Non-Gaussianity and the Adiabatic Limit

Joel Meyers

Canadian Institute for Theoretical Astrophysics

Critical Tests of Inflation Using Non-Gaussianity

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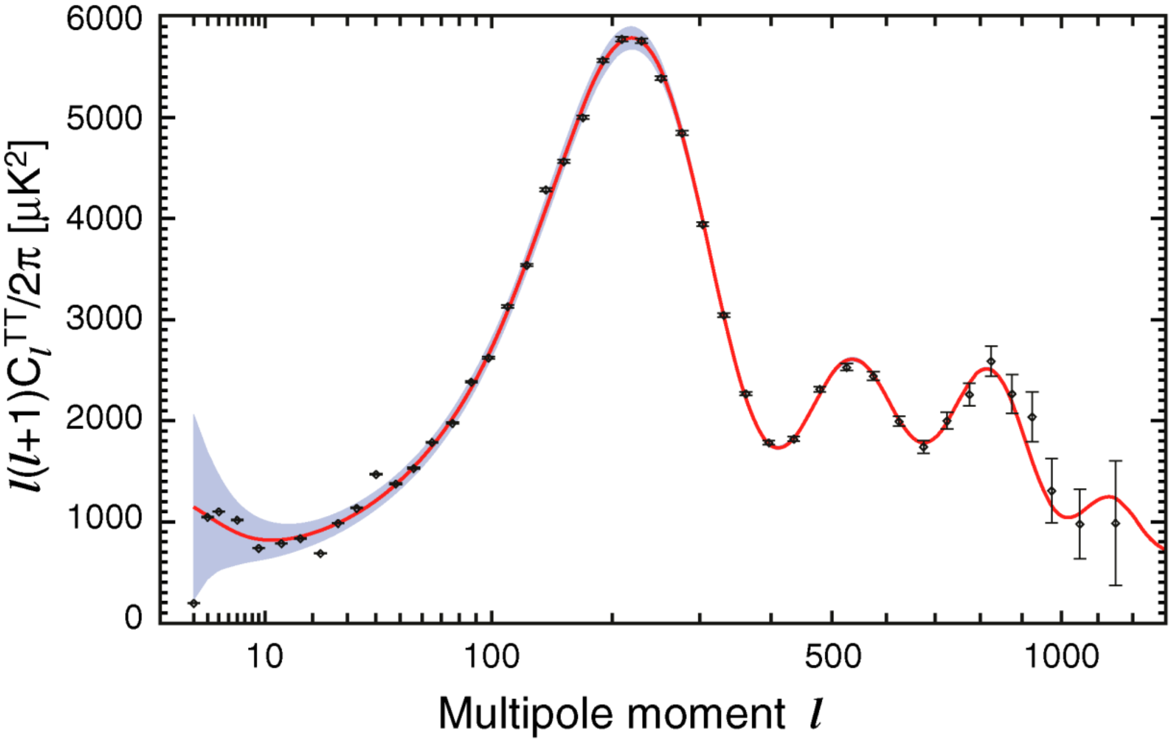
Based on arXiv:1011.4934 and 1104.5238 with Navin Sivanandam

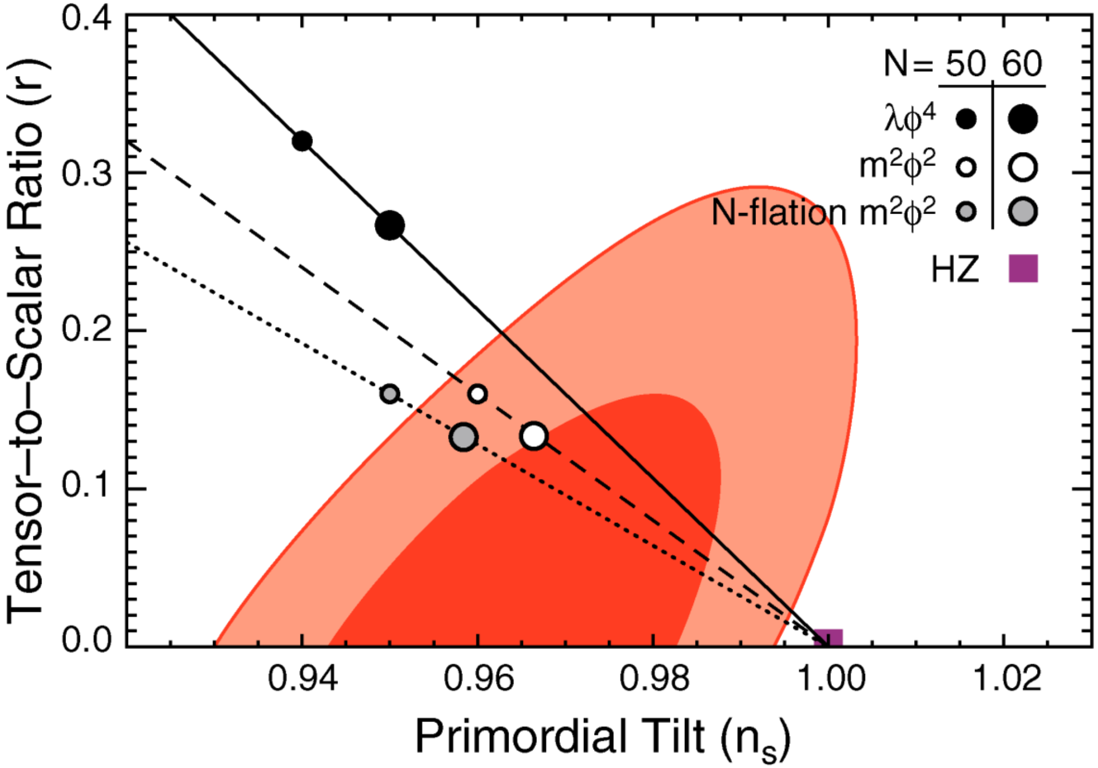
Inflation



WMAP7

- Measurements of the cosmic microwave background and large scale structure have allowed non-trivial tests of inflation

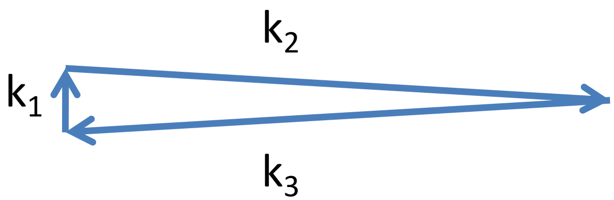
Inflation

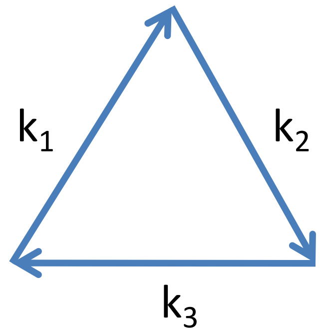


WMAP7

- Data has begun to rule out some models of inflation, but many models remain viable

Non-Gaussianity





WMAP7

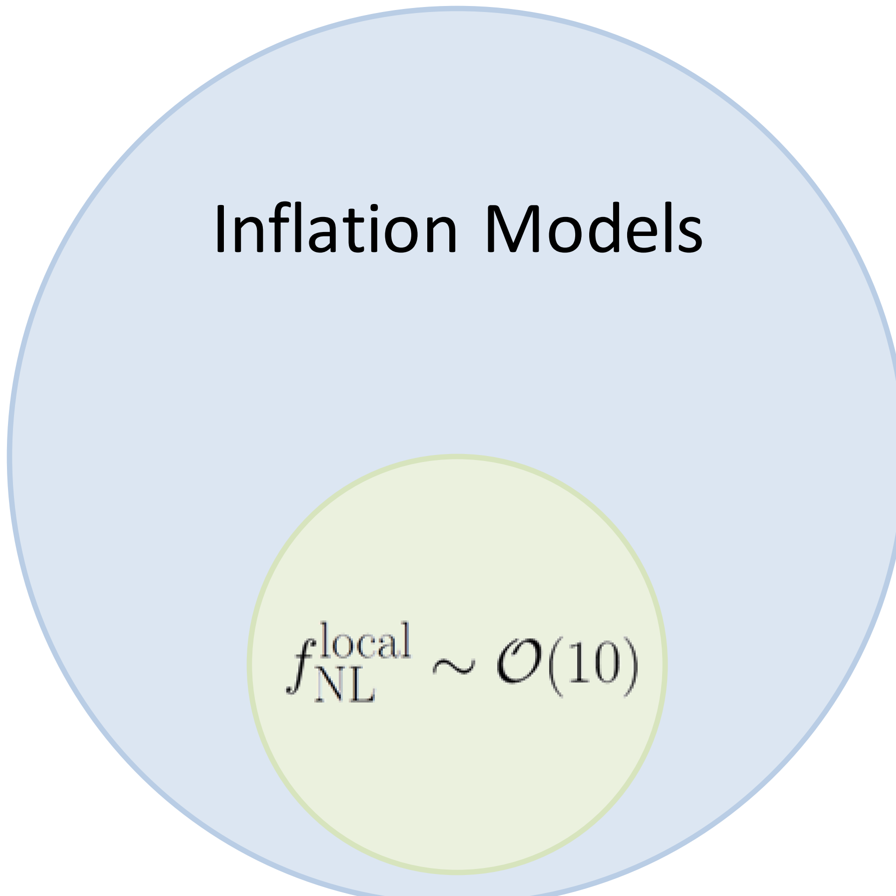
$$f_{\rm NL}^{\rm local} = 32 \pm 21 \qquad f_{\rm NL}^{\rm equil} = 26 \pm 140$$

- Experiments have already begun to constrain certain types of non-gaussianity
- The Planck satellite and future large scale structure surveys will better constrain these parameters soon

$$\Delta f_{\rm NL}^{\rm local} \sim 5$$

Komatsu, Spergel (2001)

Inflation and Non-Gaussianity

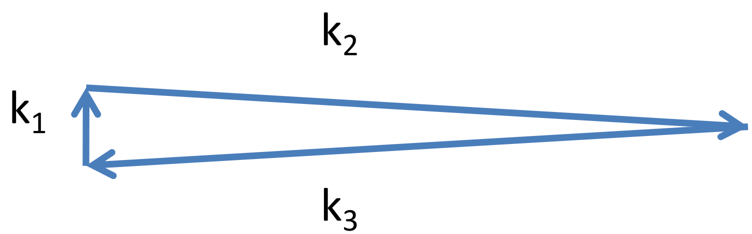


Single Field Consistency Relation

- $f_{\rm NL}^{\rm local}$ is always small in single field inflation

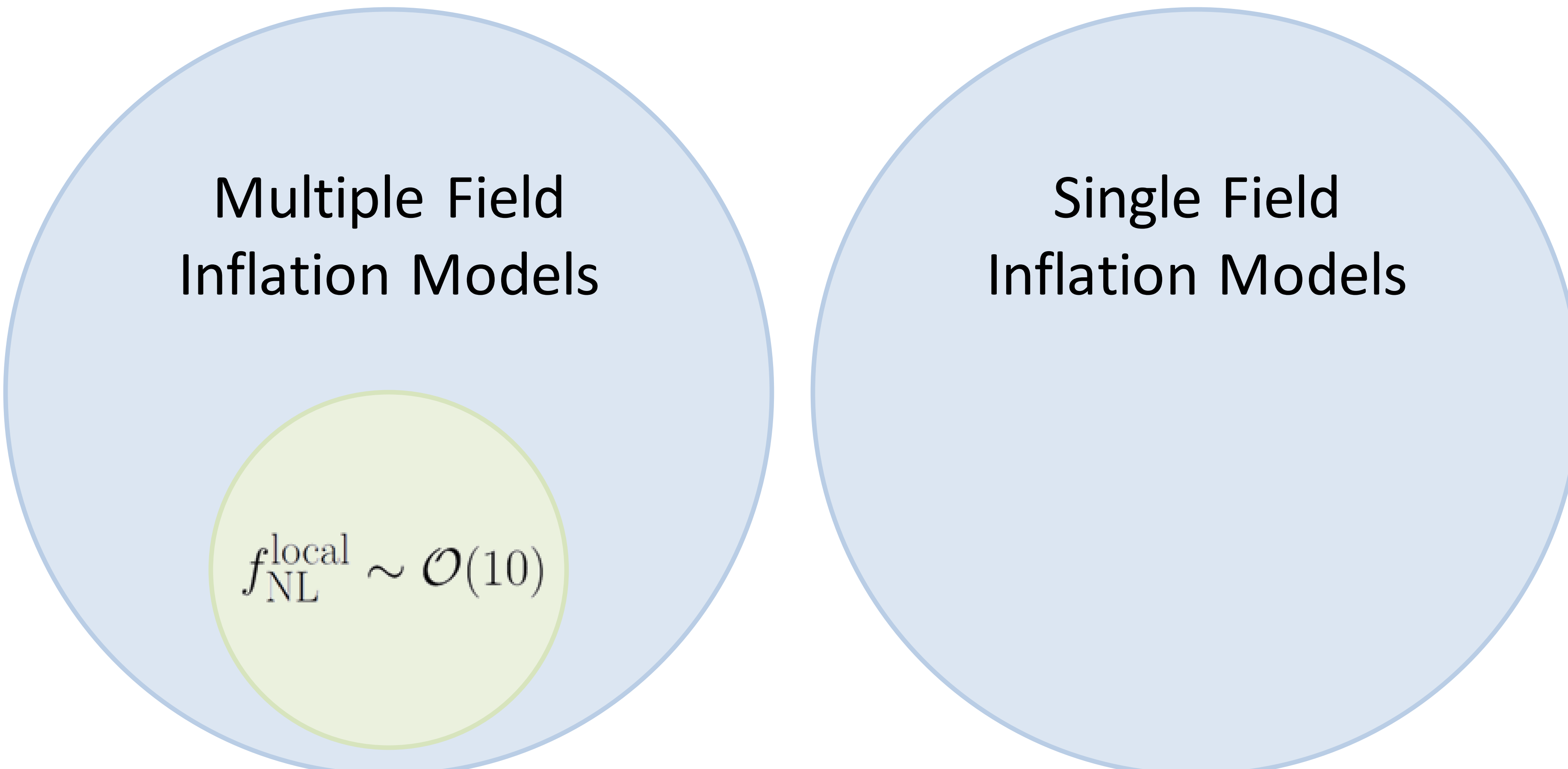
$$f_{\rm NL}^{\rm local} = \frac{5}{12}(1 - n_s)$$

Maldacena (2002)
Creminelli, Zaldarriaga (2004)
Ganc, Komatsu (2010)

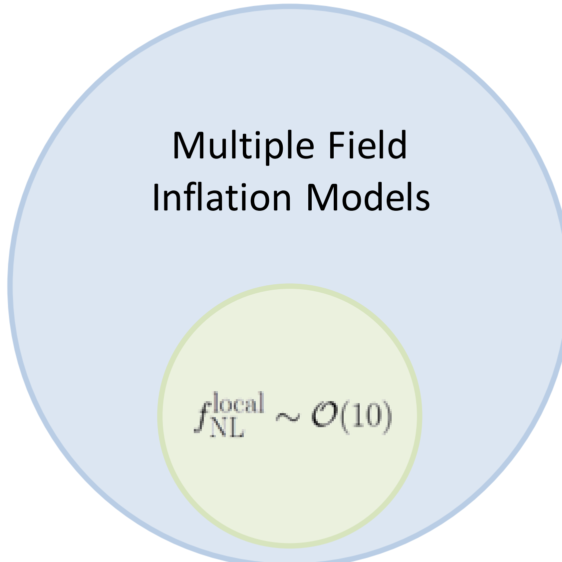


- A convincing detection of $f_{\rm NL}^{\rm local}$ would rule out *all* models of single field inflation

Inflation and Non-Gaussianity

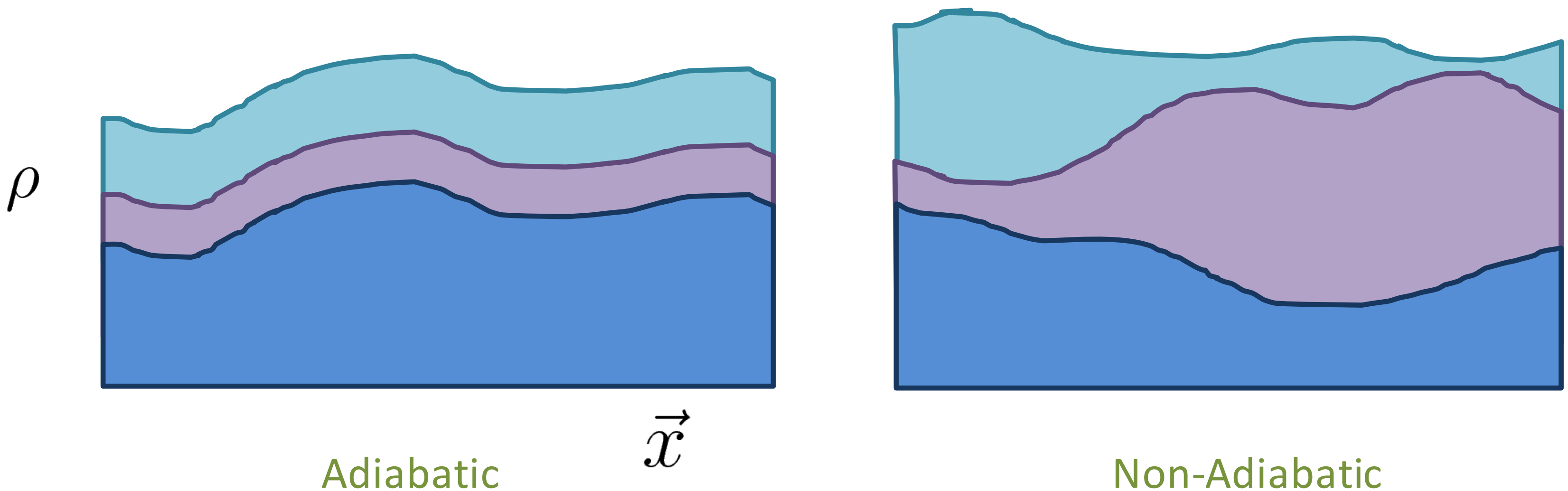


Inflation and Non-Gaussianity



- A convincing detection of $f_{\rm NL}^{\rm local}$ would rule out *all* models of single field inflation, and many models of multiple field inflation

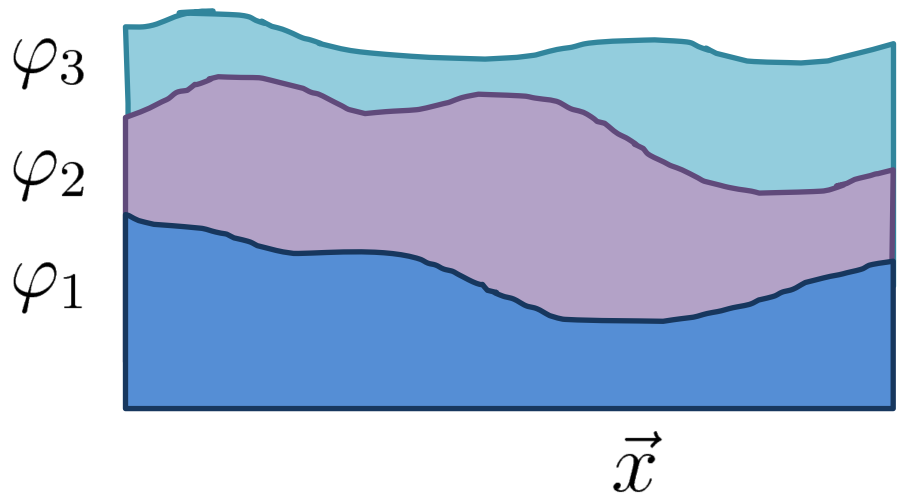
Adiabaticity



- The curvature perturbation is conserved outside the horizon when the fluctuations are adiabatic
- Single field inflation always produces purely adiabatic fluctuations

Weinberg (2003), (2004a), (2008)

Adiabaticity

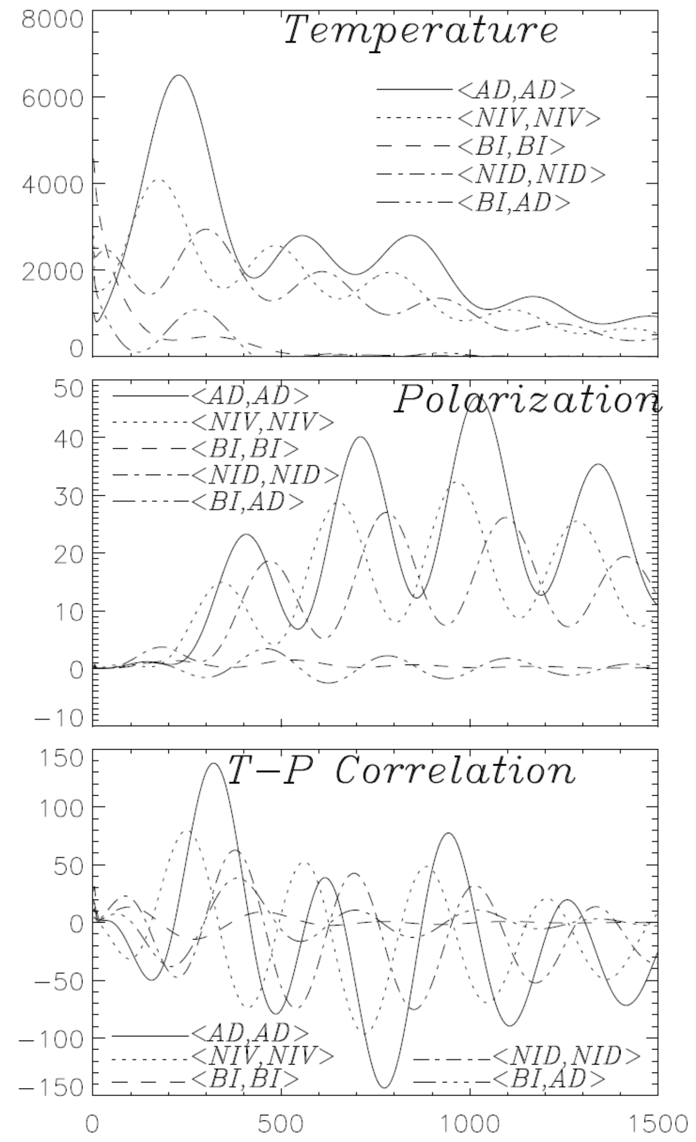


- The curvature perturbation can evolve on superhorizon scales in the presence of non-adiabatic fluctuations
- Multiple field inflation naturally produces non-adiabatic fluctuations

Adiabaticity

- Non-adiabatic fluctuations which persist through the radiation-dominated era produce observable effects
- Observations show no evidence for non-adiabatic fluctuations
  - Uncorrelated: $\alpha_0 < 0.077$
  - Anti-correlated: $\alpha_{-1} < 0.0047$

Bucher, Moodley, Turok (2001); Komatsu, et al. (2010)



Approach to Adiabaticity

- Non-adiabatic fluctuations may become adiabatic in at least two ways:

Effectively single field inflation

Local thermal equilibrium

Weinberg (2004b), (2008a), (2008b); JM (2012)

Approach to Adiabaticity

Any model with multiple dynamical fields is *incomplete* without an understanding of the evolution of the cosmological perturbations until they become adiabatic, or until they are observed.

Adiabaticity and Non-Gaussianity

- Are there general features of multiple field inflation models which predict $f_{\rm NL}^{\rm local} \sim \mathcal{O}(10)$ and a purely adiabatic power spectrum?

- We will focus on two field inflation models which pass through a short phase of effectively single field inflation before reheating

δN Formalism

- We use the δN formalism to calculate the evolution of observables outside the horizon

$$N = \int_*^c H\, dt \qquad\qquad N_I \equiv \frac{\partial N}{\partial \phi_*^I}$$

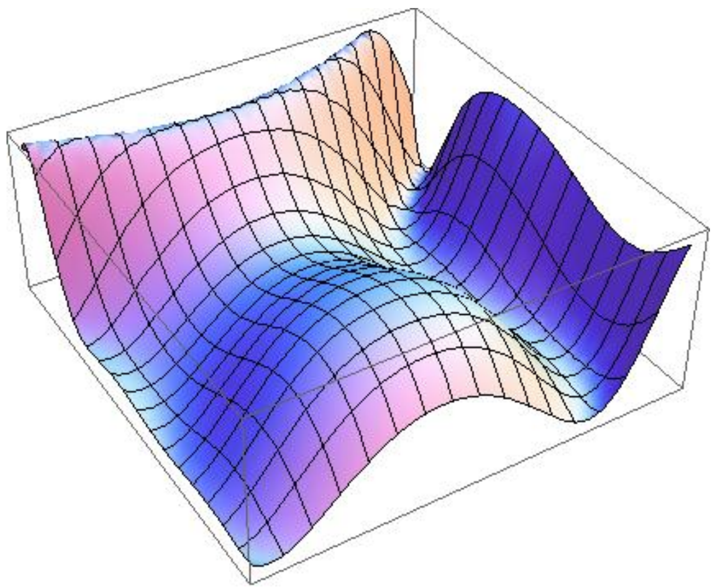
$$\zeta = \delta N \simeq \sum_I N_{,I}\delta\phi_*^I + \sum_{IJ} N_{,IJ}\delta\phi_*^I\delta\phi_*^J$$

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,I}N_{,J}N_{,IJ}}{\left(\sum_K N_K^2\right)^2}$$

Sasaki, Stewart (1996); Lyth, Rodriguez (2005)

Two Field Models

- Potentials admitting analytic treatment are of the form: $W(\phi, \chi) = F(U(\phi) + V(\chi))$



- For simplicity, focus on $W(\phi, \chi) = U(\phi) + V(\chi)$

JM, Sivanandam (2010,2011)

Results

- We find for a sum-separable potential

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\frac{x^2}{\epsilon_*^\phi}\left(2 - \frac{x\eta_*^\phi}{\epsilon_*^\phi}\right) + \frac{y^2}{\epsilon_*^\chi}\left(2 - \frac{y\eta_*^\chi}{\epsilon_*^\chi}\right)}{\left(\frac{x^2}{\epsilon_*^\phi} + \frac{y^2}{\epsilon_*^\chi}\right)^2} + \frac{2\frac{(U_c+V_c)^2}{(U_*+V_*)^2}\left(\frac{x}{\epsilon_*^\phi} - \frac{y}{\epsilon_*^\chi}\right)^2 \frac{\epsilon_c^\phi \epsilon_c^\chi}{\epsilon_c}\left(\frac{\eta_c^{ss}}{\epsilon_c} - 1\right)}{\left(\frac{x^2}{\epsilon_*^\phi} + \frac{y^2}{\epsilon_*^\chi}\right)^2}$$

- Where we have used the definitions

$$x \equiv \frac{1}{U_* + V_*}\left(U_* + \frac{V_c \epsilon_c^\phi - U_c \epsilon_c^\chi}{\epsilon_c}\right) \qquad y \equiv \frac{1}{U_* + V_*}\left(V_* - \frac{V_c \epsilon_c^\phi - U_c \epsilon_c^\chi}{\epsilon_c}\right)$$

$$\eta^{ss} \equiv \frac{\epsilon^\chi \eta^\phi + \epsilon^\phi \eta^\chi}{\epsilon}$$

Vernizzi, Wands (2006)

Effectively Single Field Inflation

- As the inflaton rolls through a steep valley, characterized by $(\eta^{ss} > 1)$ we find:
  - Entropy perturbations: $|\delta s| \sim \exp\left[-\frac{3}{2}\int H\, dt\right]$
  - Non-Gaussianity:

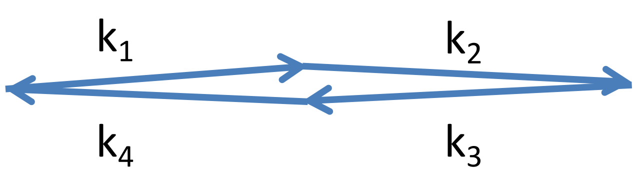
$$f_{\rm NL}^{\rm local} \sim \mathcal{O}(\varepsilon_*) + \mathcal{O}(1) \times \eta^{ss} \exp\left[-2\int C_\eta H \eta^{ss}\, dt\right]$$

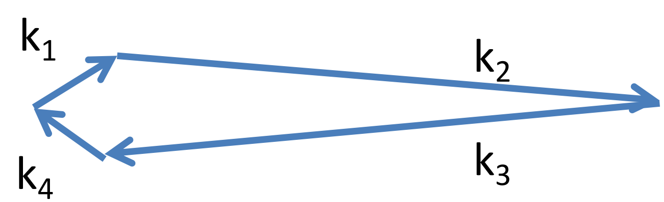
JM, Sivanandam (2010); Watanabe (2012)

Higher Point Statistics

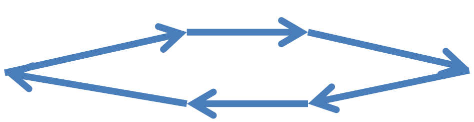
- Similar results also apply to the trispectrum after passing through a steep valley:

$$\tau_{\rm NL} \sim \mathcal{O}(\varepsilon_*) \qquad g_{\rm NL} \sim \mathcal{O}(\varepsilon_*)$$

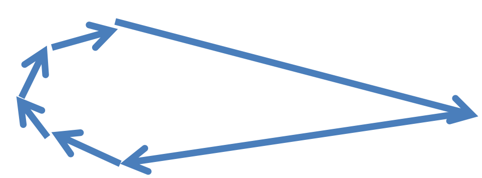




- And in fact to all local form n-point statistics:



$$F^{(n)}_{{\rm NL},i} \sim \mathcal{O}(\varepsilon_*)$$



JM, Sivanandam (2011)

Observables in Adiabatic Limit

- After adiabaticity is achieved, the observables take the following form

$$\frac{6}{5} f_{\rm NL}^{\rm local} \simeq \left(\frac{rx}{16\epsilon_*^\phi} \right)^2 (2\epsilon_*^\phi - x\eta_*^\phi) + \left(\frac{ry}{16\epsilon_*^\chi} \right)^2 (2\epsilon_*^\chi - y\eta_*^\chi)$$

$$n_s - 1 = -2\epsilon_* - 2\left(\frac{rx}{16\epsilon_*^\phi} \right) (2\epsilon_*^\phi - x\eta_*^\phi) - 2\left(\frac{ry}{16\epsilon_*^\chi} \right) (2\epsilon_*^\chi - y\eta_*^\chi)$$

- Recall the definitions

$$x \equiv \frac{1}{U_* + V_*} \left(U_* + \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c} \right) \qquad y \equiv \frac{1}{U_* + V_*} \left(V_* - \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c} \right)$$

Conditions for Observable f_{NL}

- Generating local non-gaussianity which is preserved in the adiabatic limit seems to require (at least for simple potentials):
  - One *very* slowly rolling field at horizon exit
  - A finely-tuned trajectory through field space
  - One field with negligible contribution to the energy density at horizon exit

Byrnes, Choi, Hall (2008); Kim, Liddle, Seery (2010); Elliston, et al. (2011)

Open Questions

- How general are these results?

- How does reheating affect non-gaussianity?
  - See talk by Chris Byrnes Leung et al. (2102)

- Can we understand the curvaton mechanism and modulated reheating in similar terms?

Conclusions

- Detection of local non-gaussianity rules out more than just single field inflation
- Sharp predictions require an understanding of the evolution until fluctuations become adiabatic, or until they are observed
- Two-field inflation models which pass through an effectively single field phase require tuning of parameters at horizon exit to produce local form non-gaussianity