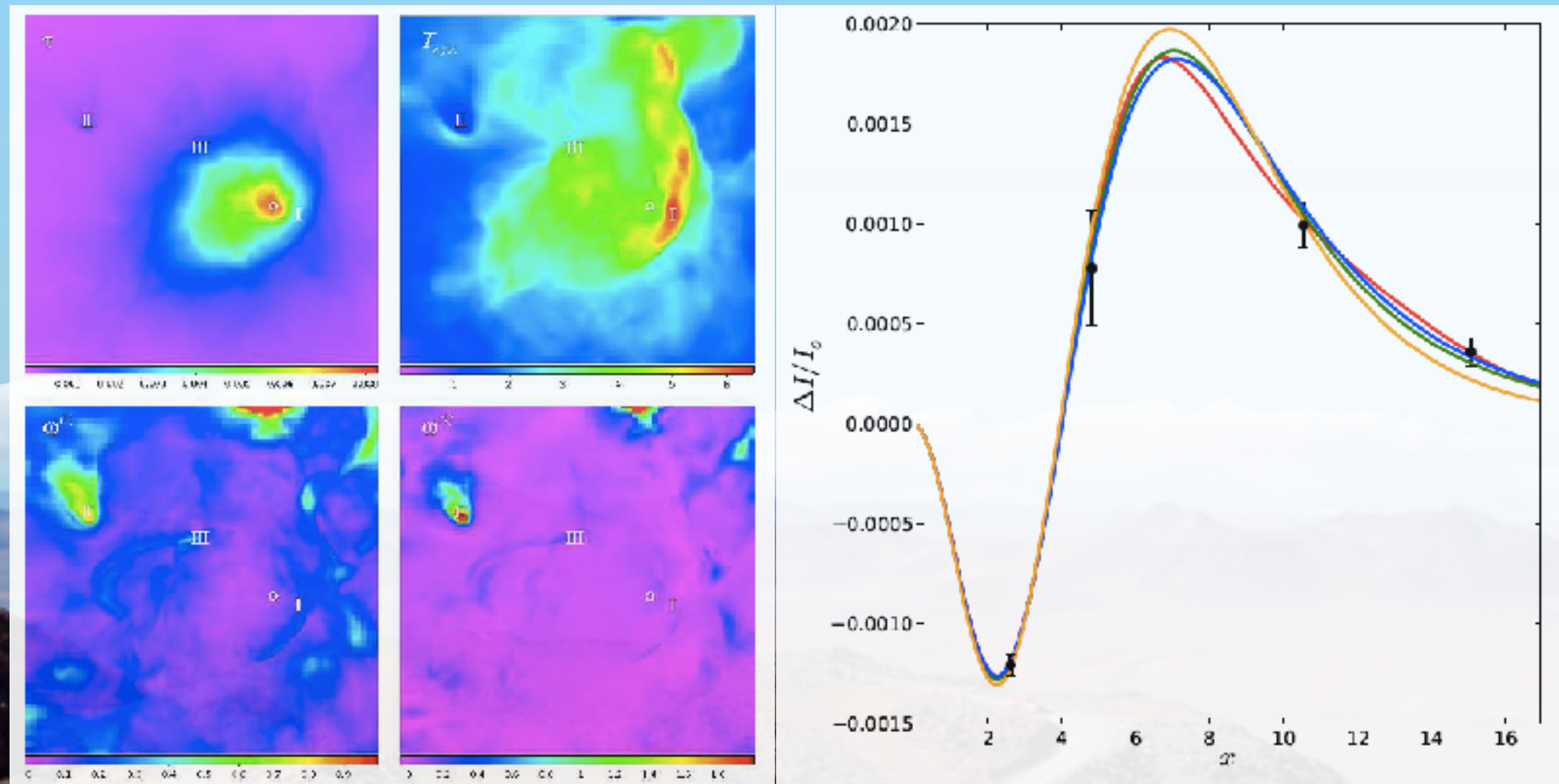


Efficient Computation of the SZ Signal Using Temperature and Velocity Moments



MANCHESTER
1824

The University of Manchester

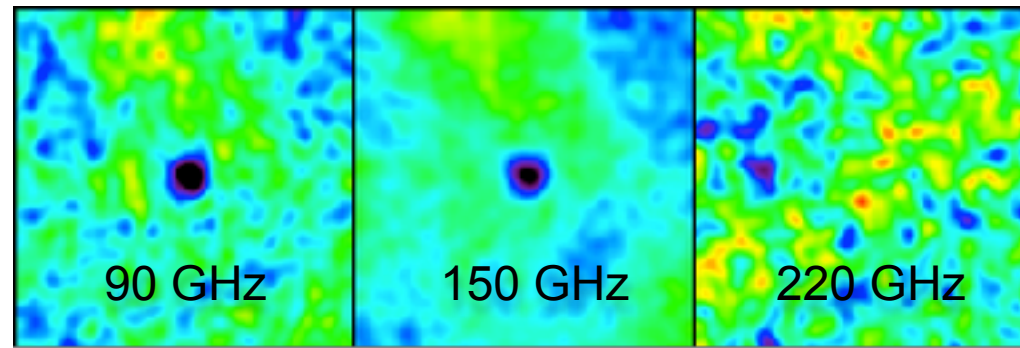
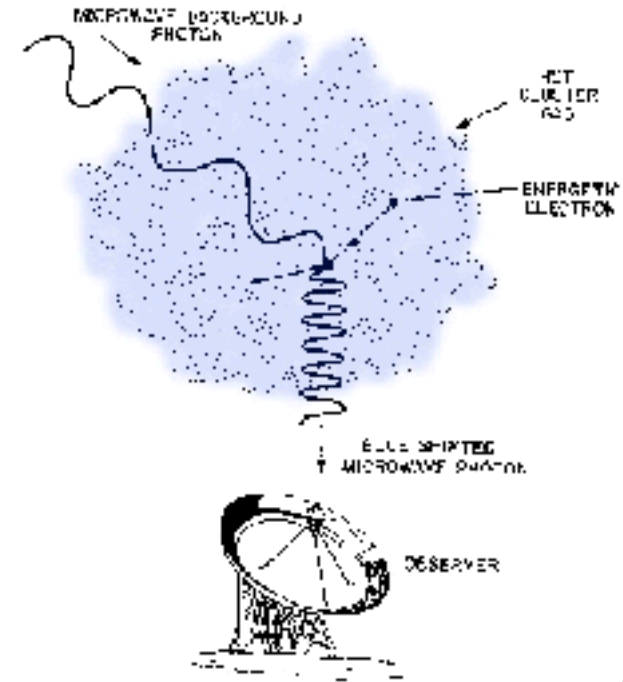
Jens Chluba

CMB in Germany - MPA

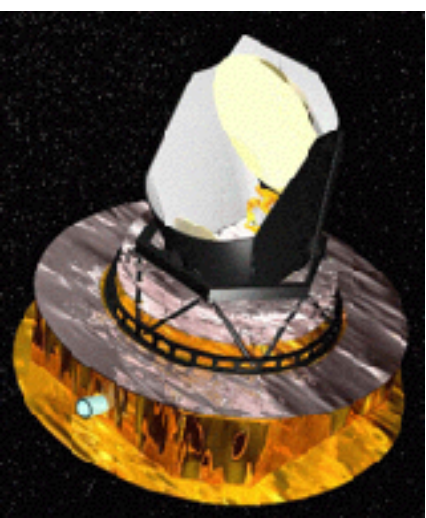
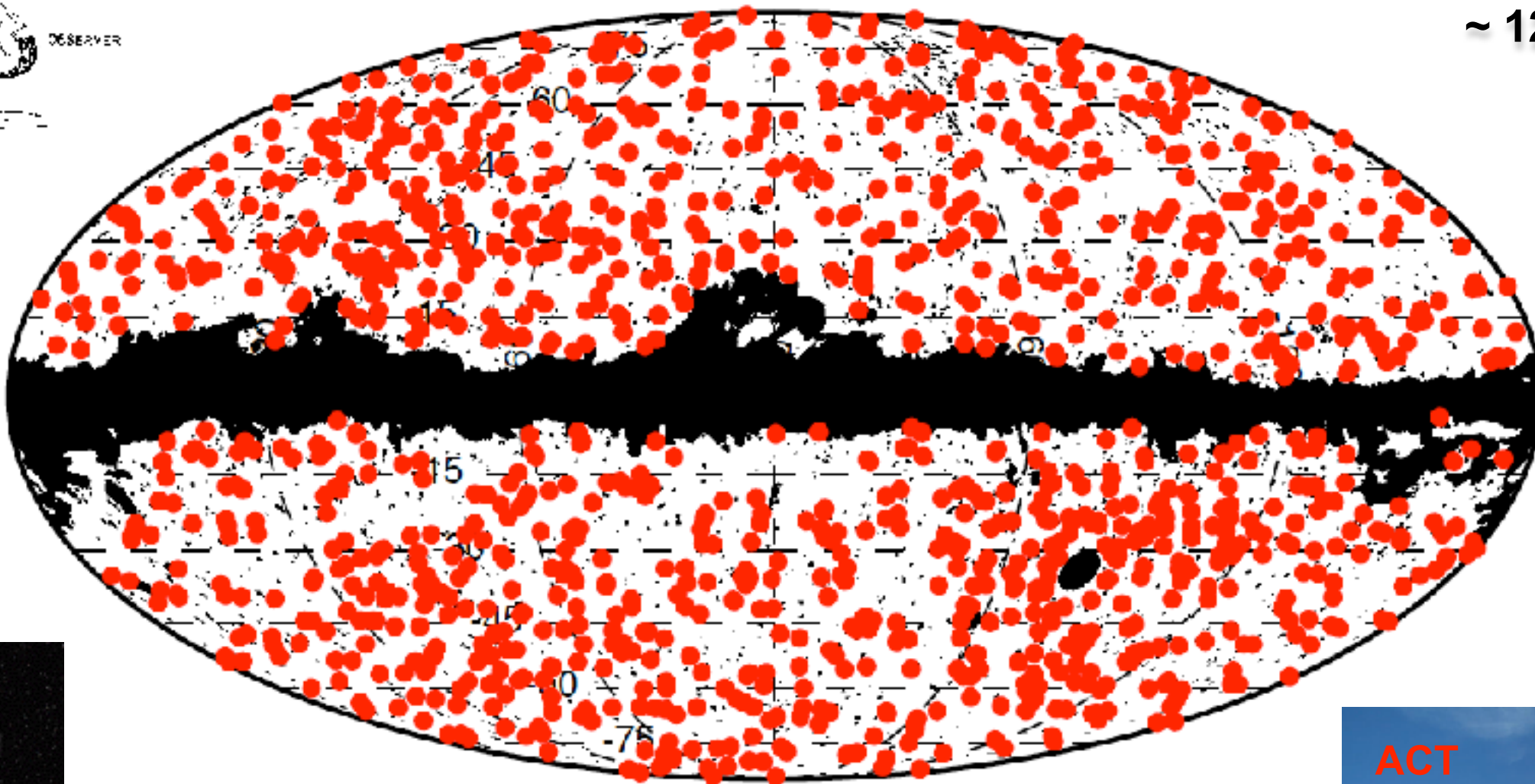
Garching, Germany, Jan 31st-Feb 1st, 2018



Thermal SZ effect is now routinely observed!



~ 1230 objects

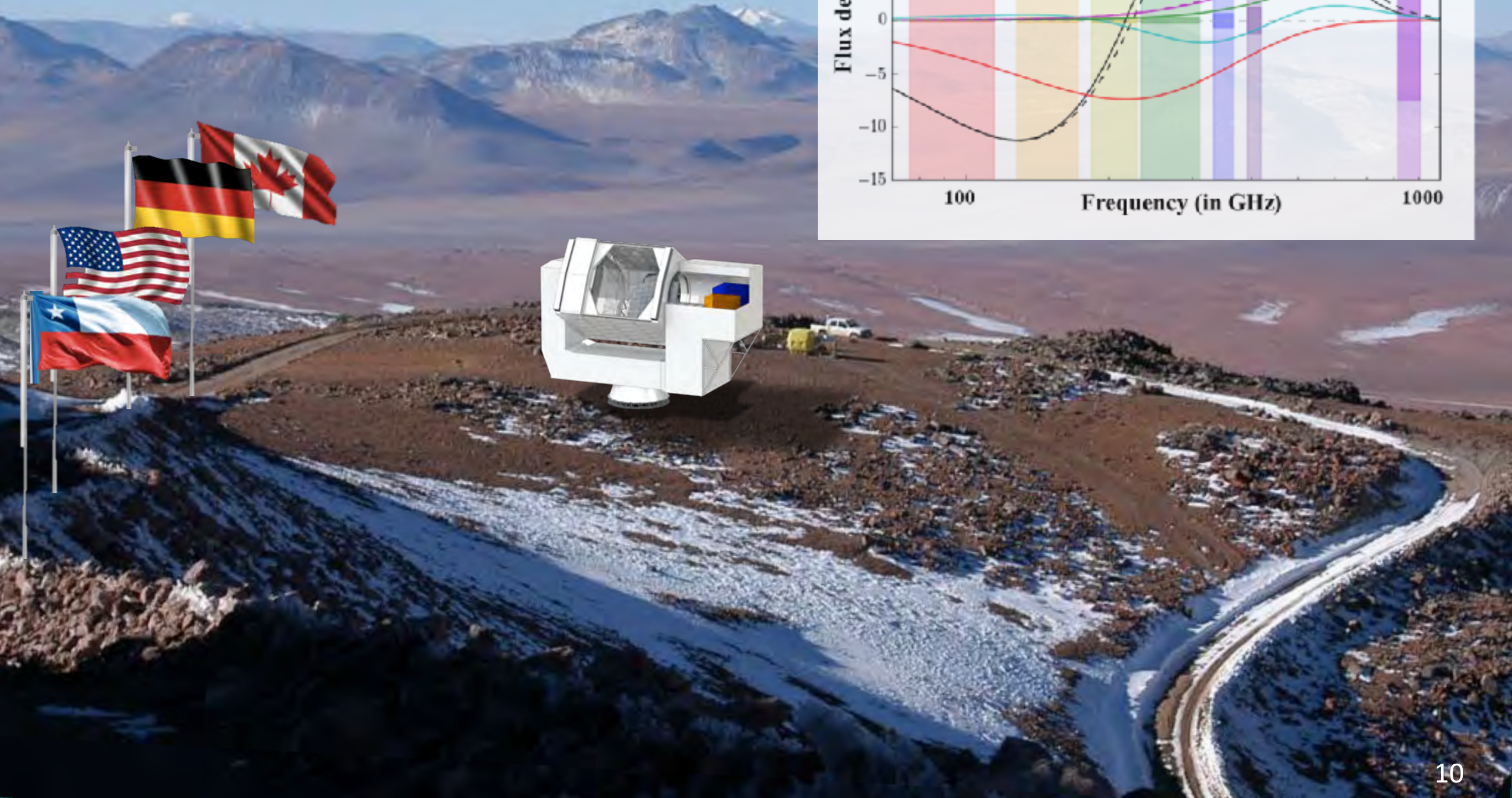
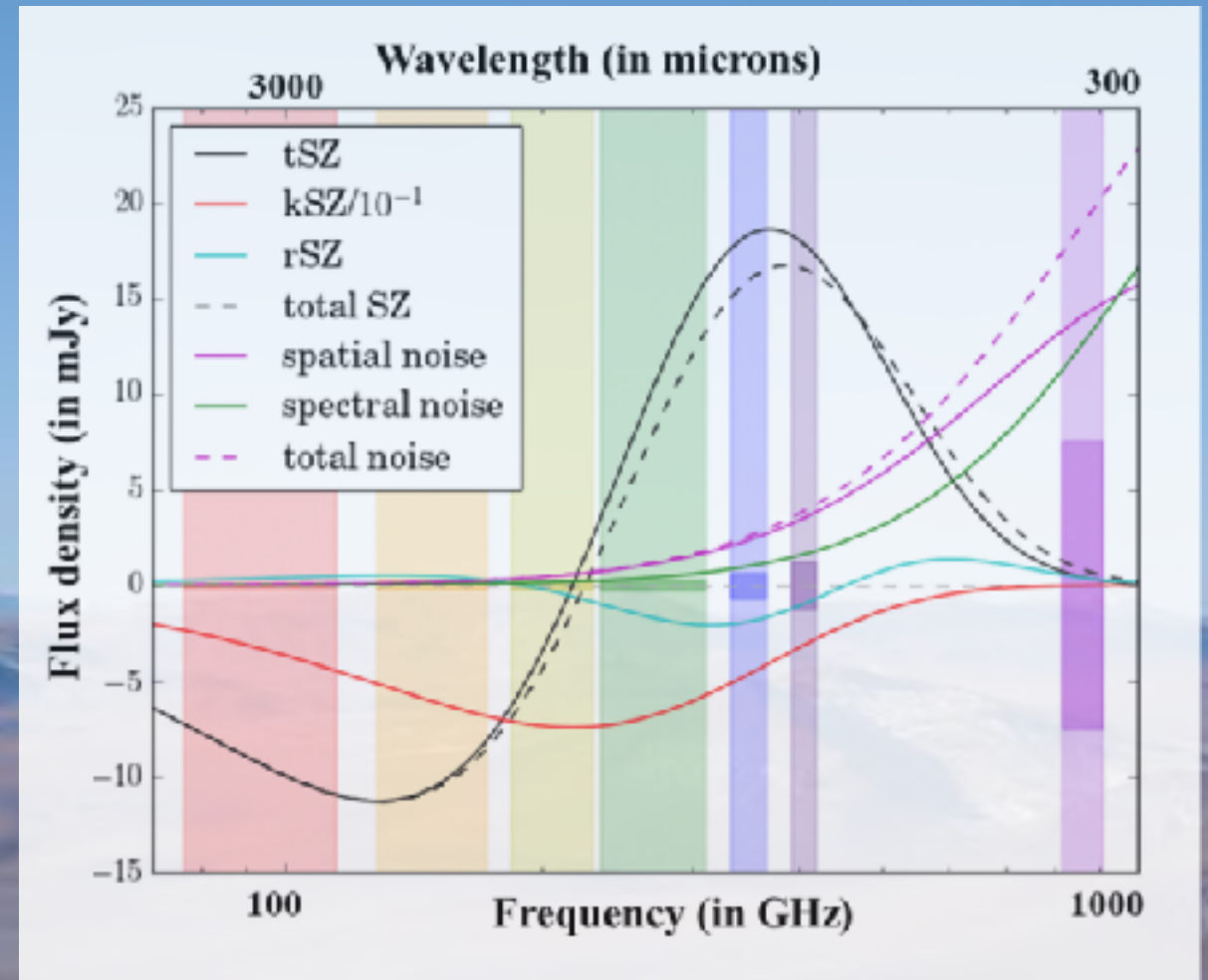


Planck Collaboration, 2013, paper XXIV
Planck Collaboration, 2015, papers



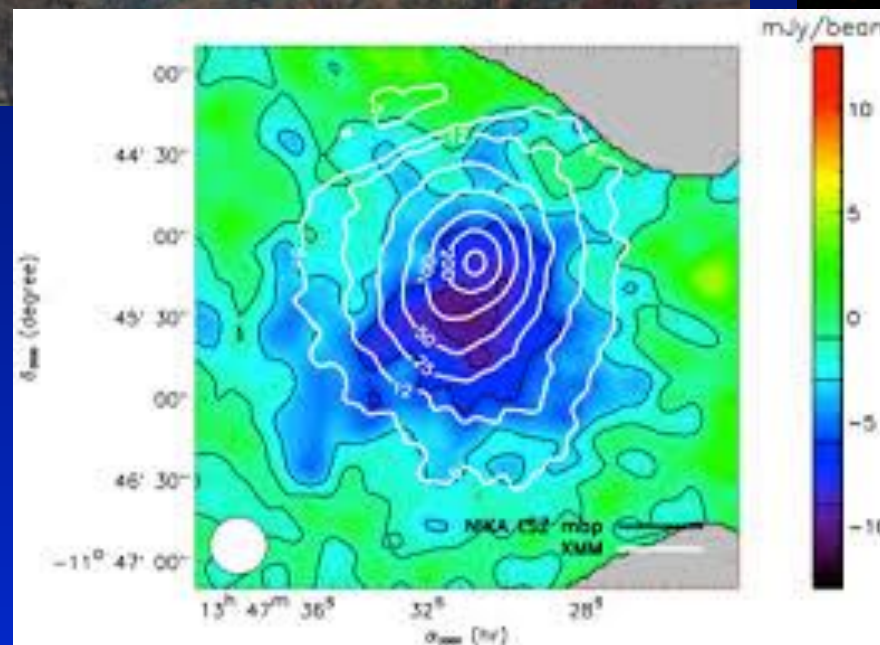
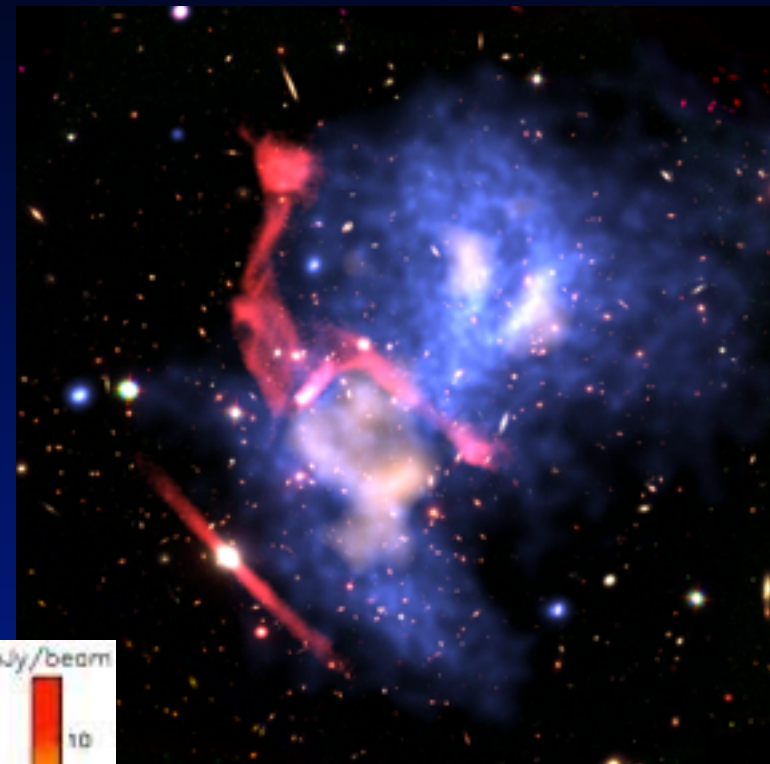


Frank's talk...



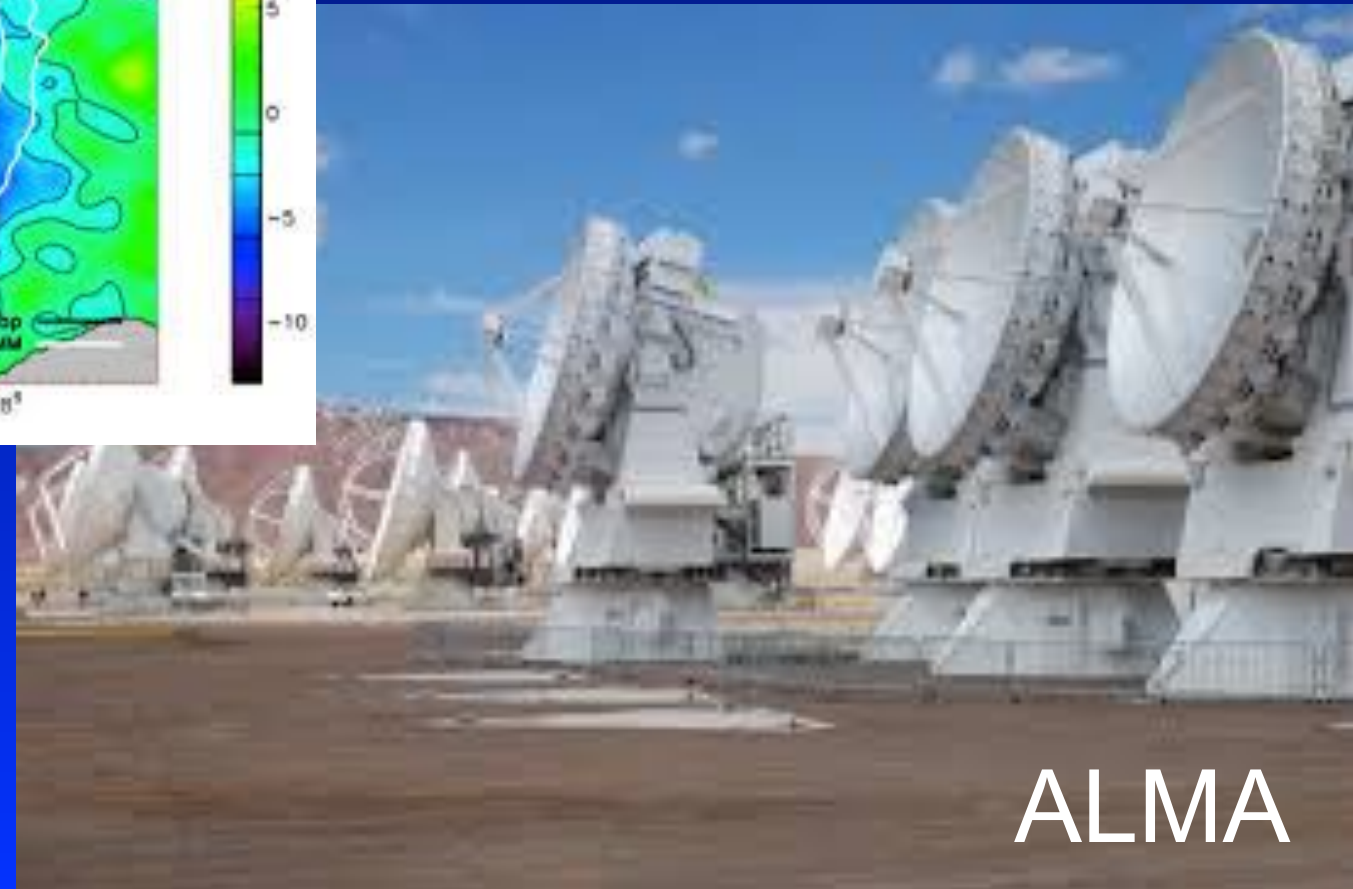
Mustang 2

Tony's talk....



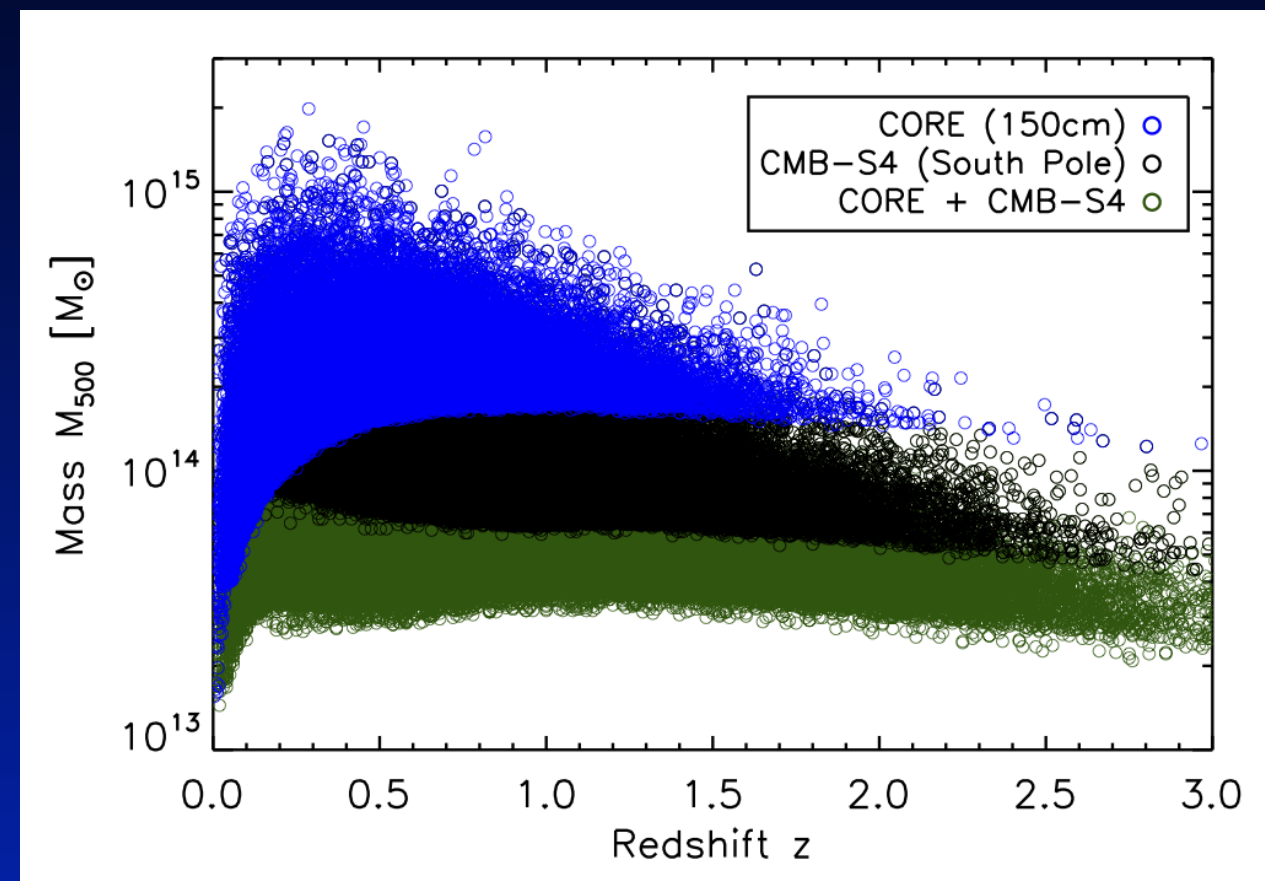
Millimetron?

AtLAST



ALMA

Future SZ science opportunities



Individual cluster studies

- \leftrightarrow 'Gastrophysics'
- \leftrightarrow state of the ICM, turbulence, profile reconstruction
- \leftrightarrow feedback mechanisms and formation history

Cluster samples / stacking

- \leftrightarrow still kind of 'Blobology'
- \leftrightarrow scaling relations
- \leftrightarrow average properties
- \leftrightarrow cosmology, lensing masses

What is the problem?

- Future high resolution & high sensitivity SZ observations
 - will allow us to address detailed questions about the state of the ICM
 - require accurate model for the SZ signal (e.g., *along different lines of sight...*)
 - which parameters actually determine the SZ signal?

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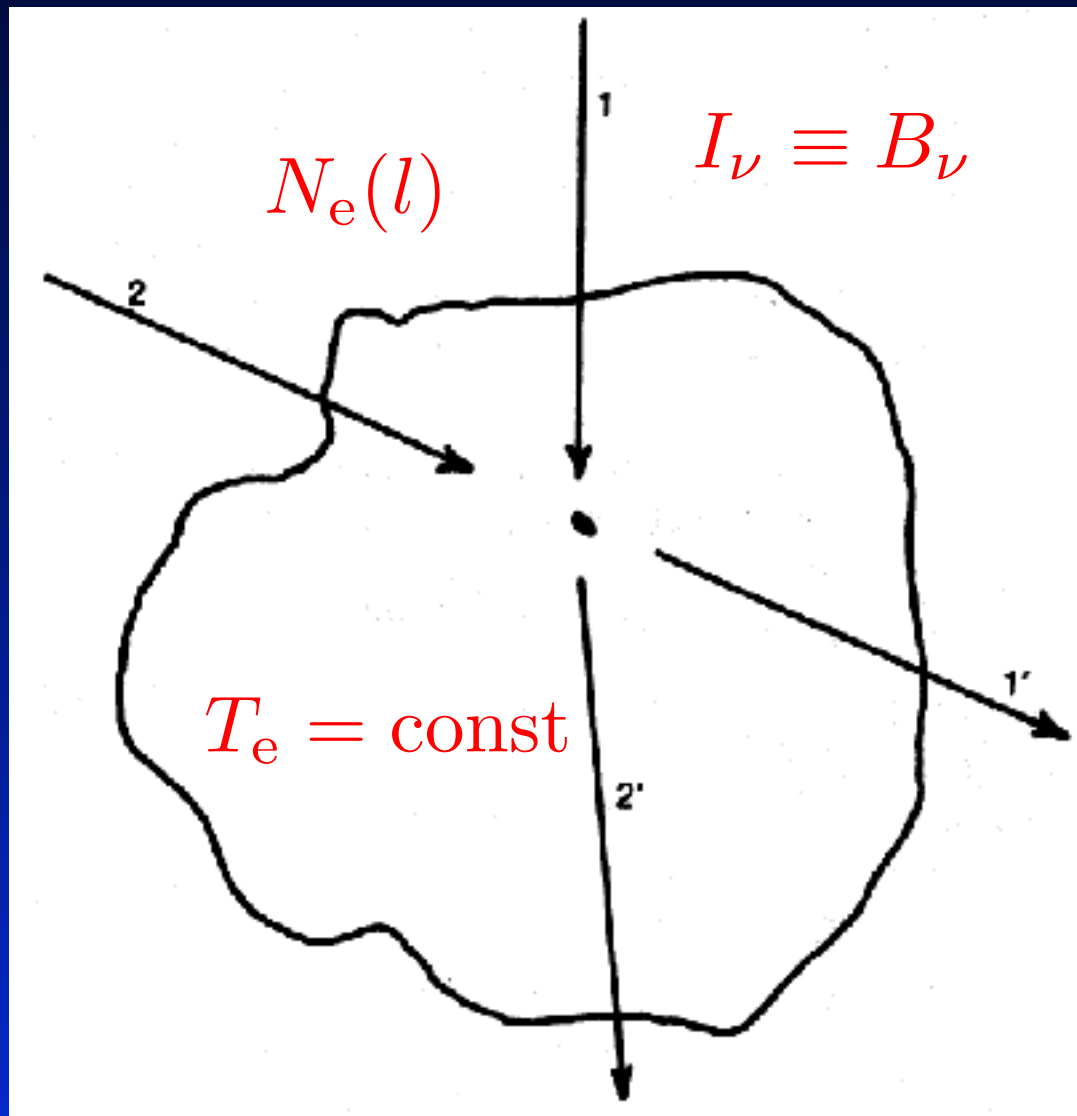
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SZpack overcomes all these problems!

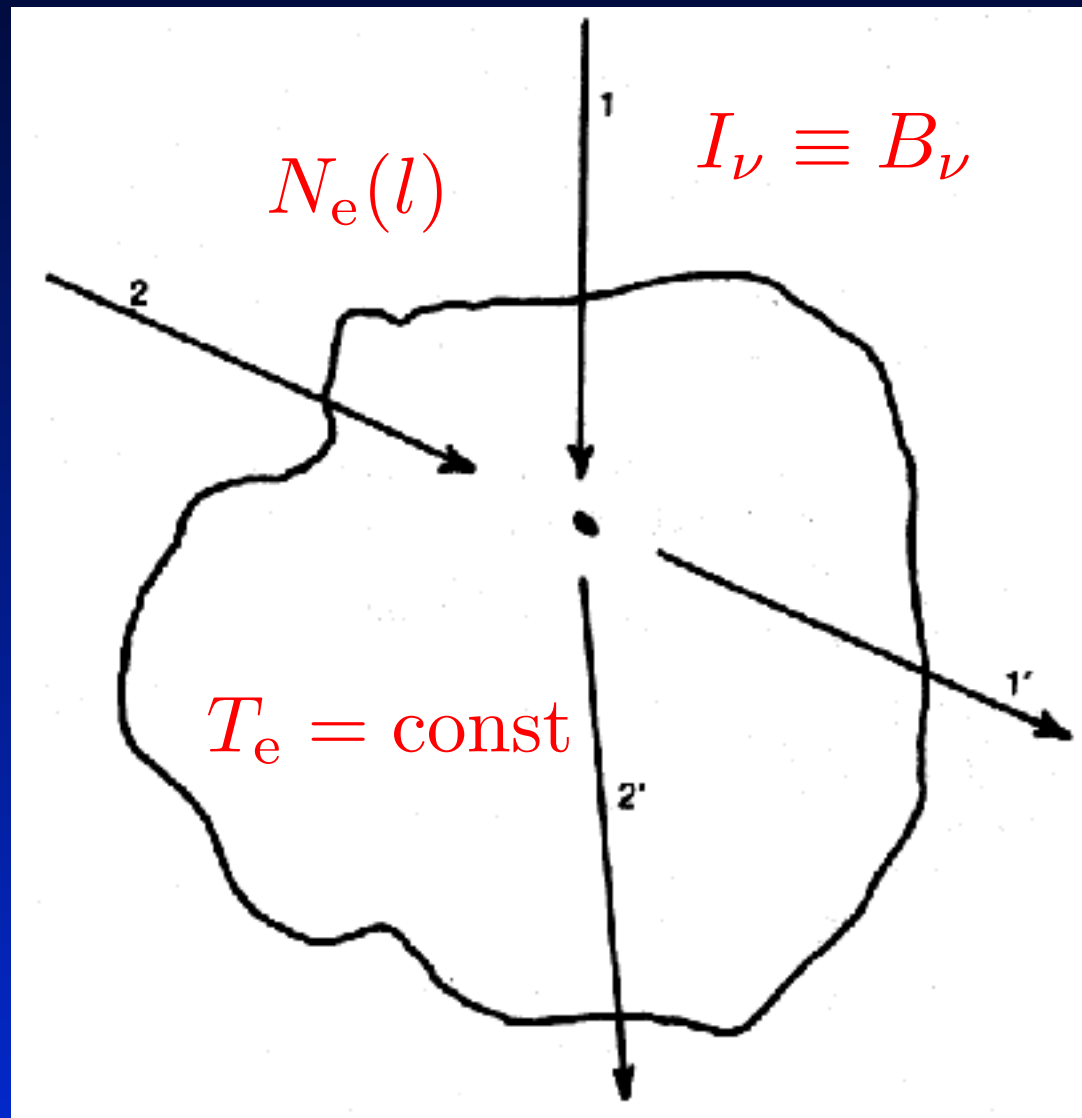
Thermal SZ effect for isothermal cluster

Scattering physics

- SZ signal depends on # of photons scattering *in* and *out* of the line-of-sight
- optically thin case $\Delta I_\nu \sim \tau S(\nu, T_e)$
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Thermal SZ effect for isothermal cluster



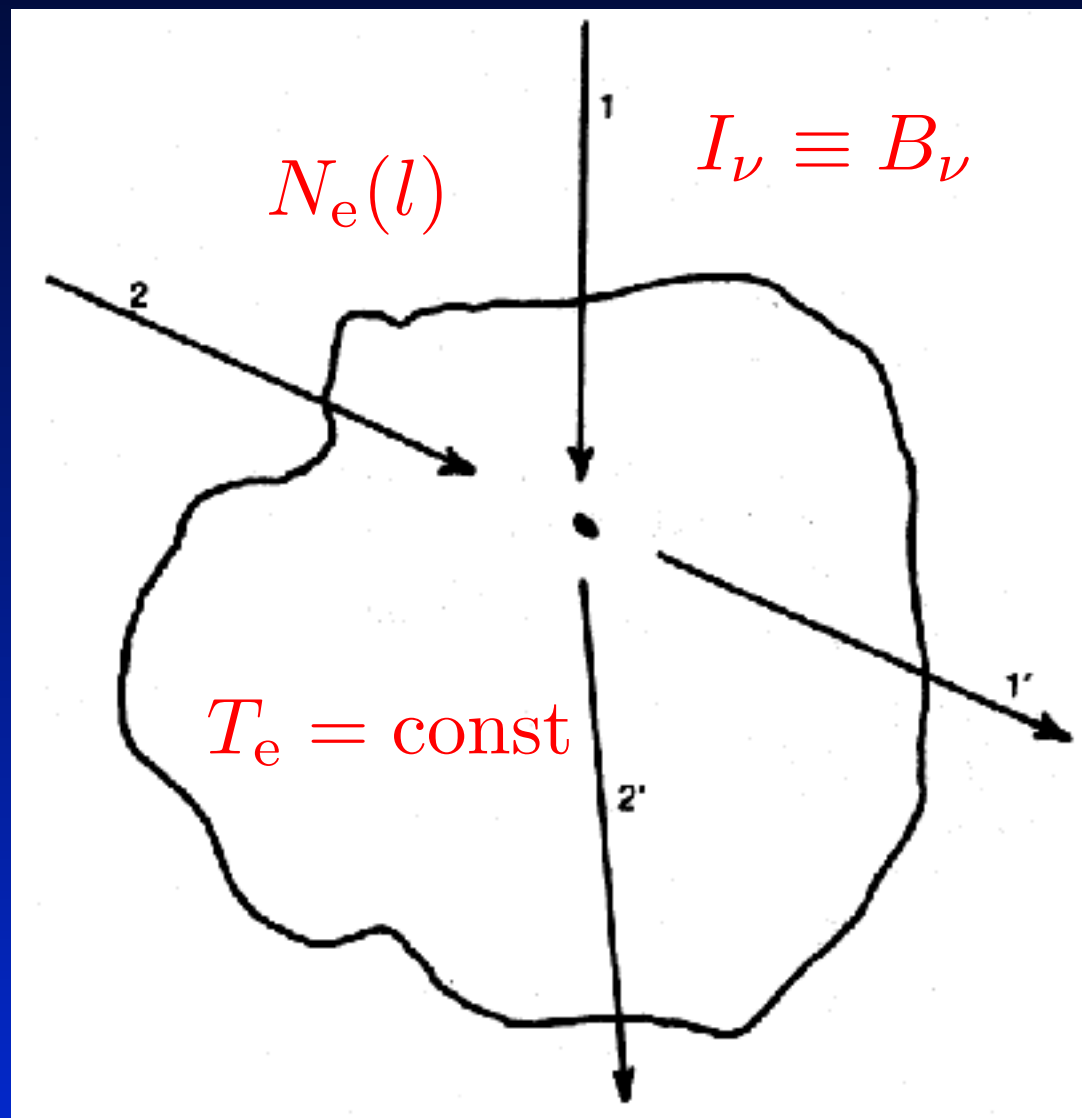
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- General for small # of scatterings
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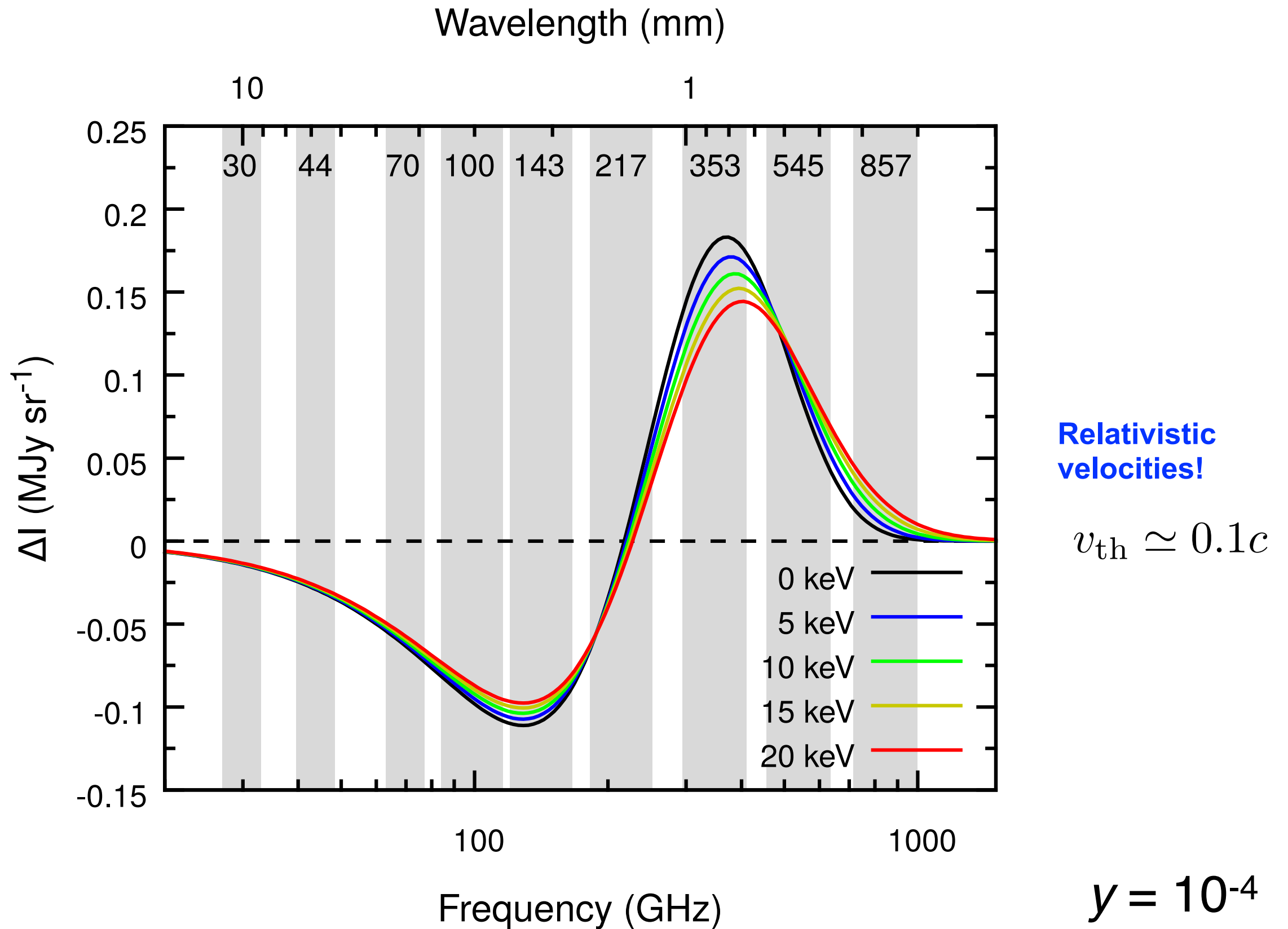
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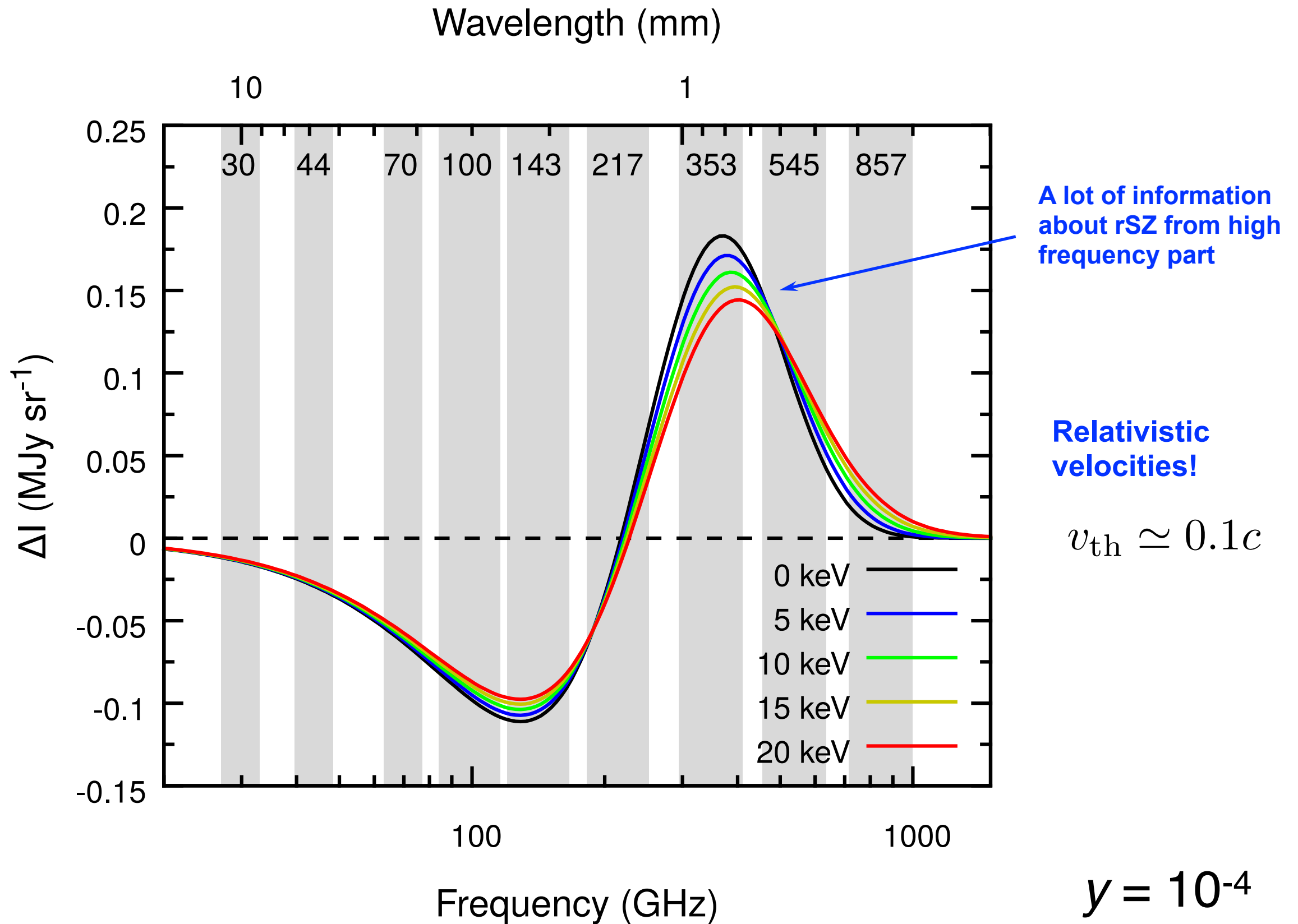
Approximate schemes

- Convergence issues
- Accuracy and user-interface
- flexibility

Effect of relativistic temperature corrections



Effect of relativistic temperature corrections



RELATIVISTIC CORRECTIONS TO THE SUNYAEV-ZELDOVICH EFFECT FOR CLUSTERS
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NAOKI ITOH

Department of Physics, Sophia University, 7-1 Kioi-cho, Chiyoda-ku, Tokyo, 102, Japan; n_ito@hoffman.cc.sophia.ac.jp

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Fuji Research Institute Corporation, 2-3 Kanda-Nishiki-cho, Chiyoda-ku, Tokyo, 101, Japan; kohyama@crab.fuji-ric.co.jp

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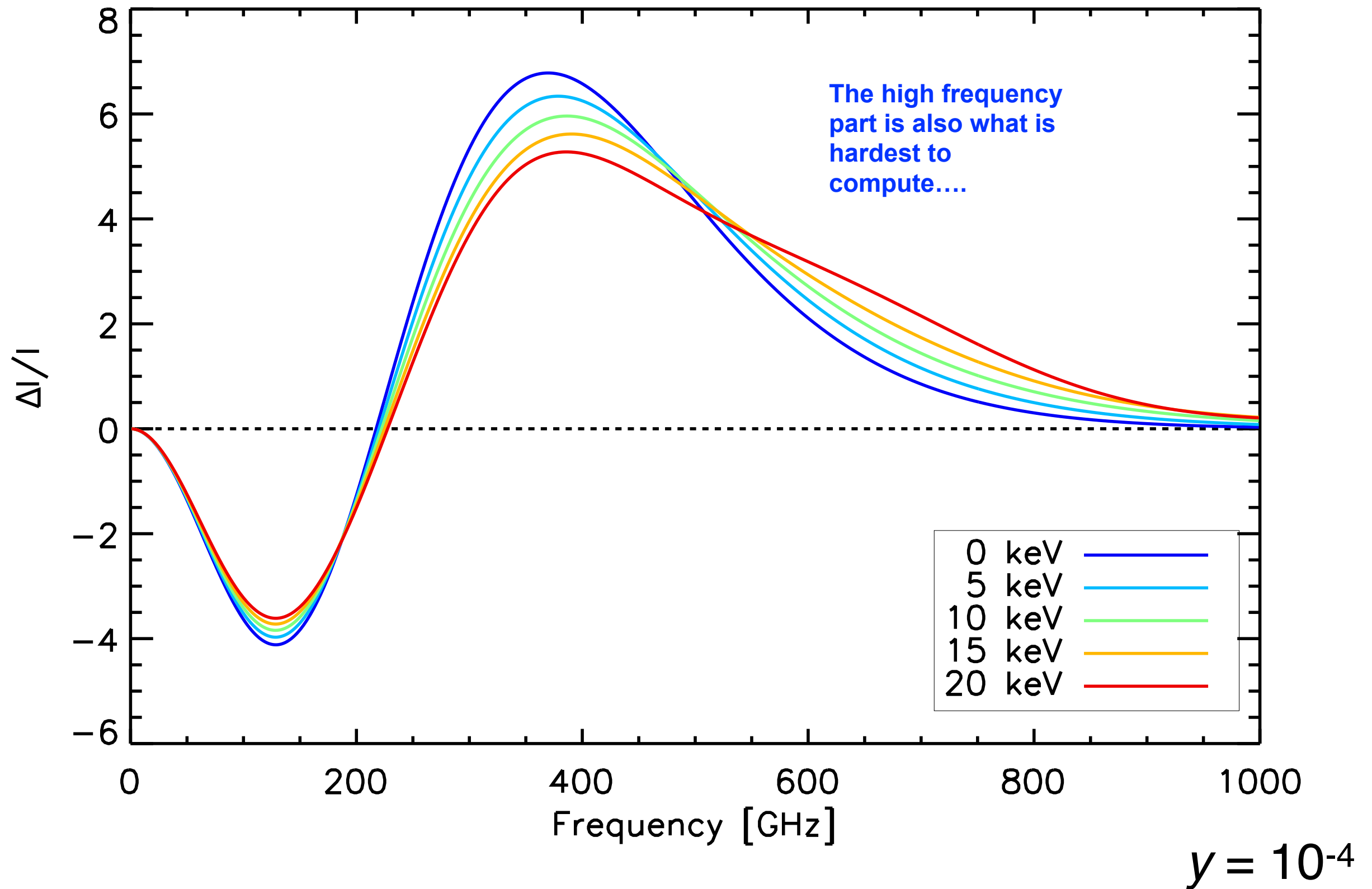
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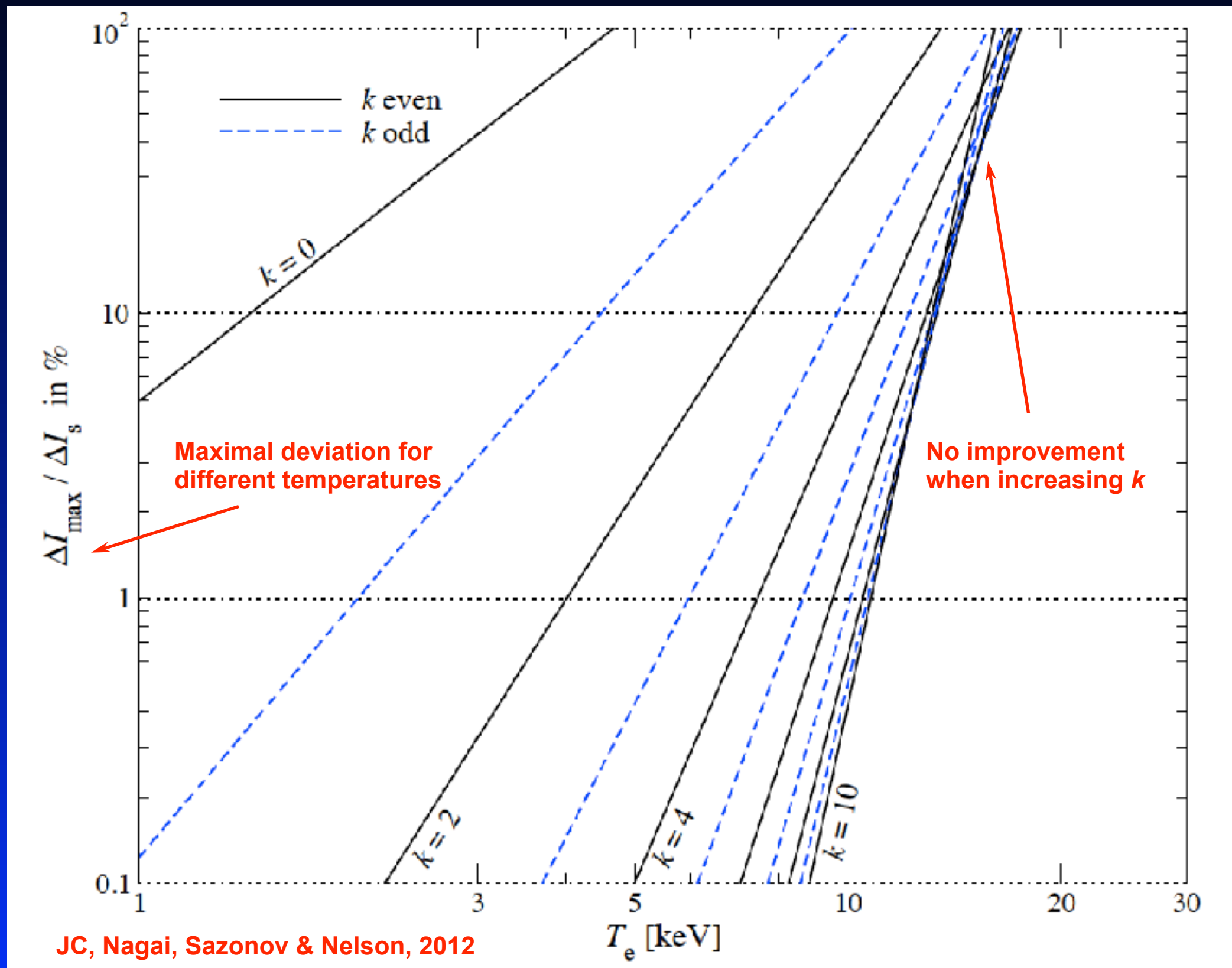
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- Velocity terms add another dimension to the expansion
- All quite complicated and not very illuminating expressions

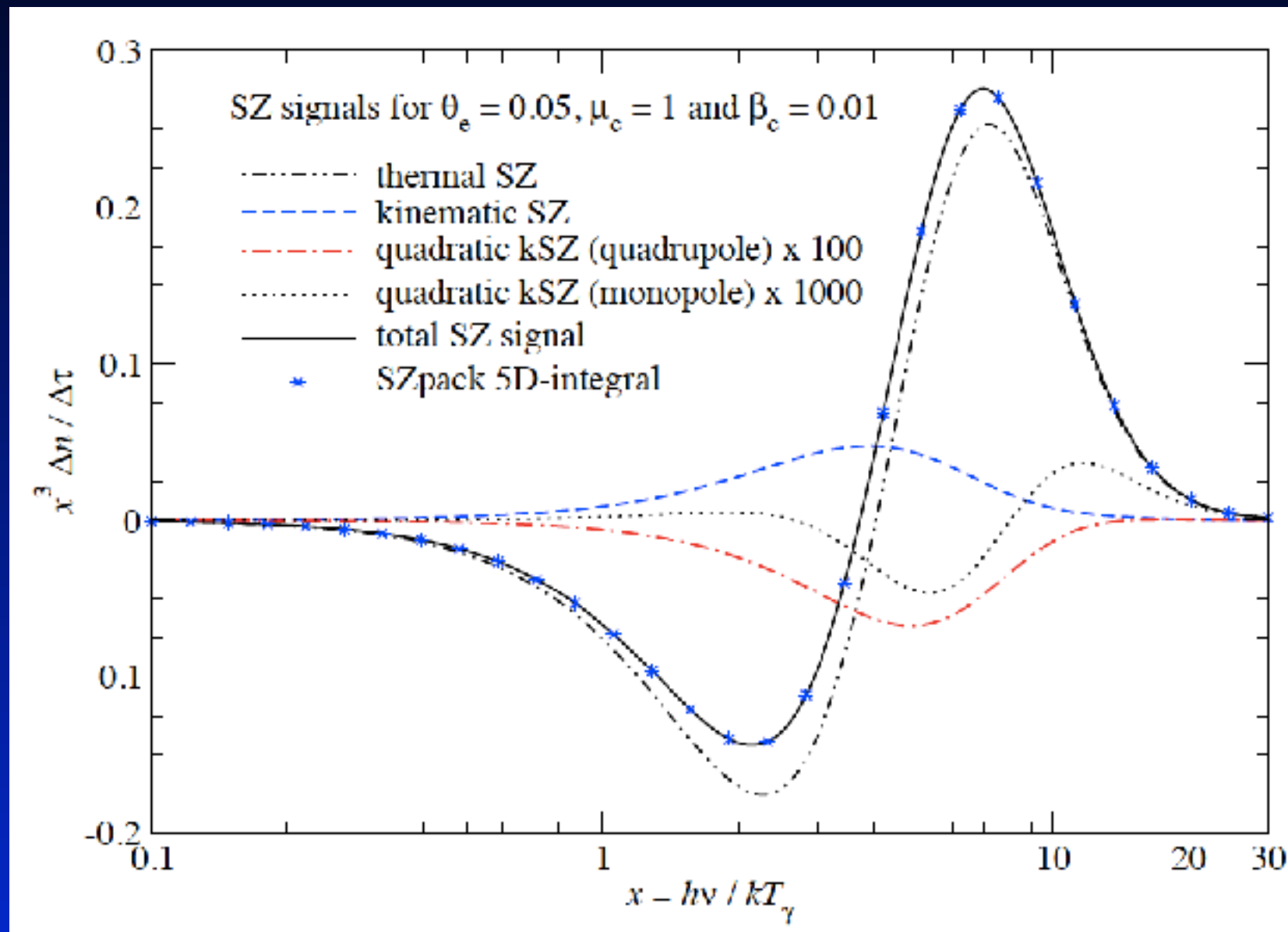
SZpack (JC, Nagai, Sazonov & Nelson, 2012)

- Asymptotic expansions up to 10th order in T_e
- Motion of *cluster* and *observer* (JC, Huetsi & Sunyaev, 2005)
- Higher orders easy to add (but kind of pointless...)

Convergence of asymptotic expansion



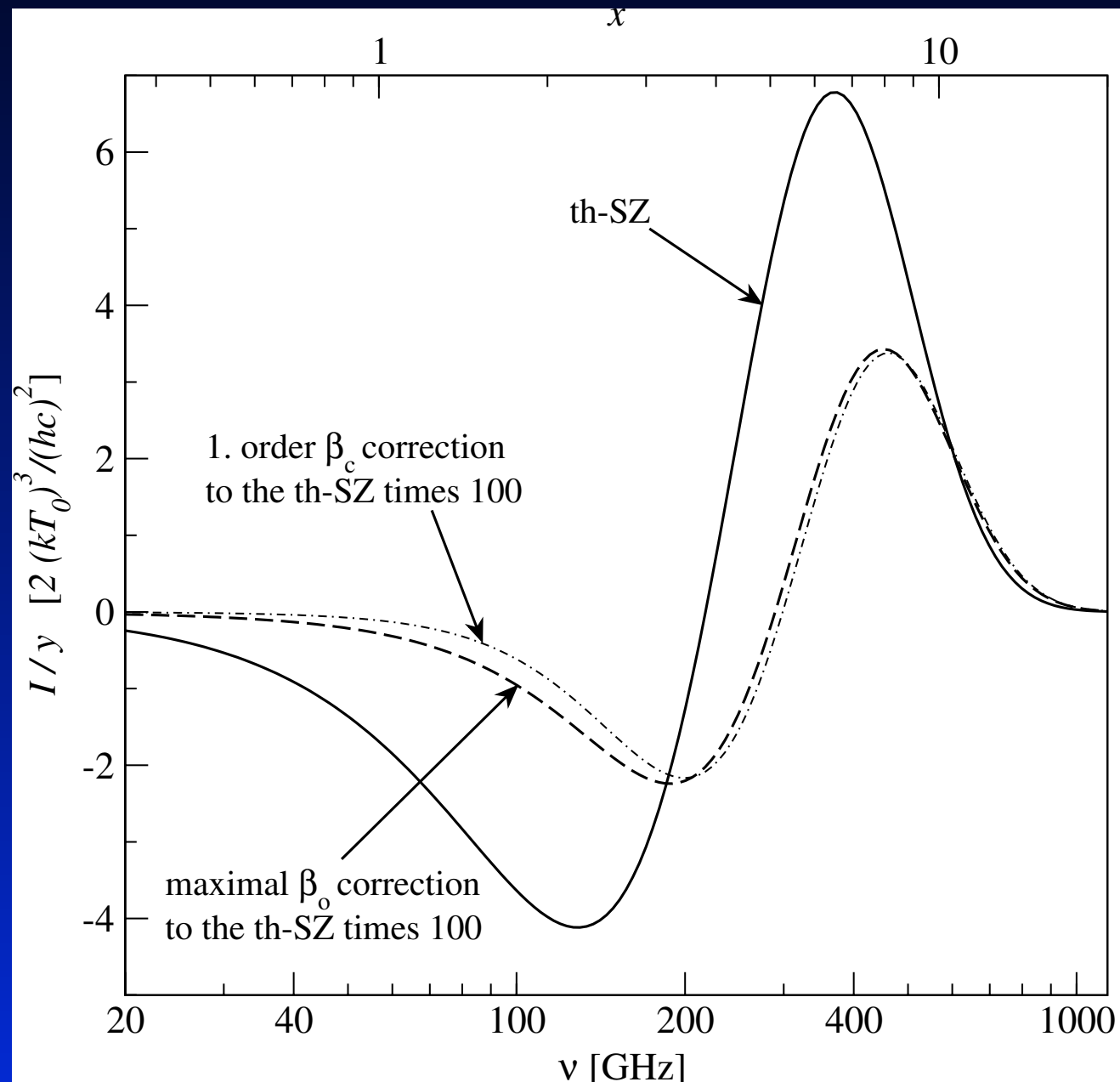
Thermal and kinematic SZ effect for isothermal cluster



New SZpack approach

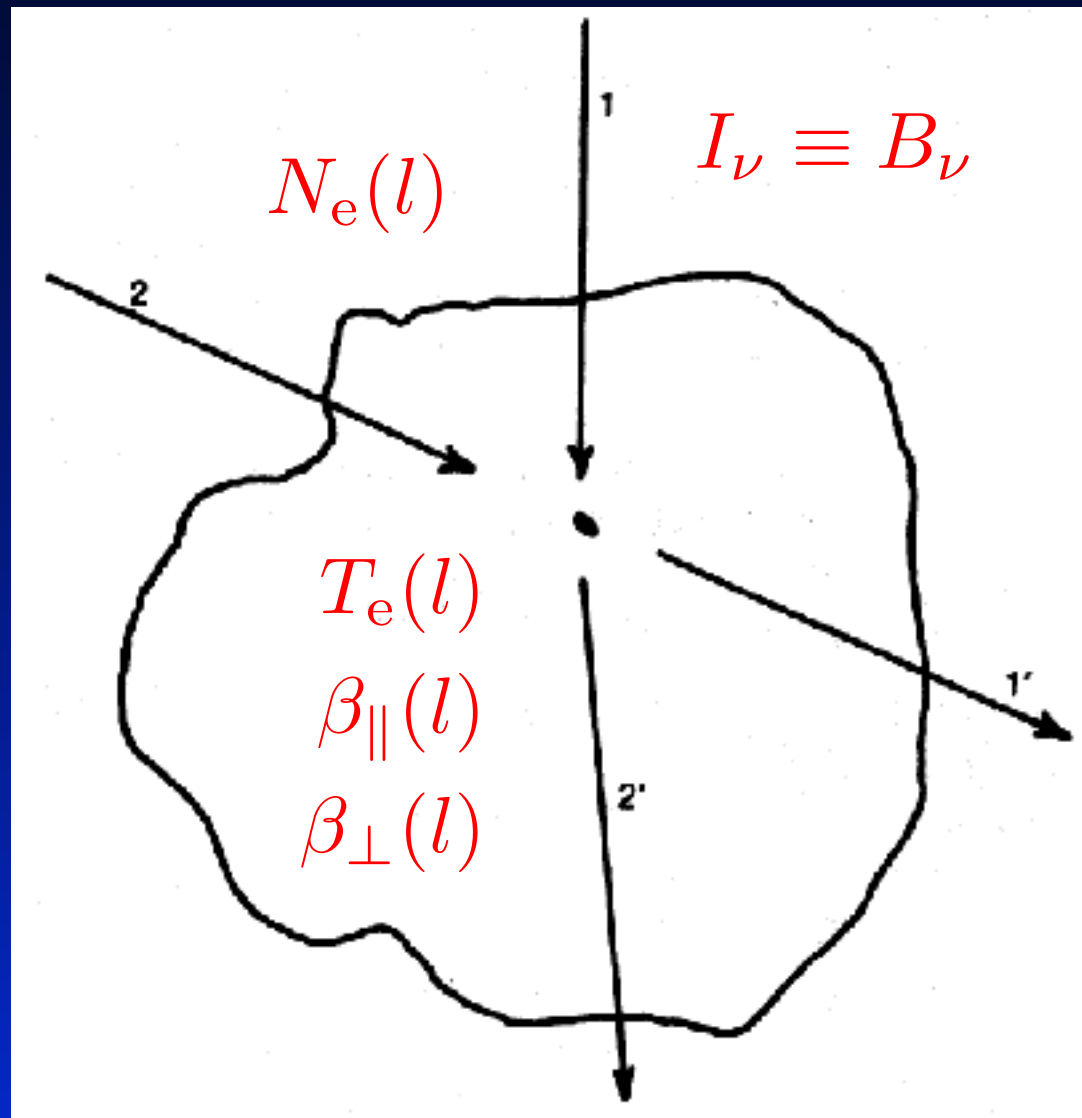
- T_e derivatives analytically
- collision integral numerically for given reference temperature
- set of smooth basis functions that is motivated by scattering physics
- excellent convergence properties
- precise representation of SZ signal for $kT_e < 75$ keV and $x < 30$

Motion of the observer



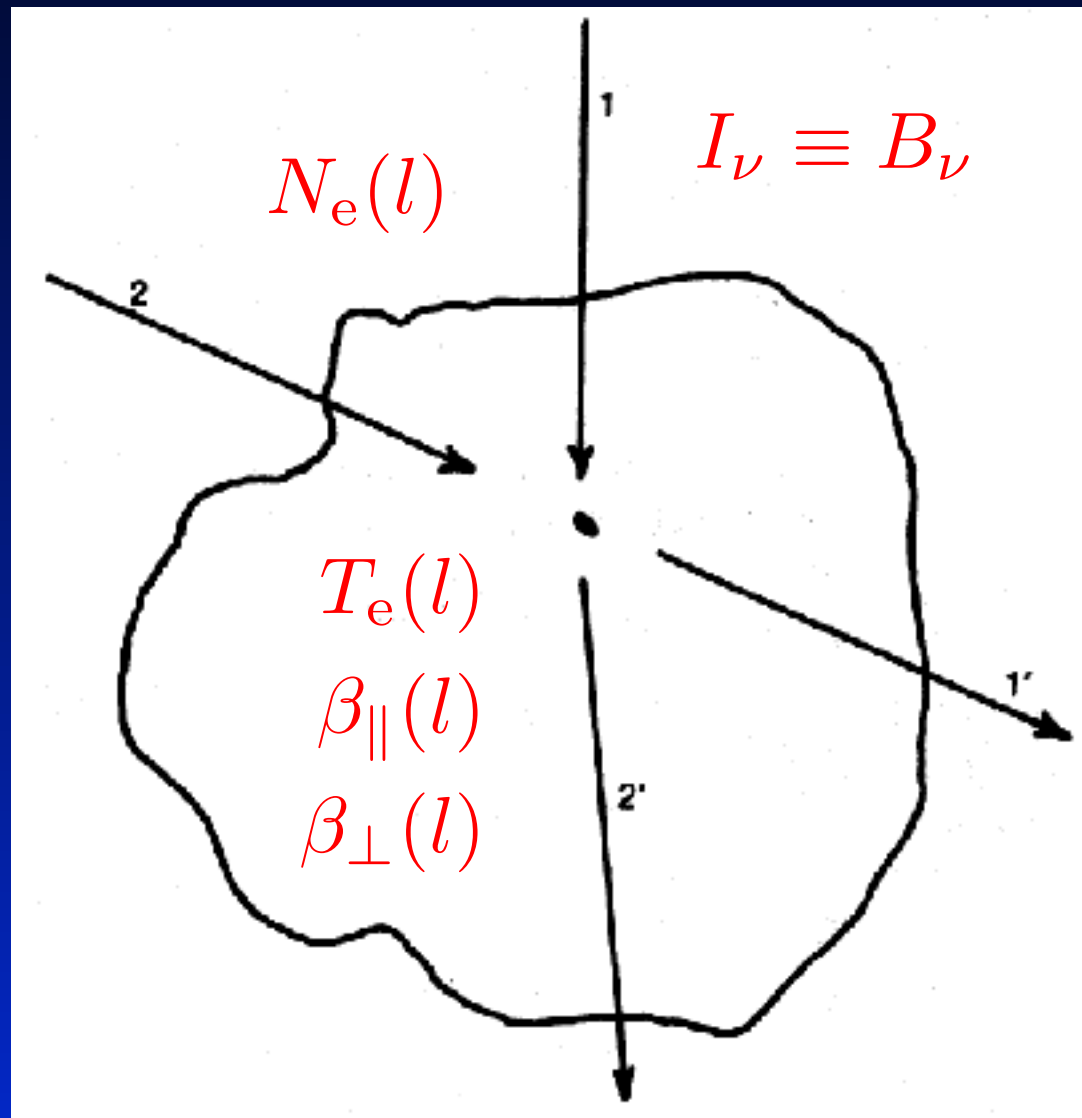
- Can be included by simple Lorentz-boost (SZpack)
- Well-known dependence on the sky
- Asymmetry in the cluster number counts
- Alternative way to constrain dipole
- Dominik's talk re systematic effects on observables

Non-isothermality and effect of internal motions



- SZ signal given by additional integral over line-of-sight
- problem becomes very demanding
- *Question*: what are the real parameters/observables?

Non-isothermality and effect of internal motions



- SZ signal given by additional integral over line-of-sight
- problem becomes very demanding
- **Question:** what are the real parameters/observables?

Simple Solution:

- perform expansion around mean values of main variables!
- **temperature-velocity moments** define new set of parameters:

$$p = \{\tau, T_e, \omega^{(k)}, \sigma^{(k)}, \kappa^{(k)}, \beta_{\parallel}, \beta_{\perp}^2\}$$

$$S \approx S_{\text{iso}}^{(0)} + S_{\text{iso}}^{(2)} \omega^{(1)} + C_{\text{iso}}^{(1)} \sigma^{(1)} + D_{\text{iso}}^{(2)} \kappa^{(1)} + E_{\text{iso}}^{(2)} \beta_{c,\perp,\text{SZ}}^2 + \dots,$$

all functions of T_e , τ and β_{\parallel}

Average CMB spectral distortions in Λ CDM

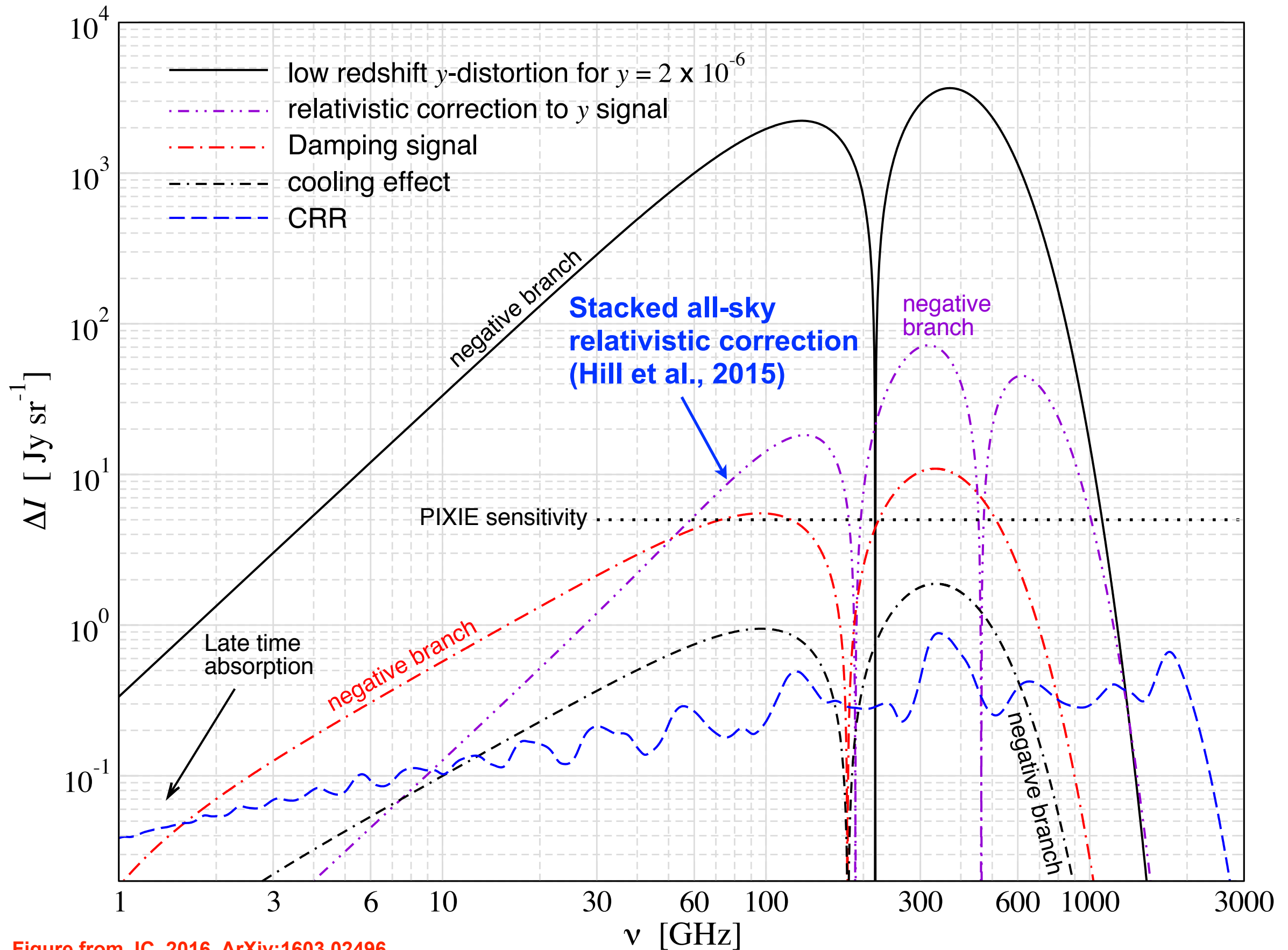
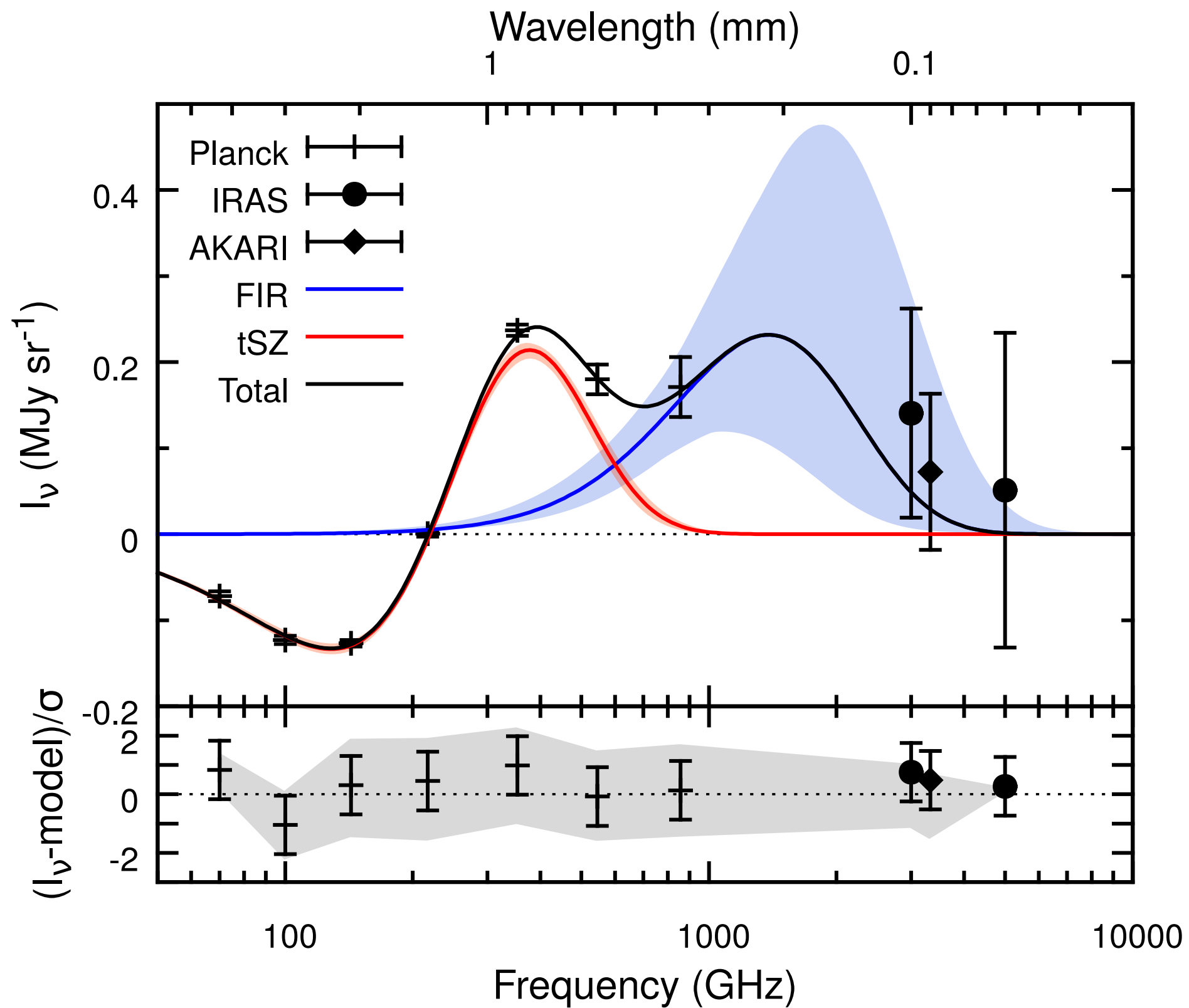
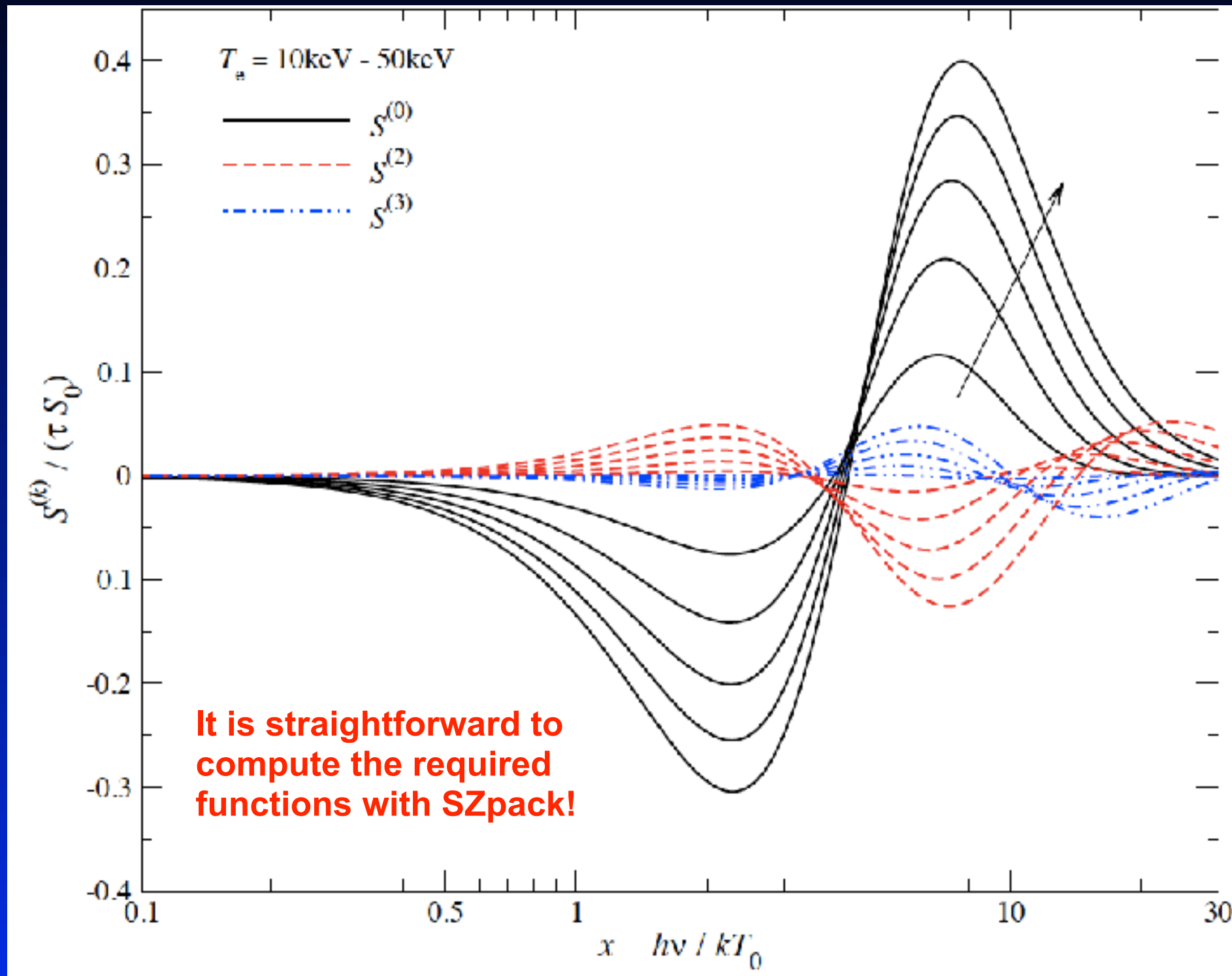


Figure from JC, 2016, ArXiv:1603.02496

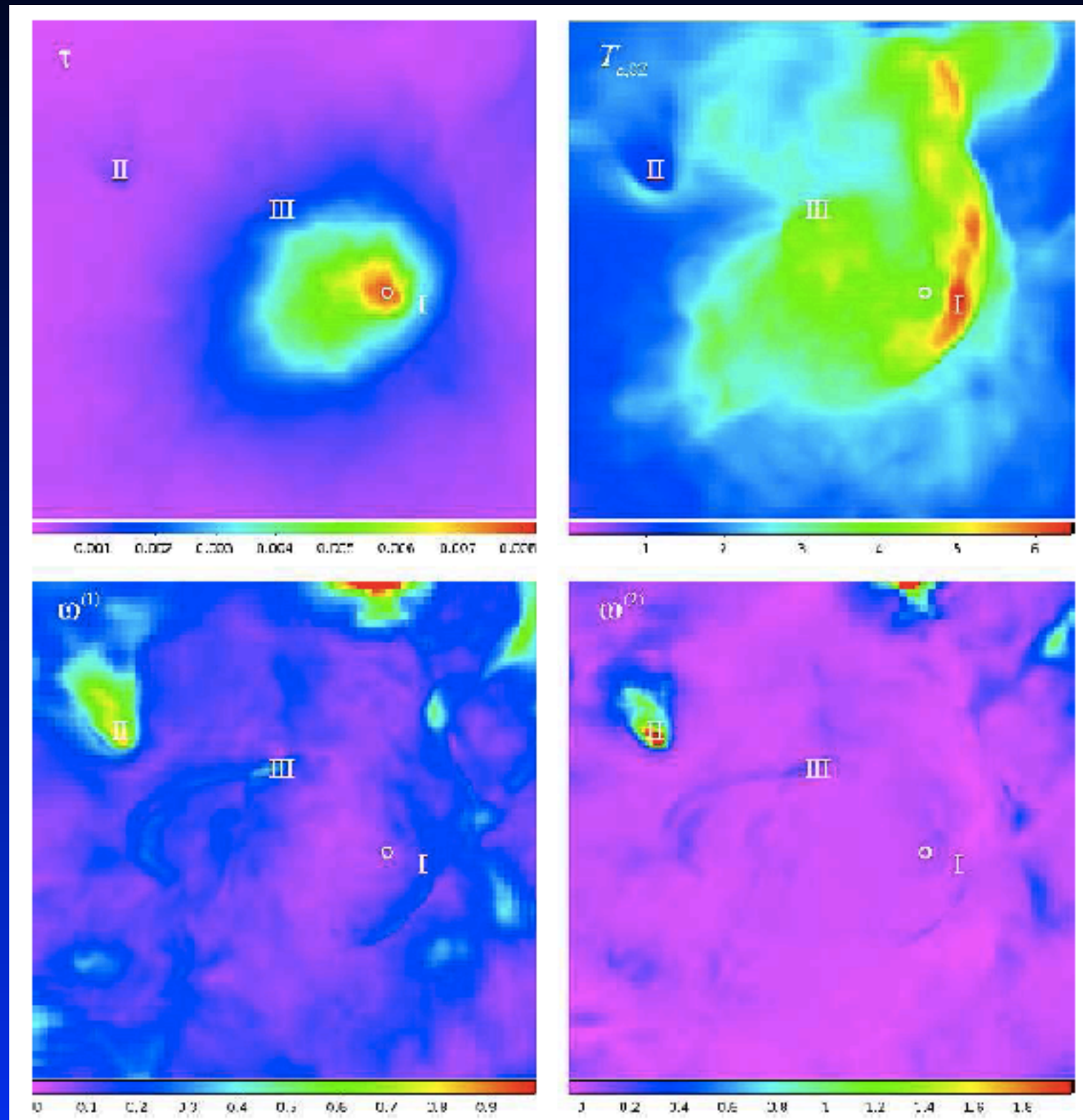
Stacked SZ signal + foregrounds



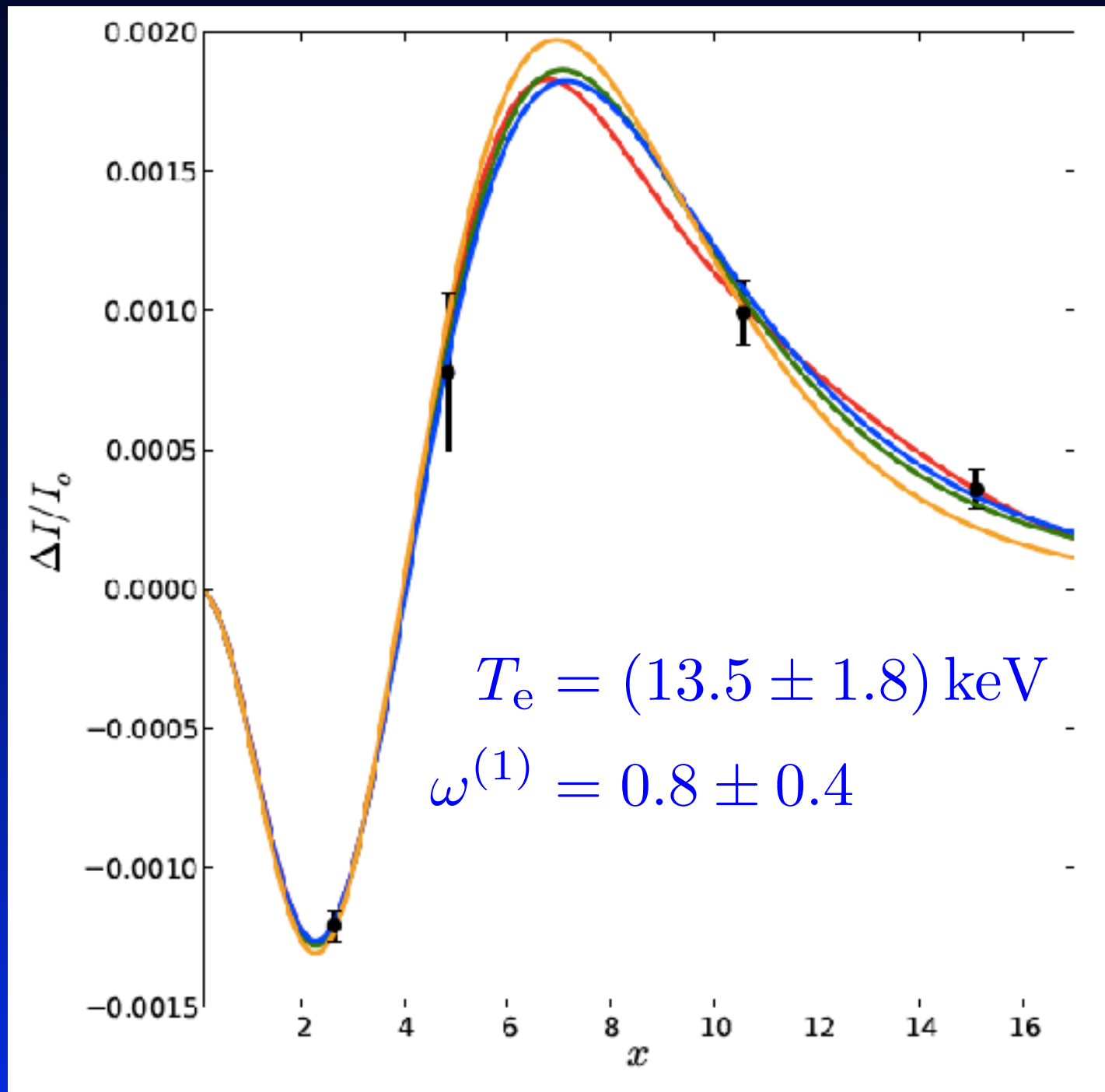
Basis functions to include non-isothermality



Can be directly used to compute signal morphologies



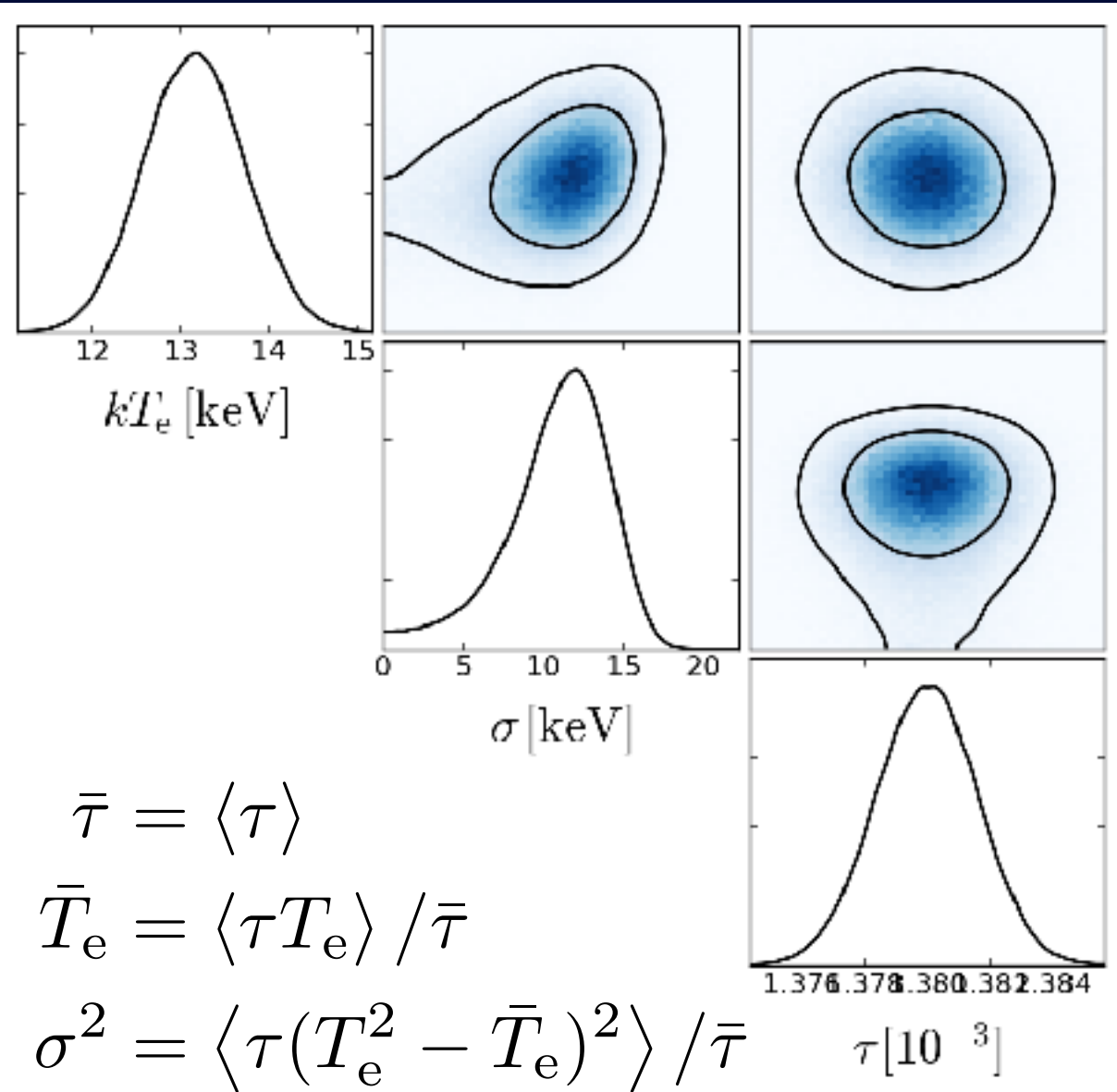
Analysis of Bullet cluster data (Prokhorov et al)



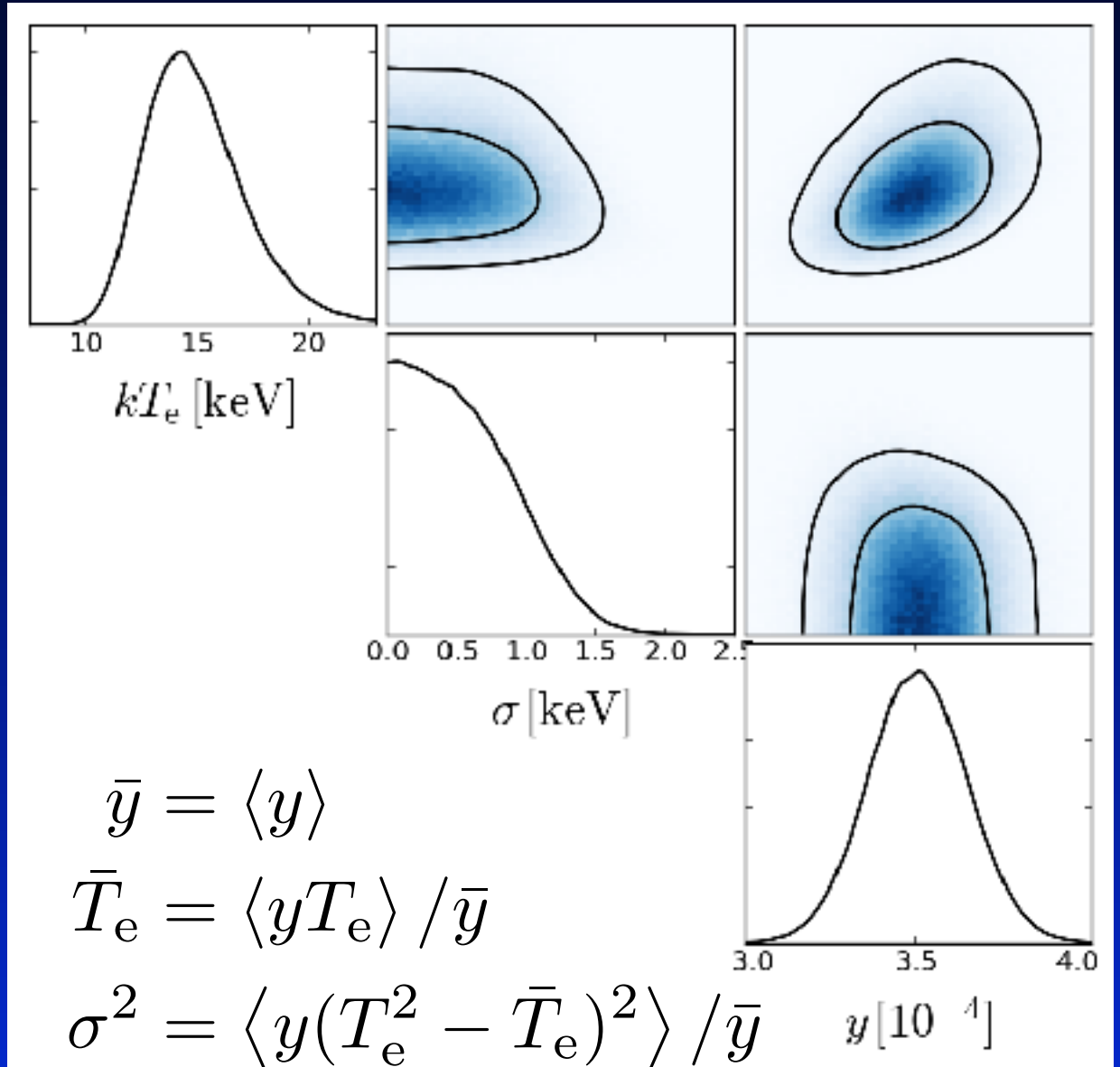
- MCMC analysis with *extended* set of parameters
- consistent analysis for mean and dispersion
- isothermal model in tension with data (*orange*)
- two-temperature (*green*) and simple dispersion (*blue*) models indistinguishable
- high frequency spectrum very important
- Itoh expansion (*red*) in this case not meaningful although at current level of precision consistent with data...

Dependence of weighting scheme (Bullet Cluster)

τ -weighted analysis



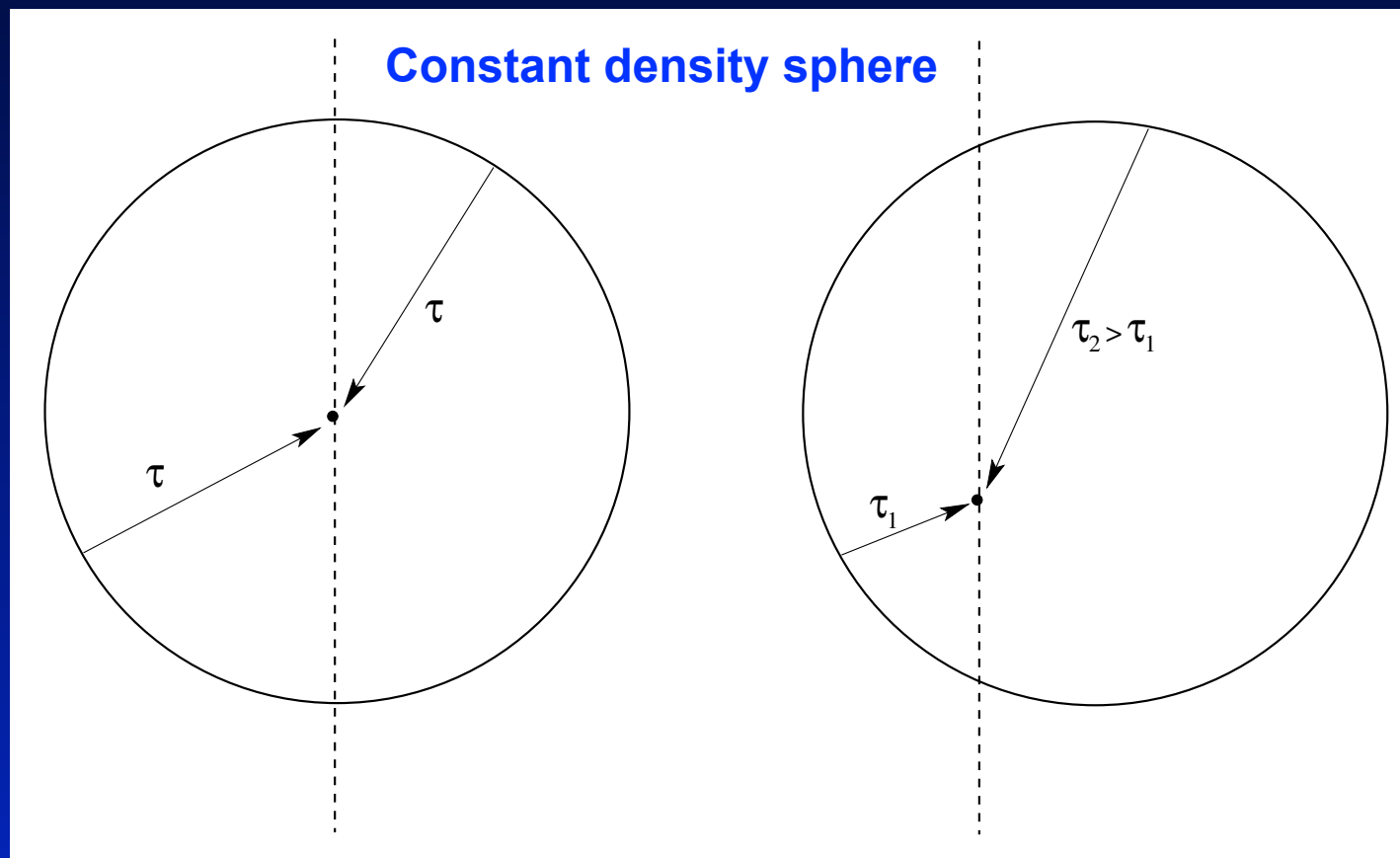
y -weighted analysis



- Prior on τ -needed (from X-ray)
- Indication for dispersion

- No external prior
- upper limit on dispersion

Multiple-scattering SZ signal



- previous analysis neglected anisotropy of local radiation field
- even in simplest cases this is inconsistent
- treatment using *anisotropic scattering*
- local *monopole* through *octupole* relevant
- relevant correction remains very small....
- One could learn about the *local anisotropy* in the density of the medium

Conclusions/Question

- SZpack can deal with physically relevant cases
- what is the typical temperature dispersion etc?
 - use cosmological sims to estimate these effects
 - quantify possible biases to temperature measurements
- how feasible will it be to use *multiple scattering* correction to learn about structure of the ICM?
- combination with X-ray lines (*Athena*)?
- What does *polarization* tell us?
- add *non-thermal electron* populations
- *Extension* of method to X-rays
- Already applied to *CMB foregrounds* (see JC, Hill & Abitbol, 2017)