Inflationary Models after Planck

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The Return of de Sitter II
Max Planck-Institut fur Astrophysik
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Outline

- Introduction: inflation in very very brief (my connexion with de Sitter: inflation \sim an almost de Sitter phase)
- Which class of models is favored after Planck? Single field slow-roll models!
- Computing the observable predictions of single field slow roll models. Which accuracy do we need after Planck?
- Model comparison: what is the best model of inflation? The encyclopedia inflationaris and the ASPIC library
- Conclusions & summary
Quantum fluctuations as seeds of CMB anisotropy and large scale structures
Planck results in brief:

\[ 100 \Omega_k = -0.05^{+0.65}_{-0.66} \]
\[ \alpha_{RCDI}^{(2,2500)} \in [-0.093, 0.014] \]
\[ n_S = 0.9603 \pm 0.0073 \]
\[ \frac{d n_S}{d \ln k} = -0.0134 \pm 0.009 \]
\[ f_{NL}^{\text{loc}} = 2.7 \pm 5.8 \]
\[ f_{NL}^{\text{eq}} = -42 \pm 75 \]
\[ f_{NL}^{\text{ortho}} = -25 \pm 39 \]

Flat universe with adiabatic, Gaussian and almost scale invariant fluctuations
<table>
<thead>
<tr>
<th>Observables</th>
<th>Physical Models</th>
<th>Single Field slow-roll</th>
<th>Single Field with Features (ie non slow-roll)</th>
<th>Single Field with non-canonical kinetic terms</th>
<th>Multi field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar power spectrum</td>
<td><strong>Entropic &amp; adiabatic perturbations</strong> $I \ll R$</td>
<td>$n_S \sim 1$</td>
<td><strong>DANGER</strong></td>
<td>$\checkmark$</td>
<td><strong>DANGER</strong></td>
</tr>
<tr>
<td>$\alpha_S \sim 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravity waves</td>
<td></td>
<td>$r &lt; 1$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>Non-Gaussianities compatible with zero</td>
<td></td>
<td></td>
<td><strong>DANGER</strong></td>
<td><strong>DANGER</strong></td>
<td><strong>DANGER</strong></td>
</tr>
</tbody>
</table>
What remains are models that can be described as single field inflationary models. There are just characterized by one function, the scalar potential (up to subtelties for the reheating in case of scalar tensor scenarios)

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_{Pl}^2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right] \]

\[ \ddot{\phi} + 3H \dot{\phi} + V(\phi) = 0 \]

Goal: find the correct single field scenario from the measurement of the two point correlation function (the fluctuations are Gaussian)

What is the shape of the potential??
One needs a general calculation of the two-point correlation function for single field inflation.

From COBE to Planck…
The slow-roll parameters are the “small parameter” of a perturbative calculation of the power spectrum.

\[ \epsilon_0 \propto H^{-1} \simeq \text{constant} \]

\[ \epsilon_{n+1} = \frac{d \ln |\epsilon_n|}{dN}, \quad n \geq 0 \]

\[ \epsilon_1 \simeq \frac{1}{2M_{Pl}^2} \left( \frac{V_\phi}{V} \right)^2 \]

\[ \epsilon_2 \simeq \frac{2}{M_{Pl}^2} \left[ \left( \frac{V_\phi}{V} \right)^2 - \frac{V_{\phi\phi}}{V} \right] \]
The calculation of the power spectrum amounts has the following structure

- an expansion around a "pivot scale"

\[ P_\zeta(k) = A_\zeta(k_P) \sum_{n=0}^{+\infty} \frac{a_n}{n!} \ln^n \left( \frac{k}{k_P} \right) \]

- The coefficients \( a_n \) will be functions of the slow-roll parameters, the small parameters of the problem.

- \( a_n \) starts at order \( n \) in the slow-roll parameters.
The ratio of dp to gw amplitudes is given by
\[ r \equiv \frac{P_\zeta}{P_h} = 16\epsilon_1 \]
Gravitational waves are subdominant

The spectral indices are given by
\[ n_S - 1 \equiv \frac{d \ln P_\zeta}{d \ln k}, \quad n_T \equiv \frac{d \ln P_h}{d \ln k} \]
\[ n_S - 1 = -2\epsilon_1 - \epsilon_2, \quad n_T = -2\epsilon_1 \]

The running, i.e. the scale dependence of the spectral indices, of dp and gw are
\[ \alpha_s \equiv \frac{d^2 \ln P_\zeta}{d (\ln k)^2}, \quad \alpha_T \equiv \frac{d^2 \ln P_h}{d (\ln k)^2} \]
\[ \alpha_s = O(\epsilon^2, \cdots), \quad \alpha_T = O(\epsilon^2, \cdots) \]
The inflationary predictions can be represented in the slow-roll plane.

For different values of the parameter(s) characterizing the potential, we have different points in the slow-roll space.

Different values of the parameter “$p$”
Instead of working in the slow-roll plane, one can also work in the observable plane.

\[ r = 16\epsilon_1 \]
\[ n_S - 1 = -2\epsilon_1 - \epsilon_2 \]

Different values of the parameter “p”
The calculation of the inflationary predictions involves two steps:

1 - Expressing the power spectrum in terms of the slow-roll parameters

2 - Expressing the slow-roll parameters in terms of the parameters characterizing the potential; this step requires

- The slow-roll trajectory

- An accurate estimation of the time at which slow-roll breaks down
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The predictions depend on what happens during reheating.
Understanding the \((n_s,r)\) space

\[
\frac{d \left( \frac{\dot{\phi}^2}{2} \right)}{dt} = H \frac{\dot{\phi}^2}{2} (\epsilon_2 - 2\epsilon_1)
\]

\[
\epsilon_2 > 2\epsilon_1 : \quad \frac{\dot{\phi}^2}{2} \uparrow
\]

\[
\epsilon_2 < 2\epsilon_1 : \quad \frac{\dot{\phi}^2}{2} \downarrow
\]

\[
\frac{d}{dt} \left( \frac{\dot{\phi}^2}{2\rho} \right) = \frac{2H}{3} \epsilon_1 \epsilon_2
\]

\[
\epsilon_2 < 0 : \quad \frac{\dot{\phi}^2}{\rho} \downarrow
\]

\[
\epsilon_2 > 0 : \quad \frac{\dot{\phi}^2}{\rho} \uparrow
\]
Understanding the \((n_s, r)\) space
Constraining models: from WMAP to Planck

\[ V(\phi) = M^4 \left( \frac{\phi}{M_{P1}} \right)^p \]
Constraining models: from WMAP to Planck
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From WMAP to Planck

Constraining models: from WMAP to Planck

Diagram showing constraints on model parameters $\epsilon_1$ and $\epsilon_2$ from WMAP3, WMAP7, WMAP9, and Planck 2013.
Category 1 is the category chosen by Planck

Plateau inflation
So … where do we stand?

- In order to find the best model, we have to
  - Define “model 1 is better than model 2”: Bayesian evidence.
  - Apply this definition to the complete slow-roll landscape, ie we have to scan all single field slow-roll models, one by one, in an industrial way and study their predictions and how they perform: Planck data = big data era
  - Establish a complete ranking of all these models: model comparison

- Single field slow-roll models is the favored class of models given the Planck data and the data prefers category 1.

- But this still leaves us with hundreds of scenarios and this does not tell us what is THE best model among those scenarios?
The encyclopedia contains the slow-roll treatment and comparison to the Planck data for all slow-roll models: this is not a review paper!

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Keywords: Cosmic Inflation, Slow-Roll, Reheating, Cosmic Microwave Background, Aspic

Source: http://www.stanford.edu/spires/

\[ N_{\text{papers}} = 4077 \]
The ASPIC library provides all the numerical codes for all models
A few examples
A few examples
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For model comparison, we compute the Bayesian evidence (integral of the likelihood over all parameter priors~probability of a model), i.e., the probability of a model, for each inflationary scenario.

\[
\frac{p(M_i | D)}{p(HI | D)} = B_{i-HI}
\]

Bayesian evidence of the model “i”

Bayesian evidence of the reference model=Starobinsky model

Posterior odds

\[
B_{i-HI} > 1 \quad \text{Model “i” is better than HI}
\]

\[
B_{i-HI} < 1 \quad \text{HI is better than model “i”}
\]
First calculation of inflationary Bayesian evidence in 2010

First calculation of inflationary Bayesian evidence in 2010

\[
\frac{p(M_i|D)}{p(HI|D)} = B_{i-HI} \quad \ln(B_{i-HI}) = 0
\]

LFI₄

\[
\ln(B_{i-HI}) < 0 \quad B_{i-HI} < 1
\]

\[
\ln(B_{i-HI}) > 0 \quad B_{i-HI} > 1
\]

HI is better than model "i"

Model "i" is better than HI

Jeffrey's scale

Strong evidence

Weak evidence

Inconclusive

Bad ← Good

HI: Starobinsky model

Different models
Planck calculation of Bayesian evidence

Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{HI})$ for the inflationary models

One more model: Natural inflation

Schwarz-Terrer-Escalante Classification:

J. Martin, C. Ringeval, V. Vennin

ASPIC project - Encyclopaedia Inflationaria

Planck collaboration
arXiv:1303.5082
Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{HI})$ for the inflationary models
Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{HI})$ for the inflationary models

Schwarz-Terredo-Escalante Classification:
- Green: 1
- Yellow: 2
- Red: 3
- Pink: 4-6
- Purple: 1-2-3

J. Martin, C. Ringeval, V. Vermin
ASPIC project - Encyclopedia Inflationaria
And the winners are …

Planck has identified the shape of the inflaton potential:

- **Plateau inflation**

NB: the difference between these models is “inconclusive”.

- **MHI**
- **HI**
- **RGI**
- **ESI**
- **KMIII**
- **SFI**
And the winners are ...
And the winners are ...
ASPIC evidences: examples of not so good models

Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{HI})$ for the inflationary models

Schwarz-Terredo-Escalante Classification:

Displayed Evidences: 178
And the loosers are ...

They are loosers too ... for instance, inflexion point inflation (models based on the MSSM) are clearly strongly disfavored by Planck
**Conclusions**

- Planck favors single field slow-roll scenarios: simplest but non trivial models

- Within this class, Planck data indicates that Plateau inflation is the correct shape of the potential (category I)

- There are a dozen of models that have a better Bayesian evidence (beyond the inconclusive level). We have come a long road ... from hundreds of models, Planck has identified a dozen of favored scenarios!

- Models are clearly disfavored, ie MSSM inflation (for instance)

- More to come ...
  - Constraints on the reheating temperature for each model
  - Bayesian complexity
  - Evidence for categories (string models, phenomeno models etc ...) 
  - Update this program with Planck2014 & polarization measurement
Planck & Inflation in a nutshell

Summarizing the summary ...

\[ \frac{\langle \phi \rangle}{\Lambda} \]

\[ \phi \]

YES!!!!!!!!

NO!!!!!!!!

NO!!!!!!!!