Relativistic effects in large-scale structure surveys

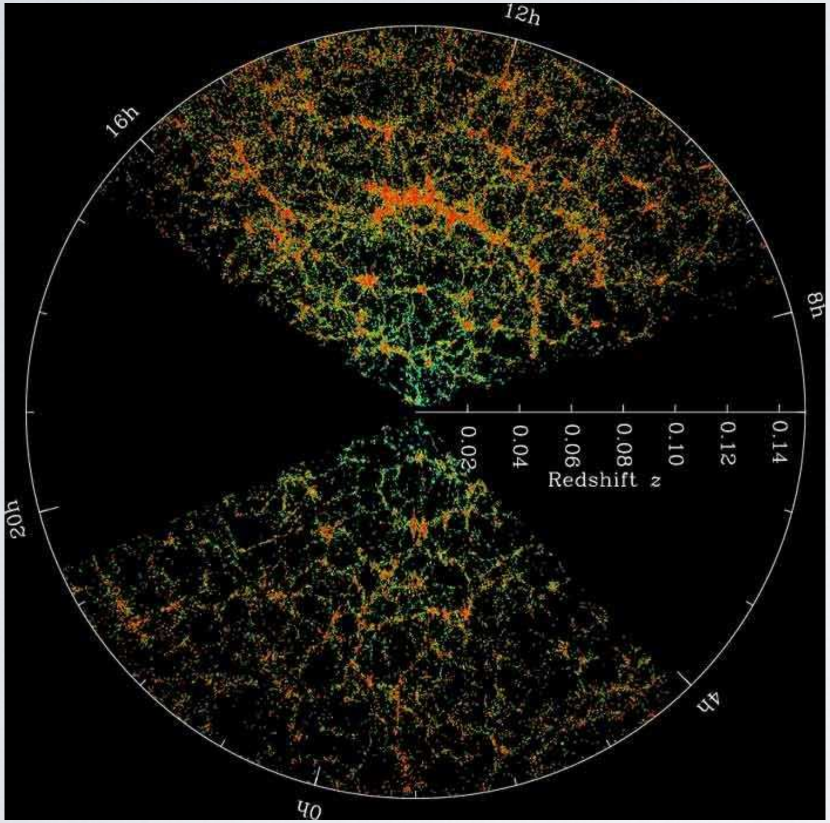
Camille Bonvin
CERN, Switzerland

Theoretical and Observational Progress on Large-scale Structure of the Universe
ESO, July 2015

Galaxy surveys

Galaxies are not randomly distributed in our sky.

Credit: M. Blanton, SDSS



To exploit the **information** present in the large-scale structure we measure fluctuations in the **number counts** of galaxies:

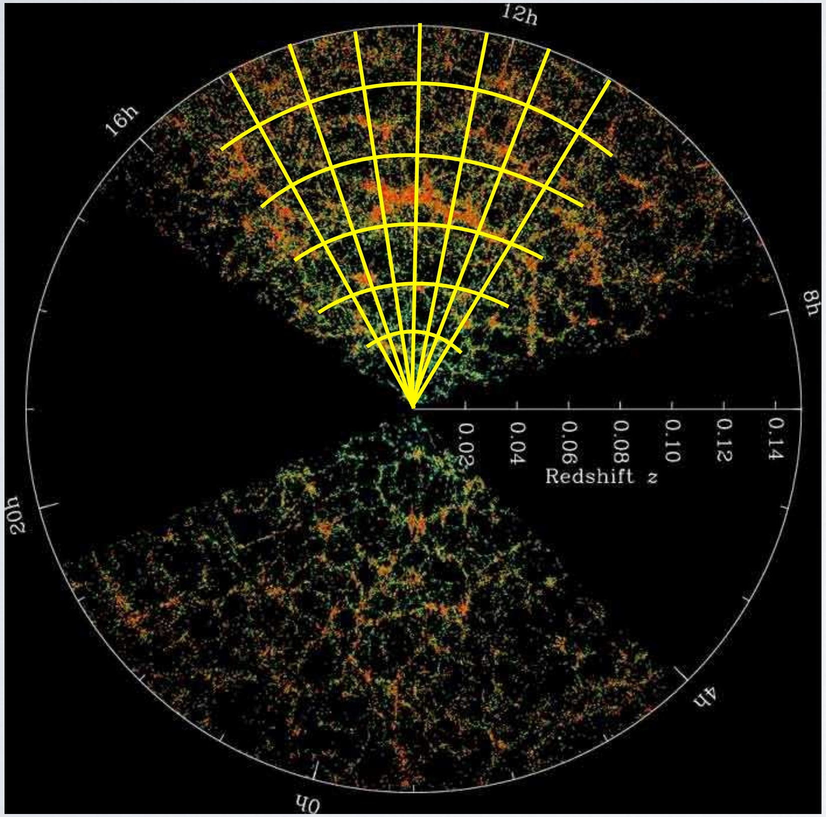
$$\Delta = \frac{N - \bar{N}}{\bar{N}}$$

Why does Δ fluctuate over the sky?

Galaxy surveys

Galaxies are not randomly distributed in our sky.

Credit: M. Blanton, SDSS



To exploit the **information** present in the large-scale structure we measure fluctuations in the **number counts** of galaxies:

$$\Delta = \frac{N - \bar{N}}{\bar{N}}$$

Why does Δ fluctuate over the sky?

Fluctuations in the galaxy number counts

First approximation: galaxies are a **tracer** of the dark matter

$$\Delta = \frac{\delta\rho}{\bar{\rho}} \equiv \delta$$

Three well-known sources of distortions:

- **Bias**: the distribution of galaxies is a biased tracer.
- We observe in redshift space: the redshift is affected by galaxies' velocity → **redshift-space distortions**. Kaiser 1987
- **Magnification bias**: gravitational lensing changes the solid angle and the threshold of observation. Broadhurst, Taylor and Peacock 1995

These distortions have already been measured.

Fluctuations in the galaxy number counts

First approximation: galaxies are a **tracer** of the dark matter

$$\Delta = b \cdot \delta$$

Three well-known sources of distortions:

- **Bias**: the distribution of galaxies is a biased tracer.
- We observe in redshift space: the redshift is affected by galaxies' velocity → **redshift-space distortions**. Kaiser 1987
- **Magnification bias**: gravitational lensing changes the solid angle and the threshold of observation. Broadhurst, Taylor and Peacock 1995

These distortions have already been measured.

Fluctuations in the galaxy number counts

First approximation: galaxies are a **tracer** of the dark matter

$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

Three well-known sources of distortions:

- **Bias**: the distribution of galaxies is a biased tracer.
- We observe in redshift space: the redshift is affected by galaxies' velocity → **redshift-space distortions**. Kaiser 1987
- **Magnification bias**: gravitational lensing changes the solid angle and the threshold of observation. Broadhurst, Taylor and Peacock 1995

These distortions have already been measured.

Fluctuations in the galaxy number counts

First approximation: galaxies are a **tracer** of the dark matter

$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) + (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$

Three well-known sources of distortions:

- **Bias**: the distribution of galaxies is a biased tracer.
- We observe in redshift space: the redshift is affected by galaxies' velocity → **redshift-space distortions**. Kaiser 1987
- **Magnification bias**: gravitational lensing changes the solid angle and the threshold of observation. Broadhurst, Taylor and Peacock 1995

These distortions have already been measured.

Relativistic distortions

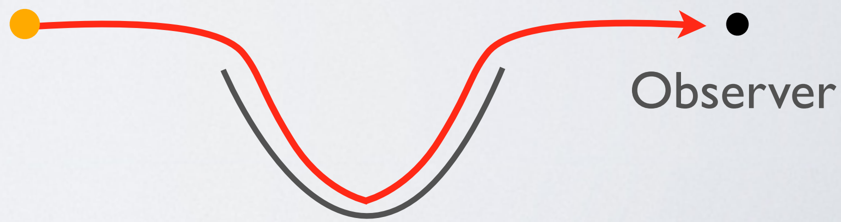
Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

Besides these three well-known sources of distortions, there are a lot of **other** subdominant **distortions**.

Examples:

- Gravitational redshift:

- ISW and Shapiro time delay



Change the redshift

Change the radial size of the bin

Full calculation

The observed **over-density** is: $\Delta(z, \mathbf{n}) = \dfrac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}$

$N(z, \mathbf{n}) = \rho(z, \mathbf{n}) \cdot V(z, \mathbf{n})$ and $\bar{N}(z) = \bar{\rho}(z) \cdot \bar{V}(z)$

At linear order in perturbation theory:

$$\Delta(z, \mathbf{n}) = b \cdot \delta(z, \mathbf{n}) + \frac{\delta V(z, \mathbf{n})}{V} - 3\frac{\delta z}{1+z}$$

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density — redshift space distortion — lensing — Doppler — gravitational redshift

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
$$- \int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$
$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$
$$+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$
$$+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$

(last two lines) → potential

Outline

- I will discuss the **impact** of the relativistic distortions on our **observables**.

- I concentrate on the two-point **correlation** function:

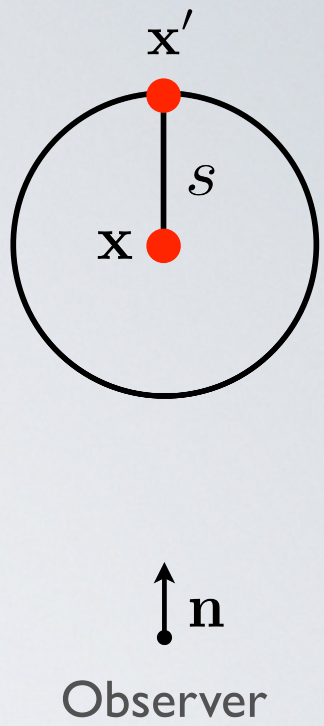
$$\xi = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}') \rangle$$

- Some of the relativistic distortions break the **symmetry** of the correlation function → dipole.

- What is the **optimal** way of measuring the **dipole**.

Density

The **density** contribution $\Delta = b \cdot \delta$, generates an **isotropic** correlation function.

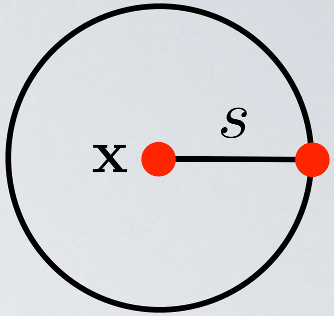


$\xi(s) = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$ depends only on the **separation** $s = |\mathbf{x} - \mathbf{x}'|$

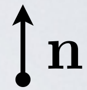
$$\xi(s) = \frac{1}{2\pi^2} \int dk k^2 P(k,z)\, j_0(k \cdot s)$$

Density

The **density** contribution $\Delta = b \cdot \delta$, generates an **isotropic** correlation function.



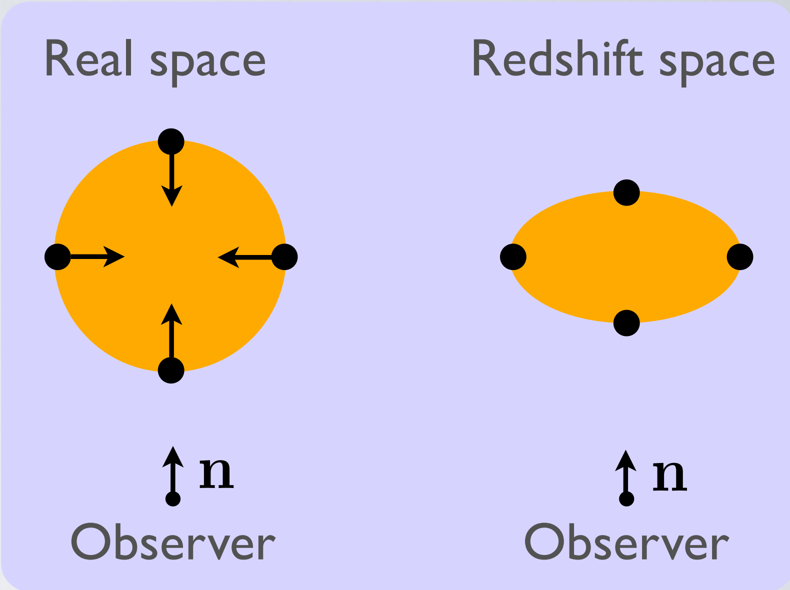
$\xi(s) = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}')\rangle$ depends only on the **separation** $s = |\mathbf{x} - \mathbf{x}'|$



$$\xi(s) = \frac{1}{2\pi^2}\int dk k^2 P(k,z)\, j_0(k \cdot s)$$

Redshift distortions

Redshift distortions **break** the **isotropy** of the correlation function.



$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

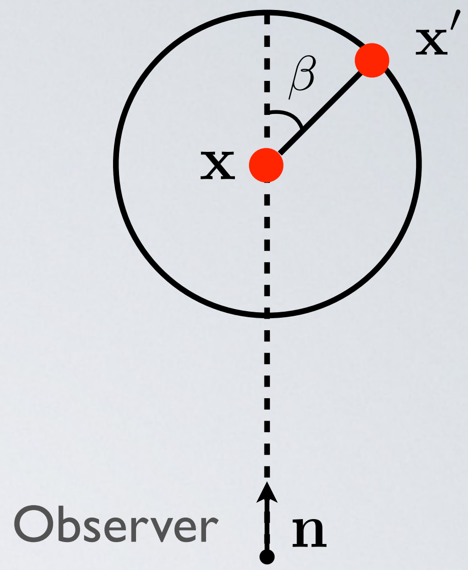
They generate a **quadrupole** and an **hexadecapole** Lilje and Efstathiou (1989), McGill (1990), Hamilton (1992)

$$\xi_2 = -\left(\frac{4f}{3}+\frac{4f^2}{7}\right)\frac{1}{2\pi^2}\int dk k^2 P(k,z) j_2(k\cdot s)\, P_2(\cos\beta)$$

$$\xi_4 = \frac{8f^2}{35}\frac{1}{2\pi^2}\int dk k^2 P(k,z) j_4(k\cdot s)\, P_4(\cos\beta)$$

Redshift distortions

Redshift distortions **break the isotropy** of the correlation function.



$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

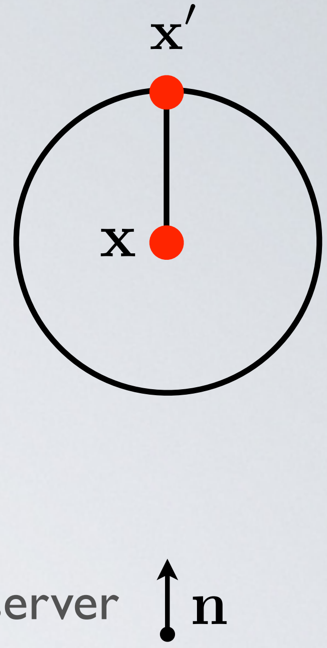
They generate a **quadrupole** and an **hexadecapole** Lilje and Efstathiou (1989), McGill (1990), Hamilton (1992)

$$\xi_2 = -\left(\frac{4f}{3} + \frac{4f^2}{7}\right) \frac{1}{2\pi^2} \int dk k^2 P(k,z) j_2(k \cdot s)\, P_2(\cos\beta)$$

$$\xi_4 = \frac{8f^2}{35} \frac{1}{2\pi^2} \int dk k^2 P(k,z) j_4(k \cdot s)\, P_4(\cos\beta)$$

CB, Hui and Gaztanaga (2013)
McDonald (2009), Croft (2013)

Relativistic distortions



The relativistic distortions break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.

This differs from the breaking of **isotropy**, which is symmetric: the squeezing is the same for galaxies in front and behind the centre of the over-density.

→ Redshift distortions have **even** powers of $\cos\beta$

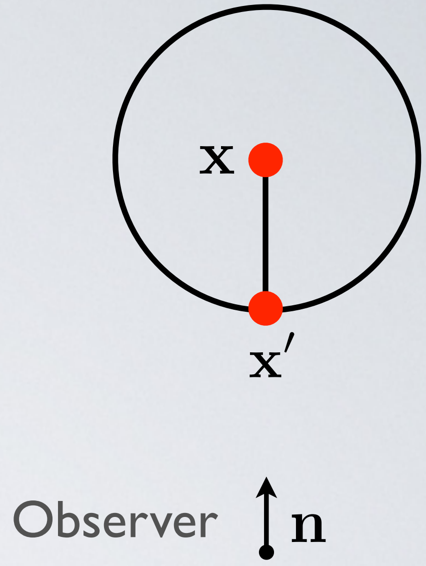
To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

Relativistic distortions

CB, Hui and Gaztanaga (2013)
McDonald (2009), Croft (2013)

The relativistic distortions break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.



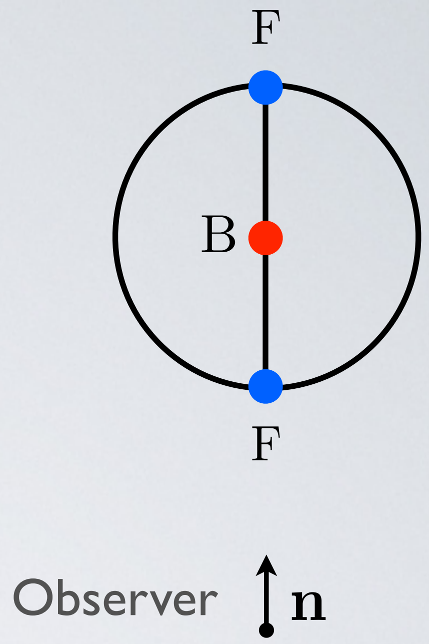
This differs from the breaking of **isotropy**, which is symmetric: the squeezing is the same for galaxies in front and behind the centre of the over-density.

→ Redshift distortions have **even** powers of $\cos\beta$

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

CB, Hui and Gaztanaga (2013)
McDonald (2009), Croft (2013)

Relativistic distortions



The relativistic distortions break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.

This differs from the breaking of **isotropy**, which is symmetric: the squeezing is the same for galaxies in front and behind the centre of the over-density.

→ Redshift distortions have **even** powers of $\cos\beta$

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

Anti-symmetries

density

redshift space distortion

lensing

Doppler

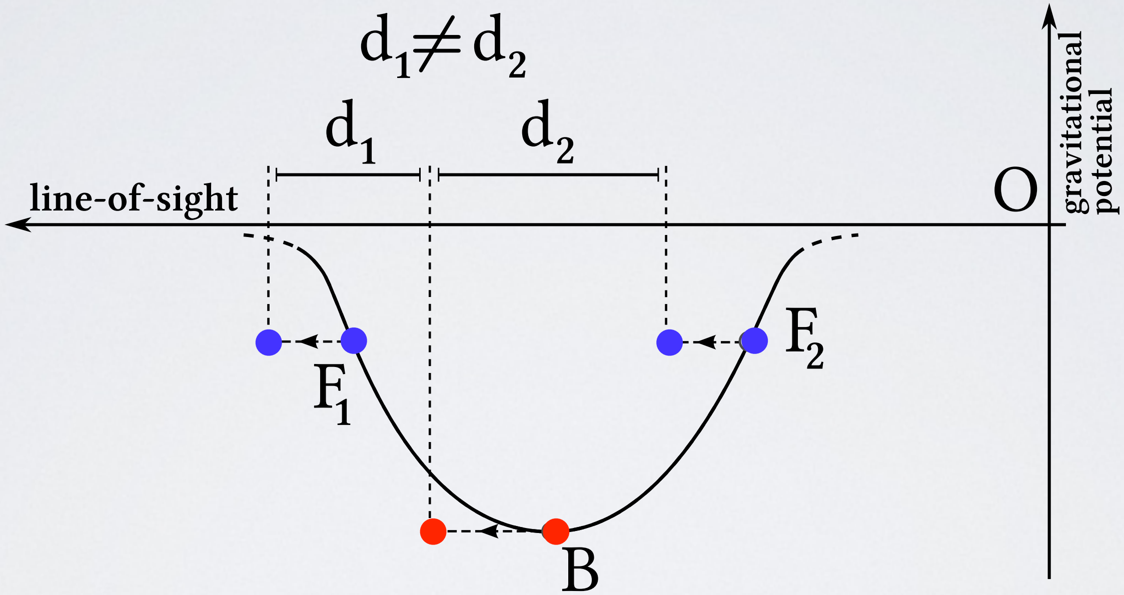
gravitational redshift

$$
\begin{aligned}
\Delta(z,\mathbf{n}) = \; & b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
& - \int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi) \\
& + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
& \left.\begin{aligned} & + \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\
& + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \end{aligned}\right] \rightarrow \text{potential}
\end{aligned}
$$

Cross-correlation

The following terms **break** the **symmetry**:

$$\Delta_{\rm rel} = \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$



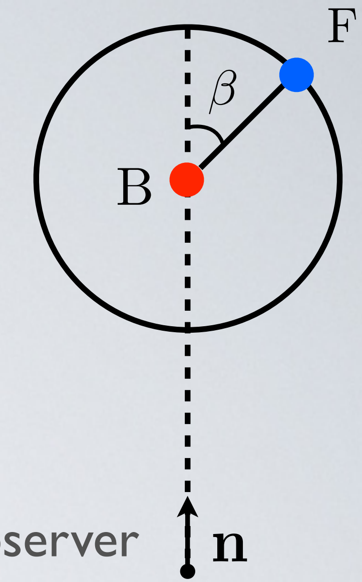
Similar to measurements of gravitational redshift in **clusters**.

Wojtak, Hansen and Hjorth (2011), Sadeh, Feng and Lahav (2015)

See also Croft's talk on Monday

Dipole in the correlation function

CB, Hui and Gaztanaga (2013)



$$\xi(s,\beta) = D_1^2\, f\,\frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)(b_{\rm B}-b_{\rm F})\nu_1(s)\cdot\cos(\beta)$$

$$\nu_1(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k)T_\Psi(k)\, j_1(k\cdot s)$$

By fitting for a **dipole** in the correlation function, we **isolate** the relativistic effects. We get rid of the dominant monopole and quadrupole generated by density and velocities.

Optimising the measurement

What is the optimal way of measuring the dipole?

Naive try:

- We split the populations into **two** populations (bright and faint)
- We measure the **cross-correlation** function
- We **weight** each pair by $\cos\beta_{ij}$

Problem: we **loose** a lot of pairs (all the auto-correlations).

Can we do better?

CB, Gaztanaga and Hui, in preparation

Generic kernel

- We split the population of galaxies according to their **luminosity**
- In each **pixel** we count $n_{L_i}(\mathbf{x}_i)$ and fluctuations $\delta n_{L_i}(\mathbf{x}_i)$
- We **combine** all pixels and populations into

$$\hat{\xi} = \sum_{ij} \sum_{L_i L_j} w_{\mathbf{x}_i \mathbf{x}_j L_i L_j} \delta n_{L_i}(\mathbf{x}_i) \delta n_{L_j}(\mathbf{x}_j)$$

The kernel w tells us how to combine the pairs.

- **Example:**

To isolate the monopole: $w_{\mathbf{x}_i \mathbf{x}_j L_i L_j} \propto \delta_K(s_{ij} - s)$

To isolate the quadrupole: $w_{\mathbf{x}_i \mathbf{x}_j L_i L_j} \propto \delta_K(s_{ij} - s) P_2(\cos\beta_{ij})$

Anti-symmetric kernel

- To isolate the relativistic effects, the kernel must depend on the **luminosity**.

- It must be **anti-symmetric** in $L_i \leftrightarrow L_j$

  For two populations the signal is proportional to $b_{\rm B} - b_{\rm F}$

$$w_{\mathbf{x}_i\mathbf{x}_j L_i L_j} = -w_{\mathbf{x}_i\mathbf{x}_j L_j L_i}$$

- It must be **anti-symmetric** in $\mathbf{x}_i \leftrightarrow \mathbf{x}_j$

  The signal is proportional to $\cos\beta_{ij}$

$$w_{\mathbf{x}_i\mathbf{x}_j L_i L_j} = -w_{\mathbf{x}_j\mathbf{x}_i L_i L_j}$$

Variance

$$\mathrm{var}(\hat{\xi}) = \sum_{ijL_iL_j}\sum_{abL_aL_b} w_{\mathbf{x}_i\mathbf{x}_i L_i L_j} w_{\mathbf{x}_a\mathbf{x}_b L_a L_b}$$
$$\times\Big[\big\langle\delta n_{L_i}(\mathbf{x}_i)\delta n_{L_a}(\mathbf{x}_a)\big\rangle\big\langle\delta n_{L_j}(\mathbf{x}_j)\delta n_{L_b}(\mathbf{x}_b)\big\rangle + i\leftrightarrow j\Big]$$

Three contributions:

- Poisson noise
- Cosmic variance
- Mixed term

Important property: the **cosmic variance** of the density exactly **vanishes**.

Minimising the variance

We **minimise** the variance under the constraints:

$$L = \mathrm{var}(\hat{\xi}) + \lambda_0\left[\langle\hat{\xi}\rangle - \xi_{\mathrm{true}}\right] + \sum_{ijL_iL_j} \lambda_{ijL_iL_j}\left(w_{\mathbf{x}_i\mathbf{x}_jL_iL_j} - w_{\mathbf{x}_j\mathbf{x}_iL_jL_i}\right)$$

$$\rightarrow \quad \boxed{w = (\mathbb{1} + N^T)^{-1} B (\mathbb{1} + N)^{-1}}$$

$$B_{ij} = \frac{\lambda_0}{4}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{L_i} - b_{L_i})\langle\delta_i\,(\mathbf{V}\cdot\mathbf{n})_j\rangle$$

$$N_{ij} = \frac{1}{2} d\bar{n}_{L_i} b_{L_i} b_{L_j} C_{ij}$$

Minimising the variance

In the regime where the Poisson noise dominates:

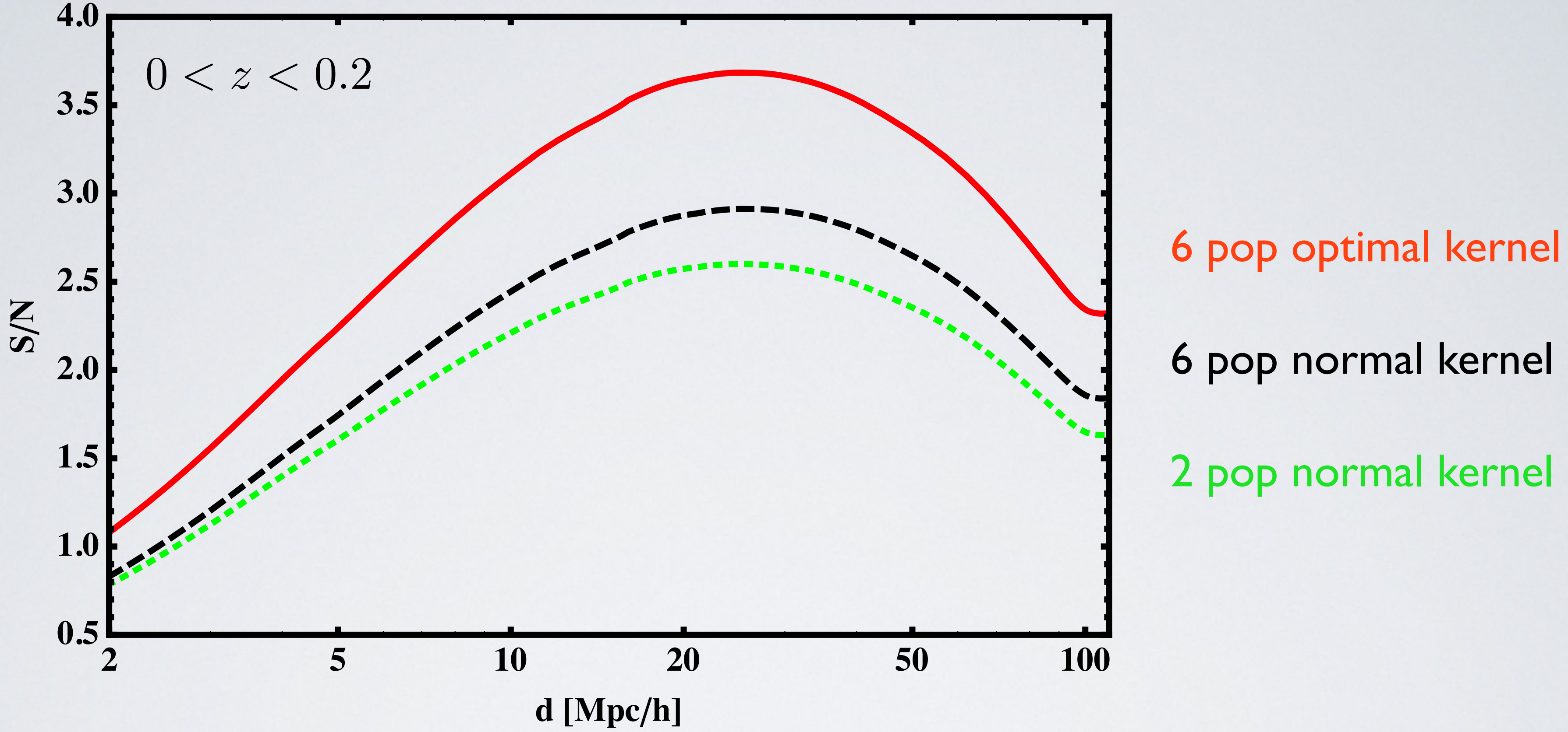
$$w_{\mathbf{x}_i\mathbf{x}_j L_i L_j} = \frac{3}{8\pi}(b_{L_i} - b_{L_j})\cos\beta_{ij}\delta_K(s_{ij} - s)$$

We calculate the signal-to-noise with this kernel. It depends on the characteristics of the survey and on the populations of galaxies.

Example: millenium simulation Jennings, Baugh and Hatt (2015)

Measurement of the bias and the number density for 6 populations of halos.

Result



Using the optimal kernel **increases** the signal-to-noise by **40 percents**.

Future

- **Measurements**: in BOSS, signal compatible with zero within error bars for two populations of galaxies. Signal-to-noise smaller than one.

- Try with **more populations** and the optimal kernel.

- Try at **lower redshifts**, main sample of SDSS.

- Try with the full kernel: $w = (\mathbb{1} + N^T)^{-1} B (\mathbb{1} + N)^{-1}$

Conclusion

- Our **observables** are affected by relativistic effects.

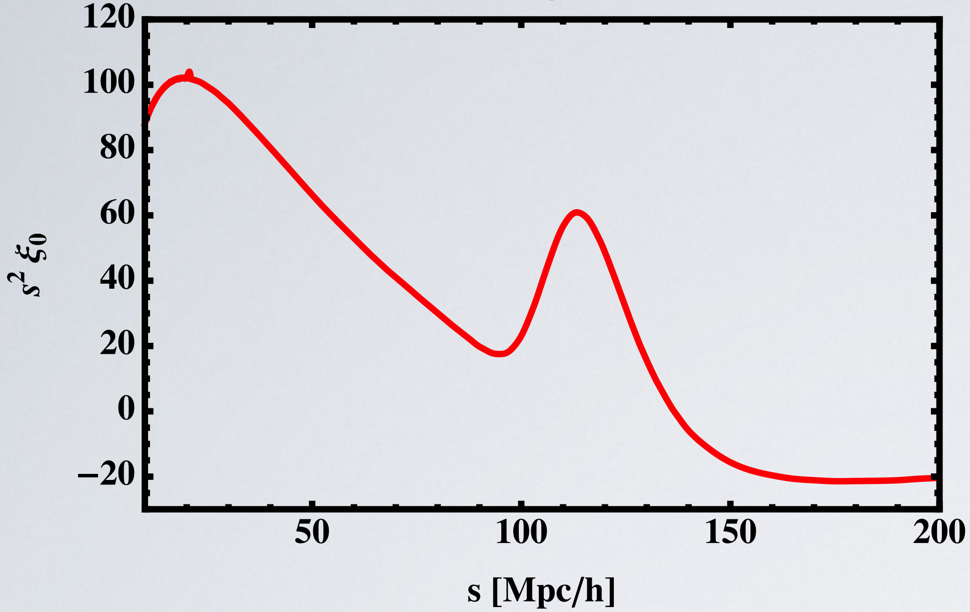
- These effects have a different **signature** in the **correlation** function: they induce anti-symmetries.

- We can construct an **optimal kernel** to measure the dipole in the correlation function.

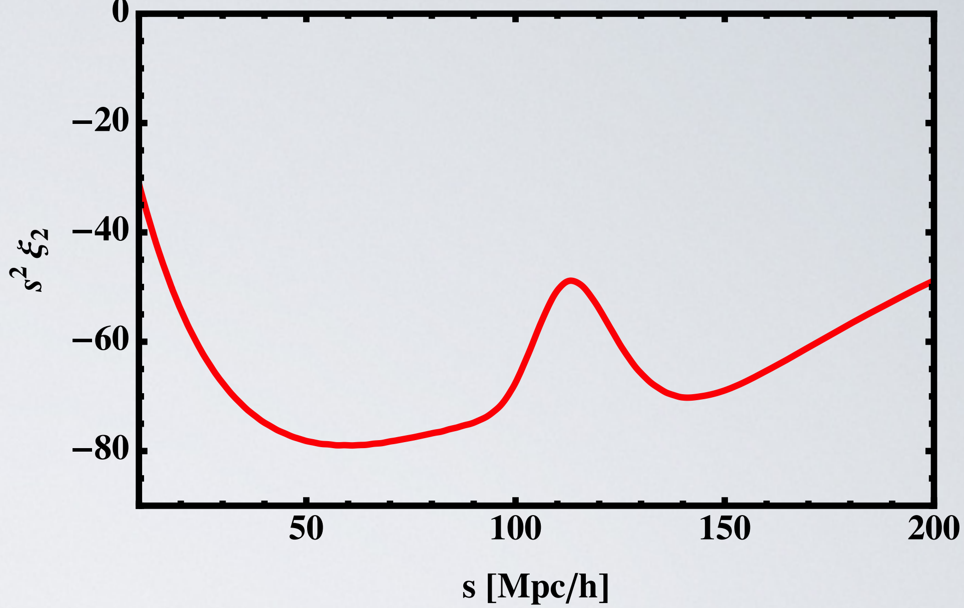
$z = 0.25$

Multipoles

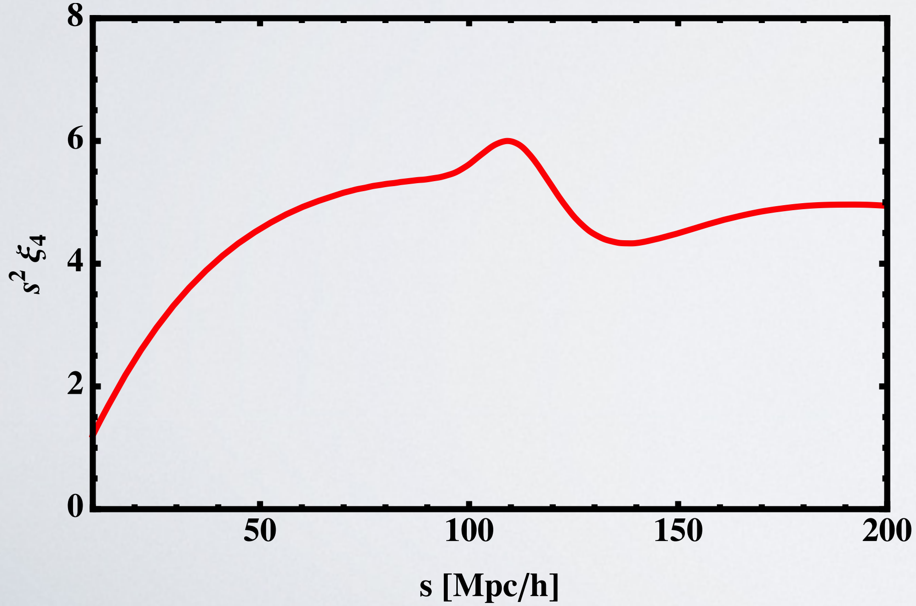
Monopole



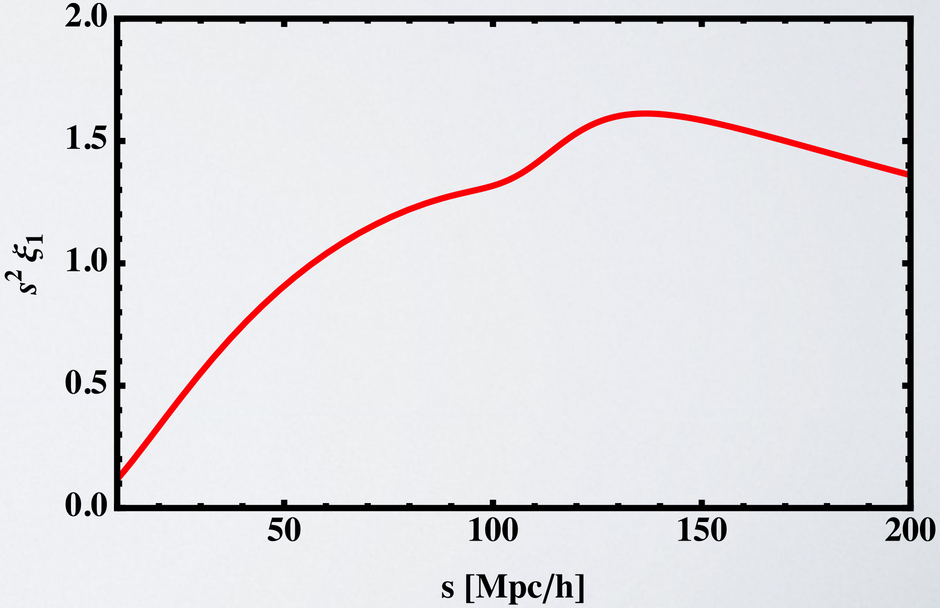
Quadrupole



Hexadecapole

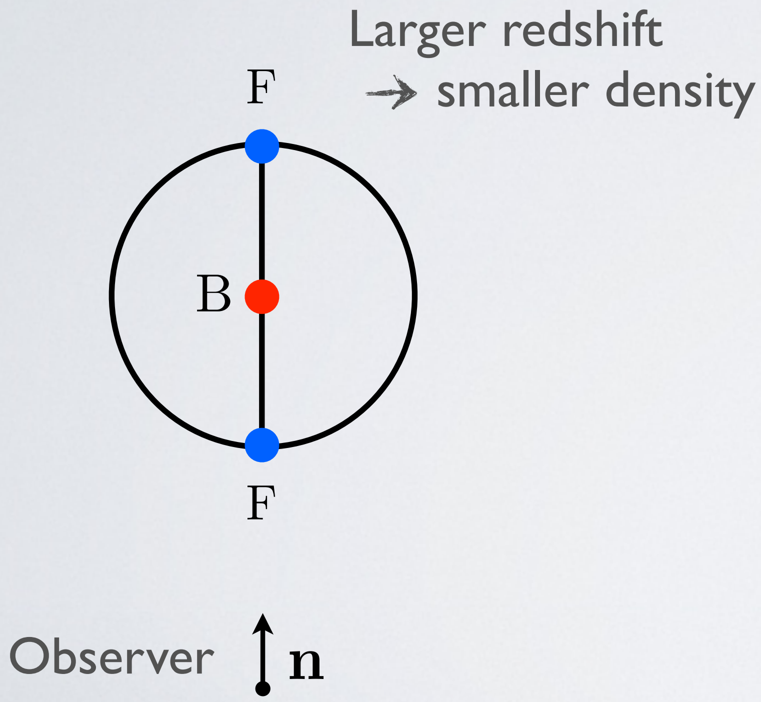


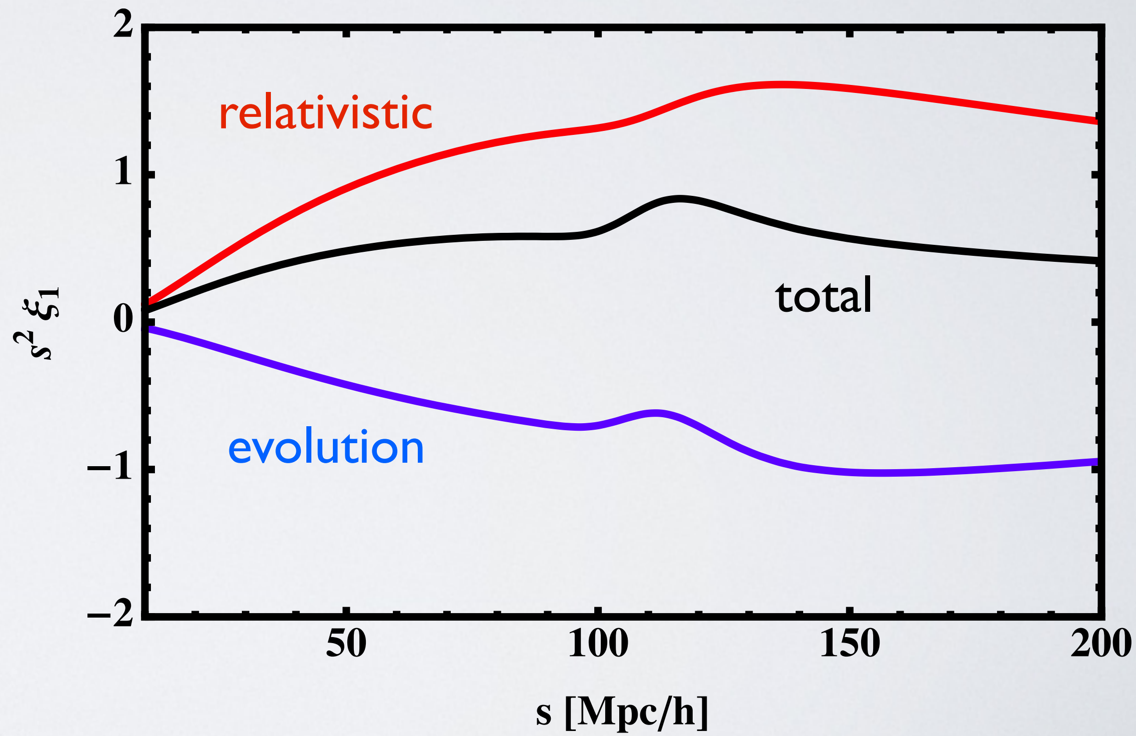
Dipole



Contamination

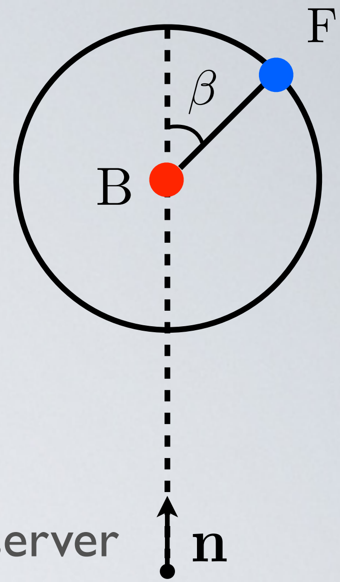
The density and velocity **evolve** with time: the density of the faint galaxies in front of the bright is larger than the density behind. This also induces a **dipole** in the correlation function.



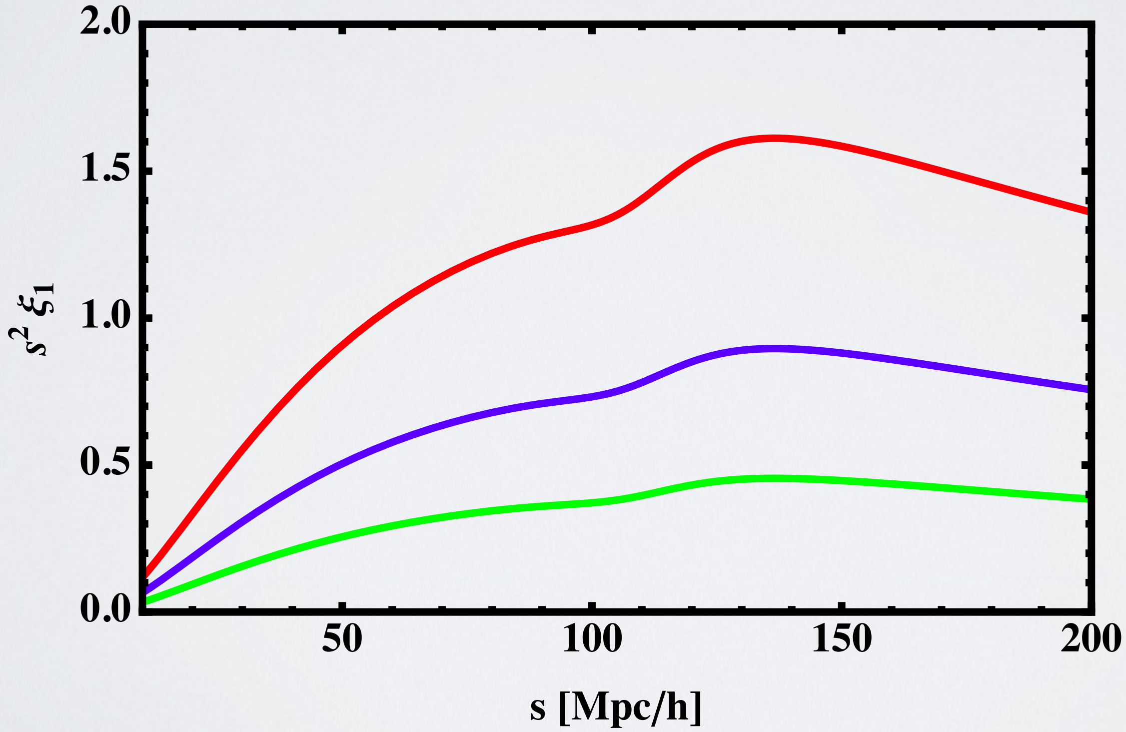


Dipole in the correlation function

$$\xi(s,\beta) = D_1^2\, f\,\frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)(b_\mathrm{B}-b_\mathrm{F})\nu_1(s)\cdot\cos(\beta)$$



$$\nu_1(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k)T_\Psi(k)\, j_1(k\cdot s)$$



$z=0.25$

$z=0.5$

$z=1$

$b_\mathrm{B}-b_\mathrm{F}\simeq 0.5$