Clustering fossils
chasing inhomogeneities
to exploit 2PCF

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Theoretical and Observational Progress on Large-scale structure of the Universe
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• Two-point correlation function $\xi(r) = $ excess number of pairs beyond random at separation $r$
  \[
  \xi(r) = \langle \delta(x)\delta(x + r) \rangle
  \]
  statistical homogeneity (translational invariance)
  where $\delta(x)$ is the density contrast, excess number density beyond mean:
  \[
  \delta(x) = \text{density}(x)/(\text{mean density}) - 1
  \]

• Power spectrum is the Fourier transform of it:
  \[
  \langle \delta(k)\delta(k') \rangle = (2\pi)^3 P(k)\delta^D(k + k')
  \]
Parameterizing inhomogeneity

- Deviation from statistical homogeneity in the two-point functions will be evident in the off-diagonal correlation:
  \[ \langle \delta(k_1)\delta(k_2) \rangle |_{k_1+k_2 \neq 0} \neq 0 \]

- Q: How does the inhomogeneity appear?
  - A way to organize the off-diagonal correlations: \( K = -(k_1 + k_2) \)
    \[ \langle \delta(k_1)\delta(k_2) \rangle = VP(k_1)\delta^D_{k_1+k_2} + \sum_K f(k_1, k_2, K)\delta^D_{k_1+k_2+K} \]
  - The pattern of inhomogeneity is encoded in the function \( f \)!
cf. parameterizing anisotropy

- This is analogous to the BiPoSH (bipolar spherical harmonic) expansion to characterize the statistical anisotropy:

\[ \langle a_{\ell m} a_{\ell' m'} \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'} + \sum_{J M} (-1)^{m'} \langle \ell, m; \ell', -m' | J, M \rangle A^{JM}_{\ell \ell'} \]

Example:
If we were to move with \( \beta \sim 1 \) w.r.t. CMB rest frame, CMB would be statistically anisotropic (\( J=1, M=0 \)) with \( A^{10}_{\ell \ell'} > 0 \).
What makes $\xi(r)$ inhomogeneous?

- Unknown systematics of the survey
  - If something varies over the survey volume and that something modulates the amplitude of clustering
- Our Universe might be intrinsically inhomogeneous
  - No compelling evidence so far, therefore, must be small!
- higher-order correlation functions
  - Non-linearities (e.g. position dependent power spectrum)
- Primordial three-point function $\rightarrow$ clustering fossil
Non-Gaussianity and homogeneity

- **IF** we have a non-linear coupling between primordial density fluctuations and a spectator field $h_p$ (JK coupling):
  \[
  \langle \delta_i(k_1)\delta_i(k_2)h_p(K) \rangle = VP_p(K)f_p(k_1, k_2)\varepsilon_{ij}^p k_{1i} k_{2j}^* \delta_{k_1+k_2+K}^D
  \]
  power spectrum of new field
  coupling amplitude
  polarization basis (scalar, vector, tensor, ...)

- **THEN**, density power spectrum we observe now has non-zero off-diagonal components: **Fossil equation**
  \[
  \langle \delta_i(k_1)\delta_i(k_2) \rangle |_{h_p(K)} = VP_i(k_1)\delta_{k_1+k_2}^D + h^*_p(K)f_p(k_1, k_2)\varepsilon_{ij}^p k_{1i} k_{2j}^* \delta_{k_1+k_2+K}^D
  \]
Why called clustering fossils?

- Inflaton(s): a scalar field(s) responsible for inflation
- But, inflaton might not be alone. Many inflationary models need/introduce additional fields. But, direct detection of such fields turns out to be very hard:
  - Additional Scalar: may not contribute seed fluctuations
  - Vector: decays as $1/\text{[scale factor]}$
  - Tensor: decays after coming inside of comoving horizon
- Clustering fossils may be the only way of detecting them!
SVT can be distinguished with $\varepsilon^{p}_{ij}$

- In a symmetric 3x3 tensor, we have 6 degrees of freedom, which are further decomposed by Scalar, Vector and Tensor polarization modes.

- They are orthogonal: $\varepsilon^{p}_{ij} \varepsilon^{p'}_{,ij} = 2\delta_{pp'}$

- Scalar (p=0, z): $\varepsilon^{0}_{ij} \propto \delta_{ij} \quad \varepsilon^{z}_{ij}(K) \propto K_i K_j - K^2 / 3$

- Vector (p=x,y): $\varepsilon^{x,y}_{ij}(K) \propto \frac{1}{2} (K_i e_j + K_j e_i)$ where $K_i e_i = 0$

- Tensor [Gravitational Waves] (p=x, +): transverse and traceless

  $K_i \varepsilon^{+,x}_{ij}(K) = 0 \quad \delta_{ij} \varepsilon^{+,x}_{ij}(K) = 0$
Effect of fossils on 2PCF

\[ \langle \delta_i(k_1)\delta_i(k_2) \rangle |_{h_p(K)} = VP_i(k_1)\delta^D_{k_1+k_2} + h_p^*(K)f_p(k_1, k_2)\varepsilon_{ij}k^i_1k^j_2\delta^D_{k_1+k_2+K} \]

- Statistical homogeneity is broken in the presence of the spectator field \( h_p(K) \).

- Depending on the polarization, the way that the spectator affects clustering is different. How?

- I will show a rotation view of equi-correlation-function surface when \( h_p(K) \) propagates upward.

- Without \( h_p(K) \), we expect that it should be spherical.
\( \xi(\mathbf{r}) \) with single scalar mode \((p=0,z)\)

Scalar mode propagation

\( h_0 \) at \( z=0 \)

\( h_z \) at \( z=0 \)

Jeong & Kamionkowski (2012)
\( \xi(r) \) with single vector mode \((p=x,y)\)

Jeong & Kamionkowski (2012)
$\xi(r)$ with single tensor mode ($p=+,x$)

Jeong & Kamionkowski (2012)
Example: tensor clustering fossils

• For the single-field slow-roll inflation models \( k_t = k_1 + k_2 + k_3 \),
  Maldacena (2003)

\[
B_{\zeta \zeta p}(k_1, k_2, k_3) = \frac{1}{2} \left[ \frac{P_{\zeta}(k_1)}{k_2^3} + \frac{P_{\zeta}(k_2)}{k_3^3} \right] P_{h_p}(k_3) \varepsilon_{ij} k_1^i k_2^j \left[ -k_t + \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{k_t} + \frac{k_1 k_2 k_3}{k_t^2} \right]
\]

\[\text{squeeze limit (} k_1 \approx k_2 \gg k_3 \text{)}\]

\[
\left( \frac{4 - n_s}{2} \right) P_{\zeta}(k_1) P_{h_p}(k_3) \frac{\varepsilon_{ij} k_1^i k_1^j}{k_1^2} \equiv -\frac{1}{2} \frac{d \ln P_{\zeta}(k)}{d \ln k} P_{\zeta}(k_1) P_{h_p}(k_3) \varepsilon_{ij} \hat{k}_1^i \hat{k}_1^j
\]

• In the squeeze limit, long-wavelength tensor field rescales small scale wave-vector: \( k^2 \rightarrow k^2 - h_{ij} k_i k_j \) (or length \( x^2 \rightarrow x^2 + h_{ij} x_i x_j \))!

• Note: the local observer (use physical ruler, not the coordinate ruler) will not see the effect!
Interaction @ horizon crossing

- After inflation, tensor (long) modes re-enters horizon, and interact with density (small) modes:

\[
\delta_{\text{int.}}(k) = -2S(K)h_p(K)\varepsilon_{ij}^p(\mathbf{k})\mathbf{k}_i\mathbf{k}_jT(k)\zeta_p(k)
\]

- Note that the influence dies out as tensor mode itself decays after horizon re-entry.

\[
S(K) \simeq \frac{3}{5} \left[1 - \exp \left(-\frac{5}{42}K^2\eta^2\right)\right]
\]
Light deflection due to GW

- Deflection of photon changes the observed location of galaxies.
- Geodesic equation gives $\Delta x^\mu$
- On large scales ($K \to 0$), the displacement field is
  $$\Delta t \to 0$$
  $$\Delta x^i \to -\frac{1}{2} h_{ij}^0 x_j$$
- This effect cancels the super-horizon contributions!

Mathematical expressions:

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

Equation references:

Jeong & Schmidt (2012)
Observable fossil amplitude

\[ \langle \delta_g(k_1)\delta_g(k_2) \rangle \simeq P_g(k_1)\delta^D_{k_1+k_2} + \left[ \frac{1}{2} (1 - T_\gamma) \frac{d \ln P_\delta(k_1)}{d \ln k_1} + 2 S(K) \right] P_g(k_1) h_p(K) \varepsilon_{ij} k_1^i k_2^j \delta^D_{k_1+k_2+K} \]

- Quadrupole power spectrum contribution (when \( K \ll k_F \)) from single-field slow-roll inflation
- **large-scale** (super-horizon) fossils cancel *completely* with projection
- **small-scale** fossil cancels *partially* with tensor-scalar interaction around horizon crossing
Fossils from other inflation models

- The large-scale cancelation happens only for the SFSR models
- With scalar-scalar-tensor correlation different from SFSR
- Power quadrupole can constrain $k_{\text{min}}$ (beginning of inflation)
- Clustering fossil signal can be big!

- E.g.
  - Solid inflation
    Dimastrogiovanni, Fasiello, Jeong & Kamionkowski (2014)
  - Quasi-single field inflation
    Dimastrogiovanni, Fasiello & Kamionkowski (2015)

\[
B_{\phi c} = \frac{3}{2} \frac{\mathcal{R}}{\epsilon} P_{\zeta}(k) P_h(K)
\]

\[
B_{\phi c} = -\frac{\pi^2}{2} w(\nu) \frac{\dot{\theta}_0^2}{H^2} P_{\zeta}(k) P_h(K)
\]
Let's start from Fossil equation

\[ \langle \delta(k_1)\delta(k_2) \rangle |_{h_p(K)} = h_p(k_1 + k_2) f_p(k_1, k_2) \epsilon_{ij}^p k_1^i k_2^j \delta^D_{k_1+k_2+K} \]

Rearranging it a bit, we get a naive estimator for the new field, which is far from optimal:

\[ \hat{h_p(K)} = \sum_{k_1+k_2=K} \frac{\delta(k_1)\delta(k_2)}{f_p(k_1, k_2) \epsilon_{ij}^p k_1^i k_2^j} \]

Azimuthal(\(\varphi\))-dependence, [cos(s\(\varphi\))] s=spin, distinguishes scalar from vector from tensor geometrically!
Optimal estimator (single mode)

- **Inverse-variance weighting** gives an optimal estimator for a single mode

\[
\hat{h_p}(\mathbf{K}) = P^{n}_p(\mathbf{K}) \sum_k \frac{f^*_p(\mathbf{k}, \mathbf{K} - \mathbf{k}) \epsilon_{ij}^p k^i (K - k)^j}{2V P^{\text{tot}}(k) P^{\text{tot}}(|\mathbf{K} - \mathbf{k}|)} \delta(\mathbf{k}) \delta(\mathbf{K} - \mathbf{k})
\]

- With a noise power spectrum \((P_{\text{tot}} = P_{\text{galaxy}} + P_{\text{noise}})\)

\[
P^{n}_p(K) = \left[ \sum_k \frac{|f_p(\mathbf{k}, \mathbf{K} - \mathbf{k}) \epsilon_{ij}^p k^i (K - k)^j|^2}{2V P^{\text{tot}}(k) P^{\text{tot}}(|\mathbf{K} - \mathbf{k}|)} \right]^{-1}
\]
Optimal estimator for the power amplitude $A_h$

- For a stochastic background of new fields with power spectrum $P_p(K) = A_h P_h^f(K)$, we optimally summed over different $K$-modes to estimate the amplitude by (w/ NULL hypothesis):

$$\hat{A}_h = \sigma_h^{-2} \sum_{K,p} \left[ P_h^f(K) \right]^2 \left( \frac{\left| h_p(K) \right|^2}{V} - P_p^p(K) \right)$$

- Here, the minimum uncertainty of measuring amplitude is

$$\sigma_h^{-2} = \sum_{K,p} \left[ P_h^f(K) \right]^2 / 2 \left[ P_p^p(K) \right]^2$$
Order-of-magnitude calculation

- For the SFSR inflation models (Maldacena, 2003)

- projected 3-sigma (99% C.L.) detection limit with galaxy survey parameters

- To detect the gravitational wave, we need a large dynamical range

- Current and future survey should set a limit on primordial V and T (and higher-spin fields)!
Conclusion

• Off-diagonal correlators are the place to look at the signature for the spatial inhomogeneity.

• “Clustering fossil” is a way to look at primordial spectator fields that existed during the early time
  • requires large dynamical range to beat the small signal (e.g. 21cm). We can distinguish scalar/vector/tensor fossils.

• Also, interesting potential to probe higher spin field.

• *We already have data, shall we dig for clustering fossils?*

• Systematics: survey systematics, non-linearities, non-Gaussianities