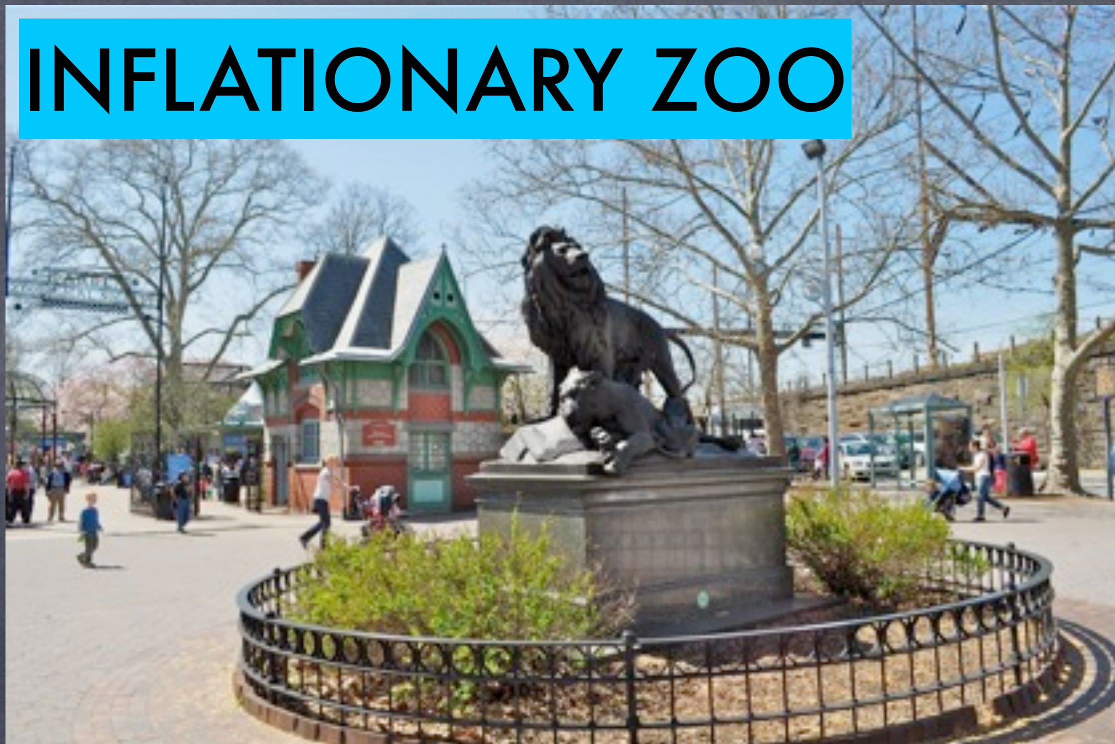
# Symmetries and Ward Identities in Cosmology

Justin Khoury (UPenn)


THE RETURN
OF DE SITTER

# THE RETURN OF DE SITTER

# INFLATIONARY ZOO



A-term Inflation

Old Inflation

Extended Inflation

N-flation

Hyper-Extended Inflation

New Inflation

Natural Inflation

k-Inflation

Hybrid Inflation

Chaotic Inflation

Pseudo-Natural Inflation

Galileon Inflation

Roulette Inflation

Chain Inflation

Extra-Natural Inflation

D-term Inflation

F-term Inflation

Ghost inflation

DBI Inflation

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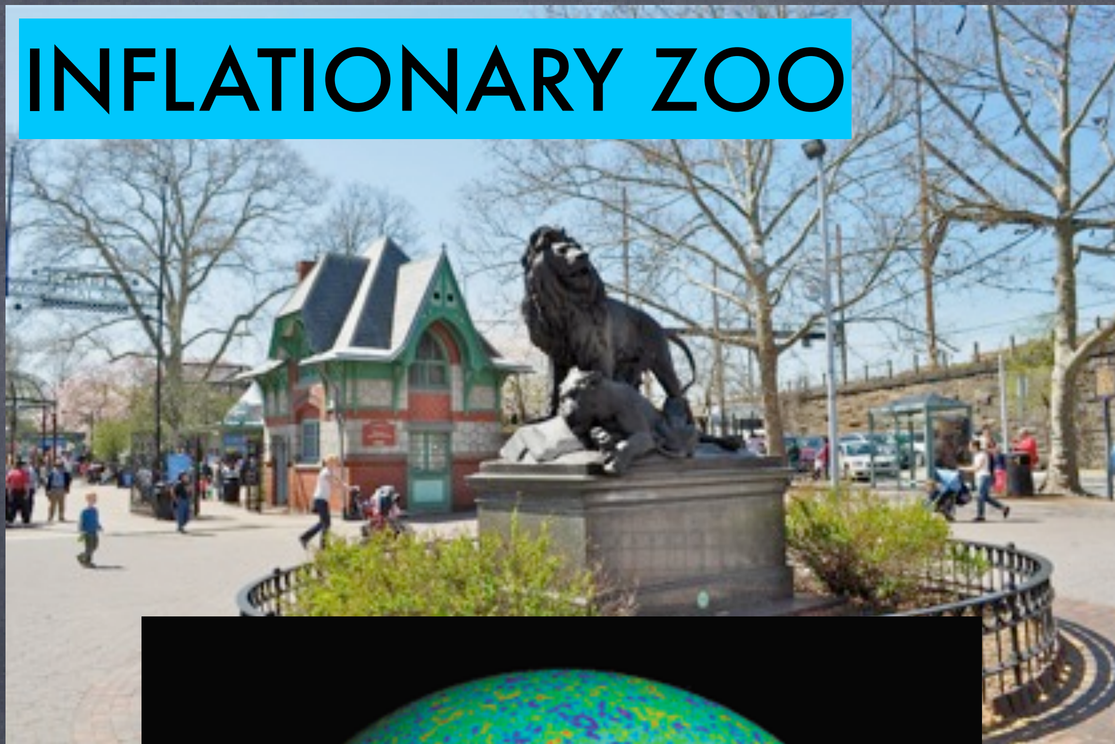

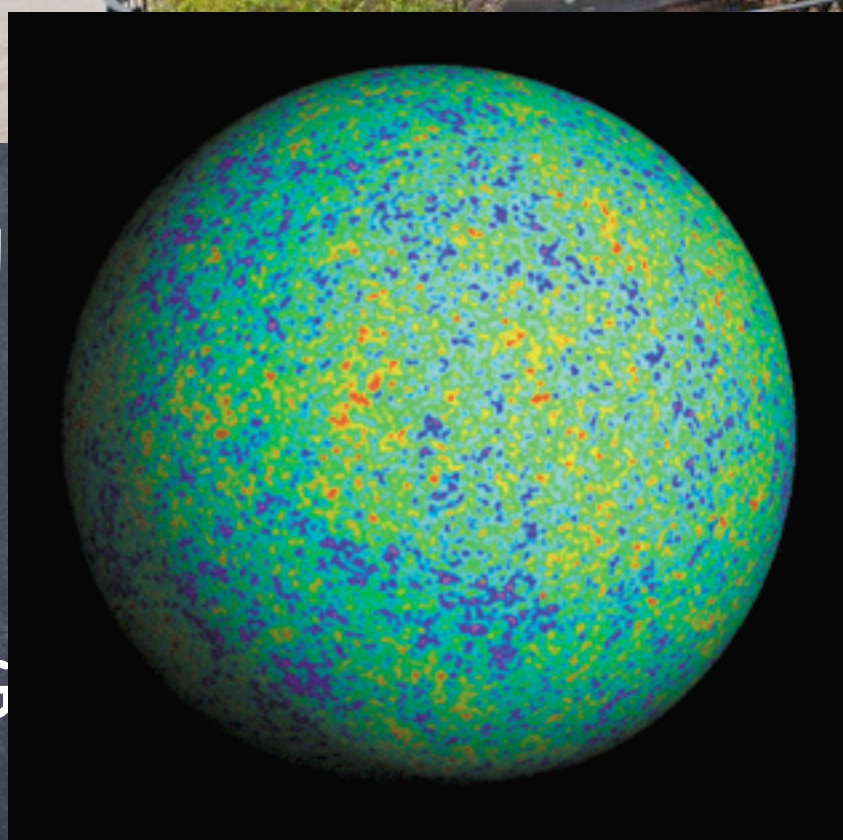

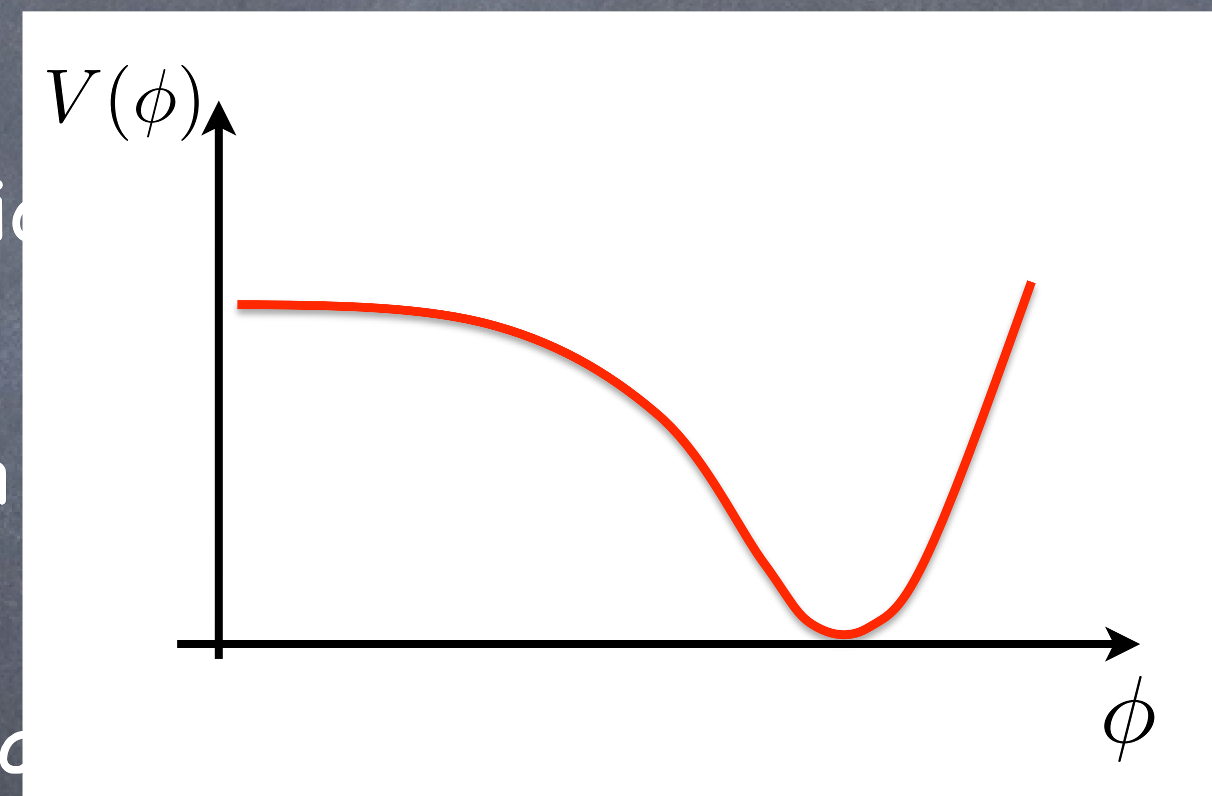


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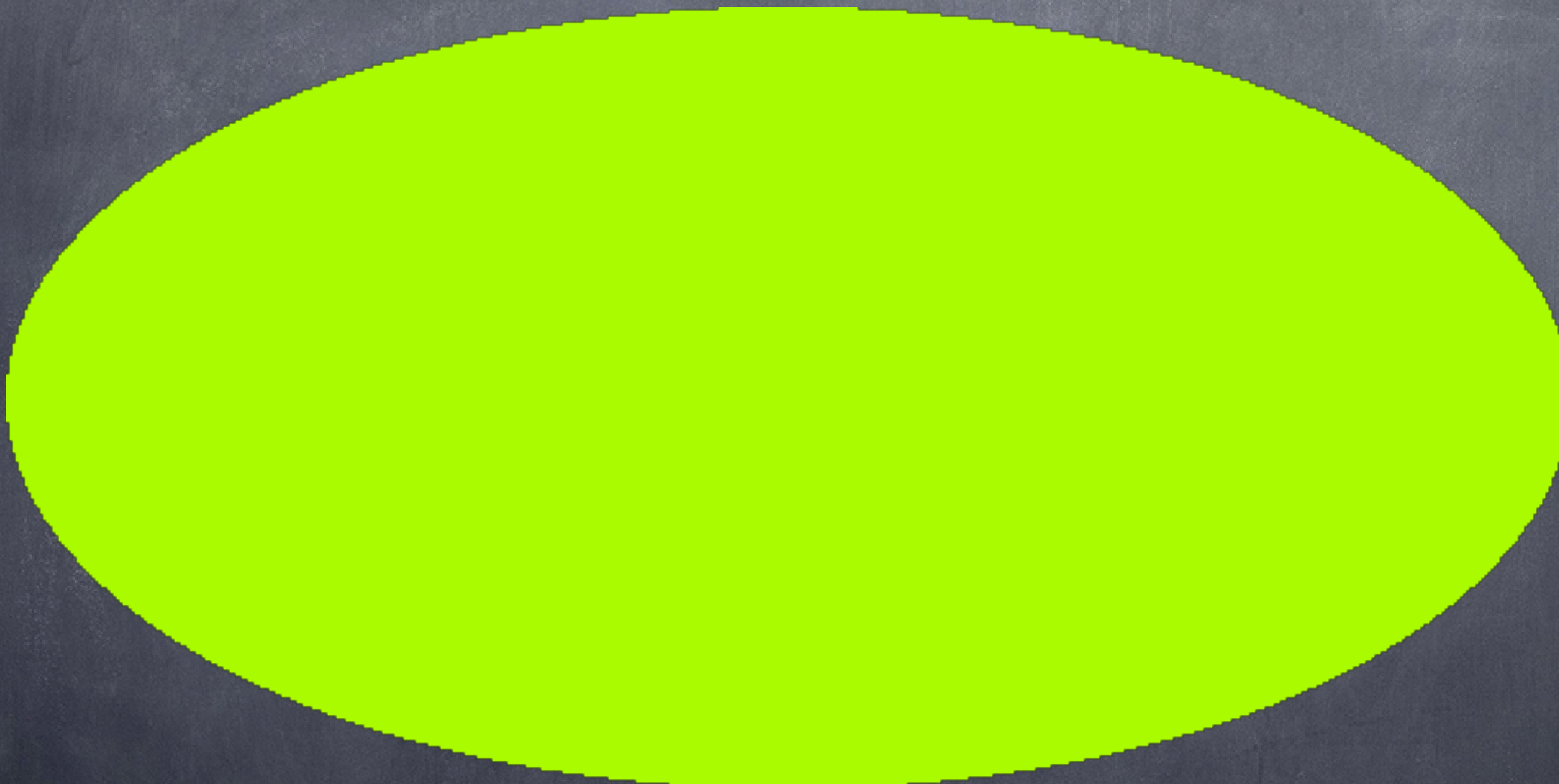
# Paradigm Shift

- More basic questions: Did inflation really occur? If so, how many fields were participating? Or is something else responsible for the observations? How can we tell the difference?

- Focus on broad class of theories, distinguished by their symmetries.

Fortunately, our universe enjoys a lot of symmetries.

# Symmetries of the Background



# Cosmic No-Hair Theorem

Gratton, Khoury, Steinhardt and Turok (2003)

Why inflation explains flatness, homogeneity and isotropy?

$$\frac{\dot{a}^2}{a^2} = \frac{C_{\rm dust}}{a^3} + \frac{C_{\rm radn}}{a^4} - \frac{k}{a^2} + \frac{C_{\rm aniso}}{a^6} + \ldots + \frac{C_\phi}{a^{3(1+w)}}$$

where $w = P/\rho$ is the equation of state parameter

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- Most dangerous term is curvature
- Need $w < -1/3$, which means accelerated expansion:

$$\frac{\ddot{a}}{a} = -\frac{\rho}{6M_{\rm Pl}^2}(1+3w) > 0$$

$\Longrightarrow$ Inflation wins over everything, and is dynamical attractor

Wald (1983), Jensen & Stein-Shabes (1986)

# Another logical possibility

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- Need $w > 1$, which means slow contraction:

$$a(t) \sim (-t)^{\frac{2}{3(1+w)}}$$

$\Longrightarrow$ Slow contraction equally powerful smoothing mechanism, and dynamical attractor

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- Supp
(before

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} > 1$$

whenever $V(\phi) < 0$

- Now

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# Ekpyrotic Cosmology

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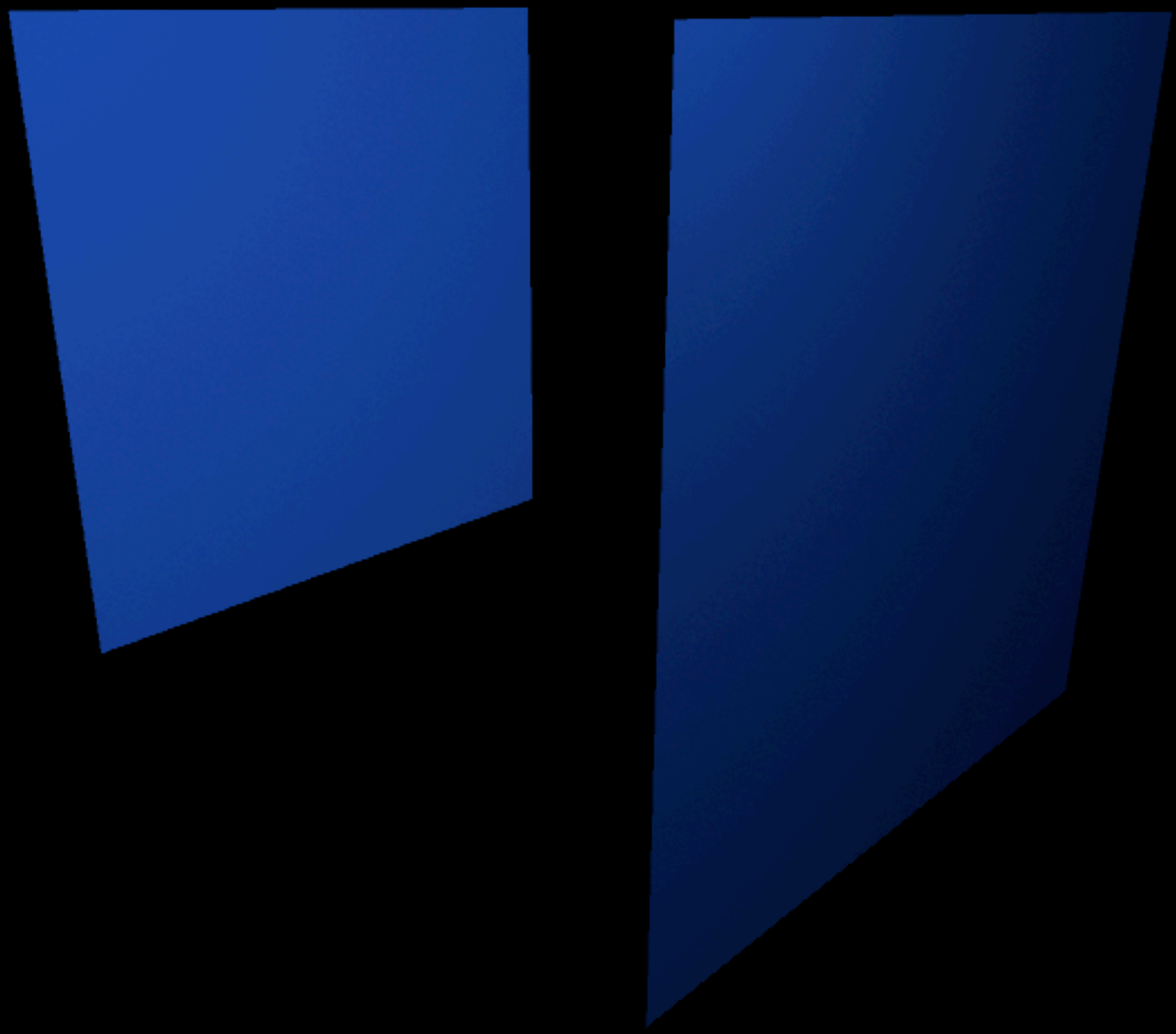

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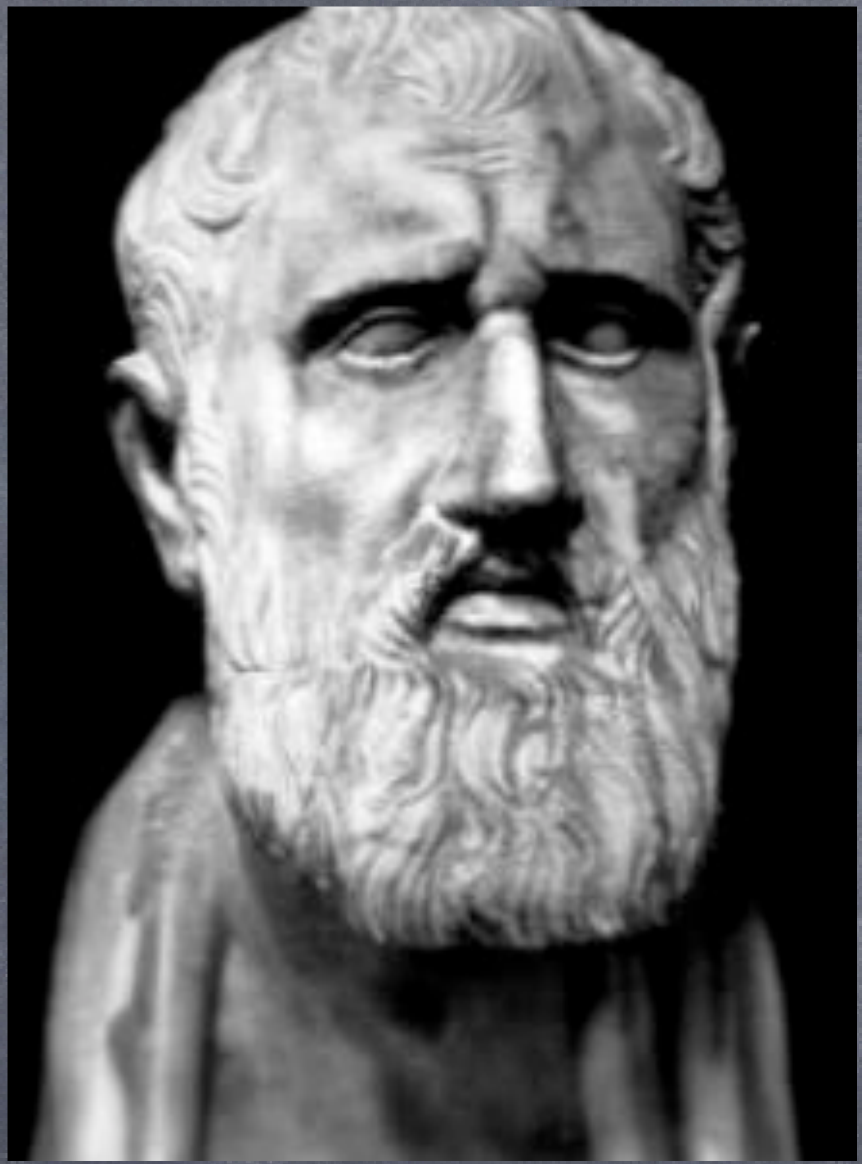
# Ekpyrosis?



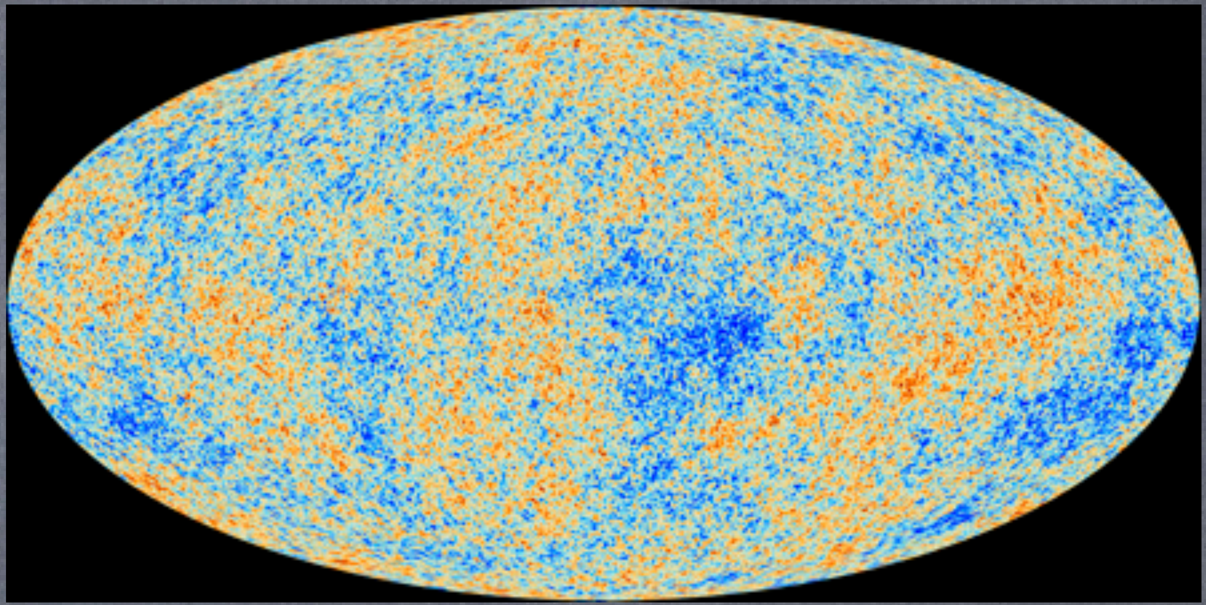
# Ekpyrosis?


Suffer
Doom
Return to Flame...

# Symmetries of Density Perturbations

# Primordial inhomogeneities are the simplest imaginable

- Approximately scale invariant



$$\left\langle \frac{\delta T}{T}(\vec{x}_1)\frac{\delta T}{T}(\vec{x}_2)\right\rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3 k^3} e^{-i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} P(k)$$

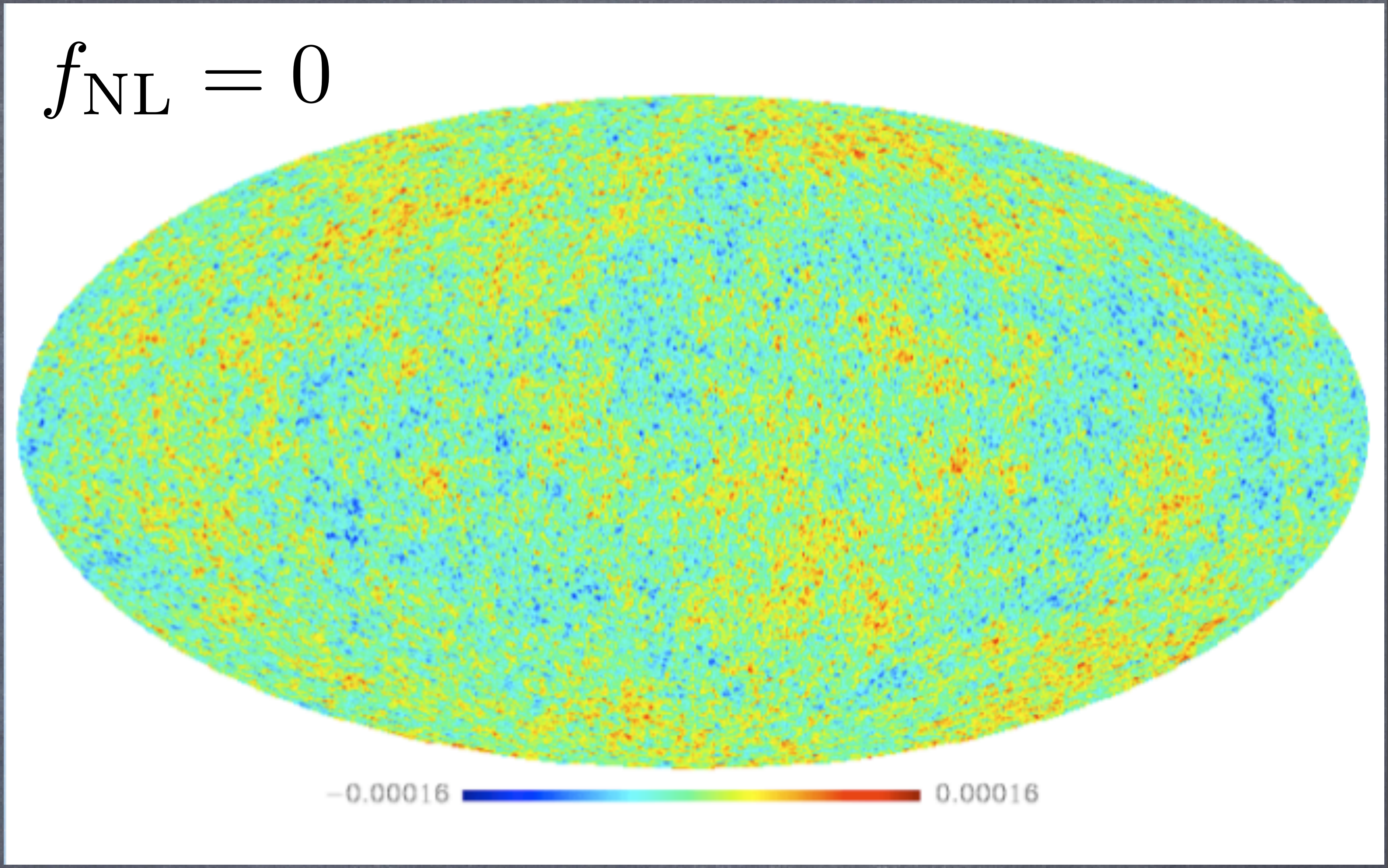
with $\boxed{P(k) \sim k^{2(n_s-1)}}$

$$\Longrightarrow \quad \boxed{n_s = 0.9603 \pm 0.0073} \quad \text{Planck (2013)}$$

- Approximately Gaussian

$$f_{\rm NL} \equiv -\frac{\langle \frac{\delta T}{T} \frac{\delta T}{T} \frac{\delta T}{T} \rangle}{\langle \frac{\delta T}{T} \frac{\delta T}{T} \rangle^2}$$

Liguori et al. (2007)



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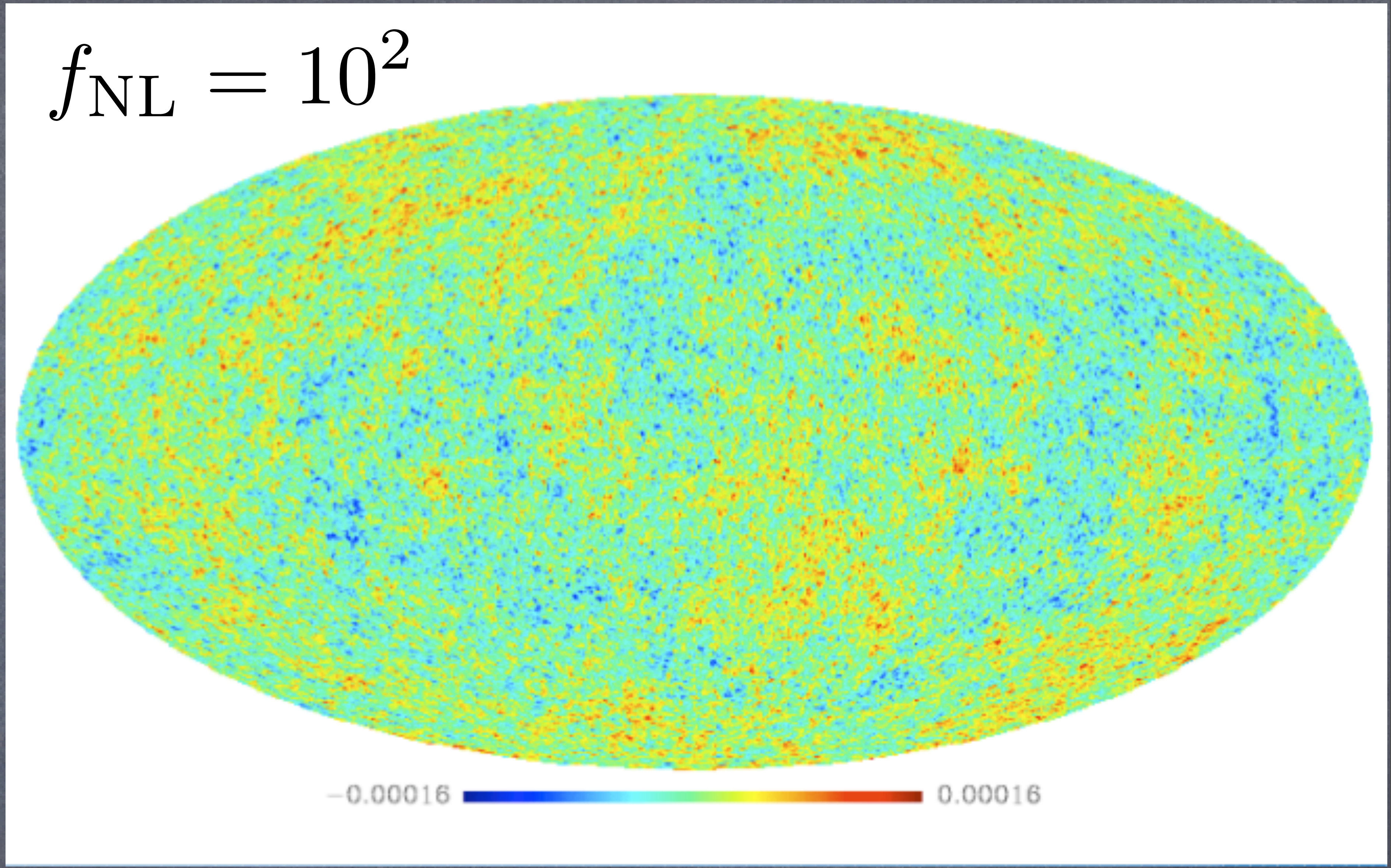

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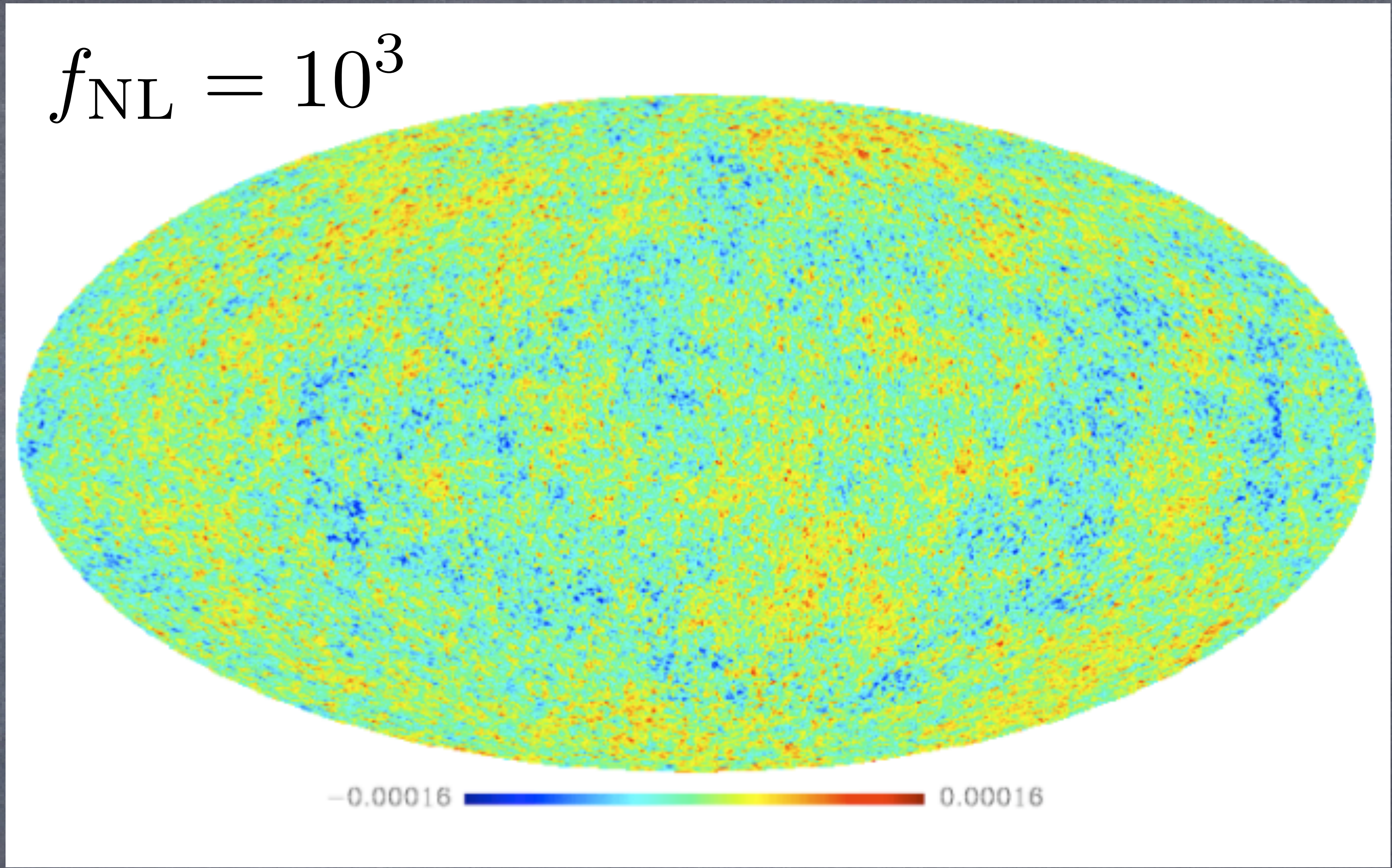

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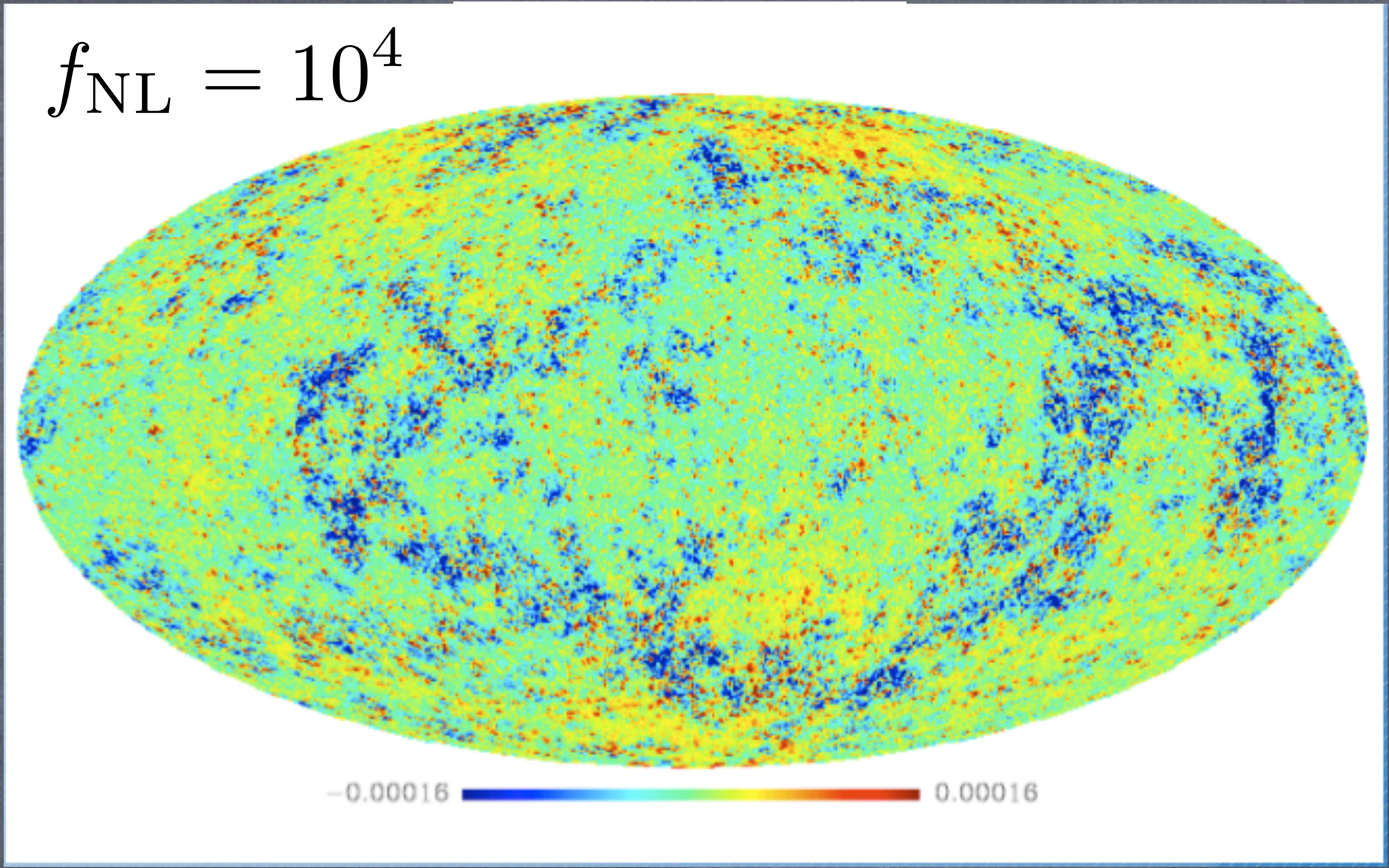

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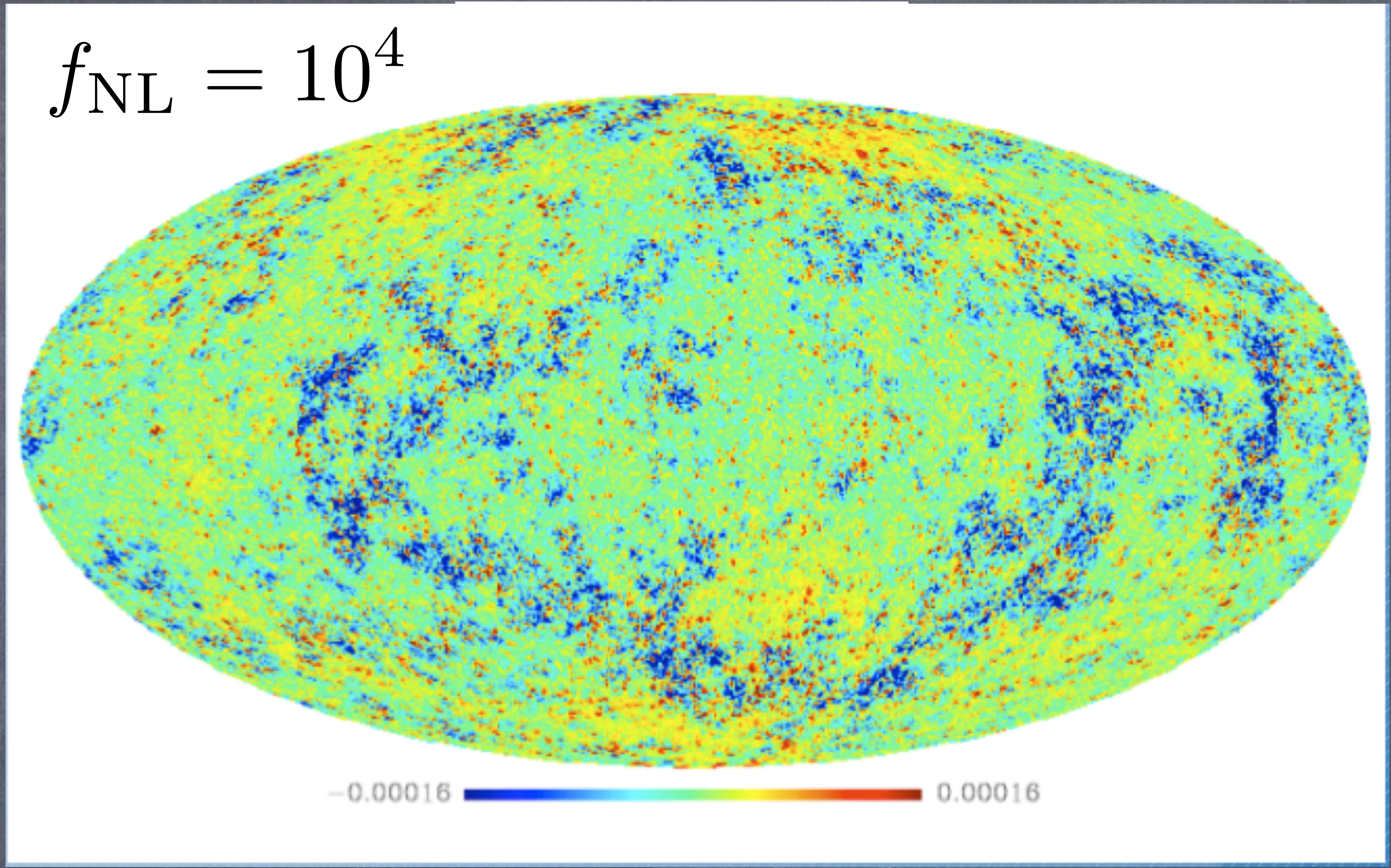
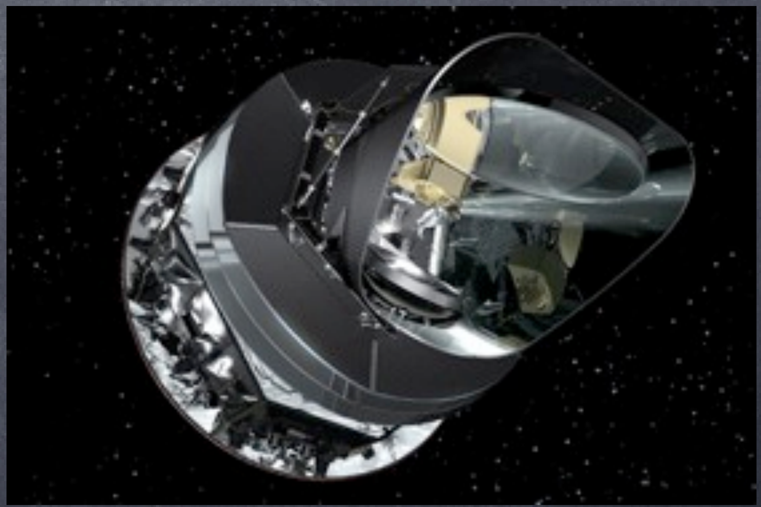



Liguori et al. (2007)

- Planck:

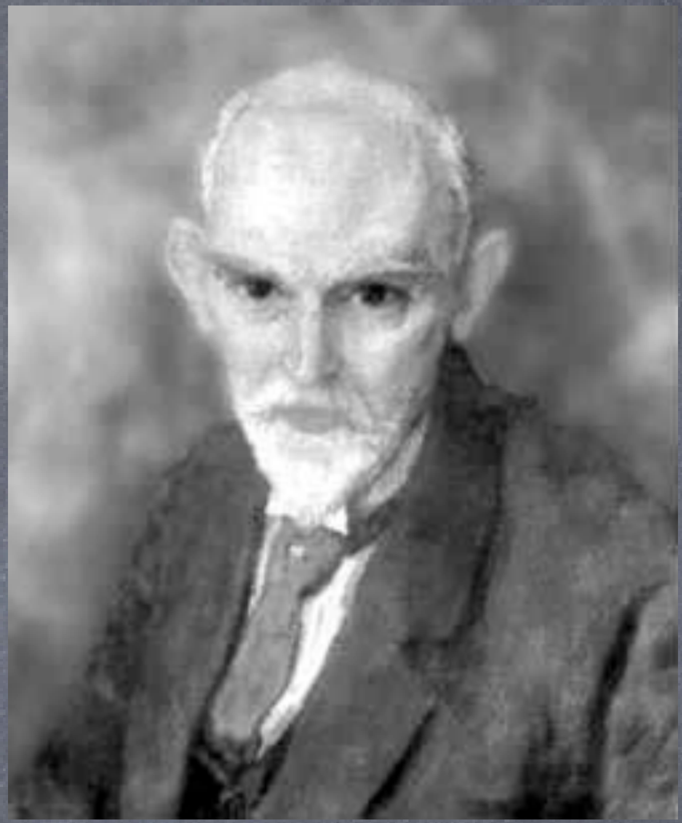
$$\boxed{f_{\rm NL}^{\rm local} = 2.7 \pm 5.8}$$



Gaussian at the 0.01% level !

# Inflation $\simeq$ de Sitter space

$$\boxed{\mathrm{d}s^2 = \frac{1}{H^2\tau^2}\left(-\mathrm{d}\tau^2 + \mathrm{d}\vec{x}^2\right)}$$



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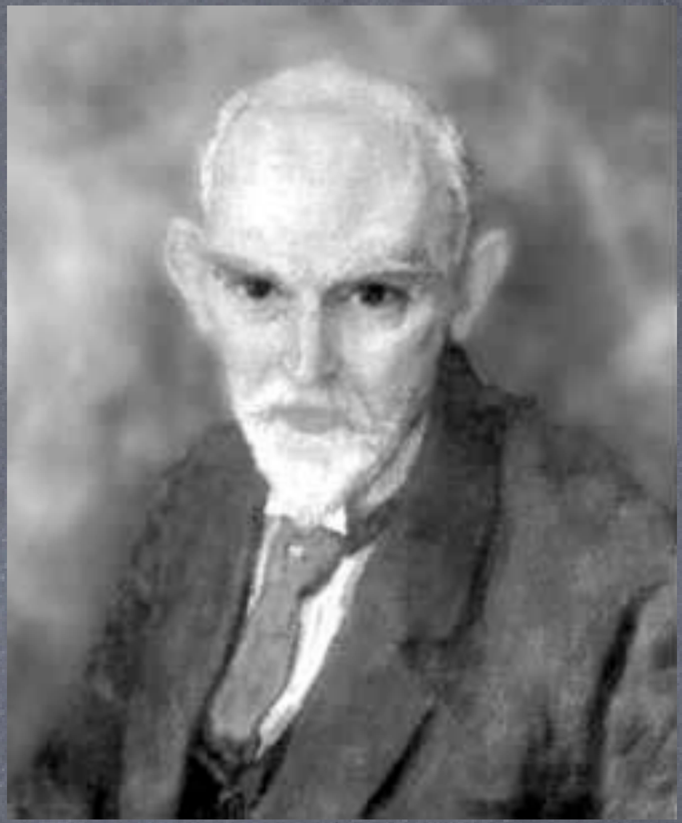

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- Transl'ns + Rot'ns
- Dilation:
- Special conformal transformations:

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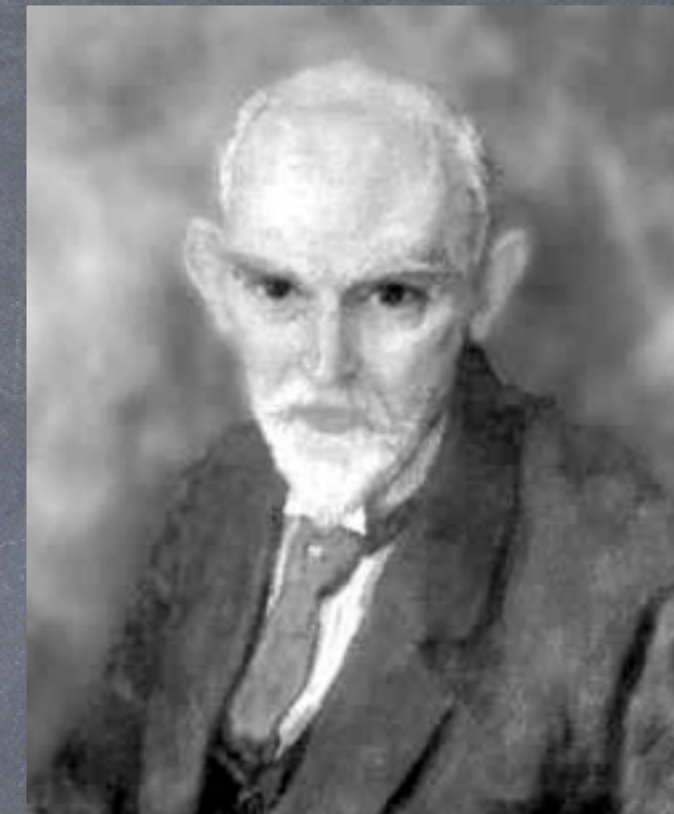

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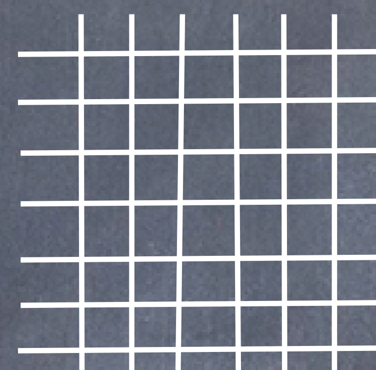

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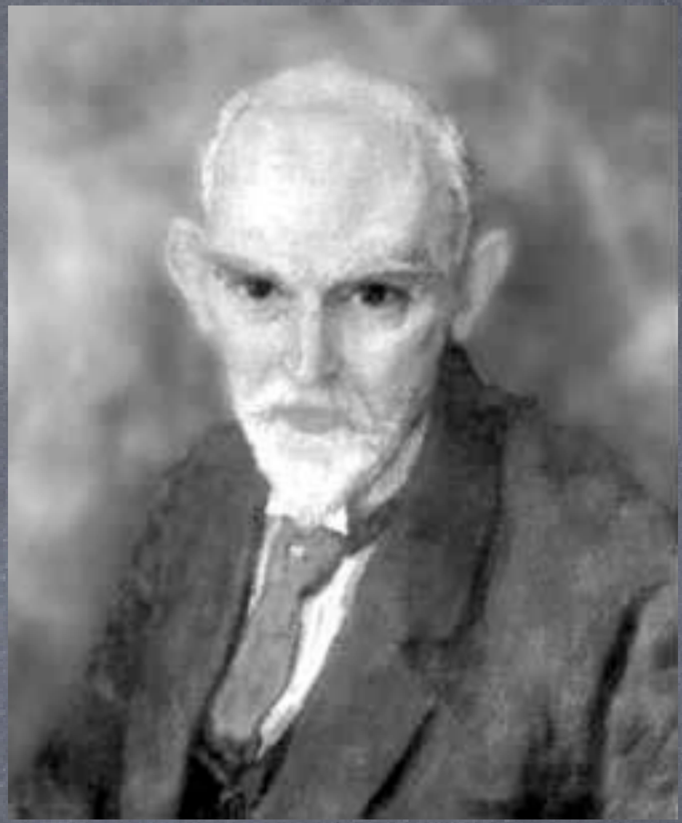

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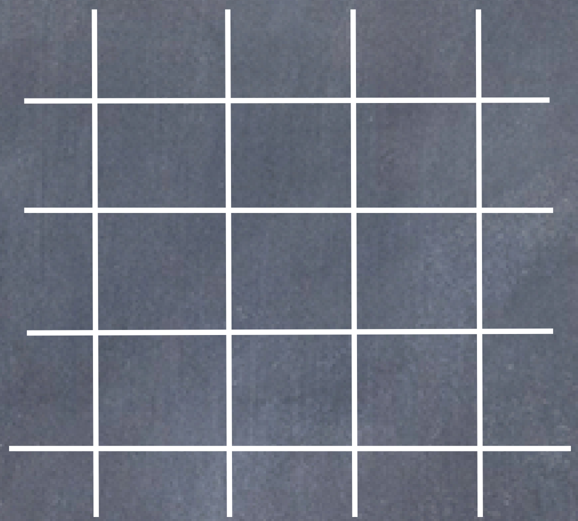

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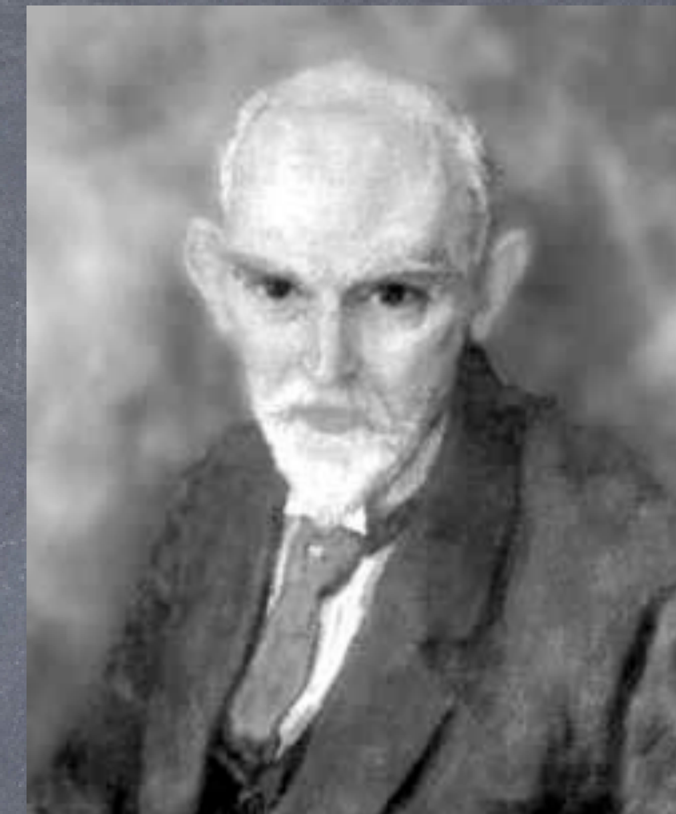

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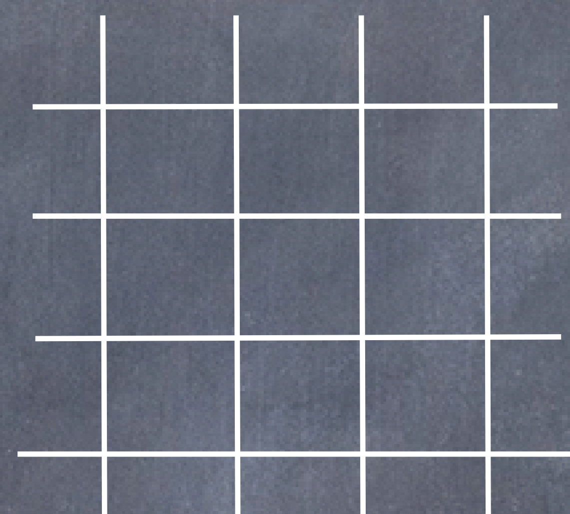

- Special conformal transformations:

| Inversion | | Translation | | Inversion |
|---|---|---|---|---|
| $\vec{x}' = \dfrac{\vec{x}}{x^2}$ | $\Rightarrow$ | $\vec{y} = \vec{x}' - \vec{b}$ | $\Rightarrow$ | $\vec{y}' = \dfrac{\vec{y}}{y^2}$ |

Their commutation relations form the $so(4,1)$ algebra

# Conformal correlators

Antoniadis, Mazur & Mottola (1997); Maldacena (2011); Creminelli (2011).

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2-point function (EXCEPT inflaton):

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with scaling dimension $\Delta = \dfrac{m_\chi^2}{3H^2}$

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3-point function also fixed by conformal invariance, up to overall normalization.

# Single-Field Inflation

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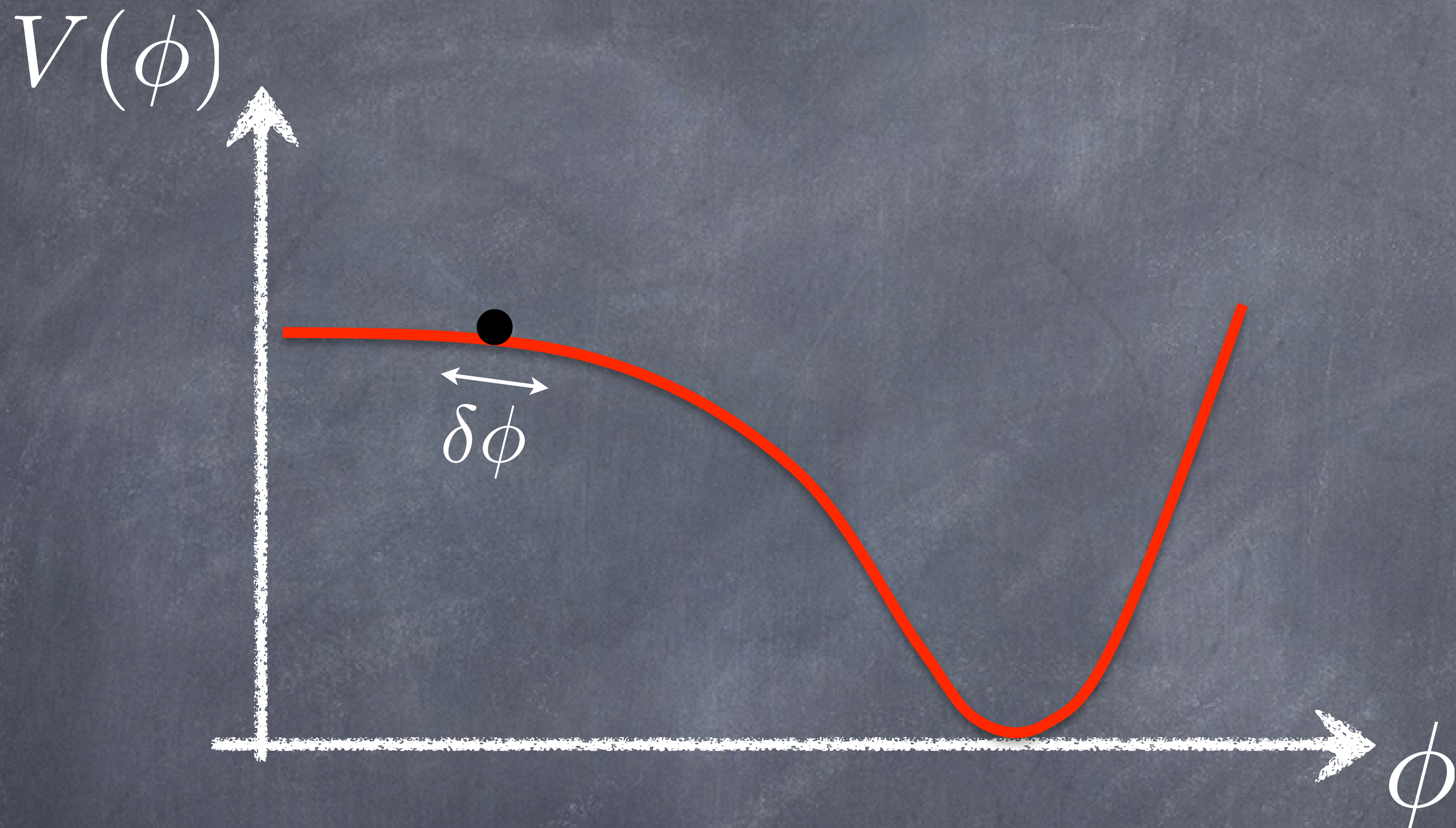


- Economical $\Longrightarrow$ Predictive

- Constrained by infinitely-many relations
(indep. of slow-roll, $c_s$ , $\phi$ fundamental or not)

# Inflaton perturbations

Hinterbichler, Hui & Khoury, JCAP (2012);
Creminelli, Norena & Simonovic, JCAP (2012);
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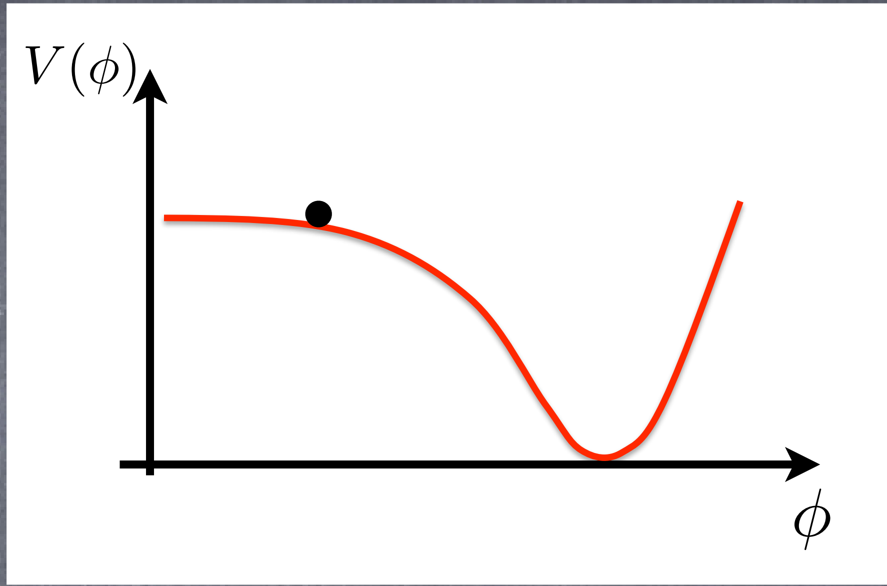


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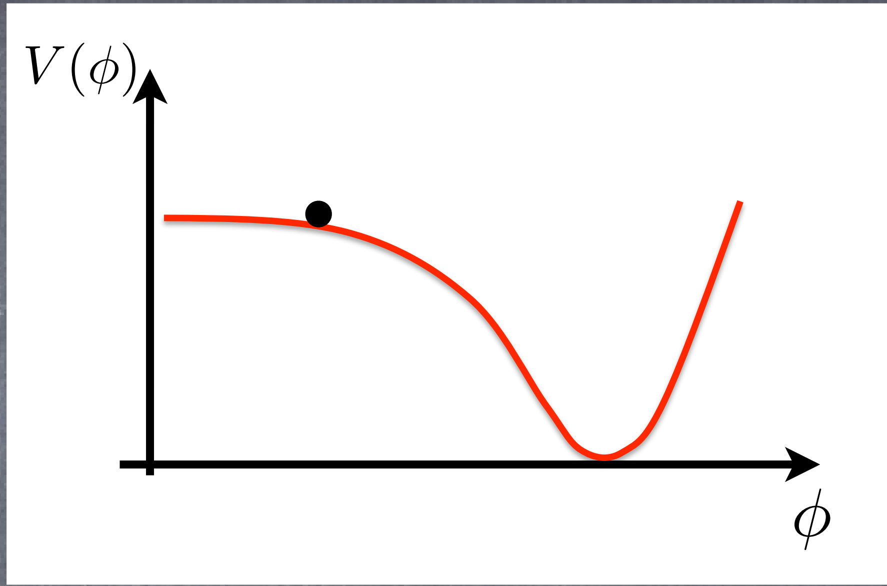


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Goldstone boson for the broken symmetries is the inflaton:

$$\boxed{so(4,1) \to \text{rotations} + \text{translations}}$$

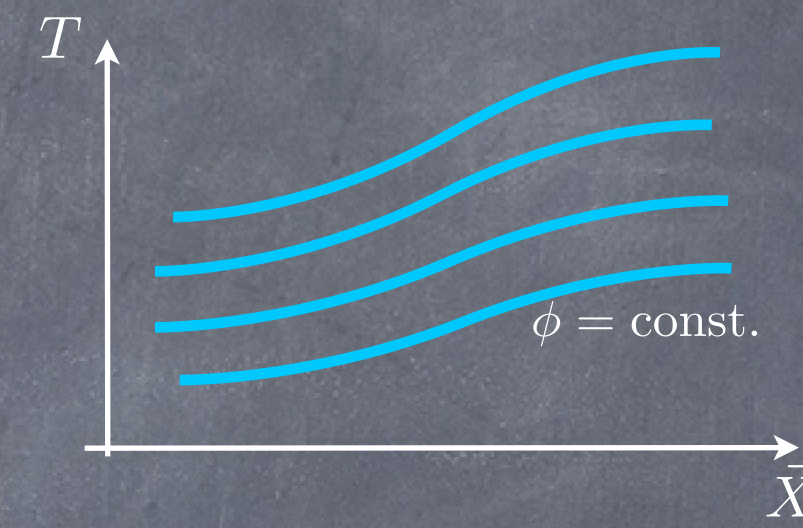
(Note: Usual counting of Goldstone bosons does not apply.)

# Scalar Perturbations

Bardeen, Steinhardt & Turner (1982);
Bond & Salopek (1990)

Comoving gauge:

$$\phi = \phi(t)\,; \qquad h_{ij} = a^2(t)e^{2\zeta(t,\vec{x})}\delta_{ij}$$



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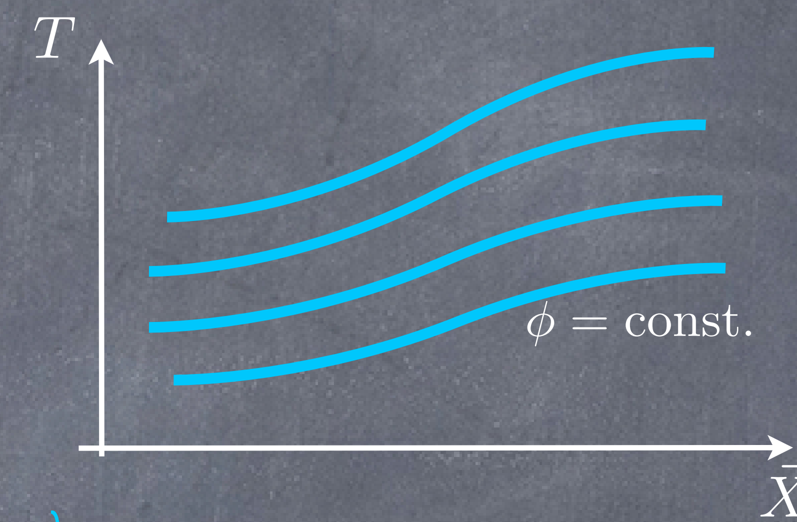


By shifting $\zeta$ , metric in this gauge is invariant under conformal transformations $\delta_{ij} \to e^{2\Omega(x)}\delta_{ij}$

- Rotations + Translations

Unbroken

- Dilation

$$x^i \to (1+\lambda)x^i$$

$$\delta\zeta = \lambda$$

- Special conformal transformations (SCTs)

$$x^i \to x^i + 2\,\vec{x}\cdot\vec{b}\,x^i - b^i\vec{x}^2$$

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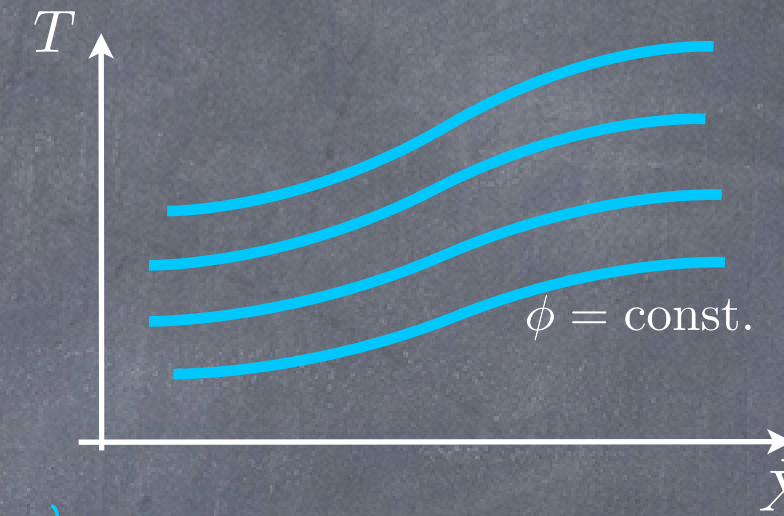


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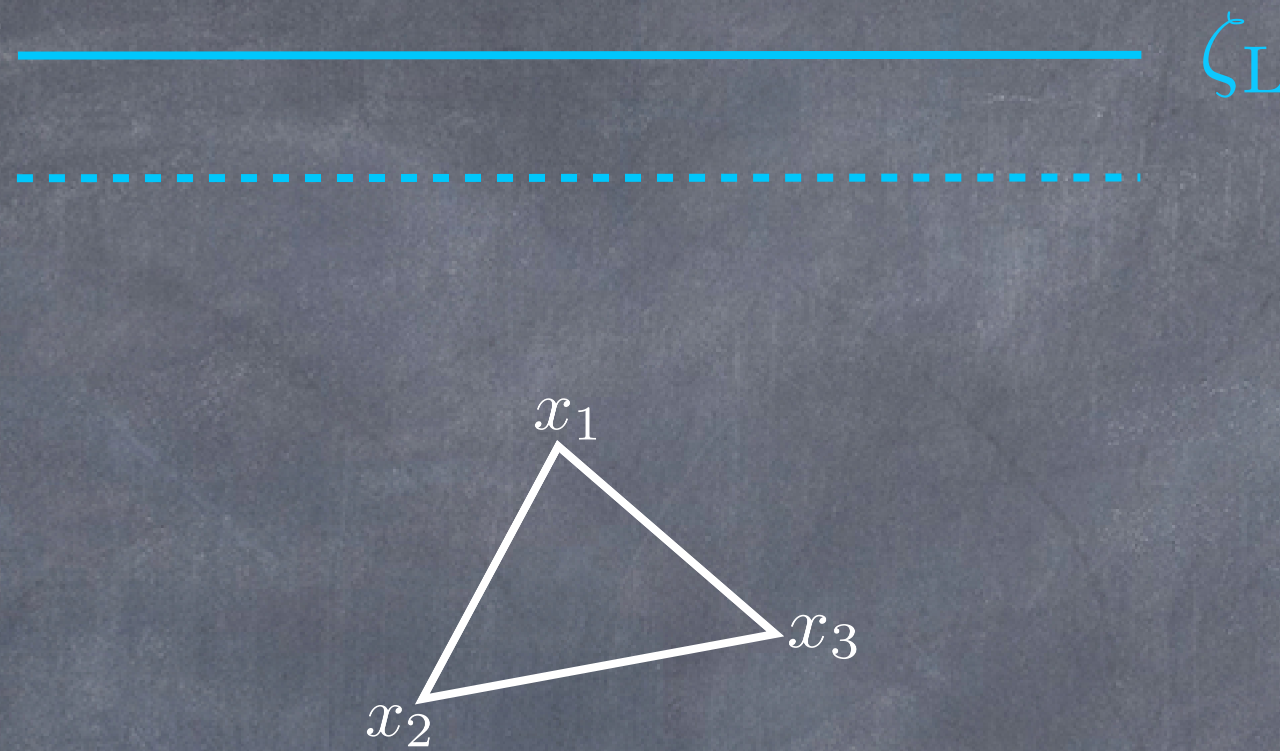
Dilation and SCTs: Spontaneously broken

$$\therefore \quad \boxed{so(4,1) \to \text{rotations} + \text{translations}}$$

$\zeta$ is Goldstone boson (dilaton) for the broken symmetries

Creminelli, Norena & Simonovic, 1203.4595; Hinterbichler, Hui & Khoury, 1203.6351

- Dilation



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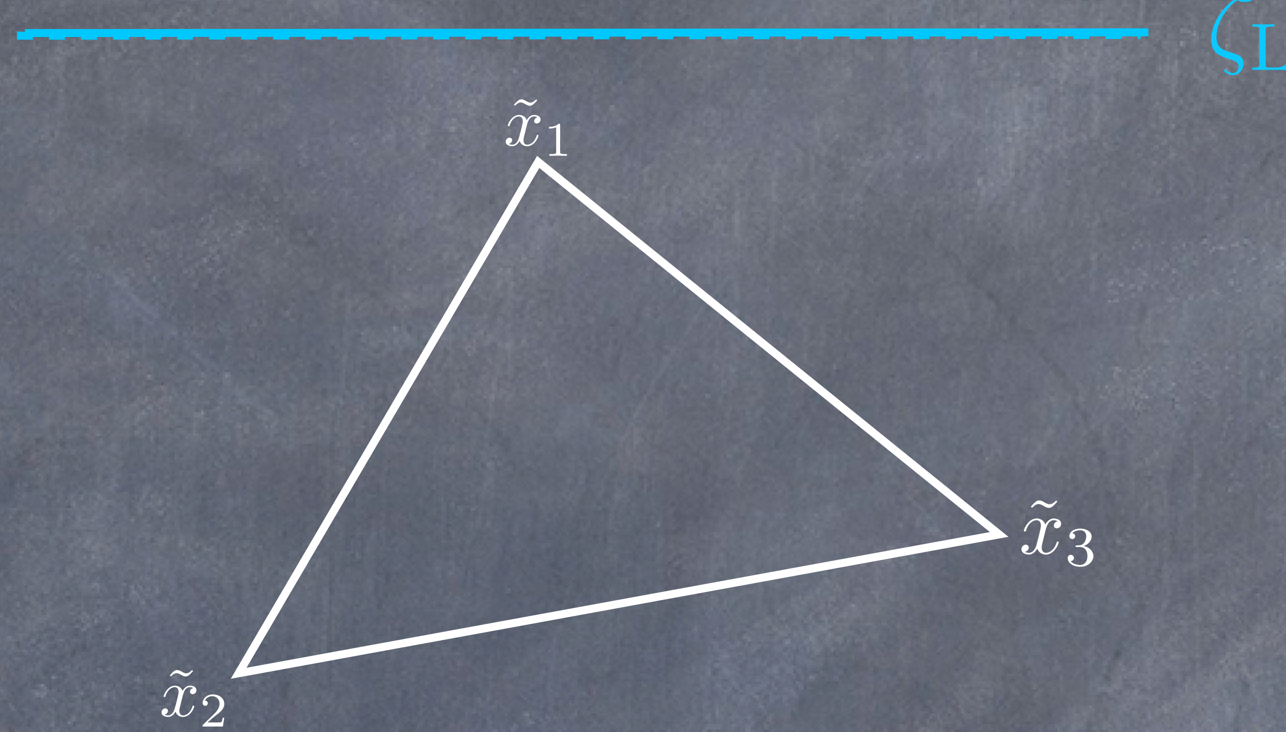

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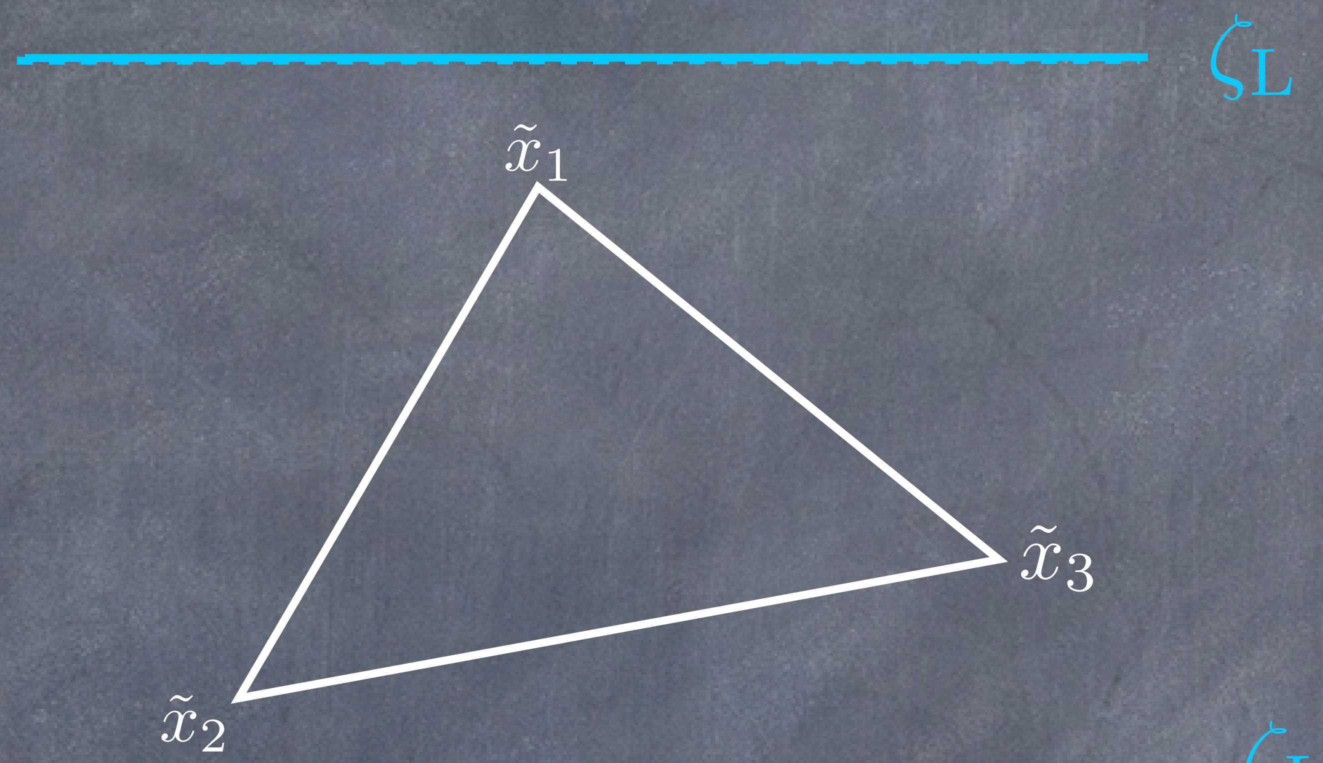
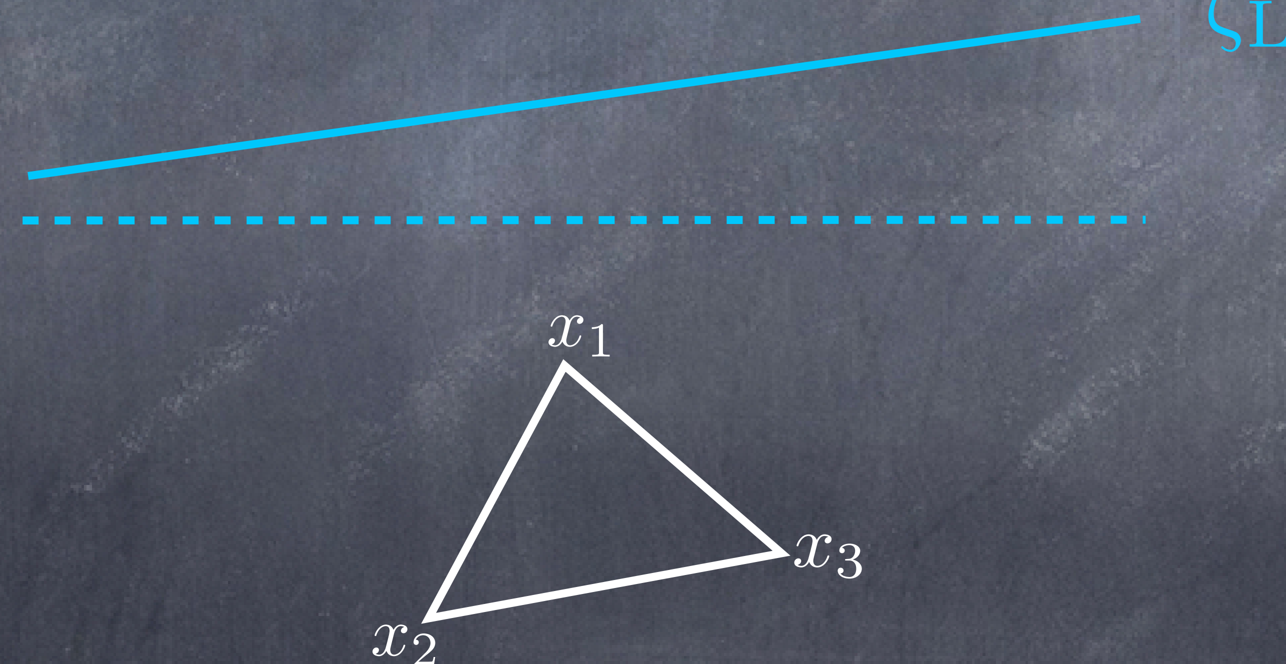



- SCT



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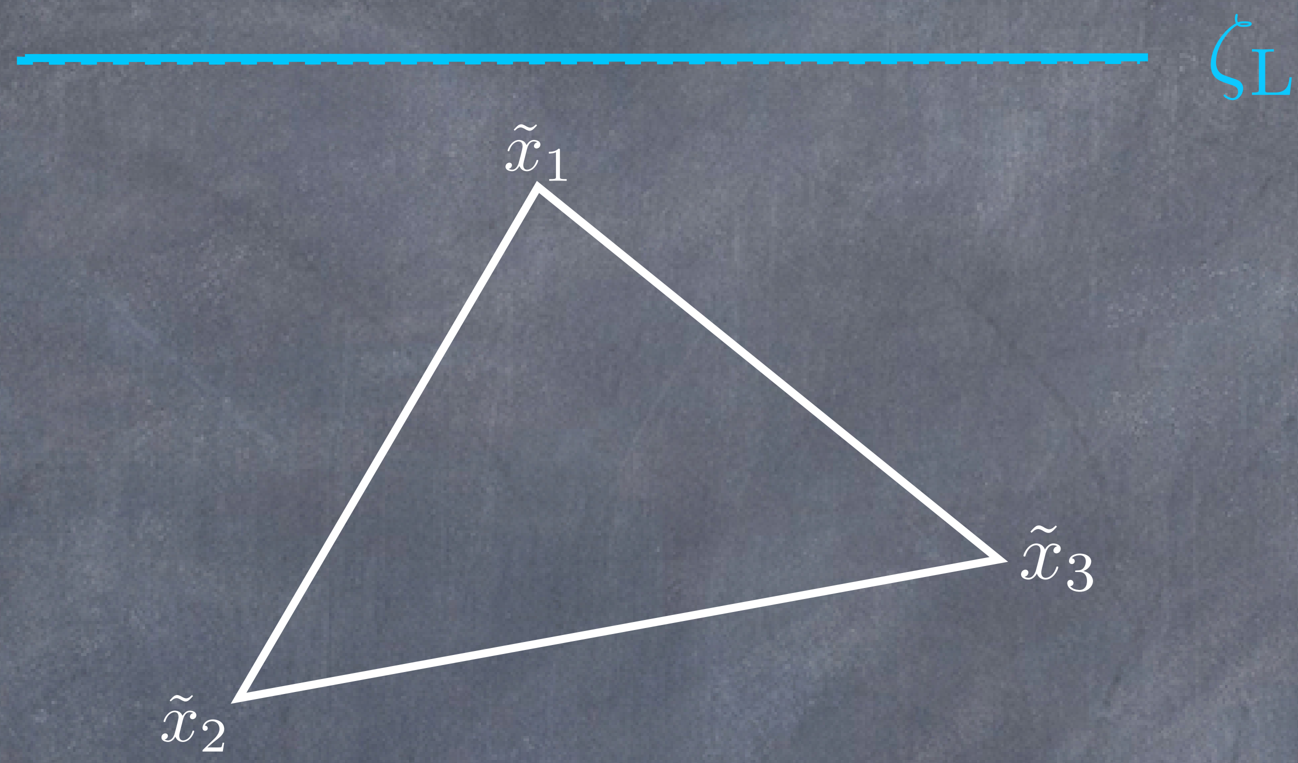


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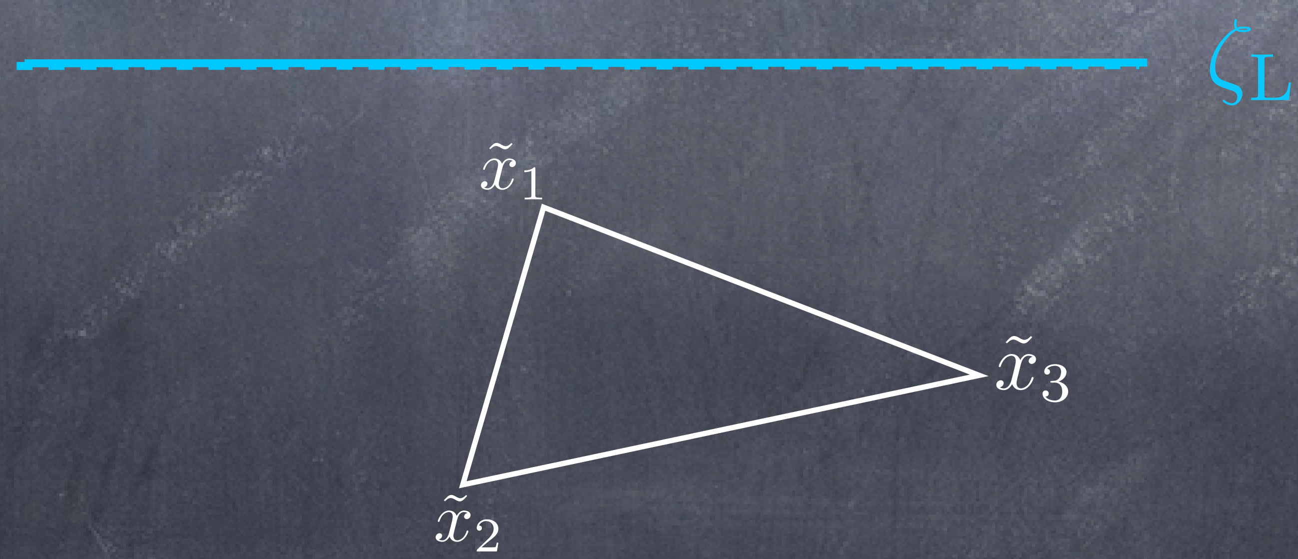

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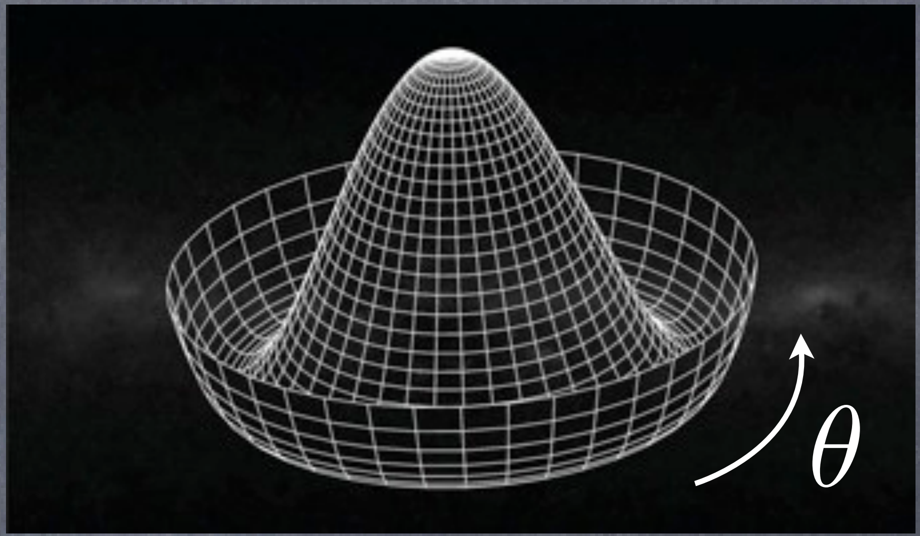


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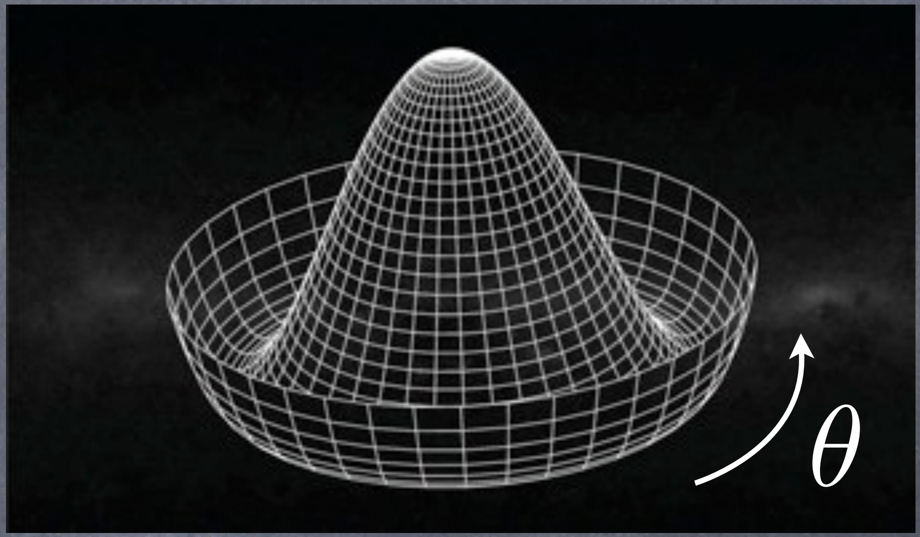


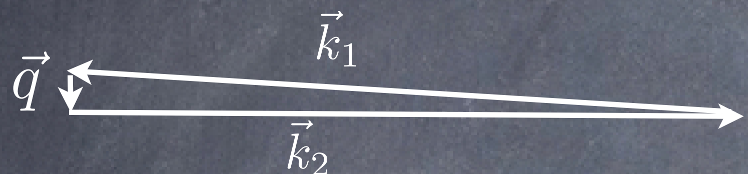

Soft Goldstone relations:

$$\lim_{\vec{q}\to 0} \underbrace{\langle \theta(\vec{q})\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle}_{N+1\text{-point}} \sim \underbrace{\langle \delta\mathcal{O}(\vec{k}_1,\ldots,\vec{k}_N)\rangle}_{N\text{-point}}$$

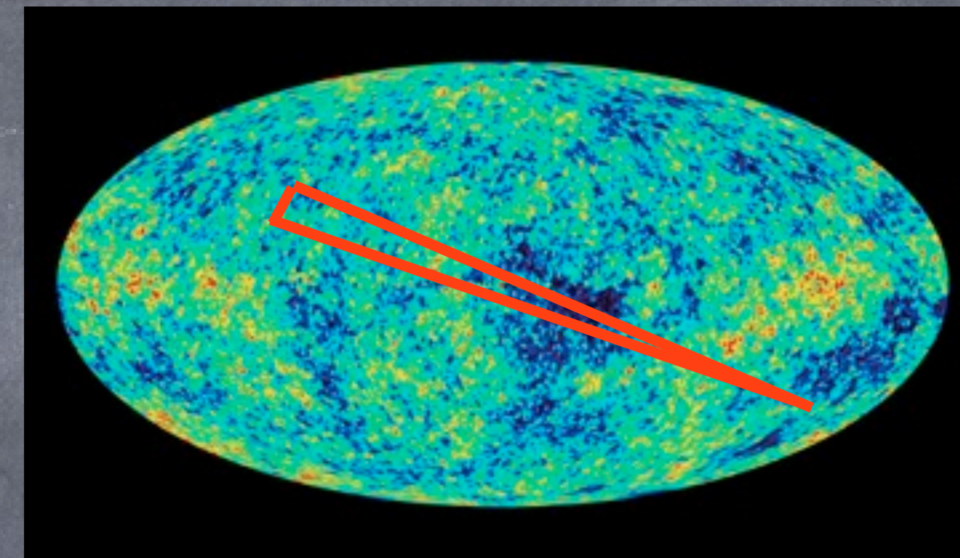
e.g. Soft pion theorems of the strong interactions

# Single-field inflation: consistency relations

$$\boxed{\lim_{\vec{q}\to 0} \frac{\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle}{P_\zeta(q)} = -(n_s - 1) P_\zeta(k_1)}$$




Maldacena (2002)



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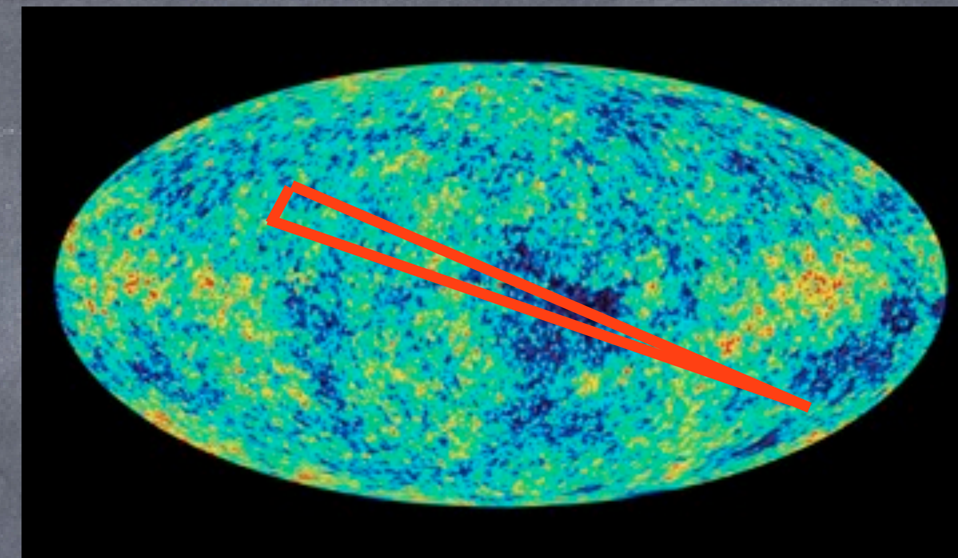

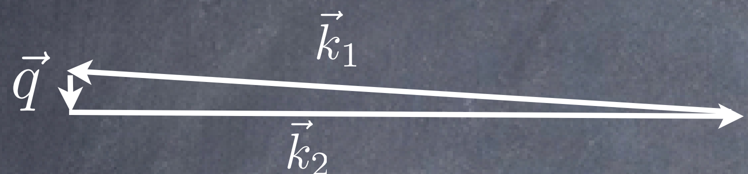


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- Holds in all inflationary models, under the assumptions:
  - single field
  - adiabatic (Bunch-Davies) vacuum
- Measuring (primordial) 3-point function in this limit

$\Longrightarrow$ automatically rules out all standard single-field models

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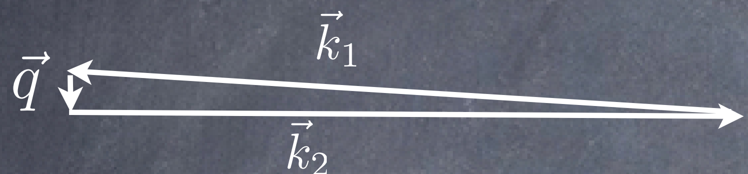


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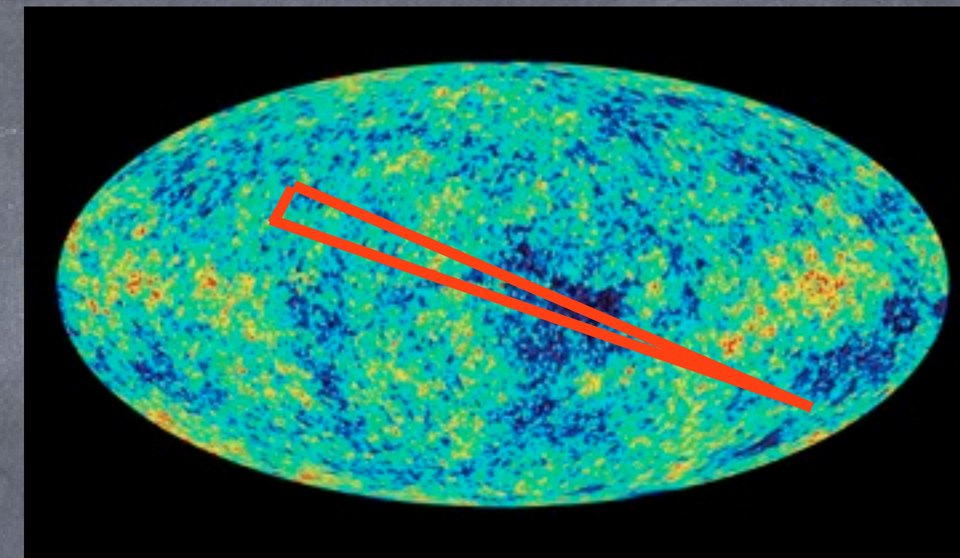

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  - Planck: $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$
- Understood as Ward identity for dilation

  Assassi, Baumann & Green (2012); Hinterbichler, Hui & Khoury, 1304.5527; Goldberger, Hui & Nicolis (2013)

# Generalized inflationay consistency relations

Hinterbichler, Hui and Khoury, 1304.5527

Single-field inflation constrained by infinite number of symmetries, corresponding to an infinite number of consistency relations:

$$\boxed{\lim_{\vec{q}\to 0} \frac{\partial^n}{\partial q^n}\left(\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\mathcal{O}(\vec{k}_a)\rangle + \frac{1}{P_\gamma(q)}\langle\gamma(\vec{q})\mathcal{O}(\vec{k}_a)\rangle\right) \sim \frac{\partial^n}{\partial k_a^n}\langle\mathcal{O}(\vec{k}_a)\rangle}$$

- $q^0$ and $q$ behavior completely fixed
- $q^n$ , $n \geq 2$ , behavior partially fixed
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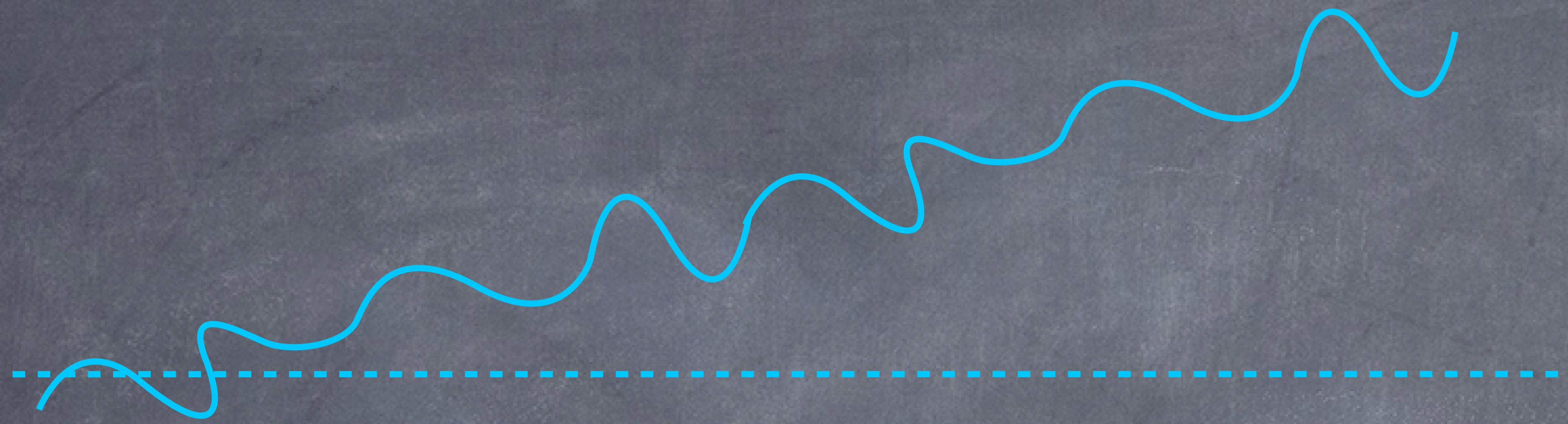
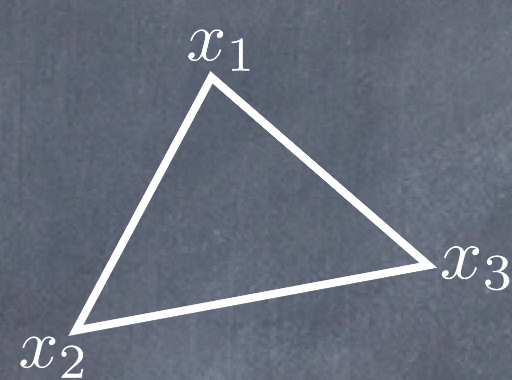
## Master consistency relation

Berezhiani and Khoury, 1309.4461

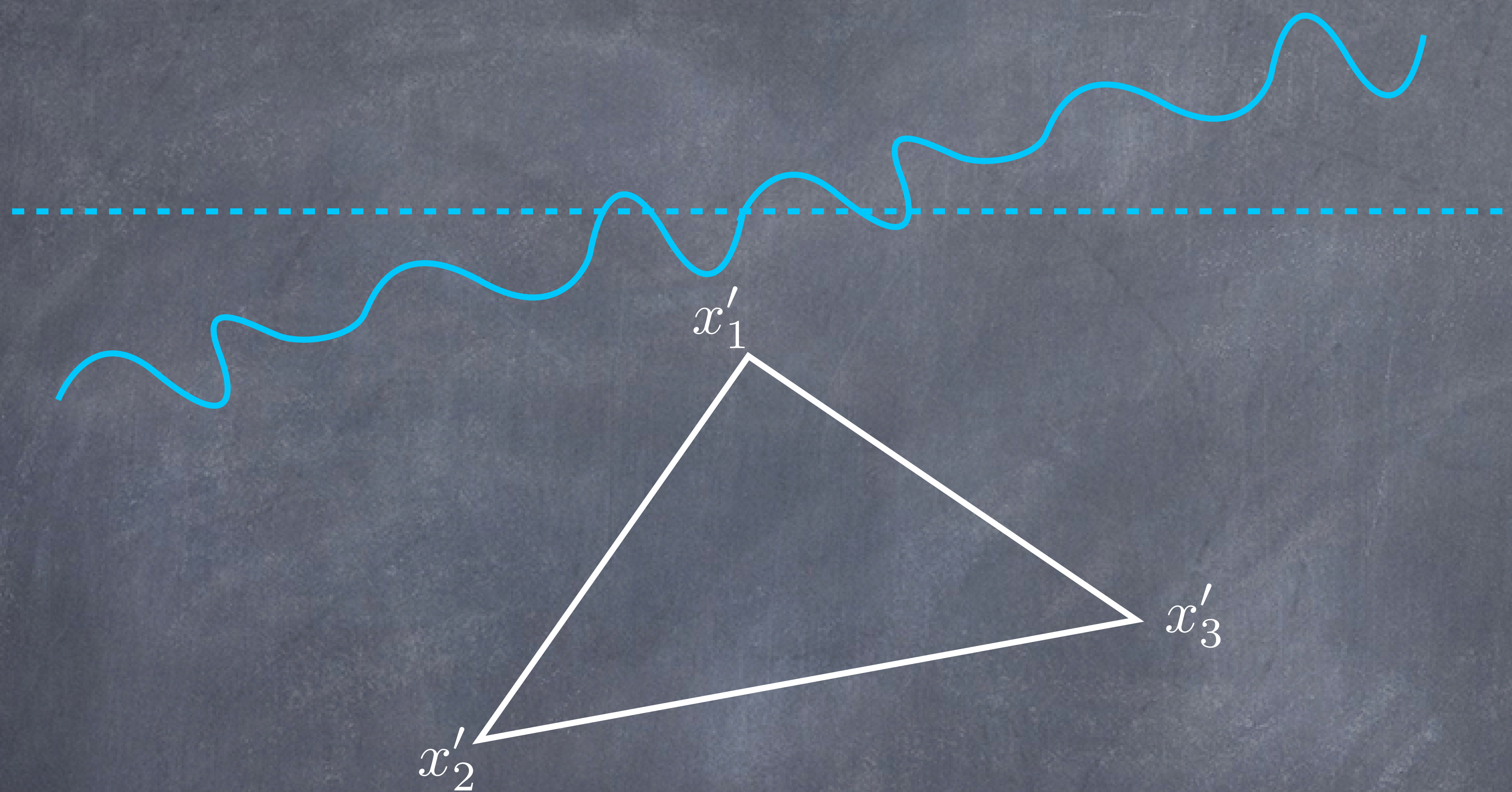
Spatial diffeomorphisms imply the Slavnov-Taylor identity:

$$\boxed{\frac{1}{3}q_i\Gamma^{\zeta\zeta\zeta}(\vec{q},\vec{p},-\vec{q}-\vec{p}) + 2q^j\Gamma^{\gamma\zeta\zeta}_{ij}(\vec{q},\vec{p},-\vec{q}-\vec{p}) = q_i\Gamma_\zeta(p) - p_i\Big(\Gamma_\zeta(|\vec{q}+\vec{p}|) - \Gamma_\zeta(p)\Big)}$$

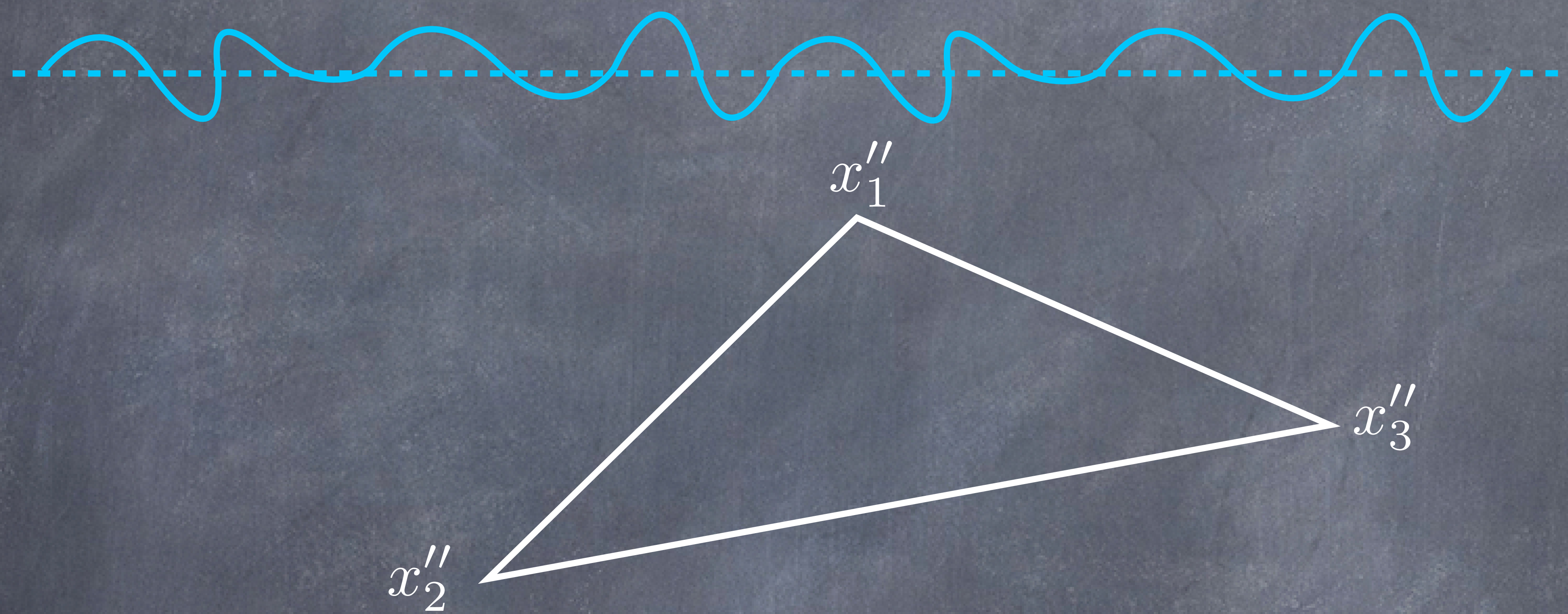




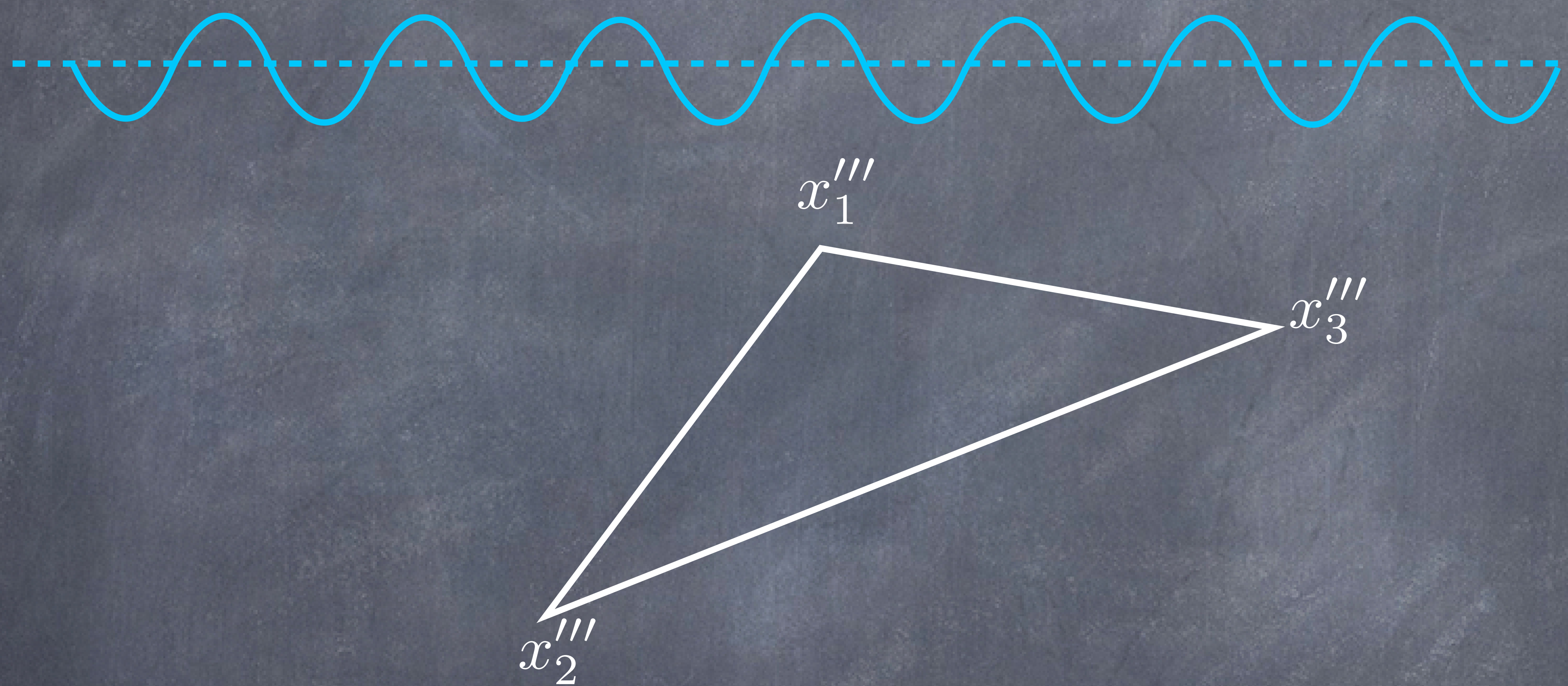

$$h_{ij} = \bar{h}_{ij} + \partial_k \bar{h}_{ij} x^k + \frac{1}{2} \partial_k \partial_\ell \bar{h}_{ij} x^k x^\ell + \dots$$




$$h_{ij} = \delta_{ij} + \partial_k \bar{h}_{ij} x'^k + \frac{1}{2} \partial_k \partial_\ell \bar{h}_{ij} x'^k x'^\ell + \ldots$$




$$h_{ij} = \delta_{ij} + 0 + \frac{1}{2}\partial_k \partial_\ell \bar{h}_{ij} x''^k x''^\ell + \dots$$




$$h_{ij} = \delta_{ij} \quad - \quad \frac{1}{3} R_{ijk\ell} x'''^k x'''^\ell \quad + \ldots$$

Geometric counting matches # of symmetries

## Multiple soft limits

Joyce, Khoury and Simonovic, to appear

Another probe of $\mathcal{O}(q^n)$ behavior is to consider multiple soft limits

e.g. $$\lim_{\vec{q}_1,\vec{q}_2\to 0} \frac{\partial^2}{\partial q_1 \partial q_2} \frac{\langle \zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle}{P_\zeta(q_1)P_\zeta(q_2)} = D^2 P_\zeta(k) - \frac{\partial}{\partial q_1}\frac{\langle \zeta_{\vec{q}_1}\zeta_{\vec{q}_2}\zeta_{-\vec{q}_1-\vec{q}_2}\rangle}{P_\zeta(q_1)P_\zeta(q_2)} D P_\zeta(k)$$

Constrains $\vec{q}_1 \cdot \vec{q}_2$ part of the 4-pt function.

# The Conformal Scenario

## An Old Idea...

Could scale invariance observed in CMB/LSS have originated from conformal invariance in early universe?

# Conformal Scenario

Rubakov (2009); Creminelli, Nicolis & Trincherini (2010);
Hinterbichler & Khoury (2011) ; Hinterbichler, Khoury & Joyce (2012)

- Non-inflationary scenario, takes place before the big bang
- Space-time is nearly static, i.e. $\approx$ flat, Minkowski space

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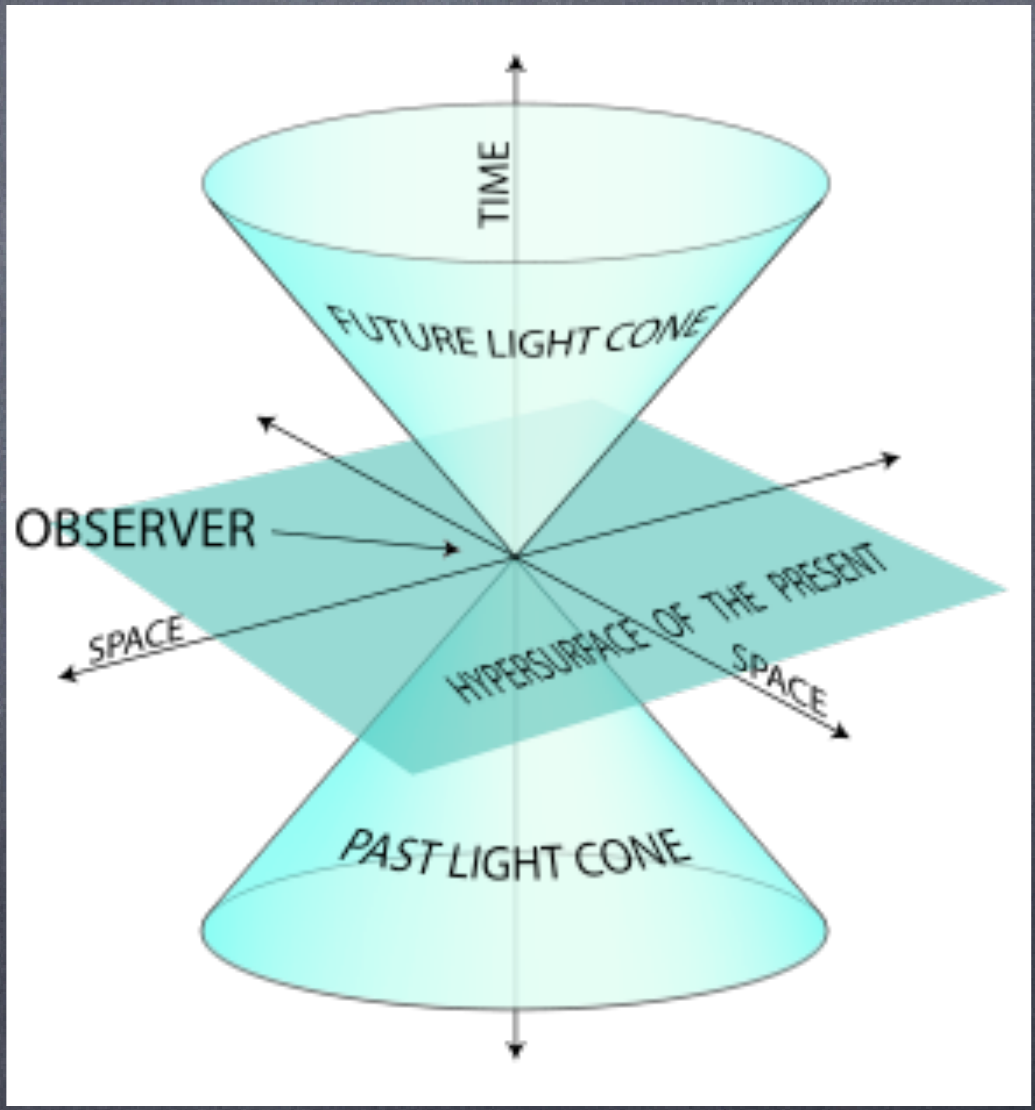
Minkowski space

- 4 space-time translations
- 6 Lorentz transformations



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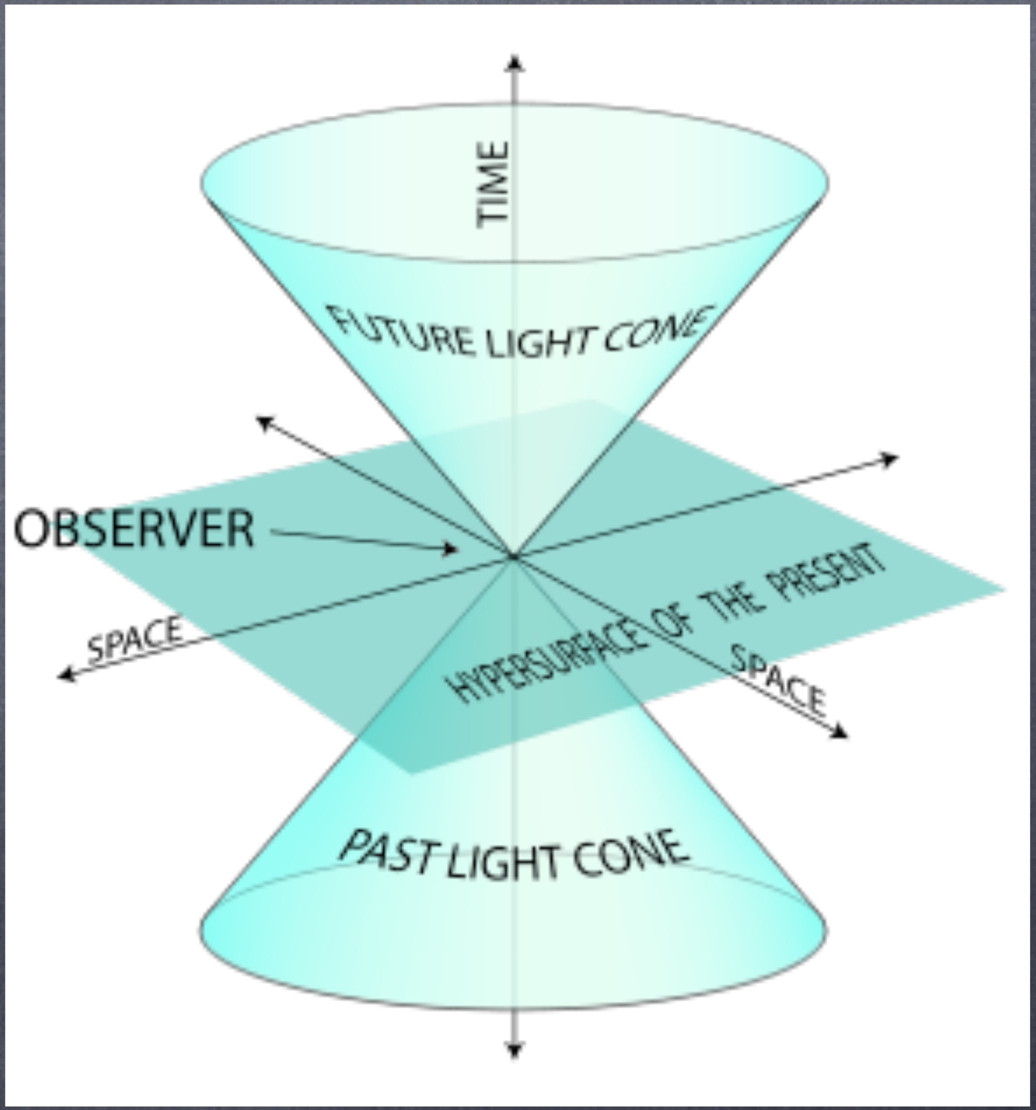
\+ conformal symmetries:

- 1 dilation
- 4 special conformal transf'ns

= 15 symmetries $(so(4,2))$

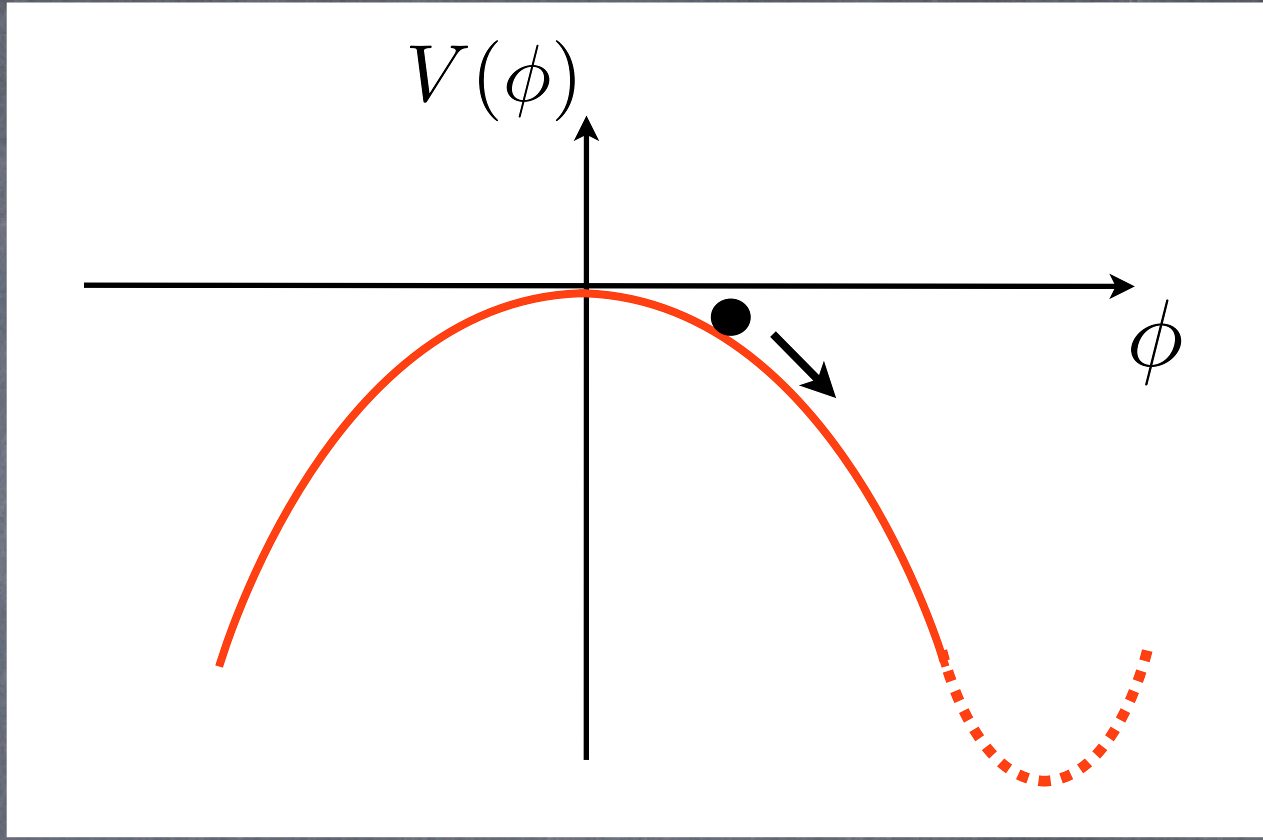


# Simplest Example

Rubakov (2009); Craps, Hertog & Turok (2007);
Hinterbichler & Khoury, 1106.1428

$$\boxed{V(\phi) = -\frac{\lambda}{4}\phi^4}$$

$\lambda > 0 \implies$ asymptotically free



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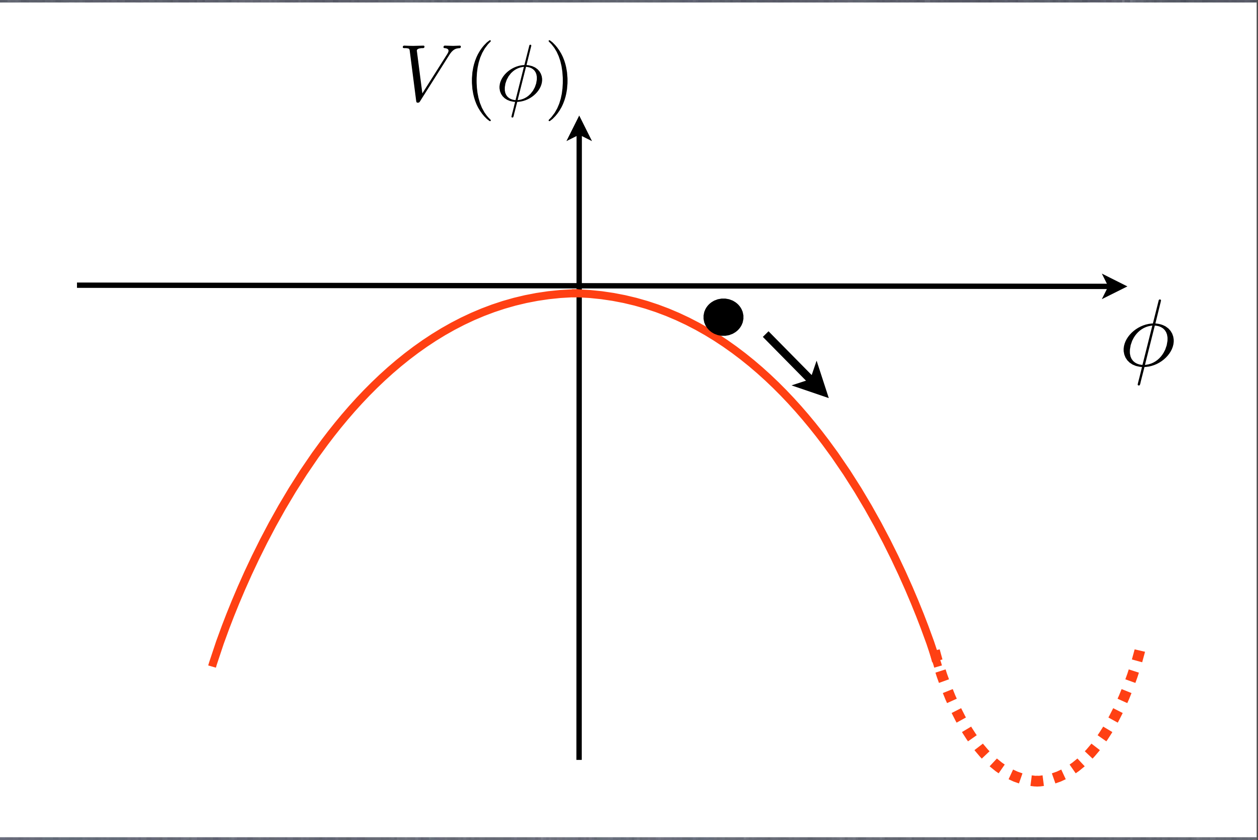



As time goes on, $\phi$ rolls off:

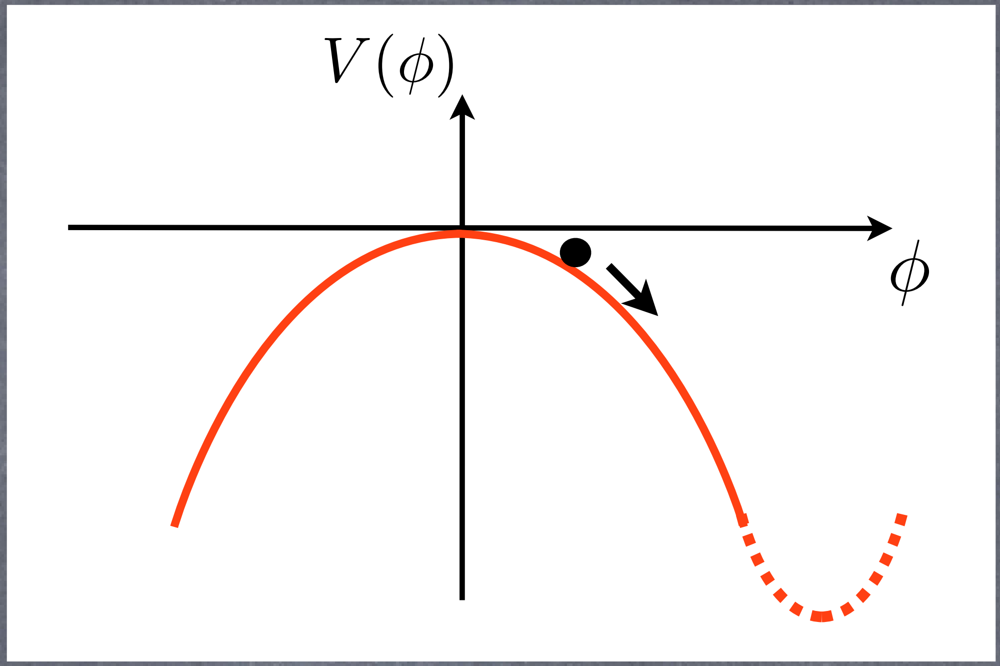
$$E = \frac{1}{2}\dot{\phi}^2 - \frac{\lambda}{4}\phi^4$$

Particular solution is $E = 0$ : $\boxed{\phi(t) = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}}$ $\quad -\infty < t < 0$

This is an <u>attractor</u>: Growing mode = time shift.

$$\boxed{\phi(t) = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}}$$

Preserves dilation



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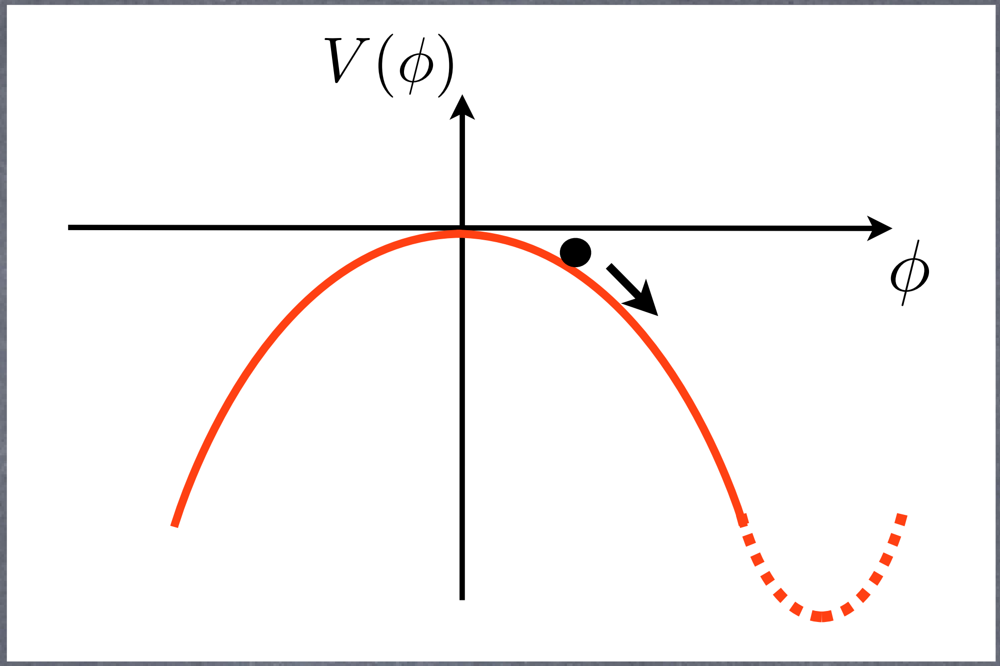


$$\underbrace{\text{15 original symmetries}}_{so(4,2)} \longrightarrow \underbrace{\text{10 unbroken symmetries}}_{so(4,1)}$$

(de Sitter symmetries)

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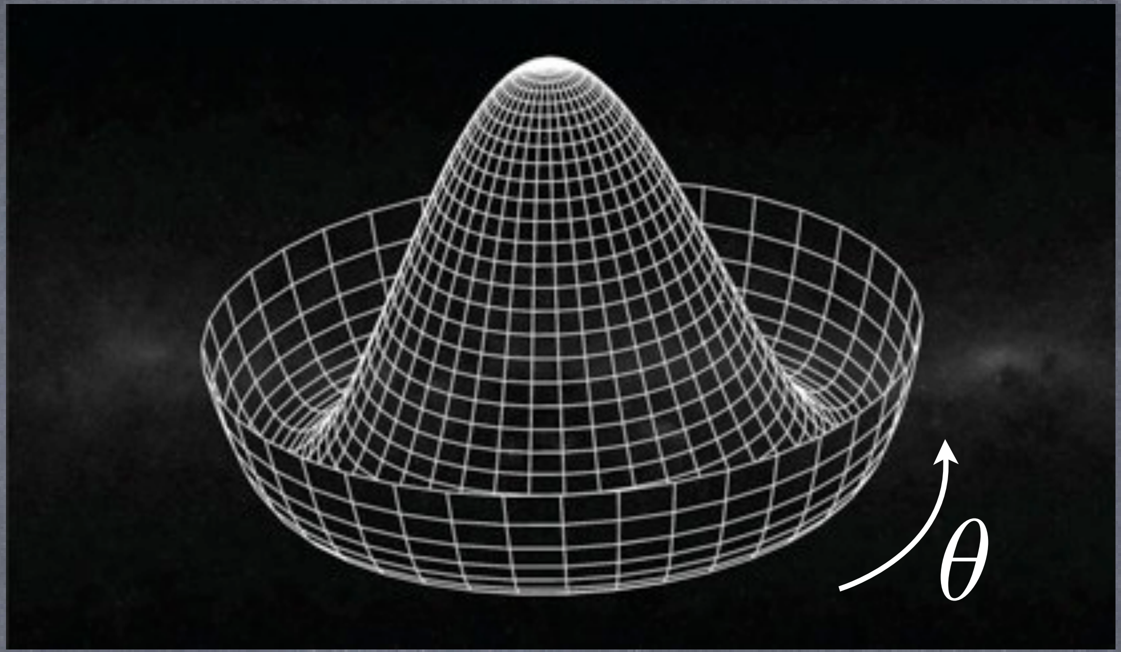



$$\underbrace{\text{15 original symmetries}}_{so(4,2)} \longrightarrow \underbrace{\text{10 unbroken symmetries}}_{so(4,1)}$$

(de Sitter symmetries)

Angular field acquires scale invariant spectrum:

$$\mathcal{L}_\theta = -\frac{1}{2}\phi^2(\partial\theta)^2 \sim \frac{1}{t^2}(\partial\theta)^2 + \ldots$$

Exactly like massless field in de Sitter!

# Other Realizations

$$\boxed{\bar{\phi}_I(t) \sim \frac{1}{(-t)^{\Delta_I}}}$$

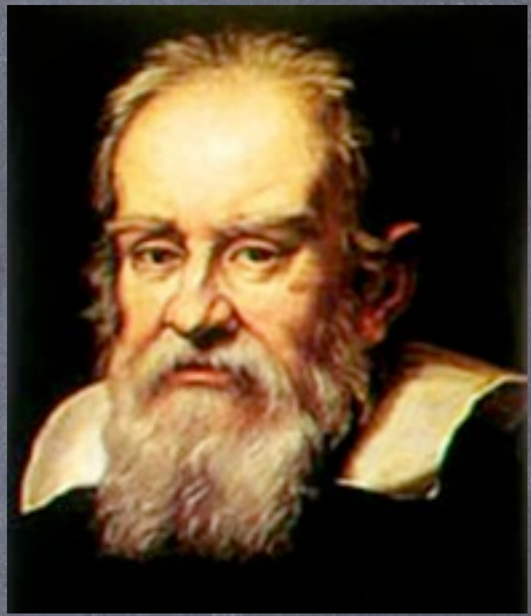
$\Delta_I =$ conformal weight

$$\Longrightarrow \quad so(4,2) \to so(4,1)$$

- Galilean Genesis

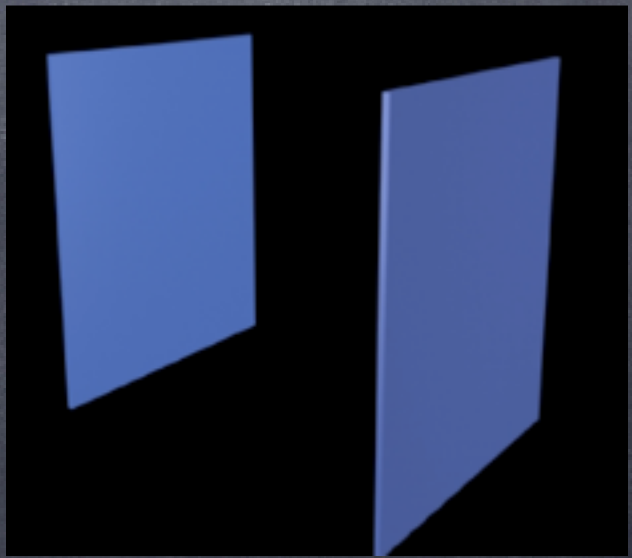
  Creminelli, Nicolis & Trincherini (2010); Creminelli, Hinterbichler, Khoury, Nicolis & Trincherini (2012)

  Universe is slowly expanding from asymptotically static past.



- Brane-world (DBI) realizations

  Hinterbichler & Khoury (2011); Hinterbichler, Joyce, Khoury & Miller (2012)



As usual in spontaneous symmetry breaking, much of the physics derives from symmetry breaking pattern,

$$so(4,2) \rightarrow so(4,1)$$

irrespective of underlying microphysical theory.

Hinterbichler, Joyce & Khoury, 1202.6056

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irrespective of underlying microphysical theory.

e.g., Goldstone action: $\phi = \bar{\phi}(t) + \pi$

$$\boxed{\begin{aligned}\mathcal{L}_\pi &= M_0^2\left(-\frac{1}{2}e^{2\pi}(\partial\pi)^2 - H^2e^{2\pi} + \frac{H^2}{2}e^{4\pi}\right) + M_1\Big((\bar{\Box}\pi)^2 + 2\bar{\Box}\pi(\partial\pi)^2 + (\partial\pi)^4 - 4H^2(\partial\pi)^2\Big) \\ &+ M_2\Big((\partial\pi)^4 + 2\bar{\Box}\pi(\partial\pi)^2 + 6H^2(\partial\pi)^2\Big) + \ldots\end{aligned}}$$

# Cosmology

Hinterbichler, Joyce, Khoury & Miller, 1209.5742

By symmetries, $$\boxed{\rho_{\rm CFT} \simeq 0 \ ; \qquad P_{\rm CFT} \simeq \frac{\beta}{t^4}}$$

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Solve Einstein's eqns: $$\dot{H} = -\frac{1}{2M_{\rm Pl}^2}\left(\rho_{\rm CFT} + P_{\rm CFT}\right)$$

$$\Longrightarrow \quad \boxed{H(t) \approx \frac{\beta}{6t^3 M_{\rm Pl}^2}} \quad \text{and} \quad \boxed{a(t) \simeq 1 - \frac{\beta}{12t^2 M_{\rm Pl}^2}}$$

Nearly static universe

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Nearly static universe

- Universe becomes increasingly flat and homogeneous

(Akin to ekpyrotic cosmologies Gratton, Khoury, Steinhardt & Turok (2003) )

# Symmetries and Consistency Relations

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Correlation fcns are $so(4,1)$ invariant (i.e. conformally inv.)

  e.g. $\langle \chi(\vec{x},\tau)\chi(\vec{x}',\tau)\rangle \sim |\vec{x}-\vec{x}'|^{-2\Delta}$

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- Prediction for $f_{\rm NL}$ is model-dependent

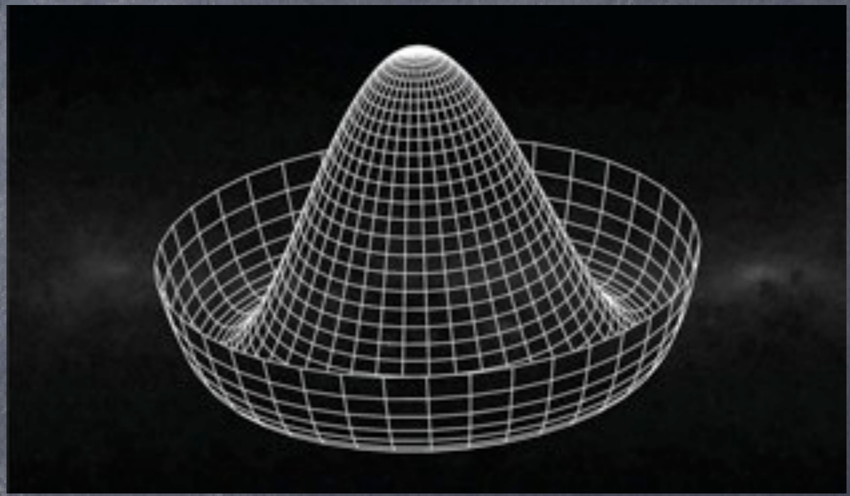
  Rubakov's model: $\langle \theta\theta\theta \rangle = 0$

  Contribution to $f_{\rm NL}$ comes from conversion

  $$\theta \to \frac{\delta\rho}{\rho}$$

  e.g. Modulated reheating with $\Gamma \ll H$ : $f_{\rm NL} = 3$

  Zaldarriaga (2003)



# Model-independent predictions

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

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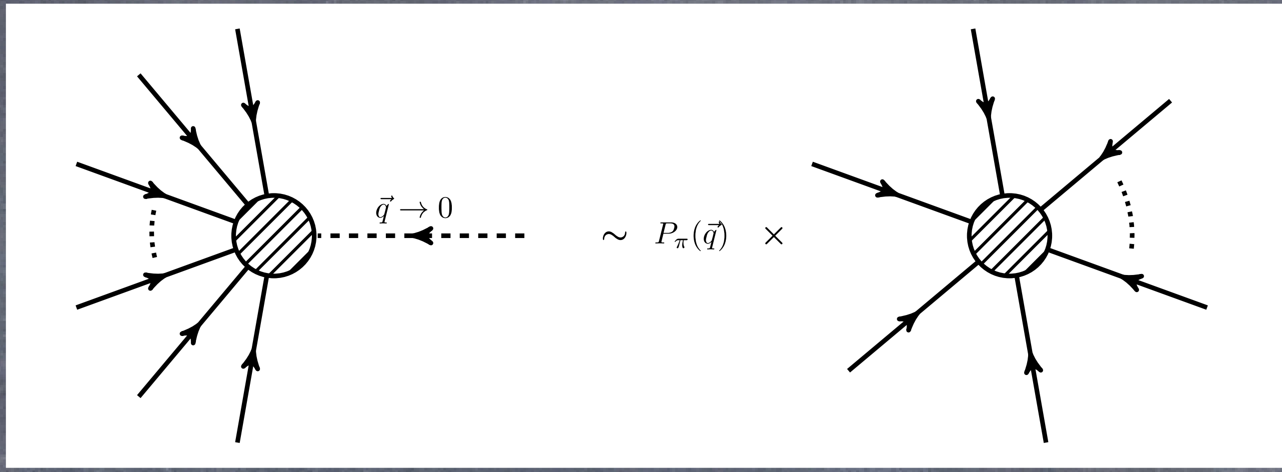


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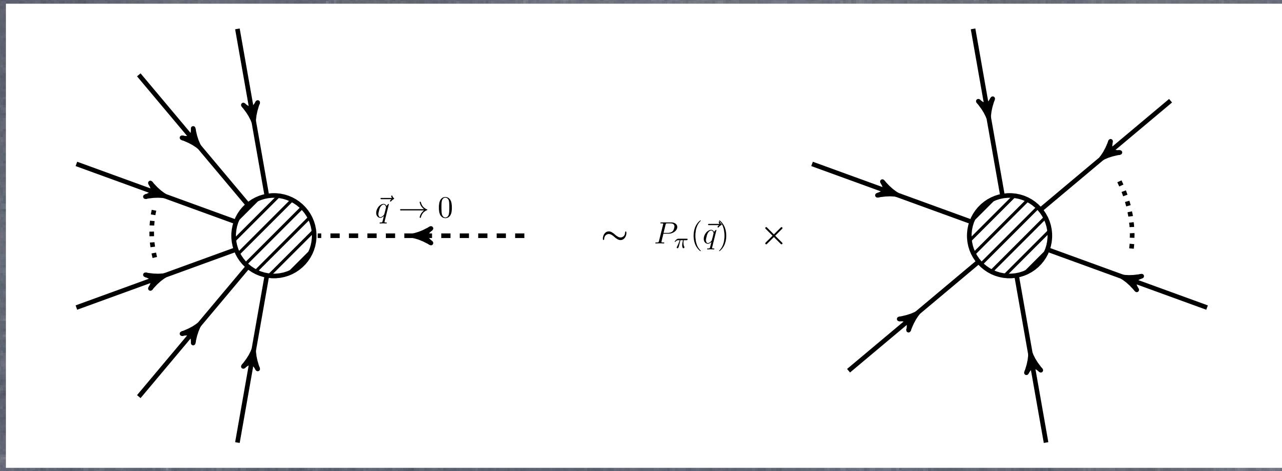


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- Goldstone spectrum is very red:

$$\boxed{q^3 P_\pi(q) = \frac{A_\pi^2}{q^2 t^2}}$$

# Observational Signatures

Creminelli, Joyce, Khoury & Simonovic, 1212.3329

- Soft internal lines: Libanov, Mironov & Rubakov (2011)

$$\langle \chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0} = \frac{1}{P_\pi(q)}\langle \pi_{-\vec{q}}\chi_{\vec{k}_1}\chi_{\vec{k}_2}\rangle_{q\to 0}\langle \pi_{\vec{q}}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0}$$
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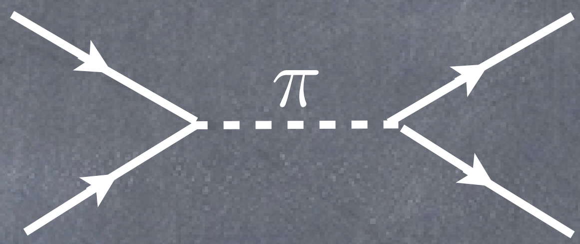

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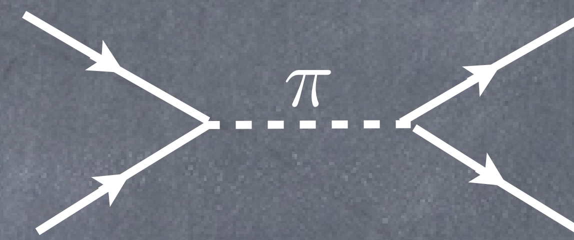


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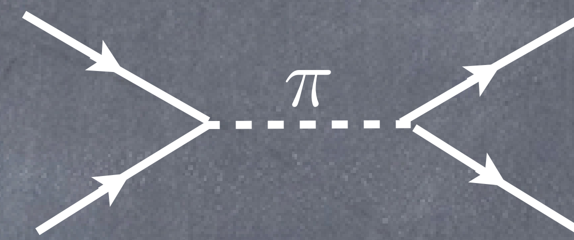


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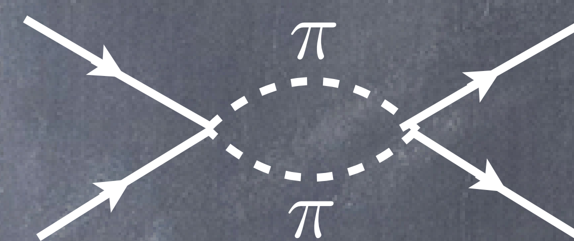
- Loop contribution:

$$\boxed{\tau_{\rm NL}\sim \log\frac{q}{\Lambda}}$$




$\Longrightarrow$ Stochastic bias, $\mu$-distortion of CMB

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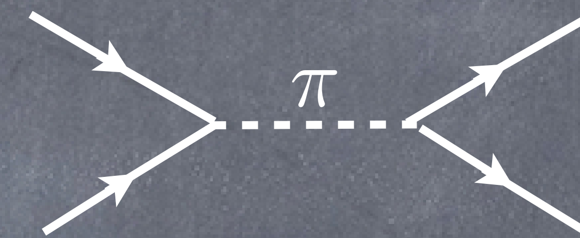
$$\langle \chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0} = \frac{1}{P_\pi(q)}\langle \pi_{-\vec{q}}\chi_{\vec{k}_1}\chi_{\vec{k}_2}\rangle_{q\to 0}\langle \pi_{\vec{q}}\chi_{\vec{k}_3}\chi_{\vec{k}_4}\rangle_{q\to 0}$$
$$\sim \frac{1}{q}\left(3(\hat{k}_1\cdot\hat{q})^2-1\right)\left(3(\hat{k}_3\cdot\hat{q})^2-1\right).$$

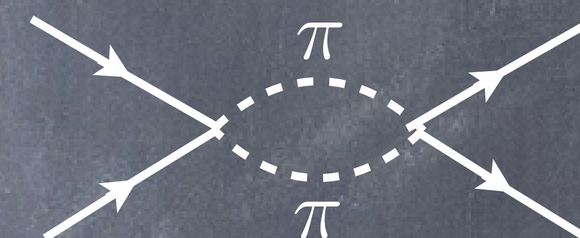



Diverges as $q\to 0$

(Vanishes as $q^2$ in inflation)

- Loop contribution:

$$\boxed{\tau_{\rm NL}\sim \log\frac{q}{\Lambda}}$$



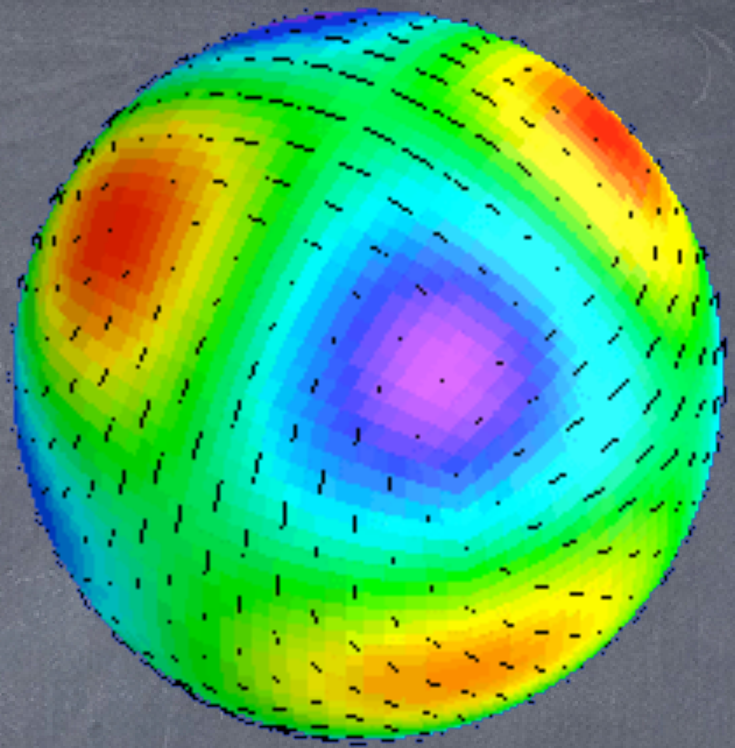

$\Longrightarrow$ Stochastic bias, $\mu$-distortion of CMB

- Anisotropy: Realization-dependent from super-Hubble $\pi$ mode

Libanov & Rubakov (2010)

$$\boxed{\langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle_{\pi_{\vec{q}}} = \langle \chi_{\vec{k}}\chi_{-\vec{k}}\rangle\left(1+c_1\frac{A_\pi}{2\pi}\frac{H_0}{k}\left(3\cos^2\theta-1\right)+c_2\frac{3A_\pi^2}{4\pi^2}\cos^2\theta\log\frac{H_0}{\Lambda}\right)}$$

## Ultimate Smoking Gun



- Inflation: - Rapid background expansion

  - All light fields are excited, including gravitational waves

$\Longrightarrow$ scale invariant primordial gravity waves

- Conformal Scenario (and Ekpyrotic):

  - Very slow contraction/expansion

  - Graviton modes not appreciably excited

Brustein, Gasperini, Giovannini & Veneziano (1995)

Khoury, Ovrut, Steinhardt and Turok (2001)

Detection of primordial gravity waves, e.g. through CMB polarization, would rule out pre-big bang scenarios.

# Null Energy Condition

$$T_{\mu\nu}n^\mu n^\nu \geq 0 \qquad \Longrightarrow \qquad \boxed{\rho + P \geq 0}$$

$n^\mu =$ null vector

Assuming spatially flat universe,

$$\therefore \qquad M_{\rm Pl}^2 \dot{H} = -\frac{1}{2}(\rho + P) \leq 0$$

Forbids smooth bounce from contraction ( $H < 0$ ) to expansion ( $H > 0$ )

Violating NEC generally comes hand in hand with various pathologies:

- Ghosts (wrong-sign kinetic term)
- Gradient instabilities
- Superluminality

# The Wish List

Hinterbichler, Joyce, Khoury & Miller, 1212.3607, PRL (2013)

| | Ghost condensate | Galileon |
|---|---|---|
| Stable, Poincare vacuum | X | X |
| Analytic S-matrix | X | X |
| Subluminality around Poincare | X | X |
| Violates NEC, no ghost | ✓ | ✓ |
| Subluminality around NEC | ✓ | X |
| Radiatively stable | ✓ | ✓ |
| Black hole thermodynamics | X | ? |

# The Wish List

Hinterbichler, Joyce, Khoury & Miller, 1212.3607, PRL (2013)

| | Ghost condensate | Galileon | DBI Galileon |
|---|---|---|---|
| Stable, Poincare vacuum | ✗ | ✗ | ✓ |
| Analytic S-matrix | ✗ | ✗ | ✓ |
| Subluminality around Poincare | ✗ | ✗ | ✗ |
| Violates NEC, no ghost | ✓ | ✓ | ✓ |
| Subluminality around NEC | ✓ | ✗ | ✓ |
| Radiatively stable | ✓ | ✓ | ✓ |
| Black hole thermodynamics | ✗ | ? | ? |

# Conclusions

- Nearly scale invariant and gaussian primordial perturbations:
  - Multi-field inflation: $so(4,1)$
  - Single-field inflation: $so(4,1) \rightarrow$ rotations + translations
  - Conformal mechanism: $so(4,2) \rightarrow so(4,1)$
- Observational signatures:
  - Soft limits of correlation functions
  - Anisotropies, large 4-point function
  - Gravitational waves
  - Consistency for large scale structure