20-24 July, 2015
MPA/ESO/MPE/Excellence Cluster
Universe Joint Conference
@ Garching

A novel scheme for perturbation theory calculation of large-scale structure

Atsushi Taruya
(YITP, Kyoto Univ.)

With F.Bernardeau, K.Koyama, E.Linder, T.Nishimichi, T.Okumura, C.Sabiu, Y-S.Song, G.Zhao

Perturbation theory calculations of large-scale structure

Precision theoretical calculations for large-scale structure :
essential ingredient to pursue precision cosmology

Reducing and/or controlling *nonlinear systematics*
(gravity/redshift-space distortions/galaxy biasing)

Perturbation theory (PT) approach

- valid at large scales in weakly nonlinear regime
- tell us how nonlinear systematics are developed through the coupling between different Fourier modes

→ ***PT kernels***

Standard PT kernels

Basic eqs.

single-stream approx.

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}]=0,$$
$$\frac{\partial\boldsymbol{v}}{\partial t}+H\,\boldsymbol{v}+\frac{1}{a}(\boldsymbol{v}\cdot\nabla)\cdot\boldsymbol{v}=-\frac{1}{a}\nabla\psi$$
$$\frac{1}{a}\nabla^2\psi=\frac{\kappa^2}{2}\,\rho_{\rm m}\,\delta$$

e.g., Bernardeau et al. ('02)

Assuming $|\delta|,\ |\theta|\ll 1$

we expand $\delta=\delta^{(1)}+\delta^{(2)}+\cdots$ and $\theta=\theta^{(1)}+\theta^{(2)}+\cdots$

$$\theta\equiv\frac{\nabla\cdot\boldsymbol{v}}{a\,H}$$

$$\delta^{(n)}(\boldsymbol{k};t)=\int\frac{d^3\boldsymbol{k}_1\cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\,\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$
$$\theta^{(n)}(\boldsymbol{k};t)=\int\frac{d^3\boldsymbol{k}_1\cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)\,\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n),$$

Kernels (F_n, G_n) are analytically constructed from recursion relation (e.g., Goroff et al. '86)

Predictions with standard PT kernels

We can do many things with standard PT kernels !!

- standard PT calculations
- resummed PT scheme by Γ-expansion (RegPT, MPTbreeze)

  (Bernardeau et al. '08; AT, et al. '12; Crocce et al. '12)
- modeling redshift-space distortions (RSD)

  (e.g., AT, Nishimichi & Saito '10; Reid & White '11; Vlah et al. '12;…)
- modeling galaxy bias (e.g., McDonald '06; McDonald & Roy '08; Saito et al. '14)

However,

Calculations relies on the analytic expressions for kernels

A slight change in basic eqs. makes calculation intractable

(e.g., modified gravity, massive neutrinos, …) ⟶ *numerical treatment*

Previous works

Time-RG Pietroni ('08); Lesgourgues et al. ('09)

✓ Application to massive neutrinos & redshift-space distortions

✓ Public codes (Copter, CLASS, redTime)

Carlson et al. ('09); Audren & Lesgourgues ('11); Upadhye et al. ('14, '15)

Numerical scheme to solve Closure eqs. Valageas ('07); Hiramatsu & AT ('09)

✓ Application to modified gravity models

Koyama, AT & Hiramatsu ('09); Brax & Valageas ('12,'13, '14); AT et al. ('13, '14)

In this talk,

As yet another approach, I develop a simple numerical method to reconstruct standard PT kernels (F_n, G_n)

Kernel reconstruction approach

Standard PT kernels as building blocks for various PT predictions

Solving evolution eqs. for PT kernels numerically:

Linear operator

nonlinear source term

$$\widehat{\mathcal{L}}(k_{1\cdots n})\begin{pmatrix} F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \\ G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \end{pmatrix} = \begin{pmatrix} S_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \\ T_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;a) \end{pmatrix}$$

$$\sum_{j=1}^{n-1}\begin{pmatrix} -\alpha(\boldsymbol{k}_{1\cdots j},\boldsymbol{k}_{j+1\cdots n})\,G_j(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_j)\,F_{n-j}(\boldsymbol{k}_{j+1},\cdots,\boldsymbol{k}_n) \\ -\frac{1}{2}\beta(\boldsymbol{k}_{1\cdots j},\boldsymbol{k}_{j+1\cdots n})\,G_j(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_j)\,G_{n-j}(\boldsymbol{k}_{j+1},\cdots,\boldsymbol{k}_n) \end{pmatrix} + \square$$

$$\widehat{\mathcal{L}}(k) \equiv \begin{pmatrix} a\frac{d}{da} & 1 \\ \frac{3}{2}\left(\frac{H_0}{H(a)}\right)^2\frac{\Omega_{\rm m,0}}{a^3} + \square & a\frac{d}{da} + \left(2+\frac{\dot{H}}{H^2}\right) + \square \end{pmatrix}$$

modification is easy

scale factor as time variable

Kernel reconstruction approach

Standard PT kernels as building blocks for various PT predictions

Recipes

1. Solve these equations with initial conditions at a_i <<1:

$$F_1 = a_i, \quad G_1 = -a_i, \quad \textit{otherwise zero}$$

2. Symmetrized : $$F_n^{(\mathrm{sym})}(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) = \frac{1}{n!} \sum_{\{n\}} \{F_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) + \mathrm{perm}\}$$

3. Store the output in multi-dim arrays

special technique is unnecessary
it can be parallelized

For power spectrum at 1-loop order,
what we need is just the 3D arrays of kernels up to 3rd order
(typical size ~100x100x10)

kernels up to 3rd order → application to → resmmed PT and/or RSD calculations

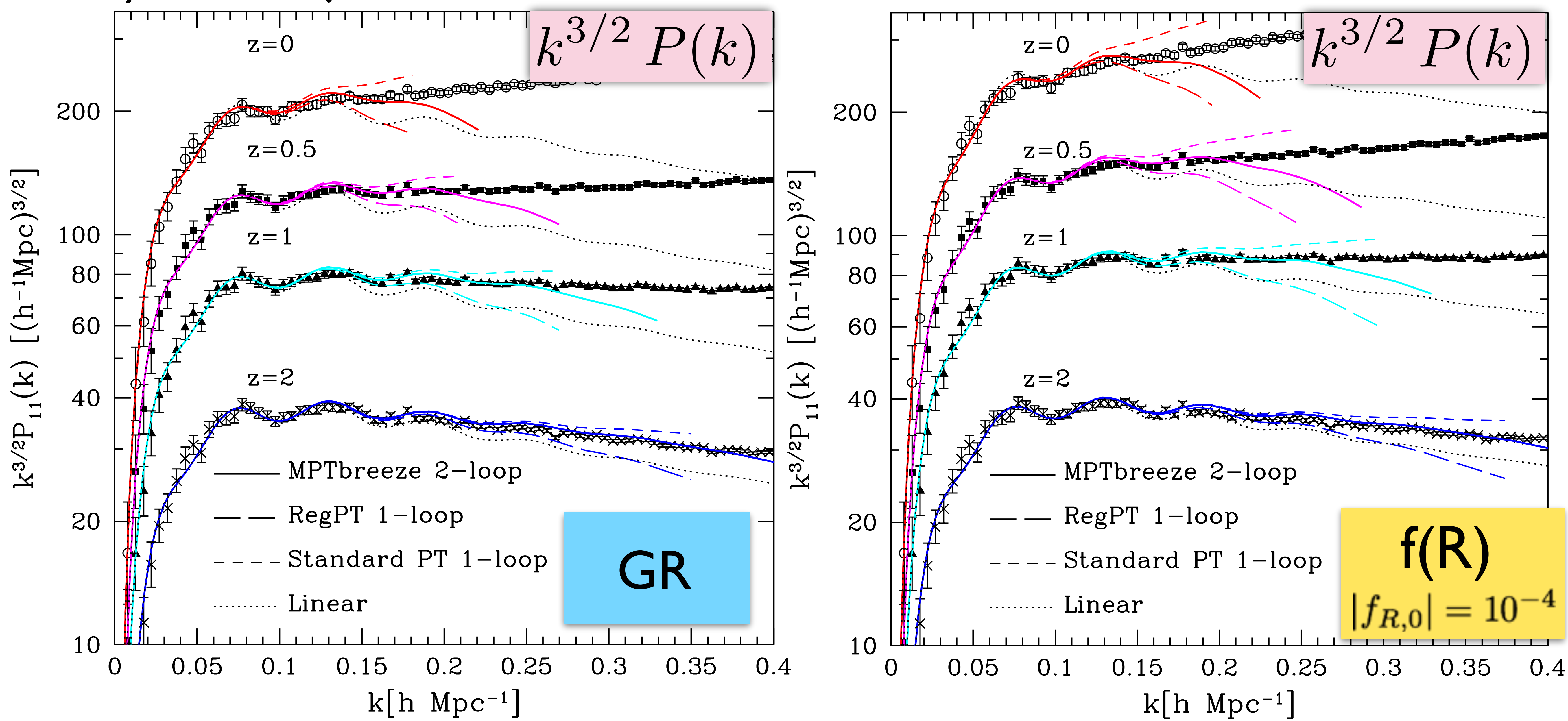
Application: f(R) gravity

All predictions are made from standard PT
kernels up to 3rd order (i.e., F_2, F_3)

$$f(R) \simeq -16\pi G \rho_\Lambda + |f_{R,0}| \frac{R_0^2}{R}$$

$|f_{R,0}| = 10^{-4}$

N-body data: Baojiu Li



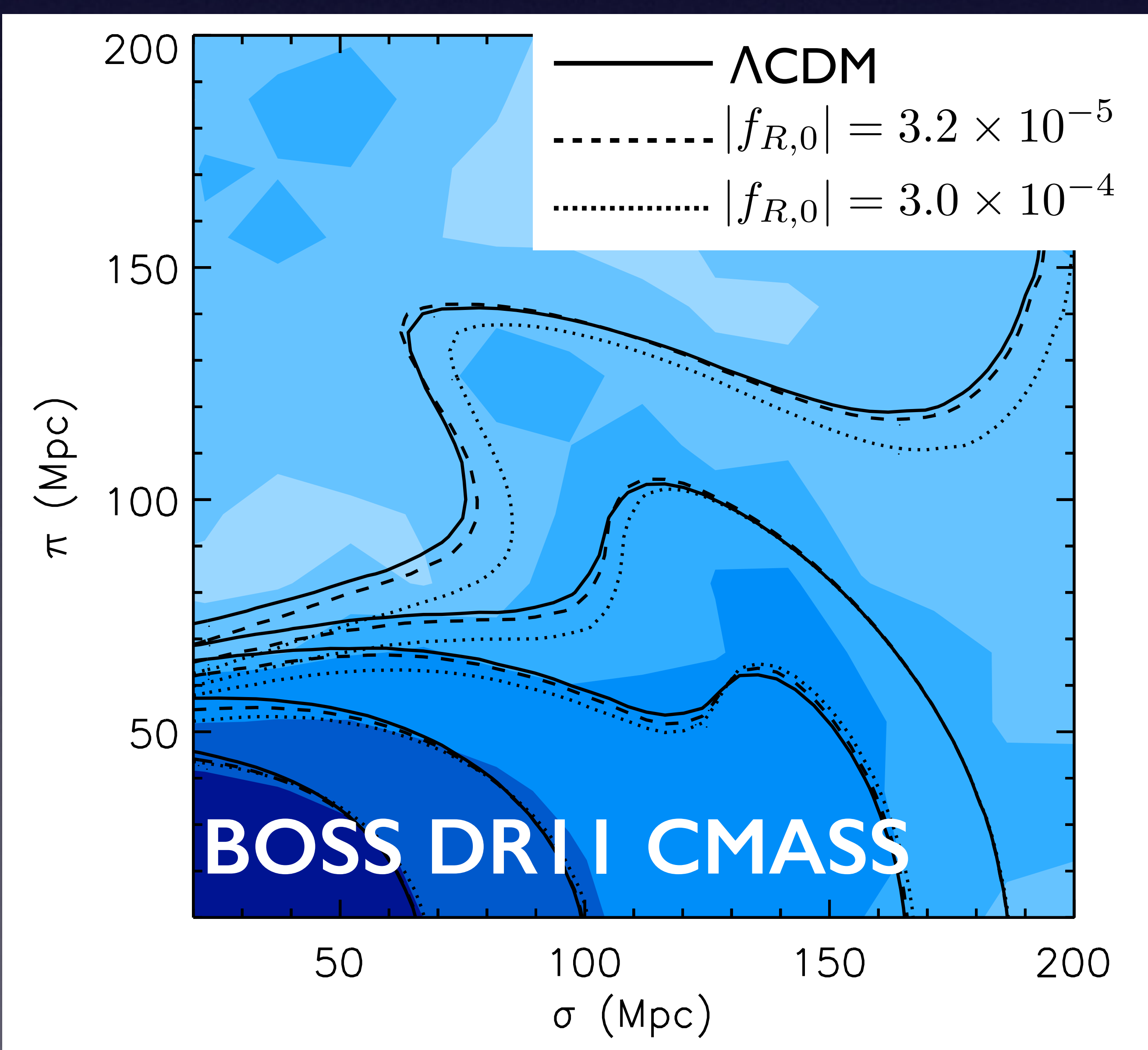
Consistent modified gravity analysis

Y-S.Song, AT, Linder, Koyama et al.

arXiv:1507.01592

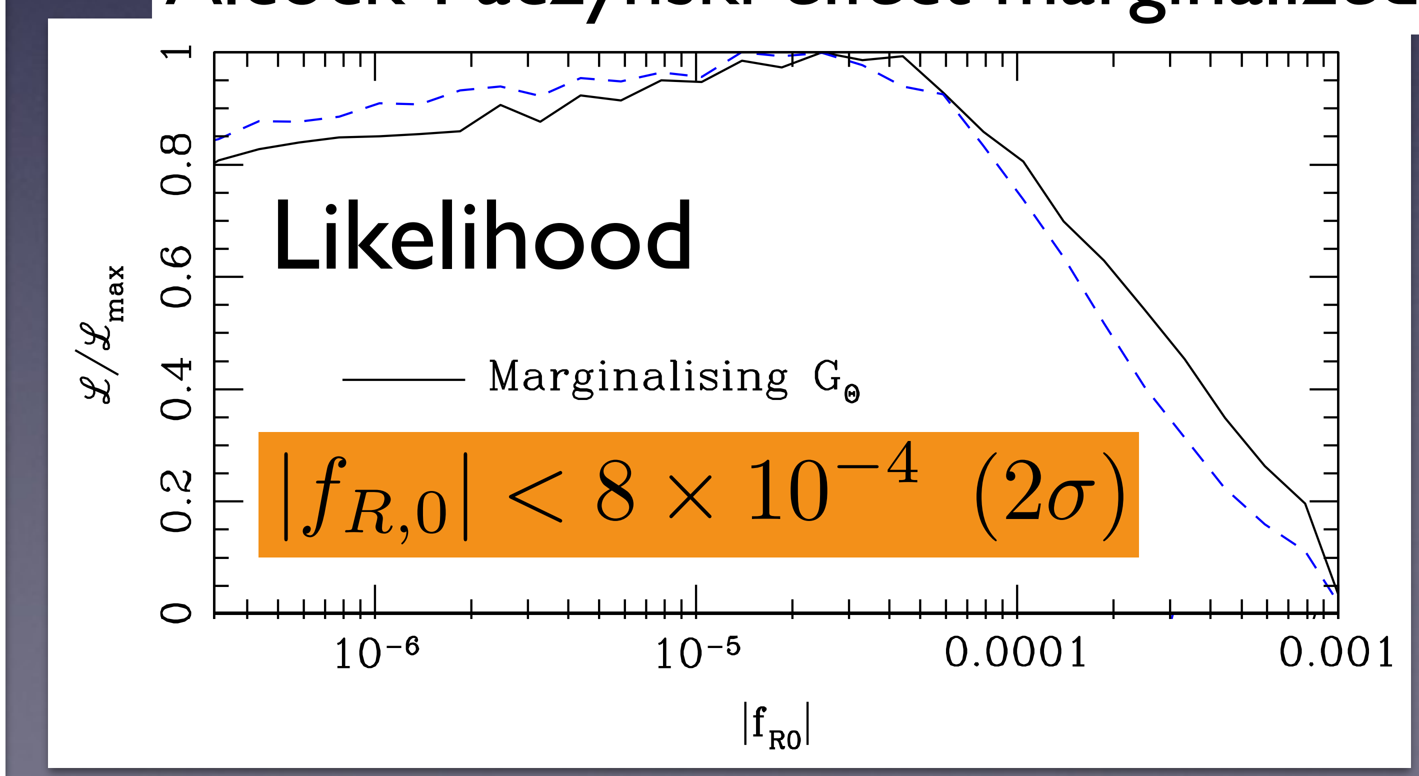
Combining TNS model of RSD,

anisotropic correlation function is consistently computed in f(R) gravity → BOSS DR11 CMASS



$$f(R) \simeq -16\pi\, G\, \rho_\Lambda + |f_{R,0}|\, \frac{R_0^2}{R}$$

Alcock-Paczynski effect marginalized



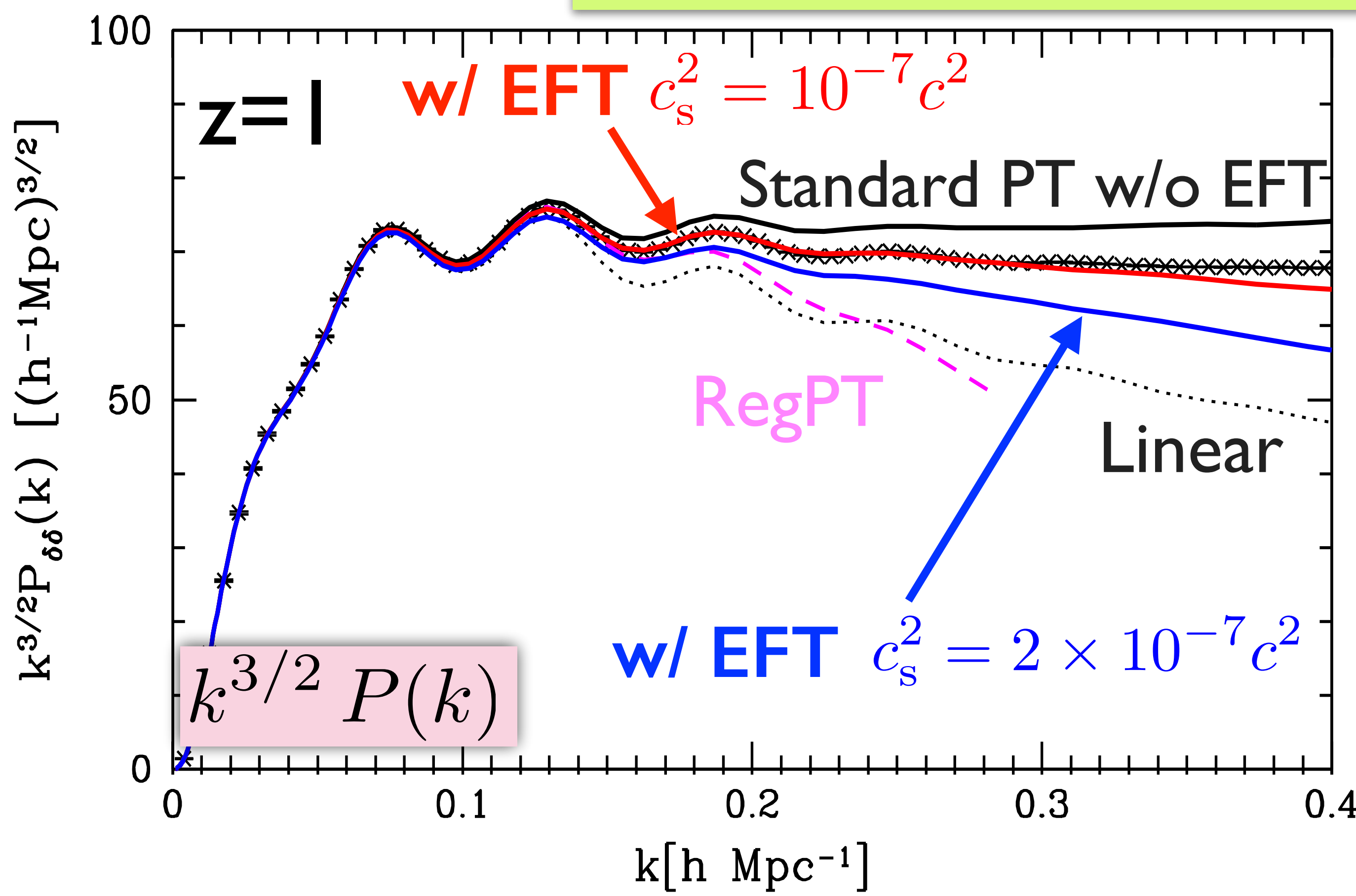
Application: effective-field theory (EFT)

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}] = 0,$$
$$\frac{\partial \boldsymbol{v}}{\partial t} + H\,\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\cdot\boldsymbol{v} = -\frac{1}{a}\nabla\psi - \frac{1}{\rho_{\rm m}}\frac{1}{a}\nabla\tau_{ij}$$
$$\frac{1}{a^2}\nabla^2\psi = \frac{\kappa^2}{2}\,\rho_{\rm m}\,\delta$$

Baumann et al. ('12), Carrasco, Herzberg & Senatore ('12), Carrasco et al. ('13ab), Porto, Senatore & Zaldarriaga ('14), …

*Leading-order **EFT** corrections* e.g., Herzberg ('14)

$$\tau_{ij} = \rho_{\rm m}\left[\left(c_{\rm s}^2\,\delta - \frac{c_{\rm bv}^2}{aH}\,\nabla\cdot\boldsymbol{v}\right)\delta_{ij} - \frac{3}{4}\frac{c_{\rm sv}^2}{aH}\left\{\partial_j v_i + \partial_i v_j - \frac{2}{3}(\nabla\cdot\boldsymbol{v})\delta_{ij}\right\}\right]$$



At 1-loop order,
corrections are approximately described by single-parameter:

$$c_{\rm s}^2 + f\,(c_{\rm bv}^2 + c_{\rm sv}^2)$$

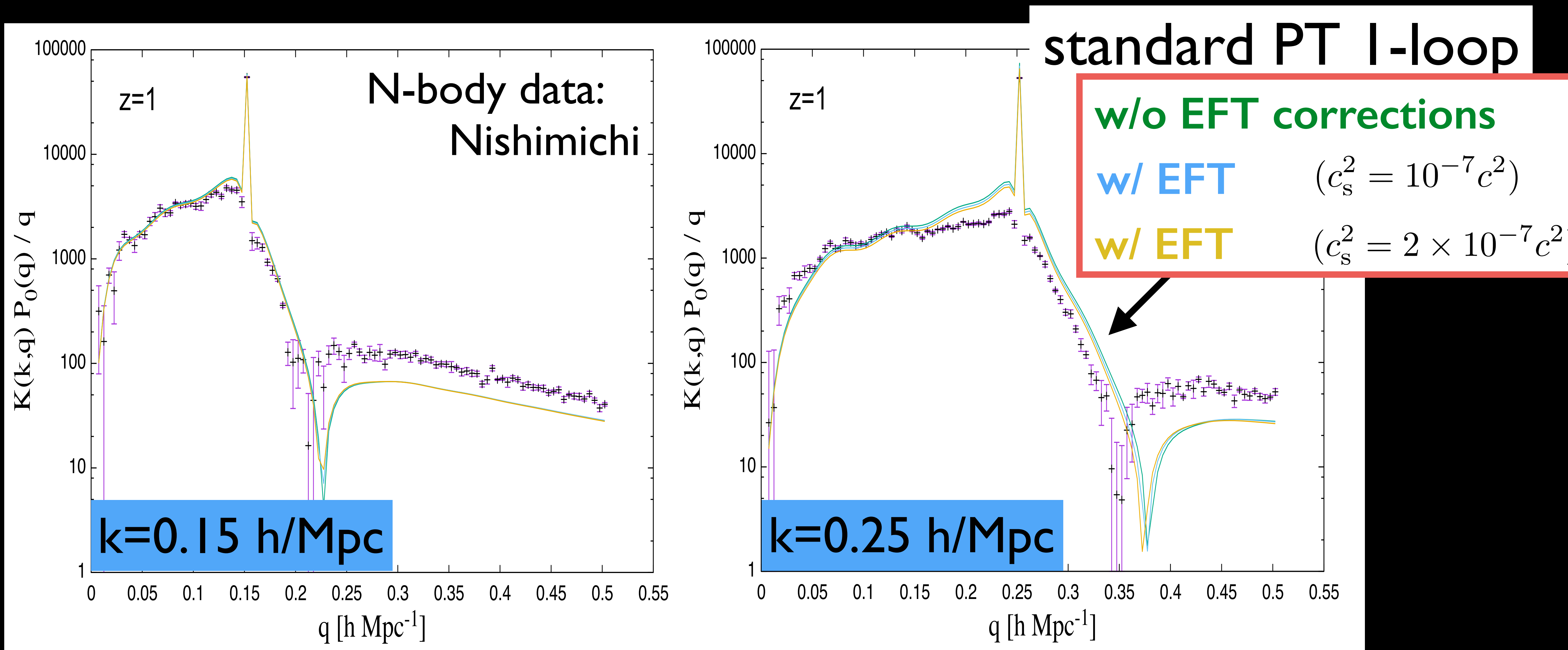
Allowing c_s to be free, EFT 1-loop reproduce N-body results well, but

Application: effective-field theory

Response function of P(k): $\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$

nonlinear — linear (initial)

Nishimichi, Bernardeau & AT
arXiv:1411.2970



At 1-loop, PT predictions with EFT do not so much differ from the one w/o EFT, which does not perfectly match simulations

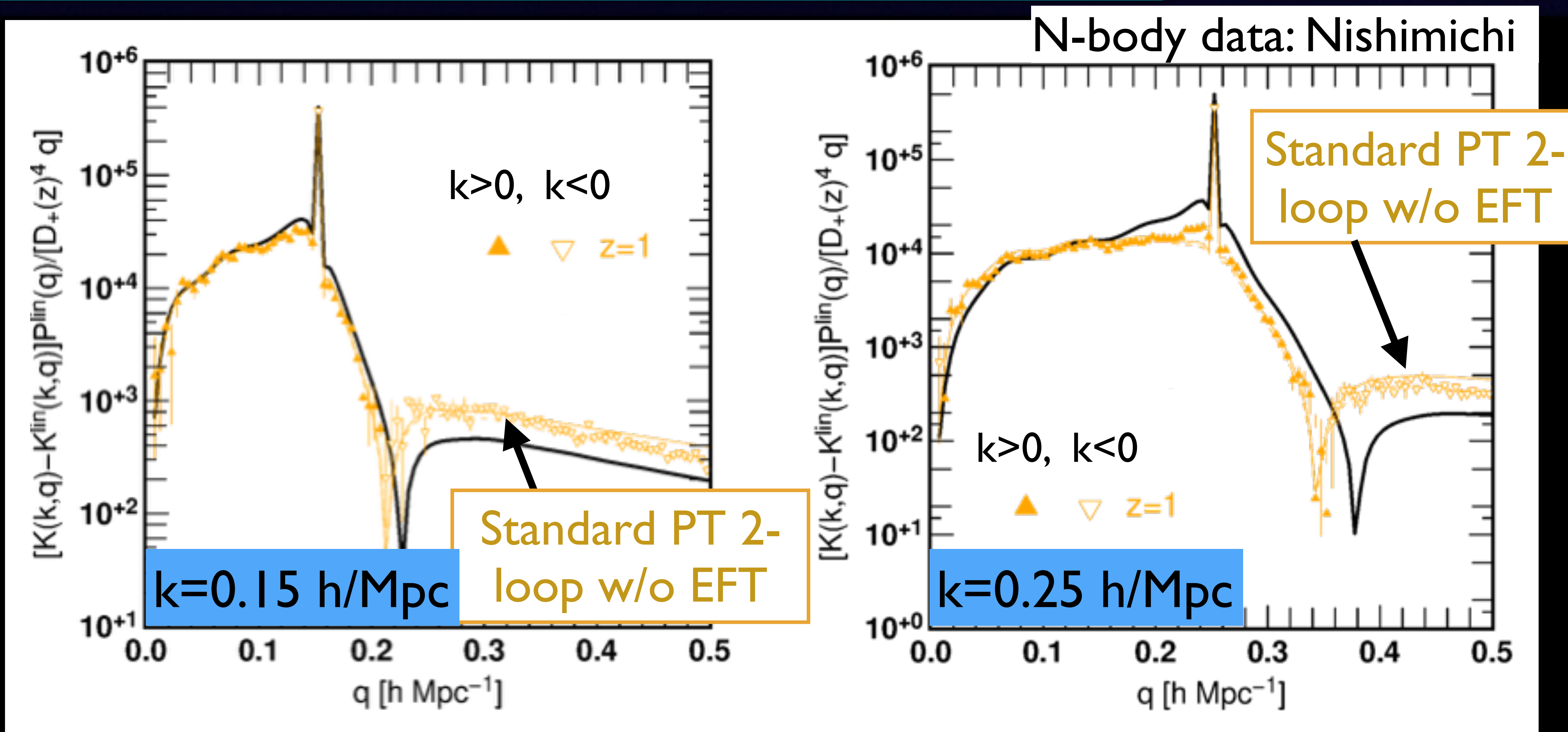
Application: effective-field theory

nonlinear *linear (initial)*

Response function of P(k)

$$\delta P_{\rm nl}(k) = \int d\ln q\, K(k,q)\, \delta P_0(q)$$

Nishimichi, Bernardeau & AT
arXiv:1411.2970



Simply adding standard PT 2-loop w/o EFT apparently looks better (although it starts to fail at k>0.4 h/Mpc)

Summary

A numerical method for PT calculation of LSS, even applicable to analytically intractable models of structure formation

Solving numerically the evolution eps. for PT kernels up to 3rd order (F_2, F_3, G_2, G_3)

⟶ (resummed) power spectrum in real & redshift spaces

Application

✓ f(R) gravity : consistent modified gravity analysis using BOSS DR 11

$|f_{R,0}| < 8 \times 10^{-4} \quad (2\sigma)$

✓ Effective-field theory :

full-numerical treatment of power spectrum & response function at 1-loop order

Calculation should be accelerated with parallel computation, and it can be applied to a practical parameter estimation study