Signatures of primordial NG in CMB and LSS

Kendrick Smith (Princeton/Perimeter)
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Primordial non-G + observations

1. CMB: can we independently constrain every interesting non-Gaussian signal?

2. Large-scale structure: what non-Gaussian signals can be constrained, and what are the degeneracies?

EFT of inflation

π = Goldstone boson of spontaneously broken time translations

1-1 correspondence between operators in S_π and f_{NL}-like parameters (Degree-N operator shows up in N-point CMB correlation function)

$$S_\pi = \int d^4x \sqrt{-g}(-\dot{H} M_{\rm pl}^2) \left[\frac{\dot\pi^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} \right.$$

$$+ \frac{A}{c_s^2} \pi_t^3 + \frac{1-c_s^2}{c_s^2} \frac{\pi_t (\partial_i \pi)^2}{a^2}$$

Equilateral+orthogonal 3-point shapes (Senatore, KMS & Zaldarriaga 2009)

$$+ B \pi_{ttt}^3 + C \pi_{ttt} \pi_{ijk}^2 + \cdots$$

Higher-derivative 3-point shapes (Behbahani, Mirbabayi, Senatore & KMS to appear)

$$+ D \dot\pi^4 + E \dot\pi^2 (\partial_i \pi)^2 + F (\partial_i \pi)^2 (\partial_j \pi)^2 + \cdots$$

4-point shapes (Senatore & Zaldarriaga 2009)

$$\left. + \rho \dot\pi \sigma + G \sigma^3 + \cdots \right]$$

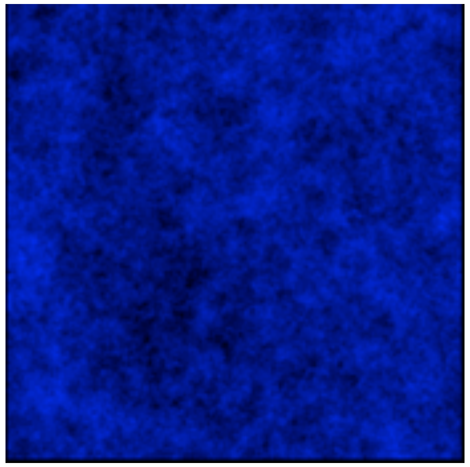
Quasi single-field inflation (Chen & Wang 2009)

CMB data analysis

Degree-N operator $\mathcal{O}$ (e.g. $\mathcal{O} = \dot{\pi}^3$ or $\mathcal{O} = \dot{\pi}^4$)

↓

Curvature N-point function $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \zeta_{\mathbf{k}_N} \rangle$



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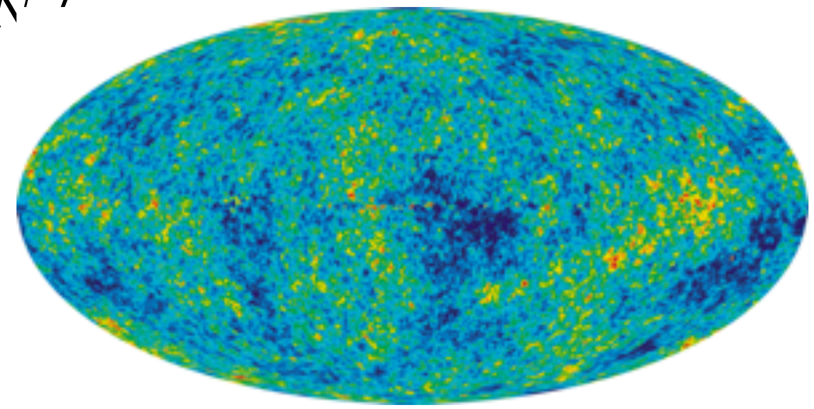
CMB N-point function $\langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle$



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CMB estimator

$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} \cdots a_{\ell_N m_N} \rangle \prod_{i=1}^{N} \tilde{a}_{\ell_i m_i} + \cdots$$

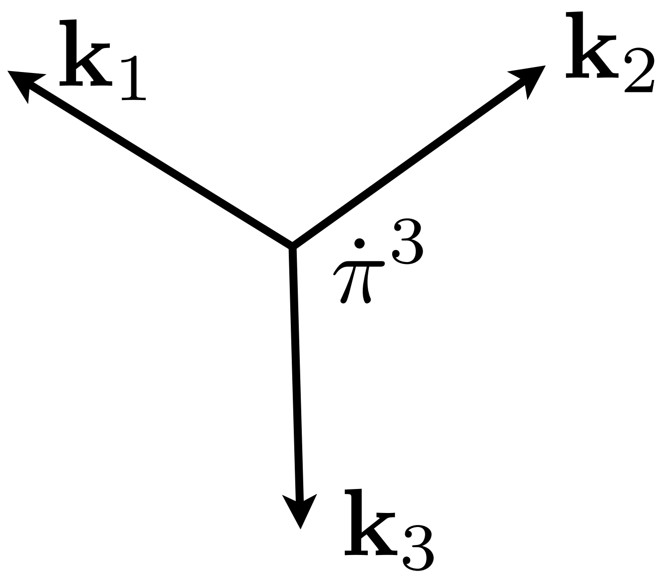
Computational difficulties

Example: $\dot{\pi}^3$ interaction

Computing the curvature 3-point function is straightforward....

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \int_{-\infty}^{0} d\tau \, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1 k_2 k_3}$$

$$= \frac{2}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$$



Computational difficulties

...but subsequent steps look intractable in full generality:

CMB three-point function: 4D oscillatory integral for each (ℓ_i, m_i)

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

$$\times \int dr\, dk_1\, dk_2\, dk_3 \left(\prod_{i=1}^{3} \frac{2k_i^2}{\pi} j_{\ell_i}(k_i r) \Delta_{\ell_i}(k_i) \right) \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$$

CMB transfer function (computed numerically)

CMB estimator: number of terms in sum is $\mathcal{O}(\ell_{\max}^5)$

$$\mathcal{E} = \sum_{\ell_i m_i} \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle\, \tilde{a}_{\ell_1 m_1} \tilde{a}_{\ell_2 m_2} \tilde{a}_{\ell_3 m_3} + \cdots$$

observed CMB multiples (appropriately filtered)

Factorizability = computability

Suppose the curvature 3-point function is factorizable

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = f_1(k_1) f_2(k_2) f_3(k_3) + 5 \text{ perm.}$$

Define (and precompute) $\alpha_\ell^{(i)}(r) = \int \frac{2k^2\, dk}{\pi} f_i(k) j_\ell(kr)$

CMB three-point function is fast to compute:

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$
$$\times \int dr\, r^2\, \alpha_{\ell_1}^{(1)}(r) \alpha_{\ell_2}^{(2)}(r) \alpha_{\ell_3}^{(3)}(r) + 5 \text{ perm.}$$

CMB estimator is fast to evaluate:

$$\mathcal{E} = \int r^2 dr \int d^2\mathbf{n} \prod_{i=1}^{3} \left(\sum_{\ell m} \alpha_\ell^{(i)} \tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)$$

Komatsu, Spergel & Wandelt 2003
Creminelli, Nicolis, Senatore, Tegmark & Zaldarriaga 2005
KMS & Zaldarriaga 2006

Making shapes factorizable

Two possibilities for making shape factorizable

$$\text{e.g. } \dot{\pi}^3 \text{ shape: } \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{2}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$$

1. approximate by a factorizable shape ("template shape")

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \approx \frac{-\sum_i k_i^3 + \sum_{i \neq j} k_i k_j^2 - 2 k_1 k_2 k_3}{k_1^3 k_2^3 k_3^3}$$

equilateral template

2. perform an algebraic magic trick, e.g. find integral representation

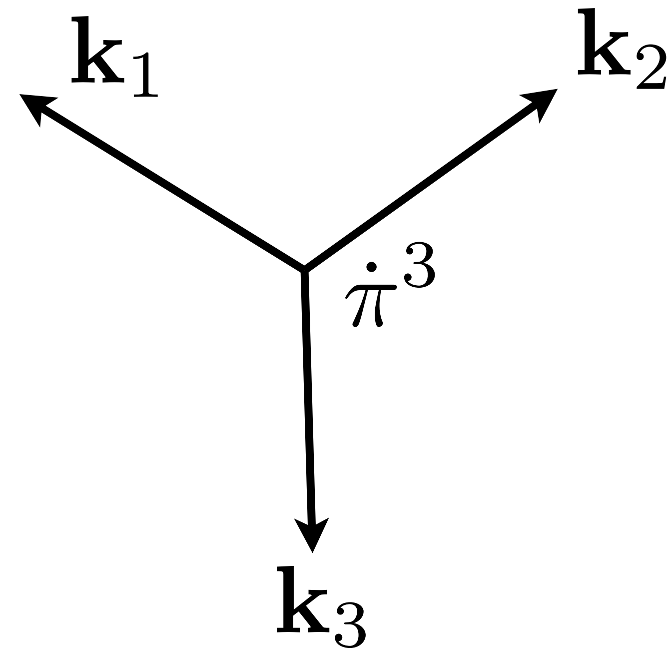
$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \int_{-\infty}^{0} t^2 dt \left(\frac{e^{tk_1}}{k_1} \right) \left(\frac{e^{tk_2}}{k_2} \right) \left(\frac{e^{tk_3}}{k_3} \right)$$

KMS & Zaldarriaga 2006

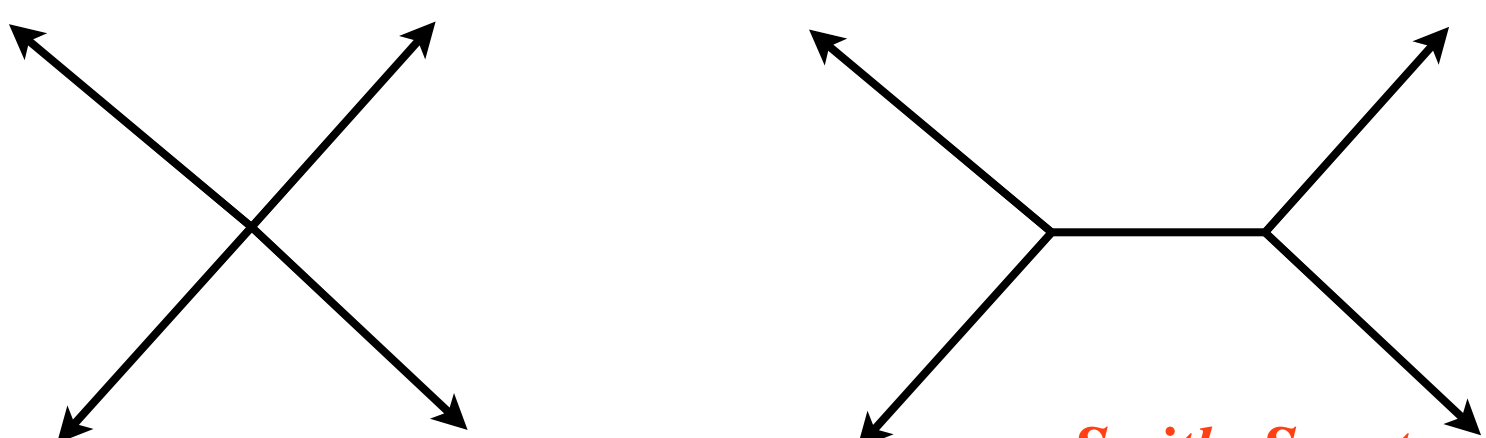
Factorizability + Feynman diagrams

Observation: for $\dot{\pi}^3$ shape, the integral representation is just undoing the last step of the Feynman diagram calculation

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \int_{-\infty}^{0} d\tau \, \frac{\tau^2 e^{(k_1+k_2+k_3)\tau}}{k_1 k_2 k_3}$$
$$= \frac{2}{k_1 k_2 k_3 (k_1+k_2+k_3)^3}$$



Generalizes to any tree diagram, e.g. 4-point estimators:

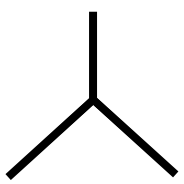


Smith, Senatore & Zaldarriaga, to appear

Factorizability + Feynman diagrams

Ultimate generalization of KSW construction: "Estimator" Feynman rules which go directly from the diagram to the CMB estimator

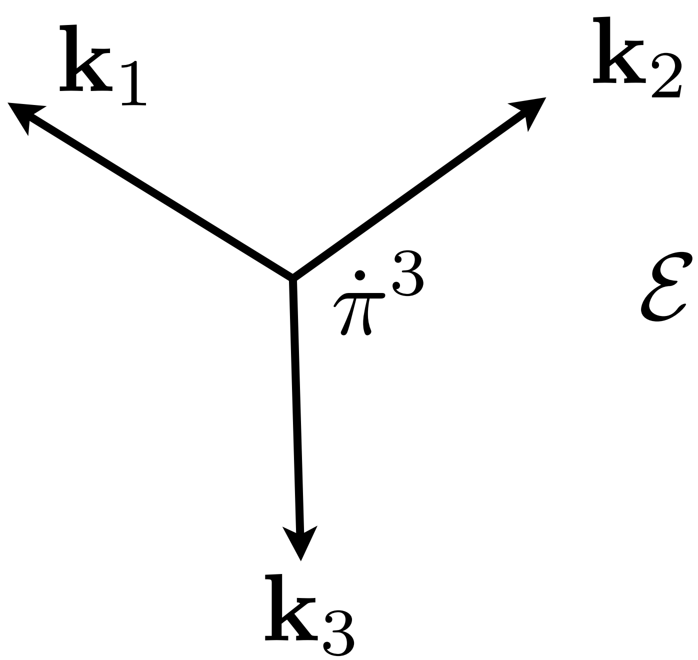
external line = CMB + harmonic-space factor $\alpha_\ell(r,t)\tilde{a}_{\ell m}$



vertex = $\int r^2 dr\, dt \Big($ N-way real-space product $\Big)$

internal line = harmonic-space factor $A_\ell(r,t,r',t')$

e.g.



$$\mathcal{E} = \int r^2 dr\, dt \left(\sum_{\ell m} \alpha_\ell(r,t)\tilde{a}_{\ell m} Y_{\ell m}(\mathbf{n}) \right)^3$$

Example: resonant NG

Just an example to illustrate the power of this method in finding a factorizable representation...

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \frac{1}{k_1^2 k_2^2 k_3^2} \left[\sin\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) + A^{-1} \sum_{i \neq j} \frac{k_i}{k_j} \cos\left(A \log \frac{k_1 + k_2 + k_3}{k_*} \right) \right]$$

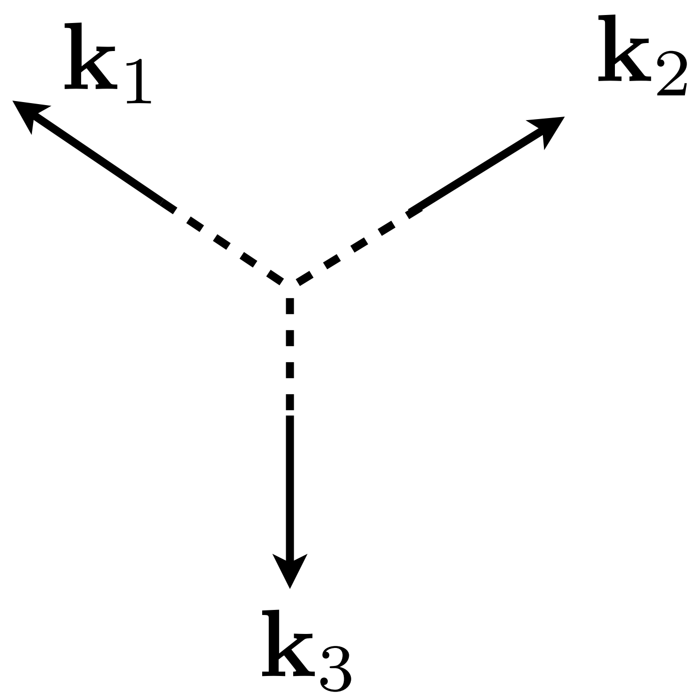
Hard to see how this could ever be made factorizable, but going back to the physics gives the following factorizable representation!

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \propto \mathrm{Re} \left[\frac{e^{(1+iA)\log(i+iA) + iA \log k_*}}{A \Gamma(1+iA)} \int_{-\infty}^{\infty} dx \, e^{(1+iA)x} g(k_1, x) g(k_2, x) g(k_3, x) \right.$$
$$\left. \times \left(\left(1 + \frac{iA}{2} \right) \frac{1}{k_1 k_2^2 k_3^2} + \frac{1}{k_2^2 k_3^3} + \frac{1}{k_1 k_2 k_3^3} + 5 \text{ perm.} \right) \right]$$

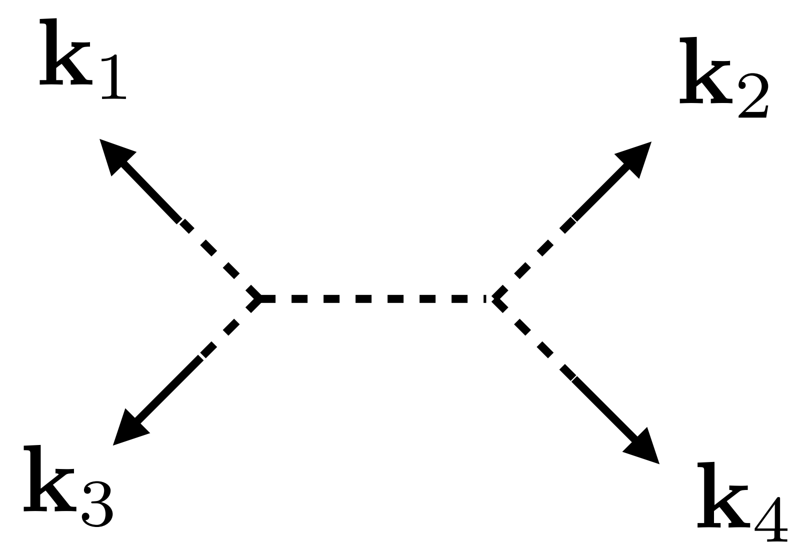
$$g(k, x) = \exp[-(1+iA) k e^x]$$

Example: quasi single field inflation

$$S_\pi = \int d^4x \sqrt{-g}\left(\frac{1}{2}(\partial\pi)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{M^2}{2}\sigma^2 + \rho\dot{\pi}\sigma - \frac{g}{3!}\sigma^3\right)$$



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \propto g\rho^3$$



$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle \propto g^2\rho^4$$

Because 3-point function and 4-point function depend on different combinations of parameters, either one can have larger signal-to-noise in different parts of the QSFI parameter space

Data analysis "to do" list...

For any "physical" shape, current machinery seems to be sufficient to do the analysis! A (possibly incomplete) to do list:

- Higher derivative shapes $(\dddot{\pi}^3, \ \dddot{\pi}\pi_{ijk}^2, \ \cdots)$
- Quartic interactions $(\dot{\pi}^4, \ \dot{\pi}^2\partial_i\pi^2, \ \partial_i\pi^2\partial_j\pi^2, \ \cdots)$
- Quasi single-field inflation
- Solid inflation
- Anything else... ?

Large-scale structure

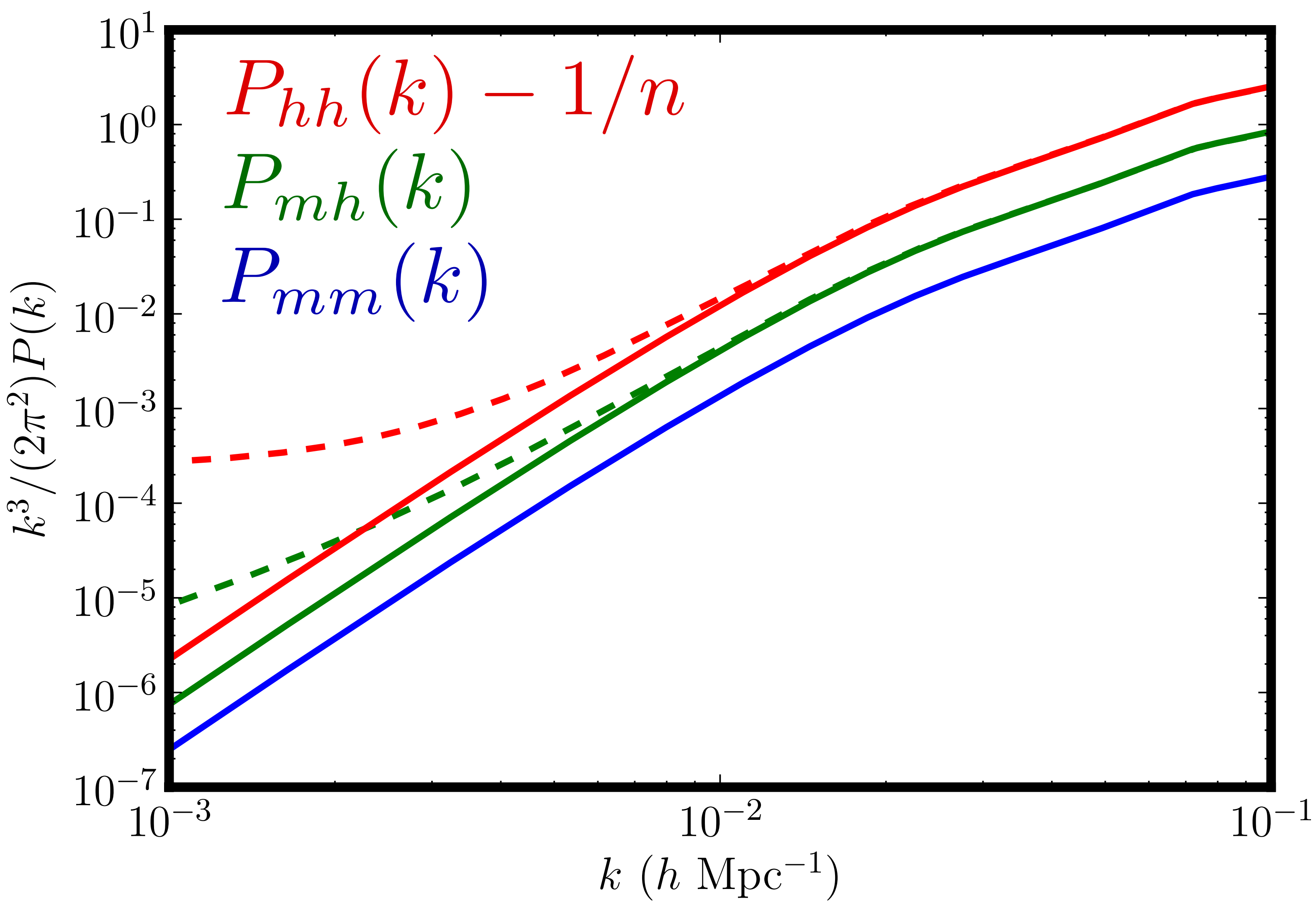
Local model: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \zeta_G(\mathbf{x})^2$

Non-Gaussian contribution to halo bias on large scales:

$$b(k) \approx b_0 + f_{NL} \frac{b_1}{(k/aH)^2} \quad \text{as } k \to 0$$

$$P_{mh}(k) \approx b(k) P_{mm}(k)$$

$$P_{hh}(k) \approx b(k)^2 P_{mm}(k)$$



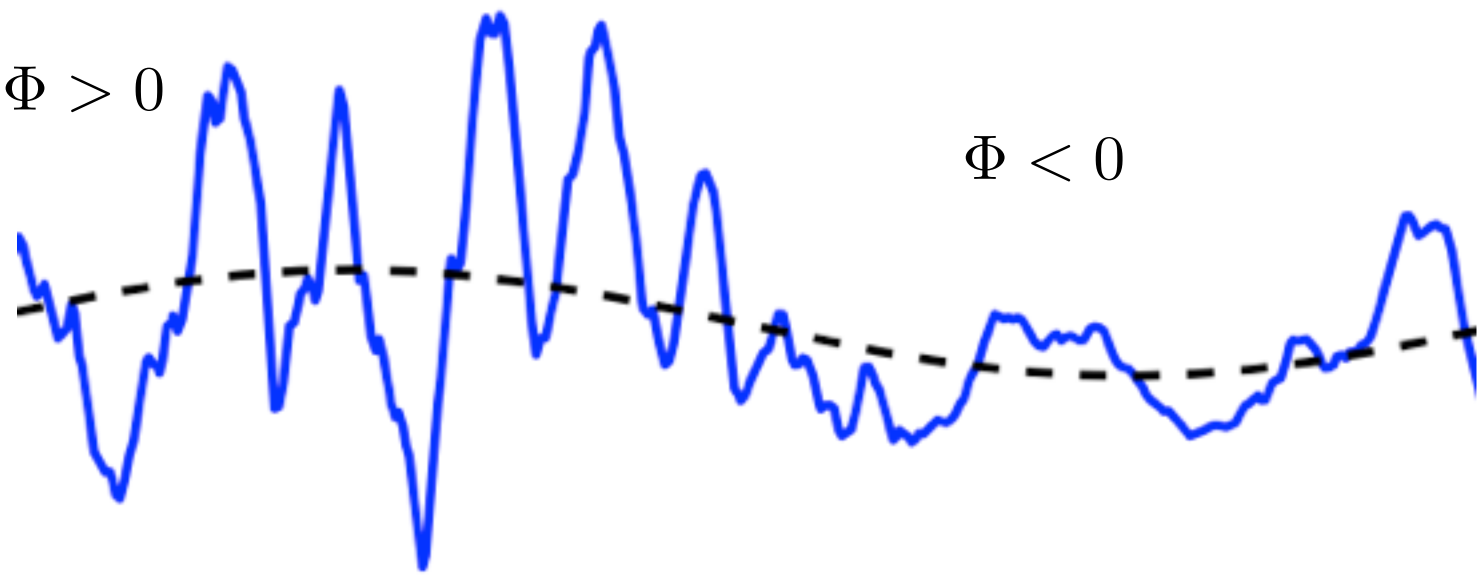
Constraints are ultimately better than the CMB

Planck: $\sigma(f_{NL}) = 5$

LSST: $\sigma(f_{NL}) \sim 1$

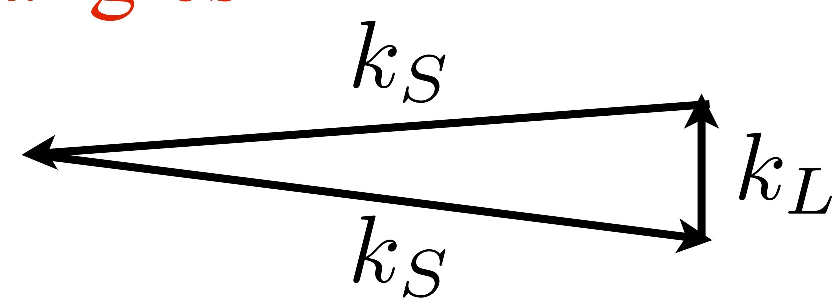
NG halo bias: interpretation

Correlation between long-wavelength mode and small-scale power



Three-point function is large in squeezed triangles

$$\langle \zeta_{\mathbf{k_L}} \zeta_{\mathbf{k_S}} \zeta_{\mathbf{k_S}} \rangle \propto f_{NL} \frac{1}{k_L^3 k_S^3}$$

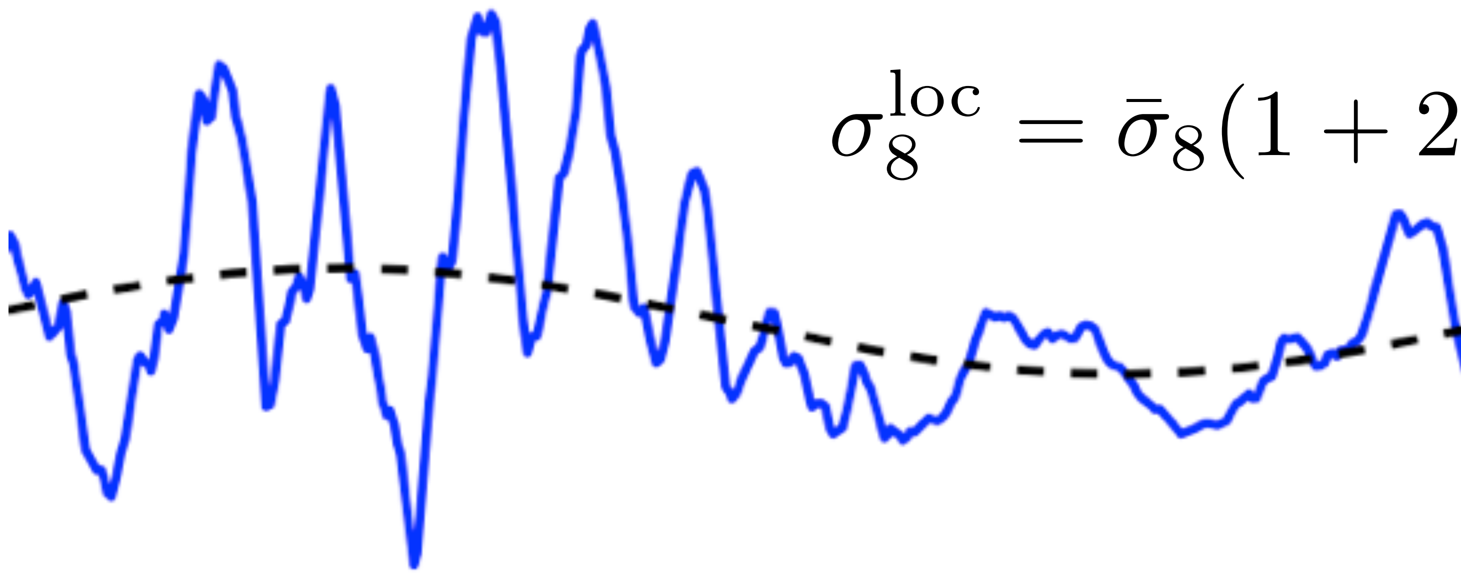


Locally measured fluctuation amplitude $\sigma_8^{\rm loc}$ near a point x depends on value of Newtonian potential $\Phi(\mathbf{x})$

$$\sigma_8^{\rm loc} = \bar{\sigma}_8 (1 + 2 f_{NL} \Phi)$$

NG halo bias: interpretation

This picture naturally leads to enhanced large-scale clustering



$$\sigma_8^{\mathrm{loc}} = \bar{\sigma}_8(1 + 2f_{NL}\Phi)$$

$$\frac{\delta n_h}{\bar{n}_h} = \underbrace{\frac{\partial \log n_h}{\partial \log \rho_m}}_{b_0} \frac{\delta \rho_m}{\bar{\rho}_m} + \underbrace{\frac{\partial \log n_h}{\partial \log \sigma_8}}_{b_1/2} \frac{\delta \sigma_8}{\bar{\sigma}_8}$$

$$= b_0 \frac{\delta \rho_m}{\bar{\rho}_m} + b_1 f_{NL} \Phi$$

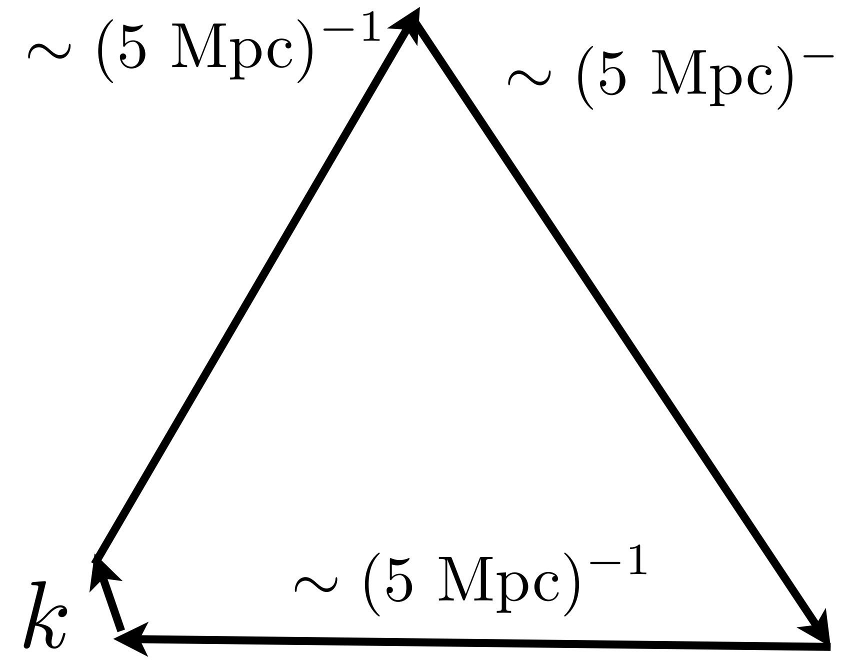
$$= \left(b_0 + b_1 \frac{f_{NL}}{(k/aH)^2} \right) \frac{\delta \rho_m}{\bar{\rho}_m}$$

General expression for non-Gaussian clustering

Schematic form:

$$P_{mh}(k) = \left(b_0 + \sum_{N=1}^{\infty} b_N f_{N+2}(k) \right) P_{mm}(k)$$

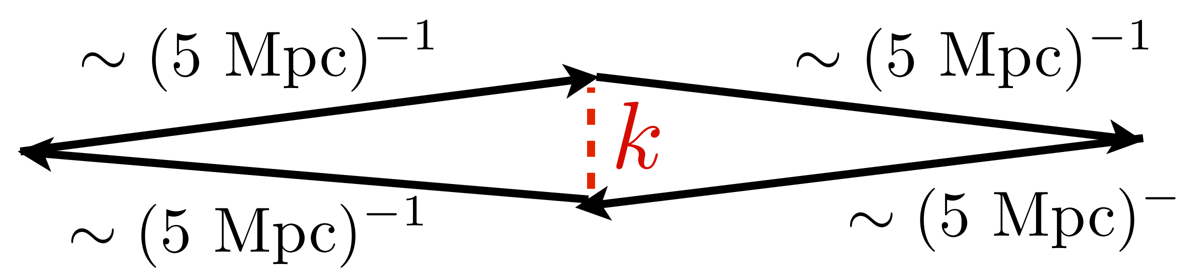
$$f_N(k) = \int_{\mathbf{k}_i} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}_1} \cdots \zeta_{\mathbf{k}_{N-1}} \rangle$$



"squeezed limit"

$$P_{hh}(k) = \left(b_0^2 + 2 \sum_{N=1}^{\infty} b_0 b_N f_{N+2}(k) + \sum_{MN} b_M b_N g_{M+1,N+1}(k) \right) P_{mm}(k)$$

$$g_{MN}(k) = \int_{\substack{\sum \mathbf{k}_i = \mathbf{k} \\ \sum \mathbf{k}'_j = -\mathbf{k}}} \langle \zeta_{\mathbf{k}_1} \cdots \zeta_{\mathbf{k}_M} \zeta_{\mathbf{k}'_1} \cdots \zeta_{\mathbf{k}'_N} \rangle$$



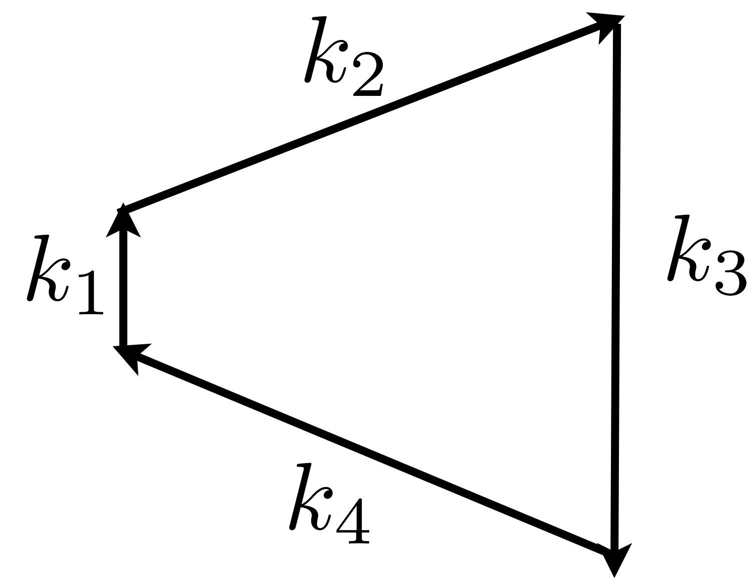
Baumann, Ferraro, Green & KMS 2012

"collapsed limit"

Example 1: bias from squeezed 4-point function

g_{NL} model:

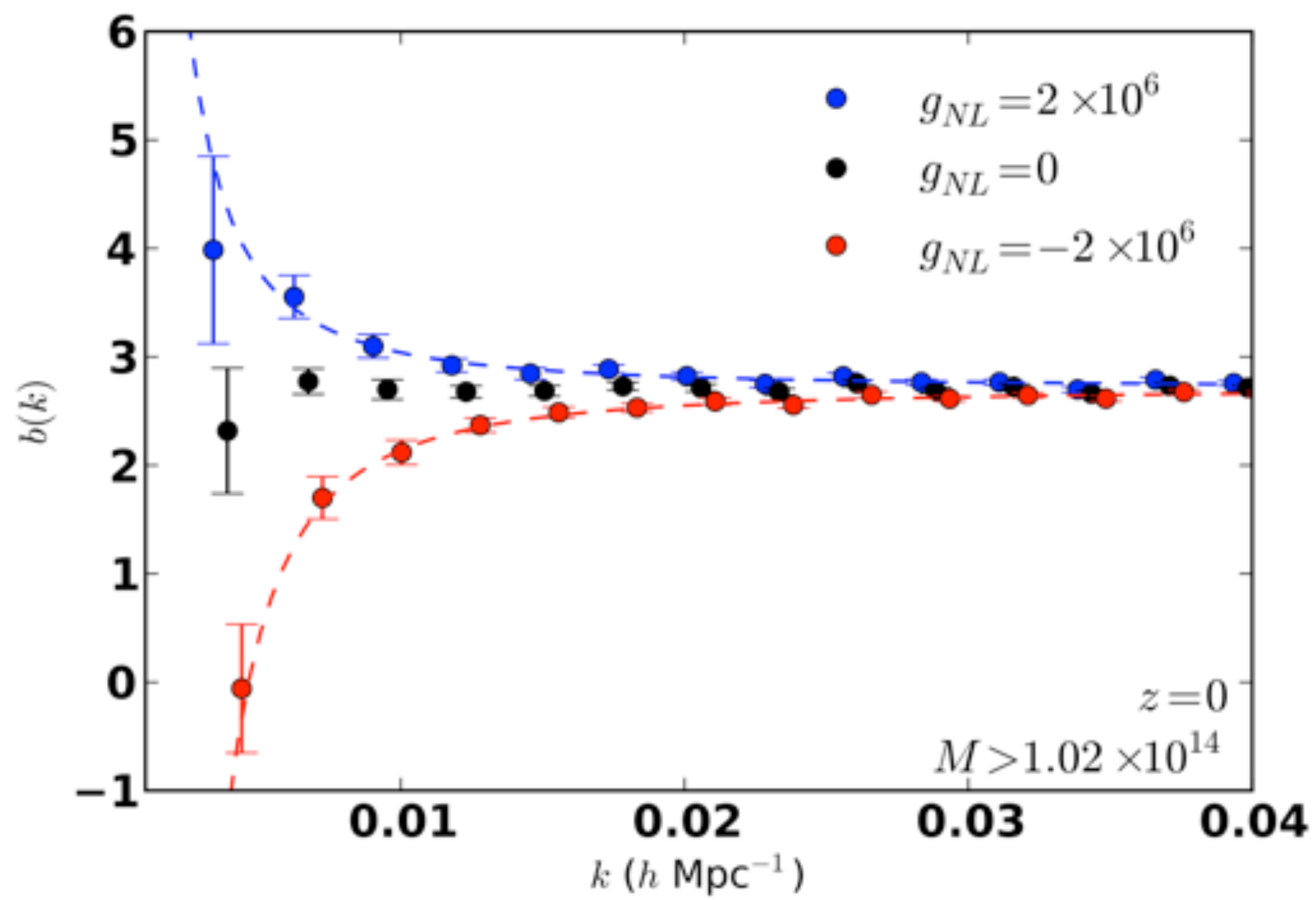
$$\zeta = \zeta_G + g_{NL}\zeta_G^3$$



Simple example of non-Gaussian model whose 4-point function $\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle$ is large in the squeezed limit $k_1 \to 0$.

Scale dependence of bias is same as f_{NL}^{loc} model

Mass and redshift dependence are different, but hard in practice to discriminate f_{NL}^{loc} and g_{NL}



Smith, Ferraro & LoVerde 2011

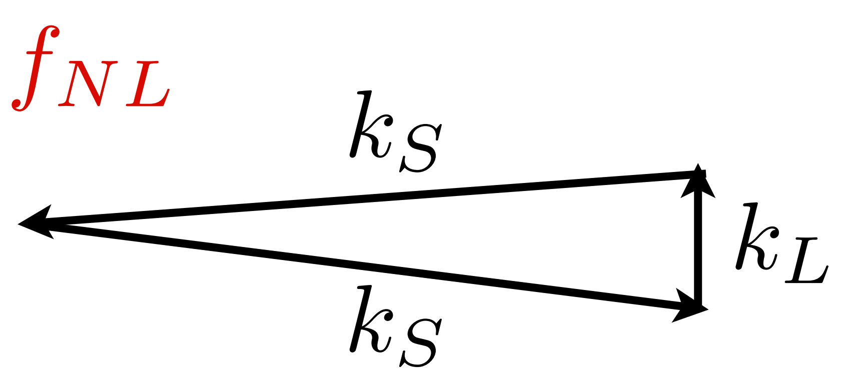
Example 2: stochastic bias from collapsed 4-pt

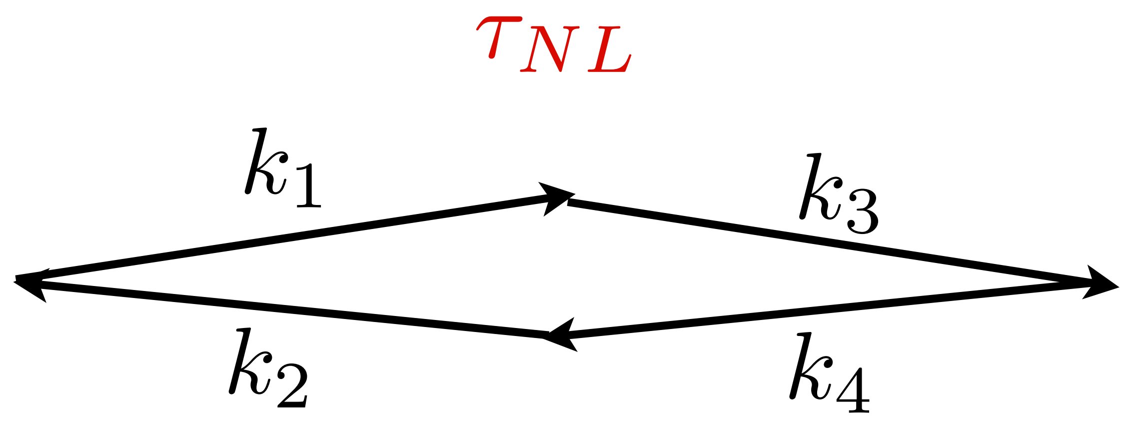
τ_{NL} model:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k_3}} \rangle = \frac{6}{5} f_{NL} (P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perm.})$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k_3}} \zeta_{\mathbf{k}_4} \rangle = 2\tau_{NL} \left[P_\zeta(k_1) P_\zeta(k_2) P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) + 23 \text{ perm.} \right]$$

In simple local model ($\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2$) one has $\tau_{NL} = (\frac{6}{5} f_{NL})^2$ but in general f_{NL}, τ_{NL} can be independent





Example 2: stochastic bias from collapsed 4-pt

Our general expression predicts the following:

$$P_{mh}(k) = \left(b_0 + b_1 \frac{f_{NL}}{(k/aH)^2} \right) P_{mm}(k)$$

$$P_{hh}(k) = \left(b_0^2 + 2b_0 b_1 \frac{f_{NL}}{(k/aH)^2} + b_1^2 \frac{\frac{25}{36}\tau_{NL}}{(k/aH)^4} \right) P_{mm}(k)$$

Qualitative prediction of τ_{NL} model: "stochastic" halo bias

Matter and halo fields are not proportional on large scales

Gives some scope for distinguishing $f_{NL},\ \tau_{NL}$:

- Different bias values inferred from $P_{mh}(k),\ P_{hh}(k)$
- Different tracer populations are not 100% correlated
- Even with a single population, can separate k^{-2}, k^{-4} terms

Smith & LoVerde 2010

Example: quasi-single field inflation

$$S_\pi = \int d^4x \sqrt{-g} \left(\frac{1}{2}(\partial\pi)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{M^2}{2}\sigma^2 + \rho\dot{\pi}\sigma - \frac{g}{3!}\sigma^3 \right)$$

Squeezed/collapsed limits (where $\alpha = \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$)

$$\lim_{k_L \to 0} \langle \zeta_{\mathbf{k}_L} \zeta_{\mathbf{k}_S} \zeta_{\mathbf{k}_S} \rangle \propto g\rho^3 \left(\frac{1}{k_L^{3-\alpha} k_S^{3+\alpha}} \right)$$

$$\lim_{|\mathbf{k}_1+\mathbf{k}_2| \to 0} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle \propto g^2\rho^4 \left(\frac{1}{|\mathbf{k}_1 + \mathbf{k}_2|^{3-2\alpha} \; k_1^{3+\alpha} k_3^{3+\alpha}} \right)$$

Prediction: non-Gaussian bias has spectral index given by

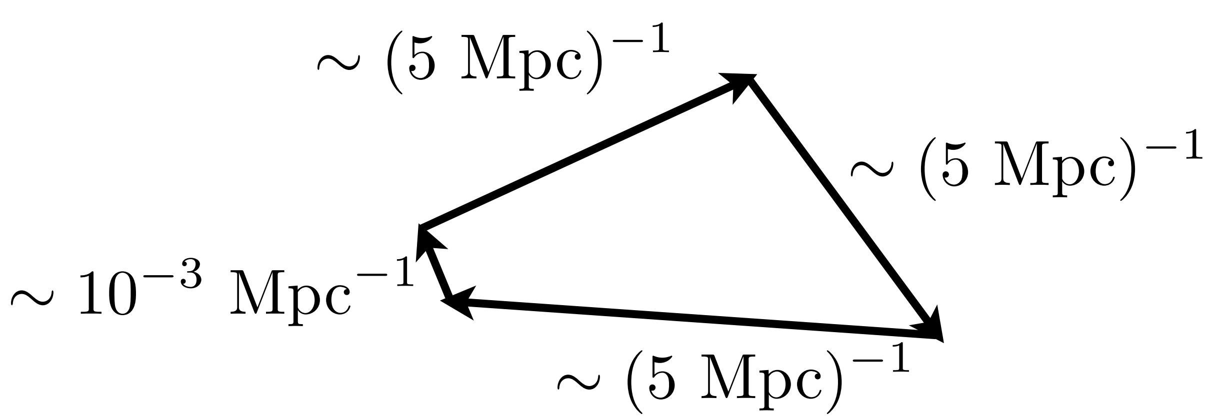
$$b(k) = b_0 + b_1 \frac{g\rho^3}{(k/aH)^{2-\alpha}}$$

Prediction: bias is mostly stochastic ("τ_{NL}"$= g^2\rho^4$ is enhanced relative to the square of "f_{NL}"$= g\rho^3$)

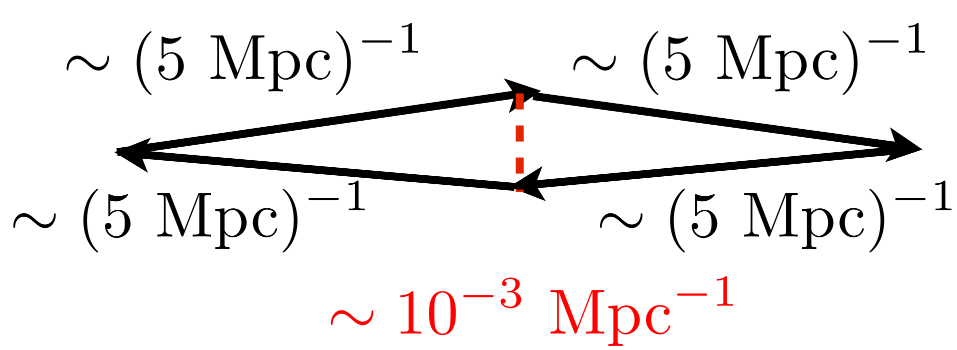
Large-scale structure: general picture

Large-scale structure constraints are best understood as precise tests of statistical homogeneity of the universe on large scales

Non-Gaussian models with large squeezed limits can be interpreted as large-scale inhomogeneity in statistics of small-scale modes, e.g:



large-scale correlation between density and small-scale skewness



large-scale inhomogeneity in small-scale power, uncorrelated to density ("stochastic")

Conclusions and future outlook

CMB:

Can measure N-point correlation function $\langle T_{\mathbf{l}_1} T_{\mathbf{l}_2} \cdots T_{\mathbf{l}_N} \rangle$ with full shape discrimination. "One estimator per diagram"

Statistical machinery is mature but many shapes unanalyzed!

Large-scale structure:

Future constraints on some models (e.g. $f_{NL}^{\rm loc}$) better than CMB

Models without squeezed limits (e.g. single field) unconstrained

Difficult to separate different N-point shapes (or different values of N) but some scope for discriminating models based on spectral index of the halo bias and stochastic vs non-stochastic bias