Non-linear structure formation in modified gravity models

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Assuming GR

\[ R = \frac{GM}{r^3 c^2} \]

\[ \Phi = \frac{GM}{rc^2} \]
Gravitational potential

Curvature $\frac{GM}{r^2}$ (cm$^{-2}$)

-20
-25
-30
-35
-40
-45
-50
-55
-60

1 AU
1 $R_{\text{sun}}$
10 kpc
10 Mpc

General Relativity
Screening mechanism
Dark Matter
Modified Gravity

Cosmological tests of gravity
General picture

- Largest scales
  Gravity is modified so that the universe accelerates without dark energy

- Large scale structure scales
  Gravity is still modified by a fifth force from scalar graviton
  *model independent tests of GR*

- Small scales (solar system)
  GR is recovered

KK: arXiv:1504.04623
How to recover GR on small scales?

On non-linear scales, the fifth force must be screened by some mechanisms

Joyce, Jain, Khoury & Trodden arXiv:1407.0059

  
The mass of a scalar mode becomes large in dense regions
  \[
  \nabla^2 \phi = \partial_\phi V + \frac{\alpha}{M_{pl}} \rho e^{\alpha \phi / M_{pl}}
  \]

- **Vainshtein mechanism**
  
  Non-linear derivative self-interactions become large in a dense region
  \[
  3 \nabla^2 \phi + r_c^2 \left\{ \left( \nabla^2 \phi \right)^2 - \partial_i \partial_j \phi \partial^i \partial^j \phi \right\} = 8\pi G a^2 \rho
  \]
Behaviour of gravity

There regimes of gravity

In most models, the scalar mode obeys non-linear equations describing the transition from the scalar tensor theory on large scales to GR on small scales

\[
\rho_{\text{crit}} \approx 10^{-29} \, \text{g} / \text{cm}^3, \\
\rho_{\text{galaxy}} \approx 10^{-24} \, \text{g} / \text{cm}^3, \\
\rho_{\text{solar}} \approx 10 \, \text{g} / \text{cm}^3
\]

Understandings of non-linear clustering require N-body simulations where the non-linear scalar equation needs to be solved
N-body simulations

- **GR**
  - superposition of forces
  \[ \vec{F}(r) = \sum_{i} m_i \vec{f}_i(r - r_i) \]

- Modified gravity models
  - the non-linear nature of the scalar field equation implies that the superposition rule does not hold

  - It is required to solve the non-linear scalar equation directly on a mesh
    - a computational challenge!
  - The breakdown of the superposition rule has interesting consequences
N-body Simulations (Puchwein’s talk)

- Multi-level adaptive mesh refinement
- solve Poisson equation using a linear Gauss-Seidel relaxation
- add a scalar field solver using a non-linear Gauss Seidel relaxation

ECOSMOG  Li, Zhao, Teyssier, KK  JCAP1201 (2012) 051
MG-GADGET  Puchwein, Baldi, Springel  MNRAS (2013) 436 348
DGPM, Schmidt  PRD80, 043001

Modified Gravity Simulations comparison project
Winther, Shcmidt, Barreira et.al. arXiv: 1506.06384
Redshift space distortions

- Power spectrum in redshift space is anisotropic
  
  \[ P(k, \mu), \quad \mu = \frac{k_\parallel}{k} \]

- Multipole decomposition

\[
P(k, \mu) = \sum_{\ell} P_{\ell}(k)L_{\ell}(\mu)
\]

\[
\left| \frac{P_2}{P_0} \right|_{\text{linear}} = \left[ \sum_{\ell} \frac{4}{3} f + \frac{4}{7} f^2 \right]
\]

\[
f = \sqrt{\frac{P_{\theta\theta}}{P_{\delta\delta}}} \quad \theta = \nabla \cdot \nu
\]

Modelling of non-linear effects is crucial to extract the differences in the linear growth rate between GR and modified gravity models

Jennings, Baugh, Li, Zhao, Koyama, 1205.2698
Perturbation theory

- Peturbation theory based template (Taruya’s talk)

\[ \tilde{P}(k, \mu) = \left\{ b^2 P_\delta(k) + 2\mu^2 b G_\Theta P_\delta(k) + \mu^4 G_\Theta^2 P_{\Theta\Theta}(k) + A(k, \mu; b, G_\Theta) + B(k, \mu; b, G_\Theta) \right\} \times D_{FOG}(k\mu\sigma_p) \]

- Scale dependent growth

BOSS DR11 constraints \( |f_{R0}| < 8 \times 10^{-4} \)

Song, Taruya, Linder, KK et.al. 1507.01592
How do we test gravity in cosmology?

- **Newton potential** \( \Psi \)
  
  controls dynamics of non relativistic particles

- **Space curvature** \( \Phi \)
  
  also deflects lights

In GR there is a special relation between the two \( \Psi = \Phi \)

\[ \text{dynamical mass} = \text{lensing mass} \]

\[ \text{in GR} \]
Where to test GR  (we consider the chameleon mechanism here)

- GR is recovered in high dense regions
  - GR is restored in massive dark matter halos

- Environmental effects
  Even if dark matter halo itself is small, if it happens to live near massive halos, GR is recovered

*Using simulations, we can develop criteria to identify the places where GR is not recovered*  
Zhao, Li, KK Phy. Rev. Lett. 107 (2011) 071303
Environmental dependence

- Difference between lensing and dynamical mass

\[ \Delta M(r) = \frac{d\Psi(r)}{dr} \left( \frac{d\Psi(r)}{dr} \right) - 1, \quad \Psi_+ = \frac{\Phi + \Psi}{2} \]

environment: \( D = d / r_{NB} \)

Zhao, Li, KK Phy. Rev. Lett. 107 (2011) 071303
Testing chameleon gravity

Outskirt of clusters

- Dynamical mass can be inferred from X-ray and SZ
- \( M_{lens} = M_{dyn} - M_\phi \)

48 X-ray clusters from XCS compared with lensing (shear) from CFHTLS

\[ | f_{R0} | < 7.9 \times 10^{-5} \]

Wilcock et al. MNRAS (1504.03937)
Creating a screening map

- It is essential to find places where GR is not recovered
  - Small galaxies in underdense regions
  - SDSS galaxies within 200 Mpc

Cabre, Vikram, Zhao, Jain, KK JCAP 1207 (2012) 034

(we consider the chameleon mechanism here)
Tests of gravity on small scales

- dwarf galaxies in voids
  - strong modified gravity effects
    - Galaxies are brighter
    - Pulsers pulsate faster
    - Various other tests

Jain & VanderPlas JCAP 1110 (2011) 032

Vikram et.al. JCAP1308 (2013) 020

\[ |f_{R0}| < 5 \times 10^{-7} \]

(we consider the chameleon mechanism here)
Constraints on chameleon gravity

- Non-linear regime is powerful for constraining chameleon gravity
- Astrophysical tests could give better constraints than the solar system tests and can be done by “piggybacking” ongoing surveys

Interaction range of the extra force that modifies GR in the cosmological background

Lombriser et.al. PRD85 102001
Jain et.al. 1309.5389
Vainshtein mechanism  (Falck’s talk)

- Vainshtein mechanism is very efficient
  - dark matter halos are all screened regardless of mass and environment  
    Schmidt PRD81 103002
  - linear/quasi non-linear scales are the best place to test the models
- Screening depends on dimensionality of the system
  - lower dimensional objects are less screened  
    Falck et.al. JCAP1407 058 1503.06673
- Voids  (Barreira, Cai’s talk)
  - Voids are unscreened by definition  
    Falck, Cautun, Zhao, KK in preparation
In the next decade, we may be able to detect the failure of GR on cosmological scales

- **Linear scales**
  model independent tests of gravity
  ![SDSS III](image1)
  ![DARK ENERGY SURVEY](image2)
  ![SUBARU](image3)

- **Non-linear scales**
  novel astrophysical tests of gravity
  (in a model dependent way)
  ![SUMIRE](image4)
  ![EUCLID](image5)

*It is required to develop theoretical models from fundamental theory*