CMB bispectrum from higher-spin non-Gaussianities

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Abstract

A key feature to probe the early Universe:
"Non-Gaussianity of the primordial fluctuations"

Good tool:
3-pt correlation function of the CMB fluctuations (CMB bispectrum)

Previously, only the effects of the simple scalar-mode fluctuations have been considered. However, depending on the situation, contributions of the vector and tensor-mode fluctuations are also important...

Our goal:
- construct the formalism for the CMB bispectrum composed of the vector and tensor fluctuations
- limit on the non-Gaussianities, which cannot be expressed by only the scalar fluctuations.

Since contractions between wave number vectors and projection operators induce complicated angular dependence, calculation of the CMB bispectrum becomes difficult...
(e.g.) tensor-scalar-scalar: $h_{ij}\partial_i R \partial_j R \rightarrow O_{ij}^{(\lambda)}(k_1) k_{2i} k_{3j}$ *???*

$$O_{ab}^{(\mp 2)}(\hat{\mathbf{k}_1})\hat{k_{2a}}\hat{k_{3b}} = \frac{(8\pi)^{3/2}}{6} \sum_{M m_a m_b} {}_{\pm 2}Y_{2M}^*(\hat{\mathbf{k}_1}) Y_{1m_a}^*(\hat{\mathbf{k}_2}) Y_{1m_b}^*(\hat{\mathbf{k}_3}) \begin{pmatrix} 2 & 1 & 1 \\ M & m_a & m_b \end{pmatrix}$$

Formulae for any CMB bispectra are given by utilizing the multipole expansion by the spin spherical harmonics and the Wigner symbols!!

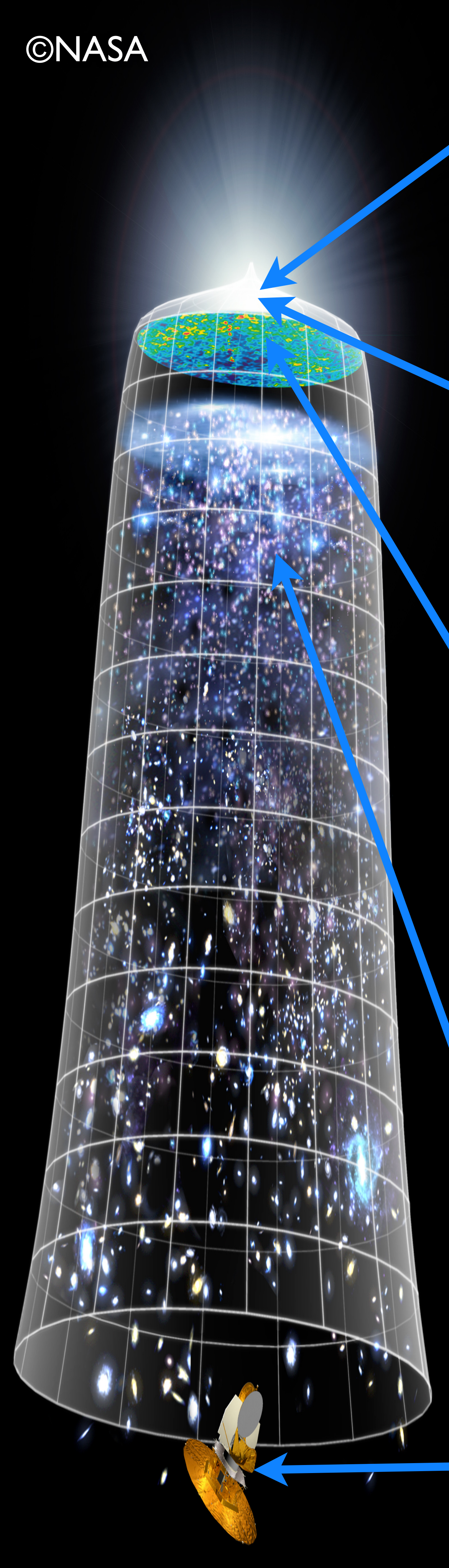
Diverse shapes due to vector and tensor modes

parity violation

statistical anisotropy

CMB bispectrum generated from primordial magnetic fields

[MS + 1009.3632, 1101.5287, 1103.4103, 1201.0376]



- inflation
- phase transition
(t < 10^{-10}sec)

nucleosynthesis
of light elements
(t ~ 1sec)

H, He
recombination
(t ~ 0.4Myr)

beginning of galaxy
formation

present

coupling between vector field
& inflaton, electroweak or
QCD phase transition?

?

Martin & Yokoyama [0711.4304],
Bamba & Sasaki [0611701], ...

from abundance of
^{4}He, ^{2}H, ^{7}Li, ...

from CMB
anisotropy

2nd order perturbation?

Ichiki + [0603631], ...

$B_{1\mathrm{Mpc}} \sim 10^{-20}\mathrm{G}$

amplified by
astrophysical processes
(dynamo mechanism?)

$B_{1\mathrm{Mpc}} \sim O(1\mu\mathrm{G})$

Magnetic field

✓PMF power spectrum: e.g., Shaw & Lewis [0911.2714]

$$\langle B_a(\mathbf{k}) B_b(\mathbf{p})\rangle = (2\pi)^3 \frac{P_B(k)}{2} P_{ab}(\hat{\mathbf{k}})\delta(\mathbf{k}+\mathbf{p})$$

☜ $f(\varphi)F^{\alpha\beta}F_{\alpha\beta}$ Talk by Prof. Peloso

$$P_B(k) = A_B k^{n_B} \qquad A_B = \frac{(2\pi)^{n_B+5} B_r^2}{\Gamma\left(\frac{n_B+3}{2}\right) k_r^{n_B+3}}$$

Projection tensor:

$$P_{ab}(\hat{\mathbf{k}}) \equiv \delta_{ab} - \hat{k}_a \hat{k}_b$$
$$= -2 \sum_{L=0,2} I_{L11}^{01-1} \sum_{M m_a m_b} Y_{LM}^*(\hat{\mathbf{k}}) \alpha_a^{m_a} \alpha_b^{m_b} \begin{pmatrix} L & 1 & 1 \\ M & m_a & m_b \end{pmatrix}$$

✓Current observational constraints:

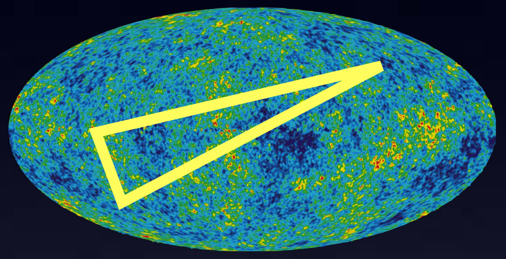
$B_{1\mathrm{Mpc}} < O(1)\mathrm{nG}$, $n_B \sim -3$ (nearly scale invariant)

CMB bispectrum from PMFs

✤PMF anisotropic stress induces curvature perturbation & gravitational wave and they drives the CMB scalar & tensor fluctuations

$$B^2 \propto \Pi_B \propto \xi \propto \Delta T$$

time scale: inflation → inflation end → rad dom ($\tau < \tau_\nu$) → recombi nation



✤Vector mode arises from the Lorentz force (proportional to Π_B)

+

✤assuming Gaussianity of the PMF...

$\Delta T \propto$ (Gaussian $B)^2$ = highly non-Gaussian field!

Bispectrum of magnetic anisotropic stress

$$\langle \Pi_{Bab}(\mathbf{k_1})\Pi_{Bcd}(\mathbf{k_2})\Pi_{Bef}(\mathbf{k_3})\rangle = (-4\pi\rho_{\gamma,0})^{-3}\left[\prod_{n=1}^{3}\int_0^{k_D} k_n'^2 dk_n' P_B(k_n')\int d^2\hat{\mathbf{k}}_\mathbf{n}'\right]$$
$$\times\delta(\mathbf{k_1}-\mathbf{k}_1'+\mathbf{k}_3')\delta(\mathbf{k_2}-\mathbf{k}_2'+\mathbf{k}_1')\delta(\mathbf{k_3}-\mathbf{k}_3'+\mathbf{k}_2')$$
$$\times\frac{1}{8}[P_{ad}(\hat{\mathbf{k}}_1')P_{be}(\hat{\mathbf{k}}_3')P_{cf}(\hat{\mathbf{k}}_2') + \{a\leftrightarrow b \text{ or } c\leftrightarrow d \text{ or } e\leftrightarrow f\}]$$

$\propto$ <(Gaussian B)^6>

scalar, vector and tensor parts of the initial perturbation:

$$\xi^{(\lambda)}(\mathbf{k}) \propto O_{ab}^{(-\lambda)}(\hat{\mathbf{k}})\Pi_{Bab}(\mathbf{k})$$

complicated angular dependence

$$O_{ab}^{(\lambda)}(\hat{\mathbf{k}}) = C_\lambda\sqrt{\frac{3}{8\pi}}\sum_{Mm_am_b} {}_{-\lambda}Y_{2M}^*(\hat{\mathbf{k}})\alpha_a^{m_a}\alpha_b^{m_b}\begin{pmatrix} 2 & 1 & 1 \\ M & m_a & m_b \end{pmatrix}$$

$$C_\lambda = \begin{cases} -2 & (\lambda=0) \\ 2\sqrt{3}\lambda & (\lambda=\pm 1) \\ 2\sqrt{3} & (\lambda=\pm 2) \end{cases} .$$

Formula for CMB bispectrum

◉*CMB anisotropy*

$$\frac{\Delta X(\hat{\mathbf{n}})}{X} = \sum_{\ell m} \left(\sum_{Z=S,V,T} a^{(Z)}_{X,\ell m} \right) Y_{\ell m}(\hat{\mathbf{n}})$$

◉*general formula for CMB all-mode bispectra* MS + [1003.2096, 1012.1079]

$$\left\langle \prod_{n=1}^{3} a^{(Z_n)}_{X_n,\ell_n m_n} \right\rangle = \left[\prod_{n=1}^{3} 4\pi (-i)^{\ell_n} \int_0^\infty \frac{k_n^2 dk_n}{(2\pi)^3} \mathcal{T}^{(Z_n)}_{X_n,\ell_n}(k_n) \sum_{\lambda_n} [\mathrm{sgn}(\lambda_n)]^{\lambda_n + x_n} \right] \left\langle \prod_{n=1}^{3} \xi^{(\lambda_n)}_{\ell_n m_n}(k_n) \right\rangle$$

- Z: scalar (= S), vector (= V), and tensor (= T)
- λ: helicities of S, V, and T (= 0, ±1, ±2)
- X: intensity (= I), linear polarizations (= E, B)
- x: parities of I, E (= 0), and B (= 1)

- transfer function (derived from CPT)

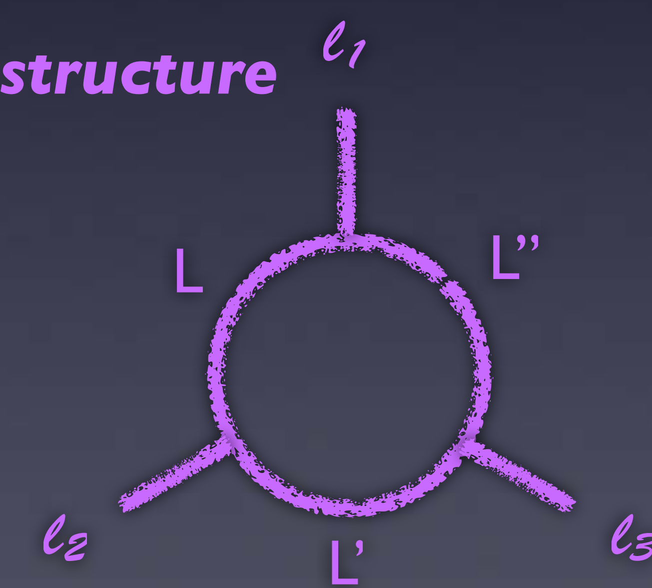
- primordial perturbation

$$\xi^{(\lambda)}(\mathbf{k}) \equiv \sum_{\ell m} \xi^{(\lambda)}_{\ell m}(k)\, {}_{-\lambda}Y_{\ell m}(\hat{\mathbf{k}})$$

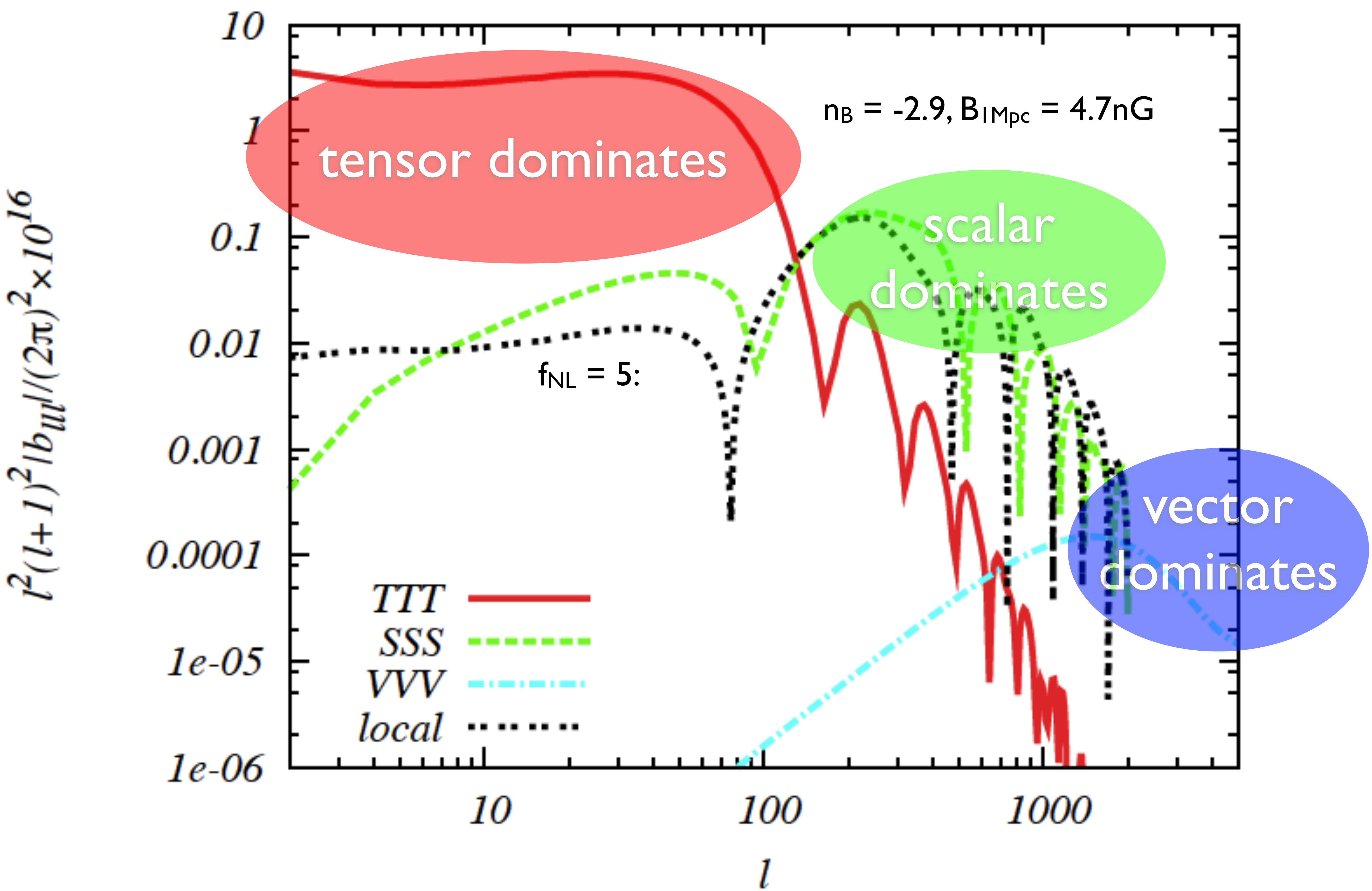
◉*primordial bispectrum generated from PMFs*

$$\left\langle \prod_{n=1}^{3} \xi^{(\lambda_n)}_{\ell_n m_n}(k_n) \right\rangle = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} (-4\pi\rho_{\gamma,0})^{-3} \left[\prod_{n=1}^{3} \int_0^{k_D} k_n'^2 dk_n' P_B(k_n') \right]$$
$$\times \sum_{LL'L''} \sum_{S,S',S''=\pm 1} \begin{Bmatrix} \ell_1 & \ell_2 & \ell_3 \\ L' & L'' & L \end{Bmatrix}$$
$$\times f^{S''S\lambda_1}_{L''L\ell_1}(k_3', k_1', k_1) f^{SS'\lambda_2}_{LL'\ell_2}(k_1', k_2', k_2) f^{S'S''\lambda_3}_{L'L''\ell_3}(k_2', k_3', k_3),$$

1-loop structure

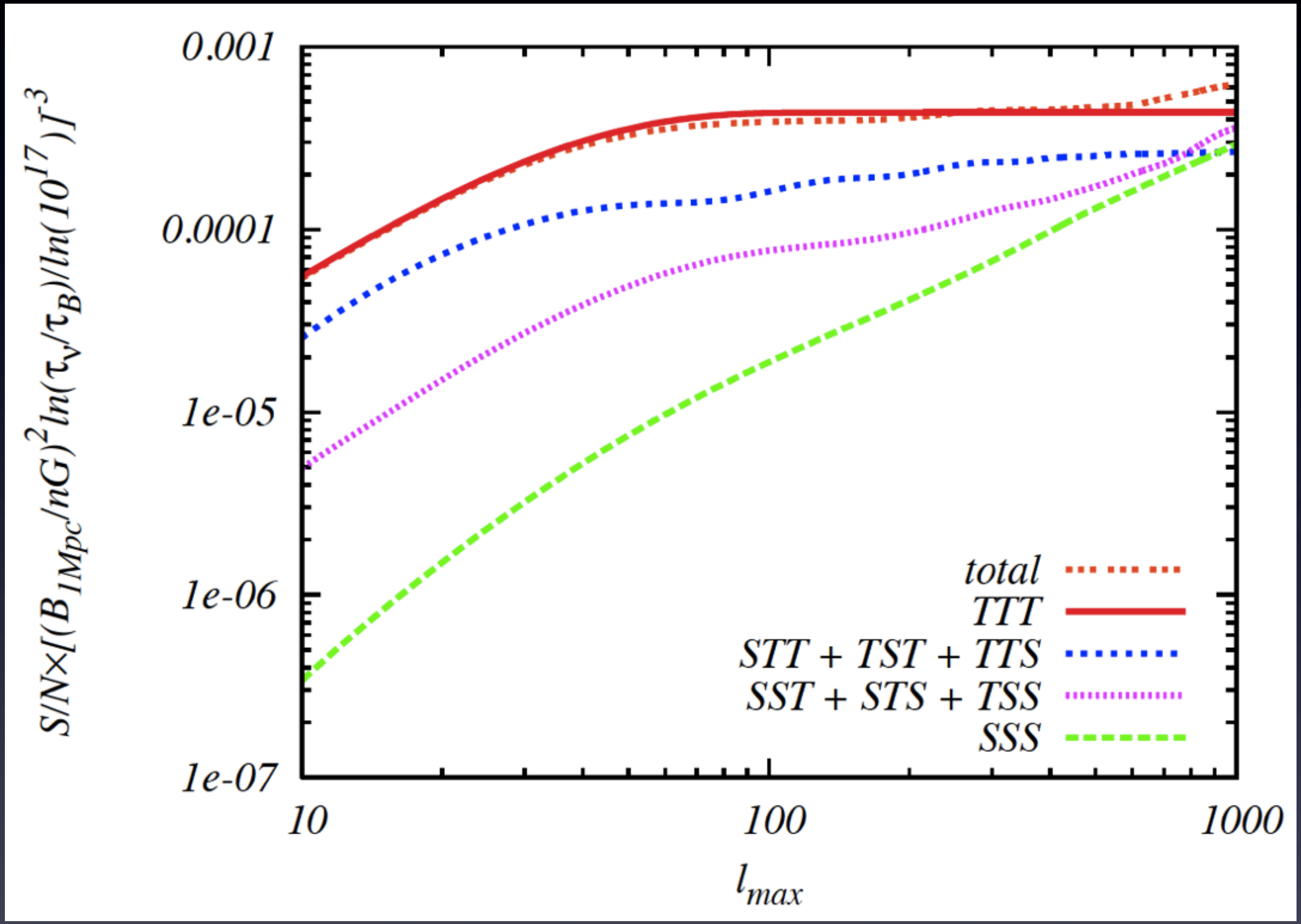


CMB reduced bispectra $b^{(Z_1Z_2Z_3)}_{III,\ell_1\ell_2\ell_3} \equiv \left(I^{0\ 0\ 0}_{\ell_1\ell_2\ell_3}\right)^{-1} B^{(Z_1Z_2Z_3)}_{III,\ell_1\ell_2\ell_3}$ of intensity mode for $\ell_1 = \ell_2 = \ell_3$



Signal-to-noise ratio

$$\left(\frac{S}{N}\right)^2 = \sum_{2\leq\ell_1\leq\ell_2\leq\ell_3\leq\ell_{\max}} \frac{\left[\sum_{Z_1Z_2Z_3} B^{(Z_1Z_2Z_3)}_{III,\ell_1\ell_2\ell_3}\right]^2}{\Delta_{\ell_1\ell_2\ell_3}\prod_{n=1}^{3}\left(C^{\rm fid}_{\ell_n}+N_{\ell_n}\right)}$$



- ΔB_{1Mpc} = 4.0nG for WMAP limit (l_{max} < 500)
- ΔB_{1Mpc} = 3.8nG for PLANCK limit (l_{max} < 1000)

Parity violation in the CMB bispectrum

[MS + 1108.0175, 1202.2847]

Parity of the CMB intensity bispectrum

intensity fluctuation:

$$\frac{\Delta I(\hat{\mathbf{n}})}{I} = \sum_{\ell m} a_{I,\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

parity flip →

$$\frac{\Delta I(-\hat{\mathbf{n}})}{I} = \sum_{\ell m} a_{I,\ell m} Y_{\ell m}(-\hat{\mathbf{n}}) = \sum_{\ell m} a_{I,\ell m} (-1)^{\ell} Y_{\ell m}(\hat{\mathbf{n}})$$

◉ CMB intensity bispectrum from the parity-even non-Gaussianity satisfies

$$\left\langle \prod_{i=1}^{3} \frac{\Delta I(\hat{\mathbf{n}}_i)}{I} \right\rangle = \left\langle \prod_{i=1}^{3} \frac{\Delta I(-\hat{\mathbf{n}}_i)}{I} \right\rangle$$

$\langle a_{\ell m} a_{\ell m} a_{\ell m} \rangle \neq 0$

only for $\ell_1 + \ell_2 + \ell_3 = \mathit{even}$

◉ CMB intensity bispectrum from the parity-odd non-Gaussianity satisfies

$$\left\langle \prod_{i=1}^{3} \frac{\Delta I(\hat{\mathbf{n}}_i)}{I} \right\rangle = - \left\langle \prod_{i=1}^{3} \frac{\Delta I(-\hat{\mathbf{n}}_i)}{I} \right\rangle$$

$\langle a_{\ell m} a_{\ell m} a_{\ell m} \rangle \neq 0$ new

only for $\ell_1 + \ell_2 + \ell_3 = \mathit{odd}$

1. graviton non-Gaussianity

MS + [1108.0175]

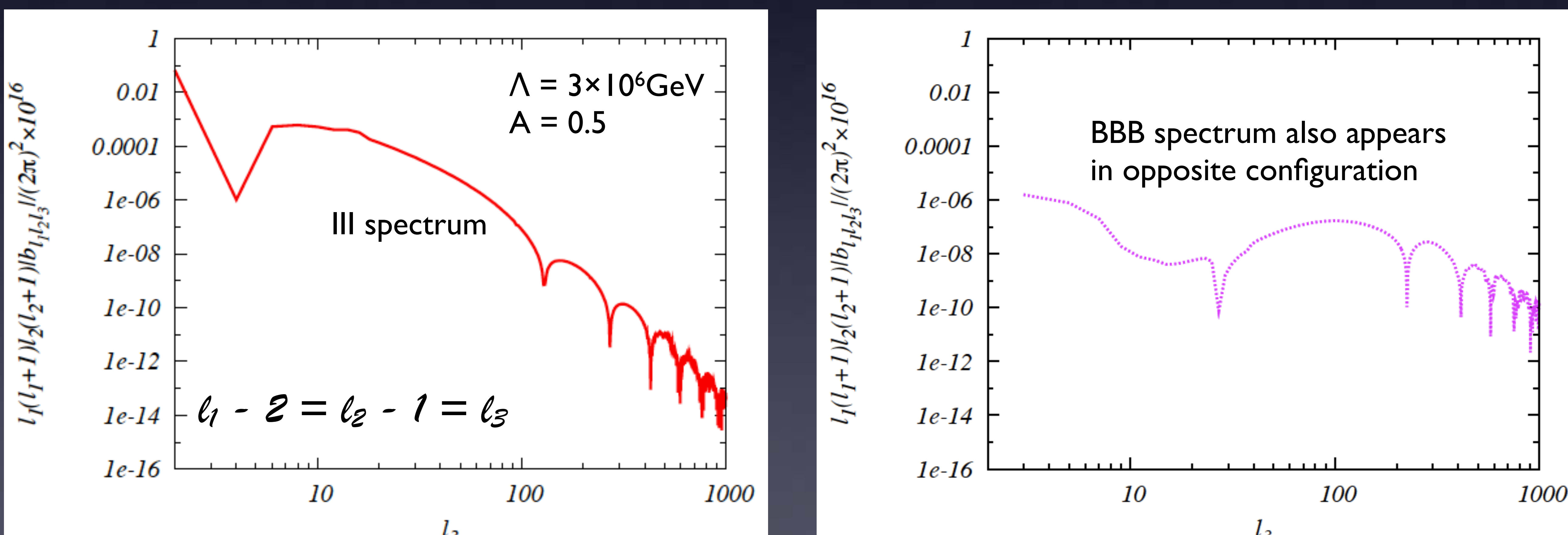
$$S \supset \int d\tau d^3x f(\tau)\Lambda^{-2}\epsilon^{\alpha\beta\mu\nu}W_{\mu\nu\gamma\delta}W^{\gamma\delta}{}_{\sigma\rho}W^{\sigma\rho}{}_{\alpha\beta}$$

Maldacena & Pimentel [1104.2846], Soda + [1106.3228]

- $W_{\mu\nu\gamma\delta}$: Weyl tensor
- $f(\tau) \propto \tau^A$: running coupling
- $\epsilon^{\alpha\beta\mu\nu}$: antisymmetric tensor
- Λ: cutoff scale

This interaction generates the parity-violating graviton non-Gaussianities and induces CMB bispectrum:

$\ell_1 + \ell_2 + \ell_3 = odd$ $\qquad$ $\ell_1 + \ell_2 + \ell_3 = even$



2. helical PMFs

MS [1202.2874]

If PMF power spectrum involves the helical term:

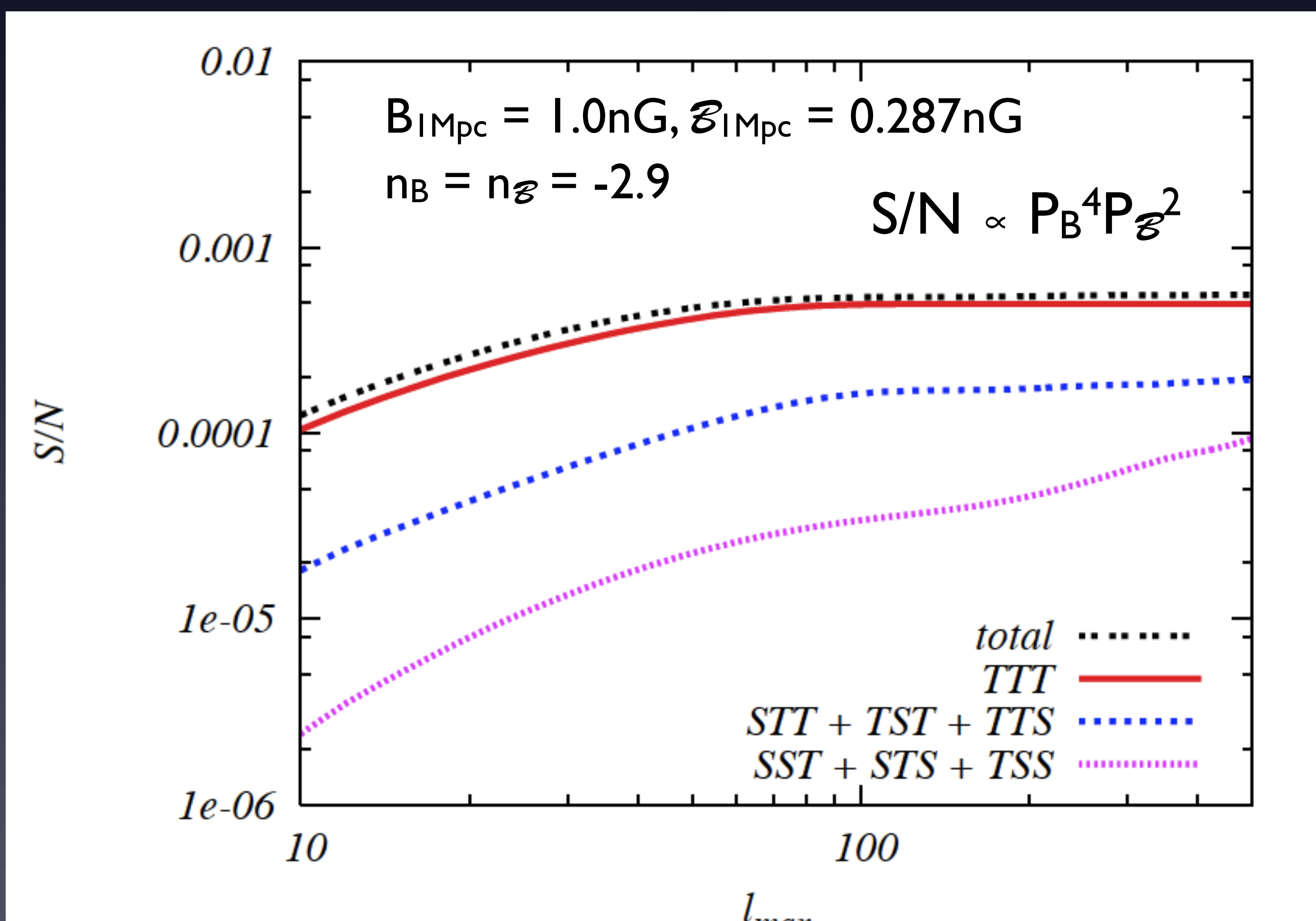
$$\langle B_a(\mathbf{k}) B_b(\mathbf{k}')\rangle = \frac{(2\pi)^3}{2}\left[P_B(k)P_{ab}(\hat{\mathbf{k}}) + i\eta_{abc}\hat{k}_c P_{\mathcal{B}}(k)\right]\delta(\mathbf{k}+\mathbf{k}')$$

☝ f(φ)FF ☝ f(φ)εFF

$$P_B(k) = A_B k^{n_B} \ , \quad P_{\mathcal{B}}(k) = A_{\mathcal{B}} k^{n_{\mathcal{B}}}$$

$$A_B = \frac{(2\pi)^{n_B+5} B_r^2}{\Gamma\left(\frac{n_B+3}{2}\right) k_r^{n_B+3}} \ , \quad |A_{\mathcal{B}}| = \frac{(2\pi)^{n_{\mathcal{B}}+5} \mathcal{B}_r^2}{\Gamma\left(\frac{n_{\mathcal{B}}+4}{2}\right) k_r^{n_{\mathcal{B}}+3}}$$

$\ell_1 + \ell_2 + \ell_3 = odd$



If $B_{1\mathrm{Mpc}}^{2/3}\mathcal{B}_{1\mathrm{Mpc}}^{1/3}$ > 2.3nG, S/N at l_{max} = 500 exceeds unity!

We need the analysis of $\ell_1 + \ell_2 + \ell_3 = odd$ components in observational intensity map!!

Violation of the rotational invariance in the CMB bispectrum

[MS + 1107.0682]

What is the rotational invariance?

power spectrum | bispectrum

k-space:

$$P(\mathbf{k}) = P(k)$$

absence of the direction dependence

$$F^{\lambda_1\lambda_2\lambda_3}(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = f^{\lambda_1\lambda_2\lambda_3}(k_1, k_2, k_3) \times [\text{polarization vectors or tensors}]$$

translate into CMB...

$$\frac{\Delta X(\hat{\boldsymbol{n}})}{X} = \sum_{\ell m} a_{X,\ell m} Y_{\ell m}(\hat{\boldsymbol{n}})$$

l-space:

$$\left\langle \prod_{i=1}^{2} a_{\ell_i m_i} \right\rangle \equiv C_{\ell_1} (-1)^{m_1} \delta_{\ell_1,\ell_2} \delta_{m_1,-m_2}$$

$$\left\langle \prod_{i=1}^{3} a_{\ell_i m_i} \right\rangle \equiv \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1 \ell_2 \ell_3}.$$

from the orthogonality of Y_{lm}

If the rotational invariance is broken, m_1, m_2 and m_3 can not confined to the Wigner-3j symbol.

CMB bispectrum with a preferred direction

System like the hybrid inflation that there are inflaton φ, waterfall field χ, and a vector field A_μ coupled with a waterfall field S. Yokoyama & J. Soda [0805.4265]

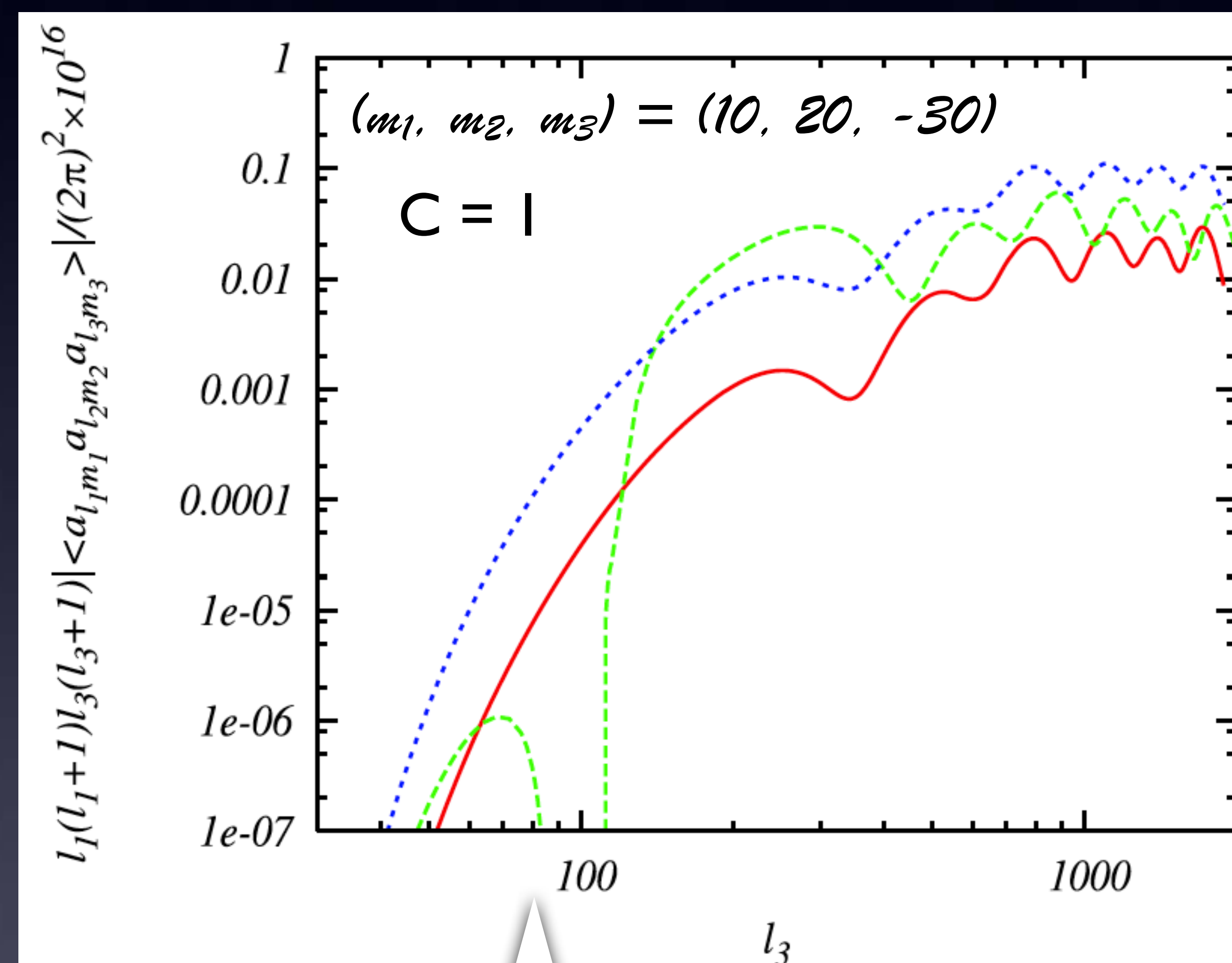
$$S = \int dx^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi) - V(\phi,\chi,A_\nu) - \frac{1}{4}g^{\mu\nu}g^{\rho\sigma}f^2(\phi)F_{\mu\rho}F_{\nu\sigma}\right].$$

The primordial bispectrum of curvature perturbations depends on the specific direction A_a:

$$\left\langle \prod_{n=1}^{3}\zeta(\boldsymbol{k_n})\right\rangle \equiv (2\pi)^3 F_\zeta(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3})\delta\left(\sum_{n=1}^{3}\boldsymbol{k_n}\right)$$

$$F_\zeta(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) = C P_\zeta^{\rm iso}(k_1)P_\zeta^{\rm iso}(k_2)\hat{A}^a\hat{A}^b\delta^{cd}P_{ac}(\hat{\boldsymbol{k_1}})P_{bd}(\hat{\boldsymbol{k_2}}) + 2 \text{ perms}$$

$$\cancel{\left\langle \prod_{i=1}^{3} a_{\ell_i m_i}\right\rangle \equiv \begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix} B_{\ell_1\ell_2\ell_3}}.$$



- $(\ell_1, \ell_2) = (102 + \ell_3, 100)$
- $(\ell_1, \ell_2) = (|100 - \ell_3| - 2, 100)$
- $(\ell_1, \ell_2) = (100 + \ell_3, 100)$

signals arise from the multipole configurations which do not satisfy the triangle inequality ($|\ell_2 - \ell_3| \leqq \ell_1 \leqq \ell_2 + \ell_3$).

Detectability

✤more general parameterization: $\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(\vec{k}_1,\vec{k}_2,\vec{k}_3)$

$$B(\vec{k}_1,\vec{k}_2,\vec{k}_3) = B^{\rm iso}(k_1,k_2)\Big[1+g_B(k_1,k_2)\Big(p(k_1)\left(\hat{k}_1\cdot\hat{N}_A\right)^2+p(k_2)\left(\hat{k}_2\cdot\hat{N}_A\right)^2 \\ +p(k_1)p(k_2)\left(\hat{k}_1\cdot\hat{k}_2\right)\left(\hat{k}_2\cdot\hat{N}_A\right)\left(\hat{k}_1\cdot\hat{N}_A\right)\Big)\Big]+2\ \text{perms.}$$

$B^{iso} = (6/5)f_{NL}P(k_1)P(k_2)$

N. Bartolo + [1107.4304]

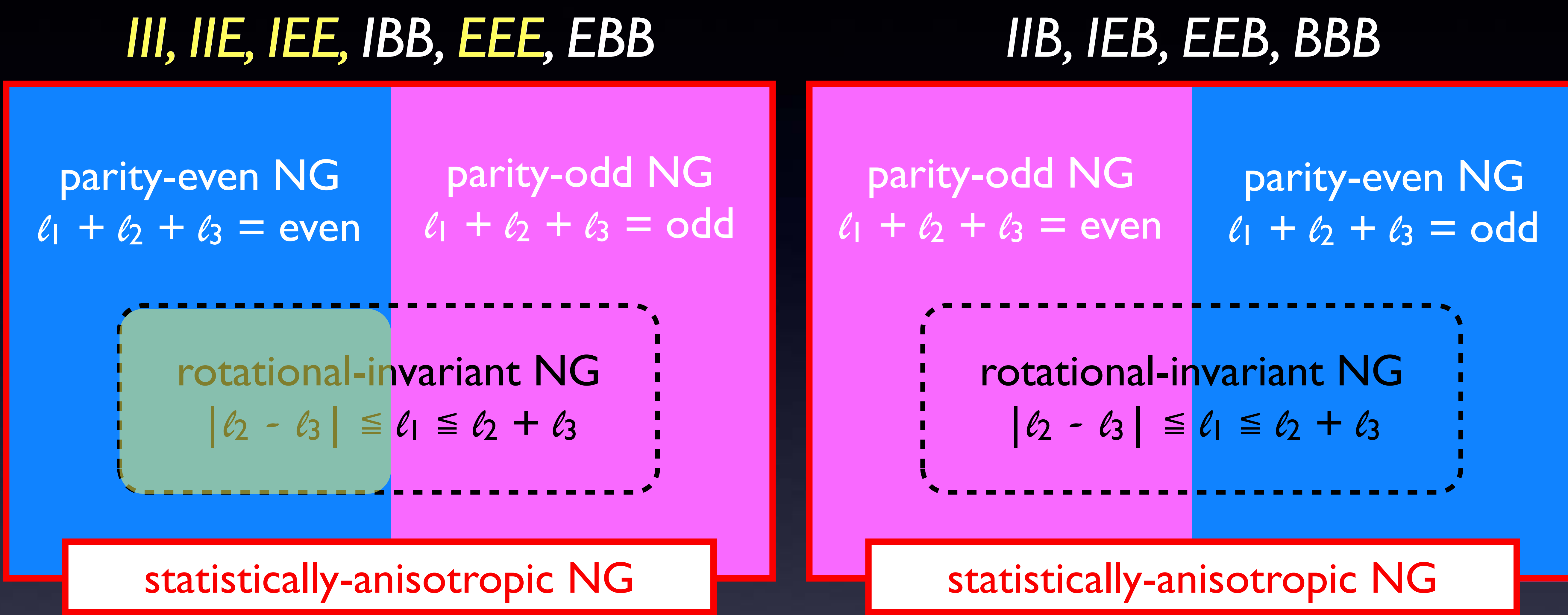
multipole coefficient of the anisotropic parameter: $\lambda_{LM} = \frac{8\pi}{15}(pg_B)Y^*_{LM}(\hat{N}_A)\delta_{L2}$

signal-to-noise ratio: $S/N \fallingdotseq 0.4 f_{NL}\lambda_{2M} \times (\ell / 2000)$

✦*Our case:* $f_{NL} = 5C/6$, $g_B = -p = 1$, $N_A{}^i = A^i$ ➡ $\lambda_{2M} \sim 0.6$

If $\ell \sim 2000$, $S/N > 1$ for $C > 5$

Summary



- constrain on the vector and tensor non-Gaussianities and the symmetry-breaking non-Gaussianities by the WMAP (and PLANCK) data
- by applying this technic, do ...