Squeezing the CMB bispectrum

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Preparing for the best...

Monday discussion by Xingang Chen

\[ f_{\text{NL}}^{\text{loc}} \sim 32 \]
Preparing for the worst...

\[ f_{NL}^{\text{loc}} \ll 1 \]
...or the not-so bad

\[ f_{NL}^{\text{loc}} \sim \text{few} \]
Even in the absence of primordial non-Gaussianity, $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = 0$, the CMB is non-Gaussian!

2\textsuperscript{nd}-order effects induce NG:

- late time: ISW-lensing (previous talk);
- at recombination: 2\textsuperscript{nd}-order perturbations in the fluid + GR nonlinearities.

All these effects are order ~ may be important to interpret Planck data!
Squeezed limit

\[ k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3 \]

\[ \langle \zeta \bar{k}_L \zeta \bar{k}_S \zeta \bar{k}_S \rangle = -P_{k_L} P_{k_S} \frac{d \ln (k_S^3 P_{k_S})}{d \ln k_S} \]

- Directly addresses the local shape (important to rule out single-field models)
- Eventually we would like to have a 2nd-order Boltzmann code: squeezed limit can be used as a consistency check

Maldacena '02, Creminelli & Zaldarriaga '04, Cheung et al. '07
Particular squeezed limit

One of the angles must subtend a scale longer than Hubble radius at recombination (but smaller than Hubble radius today):

\[ H^{-1}_{\text{at recombination}} \]
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\[ H^{-1} \text{ at recombination} \]

For the bispectrum:

\[ B_{l_1 l_2 l_3}, \quad l_1 \ll l_2 \simeq l_3 \quad \& \quad l_1 \ll 200 \]

Sachs-Wolfe effect:

\[ \frac{\delta T}{T} = \frac{1}{3} \Phi(\vec{x}_{\text{rec}}) = -\frac{1}{5} \zeta(\vec{x}_{\text{rec}}) \quad \Rightarrow \]

\[ C_l \simeq \frac{A_T}{l^2} \left( \frac{l}{l_*} \right)^{n_s - 1} \]
Physical argument

Single-field inflation: 1 clock, e.g. everything is determined by T.

Local physics is identical in Hubble patches that differ only by super-horizon modes: two observers in different places on LSS will see exactly the same CMB anisotropies (at given T).
The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due large scale $T$:

$$C_l \rightarrow C_l + \Theta_L \frac{d}{d\Theta_L} C_l \hspace{1cm} \Theta \equiv \Delta T / T \hspace{1cm} (\Theta_L = -\frac{1}{5} \zeta)$$

- Long mode changes the local average temperature: $T \rightarrow (1 + \Theta_L)T$

$$C_l \rightarrow C_l + 2\Theta_L C_l \hspace{1cm} \Rightarrow \hspace{1cm} B_{lLl_Sl_S} = 2C_{lL} C_{lS} \hspace{1cm} (f_{NL}^{\text{loc}} = -\frac{1}{6})$$

- Transverse rescaling of spatial coords $\Rightarrow$ rescaling of angles:

$$C_l \rightarrow C_l - 5\Theta_L (\hat{n} \cdot \nabla_{\hat{n}} C_l) \hspace{1cm} \Rightarrow \hspace{1cm} B_{lLl_Sl_S} = 5C_{lL} C_{lS} \frac{d \ln(l_S^2 C_{lS})}{d \ln l_S}$$

- Lensing close to last scattering displaces the 2-p function
2\textsuperscript{nd}-order evolution as a coord change

Maldacena '02; Weinberg '03; Fitzpatrick et al. '09

Locally, possible to rewrite a perturbed FRW metric as an unperturbed one by reabsorbing the long mode with a coordinate transformation. \textit{Ex, in matter dominance:}

\[ ds^2 = a^2(\eta) \left[ -(1 + 2\Phi_L) d\eta^2 + (1 - 2\Phi_L) dx^2 \right] \quad \Rightarrow \quad ds^2 = a^2(\tilde{\eta}) \left[ -d\tilde{\eta}^2 + d\tilde{x}^2 \right] \]

\[ \tilde{\eta} = \eta \left( 1 + \frac{\Phi_L}{3} \right) \]
\[ \tilde{x}^i = x^i \left( 1 - \frac{5\Phi_L}{3} \right) \]

(\( \zeta = -\frac{5}{3} \Phi_L \))

Conversely, start from a perturbed metric at 1\textsuperscript{st}-order and "generate" 2\textsuperscript{nd}-order couplings between short and long modes by the inverse coordinate transformation:

\[ ds^2 = a^2(\tilde{\eta}) \left[ -(1 + 2\tilde{\Phi}_S) d\tilde{\eta}^2 + (1 - 2\tilde{\Psi}_S) d\tilde{x}^2 \right] \Rightarrow ds^2 = a^2(\eta) \left[ -e^{2\Phi} d\eta^2 + e^{2\Psi} dx^2 \right] \]

\[ \Phi = \tilde{\Phi}_S + \Phi_L + \frac{1}{3} \Phi_L \frac{\partial \tilde{\Phi}_S}{\partial \ln \eta} - \frac{5}{3} \Phi_L x^i \frac{\partial \tilde{\Phi}_S}{\partial x^i} \]
2\textsuperscript{nd}-order evolution as a coord change

Example: radiation-to-matter transition:

\begin{align*}
\Phi_{k_S} &\rightarrow \Phi_{k_S} - \frac{5}{3} \Phi_{k_L} \left( f(\eta) \frac{\partial \Phi_{k_S}}{\partial \ln \eta} - \frac{\partial \Phi_{k_S}}{\partial \ln k} \right) \\
f(\eta) &= -\frac{20 + 15\alpha \eta + 3\alpha^2 \eta^2}{5(2 + \alpha \eta)} \\
\alpha &= (\sqrt{2} - 1)/\eta_{eq}
\end{align*}

\begin{align*}
\delta_{k_S} &\rightarrow \delta_{k_S} - \frac{5}{3} \Phi_{k_L} \left( f(\eta) \frac{\partial \delta_{k_S}}{\partial \ln \eta} - \frac{\partial \delta_{k_S}}{\partial \ln k} \right)
\end{align*}
Apply this coordinate transformation to the observed CMB:

\[ \Theta_{\text{obs}}(\hat{n}) = \frac{T_{\text{obs}}(\hat{n}) - \langle T_{\text{obs}} \rangle}{\langle T_{\text{obs}} \rangle} \]

\[ \Theta_{\text{obs}} = [\Theta + \Phi - \hat{n} \cdot \vec{v}](\eta_{\text{rec}}, \vec{x}_{\text{rec}}) \]

In pure matter dominance and instantaneous recombination.
Apply this coordinate transformation to the observed CMB:

\[ \Theta_{\text{obs}}(\hat{n}) = \frac{T_{\text{obs}}(\hat{n}) - \langle T_{\text{obs}} \rangle}{\langle T_{\text{obs}} \rangle} \]

\[ \Theta_{\text{obs}} = \Theta_{\text{obs},S} + \Theta_{\text{obs},L} \left( 1 + \frac{\partial}{\partial \ln \eta_{\text{rec}}} - 5\hat{n} \cdot \nabla \hat{n} \right) \Theta_{\text{obs},S} \]

\[ \Theta_{\text{obs},S} = [\Theta_{S} + \Phi_{S} - \hat{n} \cdot \vec{v}_{S}](\eta_{\text{rec}}, \vec{x}_{\text{rec}}) \]

\[ \Theta_{\text{obs},L} = \frac{1}{3} \Phi_{L}(\eta_{\text{rec}}, \vec{x}_{\text{rec}}) \]

Holds also when including radiation/matter transition (early Sachs-Wolfe) and finite recombination.

Check: a mode out of Hubble radius today is unobservable. Cancels out from this expression.

Time derivative is geometrically suppressed as \( \sim \frac{\eta_{\text{rec}}}{\eta_{\text{obs}}} \)

Bispectrum:

\[ B_{l_{L}l_{S}l_{S}} = C_{l_{L}} C_{l_{S}} \left( 2 + 5 \frac{d \ln (l_{S}^{2} C_{l_{S}})}{d \ln l_{S}} \right) \]

Extension of the Maldacena relation. Cf:

\[ B_{k_{L}k_{S}k_{S}} = -P_{k_{L}} P_{k_{S}} \frac{d \ln (k_{S}^{3} P_{k_{S}})}{d \ln k_{S}} \]

See also Bartolo Matarrese Riotto '11; Lewis '12
Final result

Coordinate and average temperature redefinition

\[ B_{ll_S} = C_{ll} C_{l_S} \left( 2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) + 6 C_{ll} C_{l_S} \left[ 2 \cos 2\theta - (1 + \cos 2\theta) \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right] \]

Lensing

Boubekeur et al. '09

\[ \cos \theta = \hat{l}_L \cdot \hat{l}_S \]

Lensing due to correlation between temperature at recombination and transverse displacement:

\[ \delta \vec{x}^\perp = -2 \int_{\eta_{\text{rec}}}^{\eta_0} (1 - \eta_{\text{rec}}) \vec{n} \Phi (\vec{x}) d\eta \]

\[ \Theta_{\text{lensed}} = \Theta_{\text{obs},S}(\eta_{\text{rec}}, \vec{x}_* + \delta \vec{x}^\perp) = \Theta_{\text{obs},S}(\eta_*, \vec{x}_*) + \delta \vec{x}^\perp \cdot \vec{n} \Theta_{\text{obs},S}(\eta_*, \vec{x}_*) \]

Partial cancellation between (isotropic) lensing convergence and space redefinition. Overdense regions (negative potential) give positive convergence, moving the spectrum towards larger angles, while coordinate redefinition shrinks it.
Integrating only in the squeezed limit $l_L \lesssim 60$:

$$f^\text{loc}_{NL} = -0.39, \quad l_{\text{max}} = 2000 \quad \text{See also Bartolo, Riotto '11}$$

Other effects at $l_L \gtrsim 200$  Khatri, Wandelt '08; Senatore, Tassev, Zaldarriaga '09

Seemingly negligible contamination to $f^\text{loc}_{NL}$

<table>
<thead>
<tr>
<th>From Lewis '12</th>
<th>$\sigma_{f_{NL}}$</th>
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<tbody>
<tr>
<td>$T$</td>
<td>4.3</td>
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<tr>
<td>Planck T</td>
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<tr>
<td>T+E</td>
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Boltzmann code: CMBquick

This relation can be used as **consistency check of Boltzmann codes** based on a physical limit.

There have been several contributions to development of Boltzmann numerical code at 2\textsuperscript{nd} order:

Bartolo, Matarrese, Riotto ’06; Bernardeau, Pitrou, Uzan ’08; Pitrou ’08; Bartolo, Riotto ’08; Khatri, Wandelt ’08; Senatore, Tassev, Zaldarriaga ’09; Nitta et al. ’09, Beneke and Fidler ’10

One of the most complete code is Pitrou’s CMBquick (see also next talk):
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One of the most complete code is Pitrou’s CMBquick (see also next talk):

\[ B_{l_L l_S} = C_{l_L} C_{l_S} \left( 2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) \]

The check is nontrivial! Even though analytically the squeezed limit is easy, in the code all 2nd-order effects must conspire to reproduce the simple analytical formula.
**New Boltzmann code: CosmoLib++**

by Zhiqi Huang

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![Graphs showing comparison](image-url)
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Results will appear soon!
Conclusion

In the squeezed limit (one mode longer than horizon at recombination), it is possible to compute the CMB bispectrum exactly.

Valid for adiabatic (single clock) perturbations. Already takes into account NG from single-field models. It is a consistency relation on the observable (CMB temperature) in the squeezed limit.

• Planck will (most likely) not be biased by 2nd-order effects at recombination (may be detectable).

• Test Boltzmann codes at 2nd order. Reasonable agreement with Pitrou’s CMBquick code. Need of better codes: CosmoLib++ is including 2nd-order perturbations and bispectrum computation.
Observability

Can we observe this signal? Signal-to-noise ratio is:

\[
\left( \frac{S}{N} \right)^2 = \frac{1}{\pi} \int \frac{d^2 l_2 d^2 l_3}{(2\pi)^2} \frac{[B(l_1, l_2, l_3)]^2}{6C_l_1 C_l_2 C_l_3}
\]

We are integrating only in the squeezed limit and we do not include polarization. Boltzmann code would give a better estimate. Possibly measurable effect.