Modulated Reheating Mechanism and Spectral Index of Powerspectrum

MASAHIDE YAMAGUCHI
(Tokyo Institute of Technology)

11/07/12 @MPA Workshop

\[ c = \bar{h} = M_G^2 = 1/(8\pi G) = 1 \]
Contents

• Introduction
  Basic of inflation

• Modulated reheating mechanism
  What is the modulated reheating mechanism
  Powerspectrum, non-Gaussianities

• Spectral index of a (general) light field model
  Tension with observations

• Discussion and conclusions
Introduction
Inflation

Inflation can naturally solve the problems of the standard big bang cosmology.

- The horizon problem
- The flatness problem
- The origin of density fluctuations
- The monopole problem
- ...

After inflation, inflaton decays into the standard particles to (re)heat the Universe.
Primordial density fluctuations originating from inflaton

\[ 3M_G^2 H_0^2 = \rho_0 \]

\[ \Gamma = H \]

\[ \propto a^{-3} \propto H^2 \]

\[ \propto a^{-4} \]
Other sources for primordial density fluctuations

- However, **inflaton** is not necessarily only the candidate responsible for the curvature perturbations.

- **Other light fields**, which acquire quantum fluctuations during inflation, can contribute to them.

- In fact, there can be **light fields like moduli** in string theory, which determine coupling constants.
Modulated reheating mechanism
The decay rate of the inflaton depends on a light scalar field $\sigma$.

$$\Gamma = \Gamma(\sigma)$$

e.g. $\mathcal{L} = h(\sigma)\phi\overline{\psi}\psi$

(Inflaton must couple to a light scalar field (modulus) and SM particles.)

$$\sigma = \langle \sigma \rangle + \delta\sigma, \quad \delta\sigma \simeq \frac{H}{2\pi} \quad \text{during inflation}$$

$$\Gamma(\sigma) = \Gamma(\langle \sigma \rangle) + \delta\Gamma$$

$$T_R \propto \Gamma^{1/2} \quad \Rightarrow \quad \frac{\delta T_R}{T_R} = \frac{1}{2} \frac{\delta\Gamma}{\Gamma}$$

How does this fluctuation lead to the curvature perturbation?
Curvature perturbation in the modulated reheating mechanism

\[ \rho(t) = \rho_0 \left( \frac{a_R}{a_e} \right)^{-3} \left( \frac{a(t)}{a_R} \right)^{-4} \]

\[ = \rho_0 \left( \frac{a_e}{a(t)} \right)^4 \left( \frac{a_R}{a_e} \right) \left( \frac{a_R}{a_e} = \left( \frac{\Gamma}{H_0} \right)^{-2/3} \right) \]

\[ \frac{\delta \rho(t)}{\rho(t)} = \frac{\delta a_R}{a_R} = -\frac{2}{3} \frac{\delta \Gamma}{\Gamma} \] (on flat slicings)

\[ \zeta = \frac{1}{4} \frac{\delta \rho(t)}{\rho(t)} = -\frac{1}{6} \frac{\delta \Gamma}{\Gamma} = -\frac{1}{3} \frac{\delta T_R}{T_R} \] (The minus sign reflects that higher decay rate leads to lower energy.)
More general discussions
Decay channels and potentials of inflaton

Ichikawa, Suyama, Takahashi, MY

Inflaton starts the oscillation around its minimum of the potential after inflation.

\[ V(\phi) \propto \phi^{2n}. \]

\[ \rho_\phi \propto a^{-\frac{6n}{n+1}} \propto \left( a^{-3} \text{ for } n = 1, \ a^{-4} \text{ for } n = 2, \ a^{-\frac{9}{2}} \text{ for } n = 3 \right). \]

decrease slower \quad radiation \quad decrease faster

Decay channels:

\[ \mathcal{L}_{\text{int}} \supset - \sum_a y_a(\sigma) \phi \bar{\psi}_a \psi_a - \sum_a M_a(\sigma) \phi \chi_a^2 - \sum_a h_a(\sigma) \phi^2 \chi_a^2. \]

\[ \Gamma^{(n)}(\sigma) = \sum_a A_n \frac{y_a^2(\sigma)}{8\pi} m_{\phi}^{\text{eff}} + \sum_a B_n \frac{M_a^2(\sigma)}{8\pi m_{\phi}^{\text{eff}}} + \sum_a C_n \frac{h_a^2(\sigma)}{8\pi (m_{\phi}^{\text{eff}})^2} \rho_\phi. \]

\[ (m_{\phi}^{\text{eff}})^2 = V_{\phi\phi} \big|_{\phi = \bar{\phi}} \]

<table>
<thead>
<tr>
<th>(n=1)</th>
<th>(n=2)</th>
<th>(n=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_n)</td>
<td>1</td>
<td>0.676</td>
</tr>
<tr>
<td>(B_n)</td>
<td>1</td>
<td>2.693</td>
</tr>
<tr>
<td>(C_n)</td>
<td>0.5</td>
<td>8.86</td>
</tr>
</tbody>
</table>
**e-folding number**

\[ N(t_f, t_*, \phi_*, \sigma_*) = N(t_e, t_*, \phi_*) + N(t_f, t_e, \sigma_*) \]

final time, horion exit  
negligible(assumption)  
inflation end

\[ N(t_f, t_e, \sigma_*) = \frac{1}{4} \log \frac{\rho_e}{\rho_f} + Q \left[ \Gamma_\phi(\sigma_*, t_e) \right] \frac{H_e}{H_e} \]

\[
\begin{align*}
\frac{d\rho_r}{dN} + 4 \rho_r &= \frac{\Gamma_\phi}{H} \rho_\phi \delta(N - N_R), \\
\frac{d\rho_\phi}{dN} + c_\phi \rho_\phi &= -\frac{\Gamma_\phi}{H} \rho_\phi \delta(N - N_R), \\
H^2 &= \frac{1}{3M_{pl}^2} (\rho_\phi + \rho_r).
\end{align*}
\]

\[ N(t_f, t_e, \sigma_*) = \frac{1}{4} \log \frac{\rho_e}{\rho_f} + \frac{1}{2} \log \frac{\Gamma_R}{H_e} + N_R. \]
e-folding number II

\[ N(t_f, t_e, \sigma_*) = \frac{1}{4} \log \frac{\rho_e}{\rho_f} + Q \left[ \frac{\Gamma_{\phi}(\sigma_*, t_e)}{H_e} \right] . \]

\[ Q \left[ \frac{\Gamma_{\phi}(\sigma_*, t_e)}{H_e} \right] = \frac{1}{2} \log \frac{\Gamma_{R}}{H_e} + N_R = \frac{4 - c_\phi}{2(2c_\Gamma - c_\phi)} \log \frac{\Gamma_{\phi}(\sigma_*, t_e)}{H_e} . \]

\[ Q(x) = a_0 \log x, \quad x = \frac{\Gamma_{\phi}(\sigma_*, t_e)}{H_e}, \quad a_0 = \frac{4 - c_\phi}{2(2c_\Gamma - c_\phi)} = \frac{2 - n}{2[(c_\Gamma - 3)n + c_\Gamma]} . \]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n = 1 ) (( c_\phi = 3 ))</th>
<th>( n = 2 ) (( c_\phi = 4 ))</th>
<th>( n = 3 ) (( c_\phi = 9/2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{L}_{\text{int}} )</td>
<td>( -y \phi \bar{\psi} \psi )</td>
<td>(-\frac{1}{6}(c_\Gamma = 0))</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(-M \phi \chi \chi )</td>
<td>(-\frac{1}{6}(c_\Gamma = 0))</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(-h \phi^2 \chi^2 )</td>
<td>... (cannot decay)</td>
<td>0</td>
</tr>
</tbody>
</table>

\( \propto a^{-3} \propto H^2 \)

\( \propto a^{-4} \)

\( \propto a^{-9/2} \propto H^2 \)
Non-gaussianity from $\delta N$

$\zeta(t) = \delta N = N^*_{\sigma} \delta \sigma^* + \frac{1}{2} N^*_{\sigma \sigma} \delta \sigma^2 + \frac{1}{6} N^*_{\sigma \sigma \sigma} \delta \sigma^3 + \ldots$.

$N^*_{\sigma} = \left. \frac{\partial N}{\partial \sigma} \right|_{\sigma = \sigma(t^*)}$ 

$\delta \sigma^*: \text{Gaussian}$

$N(t_f, t_e, \sigma^*) = \frac{1}{4} \log \frac{\rho_e}{\rho_f} + Q(x) = \frac{1}{4} \log \frac{\rho_e}{\rho_f} + a_0 \log x, \quad x = \frac{\Gamma(\sigma^*)}{H_e}$.

\[ \begin{align*}
N_{\sigma} &= x Q'(x) \frac{\Gamma_{\sigma}}{\Gamma} = A(x) \frac{\Gamma_{\sigma}}{\Gamma}, \\
N_{\sigma \sigma} &= x Q'(x) \frac{\Gamma_{\sigma \sigma}}{\Gamma} + x^2 Q''(x) \frac{\Gamma_{\sigma}^2}{\Gamma^2} = A(x) \frac{\Gamma_{\sigma \sigma}}{\Gamma} + B(x) \frac{\Gamma_{\sigma}^2}{\Gamma^2}, \\
N_{\sigma \sigma \sigma} &= x Q'(x) \frac{\Gamma_{\sigma \sigma \sigma}}{\Gamma} + 3 x^2 Q''(x) \frac{\Gamma_{\sigma} \Gamma_{\sigma \sigma}}{\Gamma^2} + x^3 Q'''(x) \frac{\Gamma_{\sigma}^3}{\Gamma^3} = A(x) \frac{\Gamma_{\sigma \sigma \sigma}}{\Gamma} + 3 B(x) \frac{\Gamma_{\sigma} \Gamma_{\sigma \sigma}}{\Gamma^2} + C(x) \frac{\Gamma_{\sigma}^3}{\Gamma^3}
\end{align*} \]

$(A(x) = a_0, \quad B(x) = -a_0, \quad C(x) = 2a_0)$

\[ \begin{align*}
\frac{6}{5} f_{NL} &= \frac{N^*_{\sigma \sigma}}{N^*_{\sigma}^2} = \frac{1}{a_0} \left( -1 + \frac{\Gamma_{\sigma \sigma}}{\Gamma_{\sigma}^2} \right), \\
\tau_{NL} &= \frac{36}{25} f_{NL}^2, \\
\frac{54}{25} g_{NL} &= \frac{N^*_{\sigma \sigma \sigma}}{N^*_{\sigma}^3} = \frac{1}{a_0^2} \left( 2 - 3 \frac{\Gamma_{\sigma \sigma}}{\Gamma_{\sigma}^2} + \frac{\Gamma_{\sigma}^2 \Gamma_{\sigma \sigma \sigma}}{\Gamma_{\sigma}^3} \right).
\end{align*} \]
Non-gaussianity in the modulated reheating scenario

\[ \frac{6}{5} f_{NL} = \frac{1}{a_0} \left( -1 + \frac{\Gamma \sigma \sigma}{\gamma^2} \right), \quad 54 \frac{g_{NL}}{25} = \frac{1}{a_0^2} \left( 2 - 3 \frac{\Gamma \sigma \sigma}{\gamma^2} + \frac{\Gamma^2 \sigma \sigma \sigma}{\gamma^3} \right). \]

Non-linealrity between \( \zeta (= \delta N) \) and \( \delta \Gamma \).

<table>
<thead>
<tr>
<th>( L_{\text{int}} )</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -y \phi \bar{\psi} \psi )</td>
<td>( -\frac{1}{6} )</td>
<td>0</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>( -M \phi \chi \chi )</td>
<td>( -\frac{1}{6} )</td>
<td>0</td>
<td>( \frac{1}{30} )</td>
</tr>
<tr>
<td>( -h \phi^2 \chi^2 )</td>
<td>( \ldots )</td>
<td>0</td>
<td>( \frac{1}{18} )</td>
</tr>
</tbody>
</table>

Inflaton potential for the oscillation:

\[ V(\phi) \propto \phi^{2n} \]

\( a_0: \)

- **n=1:** Non-linealrity between \( \zeta \) & \( \delta \Gamma \) generates \( f_{NL} = 5 \), but non-linealrity \( \Gamma \) & \( \sigma \) can generate large (either sign) \( f_{NL} \).
- **n=2:** No perturbation is generated.
- **n=3:** Non-linealrity between \( \zeta \) & \( \delta \Gamma \) as well as \( \Gamma \) & \( \sigma \) can generate large (minus) \( f_{NL} \),

Large (local type) non-Gaussianity can be easily generated in the modulated reheating scenario.
Relation between bispectrum & trispectrum

Suyama, Takahashi, MY, Yokoyama

\[
\frac{6}{5} f_{NL} = \frac{1}{a_0} \left( -1 + \frac{\Gamma_{\sigma\sigma}}{\gamma_{\sigma}^2} \right), \quad \frac{54}{25} g_{NL} = \frac{1}{a_0^2} \left( 2 - 3 \frac{\Gamma_{\sigma\sigma}}{\gamma_{\sigma}^2} + \frac{\Gamma_{\sigma\sigma\sigma}^2}{\gamma_{\sigma}^3} \right).
\]

\[ g_{NL} = -\frac{5}{3a_0} f_{NL} - \frac{25}{54a_0^2}. \]

\( (\Gamma_{\sigma\sigma\sigma} = 0) \)

\[ g_{NL} = 10 f_{NL} - \frac{50}{3}. \]

\( (f_{NL} \text{ & } g_{NL} \text{ have the same sign, while the curvaton predicts opposite in general.}) \)

\[ \tau_{NL} = \frac{36}{25} f_{NL}^2. \]
Spectral index of a light field model
**Spectral index in a light field model**

Kobayashi, Takahashi, Takahashi, MY

\[ n_s - 1 = \frac{d \ln P_\zeta}{d \ln k} \sim \frac{2}{3} \frac{V''(\sigma^*_e)}{H^2_*} + 2 \frac{\dot{H}_*}{H^2_*} \]

**WMAP7 :** \( n_s \sim 0.967, \quad \text{SPT : } n_s \sim 0.955 \ldots \)**

Generally speaking, a light field mass is really light and \( \varepsilon \) is small. But, \( \varepsilon \sim 0.016 \) can reproduce \( n_s \sim 0.967 \), which requires large field inflation.

\[ \varepsilon = -\frac{\dot{H}}{H^2} = -\frac{1}{2M^2_{\text{pl}}} \left( \frac{d\phi}{dN} \right)^2 \implies \Delta \phi \sim \sqrt{1 - n_s N M_{\text{pl}}} \gg M_{\text{pl}} \text{ for } N \gtrsim 60. \]

\text{e.g.} \quad V(\phi) \propto \phi^n \implies 2\varepsilon \sim \frac{n^2}{\phi^2_N} \sim \frac{n}{2N}.

\[ n_s \simeq 0.967 \iff n \sim 2(1 - n_s)N \sim 4 \text{ for } N \sim 60. \]

The quartic potential is favored in comparison to the conventional wisdom.

\( V'' \sim -0.05 H^2 \), that is, a hill-top type potential is also good. (Kawasaki et al. 2011)

\text{e.g.} \quad V(\sigma) \simeq V_0 - \frac{1}{2}m^2(\sigma - \sigma_0)^2 \quad \rightarrow \quad f_{NL} \simeq \frac{5(4 + 3f)}{18f} \frac{\sigma_{\text{osc}}}{\sigma_0 - \sigma_{\text{osc}}} \quad \text{, } f = \frac{\rho_\sigma}{\rho_r}.

Even if the curvaton dominates at its decay, the sizable \( f_{NL} \) can be obtained.
Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \approx 1$ because they provide powers only for large scales or reduce small scale powers.

① Addition of isocurvature perturbations:
residual isocurvature modes in curvaton (Lyth & Wands)
intrinsic isocurvature modes of axion …

② Additional relativistic degree of freedoms (dark radiation):
inflaton must decay into a modulus in modulated reheating, which can behave like dark radiation. (Kobayashi, Takahashi, Takahashi, MY)

③ Tensor perturbations:
multibrid model (Naruko & Sasaki) …
Addition of isocurvature perturbations

Addition of uncorrelated or positively correlated CDM isocurvature perturbations can still accommodate an extremely scale invariant spectral index $n_s \sim 1$. 

Figure 5: $1\sigma$ and $2\sigma$ limits in the $n_s-\sigma_0$ plane for uncorrelated CDM isocurvature perturbations (WMAP+BAO+H0).

Figure 6: $1\sigma$ and $2\sigma$ limits in the $n_s-\sigma_1$ plane for positively-correlated CDM isocurvature perturbations (WMAP+BAO+H0).

Figure 7: $1\sigma$ and $2\sigma$ limits in the $n_s-\sigma_1$ plane for negatively-correlated CDM isocurvature perturbations (WMAP+BAO+H0).
Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \approx 1$ because they provide powers only for large scales or reduce small scale powers.

1. **Addition of isocurvature perturbations:**
   - residual isocurvature modes in curvaton (Lyth & Wands)
   - intrinsic isocurvature modes of axion …

2. **Additional relativistic degree of freedom** (dark radiation):
   - inflaton must decay into a modulus in modulated reheating, which can behave like dark radiation. (Kobayashi, Takahashi, Takahashi, MY)

3. **Tensor perturbations:**
   - multibrid model (Naruko & Sasaki) …
Additional relativistic degree of freedom

Addition of **extra relativistic degree of freedom** (dark radiation) can still accommodate an extremely scale invariant spectral index $n_s \sim 1$.

*Figure 20: 1σ and 2σ limits in the $n_s-N_{\text{eff}}$ plane (WMAP+BAO+H0).*
Spectral index in a light field model II

Extensions of minimal cosmological assumptions may allow an extremely scale invariant spectral index $n_s \approx 1$ because they provide powers only for large scales or reduce small scale powers.

① Addition of isocurvature perturbations:
   residual isocurvature modes in curvaton (Lyth & Wands)
   intrinsic isocurvature modes of axion …

② Additional relativistic degree of freedoms (dark radiation):
   inflaton must decay into a modulus in modulated reheating,
   which can behave like dark radiation. (Kobayashi, Takahashi, Takahashi, MY)

③ Tensor perturbations:
   multibrid model (Naruko & Sasaki) …
Though $n_s \sim 1$ is marginally consistent for CMB, additions of BAO and H rule out it even if we take into account tensor contributions.
Let’s look forward to the Planck results!!
The modulated reheating mechanism can provide large non-Gaussianities.

The relation between $f_{NL}$ & $g_{NL}$ can be useful to discriminate the modulated reheating mechanism from other models.

The spectral index may be a serious problem for light field models generating large local type non-Gaussianities.