Inflationary Models after Planck

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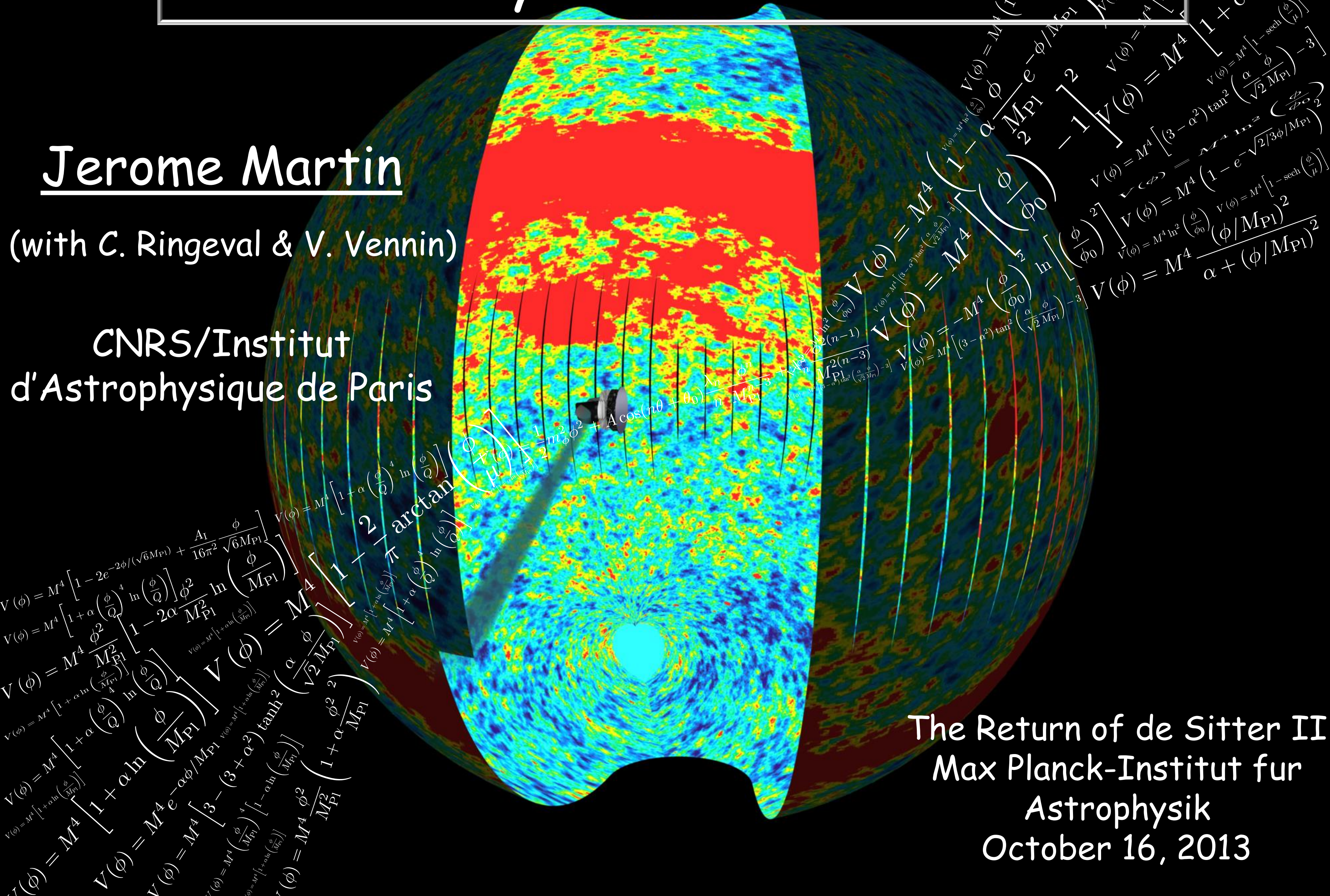


The Return of de Sitter II
Max Planck-Institut fur Astrophysik
October 16, 2013

Outline

- Introduction: inflation in very very brief (my connexion with de Sitter: inflation ~ an almost de Sitter phase)

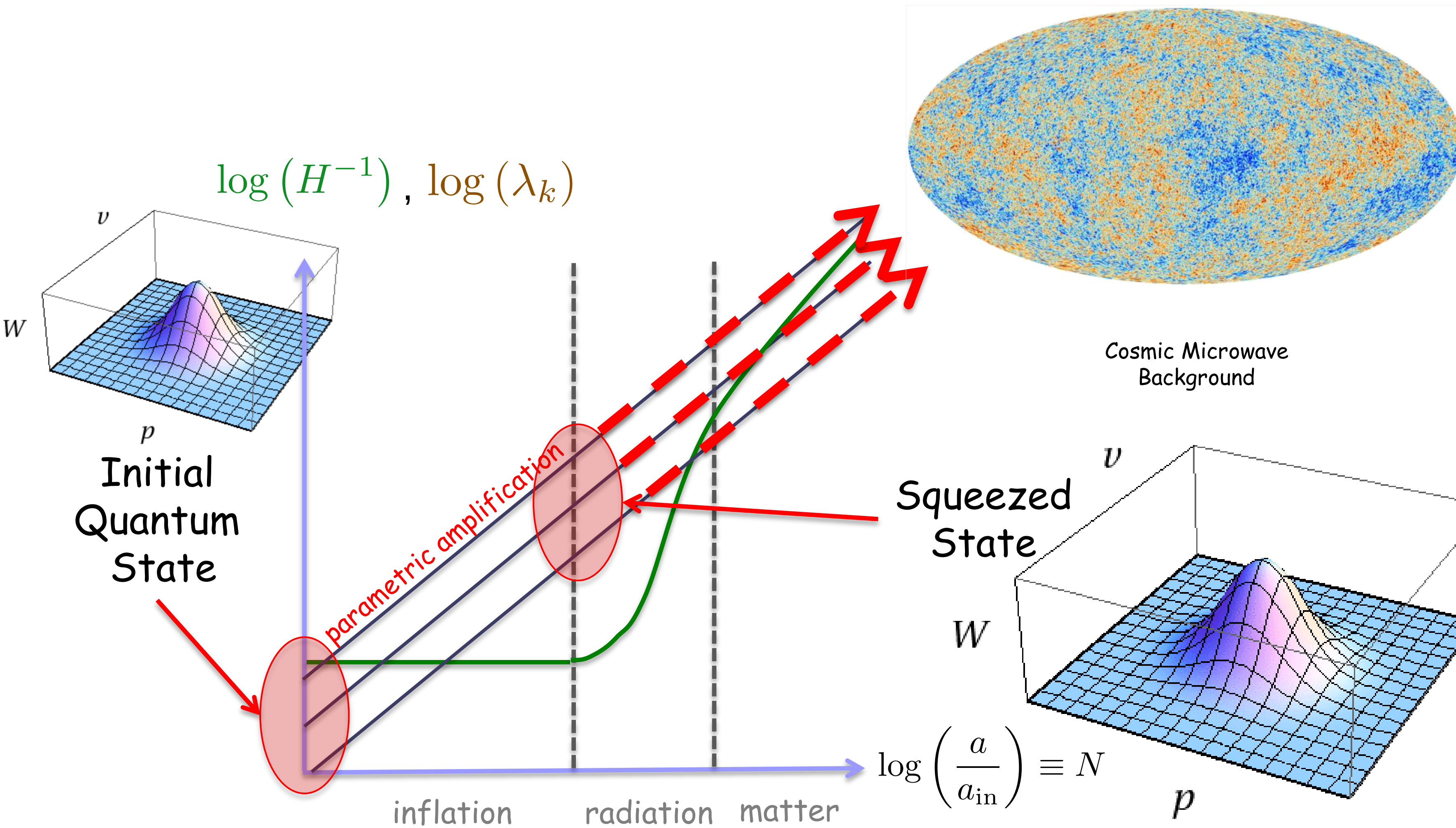
- Which class of models is favored after Planck? Single field slow-roll models!

- Computing the observable predictions of single field slow roll models. Which accuracy do we need after Planck?

- Model comparison: what is the best model of inflation? The encyclopedia inflationaris and the ASPIC library

- Conclusions & summary

Quantum fluctuations as seeds of CMB anisotropy and large scale structures



Planck results in brief:

$$100\,\Omega_{\mathcal{K}} = -0.05^{+0.65}_{-0.66}$$

$$\alpha^{(2,2500)}_{\mathcal{R}CDI} \in [-0.093, 0.014]$$

$$n_\mathrm{s} = 0.9603 \pm 0.0073$$

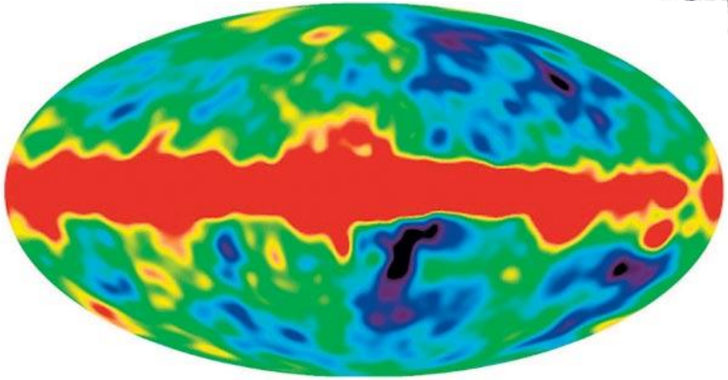
$$\frac{\mathrm{d}n_\mathrm{s}}{\mathrm{d}\ln k} = -0.0134 \pm 0.009$$

$$f_\mathrm{NL}^\mathrm{loc} = 2.7 \pm 5.8$$

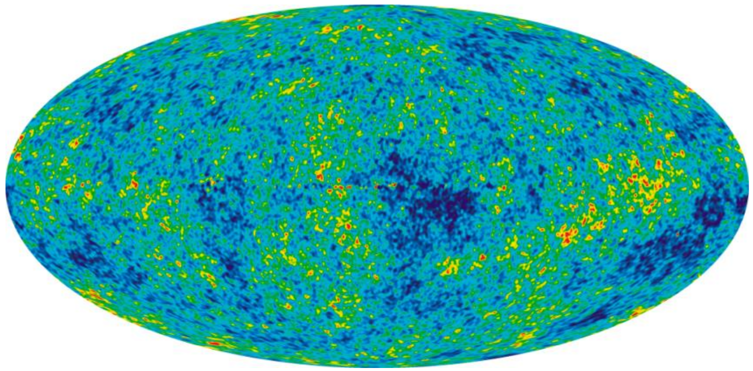
$$f_\mathrm{NL}^\mathrm{eq} = -42 \pm 75$$

$$f_\mathrm{NL}^\mathrm{ortho} = -25 \pm 39$$

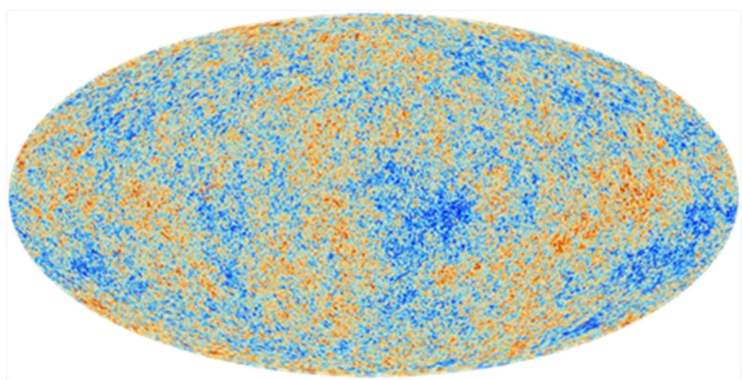
Flat universe with adiabatic, Gaussian and almost scale invariant fluctuations



COBE (1992)



WMAP (2003)



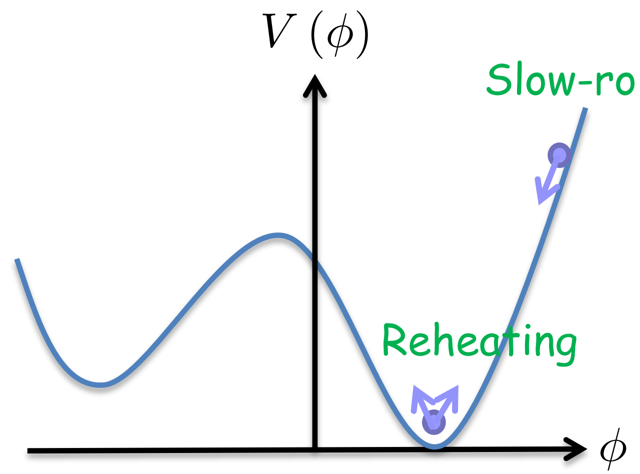
Planck (2013)

Physical Models / Observables	Single Field slow-roll	Single Field with Features (ie non slow-roll)	Single Field with non-canonical kinetic terms	Multi field	
Scalar power spectrum $n_S \sim 1$ $\alpha_S \sim 0$	✓	DANGER	✓	DANGER	...
Entropic & adiabatic perturbations $\mathcal{I} \ll \mathcal{R}$	✓	✓	✓	DANGER	...
Gravity waves $r < 1$	✓	✓	✓	✓	...
Non-Gaussianities compatible with zero	✓	DANGER	DANGER	DANGER	...

- What remains are models that can be described as single field inflationary models. There are just characterized by one function, the scalar potential (up to subtelties for the reheating in case of scalar tensor scenarios)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm Pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$$

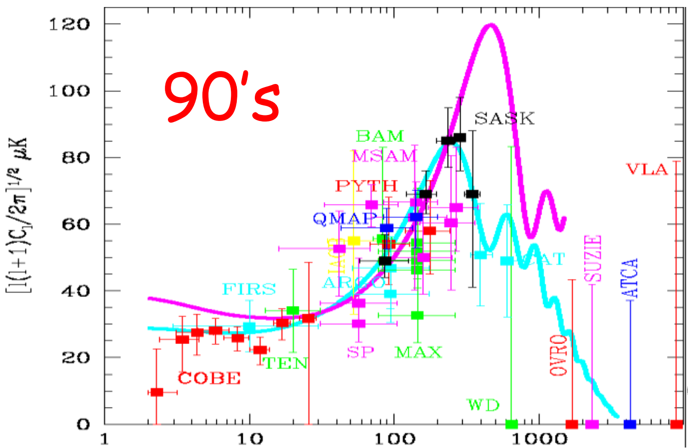
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$



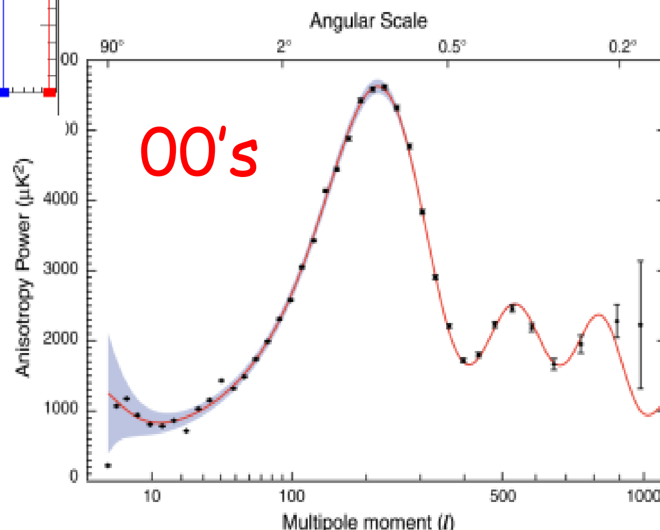
Goal: find the correct single field scenario from the measurement of the two point correlation function (the fluctuations are Gaussian)

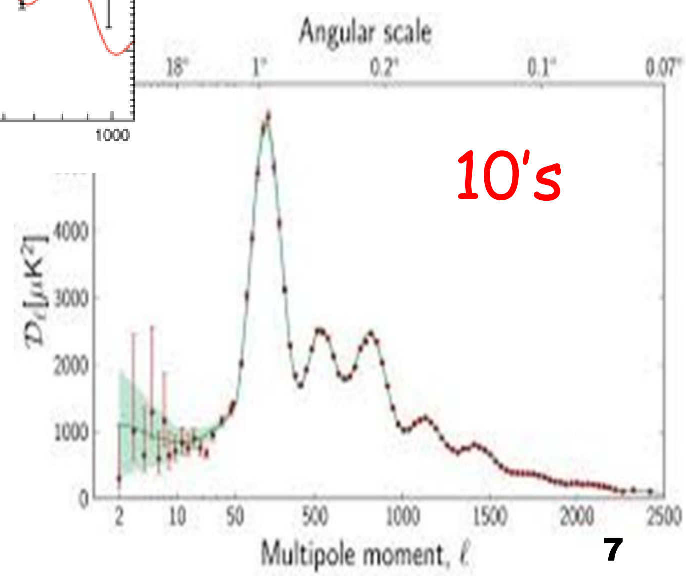
What is the shape of the potential??

CMB Power Spectrum



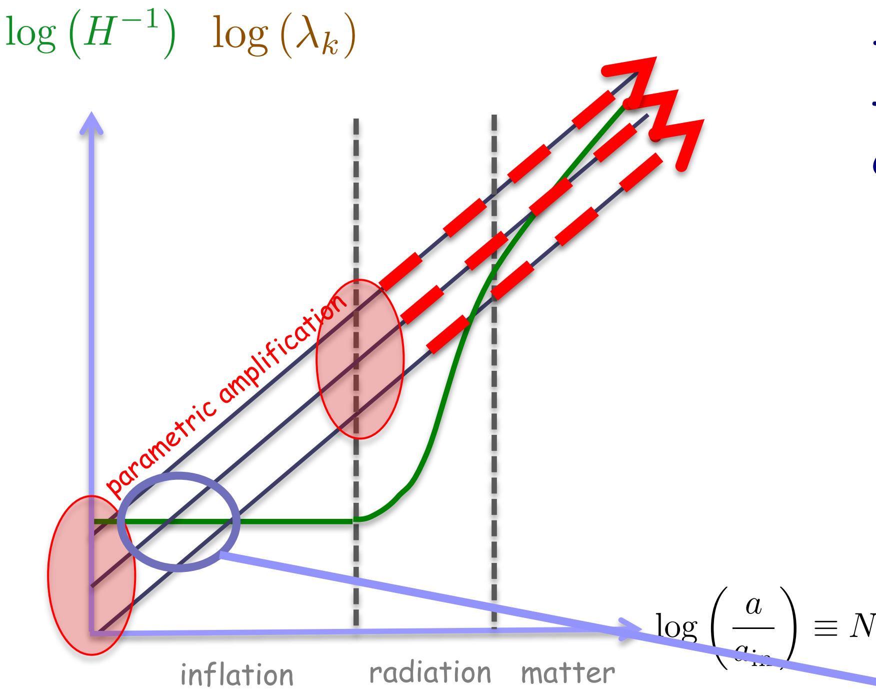
From COBE to Planck …





One needs a general calculation of the two-point correlation function for single field inflation

Slow-roll parameters



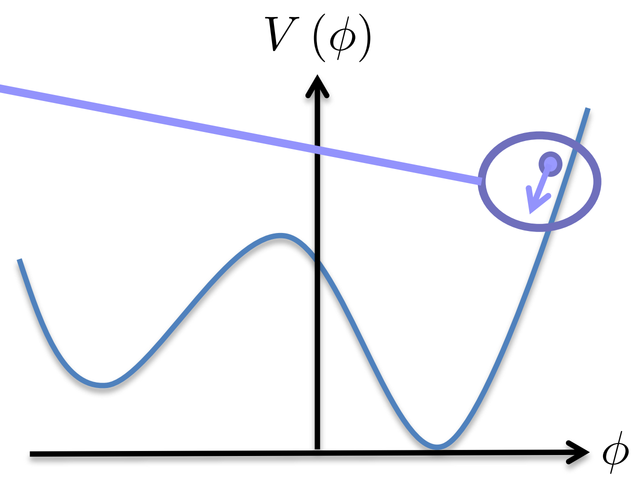
The slow-roll parameters are the "<u>small parameter</u>" of a perturbative calculation of the power spectrum

$$\epsilon_0 \propto H^{-1} \simeq \text{constant}$$

$$\epsilon_{n+1} = \frac{\mathrm{d}\ln|\epsilon_n|}{\mathrm{d}N}, \qquad n \geq 0$$

$$\epsilon_1 \simeq \frac{1}{2M_{\rm Pl}^2}\left(\frac{V_\phi}{V}\right)^2$$

$$\epsilon_2 \simeq \frac{2}{M_{\rm Pl}^2}\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right]$$



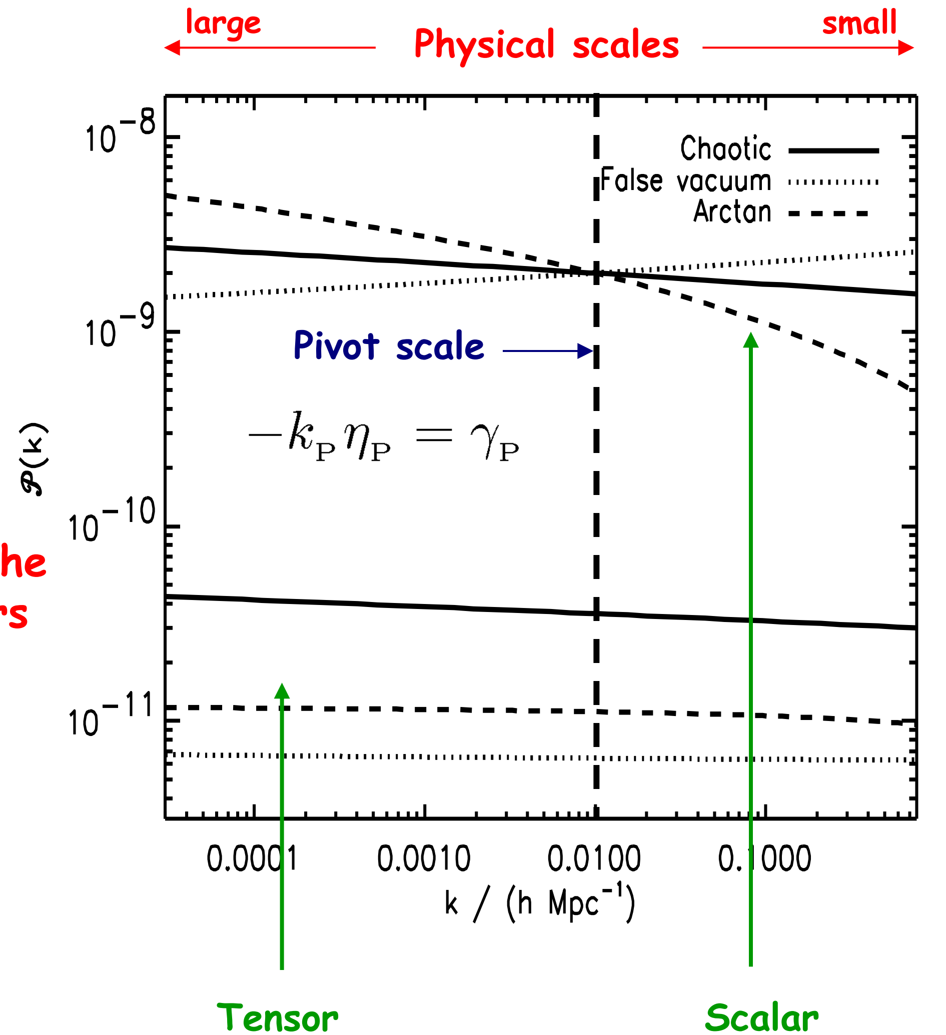
The calculation of the power spectrum amounts has the following structure

- an expansion around a "pivot scale"

$$\mathcal{P}_\zeta(k) = \mathcal{A}_\zeta\left(k_{\rm P}\right)\sum_{n=0}^{+\infty}\frac{a_n}{n!}\ln^n\left(\frac{k}{k_{\rm P}}\right)$$

- The coefficients a_n will be functions of the slow-roll parameters, the small parameters of the problem.

- a_n starts at order n in the slow-roll parameters



Inflationary predictions: the two-point correlation function

$$k^3 P_\zeta = \frac{H^2}{\pi \epsilon_1 m_{\rm Pl}^2}\left[1 - 2(C+1)\epsilon_1 - C\epsilon_2 - (2\epsilon_1 + \epsilon_2)\ln\frac{k}{k_{\rm P}}\right]$$

$$k^3 P_h = \frac{16 H^2}{\pi m_{\rm Pl}^2}\left[1 - 2(C+1)\epsilon_1 - 2\epsilon_1 \ln\frac{k}{k_{\rm P}}\right]$$

- The amplitude is controlled by H
- For the scalar modes, the amplitude also depends on ε_1

The power spectra are scale-invariant plus logarithmic corrections the amplitude of which depend on the sr parameters, ie on the microphysics of inflation

The ratio of dp to gw amplitudes is given by

$$r \equiv \frac{\mathcal{P}_\zeta}{\mathcal{P}_h} = 16\epsilon_1$$

Gravitational waves are subdominant

The spectral indices are given by

$$n_{\rm S} - 1 \equiv \frac{{\rm d}\ln \mathcal{P}_\zeta}{{\rm d}\ln k}\,, \quad n_{\rm T} \equiv \frac{{\rm d}\ln \mathcal{P}_h}{{\rm d}\ln k}$$

$$n_{\rm S} - 1 = -2\epsilon_1 - \epsilon_2\,, \quad n_{\rm T} = -2\epsilon_1$$

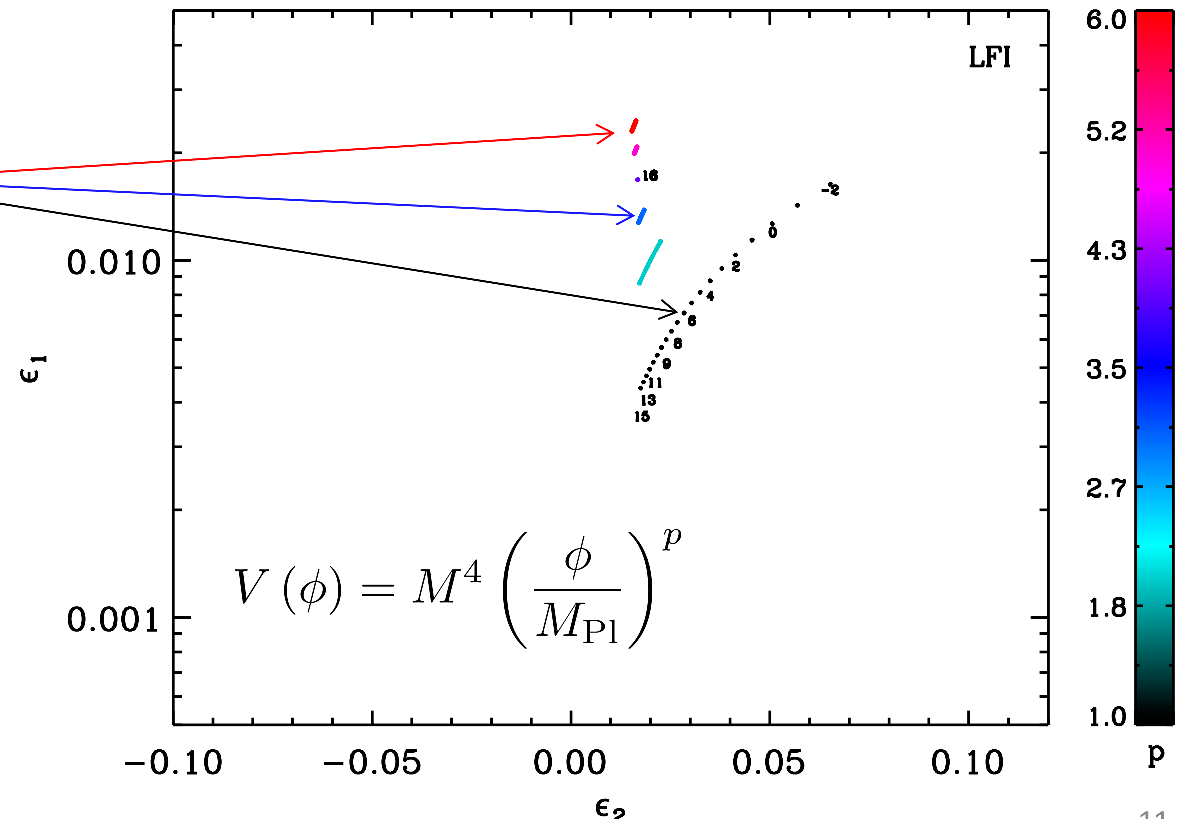
The running, i.e. the scale dependence of the spectral indices, of dp and gw are

$$\alpha_{\rm S} \equiv \frac{{\rm d}^2 \ln \mathcal{P}_\zeta}{{\rm d}\left(\ln k\right)^2} \qquad \alpha_{\rm T} \equiv \frac{{\rm d}^2 \ln \mathcal{P}_h}{{\rm d}\left(\ln k\right)^2} \qquad \alpha_{\rm S} = \mathcal{O}\left(\epsilon^2, \cdots\right) \quad \alpha_{\rm T} = \mathcal{O}\left(\epsilon^2, \cdots\right)$$

- The inflationary predictions can be represented in the slow-roll plane

- For different values of the parameter(s) characterizing the potential, we have different points in the slow-roll space

Different values of the parameter "p"



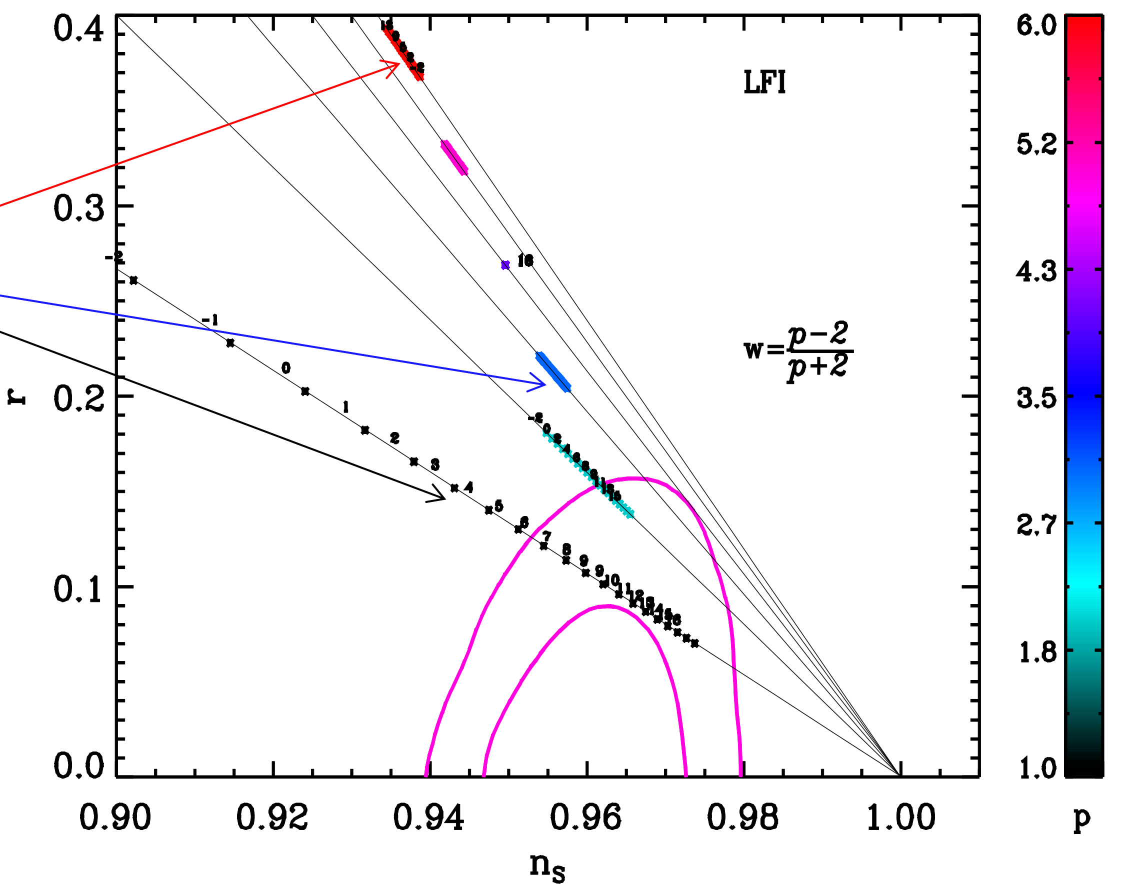
$$V(\phi) = M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$$

- Instead of working in the slow-roll plane, one can also work in the observable plane

$$r = 16\epsilon_1$$

$$n_{\mathrm{S}} - 1 = -2\epsilon_1 - \epsilon_2$$

Different values of the parameter "p"



Inflationary predictions: the need for numerical calculations

The calculation of the inflationary predictions involves two steps:

1 - Expressing the power spectrum in terms of the slow-roll parameters

2 – Expressing the slow-roll parameters in term of the parameters characterizing the potential; this step requires

- The slow-roll trajectory

- An accurate estimation of the time at which slow-roll breaks down

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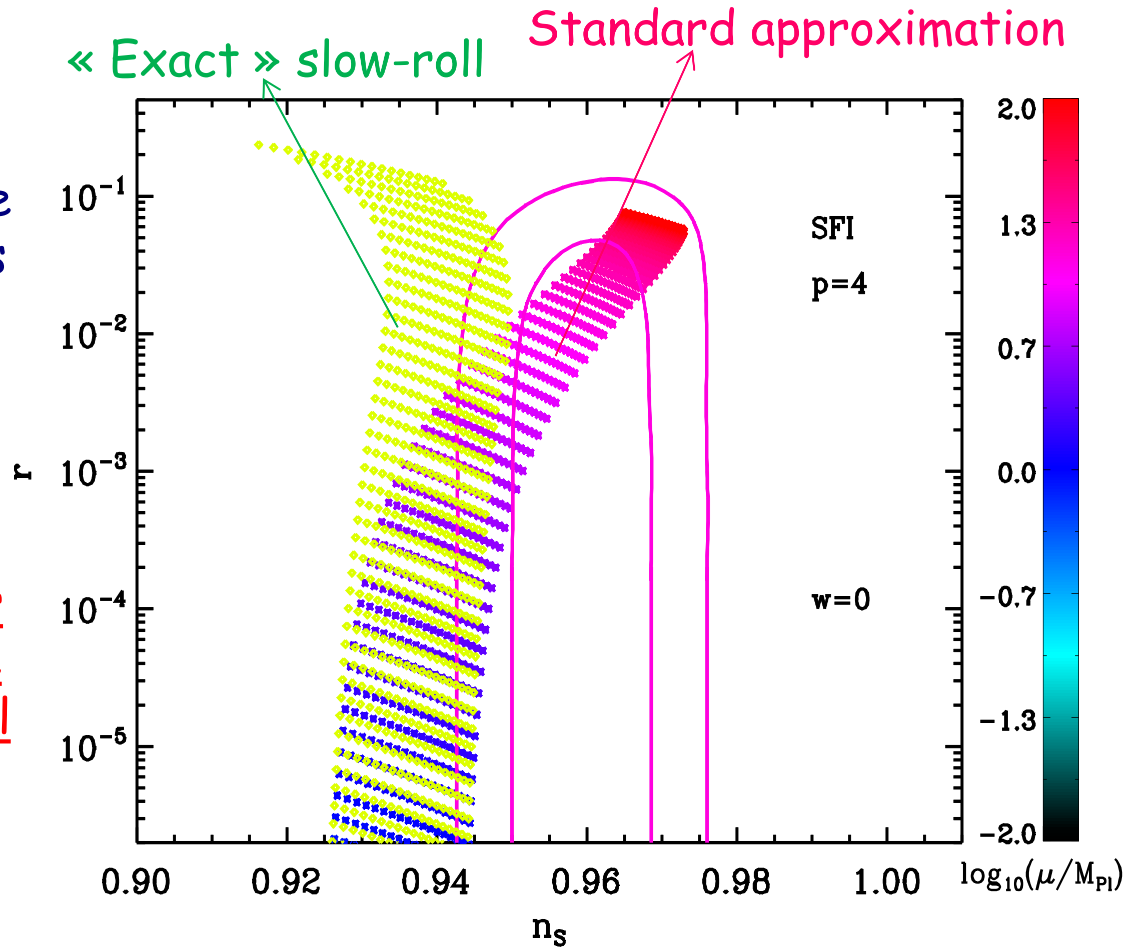

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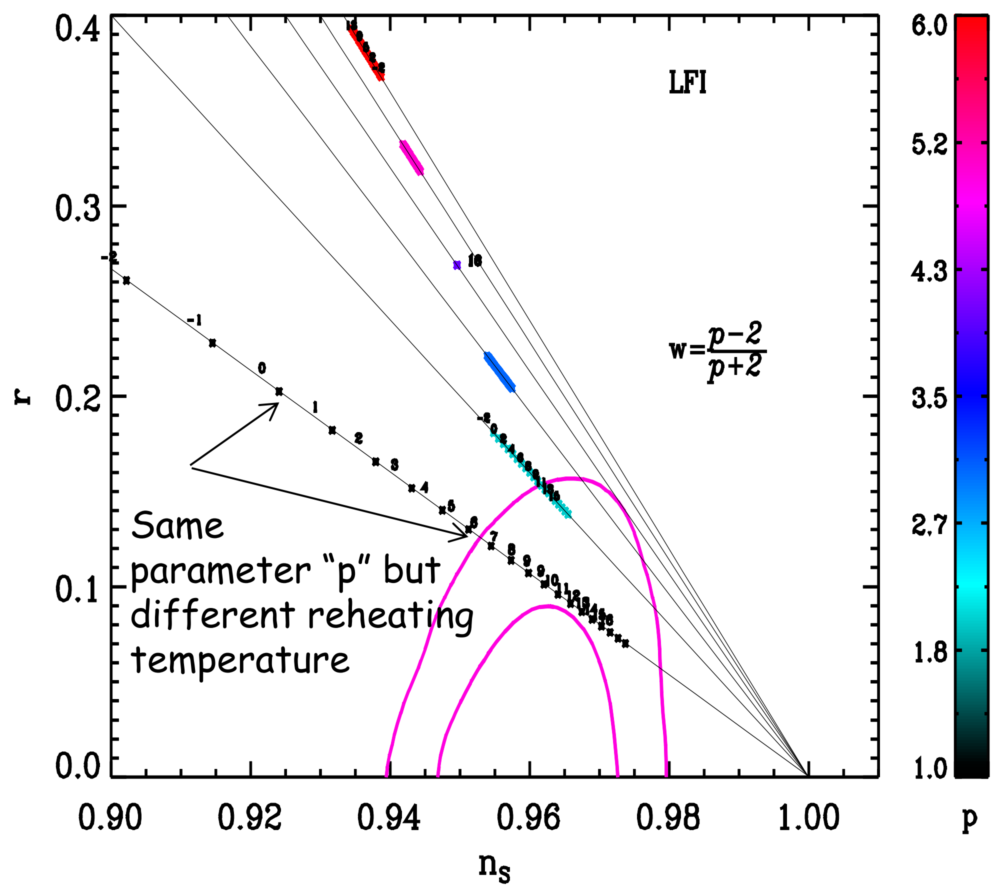


The predictions depend on what happens during reheating

Inflationary categories

Understanding the (n_S,r) space

$$\frac{\mathrm{d}\left(\dot{\phi}^2/2\right)}{\mathrm{d}t} = H\frac{\dot{\phi}^2}{2}\left(\epsilon_2 - 2\epsilon_1\right)$$

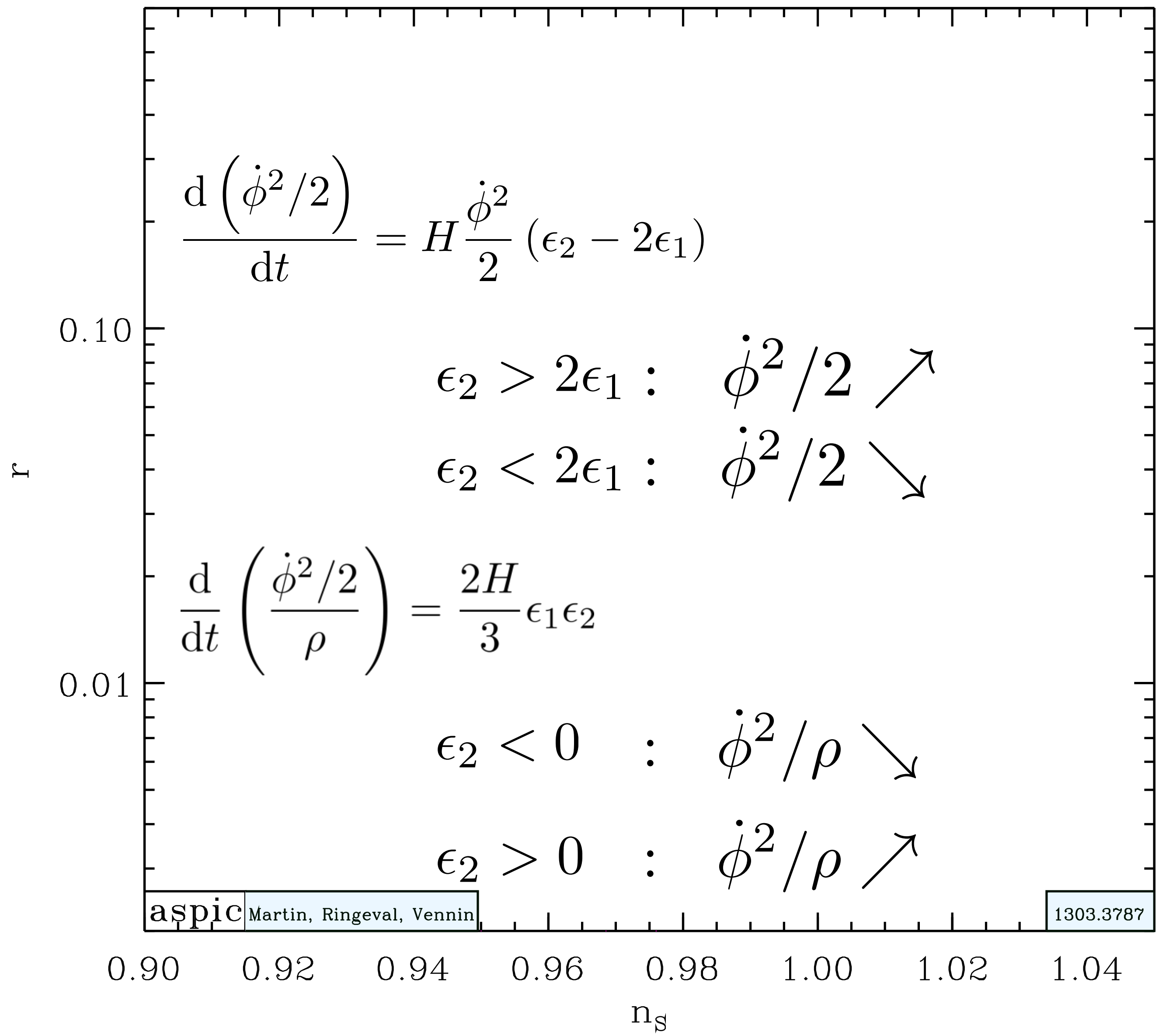
$$\epsilon_2 > 2\epsilon_1 : \quad \dot{\phi}^2/2 \nearrow$$

$$\epsilon_2 < 2\epsilon_1 : \quad \dot{\phi}^2/2 \searrow$$

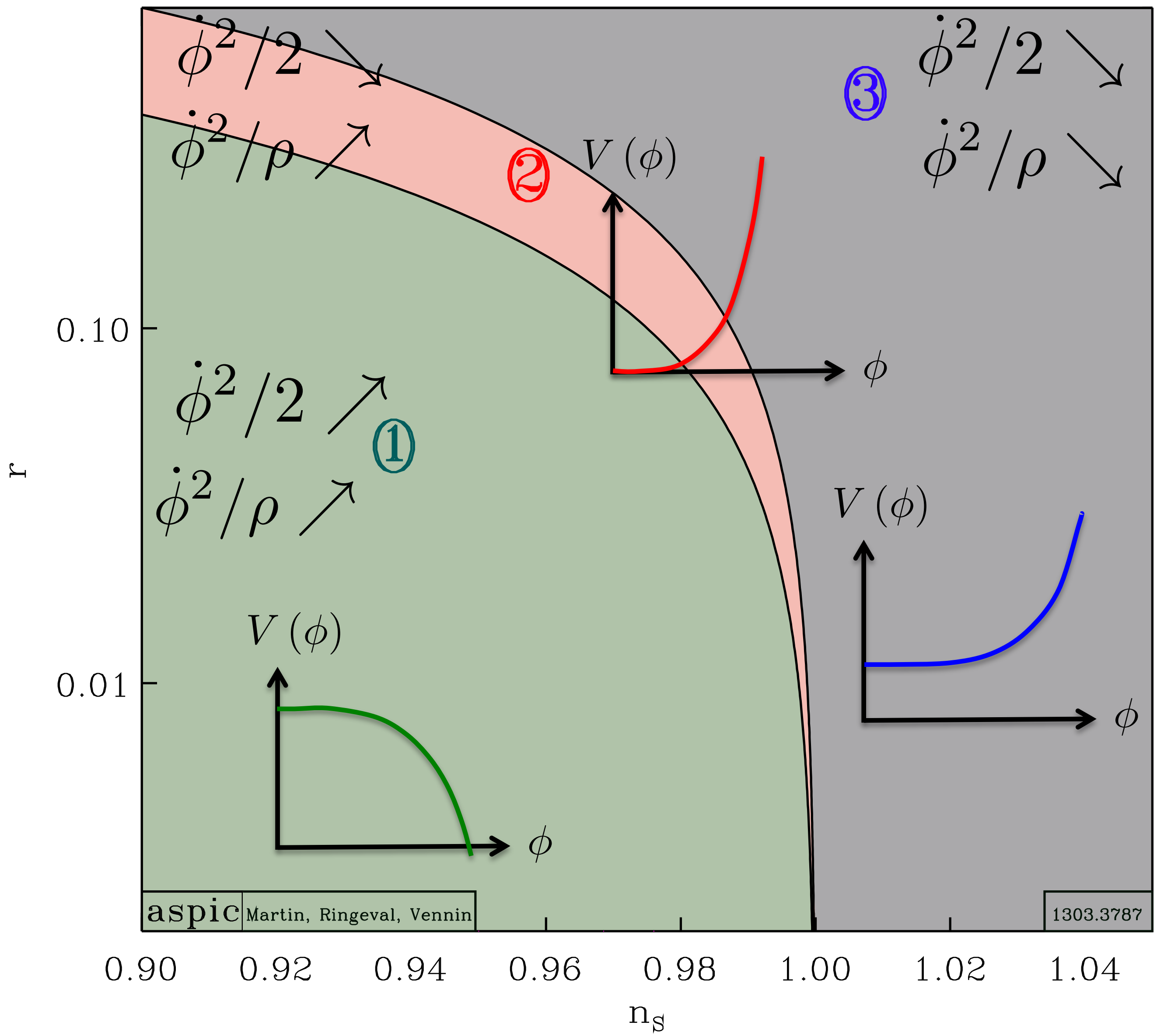
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{\phi}^2/2}{\rho}\right) = \frac{2H}{3}\epsilon_1\epsilon_2$$

$$\epsilon_2 < 0 \quad : \quad \dot{\phi}^2/\rho \searrow$$

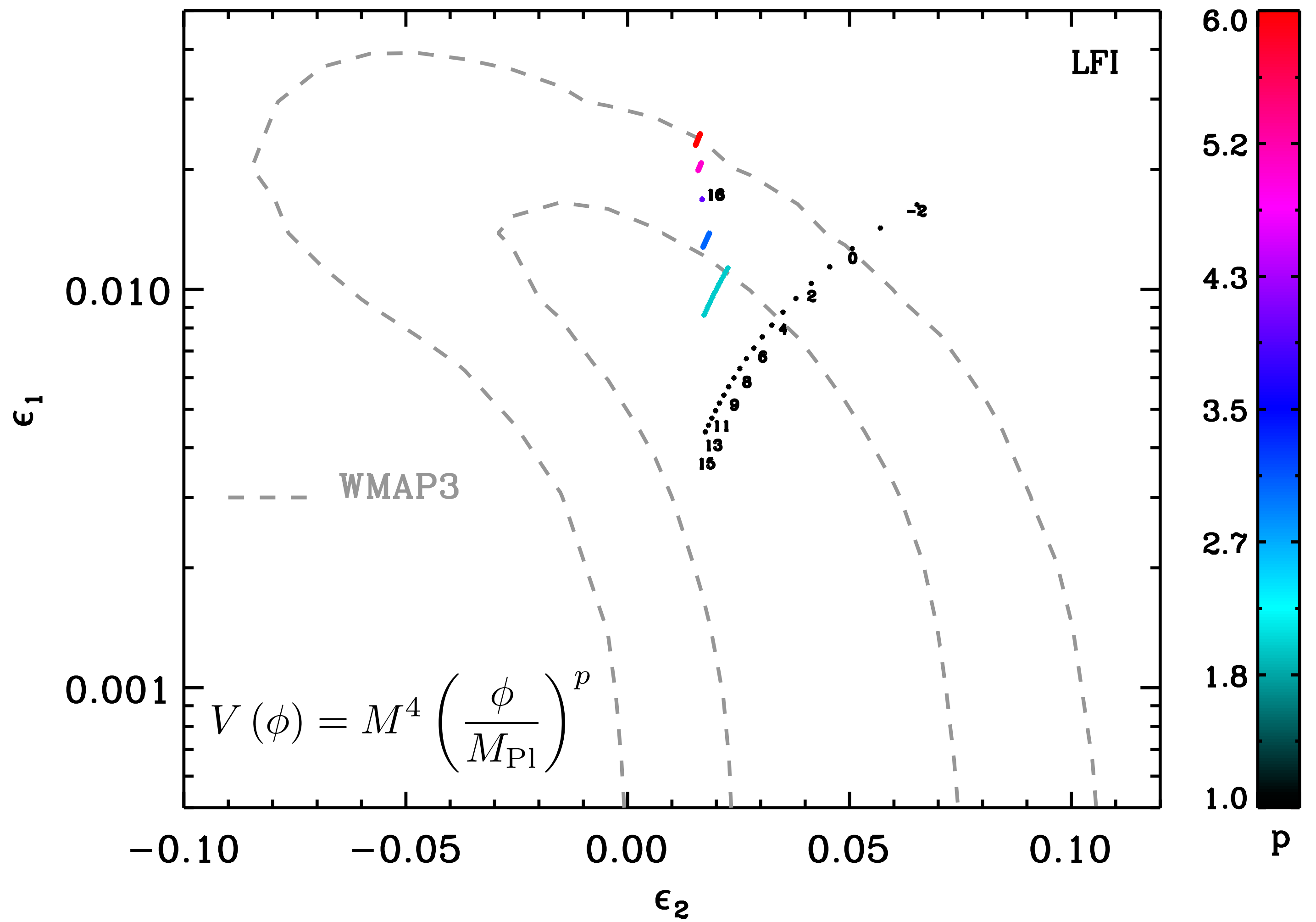
$$\epsilon_2 > 0 \quad : \quad \dot{\phi}^2/\rho \nearrow$$



Understanding the (n_S, r) space



Constraining models: from WMAP to Planck



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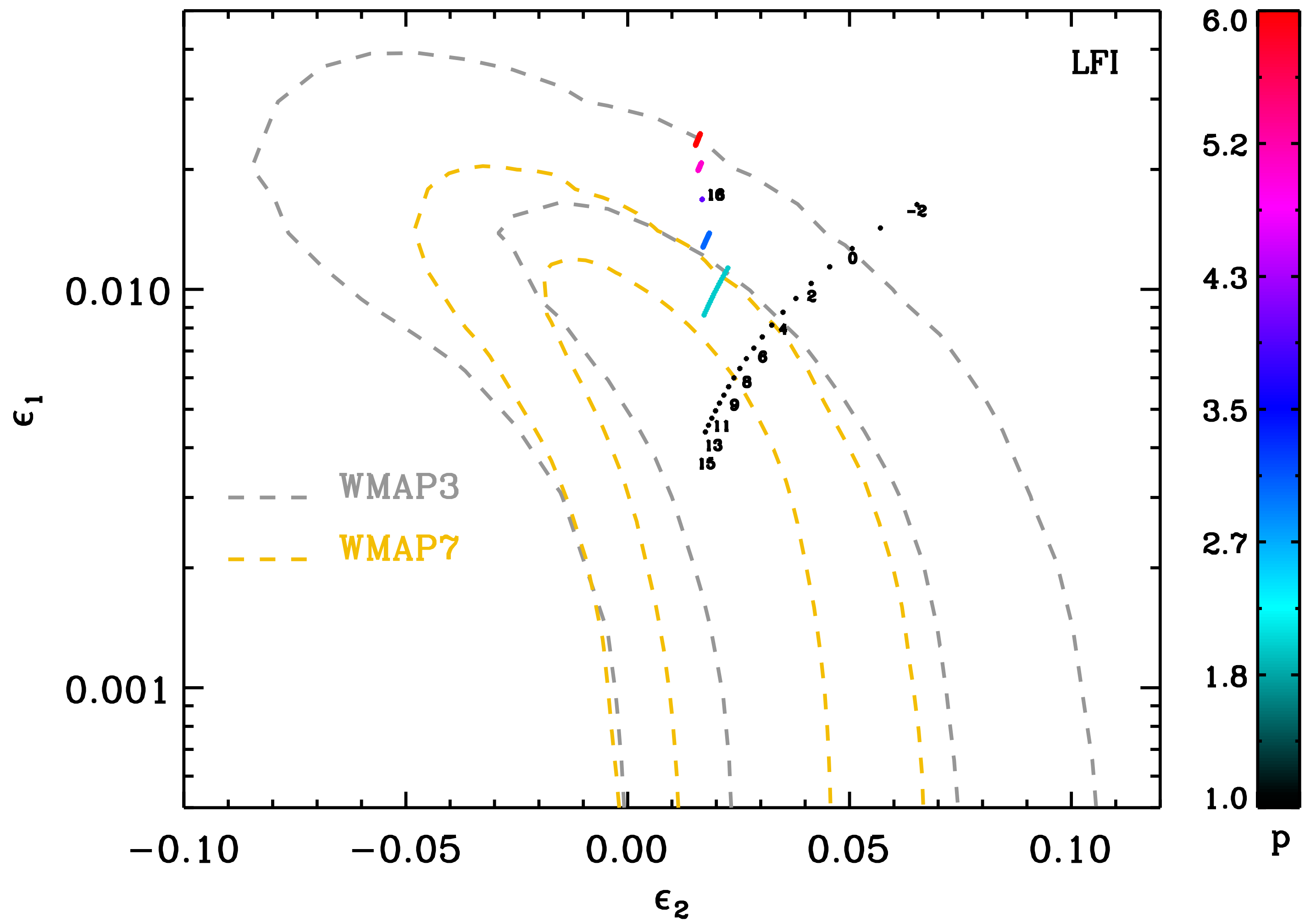

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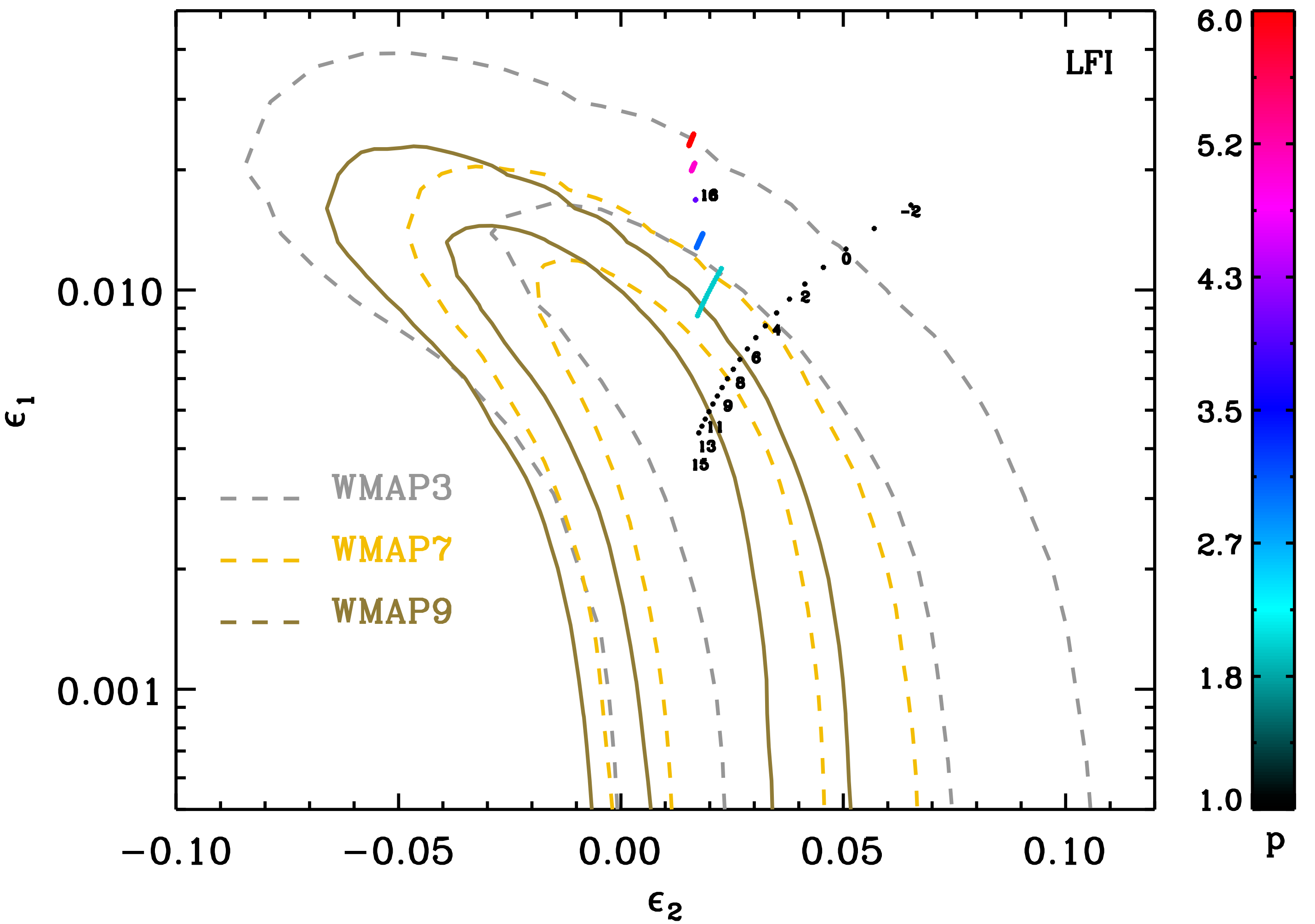

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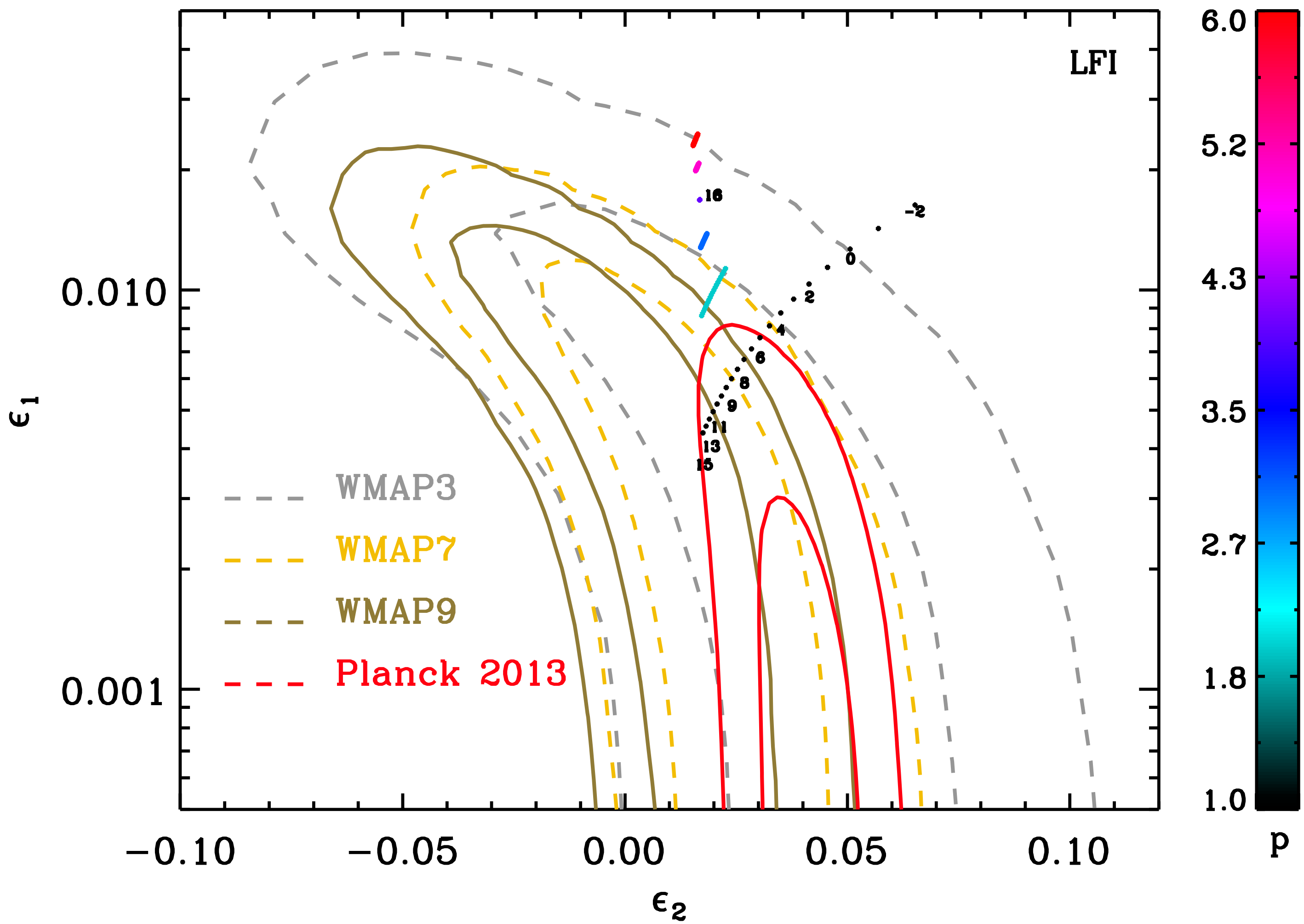
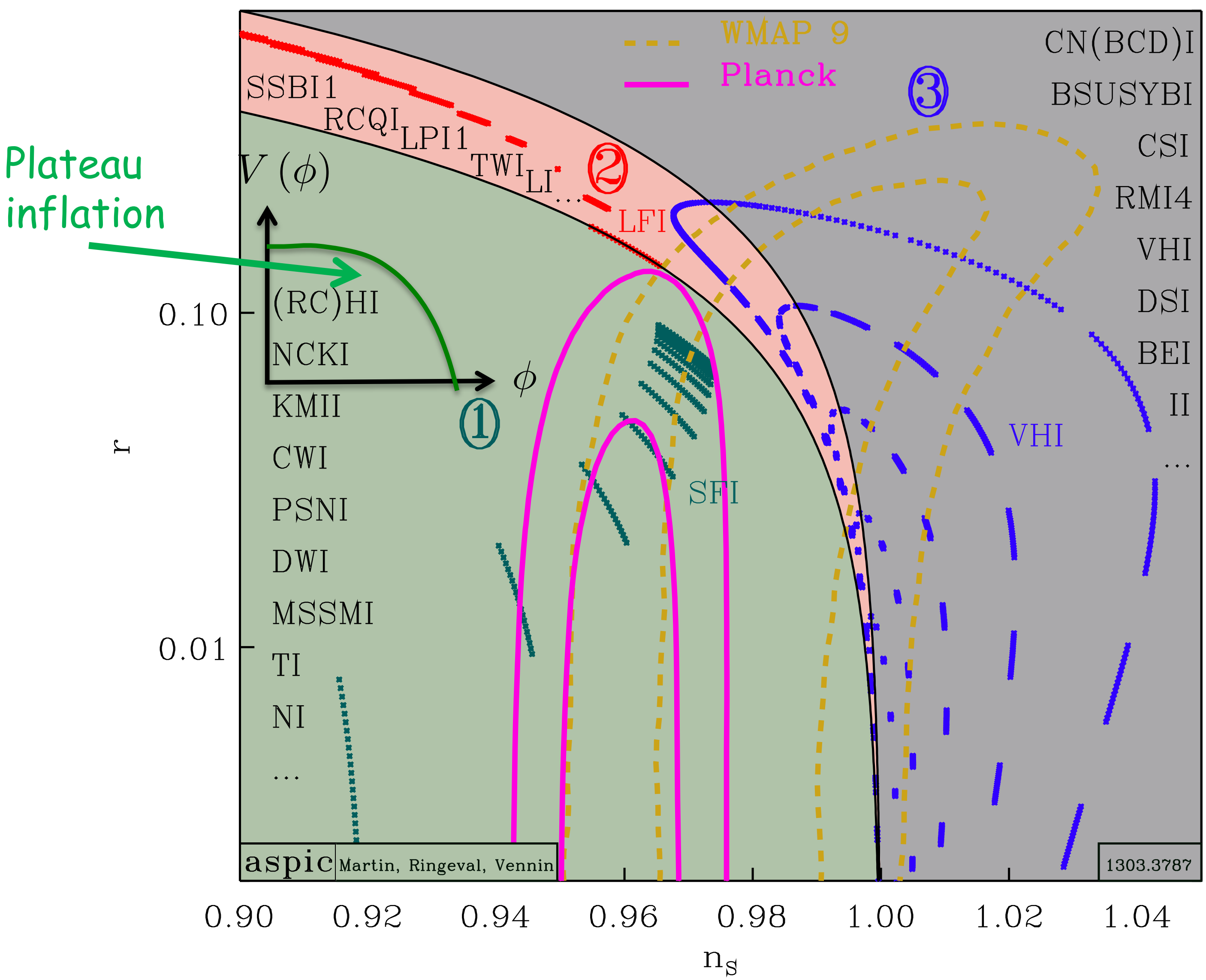
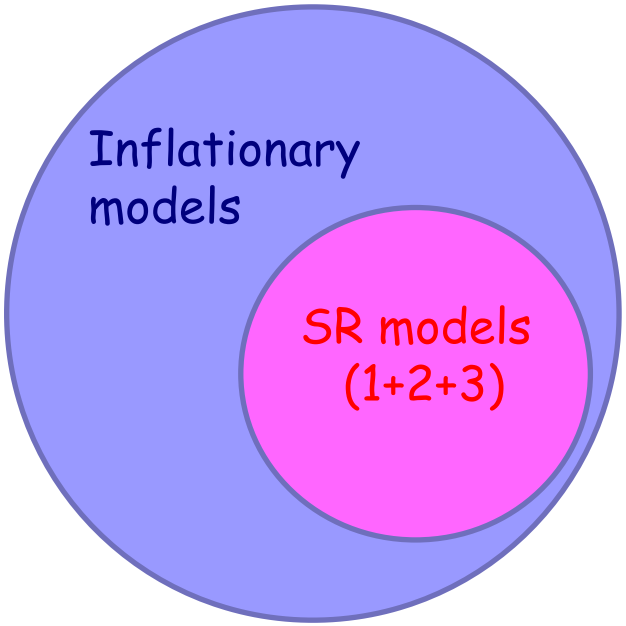


Planck likes category 1

Category 1 is the category chosen by Planck



So … where do we stand?



- Single field slow-roll models is the favored class of models given the Planck data and the data prefers category 1.
- But this still leaves us with hundreds of scenarios and this does not tell us what is THE best model among those scenarios?

- In order to find the best model, we have to

  - Define "model 1 is better than model 2": Bayesian evidence.

  - Apply this definition to the complete slow-roll landscape, ie we have to scan all single field slow-roll models, one by one, in an industrial way and study their predictions and how they perform: Planck data =big data era

  - Establish a complete ranking of all these models: model comparison

arXiv:1303.3787

≈ 74 models

≈ 700 slow roll formulas

≈ 365 pages

Encyclopædia Inflationaris

The encyclopedia contains the slow-roll treatment and comparison to the Planck data for all slow-roll models : this is not a review paper!

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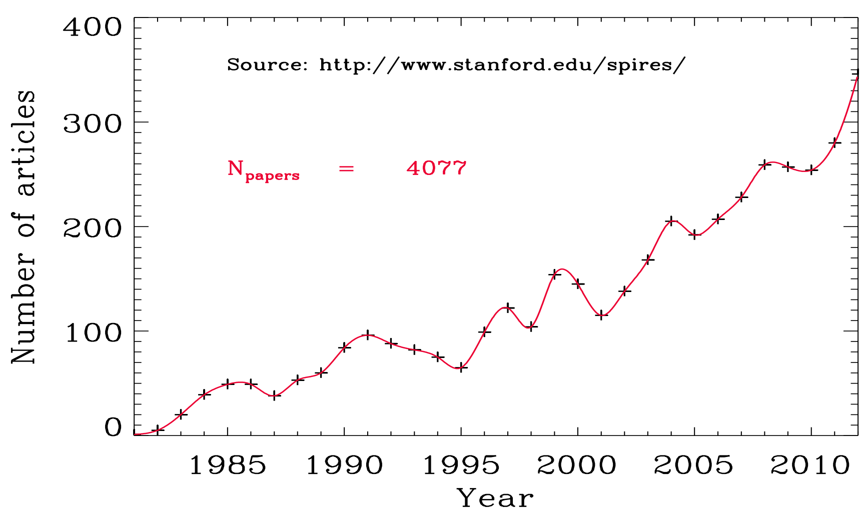
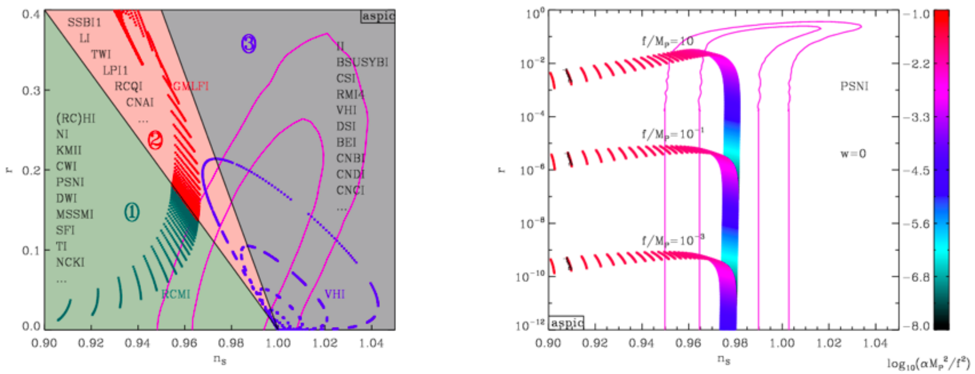
Keywords: Cosmic Inflation, Slow-Roll, Reheating, Cosmic Microwave Background, Aspic



theory.physics.unige.ch/~ringeval/aspic.html

Accurate Slow-roll Predictions for Inflationary Cosmology



Reheating consistent slow-roll predictions for a subset of inflationary models supported by **aspic** (left). The right panel features the Pseudo Natural Inflation (PSNI) predictions. The annotated values show the logarithmic energy scale, log(Ereh/GeV), at which a matter dominated reheating ends (arXiv:1303.3787).

Aspic is a collection of fast modern fortran routines for computing various observable quantities used in Cosmology from definite single field inflationary models. It is distributed as a scientific library and aims at providing an efficient, extendable and accurate way of comparing theoretical inflationary predictions with cosmological data. **Aspic** currently supports 64 models of inflation, and more to come!

By observable quantities, we currently refer to as the Hubble flow functions, up to second order in the slow-roll approximation, which are in direct correspondence with the spectral index, the tensor-to-scalar ratio and the running of the primordial power spectrum. The **aspic** library also provides the field potential, its first and second derivatives, the energy density at the end of inflation, the energy density at the end of reheating, and the field value (or e-fold value) at which the pivot scale crossed the Hubble radius during inflation. All these quantities are computed in a way which is consistent with the existence of a reheating phase.

The code is released as a GNU software which compiles itself into both a static and shared library. As the list of inflationary models is always increasing, you are encouraged to add support for any model that would n

Please, check the MAN file for a complete documentation a

download the source file.

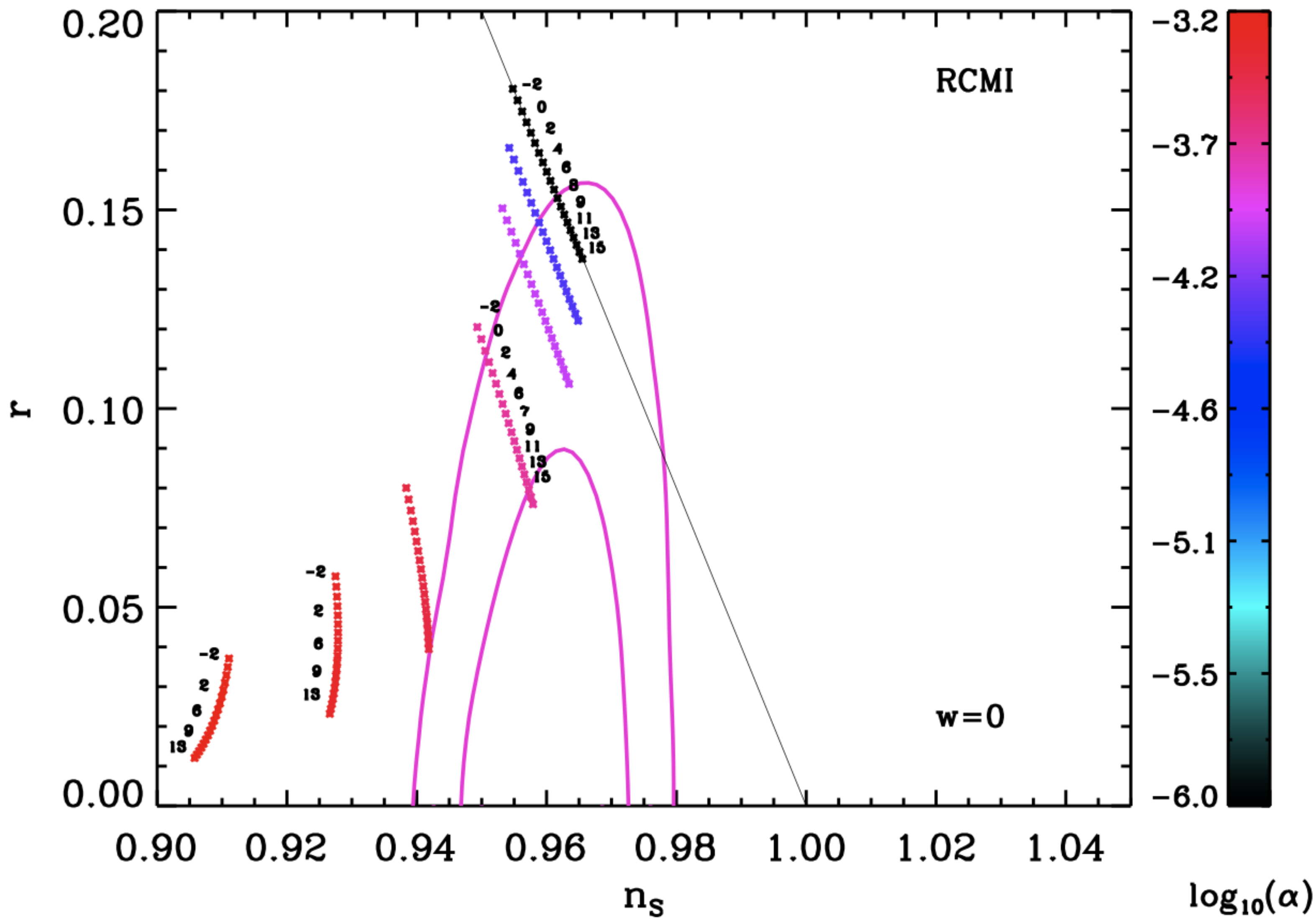
For details, please read the original paper arXiv:1303.3787

For an exact integration of any inflationary models, without assuming slow-roll, checkout the **fieldinf** code and library.

Last modif 03/2013

The ASPIC library provides all the numerical codes for all models

A few examples



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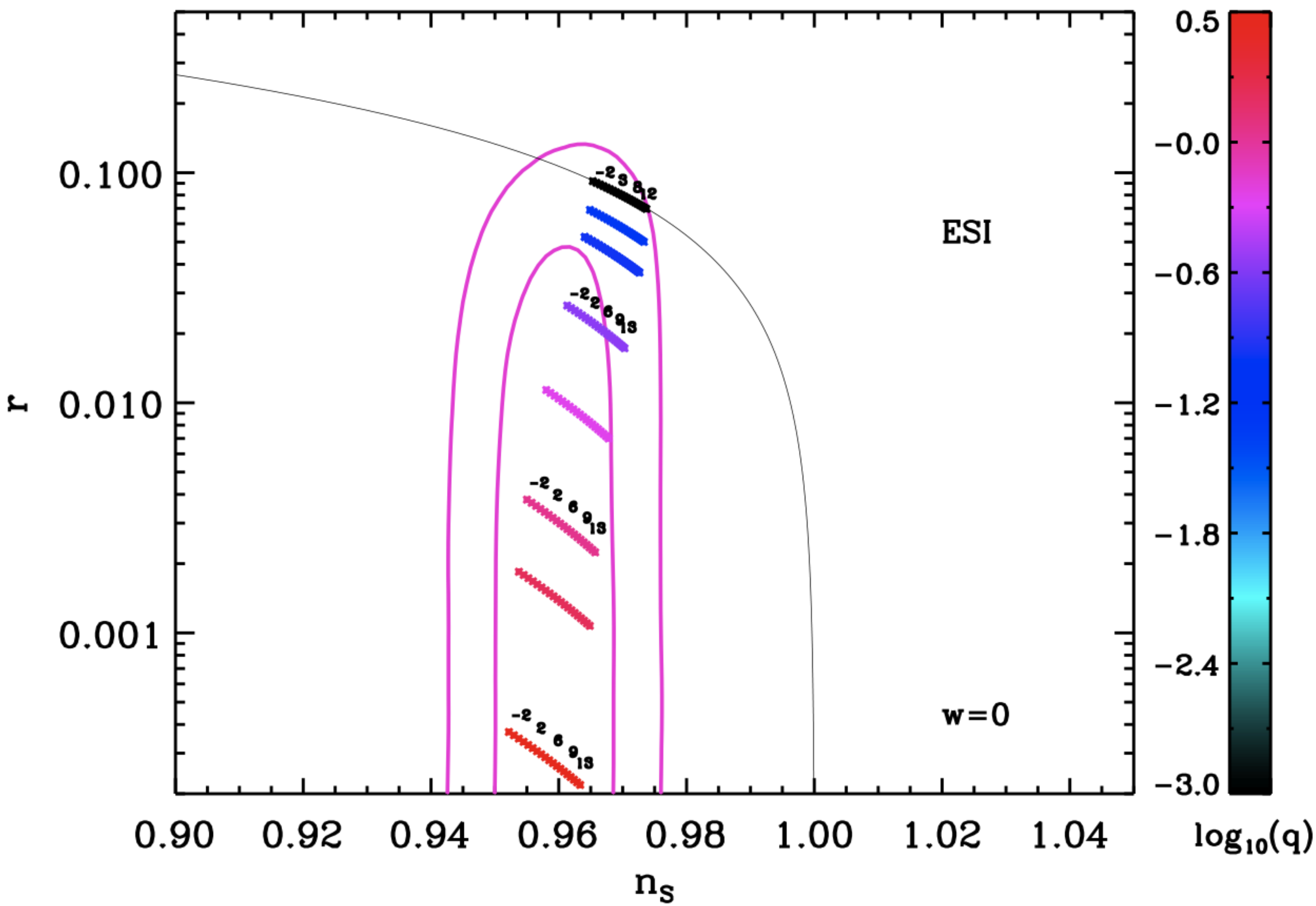

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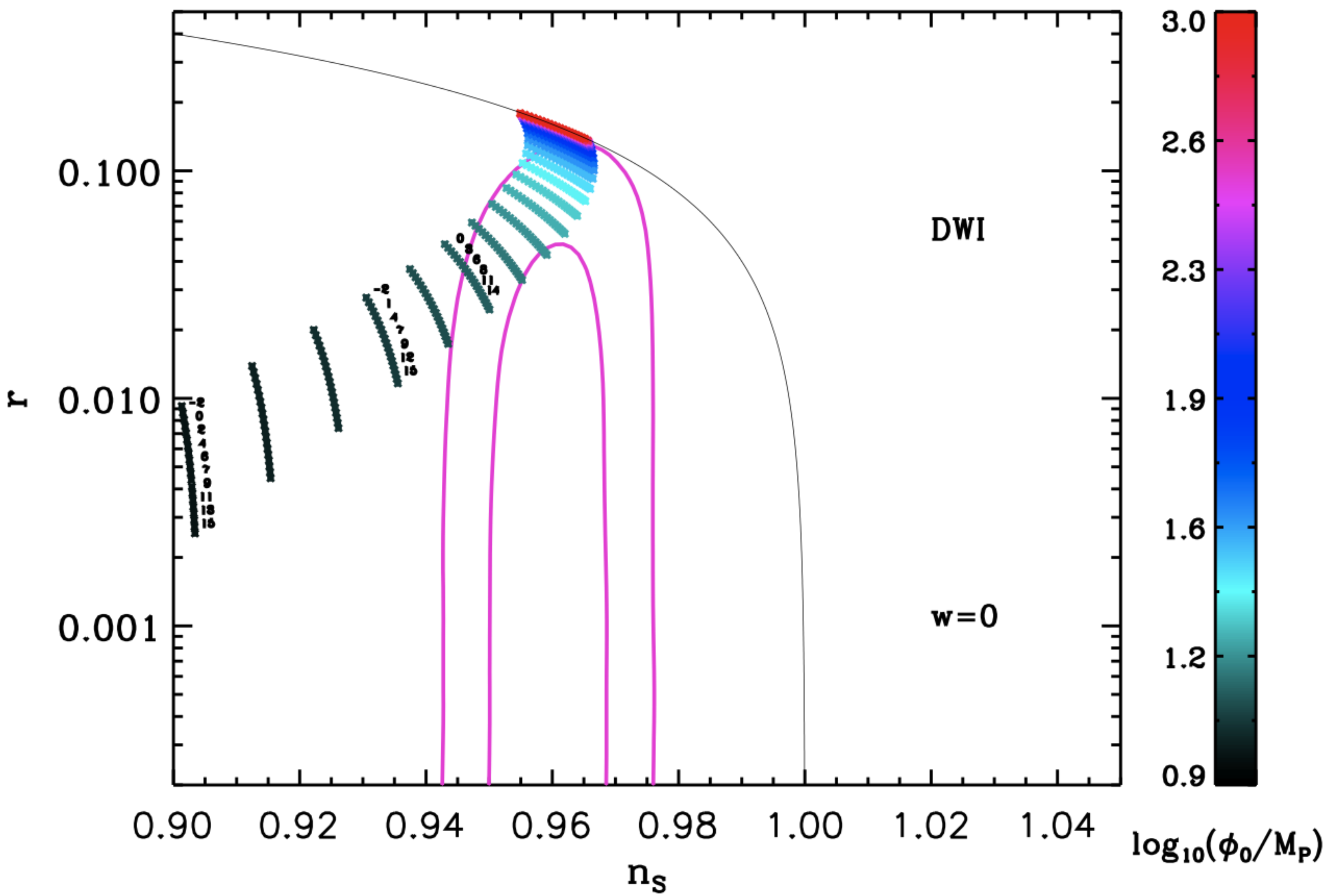

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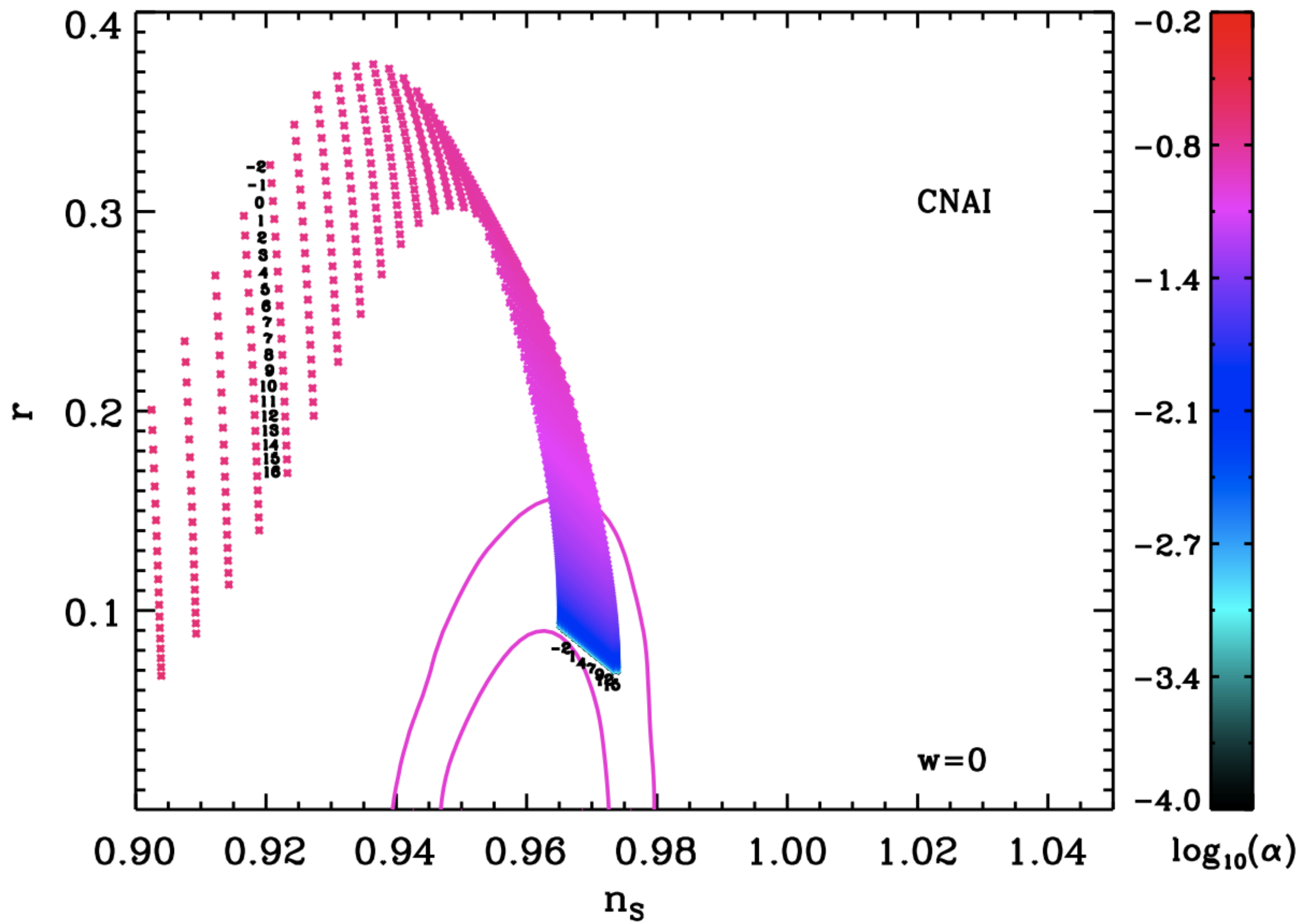

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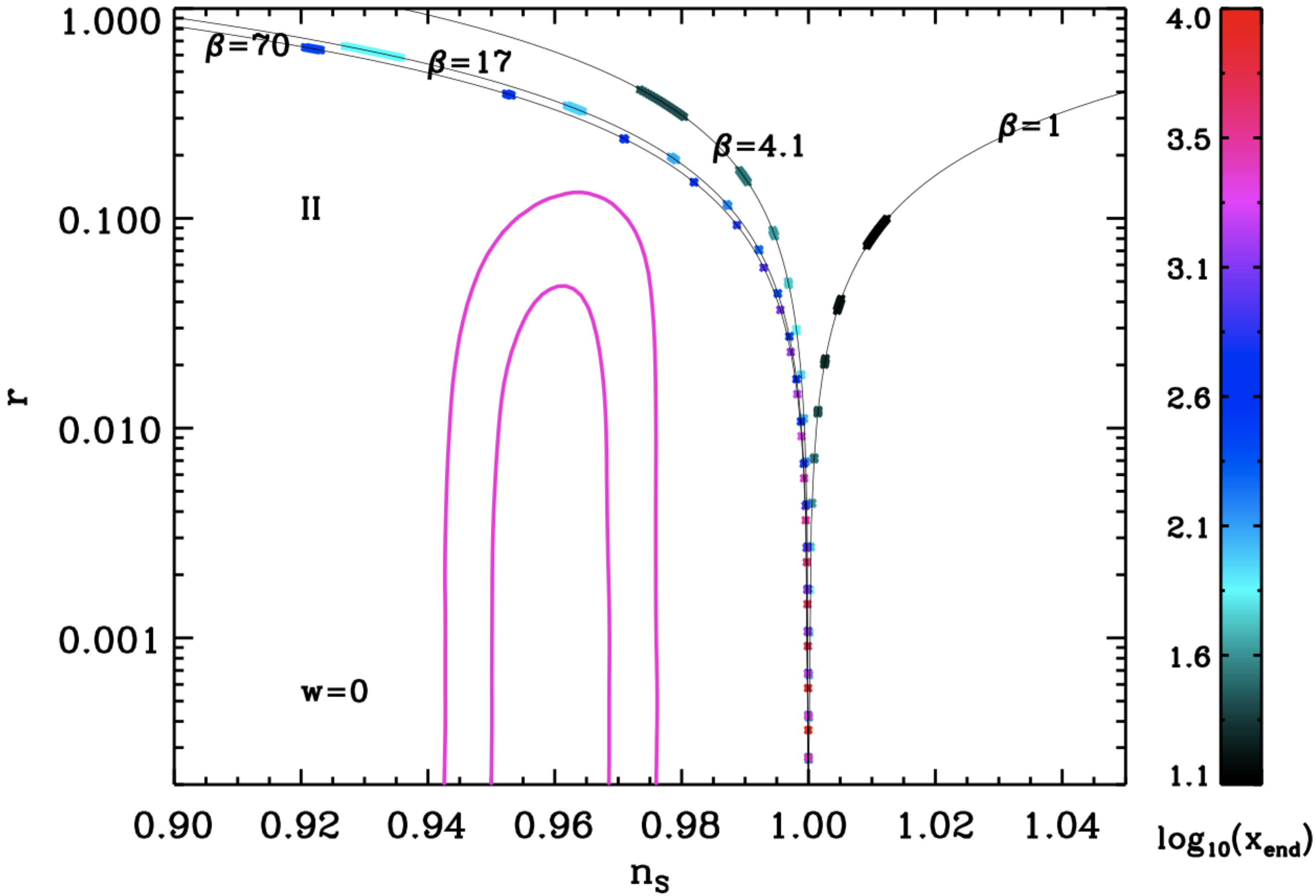

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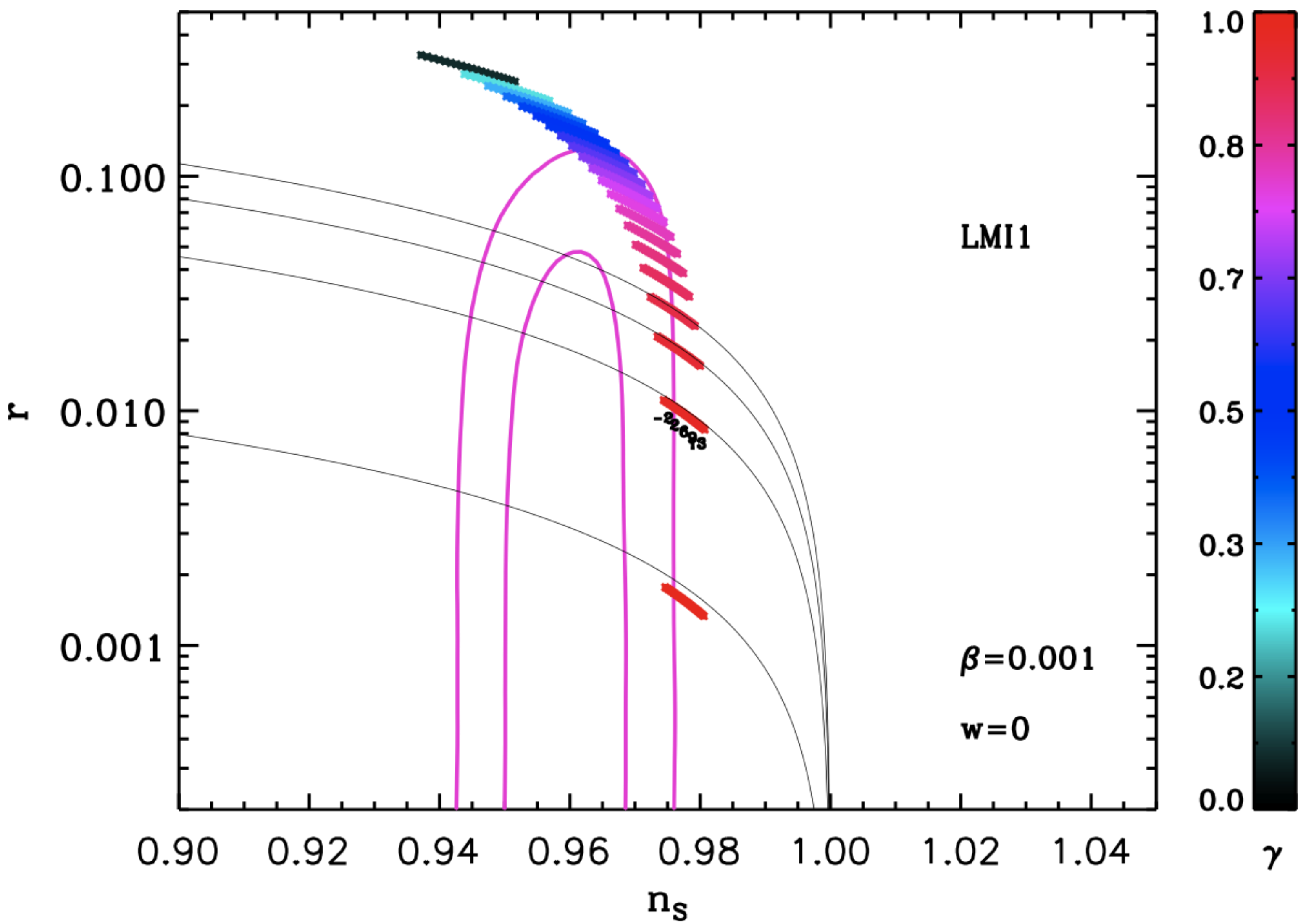

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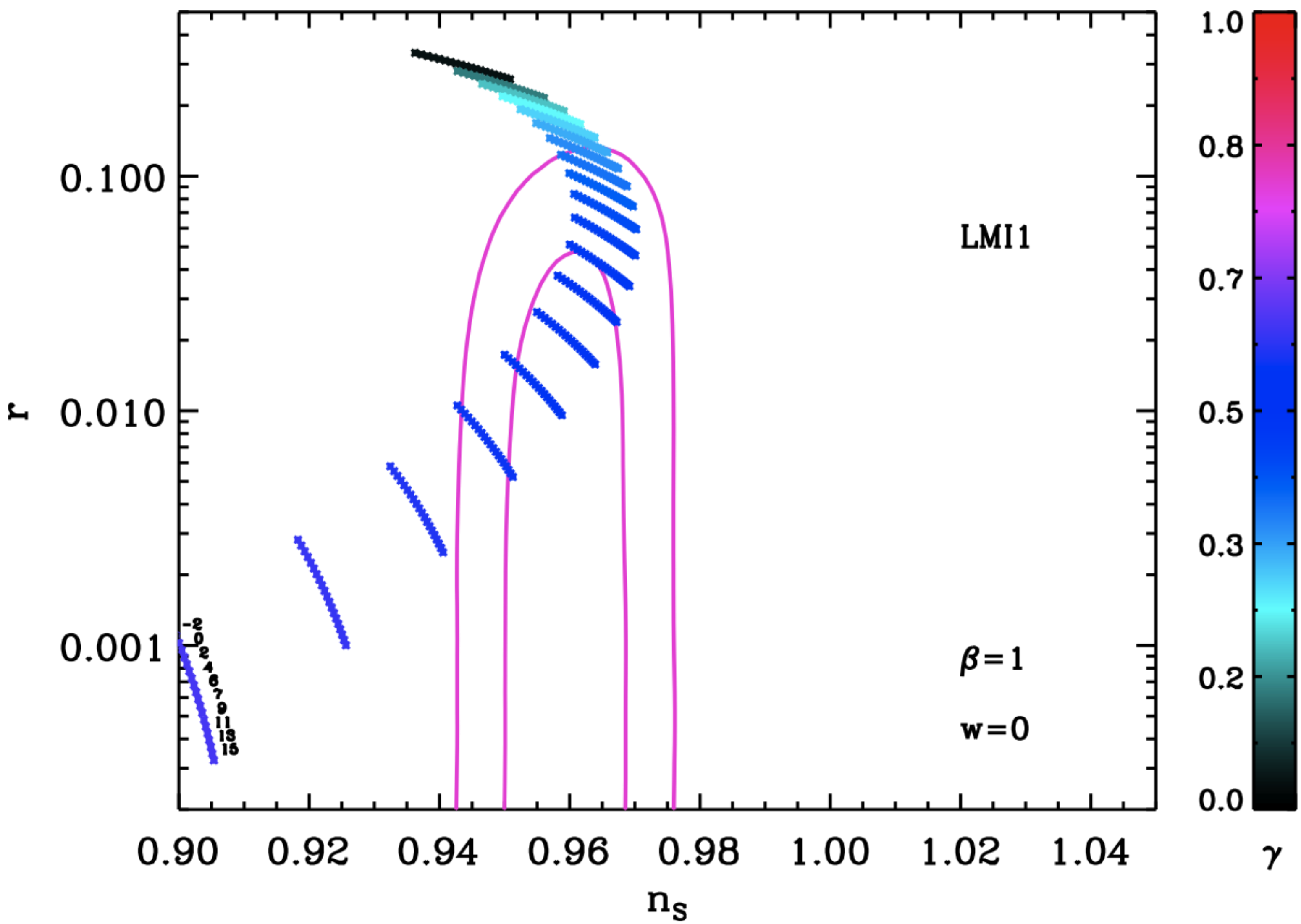

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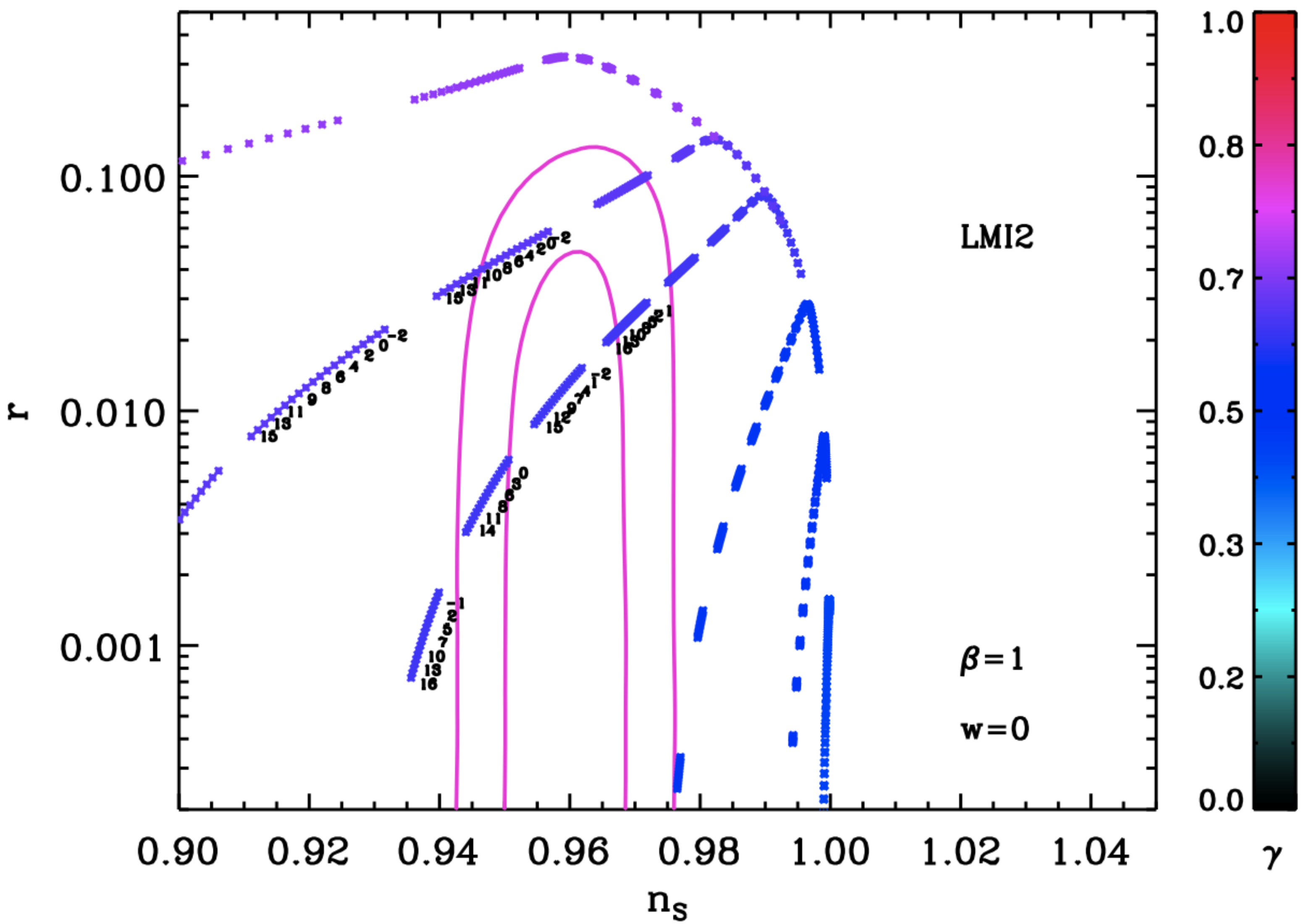

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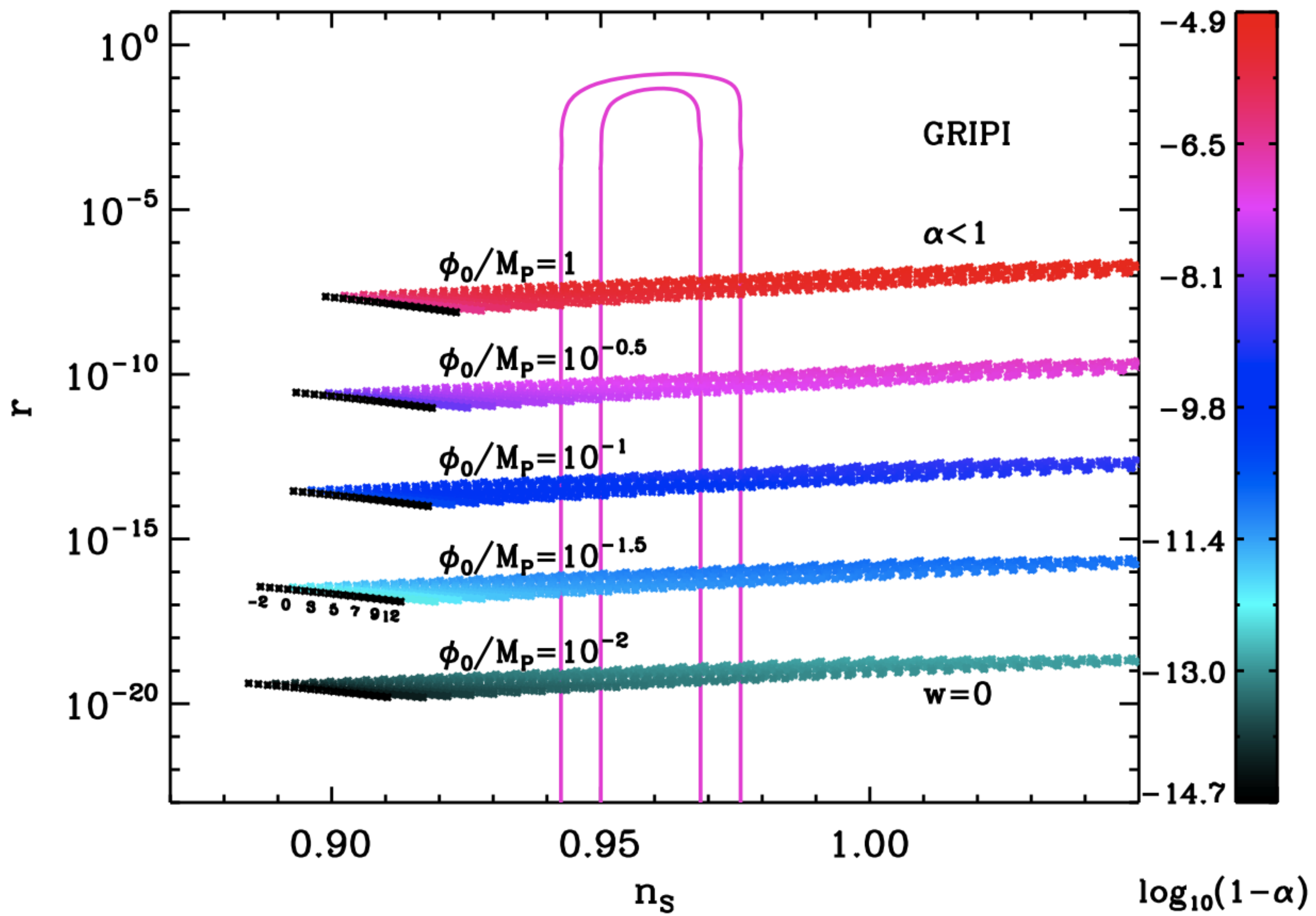

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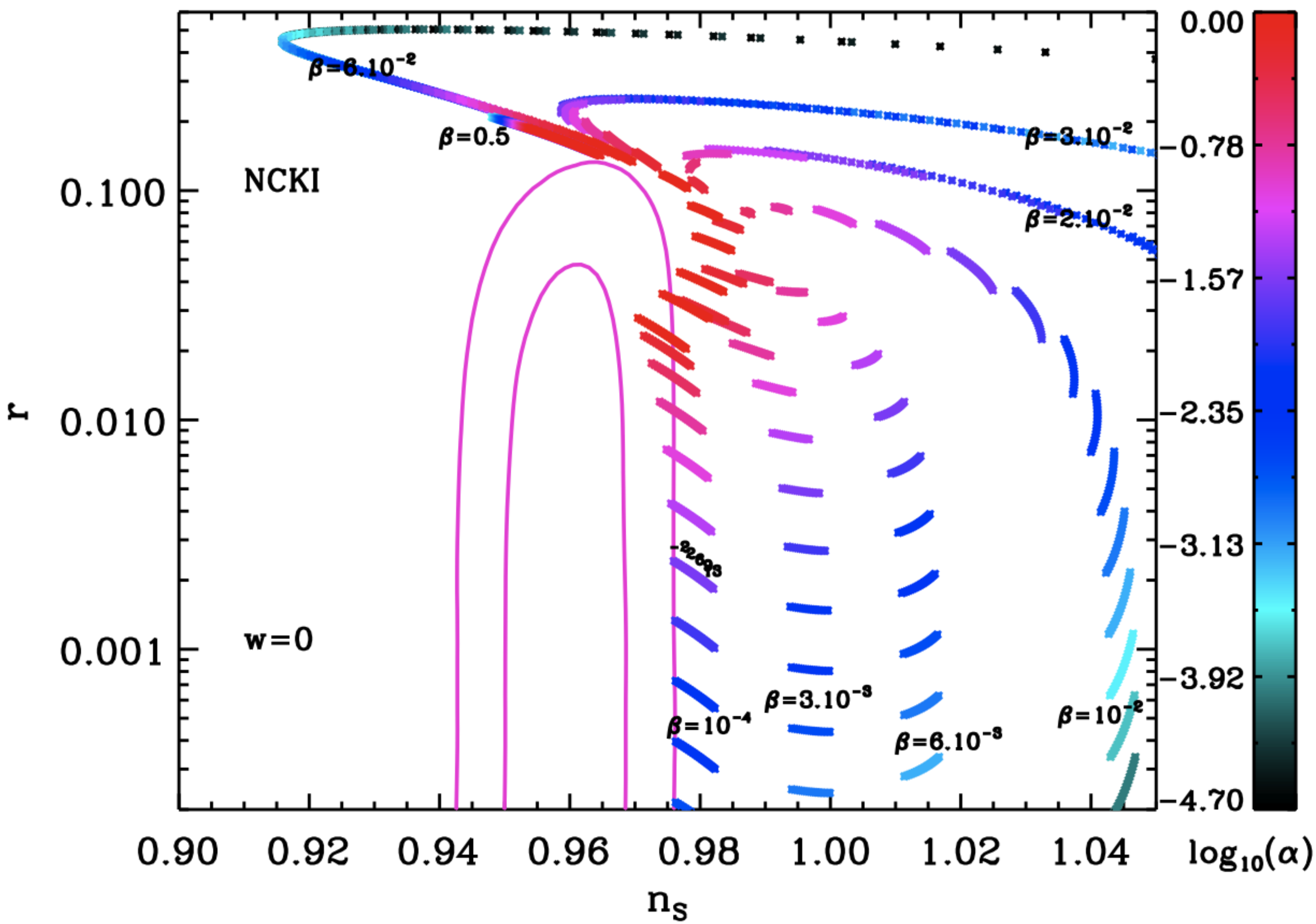

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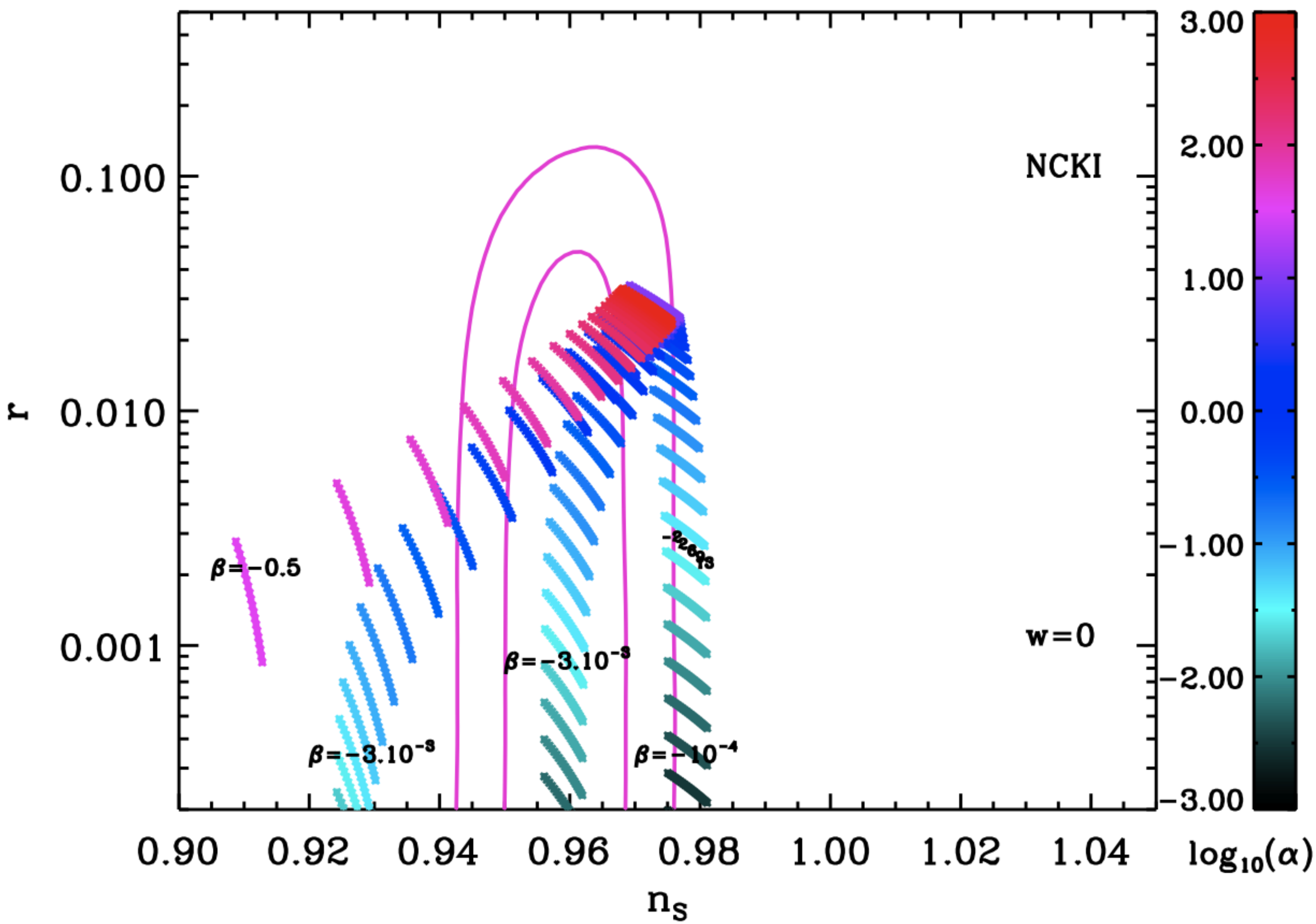

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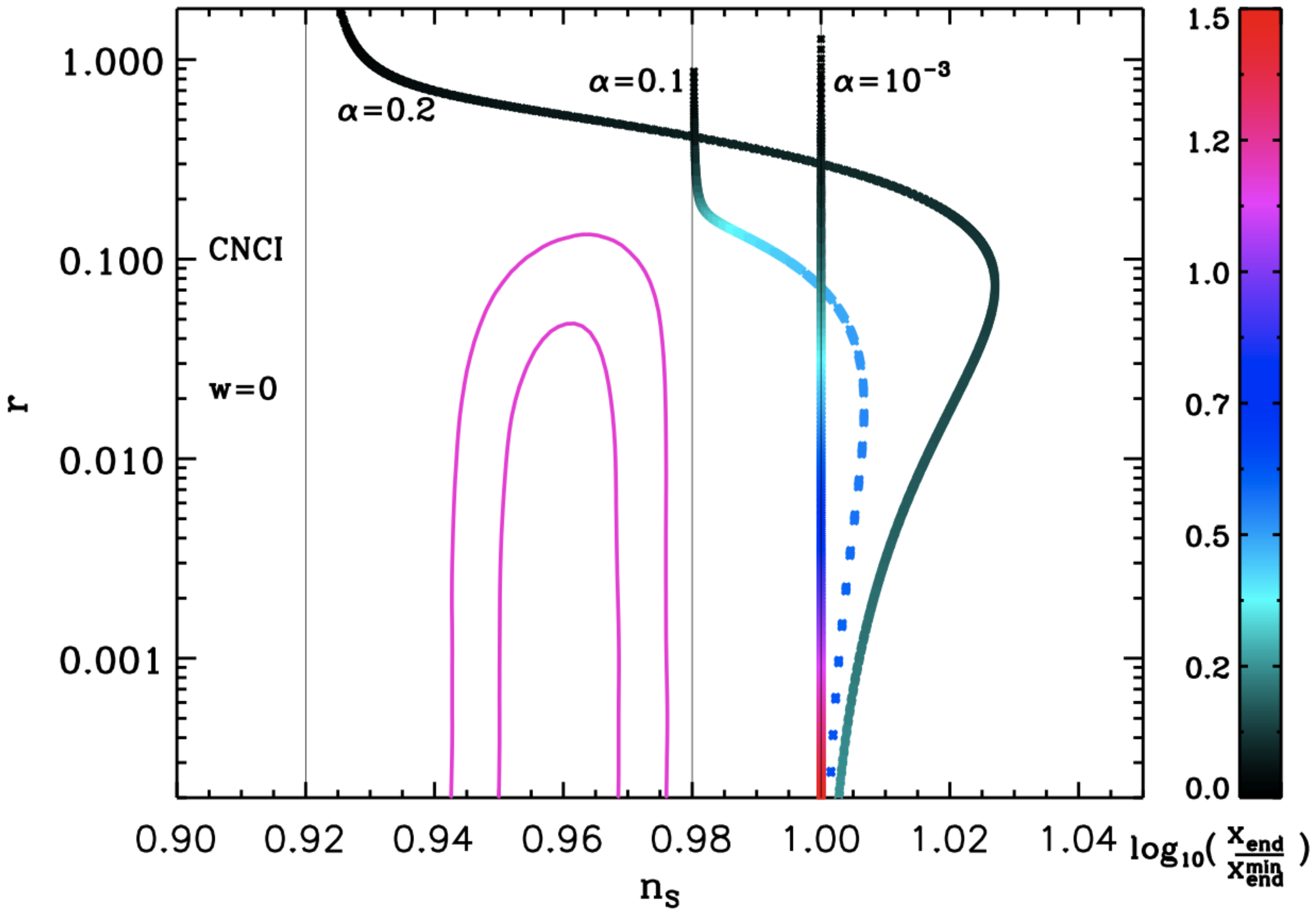

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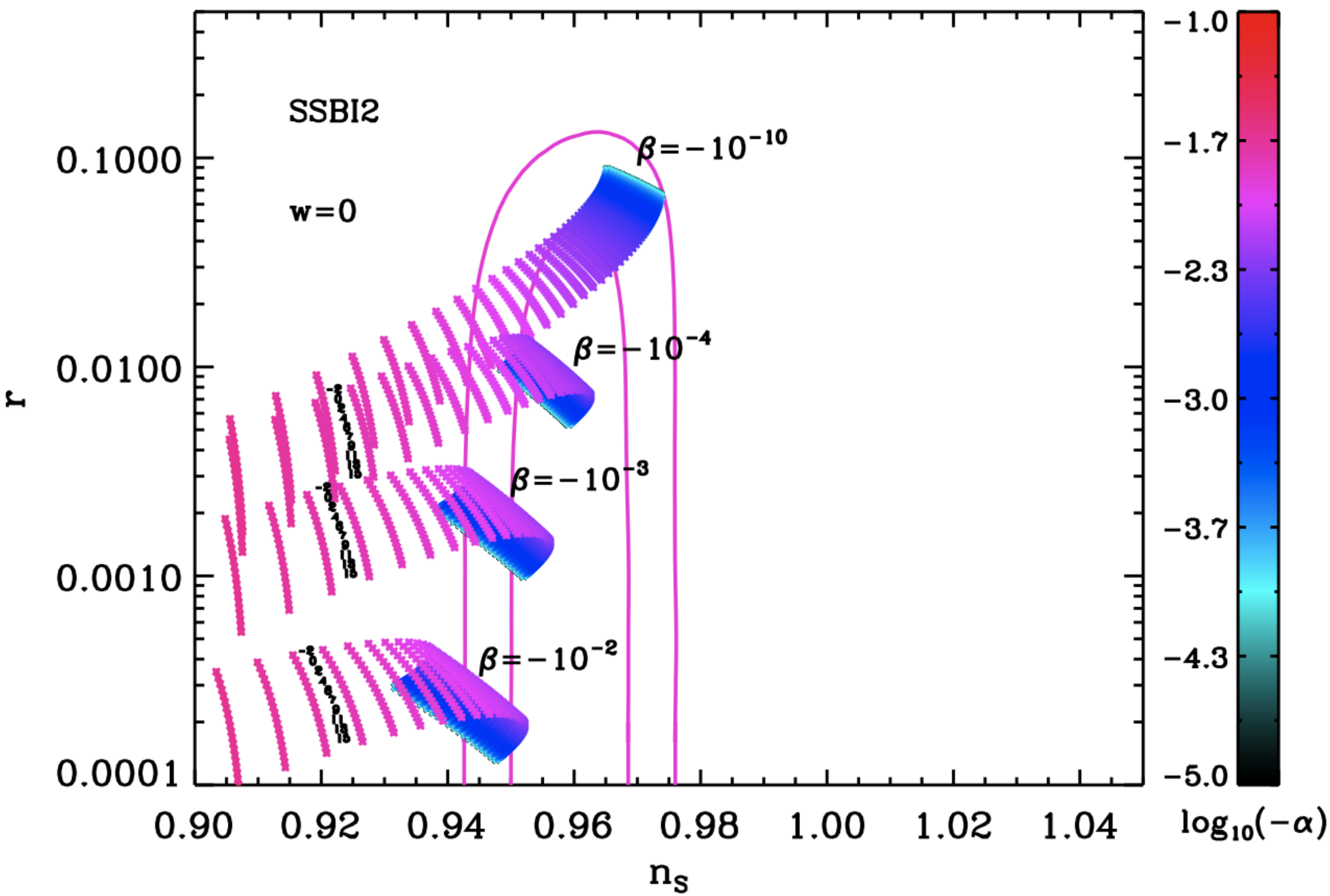

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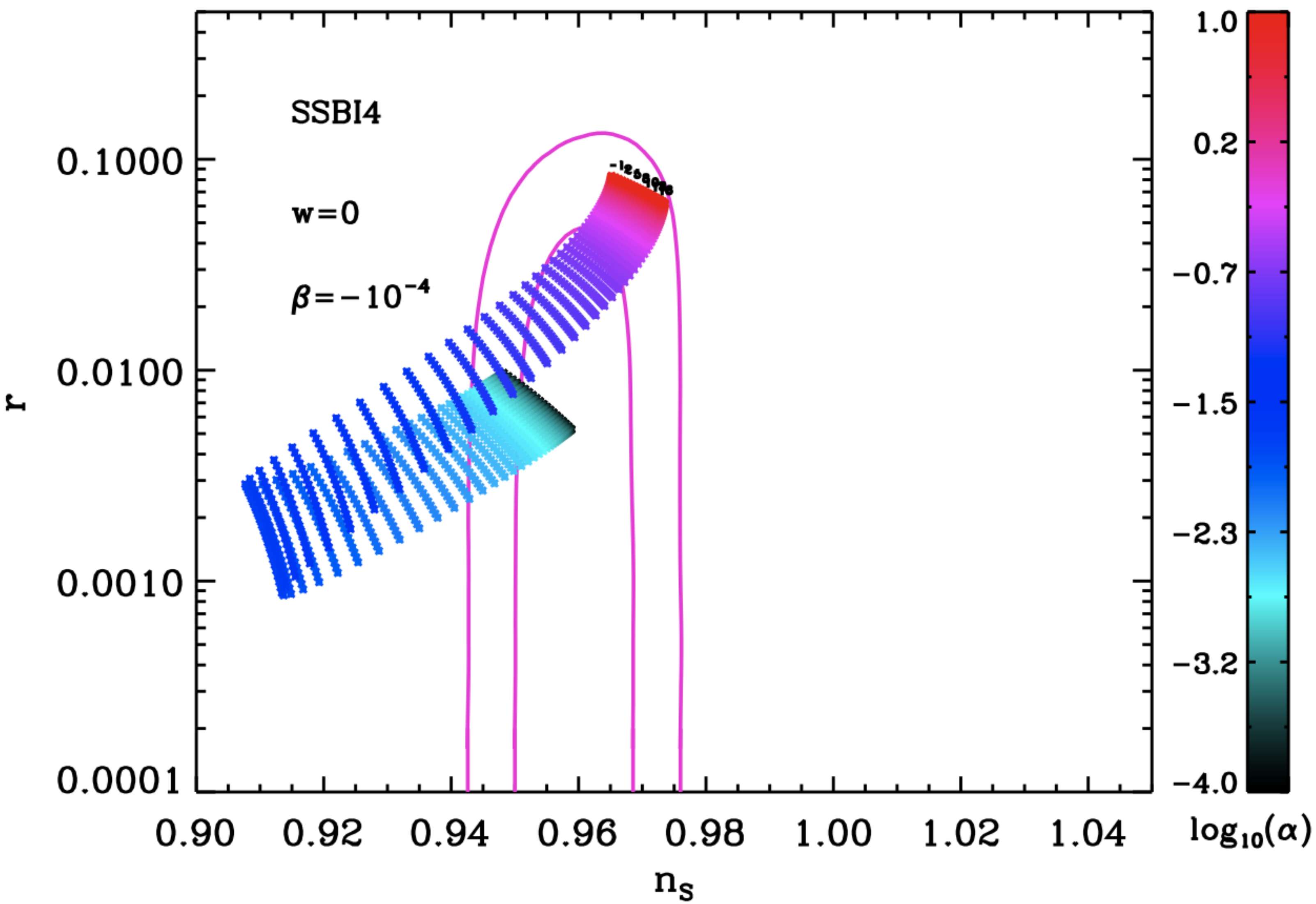

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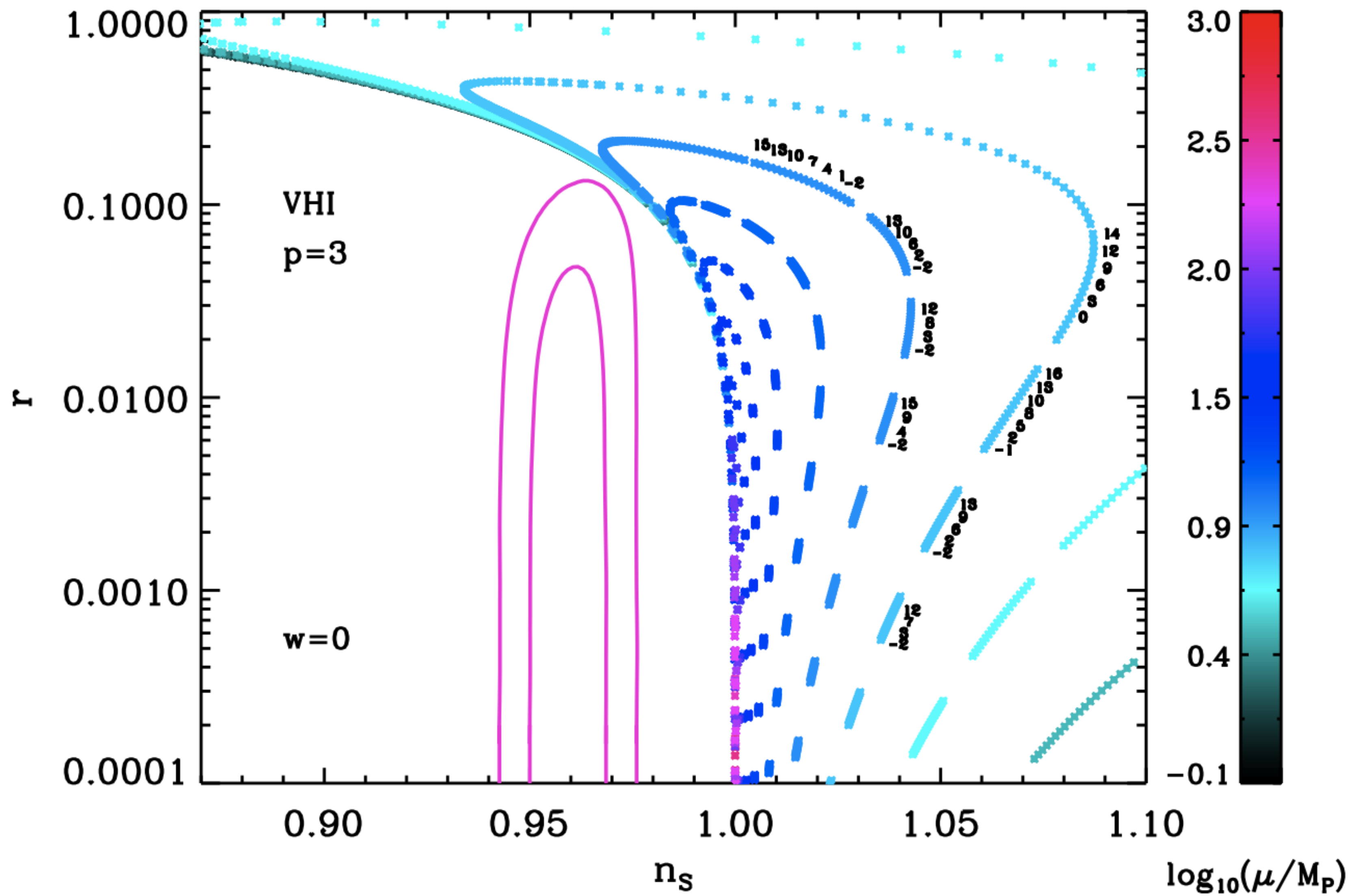

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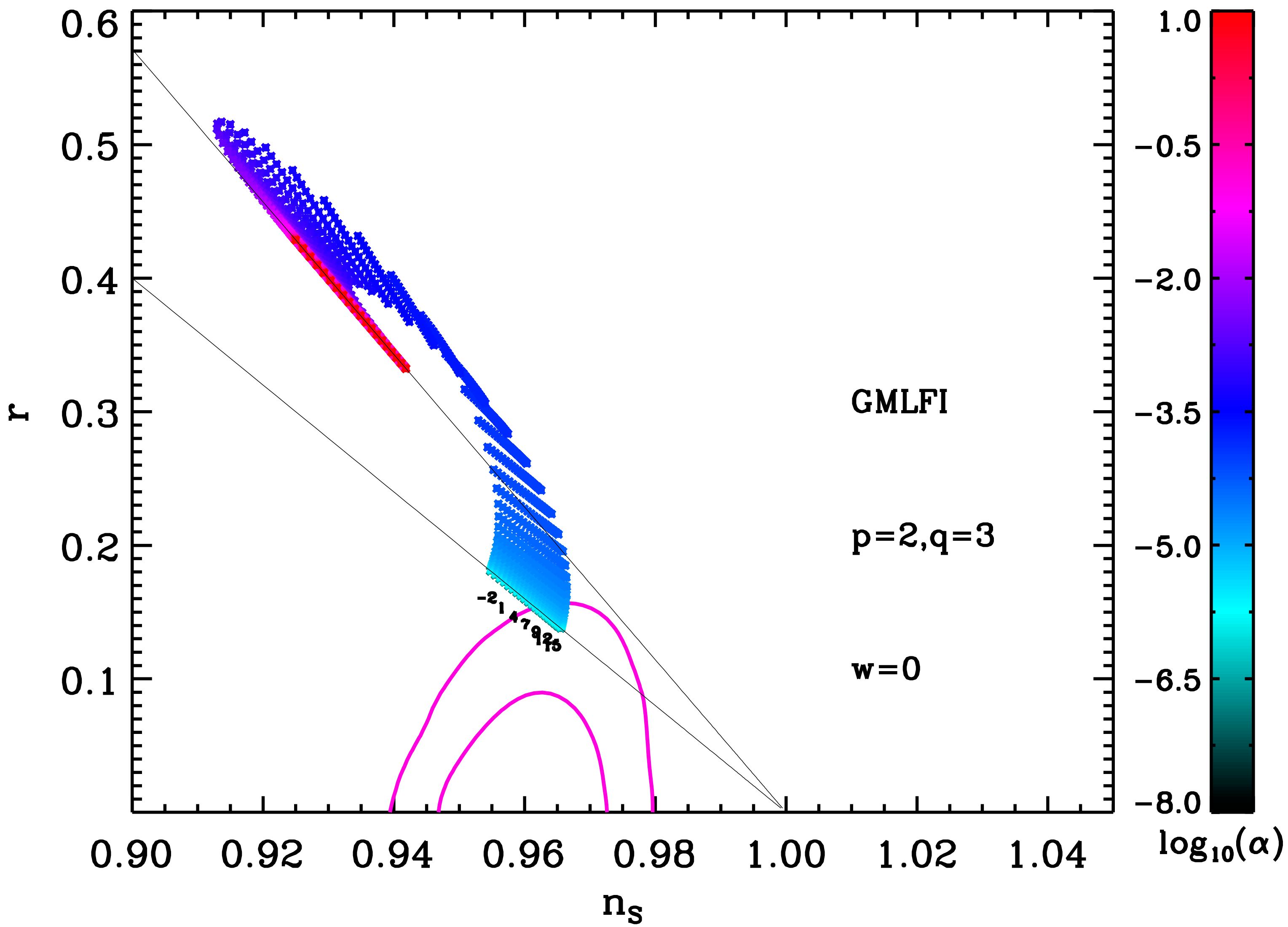


How to do model comparison?

- For model comparison, we compute the Bayesian evidence (integral of the likelihood over all parameter priors~probability of a model), ie the probability of a model, for each inflationary scenario

Bayesian evidence of the model "i"

$$\frac{p\left(\mathcal{M}_i|D\right)}{p\left(\mathrm{HI}|D\right)} = B_{i-\mathrm{HI}} \begin{cases} B_{i-\mathrm{HI}} > 1 & \text{Model "i" is better than HI} \\ B_{i-\mathrm{HI}} < 1 & \text{HI is better than model "i"} \end{cases}$$

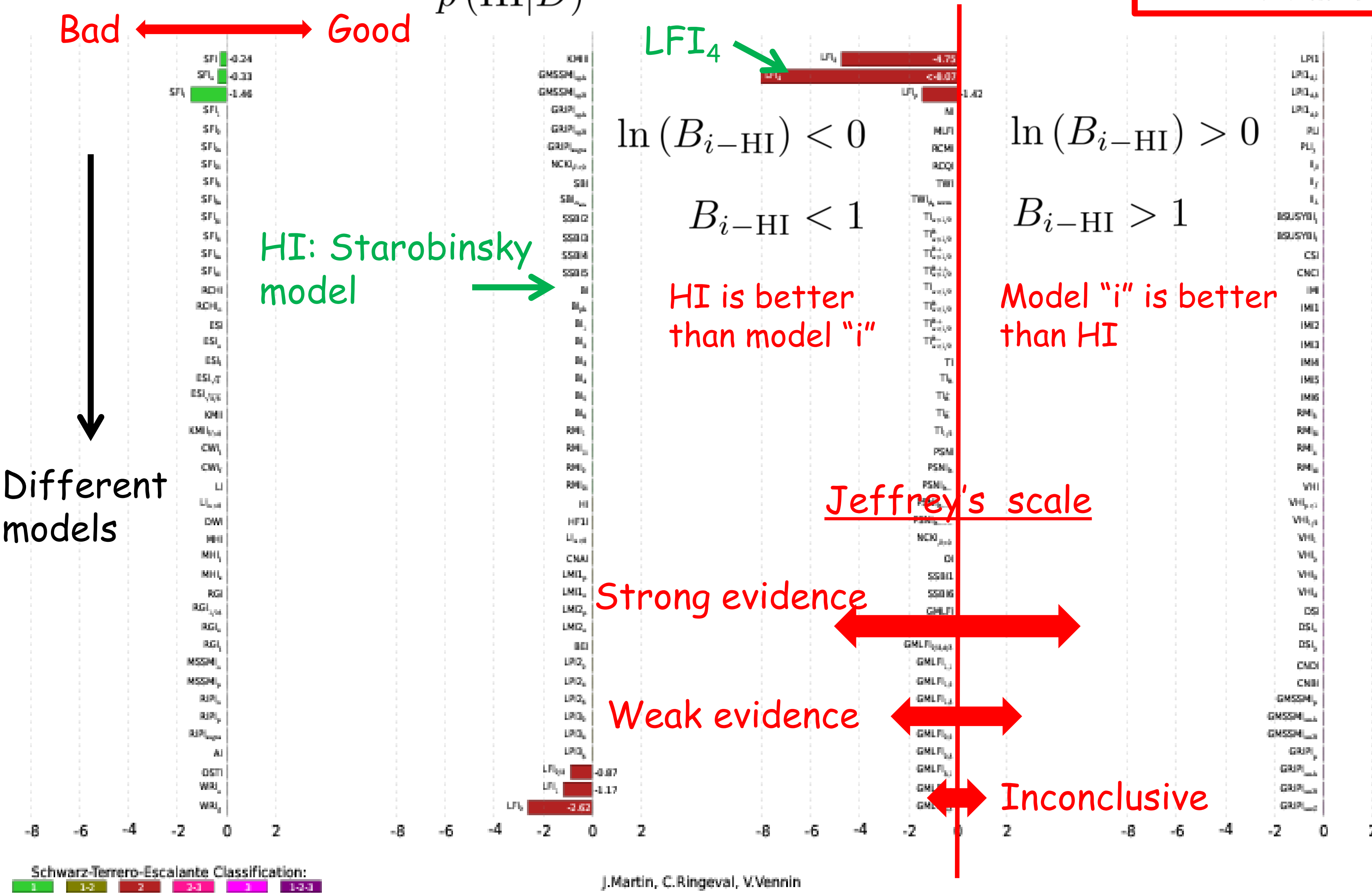
posterior odds

Bayesian evidence of the reference model=Starobinsky model

First calculation of inflationary Bayesian evidence in 2010

$$\frac{p\left(\mathcal{M}_i|D\right)}{p\left(\mathrm{HI}|D\right)} = B_{i-\mathrm{HI}} \qquad \ln\left(B_{i-\mathrm{HI}}\right) = 0$$

WMAP7: Martin, Ringeval & Trotta
arXiv:1009.4157



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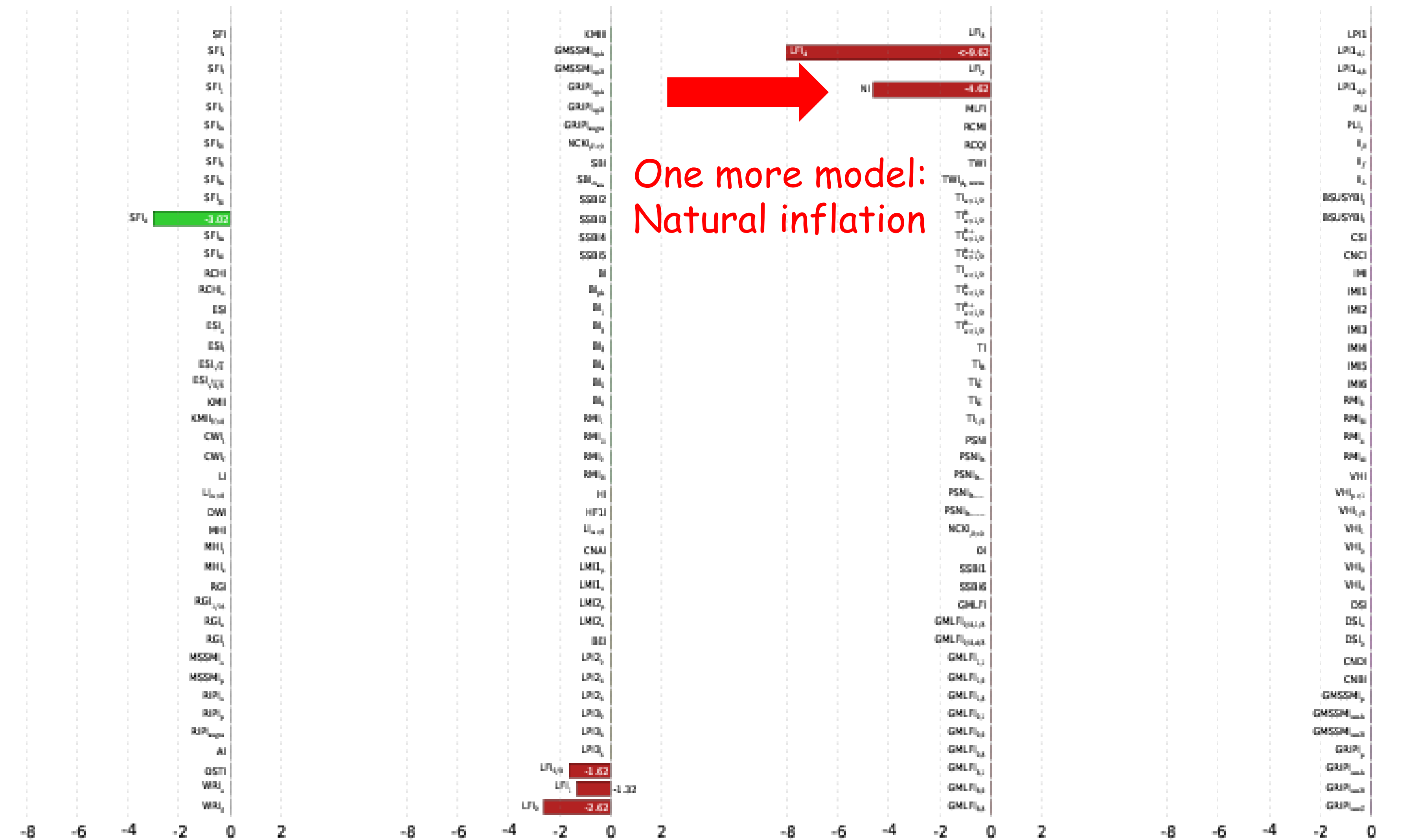
ASPIC project - *Encyclopaedia Inflationaris*

Planck calculation of Bayesian evidence

Planck collaboration
arXiv:1303.5082

Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{\mathrm{HI}})$ for the inflationary models



Schwarz-Terrero-Escalante Classification:
1 | 1-2 | 2 | 2-1 | 3 | 1-2-1

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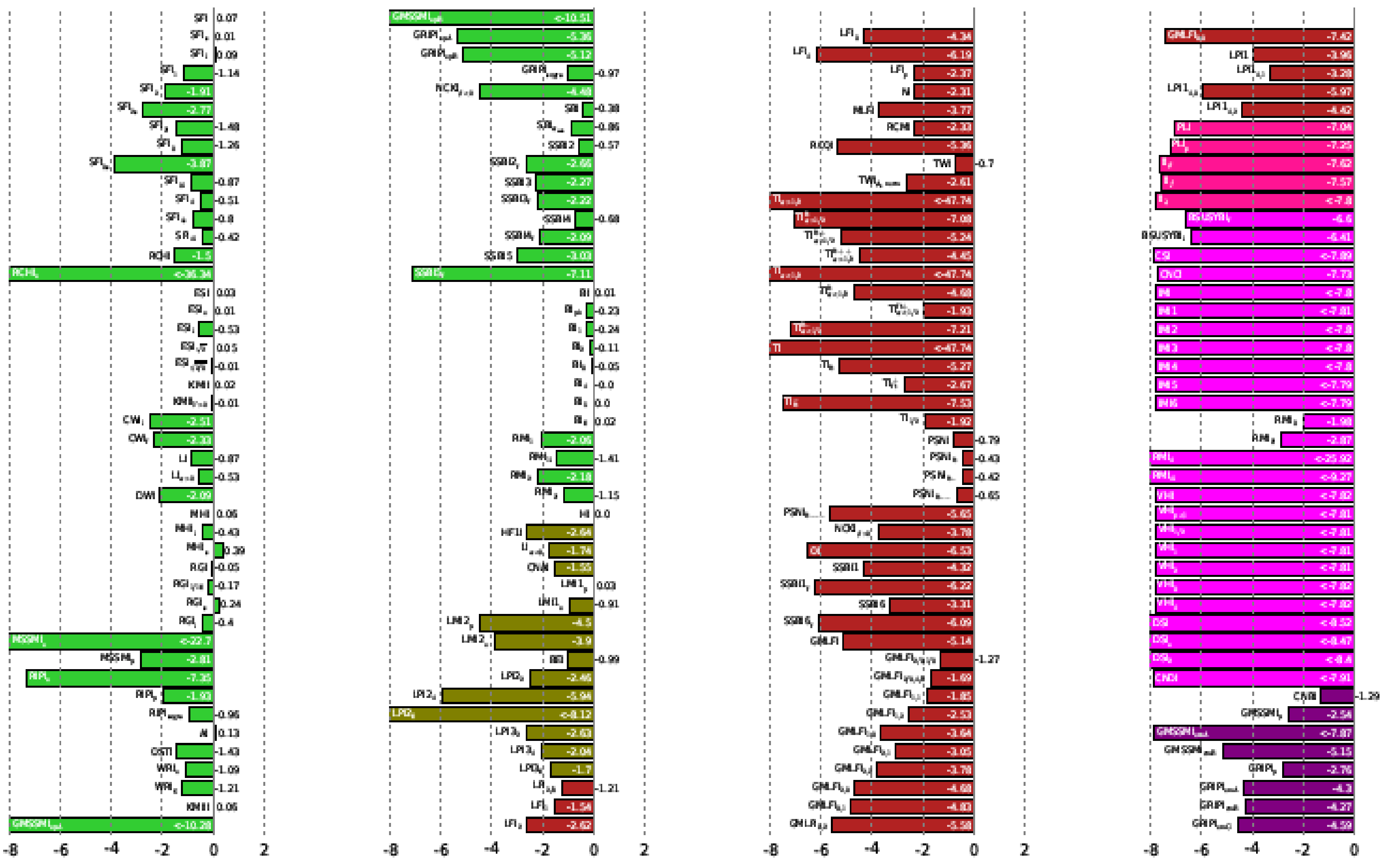
ASPIC evidence

Bayesian Evidences $\log(\mathcal{E}/\mathcal{E}_{\mathrm{HI}})$ for the inflationary models



Schwarz-Terrero-Escalante Classification:
1 | 1-2 | 2 | 2-3 | 3 | 1-2-3

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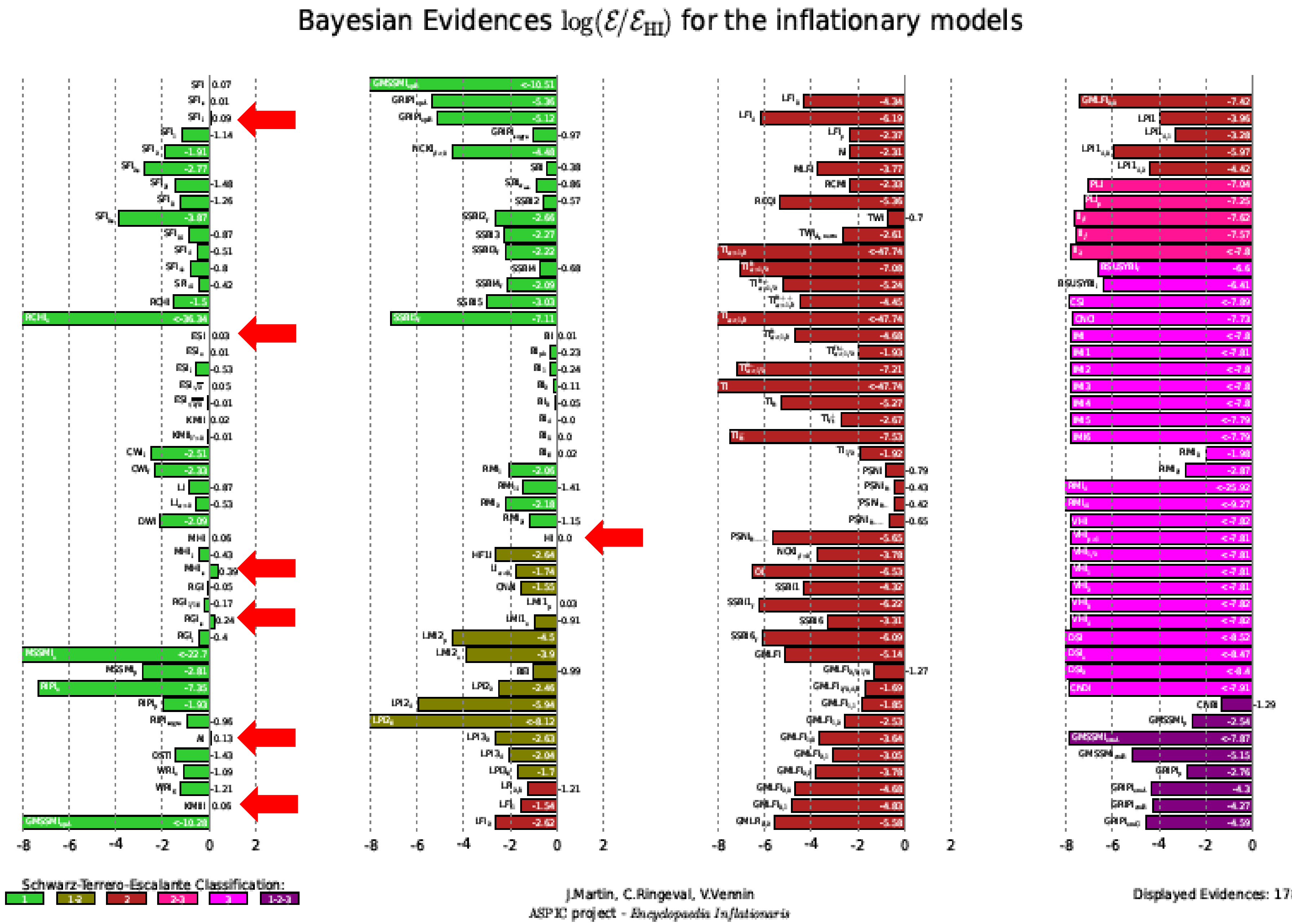
Displayed Evidences: 178

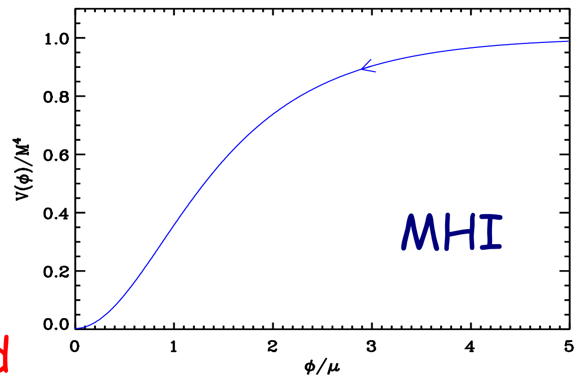
ASPIC evidence: good models

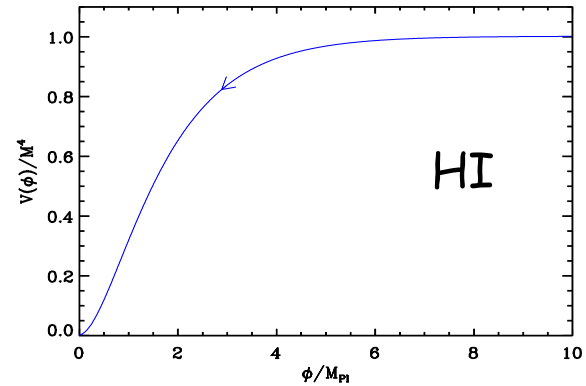


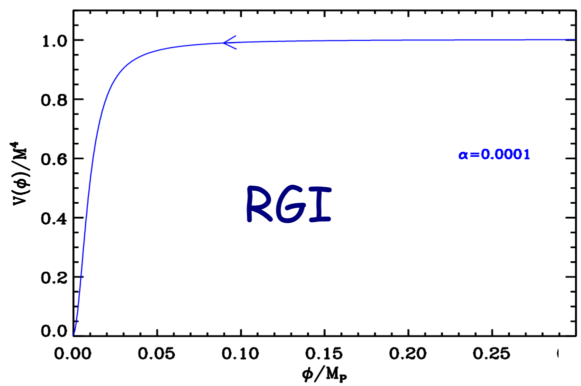
And the winners are …

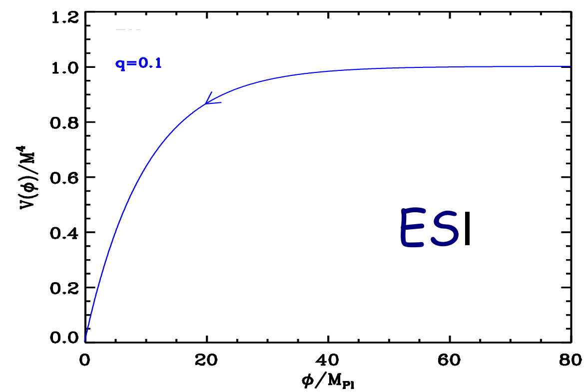
Planck has identified the shape of the inflaton potential:

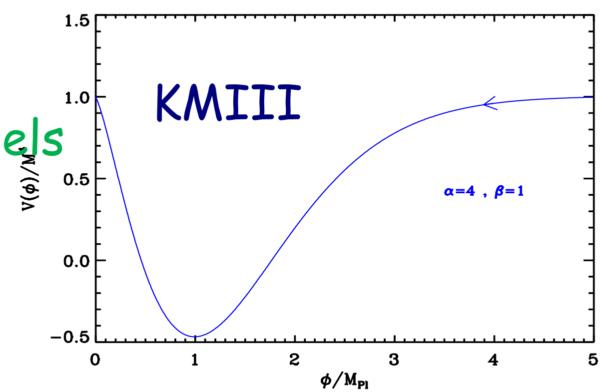
<u>Plateau inflation</u>

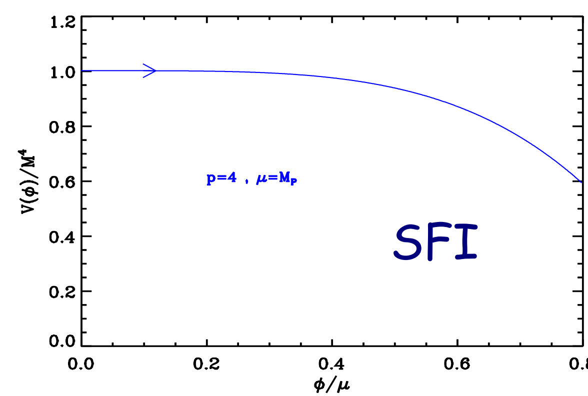






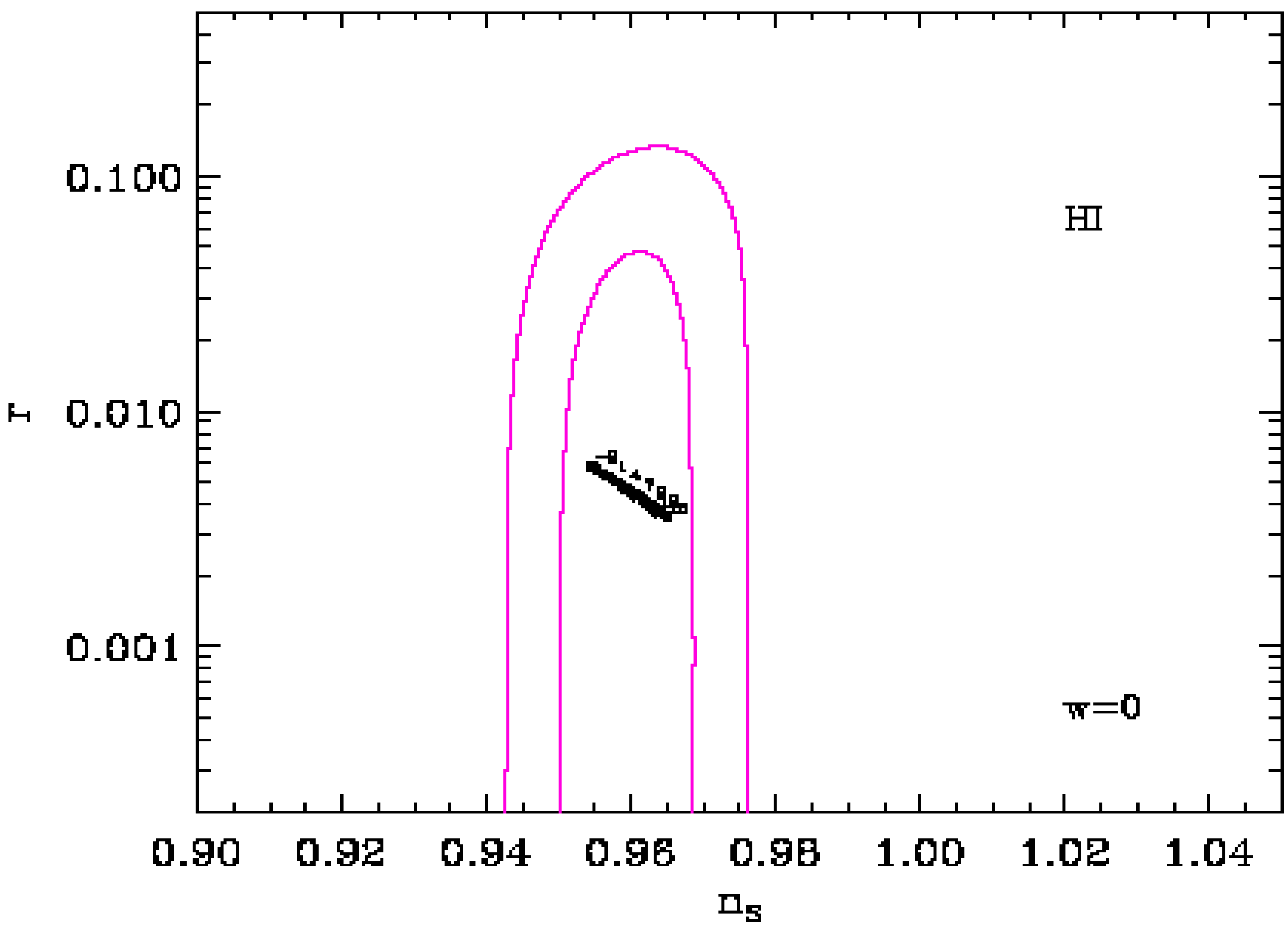




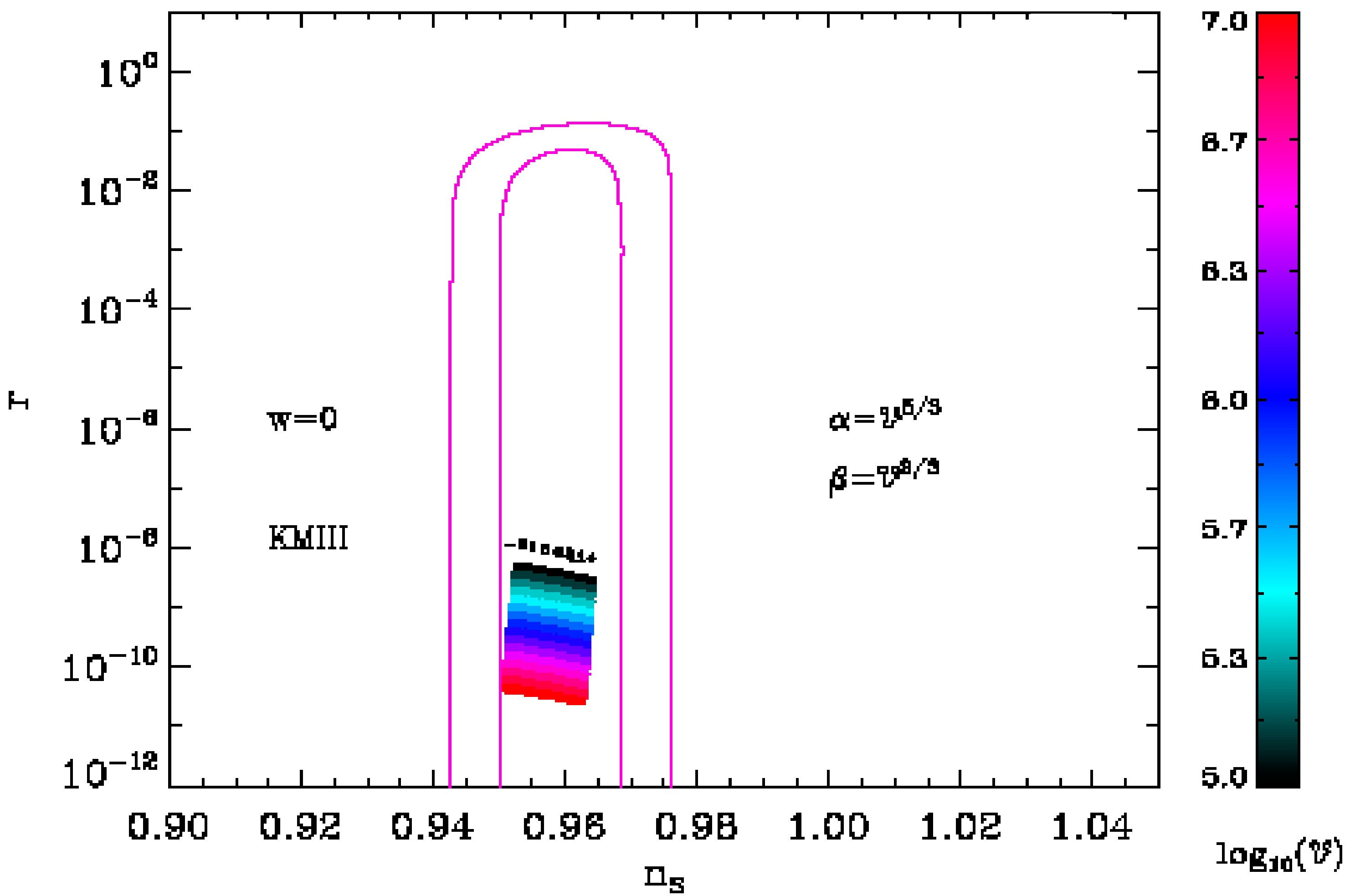


NB: the difference between these models is "inconclusive".

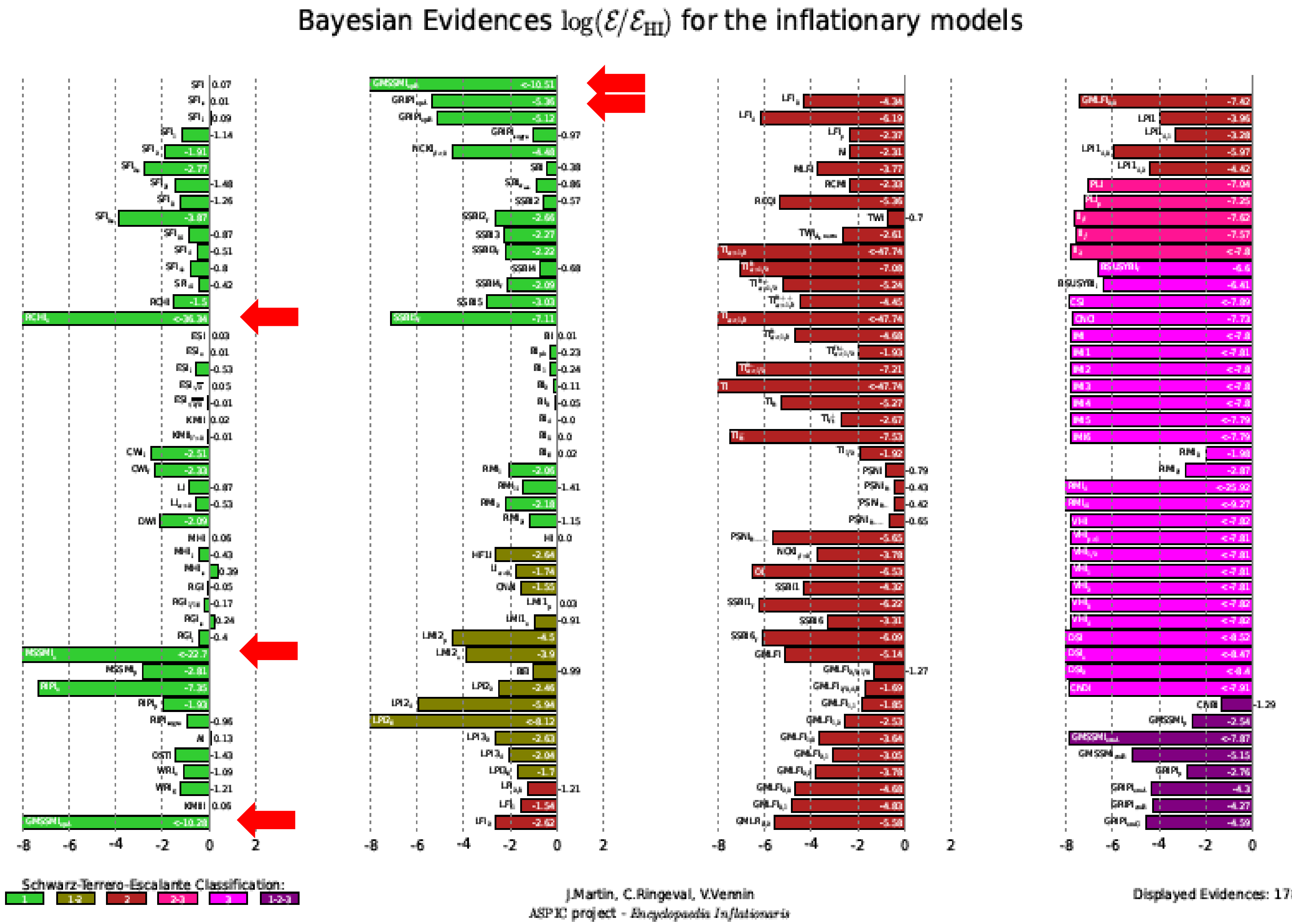
And the winners are …



And the winners are …

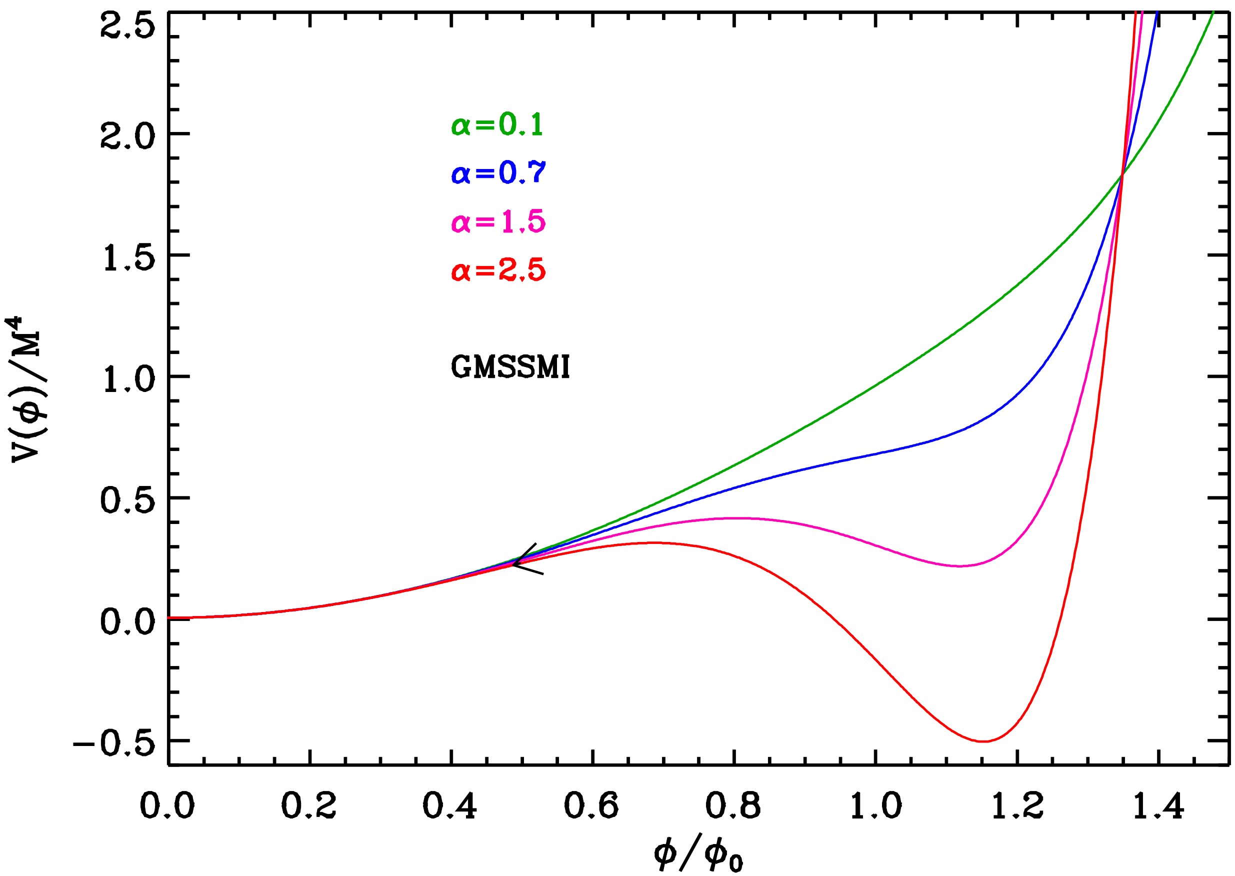


ASPIC evidences: examples of not so good models



And the loosers are …

They are loosers too … for instance, inflexion point inflation (models based on the MSSM) are clearly strongly disfavored by Planck



Conclusions

- ❑ Planck favors single field slow-roll scenarios: simplest but non trivial models

- ❑ Within this class, Planck data indicates that Plateau inflation is the correct shape of the potential (category I)

- ❑ There are a dozen of models that have a better Bayesian evidence (beyond the inconclusive level). We have come a long road … from hundreds of models, Planck has identified a dozen of favored scenarios!

- ❑ Models are clearly disfavored, ie MSSM inflation (for instance)

- ❑ More to come …

  - Constraints on the reheating temperature for each model

  - Bayesian complexity

  - Evidence for categories (string models, phenomeno models etc …)

  - Update this program with Planck2014 & polarization measurement

Summarizing the summary …