Critical tests of inflation, MPA, Garching
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Curvaton
a worked example of local-type non-Gaussianity

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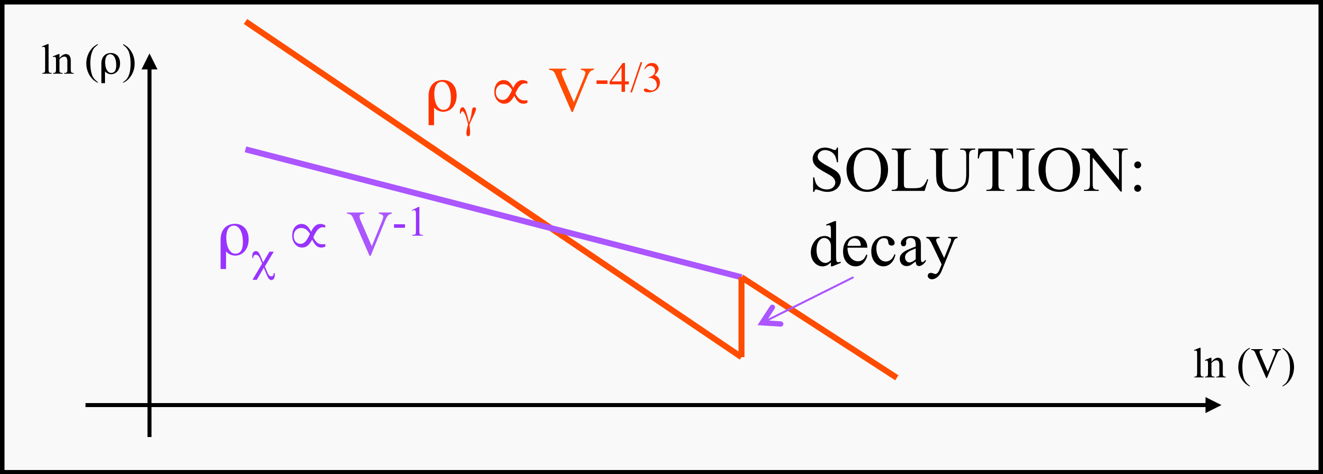
Q: what is the curvaton?

A: a light weakly-coupled scalar field

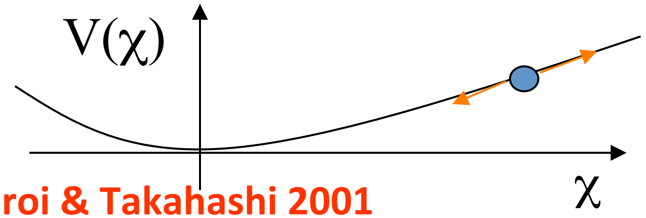
Polonyi / moduli problem:

- supersymmetric theories have many light weakly-coupled scalar fields
- after inflation they are expected to be displaced from their true vacuum
- they begin oscillating about true minimum when Hubble rate drops below effective mass of the field, $H < m$
- oscillating field has (time-averaged) equation of state, $P = 0$
- hence density grows relative to radiation / photons
- leads to early matter domination, incompatible with standard cosmology



curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001



- light field, m<<H, during inflation acquires almost scale-invariant, ***Gaussian distribution of field fluctuations δχ*** on large scales

- **quadratic energy density** for free field, $\rho_\chi = m^2\chi^2/2$

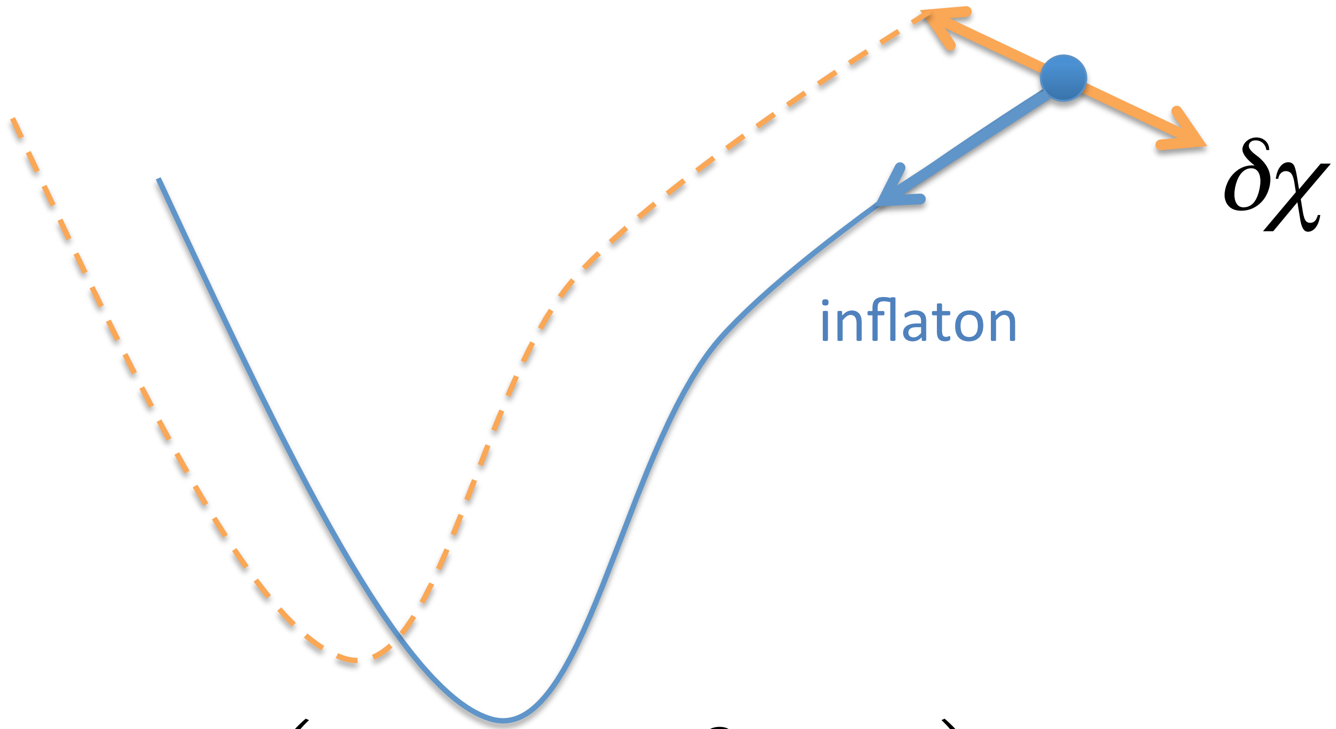
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{2}{3}\frac{\delta\chi}{\chi}$$

- transferred to radiation when curvaton decays with some **efficiency,** $0<r<1$, where $r \approx \Omega_{\chi,decay}$

$$\zeta = r\zeta_\chi \approx \frac{2r}{3}\frac{\delta\chi}{\chi}$$

non-adiabatic field fluctuations



$$\zeta \sim r\left(\frac{\delta\chi}{\chi} + \frac{\delta\chi^2}{\chi^2} + \ldots\right)$$

non-linear curvature perturbation

$$\zeta = \delta N + \int_{\bar{\rho}}^{\rho} \frac{d\rho'}{3\left[\rho' + P(\rho')\right]}$$

Lyth, Malik & Sasaki (2005),
but see also Lyth & Wands (2003),
Rigopoulos, Shellard & van Tent (2003)
Langlois & Vernizzi (2005)

where, metric $ds^2 = a^2 e^{2\delta N} \gamma_{ij} dx^i dx^j$, density $\rho = \bar{\rho} + \delta\rho$

Hence

$$\zeta = \delta N\big|_{\rho=\bar{\rho}}$$ = curvature perturbation on uniform-density slices

$$= \frac{1}{3(1+w)} \ln\left[\frac{\rho}{\bar{\rho}}\right]_{\delta N=0} \quad \Rightarrow \quad \rho = \bar{\rho}\, e^{3(1+w)\zeta}$$

= density perturbation on uniform-curvature (*δN=0*) slices

curvaton perturbation

Sasaki, Valiviita & Wands (2006)

$$\zeta_\chi = \frac{1}{3}\ln\left(\frac{\rho_\chi}{\bar{\rho}_\chi}\right)_{\delta N=0}$$

where curvaton density $\rho_\chi = \frac{1}{2}m^2\chi^2 \quad \Rightarrow \quad e^{3\zeta_\chi} = \frac{\chi^2}{\bar{\chi}^2}$

expand order by order

$$3\zeta_{\chi 1} = 2\frac{\delta_1\chi}{\bar{\chi}}$$

$$3\zeta_{\chi 2} + 9\zeta_{\chi 1}{}^2 = 2\frac{\delta_2\chi}{\bar{\chi}} + \left(\frac{\delta_1\chi}{\bar{\chi}}\right)^2 \Rightarrow \quad \zeta_2 = -\frac{3}{2}\zeta_1{}^2 \quad \Leftrightarrow \quad f_{NL}{}^{(\chi)} = -\frac{5}{4}$$

for purely Gaussian $\delta\chi$

sudden-decay approximation: $H_{decay} = \Gamma$

ζ on uniform-total-density hypersurface before curvaton decay:

$$\rho_r + \rho_\chi = \bar{\rho}_r e^{4(\zeta_r - \zeta)} + \bar{\rho}_\chi e^{3(\zeta_\chi - \zeta)} = \text{constant}$$

$$\Rightarrow \quad \left(1 - \Omega_\chi\right) e^{4(\zeta_r - \zeta)} + \Omega_\chi \, e^{3(\zeta_\chi - \zeta)} = 1$$

Sasaki, Valiviita & Wands (2006)

expand order by order (and assuming, for simplicity, $\zeta_r = 0$)

$$\left(1 - \Omega_\chi\right) + \Omega_\chi = 1$$

$$4\left(1 - \Omega_\chi\right)\zeta_1 = 3\Omega_\chi\left(\zeta_{\chi 1} - \zeta_1\right)$$

$$4\left(1 - \Omega_\chi\right)\zeta_2 - 16\left(1 - \Omega_\chi\right)\zeta_1^2 = 3\Omega_\chi\left(\zeta_{\chi 2} - \zeta_2\right) + 9\Omega_\chi\left(\zeta_{\chi 1} - \zeta_1\right)^2$$

$$4\left(1 - \Omega_\chi\right)\zeta_3 + \ldots$$

simplest quadratic curvaton: *V=m²χ²/2*

- *first-order perturbations*

$$\zeta = r\,\zeta_\chi \qquad \text{where} \qquad r = \left[\frac{3\Omega_\chi}{4-\Omega_\chi}\right]_{\rm decay} \leq 1$$

- *second-order perturbations*

$$\zeta_2 = \left[\frac{3}{2r} - 2 - r\right]\zeta_1^2 \quad \Rightarrow \quad f_{NL} = \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6} \geq -\frac{5}{4}$$

- *third-order perturbations*

$$\zeta_3 = \left[-\frac{9}{r} + \frac{1}{2} + 10r + 3r^2\right]\zeta_1^3 \quad \Rightarrow \quad g_{NL} = -\frac{25}{6r}\left[1 - \frac{r}{18} - \frac{10r^2}{9} - \frac{r^3}{3}\right] \leq \frac{9}{2}$$

- *Predictions of the simplest quadratic curvaton model:*
  - *for r ≈ 1*

$$f_{NL} = -\frac{5}{4} \quad , \quad g_{NL} = \frac{9}{2}$$

  - *for r << 1*

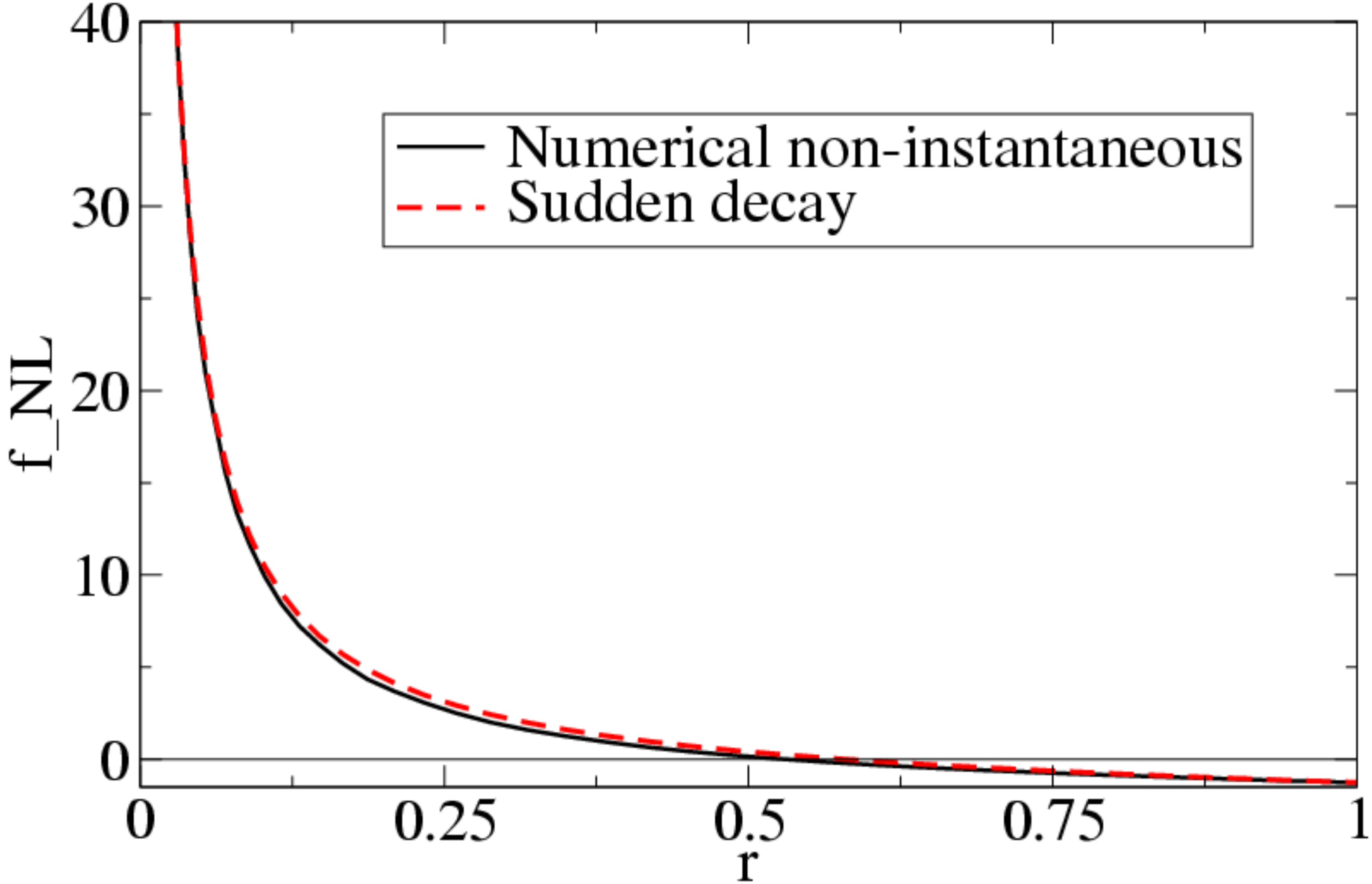
$$f_{NL} >> 1 \quad , \quad \tau_{NL} = \frac{36}{25} f_{NL}^2 \quad >> \quad g_{NL} = -\frac{10}{3} f_{NL}$$

single source obeys Suyama-Yamaguchi equality

non-linearity parameter

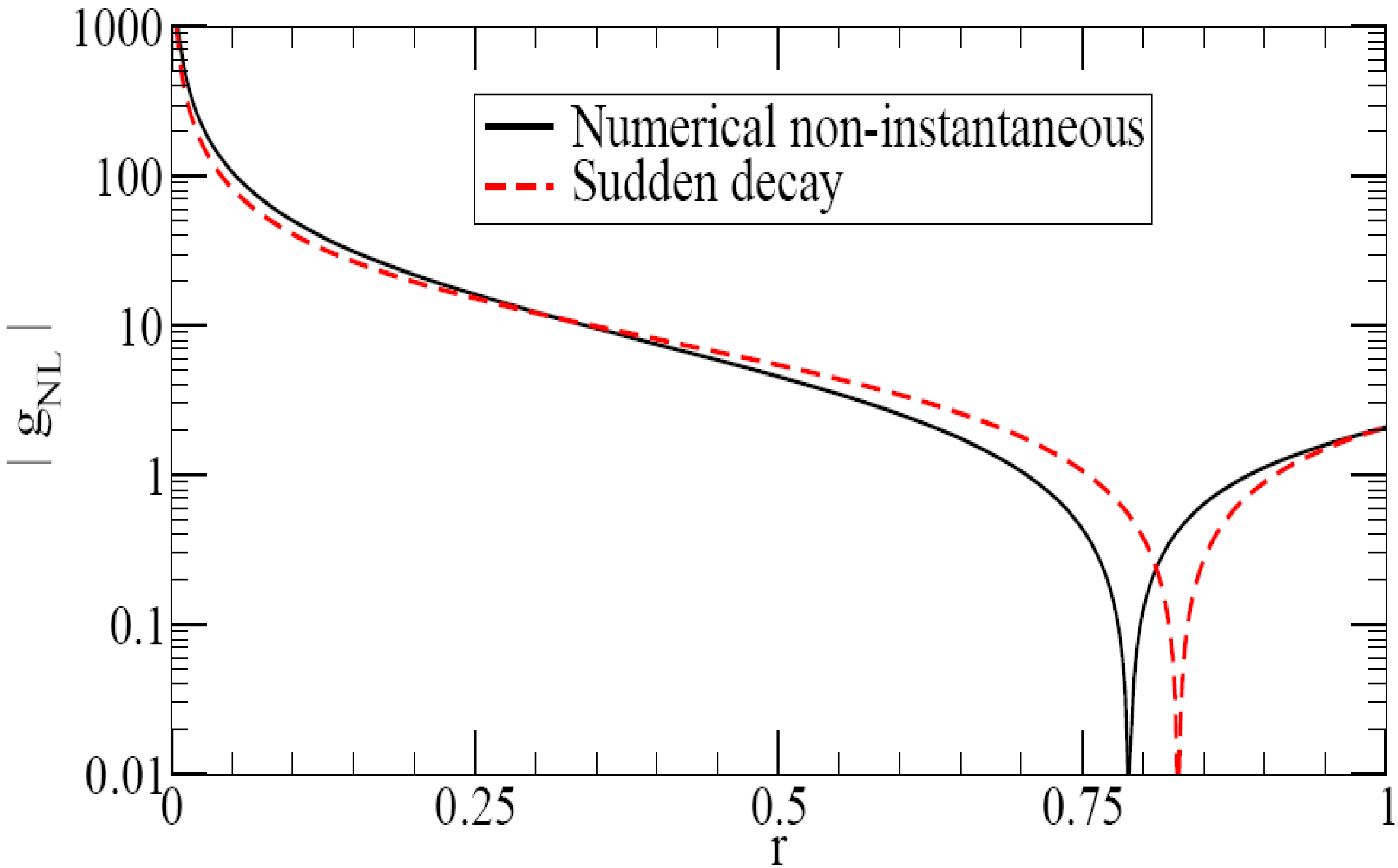
Sasaki, Valiviita & Wands (2006)
see also Malik & Lyth (2006)

c.f. exact (numerical) calculation



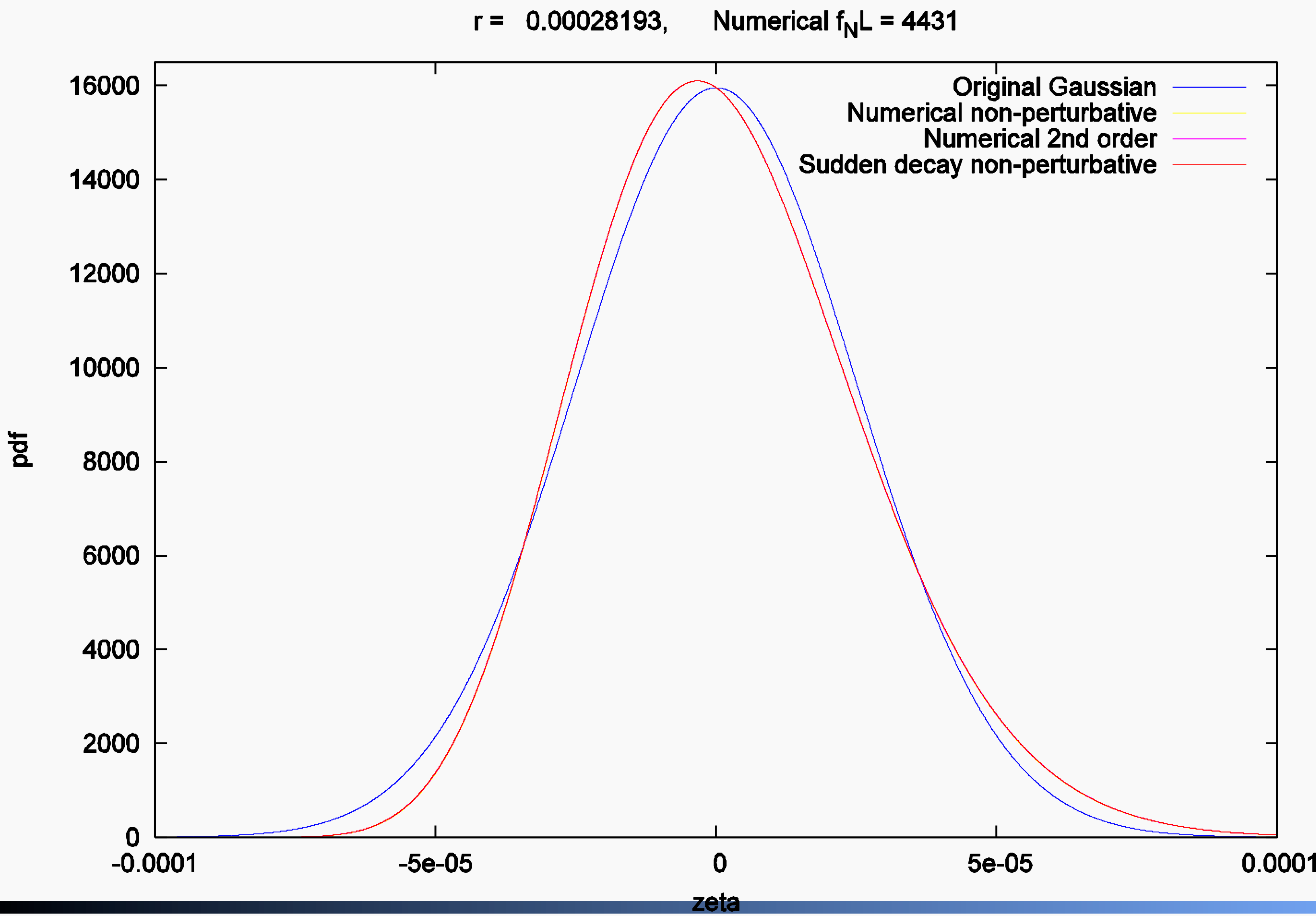
3rd order non-linearity for curvaton

Sasaki, Valiviita & Wands (astro-ph/0607627)

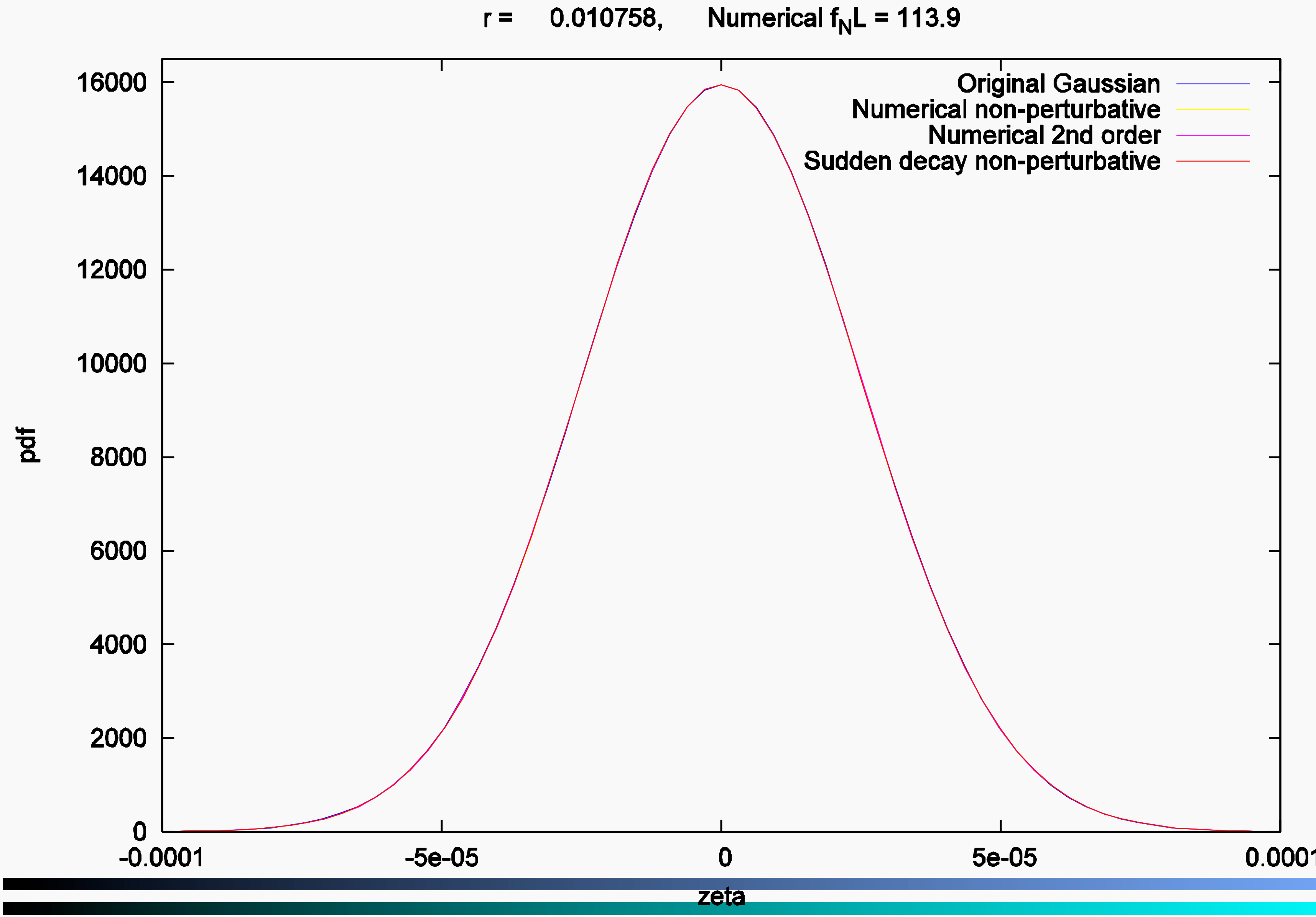


full pdf for ζ from δN

Sasaki, Valiviita & Wands (2006)

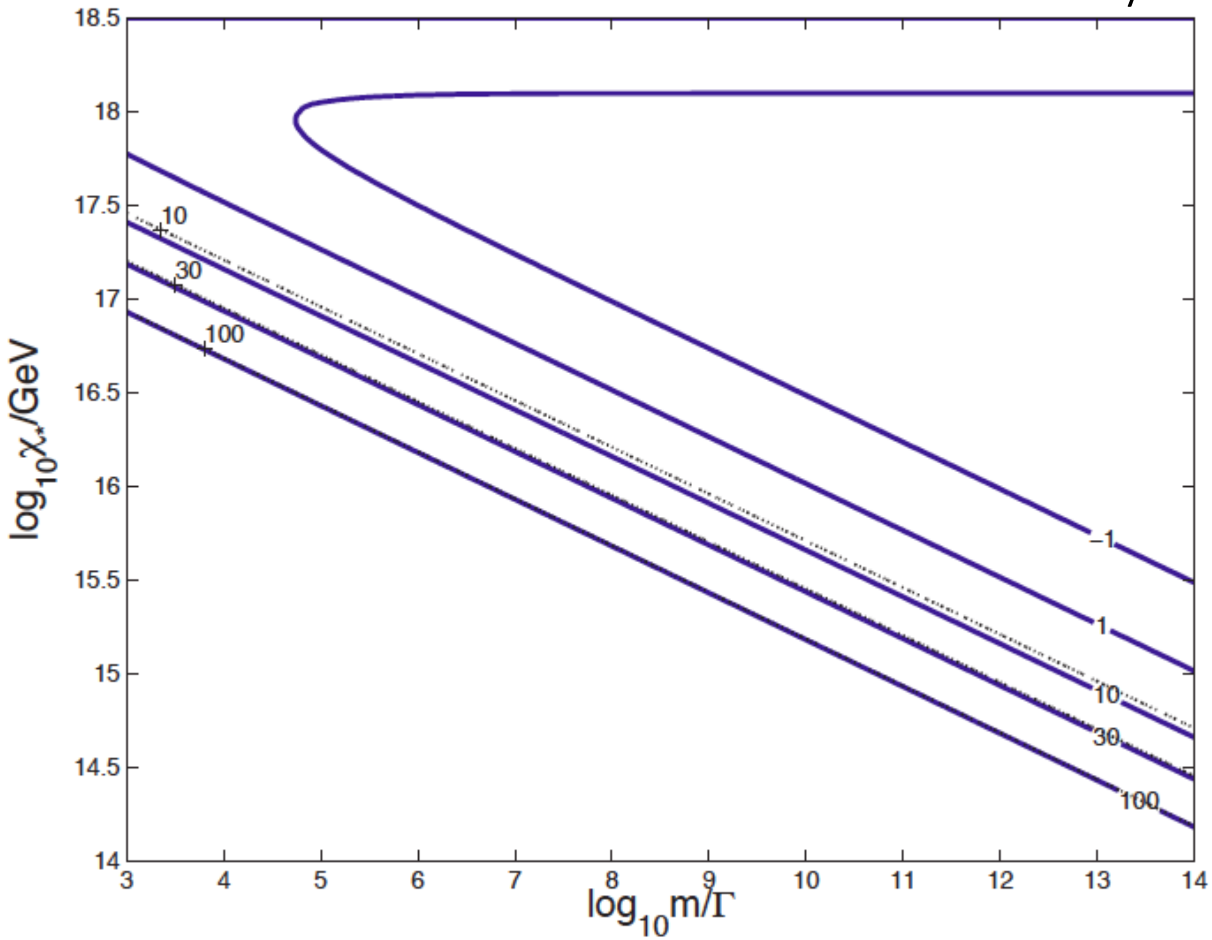


probability distribution for ζ



f_{NL} bounds on curvaton parameters

Fonseca & Wands (2011)
see also Nakayama et al (2010)

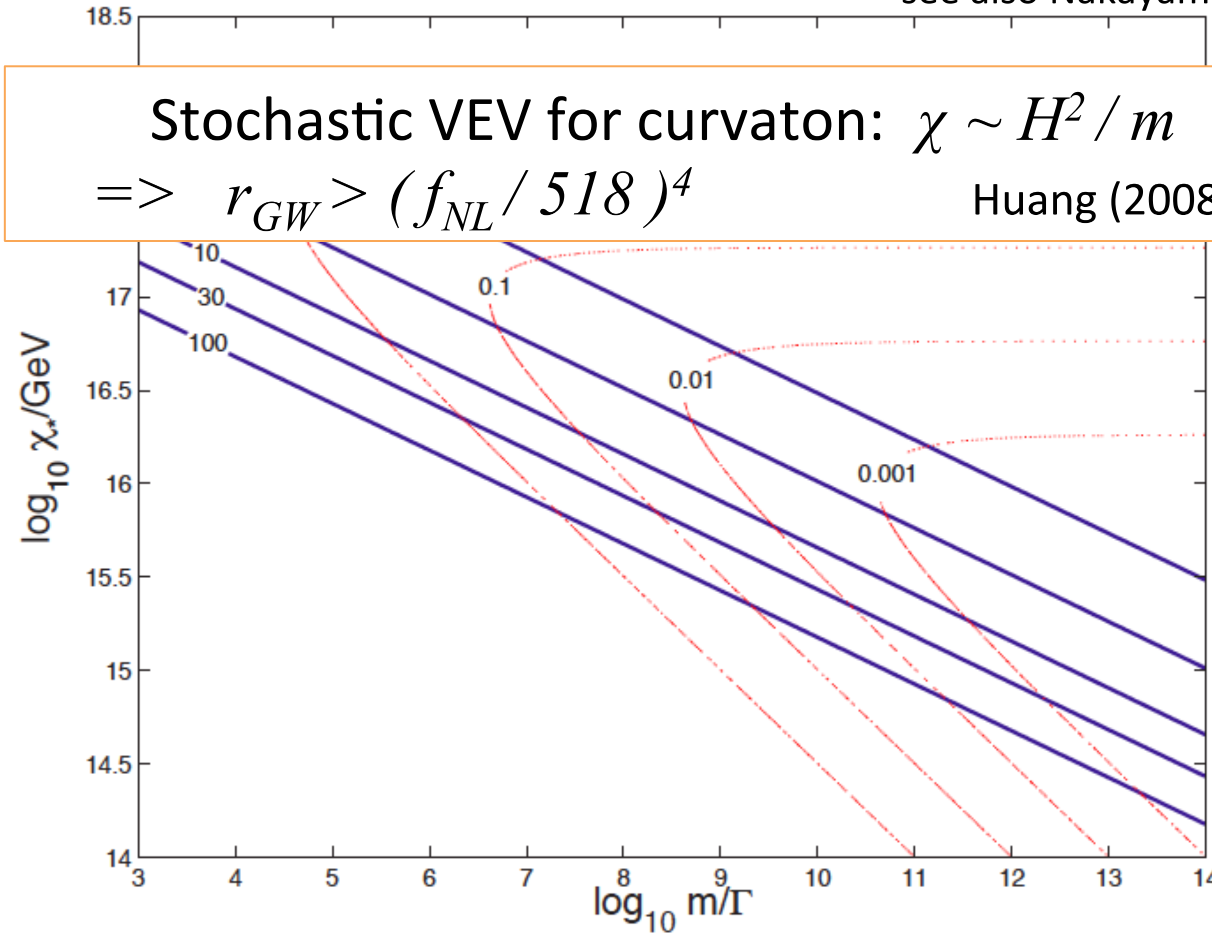


f_{NL} + r_{GW} bounds on curvaton parameters

Fonseca & Wands (2011)
see also Nakayama et al (2010)

Stochastic VEV for curvaton: $\chi \sim H^2 / m$

=> $r_{GW} > (f_{NL} / 518)^4$ Huang (2008)



self-interacting curvaton

Enqvist & Nurmi (2005)
Huang (2008)
Enqvist et al (2009)
...

$$V(\chi) = \frac{1}{2} m^2 \chi^2 + m^4 \left(\frac{\chi}{f} \right)^n$$

assume curvaton is **Gaussian at Hubble exit** during inflation, but allow for **non-linear evolution** on large scales before oscillating about quadratic minimum

$$\chi_{osc} = g(\chi_*) \quad \Rightarrow \quad \chi_{osc} = g + g'\delta\chi + \frac{1}{2} g''(\delta\chi)^2 + \ldots$$

nG from self-interacting curvaton

- *second-order perturbations*

$$f_{NL} = \frac{5}{4r}\left(\frac{g''g}{g'^2}+1\right) - \frac{5}{3} - \frac{5r}{6}$$

- *third-order perturbations*

$$g_{NL} = \frac{9}{4r^2}\left(\frac{g'''g^2}{g'^3}+3\frac{g''g}{g'^2}\right) - \frac{9}{r}\left(\frac{g''g}{g'^2}+1\right) - \frac{1}{2}\left(9\frac{g''g}{g'^2}-1\right) + 10r + 3r^2$$

- *Predictions of the simplest quadratic curvaton model:*
  - *for r ≈ 1*

$$f_{NL} = \frac{5}{4}\left(\frac{g''g}{g'^2}-1\right) \quad , \quad g_{NL} = \frac{9}{4}\left(\frac{g'''g^2}{g'^3} - \frac{1}{3}\frac{g''g}{g'^2}+2\right)$$

  - *for r << 1*

$$f_{NL} = \frac{5}{4r}\left(\frac{g''g}{g'^2}+1\right) \quad , \quad \tau_{NL}^{tree} = \frac{36}{25}f_{NL}^2 \sim \quad g_{NL} = \frac{9}{4r^2}\left(\frac{g'''g^2}{g'^3}+3\frac{g''g}{g'^2}\right)$$

for $g_{NL} >> f_{NL}^2$ => loop corrections to τ_{NL} violating SY equality…

an aside: loop corrections to SY equality

$$\tau_{NL}^{loop}(k) = \frac{36}{25}\left(f_{NL}^{loop}\right)^2\left[1 + \frac{81 g_{NL}^2}{25 f_{NL}^2} P \ln\left(\frac{k}{k_{IR}}\right)\right]$$

for $|g_{NL}| \gg f_{NL}^2$ loop corrections violate SY equality
(does not hold on all scales)

-> generalised equality: Tasinato, Byrnes, Nurmi & Wands (2012)

scale-dependence of f_{NL}?

Byrnes, Choi & Hall 2009
Khoury & Piazza 2009
Sefusatti, Liguori, Yadav, Jackson & Pajer 2009
Byrnes, Nurmi, Tasinato & Wands (2009); Byrnes, Gerstenlauer, Nurmi, Tasinato & Wands (2010)

- power spectrum $$P_\zeta(k) = \left[N'^2 P_{\delta\varphi} \right]_{k=aH}$$

  $\Rightarrow$ scale-dependence $$n_\zeta - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} = H^{-1} \frac{d \ln N'^2}{dt} - 2\varepsilon$$

- bispectrum $$f_{NL}(k) = \frac{5}{6} \left[\frac{N''}{N'^2} \right]_{k=aH}$$

  $\Rightarrow$ scale-dependence $$n_{fNL} \equiv \frac{d \ln |f_{NL}|}{d \ln k} = \frac{N'}{N''} \left(\sqrt{2\varepsilon}(4\varepsilon - 3\eta) + \frac{V'''}{3H^2} \right)$$

- e.g., **self-int. curvaton** $$n_{fNL} = \frac{N'}{N''} \left(\frac{V'''}{3H^2} \right)$$

scale-dependence probes self-interaction, not probed by power spectrum

could be observable for curvaton models where $g_{NL} \sim \tau_{NL}$ (Byrnes et al 2011)

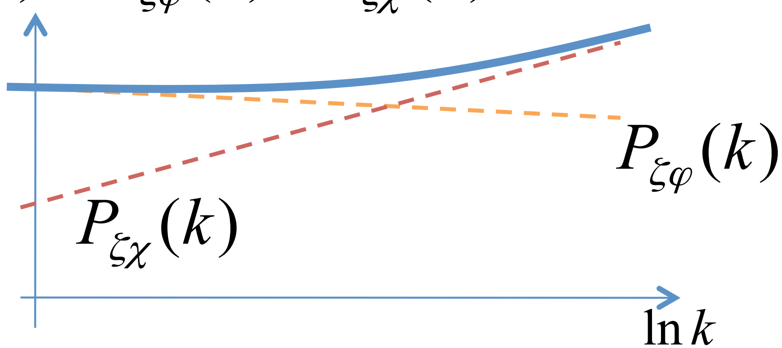
scale-dependent f_{NL} from curvaton + inflaton

Byrnes, Nurmi, Tasinato & Wands (2009)

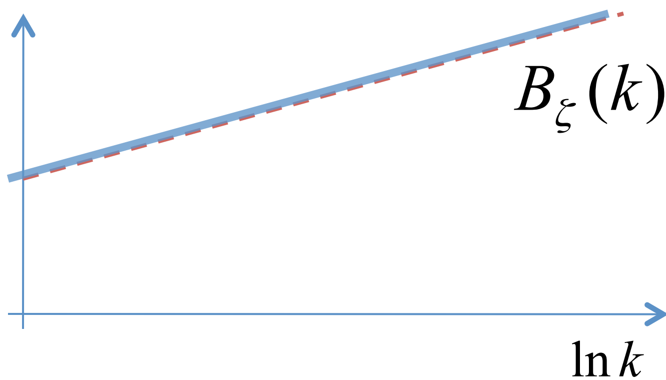
$$\zeta(x) = \zeta_\varphi(x) + \zeta_\chi(x) + \frac{3}{5} f_{\chi\chi} \zeta_\chi{}^2(x)$$

- power spectrum

$$P_\zeta(k) = P_{\zeta\varphi}(k) + P_{\zeta\chi}(k)$$



- bispectrum



$$B_\zeta(k) = B_{\zeta\chi}(k)$$

$$\rightarrow f_{NL}(k) \approx B_\zeta(k) \Big/ \left(P_\zeta(k)\right)^2$$

- **but g_{NL} small**

Conclusions:

- ***Curvaton*** provides a simple (natural) model for large primordial non-Gaussianity
- ***Quadratic (non-interacting) curvaton well-described by simplest local*** f_{NL}
- ***Many variants***
  - ***self-interactions, inflaton+curvaton, multi-curvaton…***

    offer wider range of ***observables***
  - nf_{NL}
  - *large* g_{NL}
  - ***inhomogeneous*** *non-Gaussianity*
- ***More precise data*** allows us to study more detailed models of inflation and origin of cosmological perturbations