Parametric polarized foreground removal

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MPA, Nov 27
Estimate the CMB, synchrotron, dust $Q/U$ signal in each pixel on the sky.

The method

1. \[ \mathcal{L} = \sum_{\nu} [d_{\nu} - m_{\nu}]^T N_{\nu}^{-1} [d_{\nu} - m_{\nu}], \]

2. \[ m_{\nu} = \sum_k \alpha_{k,\nu} A_k, \]

\[ \alpha_{1,\nu} = f(\nu) I, \]

\[ \alpha_{2,\nu} = \text{diag}[(\nu/\nu_K)^{\beta_2}], \]

\[ \alpha_{3,\nu} = \text{diag}[(\nu/\nu_W)^{\beta_3}]. \]

$k=1$ (CMB), $k=2$ (synch), $k=3$ (dust).

There are variations on this, which can include monopoles, multi-temp dust, spectral curvature, simultaneous spectrum estimation. Many applications to temperature maps.
Estimating parameters

Map out the joint distribution for A (amplitudes) and beta (spectral indices) vectors, and extract marginalized distribution for CMB Q/U in each pixel.

\[
p(A_1, A_2, A_3, \beta_2, \beta_3 | d) \\
p(A_1 | d) = \int p(A_1, A_2, A_3, \beta_2, \beta_3 | d) dA_2 dA_3 d\beta_2 d\beta_3
\]

- If maps have 7 degree pixels, this would give 2x3x768 A parameters.
- Synchrotron spectral indices - if they vary in e.g. 30 degree pixels, this gives 48 parameters, but can be thousands.
- \(p(A, b | d)\) is not a distribution we can draw analytic samples from
**Gibbs sampling**

*Minimal case:*

1. For fixed beta, $p(A|b,d)$ is Gaussian, so we draw a new $A$ sample.

2. For fixed $A$, $p(b|A,d)$ is not known so draw a new beta sample using Metropolis algorithm, or other sampling method.

3. Draw $A$ and beta samples in turn until mapped out full distribution

$$
\mathcal{L} = \sum_{\nu} \beta_{\nu} - \sum_{k} \alpha_{k,\nu} A_k \left[ \mathbf{N}_\nu^{-1} \left[ d_\nu - \sum_{k} \alpha_{k,\nu} A_k \right] \right]
$$
Application to data and to sims
Application 1:
WMAP

K band

Ka band

V band

Q band

W band

Hinshaw et al 2008
Estimated maps

CMB

Synchrotron

Dust

Dunkley et al 2009
Feed maps and covariance matrix into low-ell likelihood.

Gave consistent results for large scale CMB power and \( \tau \):

\[ \tau = 0.091 \pm 0.019 \quad \text{(parametric)} \]

\[ \tau = 0.086 \pm 0.017 \quad \text{(template)} \]

Dunkley et al 2009
But, needed priors

\[ Q_d(n) = 0 \pm 0.2 I_d(n) \]
\[ U_d(n) = 0 \pm 0.2 I_d(n) \]

\[ \beta_s = -3.0 \pm 0.3 \]
\[ \beta_d = 1.7 \]

In pixels of side \( \sim 30 \) degrees
Application 2: Planck-like sims
Large-Scale Polarized Foreground Component Separation for Planck

**Figure 1.**

First row: input Q CMB map (left column), Commander posterior mean output Q map (middle column), and Galclean posterior mean output Q map (right column). Second row: marginalized error, Commander difference in standard deviations per pixel (middle column), and Galclean difference in standard deviations per pixel (right column) for the Q component. Third row: input U CMB map (left column), Commander posterior mean output U map (middle column), and Galclean posterior mean output U map (right column). Fourth row: As in second row but for the U component.

The presence of the foregrounds. This effect is summarized in Table 2 which gives the average upper 95% cut-off limits on estimates of $r$ for $r=0$. We also apply the standard WMAP P06 mask (Page et al. 2007), which masks about 26% of the sky, and calculate the likelihood distributions for the masked case. For our chosen simulations and modeling, we find minimal error inflation in $\sigma_\tau$ and $\sigma_r$. $\sigma_\tau$ remains nearly constant at $\sim 0.005$ in the absence or presence of foregrounds. $\sigma_r$ increases from $\sim 0.02$ to $\sim 0.03$ with the addition of foregrounds. Our limits on $r=0$ show that it is more sensitive to the presence of foregrounds than an estimation of an $r=0$ signal. We find $\sigma_r/r=0.32$ for $r=0.1$ and $\sigma_\tau/\tau=0.05$ for $\tau=0.1$.

Using Commander and Galclean approach, Betoule et al. (2009) find values of $\sigma_r/r$ similar to ours: $\sigma_r/r=0.34$ with foregrounds and $\sigma_r/r=0.25$ with noise only. In another Fisher matrix forecast for Planck, Baumann et al. (2009) finds $\sigma_r=0.011$ for $r=0.001$ without foregrounds.

Our pixel likelihood code can be used not only to constrain parameters, but also to find the power at each multipole in the polarized power spectra. At each multipole, we compute the conditional likelihood as a function of $C_{EE}$ and $C_{BB}$ for $\ell=2-7$ with the multipole shell fixed at the fiducial $\Lambda$CDM values, using the method described.

Same results, two different codes: Galclean and Commander

Armitage-Caplan et al 2011, 1103.2554
Figure 4. Likelihood distributions for $\tau$ (left plot) and $r$ (right plot) for four simulations of CMB+foregrounds with $\tau = 0.1$ and $r = 0.1$. The four different simulations are represented by the black, blue, green, and red curves. Results from Commander are shown with a solid line and results from Galclean are shown with a dashed line. Note that the Galclean curve for the red simulation is completely overlaid by the Commander curve for the red simulation (left plot), and that the Galclean curve for the green simulation is completely overlaid by the Commander curve for the green simulation (right plot). We find $\sigma(\tau) \approx 0.004$ and $\sigma(r) \approx 0.03$.

Figure 5. $\tau = 0.1$ foreground-free case for 10 simulations. The left-hand plot shows the likelihood distributions for each of the 10 simulations, while the plot on the right-hand side is the sum of the log-likelihoods of the 10 distributions.

Figure 6. $r = 0.1$ foreground-free case for 10 simulations. The left-hand plot shows the likelihood distributions for each of the 10 simulations, while the plot on the right-hand side is the sum of the log-likelihoods for the 10 simulations.

Armitage-Caplan et al 2011

Recover input optical depth and tensor-to-scalar ratio
Figure 2. Maps of the polarization amplitude $P = \sqrt{Q^2 + U^2}$ for the synchrotron at 30 GHz (first row) and dust at 353 GHz (third row). The difference in standard deviations per pixel for the $Q$ component (second and fourth rows) indicates that the synchrotron and dust maps have been recovered to the expected statistical result.

Table 2. Average upper 95% cut-off limits on estimates of $r = 0$, and average estimates on $\sigma(r = 0)$ and $\sigma(\tau = 0)$. We find a foreground-free error on $r$ that matches the size of errors found in analogous Fisher matrix forecasts for Planck (Betoule et al. 2009; Baumann et al. 2009). The effect of foregrounds is seen to inflate the error bar in the case of $r$ but not $\tau$. The error on $r$ for the $r = 0.1$ model is amplified by a factor of $\sim 1.4$, and the 95% cut-off limit on $r$ for the $r = 0.0$ model is amplified by a factor of $\sim 3$, when foregrounds are included.

Figure 3. Spectral index, $\beta$, in input map (top), output map (second row), error map (third row), and deviation map (bottom row) for dust (left) and synchrotron (right). Note that in areas of low signal-to-noise, the error is driven to the prior value of 0.5 for dust and 0.3 for synchrotron in areas of low signal-to-noise.

Index prior $-3 + 0.3$

Armitage-Caplan et al 2011
1. Test on fg-free sims

2. Error inflation with fg included

3. Compare to template-cleaning

Armitage-Caplan et al 2011
What if we get modeling wrong?
Insert 1% polarized spinning dust or free-free

Armitage-Caplan et al 2012
Incorrect spectral model

- Assume power-law when modified grey-body
- Assume one-component when two-component
- Assume no curvature when really has curvature (0.3 from 30-100 GHz)

Armitage-Caplan et al. 2012
Wrong priors

- Assume synch prior -2.5±0.5 or -2.8±0.5 in 4 deg pixels, when really -3.0
- Same effect for dust
- Same effect for r=0
- Increasing S/N with C-BASS helps

0 < sigma < 0.5
Some observations

1. Method can return wrong answer where S/N is low, if applied blindly.

2. Be very careful when imposing priors, or over-parameterizing model.

3. Also be careful under-parameterizing model!

4. All modeling errors over-predict r

5. However, properly treated, this formalism is powerful: it inflates CMB error to account for foreground uncertainty.

6. So far, limited application beyond reionization bump to go for l~100 need to think about how to include spectral variation. In fact, spatial coherence is missing in most models.
Summary

• So far, polarized foreground removal has not required more than simple template cleaning.

• But, parameterizing the foregrounds, and marginalizing over their parameters, allows for more rigorous error propagation, which is much more important for smaller CMB signals.

• The community has codes ready to do this, but the models may not be most ‘elegant’.

• It is clear that care must be taken in how the model is set up, avoiding too much freedom in low S/N regime.