\pi)^3} \hat{A}^s_{\boldsymbol{k}}(\tau) e^s_i(\hat{\boldsymbol{k}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{A}^s_{\boldsymbol{k}} = A^s_k a^s_{\boldsymbol{k}} + A^{s*}_k a^{s\dagger}_{-\boldsymbol{k}} \qquad [a^{s1}_{\boldsymbol{k}},\ a^{s2\dagger}_{-\boldsymbol{k}'}] = (2\pi)^3 \delta^{s_1 s_2} \delta(\boldsymbol{k} + \boldsymbol{k}')$$

↑ find a solution of mode function

# Solution of mode function (1)

- EOM for the mode function: $V_k \equiv I A_k^s$

$$\partial_\tau^2 V_k + \left(k^2 - \frac{\partial_\tau^2 I}{I}\right) V_k = 0$$

- Define the following index: $n \equiv \frac{\dot{I}}{HI}$ $\rightarrow$ $I(\varphi(\tau)) \propto a(\tau)^n$

when $n = n_0 \ (\mathrm{const.}),$ EOM leads to

$$\partial_\tau^2 V_k + \left(k^2 - \frac{n_0(n_0+1)}{\tau^2}\right) V_k = 0$$

then, we obtain $$V_k(\tau) = \frac{e^{i(n_0+1)\pi/2}}{\sqrt{2k}} \sqrt{\frac{-\pi k\tau}{2}} H^{(1)}_{n_0+1/2}(-k\tau)$$

(Banch-Davies initial condition is chosen)

# Solution of mode function (2)

- The solution of the mode functions of electromagnetic field:

$$E_k = \frac{I}{a^2}\frac{d}{d\tau}\left(\frac{V_k}{I}\right) = e^{i(n-1)\pi/2}\frac{H^2}{\sqrt{2k^3}}\sqrt{\frac{\pi(-k\tau)^5}{2}}H^{(1)}_{n-1/2}(-k\tau) \ ,$$

$$B_k = \frac{kV_k}{a^2} = e^{i(n+1)\pi/2}\frac{H^2}{\sqrt{2k^3}}\sqrt{\frac{\pi(-k\tau)^5}{2}}H^{(1)}_{n+1/2}(-k\tau) \ .$$

- Then, using the following asymptotic form in the super-horizon limit:

$$H^{(1)}_{\nu<0}(x) = i(-1)^{\nu+1}\frac{\Gamma(-\nu)}{\pi}\left(\frac{2}{x}\right)^{-\nu} \qquad (x \to 0) \ ,$$

we obtain

$$E_k = e^{-i(n+1)\pi/2}\frac{H^2}{\sqrt{2k^3}}\frac{\Gamma(\frac{1}{2}-n)}{2^n\sqrt{\pi}}(-k\tau)^{n+2} \qquad (|k\tau| \to 0) \ ,$$

$$B_k = e^{-i(n+1)\pi/2}\frac{H^2}{\sqrt{2k^3}}\frac{\Gamma(-\frac{1}{2}-n)}{2^{n+1}\sqrt{\pi}}(-k\tau)^{n+3} \qquad (n < -\tfrac{1}{2},\ |k\tau| \to 0) \ .$$

# Solution of mode function (3)

Thus, the magnitude of electromagnetic fields evolves as

$$|E_k(a)| = \frac{H^2}{\sqrt{2k^3}} \frac{\Gamma(\frac{1}{2} - n_0)}{2^{n_0}\sqrt{\pi}} \left(\frac{a_k}{a}\right)^{n_0+2}$$

$$|B_k(a)| = \frac{H^2}{\sqrt{2k^3}} \frac{\Gamma(-\frac{1}{2} - n_0)}{2^{n_0+1}\sqrt{\pi}} \left(\frac{a_k}{a}\right)^{n_0+3}$$

$$(a_k \equiv k/H \ll a)$$

on super-horizon scales

- Electric (magnetic) field is amplified when $n_0 < -2 \quad (n_0 < -3)$

- This expression holds when the index "n" is a **constant** value

However,

**The index "n" generically depends on the configuration of kinetic function**

# The problem is…

- In most cases, the index n is not a constant but a dynamical value

(except for)

$$I(\varphi) = \exp\left(-\frac{n_0}{M_p^2}\int \frac{V}{V_\varphi} d\varphi\right)$$

Ex) consider the simplest configuration

$$I(\varphi) = I_0 \exp\left(\frac{\varphi}{\Lambda}\right) \quad \rightarrow \quad n = \frac{\dot{\varphi}}{H\Lambda} \neq \text{const.}$$

Since the speed of scalar field naturally increases in time,
we need to solve the EOM with dynamical "n":

$$\partial_\tau^2 V_k + \left(k^2 - \frac{n(\tau)(n(\tau)+1)}{\tau^2}\right) V_k = 0$$

→ No analytical solution. Numerical analysis has been developed

***Kawasaki, Nakatsuka, IO (2019); Fujita, Nakatsuka, IO, Young (2022);***

# New discovery???

- Once we define the following **time-averaged index**

$$\bar{n}(k,\tau) \equiv \frac{1}{N(\tau)-N_l}\int_{N_l}^{N(\tau)} n(N)dN$$

$$N_l \equiv \ln(k/H) - \ln\sqrt{n(-k^{-1})(n(-k^{-1})+1)}$$

the following expression fits well the evolution on super-horizon scales:

$$\begin{aligned} |E_k^{\mathrm{app}}(\tau)| &\equiv \frac{H^2}{\sqrt{2k^3}}\sqrt{\frac{\pi(-k\tau)^5}{2}}\left|H^{(1)}_{\bar{n}(k,\tau)-1/2}(-k\tau)\right| \\ &= \frac{H^2}{\sqrt{2k^3}}\frac{\Gamma(1/2-\bar{n}(k,\tau))}{2^{\bar{n}(k,\tau)}\pi^{1/2}}(-k\tau)^{\bar{n}(k,\tau)+2} \qquad (\tau\to 0) \end{aligned}$$

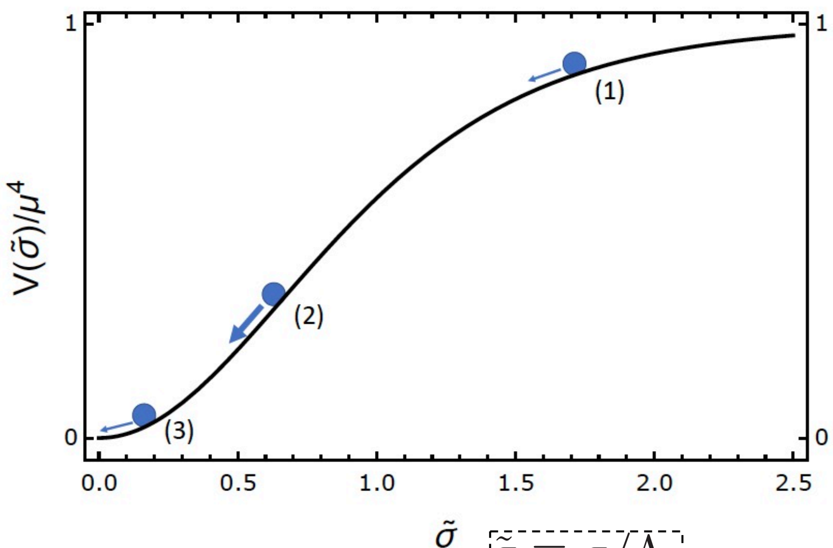
# Toy model (1)

$$\mathcal{L} = \frac{M_p^2}{2}R + \mathcal{L}_{\rm inf} - \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma) - \frac{1}{4}I(\sigma)^2 F_{\mu\nu}F^{\mu\nu}$$

$\sigma$ : spectator field

Kinetic function: $I(\sigma) = I_0 \exp\left(\frac{\sigma}{\Lambda_1}\right) \rightarrow n = \frac{dI/dN}{I} = \frac{d\sigma/dN}{\Lambda_1}$

Potential form: $V(\sigma) = \mu^4 \tanh^2\left(\frac{\sigma}{\Lambda_2}\right)$




$\tilde{\sigma} \equiv \sigma/\Lambda_2$

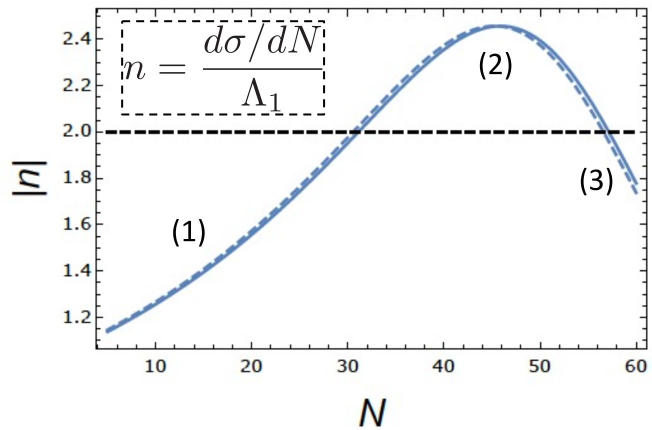
(1) At initial time (CMB scales), the velocity speed monotonically increases.

(2) At a certain time, the scalar field crosses the inflection point and the velocity gets maximized.

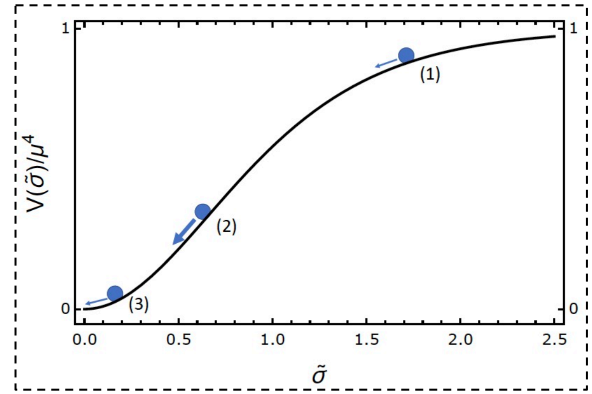
(3) After that, the speed turns to decrease towards the minimum of potential

# Toy model (2)

## Time evolution of “n”

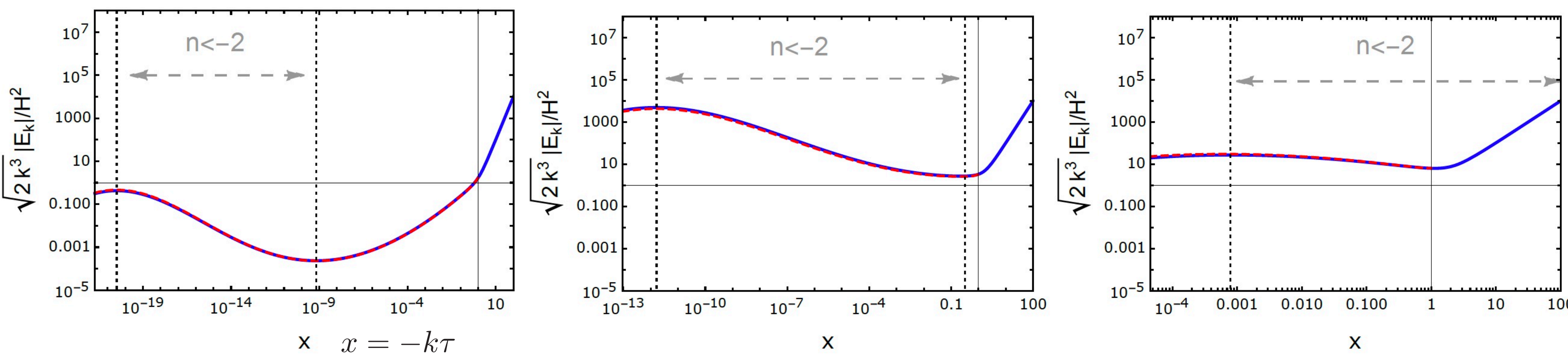



:exact solution
:slow-roll solution




## Evolutions of electric mode function on super-horizon regime

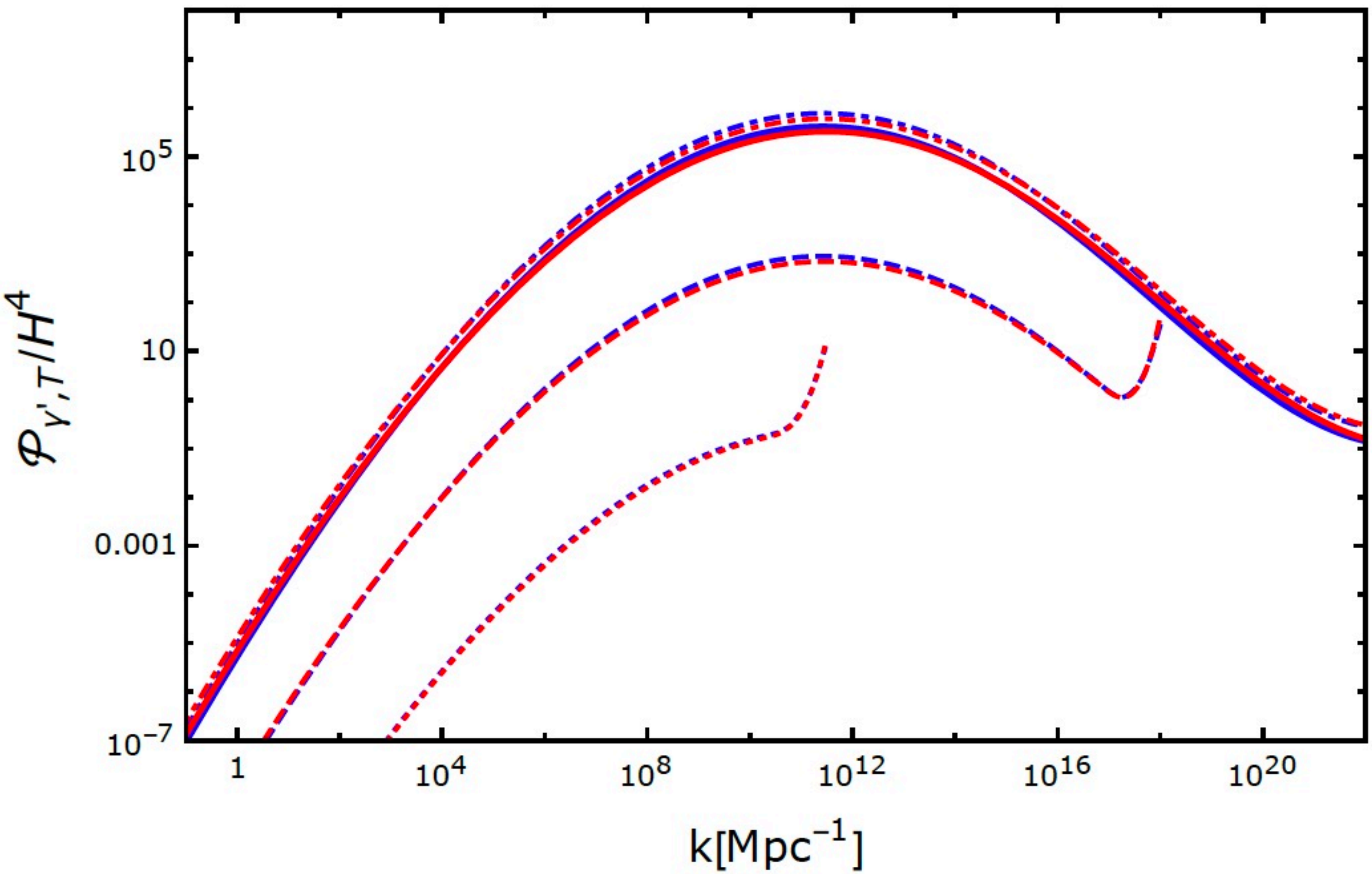
:exact solution
:approximate




Exiting horizon at $N = 10$ $N = 30$ $N = 50$

# Toy model (3)

Power spectrum of vector field at several e-folds



# Questions…

$$|E_k^{\mathrm{app}}(\tau)| \equiv \frac{H^2}{\sqrt{2k^3}}\sqrt{\frac{\pi(-k\tau)^5}{2}}\left|H^{(1)}_{\bar{n}(k,\tau)-1/2}(-k\tau)\right|$$
$$= \frac{H^2}{\sqrt{2k^3}}\frac{\Gamma(1/2-\bar{n}(k,\tau))}{2^{\bar{n}(k,\tau)}\pi^{1/2}}(-k\tau)^{\bar{n}(k,\tau)+2} \qquad (\tau\to 0)$$

$$\bar{n}(k,\tau) \equiv \frac{1}{N(\tau)-N_l}\int_{N_l}^{N(\tau)} n(N)dN$$

$$N_l \equiv \ln(k/H) - \ln\sqrt{n(-k^{-1})(n(-k^{-1})+1)}$$

- How exactly to prove this? Is it already known?
- How universal does this hold? e.g. 2-form field model
- Application to the cosmological scenarios?

***Nakai, Namba, IO; in progress***