Magnetic Fields in Cosmology

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Overview

Magnetic fields in cosmology

1

Origin?

Generation mechanisms in the very early universe

2

Effect on cosmological perturbations

Observational signature?

3

Evolution and dissipation

KK PRD 81(2010) 023006; PRD 83 (2011) 023006; PRD 85 (2012) 083004; PRD 87 (2013) 103005; PRD 89 (2014) 103016;PRD 96 (2017) 063526; JCAP 1901 (2019) 033; JCAP 11 (2021) 11.

Kandus, KK, Tsagas Phys.Rept. 505 (2011) 1.

KK, E. Komatsu, JCAP 1506 (2015) 06, 027.
KK, E. Komatsu, JCAP 1401 (2014) 01, 009 .

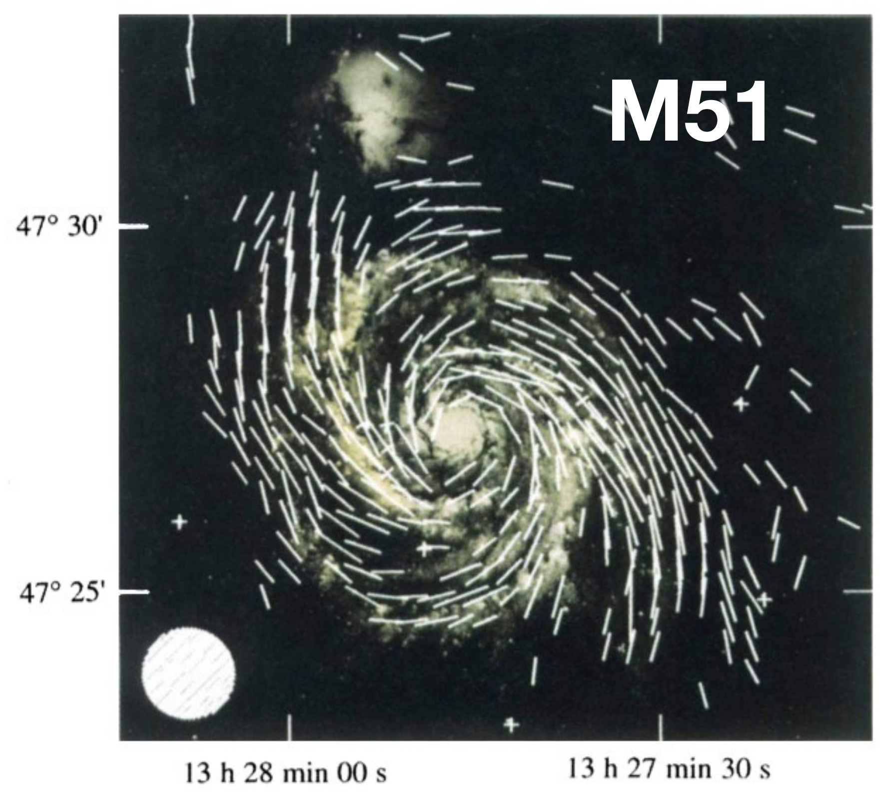
Magnetic Fields in Cosmology

1

Origin?

Generation mechanisms in the very early universe

Observations

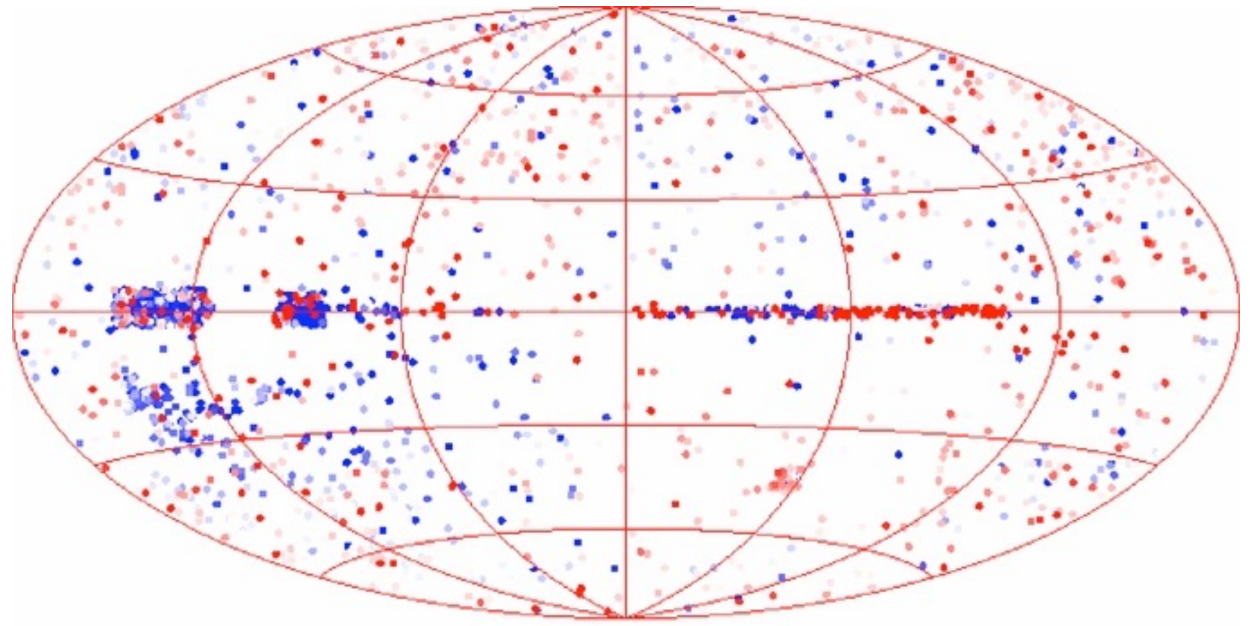


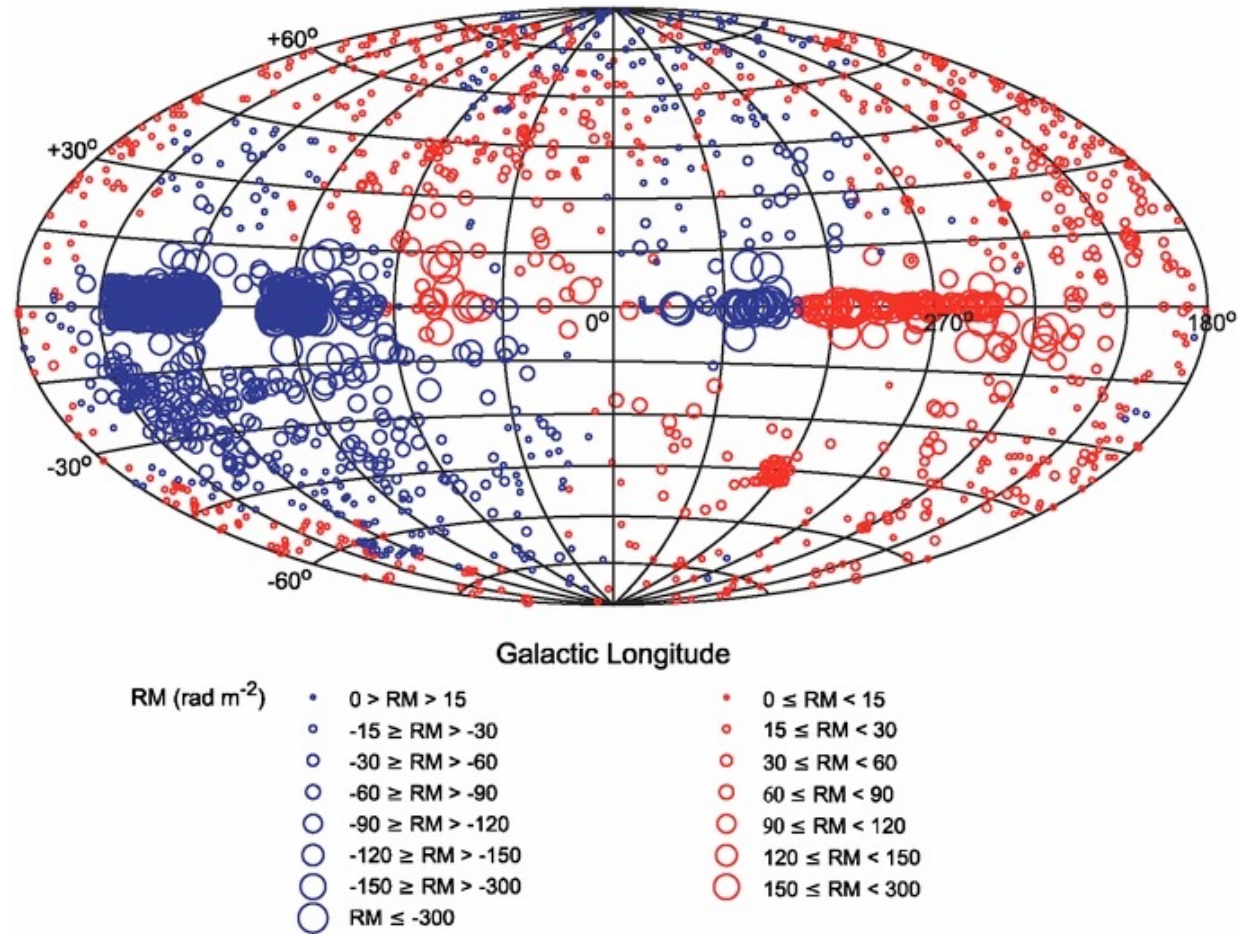
(Neininger 1992)

Total magnetic field strength

$$B_t = (13.7 \pm 2)\mu\mathrm{G}$$

- Galactic magnetic field

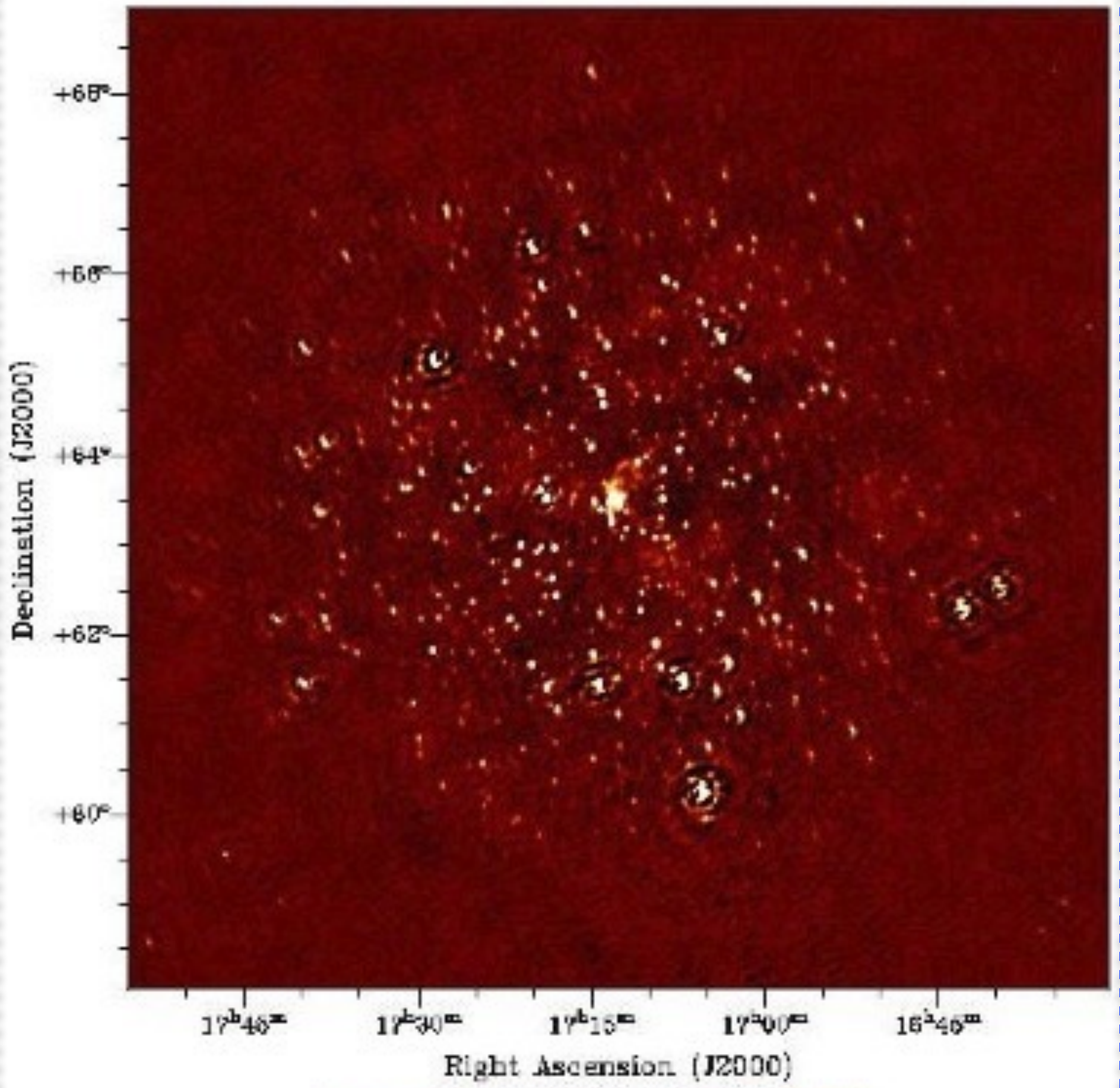


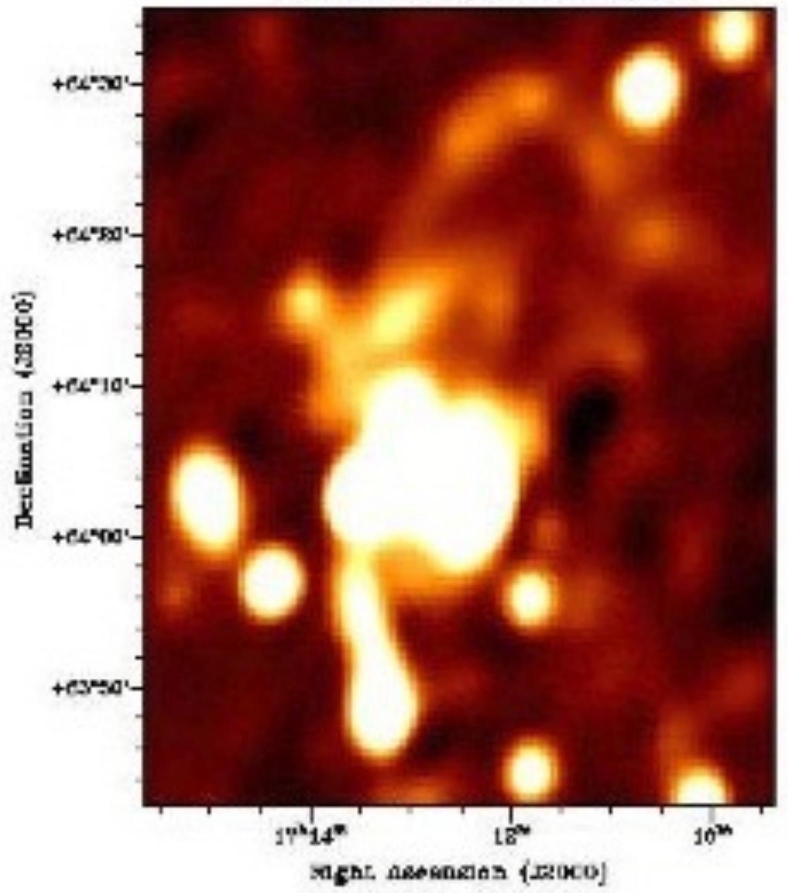


Kronberg, Newton-McGee (2011)
All-sky map of rotation measures in the Milky Way, using data of 2257 sources.

- Magnetic field strength: near sun: 2 μG
  halo (north/south): 4 (2) μG

Total synchrotron intensity around the galaxy cluster Abell 2255





(Govoni et al. 2006)

Magnetic field strength at center ~2.5 μG.

How to generate cosmological magnetic fields

- Origin of large scale magnetic fields?

- Usually a dynamo mechanism is assumed to amplify an initial seed field

- In a flat universe with $\Lambda = 0$ $\quad B_{seed} \geq 10^{-20}\mathrm{G}$ in order to explain galactic field today. $\Lambda > 0$ $\quad B_{seed} \geq 10^{-30}\mathrm{G}$

How to generate cosmological magnetic fields

- Origin of initial seed field?

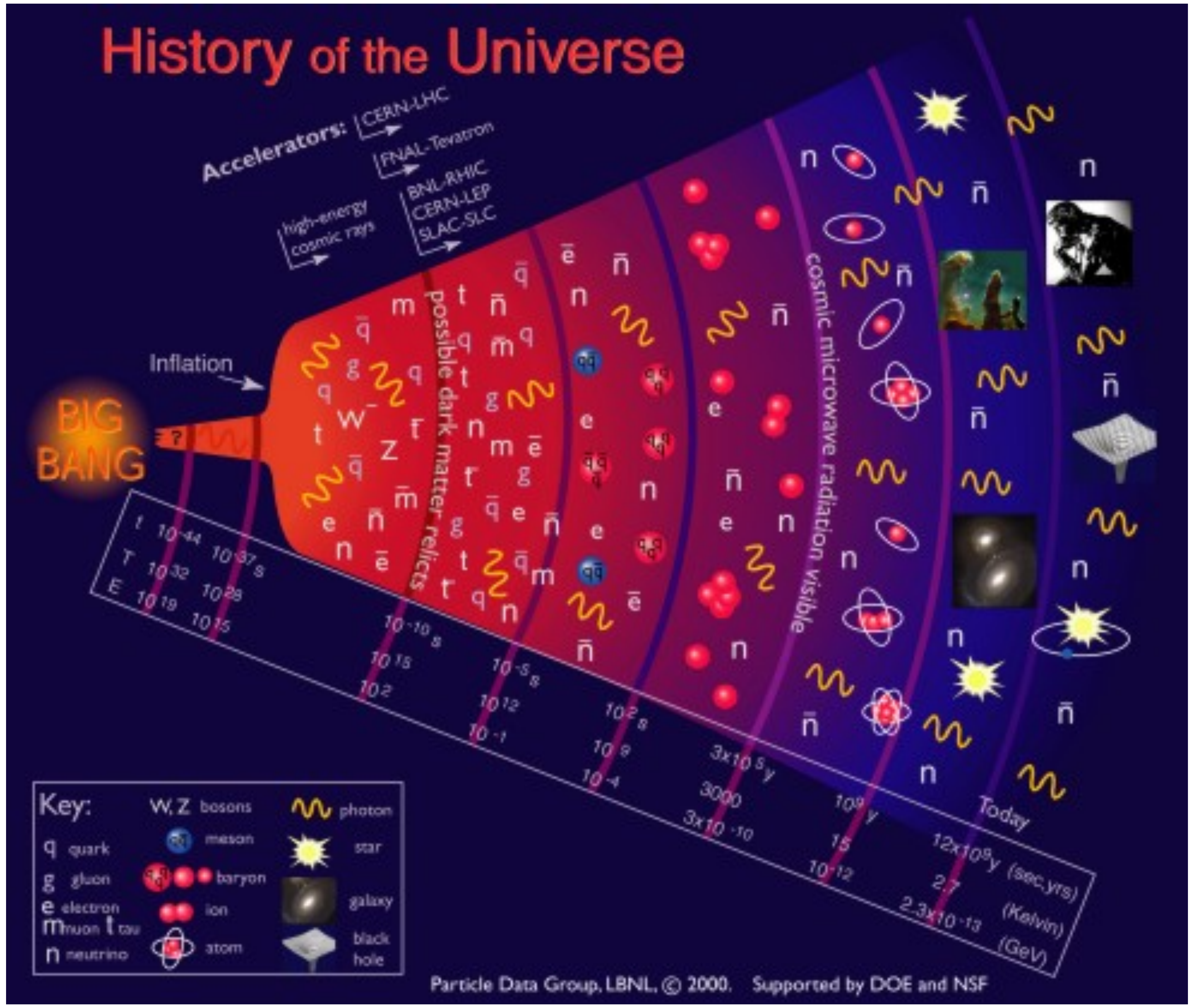
- two classes of mechanisms:

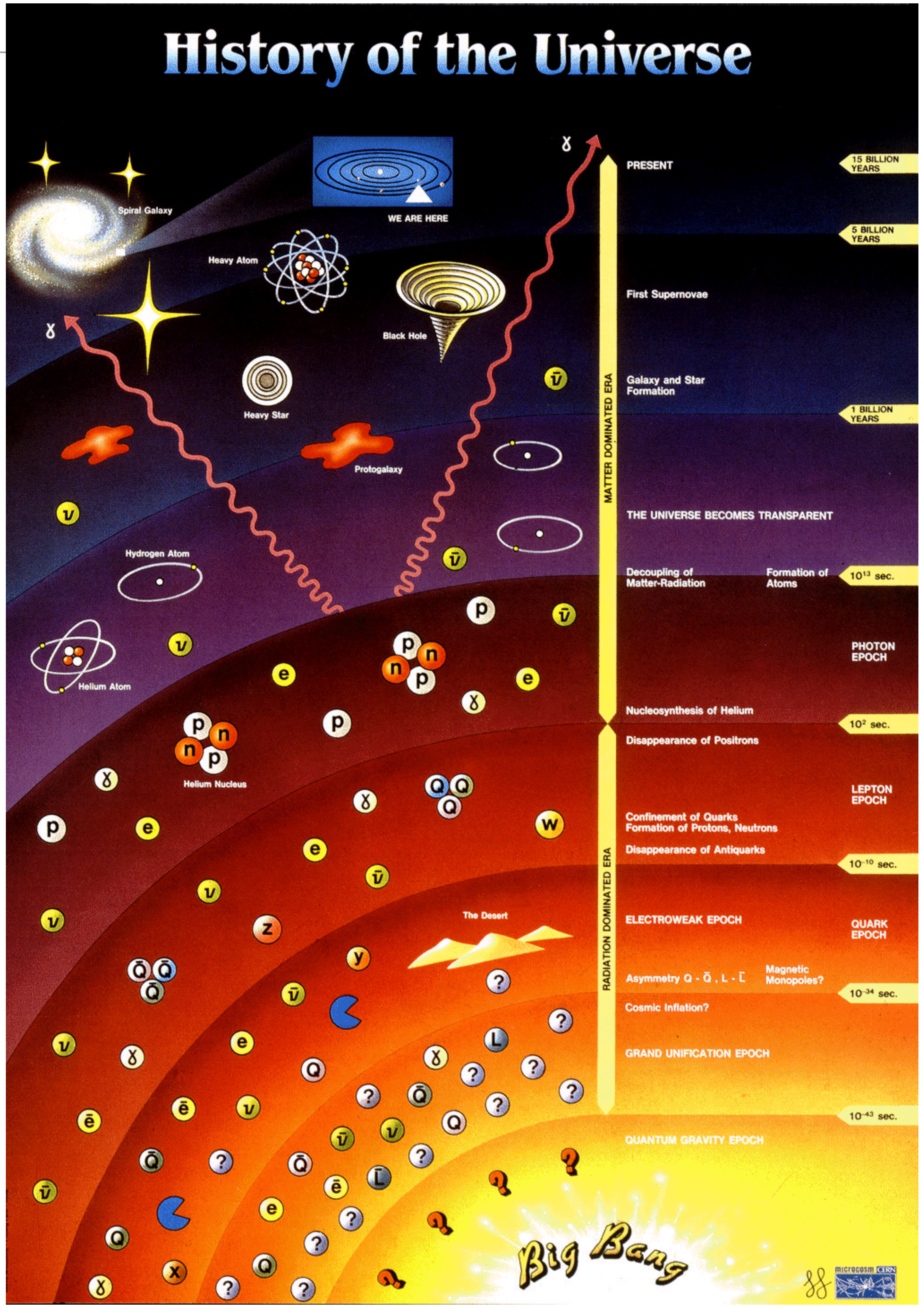
1. processes on small scales: vortical perturbations, phase transitions

2. amplification of perturbations in the electromagnetic field during inflation (Turner, Widrow 1988....)

**(Reviews: e.g.
Grasso, Rubinstein ’01;
Widrow ’02;
Kandus, KK, Tsagas ’11)**

How to generate cosmological magnetic fields

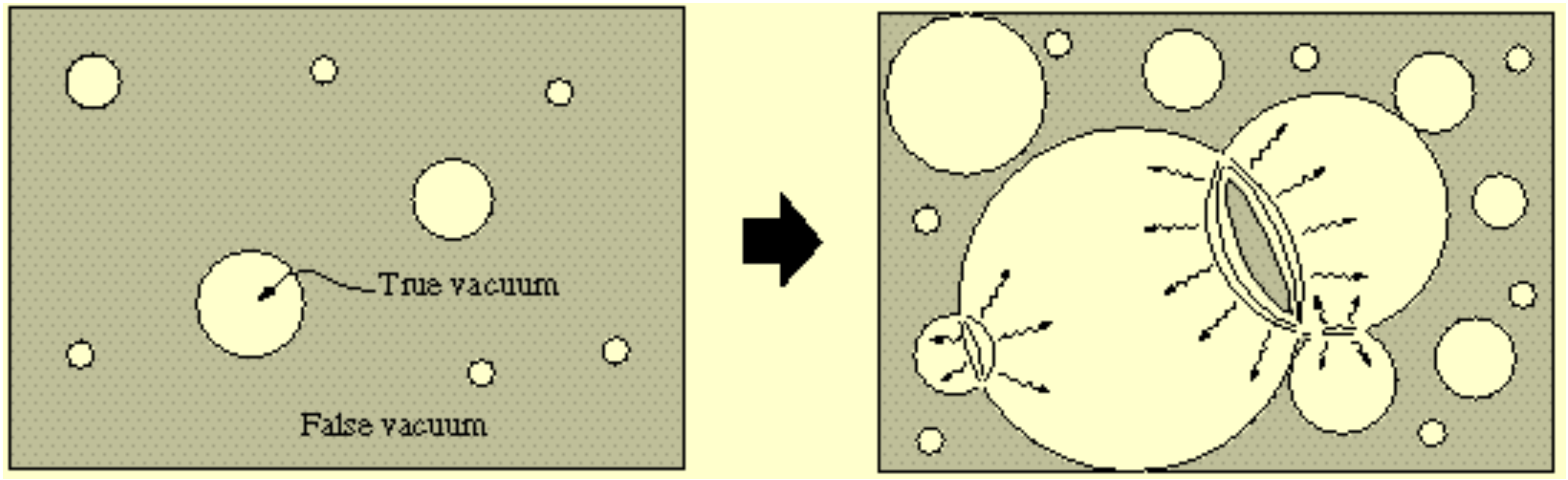




How to generate cosmological magnetic fields

- Example: Generation of magnetic fields during a phase transition

First order



DAMTP: http://www.damtp.cam.ac.uk/research/gr/public/cs_phase.html

First order phase transitions proceed by bubble nucleation. A bubble of the new phase (the true vacuum) forms and then expands until the old phase (the false vacuum) disappears. A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

Magnetic field generation

occurs during collisions

turbulent flows

helical magnetic fields

magnetic fields produced with correlation length of bubble radius, pattern of randomly oriented field lines

Hogan 1983

How to generate cosmological magnetic fields

- **SECOND ORDER PHASE TRANSITION**

- Smooth transition from old to new phase

Magnetic field generation

Phase transition creates domains of different vacuum expectation values of the Higgs field $\Rightarrow$ Gradients in the field leading to electromagnetic fields after the phase transition

Vachaspati 1991

✳ Magnetic fields produced during phase transitions can be very strong but typically have small coherence lenghts (limited by the horizon size at the time).

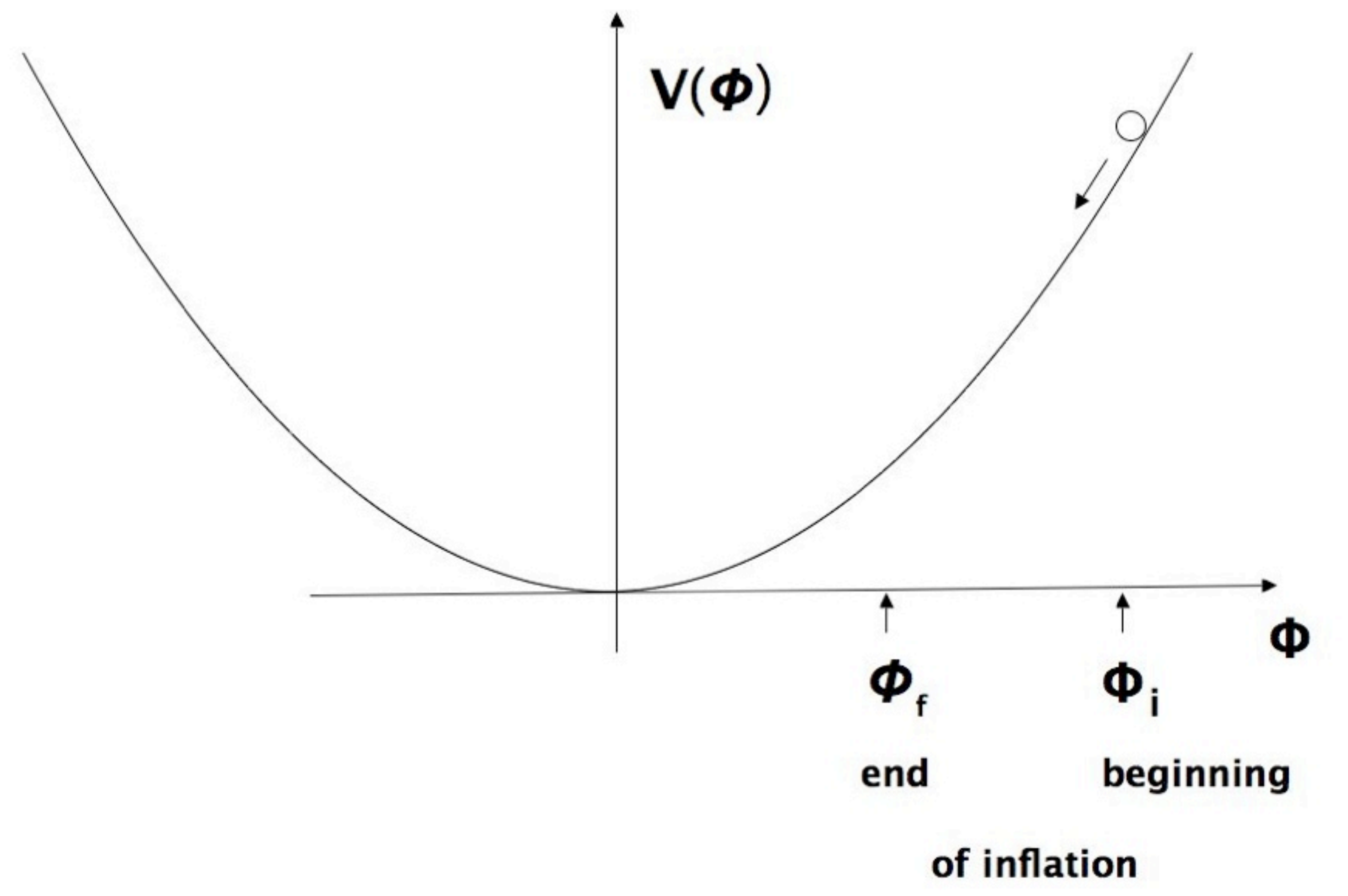
Primordial magnetic fields from inflation

- Inflation:

slow roll inflation

accelerated expansion driven by potential
energy density of scalar field (=inflaton)

$$\boxed{H^2 \simeq \frac{8\pi G}{3} V(\phi) \qquad \dot{\phi} \simeq -\frac{V'}{3H}}$$



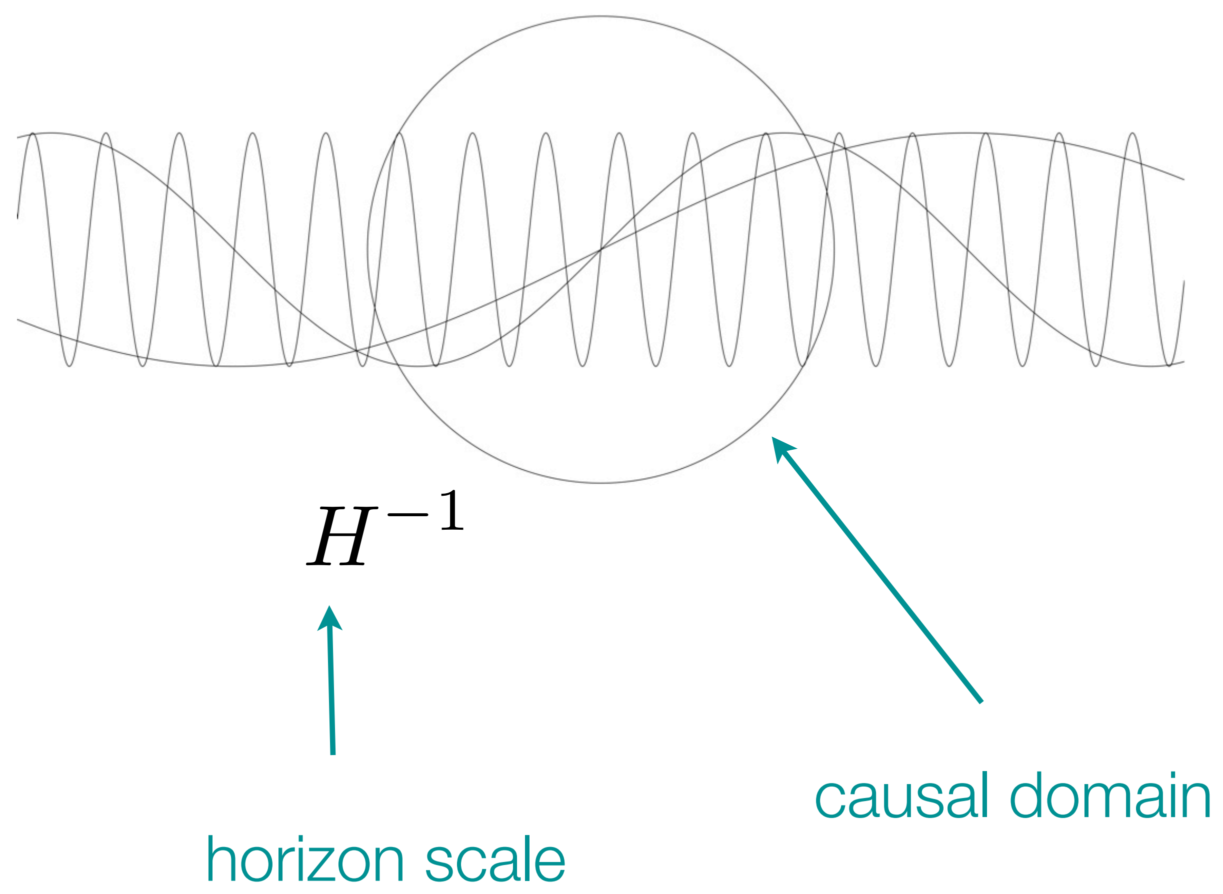
$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

Linde 1984

Primordial magnetic fields from inflation

- Fluctuations "freeze" on superhorizon scales, treat as classical contribution to classical values of inflaton field.

- Quantum fluctuations induce fluctuations with amplitude $|\delta\phi| \simeq \dfrac{H}{2\pi}$

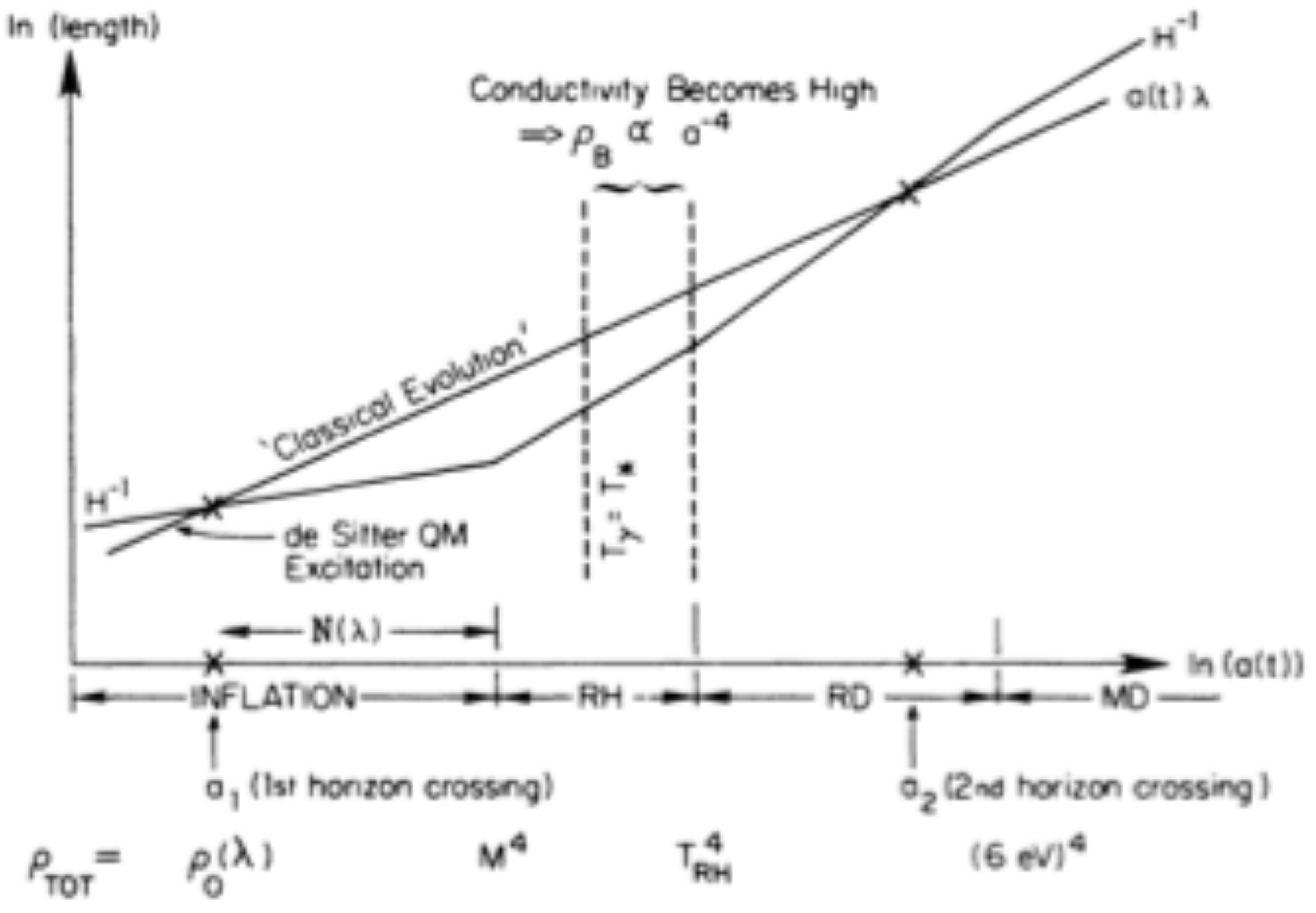


- Phases of each wave are random. Sum of all waves at a given point fluctuates, described by Brownian motion in all directions.

Linde (1986)

Primordial magnetic fields from inflation

- Generation of magnetic fields during inflation: Amplify perturbations in the electromagnetic field.



Turner, Widrow (1988)

✓Coherence lengths can be large.

- Problems with magnetic field strength.

Primordial magnetic fields from inflation

Standard Maxwell electrodynamics

flat backgrounds

resulting magnetic field too small to seed, e.g., galactic magnetic field

Large number of possibilites

Non-flat backgrounds(Tsagas, Barrow 1998; Barrow, Tsagas 2011).

go beyond standard electrodynamics

linear theory

- massive photons
- couple ED to gravity
- to other fields
- extra dimensions
- ......

key point: break conformal invariance of Maxwell ED

non linear theory

Primordial magnetic fields from inflation

line element

$$ds^2 = a^2(-d\eta^2 + d\vec{x}^{\,2})$$

$$A^2 \equiv A_m A^m$$

Turner, Widrow (1988)

$$\mathcal{L} = -\frac{1}{4}F_{mn}F^{mn} - \frac{b}{2}RA^2 - \frac{c}{2}R_{mn}A^m A^n$$

$$n \equiv \eta^2 \left(6b\frac{\ddot{a}}{a} + c\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right]\right)$$

$$F_{mn} = a^2 \begin{pmatrix} 0 & -\hat{E}_x & -\hat{E}_y & -\hat{E}_z \\ \hat{E}_x & 0 & \hat{B}_z & -\hat{B}_y \\ \hat{E}_y & -\hat{B}_z & 0 & \hat{B}_x \\ \hat{E}_z & \hat{B}_y & -\hat{B}_x & 0 \end{pmatrix}$$

$$F_\alpha(\vec{k},\eta) \equiv a^2 \int d^3x e^{i\vec{k}\cdot\vec{x}} \hat{B}_\alpha(\vec{x},\eta)$$

$$\ddot{F}_\alpha(k) + k^2 F_\alpha(k) + \frac{n}{\eta^2}F_\alpha(k) = 0$$

averaged magnetic field energy density

$$\rho_{\rm mag}(\eta) = \langle(\hat{B}_\alpha\hat{B}^\alpha)(\vec{x},\eta)\rangle/(8\pi) \longrightarrow \rho_{\rm mag}(\eta) = \frac{1}{4\pi a^4}\int d^3k |F_\mu F^\mu|$$

correlation function

$$\langle F_\alpha(\vec{k},\eta)F^*_\beta(\vec{q},\eta)\rangle = |F_\mu F^\mu|(k,\eta)\left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right)$$

spectral energy density

$$\rho_{\rm B}(k,\eta) = k\frac{d\rho_{\rm mag}}{dk} \longrightarrow \rho_{\rm B}(k,\eta) = \frac{1}{4\pi a^4}k^3|F_\mu F^\mu|$$

super horizon modes

$$\sqrt{|F_\mu F^\mu|} \propto \eta^{m_\pm} \qquad m_\pm \equiv \frac{1}{2}\left(1 \pm \sqrt{1-4n}\right)$$

Primordial magnetic fields from inflation

$$r \equiv \frac{\rho_B}{\rho_\gamma}$$

at time of galaxy formation

$$r \simeq (7\times 10^{25})^{-2(p+2)} \left(\frac{M}{M_P}\right)^{4(q-p)/3} \left(\frac{T_{\rm RH}}{M_P}\right)^{2(2q-p)/3} \left(\frac{T_*}{M_P}\right)^{-8q/3} \left(\frac{\lambda}{1{\rm Mpc}}\right)^{-2(p+2)}$$

temperature at which plasma effects become important during reheating

$$T_* \sim \min\left[\left(T_{\rm RH} M\right)^{1/2}, \left(T_{\rm RH}^2 M_P\right)^{1/3}\right]$$

$$p \equiv m_- = \frac{1}{2}\left(1 - \sqrt{1-48b-12c}\right)$$

$$q \equiv m_+ = \frac{1}{2}\left(1 + \sqrt{1-48b-24c}\right)$$

Lower limit for seed magnetic field

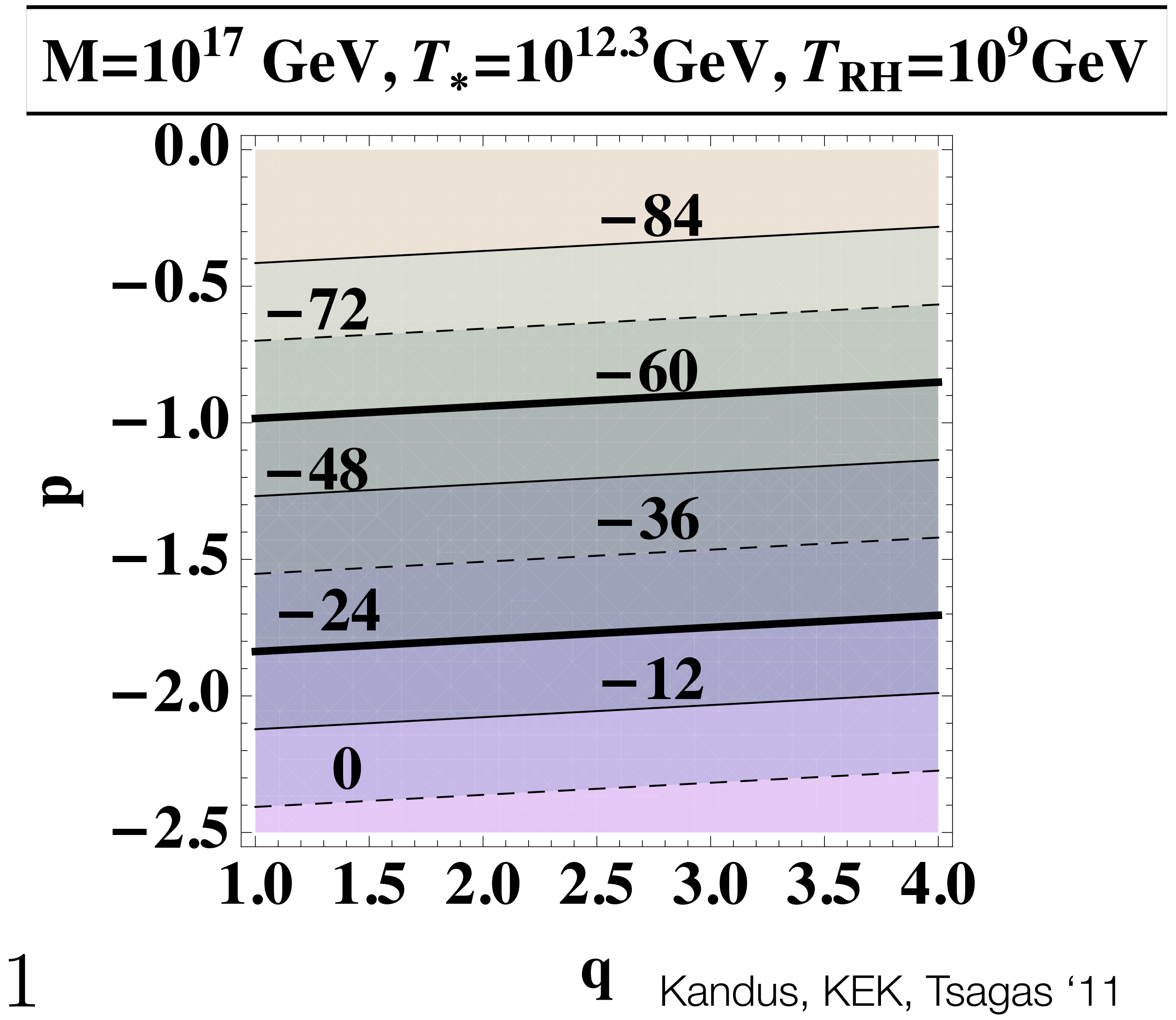
- with a galactic dynamo:

$$r > 10^{-57}$$

- without a galactic dynamo:

$$r > 10^{-8}$$

standard ED: $p = 0 \quad q = 1$



Kandus, KEK, Tsagas '11

Primordial magnetic fields from inflation

- Quantum corrections in QED in a curved background (KEK '10)

QED one-loop vacuum polarization of the photon in a general curved background gives rise to terms coupling the Maxwell tensor to the curvature
(Drummond, Hathrell 1980)

constants constants

$$\mathcal{L} = -\frac{1}{4}F_{mn}F^{mn} - \frac{1}{4m_e^2}\left(bRF_{mn}F^{mn} + cR_{mn}F^{mk}F^{n}{}_{k} + dR_{mnlk}F^{mn}F^{lk} + f(\nabla_m F^{mn})(\nabla_a F^{a}{}_{n})\right)$$

$$\Rightarrow \quad \nabla^m F_{mn} + \frac{1}{m_e^2}\nabla^m\left[bRF_{mn} + \frac{c}{2}\left(R^{l}{}_{m}F_{ln} - R^{l}{}_{n}F_{lm}\right) + dR^{lk}{}_{mn}F_{lk}\right] + \frac{f}{2m_e^2}\left(\nabla_a\nabla^a\nabla^b F_{bn} + \cancel{R^{a}{}_{n}\nabla^b F_{ba}}\right) = 0$$

neglect

Primordial magnetic fields from inflation

- Background cosmology

$$a(\eta) = \begin{cases} a_1 \left(\frac{\eta}{\eta_1}\right)^{\beta} & \eta < \eta_1 \\ a_1 \left(\frac{\eta - 2\eta_1}{-\eta_1}\right) & \eta \geq \eta_1 . \end{cases}$$

$a_1 = 1$

inflation:

de Sitter: $\beta = -1$

power-law: $\beta < -1$

radiation dominated era

- Maxwell tensor

$$F_{mn} = \partial_m A_n - \partial_n A_m$$

- radiation gauge

$$A_0 = 0, \quad \partial_\lambda A_\lambda = 0$$

$$\vec{\epsilon}^{\,(\lambda)}_{\vec{k}} \cdot \vec{k} = 0$$

$$[a^{(\lambda)}_{\vec{k}}, a^{(\lambda')}_{\vec{k}'}] = 0 = [a^{(\lambda)\,\dagger}_{\vec{k}}, a^{(\lambda')\,\dagger}_{\vec{k}'}]$$

$$[a^{(\lambda)}_{\vec{k}}, a^{(\lambda')\,\dagger}_{\vec{k}'}] = \delta_{\lambda\lambda'}\delta_{\vec{k}\vec{k}'}$$

- Expansion in Fourier modes

$$A_\mu(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}\sqrt{2k}} \sum_{\lambda=1}^{2} \epsilon^{(\lambda)}_{\vec{k}\ \mu} \left[a^{(\lambda)}_{\vec{k}} A_k(\eta) e^{i\vec{k}\cdot\vec{x}} + a^{(\lambda)\,\dagger}_{\vec{k}} A^*_k(\eta) e^{-i\vec{k}\cdot\vec{x}} \right]$$

Primordial magnetic fields from inflation

- Mode equation

$$\mu_1 = \beta\Big[6b(\beta-1) + c(\beta-2) - 2d\Big]$$

$$\mu_2 = -2(\beta+1)\mu_1$$

$$\mu_3 = \beta\Big[6b(\beta-1) + c(2\beta-1) + 2d\beta\Big]$$

$$\boxed{F_1(\eta)\ddot{A}_k + F_2(\eta)\dot{A}_k + F_3(\eta)k^2 A_k = 0}$$

$$F_1(\eta) = 1 + \frac{\mu_1}{m_e^2\eta_1^2}\left(\frac{\eta}{\eta_1}\right)^{-2(\beta+1)}$$

$$F_2(\eta) = \frac{\mu_2}{\eta_1^3 m_e^2}\left(\frac{\eta}{\eta_1}\right)^{-2\beta-3}$$

$$F_3(\eta) = 1 + \frac{\mu_3}{\eta_1^2 m_e^2}\left(\frac{\eta}{\eta_1}\right)^{-2(\beta+1)}$$

- Canonical field

$$\Psi = F_1^{\frac{1}{2}}(\eta)A_k$$

$$\boxed{\Psi'' + P\Psi = 0}$$

$$z \equiv -k\eta$$

constants depending
on the other constants

$$P = \frac{1}{4}\frac{\kappa_1 z^{-4\beta-6}}{\Big[1+\kappa_2 z^{-2(\beta+1)}\Big]^2} + \frac{1}{2}\frac{\kappa_3 z^{-2\beta-4}}{1+\kappa_2 z^{-2(\beta+1)}} + \frac{1+\kappa_4 z^{-2(\beta+1)}}{1+\kappa_2 z^{-2(\beta+1)}}$$

Primordial magnetic fields from inflation

- On superhorizon scales

$$z \ll 1$$

$$\Psi'' + (\xi_1 z^{-2} + \xi_2)\Psi = 0$$

$$\xi_1 = -(\beta+1)(\beta+2) \qquad \xi_2 = \frac{6b(\beta-1)+c(2\beta-1)+2d\beta}{6b(\beta-1)+c(\beta-2)-2d}$$

- during power law inflation

Hankel function of the 2nd kind

$$\Psi^{(\mathrm{I})} = \sqrt{\frac{\pi}{2k}}\sqrt{z}H_\nu^{(2)}(\sqrt{\xi_2}\, z), \qquad \text{where} \qquad \nu = \left|\beta + \frac{3}{2}\right|$$

- during radiation dominated stage (assuming standard ED)

$$\Psi^{(\mathrm{RD})} = \frac{1}{\sqrt{k}}\left(c_+ e^{-i(z-z_1)} + c_- e^{i(z-z_1)}\right)$$

Primordial magnetic fields from inflation

- Matching gauge potential and its first derivative at η_1 determines Bogoliubov coefficient

$$|c_-|^2 \simeq \frac{[\Gamma(\nu)]^2}{8\pi\mu_1}\left(\frac{1}{2}-\nu\right)^2\left(\frac{m_e}{H_1}\right)^2\left(\frac{\xi_2}{4}\right)^{-\nu}\left(\frac{k}{k_1}\right)^{-1-2\nu}$$

total spectral energy density

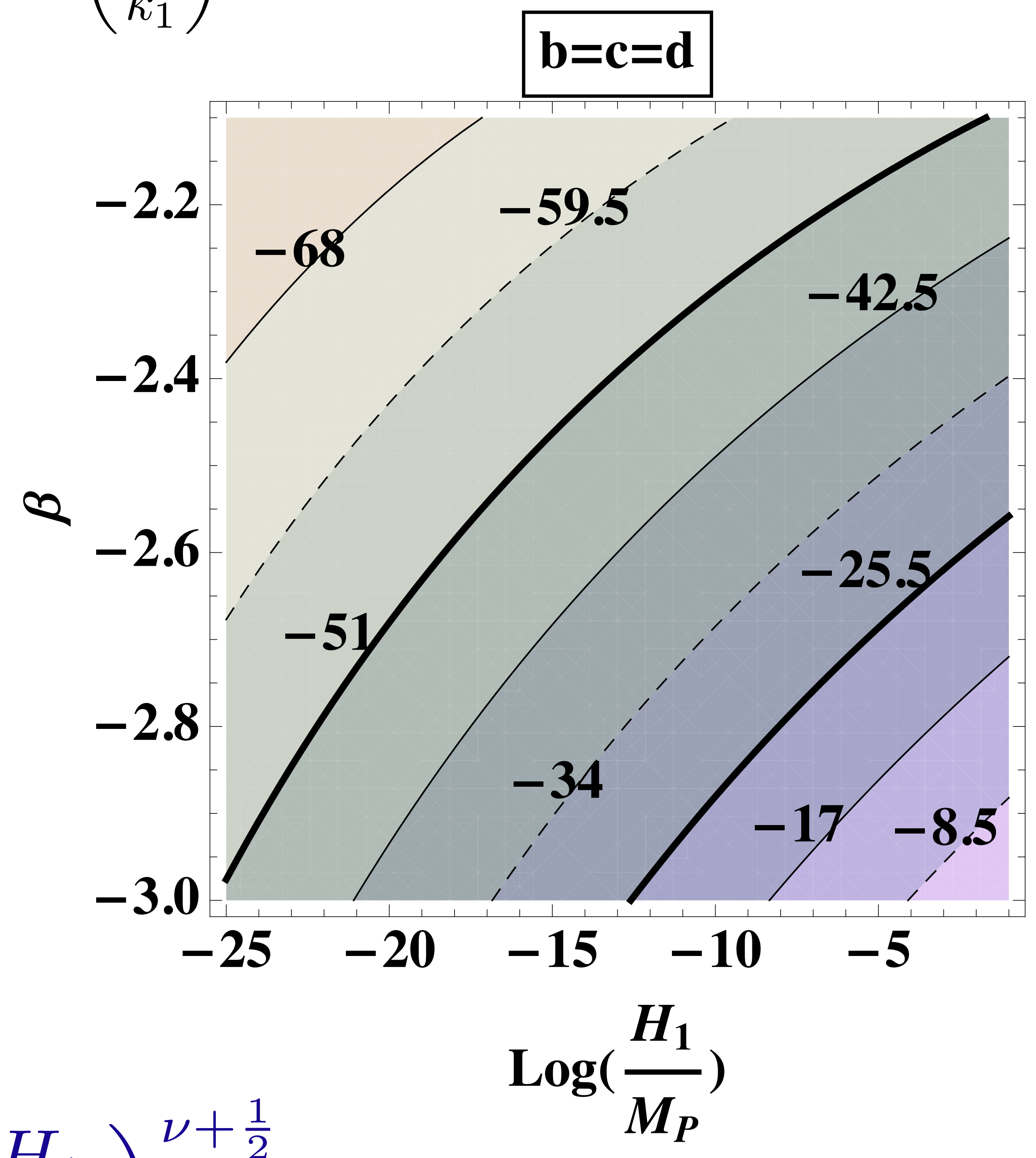
$$\rho(\omega) \equiv \frac{d\rho}{d\log k} \simeq 2\left(\frac{k}{a}\right)^4\frac{|c_-|^2}{\pi^2}$$

$$r \equiv \frac{\rho_B}{\rho_\gamma}$$

$$r \simeq \frac{2\,[\Gamma(\nu)]^2}{3\pi^2\mu_1}\left(\frac{1}{2}-\nu\right)^2\left(\frac{m_e}{M_{\rm P}}\right)^2\left(\frac{\xi_2}{4}\right)^{-\nu}\left(\frac{k}{k_1}\right)^{3-2\nu}$$

at galactic scale 1 Mpc

$$r_{\rm max}(\omega_G) = 10^{-79+52\nu}\,[\Gamma(\nu)]^2\left(\frac{1}{2}-\nu\right)^2\left(\frac{\xi_2}{4}\right)^{-\nu}\left(\frac{H_1}{M_{\rm P}}\right)^{\nu+\frac{1}{2}}.$$



Primordial magnetic fields from inflation

- Checking for back reaction

➡ Compare energy density associated with perturbations in electromagnetic field with total energy density during inflation

Energy momentum tensor

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \frac{1}{m_e^2}\left(bT^{(1)}_{\mu\nu} + \frac{c}{2}T^{(2)}_{\mu\nu} + dT^{(3)}_{\mu\nu}\right),$$

$$T^{(0)}_{\mu\nu} \simeq F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta},$$

$$T^{(1)}_{\mu\nu} \simeq RT^{(0)}_{\mu\nu} + \frac{1}{2}R_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta},$$

$$T^{(2)}_{\mu\nu} \simeq -\left(\frac{1}{2}g_{\mu\nu}R_{\alpha\beta}F^{\beta\kappa}F^{\alpha}{}_{\kappa} - F^{\alpha\beta}(R_{\mu\alpha}F_{\nu\beta} + R_{\nu\alpha}F_{\mu\beta}) - R^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}\right),$$

$$T^{(3)}_{\mu\nu} \simeq -\left(\frac{1}{4}g_{\mu\nu}R^{\alpha\beta\kappa\sigma}F_{\alpha\beta}F_{\kappa\sigma} - \frac{3}{4}F^{\kappa\sigma}(F_{\mu}{}^{\lambda}R_{\nu\lambda\kappa\sigma} + F_{\nu}{}^{\lambda}R_{\mu\lambda\kappa\sigma})\right).$$

Primordial magnetic fields from inflation

- So that

$$\frac{\langle \rho_{(\mathrm{em})} \rangle(\eta)}{M_{\mathrm{P}}^4} \simeq \frac{1}{\pi} \frac{[\Gamma(\nu)]^2}{3-2\nu} \left(\frac{\xi_2}{4}\right)^{-\nu} \left(\frac{H_1}{M_{\mathrm{P}}}\right)^4 \left(\frac{\eta}{\eta_1}\right)^{-4(\beta+1)}.$$

- During inflation total energy density

$$\frac{\rho}{M_{\mathrm{P}}^4} = \frac{3}{8\pi} \left(\frac{H_1}{M_{\mathrm{P}}}\right)^2 \left(\frac{\eta}{\eta_1}\right)^{-2(\beta+1)}.$$

Ratio

$$r^{(\mathrm{I})} \equiv \frac{\langle \rho_{(\mathrm{em})} \rangle}{\rho} \simeq \frac{8}{3} \frac{[\Gamma(\nu)]^2}{3-2\nu} \left(\frac{\xi_2}{4}\right)^{-\nu} \left(\frac{H_1}{M_{\mathrm{P}}}\right)^2 \left(\frac{\eta}{\eta_1}\right)^{-2(\beta+1)}.$$

$$r^{(I)} \sim \frac{\rho}{M_P^4} < 1$$

no strong back reaction

KK '10

Primordial magnetic fields from inflation

- What about the curvature perturbation in this model? (KK ’13)

Power law inflation $a \sim \left(\dfrac{a}{a_1}\right)^{\beta}$

Cosmologically relevant magnetic fields are generated only for $\beta<-2.8$.

$\Rightarrow$

Spectral index of curvature perturbation too small

Primordial magnetic fields from inflation

- Slow roll (power-law) inflation $\Longrightarrow$ Exponential potential of the inflaton

$$V(\phi) = V_i \exp\left[4\sqrt{\frac{\pi}{p}}\frac{\phi-\phi_i}{M_P}\right]$$

$$p = \frac{\beta}{\beta+1}$$

Power spectrum of curvature perturbation at horizon crossing

$$\mathcal{P}_\zeta^\phi = \frac{1}{\pi M_P^2}\left[2^{\mu-\frac{3}{2}}\frac{\Gamma(\mu)}{\Gamma(\frac{3}{2})}\right]^2\left(\mu-\frac{1}{2}\right)^{-2\mu+1}\left(\frac{H^2}{\epsilon}\right)_{k=aH}$$

$$\mu \equiv \frac{3}{2}+\frac{1}{p-1}$$

$\downarrow$ slow roll equations

$$\mathcal{P}_\zeta^\phi = \frac{p}{\pi}\left[2^{\mu-\frac{3}{2}}\frac{\Gamma(\mu)}{\Gamma(\frac{3}{2})}\right]^2\left(\mu-\frac{1}{2}\right)^{-2\mu+1}(5.24\times10^{-58}\Omega_{\gamma,0}^{-\frac{1}{4}})^{2(\beta+1)}\left(\frac{k_p}{\mathrm{Mpc}^{-1}}\right)^{2(\beta+1)}\left(\frac{H_0}{M_P}\right)^{-(\beta+1)}\left(\frac{H_1}{M_P}\right)^{1-\beta}$$

Primordial magnetic fields from inflation

- Spectral index (Slow roll inflation)

$$n_s = 1 + 2\eta - 6\epsilon$$

$$\epsilon = \frac{\eta}{2} = \frac{1}{p}$$

$$\beta < -2.8 \Rightarrow -1.0 < n_s < -0.29$$

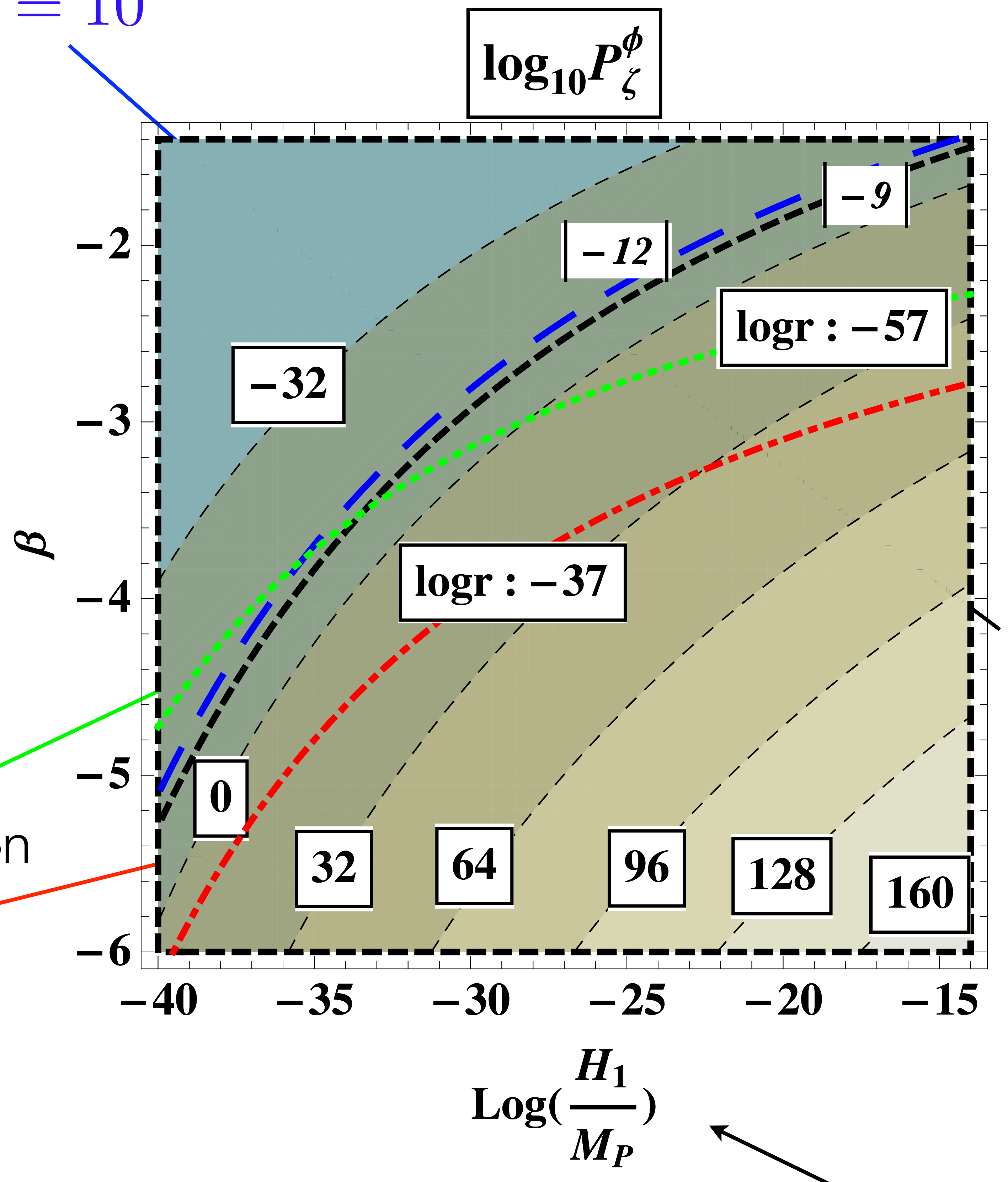
but

$$\boxed{n_s = 0.972}$$

WMAP9

Primordial magnetic fields from inflation

$\mathcal{P}_\zeta^\phi = 10^{-12}$



WMAP 9

$\mathcal{P}_\zeta = 2.41 \times 10^{-9}$

$\mathcal{P}_\zeta^\phi = 10^{-9}$

corresponding to lower bound on initial magnetic field strength

Value at beginning of radiation dominated era

Strong constraints:

$\cancel{r > 10^{-37}}$

KK '13

Primordial magnetic fields from inflation

Lyth, Wands '02
Bartolo, Liddle '02

- Generating the curvature perturbation using the simplest curvaton model

- Spectral index of curvature perturbation: $n_s = 1 + 2\eta_{\sigma\sigma} - 2\epsilon$

Dimopoulos, Lyth '04

$$V(\phi,\sigma) = V_i \exp\left[4\sqrt{\frac{\pi}{p}}\frac{(\phi-\phi_i)}{M_P}\right] + \frac{1}{2}m_\sigma^2\sigma^2$$

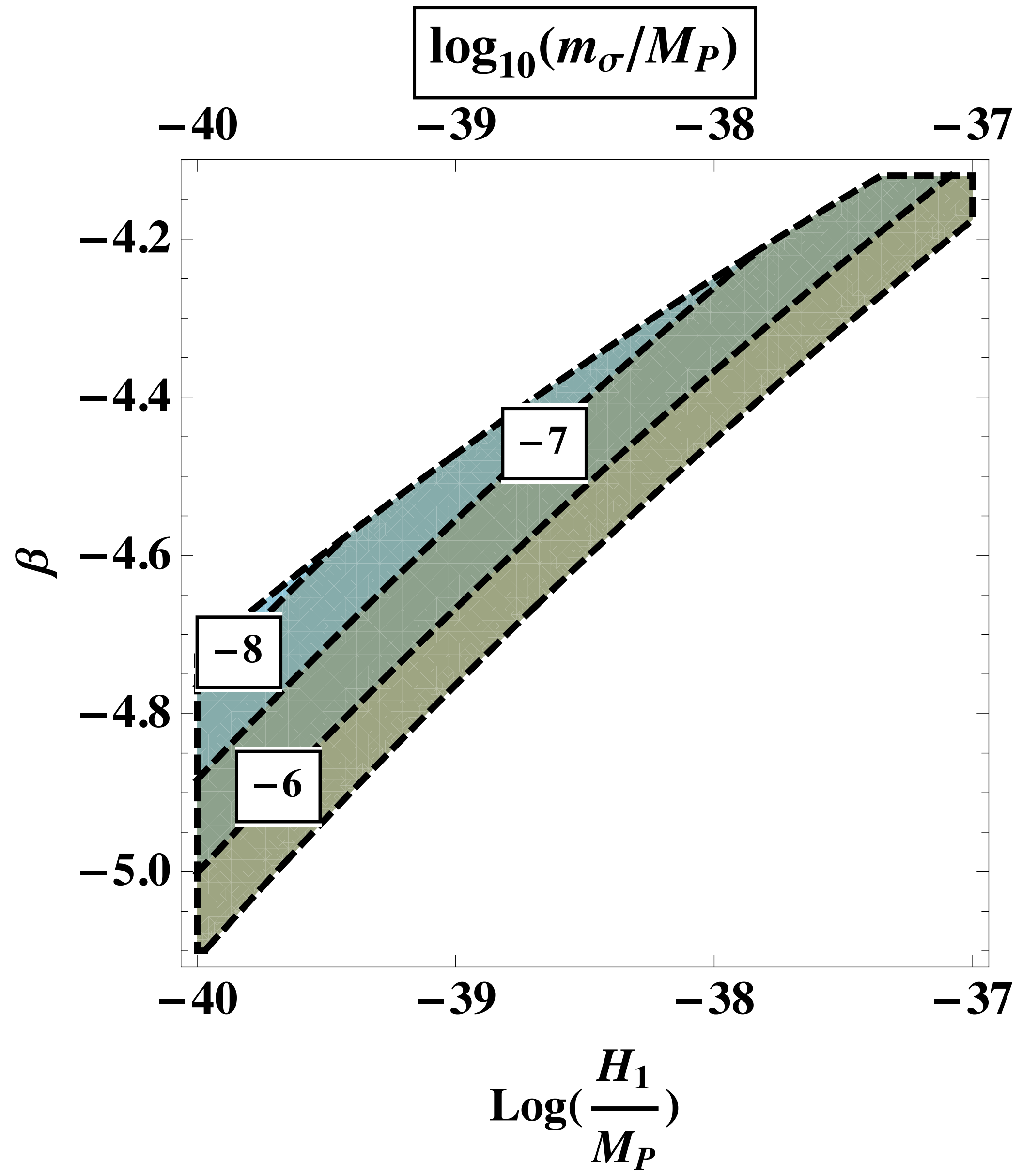
inflaton   curvaton

$$\eta_{\sigma\sigma} \equiv \frac{\bar{M}_P^2}{V}\frac{\partial^2 V}{\partial\sigma^2}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0$$

- Nearly scale invariant spectrum: $\eta_{\sigma\sigma} \simeq \epsilon$

constraint on curvaton mass



KK '13

Primordial magnetic fields from inflation

- Curvaton becomes massive when m~H after inflation.
- It decays instantaneously when $\Gamma_\sigma = H_{decay}$

$$\left(\frac{H_k}{M_P}\right)^2 = \left(\frac{H_1}{M_P}\right)^{-2\beta} \left(\frac{k}{M_P}\right)^{2(\beta+1)}$$

σ_* constant value after inflation

curvaton subdominant at decay

$$\mathcal{P}_\zeta = \frac{1}{9}\left(\frac{H_k}{M_P}\right)^2 \left(\frac{m_\sigma}{M_P}\right) \left(\frac{\sigma_*}{M_P}\right)^2 \left(\frac{\Gamma_\sigma}{M_P}\right)^{-1}$$

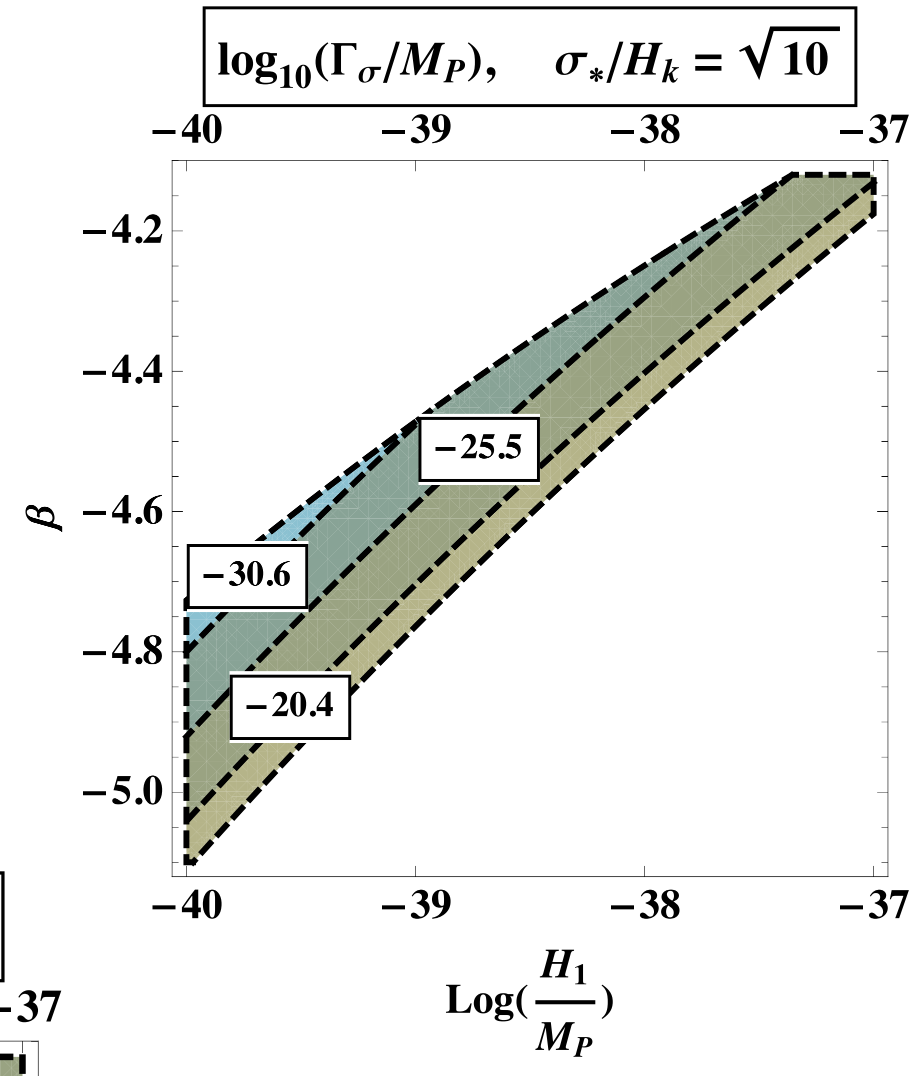
Gaussianity constraint $\quad 1 < \frac{\sigma_*}{H_k} < \frac{1}{4\pi \mathcal{P}_\zeta^{\frac{1}{2}}}$

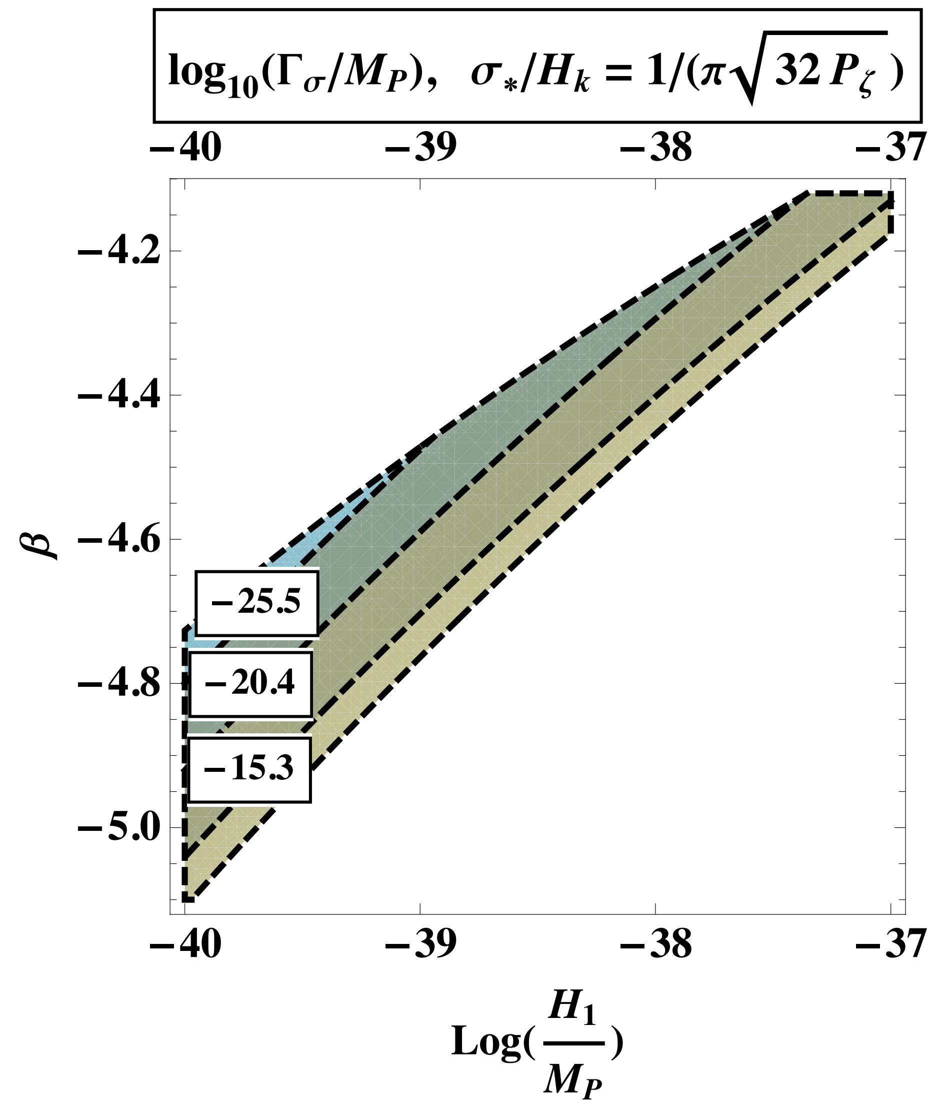
curvaton dominant at decay

$$\mathcal{P}_\zeta \simeq \frac{1}{9}\frac{H_k^2}{\pi^2 \sigma_*^2}$$

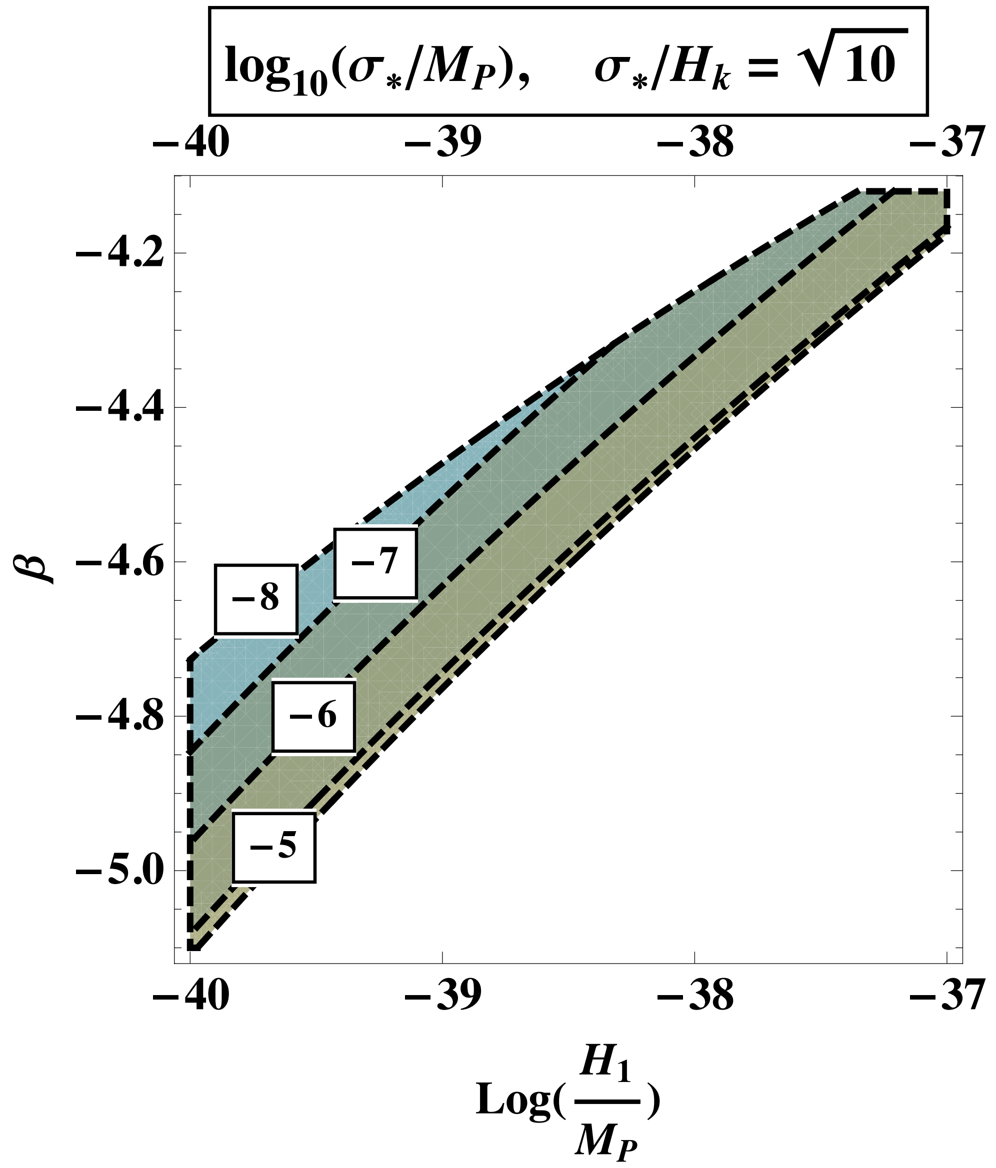
Primordial magnetic fields from inflation

curvaton subdominant at decay





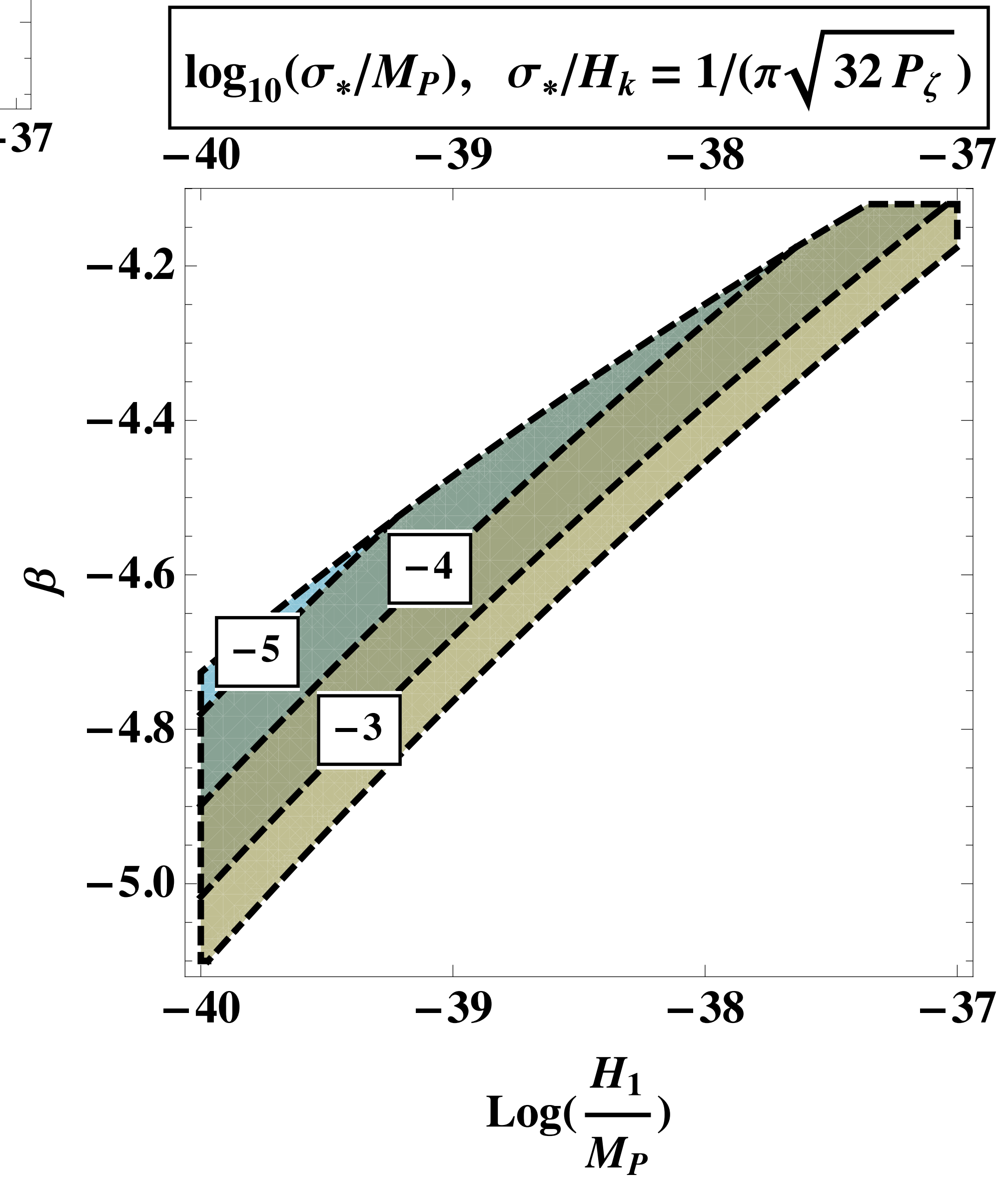
Constraints on model parameters



Regions allowed by constraints:

$$\mathcal{P}_\zeta^\phi \leq 10^{-12}$$

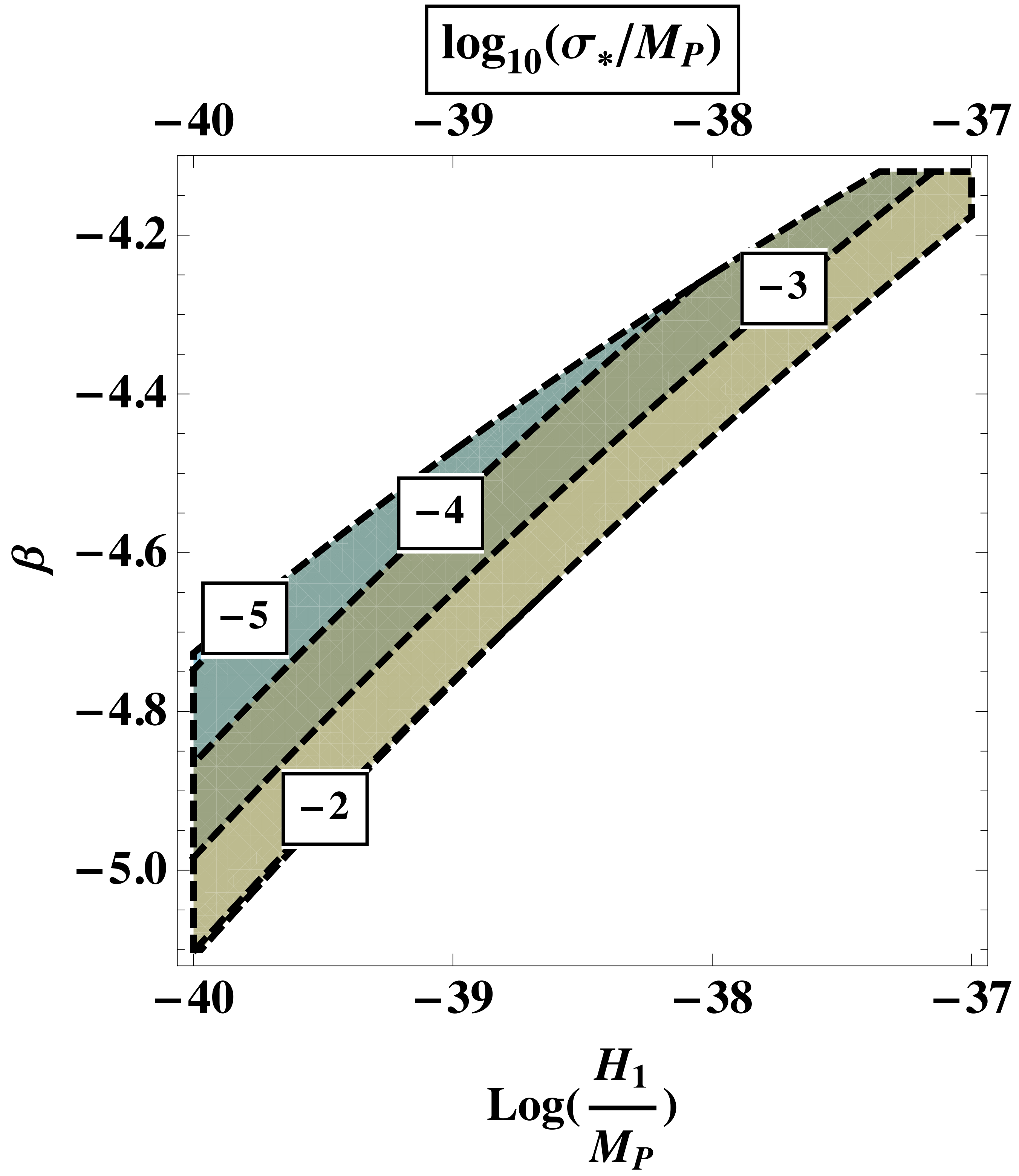
$$r > 10^{-57}$$



KK '13

Primordial magnetic fields from inflation

curvaton
dominant at
decay



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$$\mathcal{P}_\zeta^\phi \leq 10^{-12}$$

$$r > 10^{-57}$$

Constraints on model parameters

KK '13

Primordial magnetic fields from inflation

- Model building

Check list

✓ Constraints from

➡ nucleosynthesis

➡ gravitational wave production (Caprini, Durrer '01)

✓ Back reaction during inflation (magnetic as well as electric fields)

(Demozzi et al. '09)

✓ Strong coupling problem

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}f^2(\phi)F_{\mu\nu}F^{\mu\nu}$$

(Ferreira, Jain, Sloth '13)

✓ Complete model including curvature perturbations

Magnetic Fields in Cosmology

Observational signature?

2

Effect on cosmological perturbations

Cosmological magnetic fields and the CMB

Assumptions

- Origin of magnetic fields in the very early universe

- Stochastic magnetic field

Most general form:
helical magnetic field

$$\langle B_i^*(\vec{k})B_j(\vec{q})\rangle = \delta_{\vec{k},\vec{q}}\mathcal{P}_S(k)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)+\delta_{\vec{k}\vec{k}'}P_A(k)i\epsilon_{ijm}\hat{k}_m$$

where

$$\mathcal{P}_M(k,k_m,k_L) = A_M\left(\frac{k}{k_L}\right)^{n_M} W(k,k_m)$$

$$M = S, A$$

pivot scale

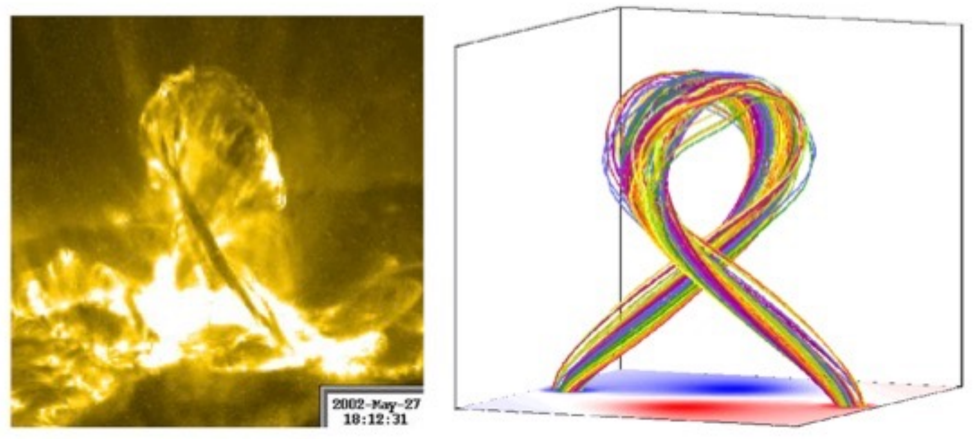
window function

$$W(k,k_m) = \pi^{-\frac{3}{2}}k_m^{-3}e^{-(k/k_m)^2}$$

damping scale

upper cut-off

Example of helical magnetic field structure: Filament eruption in solar corona modelled by twisted flux rope



Torok & Kliem (2005)

CMB anisotropies

- More on the magnetic field spectrum...

First approach to damping

Jedamzik, Katalinic, Olinto (1998): damping of linear Alfvén waves

The damping scale k_m
determined by dimensionless Alfvén velocity and
Silk damping scale (Subramanian, Barrow 1998)

(damping of nonlinear Alfvén waves)

$$\boxed{k_m^{-2} = V_{Alf}^2 k_{Silk}^{-2}}$$

largest damped scale

maximal wave number

$$\boxed{k_m \simeq 200.694 \left(\frac{B}{\mathrm{nG}}\right)^{-1} \mathrm{Mpc}^{-1}}$$

damping scale

$$\boxed{\lambda_m \simeq 30 \left(\frac{B}{\mathrm{nG}}\right) \mathrm{kpc}}$$

ΛCDM best fit WMAP7 $\Omega_b = 0.0227 h^{-2}$ $h = 0.714$

Contribution to cosmological perturbations

Assumption: magnetic energy density does not contribute to total background energy density

magnetic field

contribution to perturbations of energy density, anisotropic stress and to baryon velocity via Lorentz force

scalar, vector and tensor modes

Cosmological perturbation theory in a nutshell:

- **linear**
- **small perturbations of Friedmann-Robertson-Walker metric**
- **3 types of perturbations: scalar, vector, tensor modes**

Matter power spectrum LSS

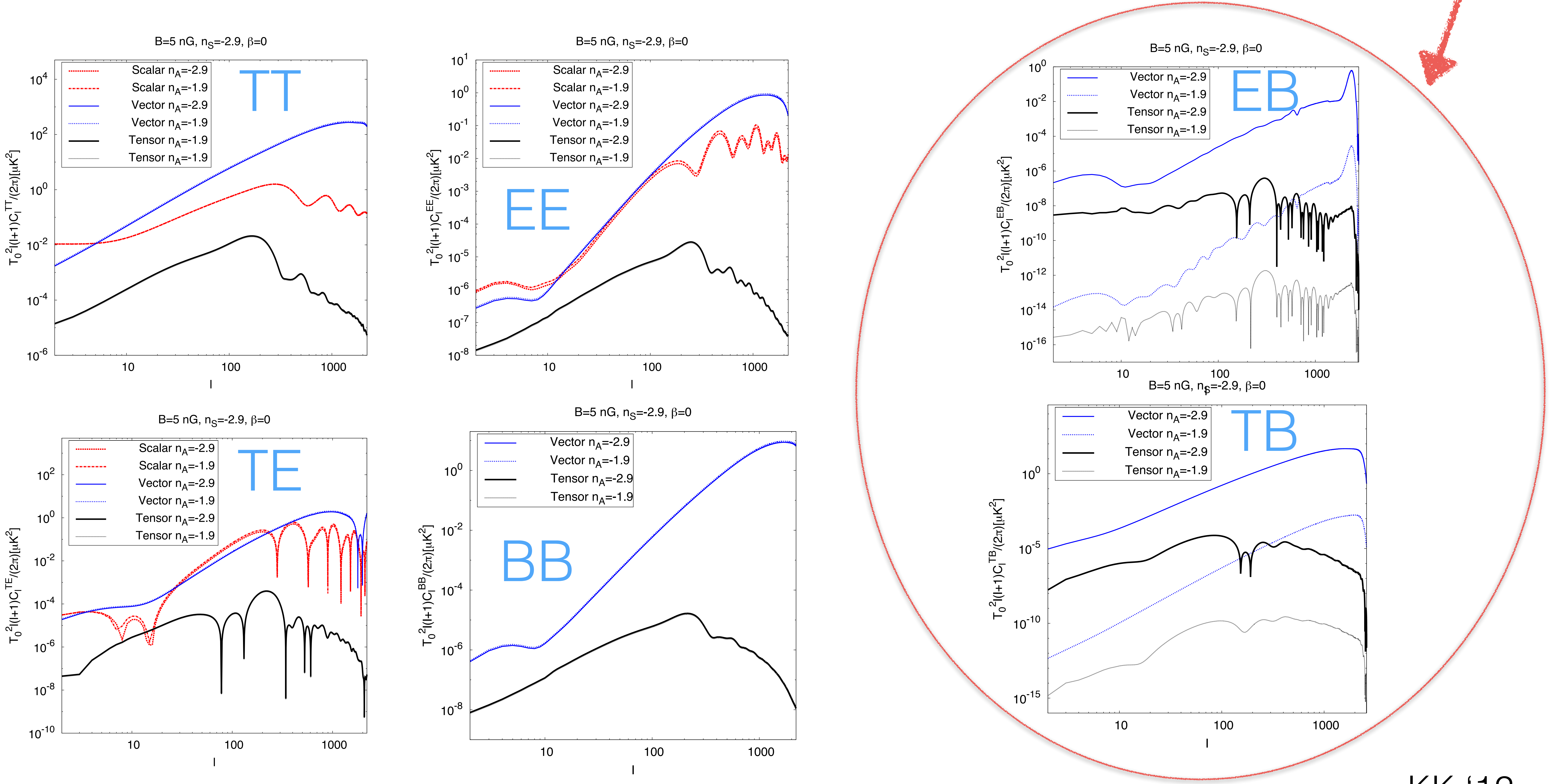
21 cm line signal

CMB temperature anisotropies and polarisation

Primary CMB anisotropies and polarisation induced by **contribution** of *helical* magnetic field

distinctive signature of helical magnetic field

ANGULAR POWER SPECTRA FOR SCALAR, VECTOR AND TENSOR MODES



KK '12

MODIFIED VERSION OF CMBEASY

- Bulk motions of electrons along the line of sight induce secondary temperature fluctuations in the postdecoupling, reionized universe.

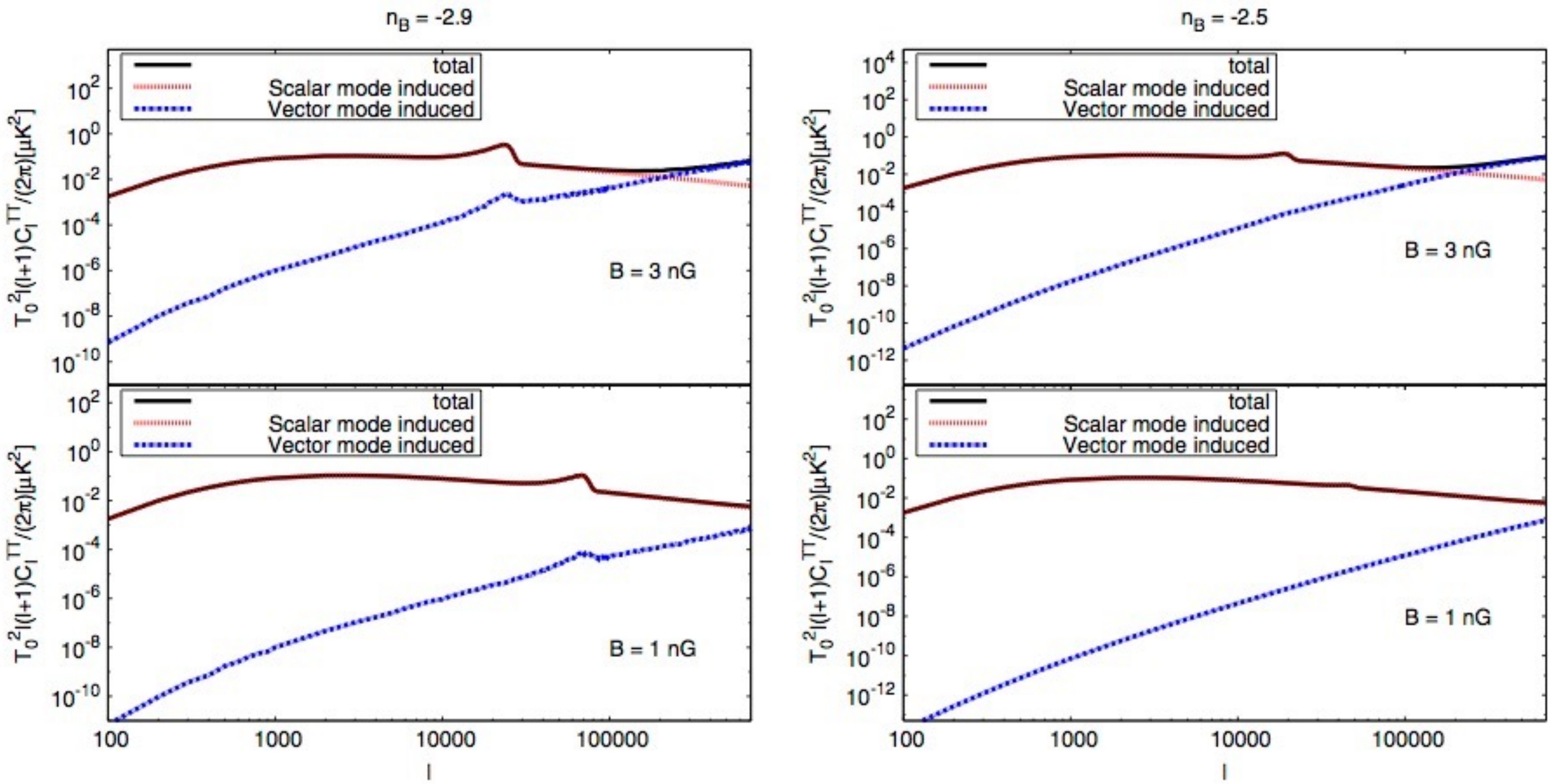
$$\Theta(\hat{\boldsymbol{n}}) = \int dD g(D) \hat{\boldsymbol{n}} \cdot \boldsymbol{V}_b(\boldsymbol{x}),$$

Fluctuations in baryon energy density along line-of-sight change number density of potential scatterers for CMB photons, thus change scattering probability and visibility function.

$$\delta \boldsymbol{V}_b(\boldsymbol{x}, \eta) = \Delta_b(\boldsymbol{x}, \eta) \boldsymbol{V}_b(\boldsymbol{x}, \eta).$$

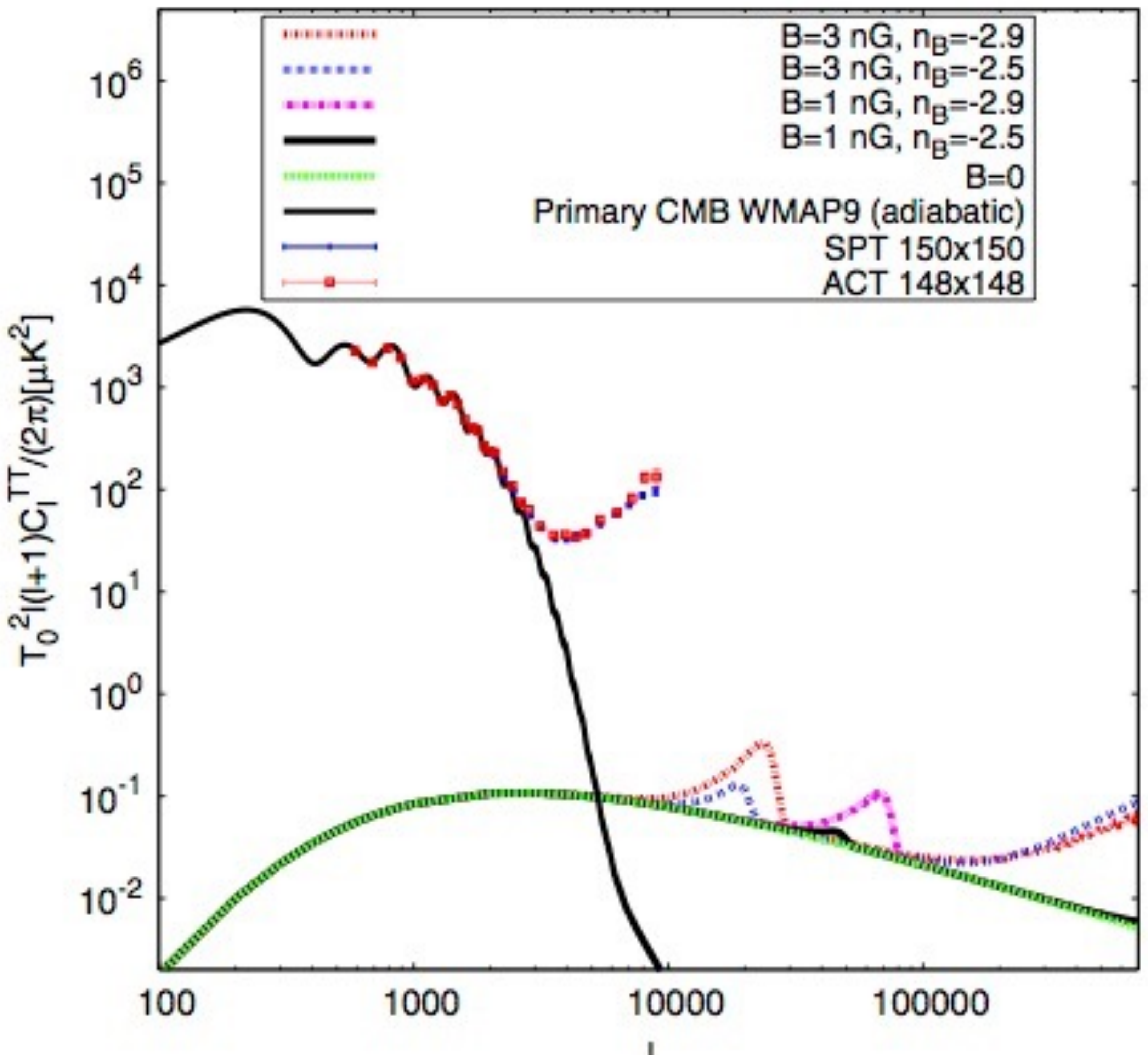
Secondary CMB anisotropies

- In the presence of a magnetic field not only the scalar mode but also the vector mode source bulk motions.



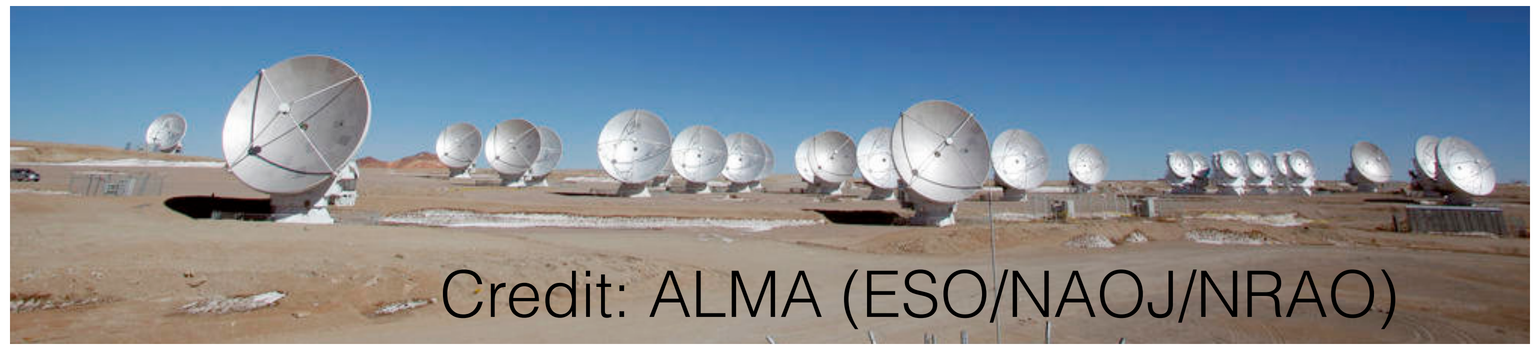
Secondary CMB anisotropies

Induced temperature anisotropies



Future observations with ALMA might reach

$$10^4 < \ell < 10^6$$

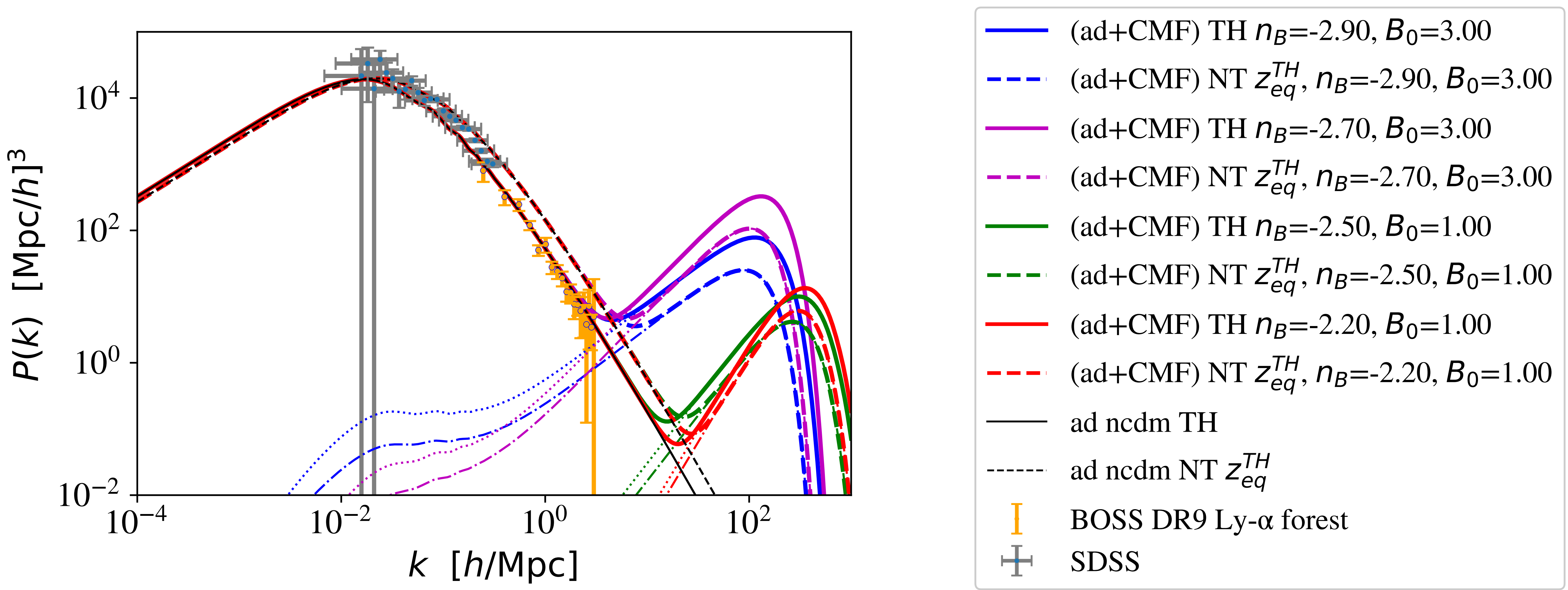


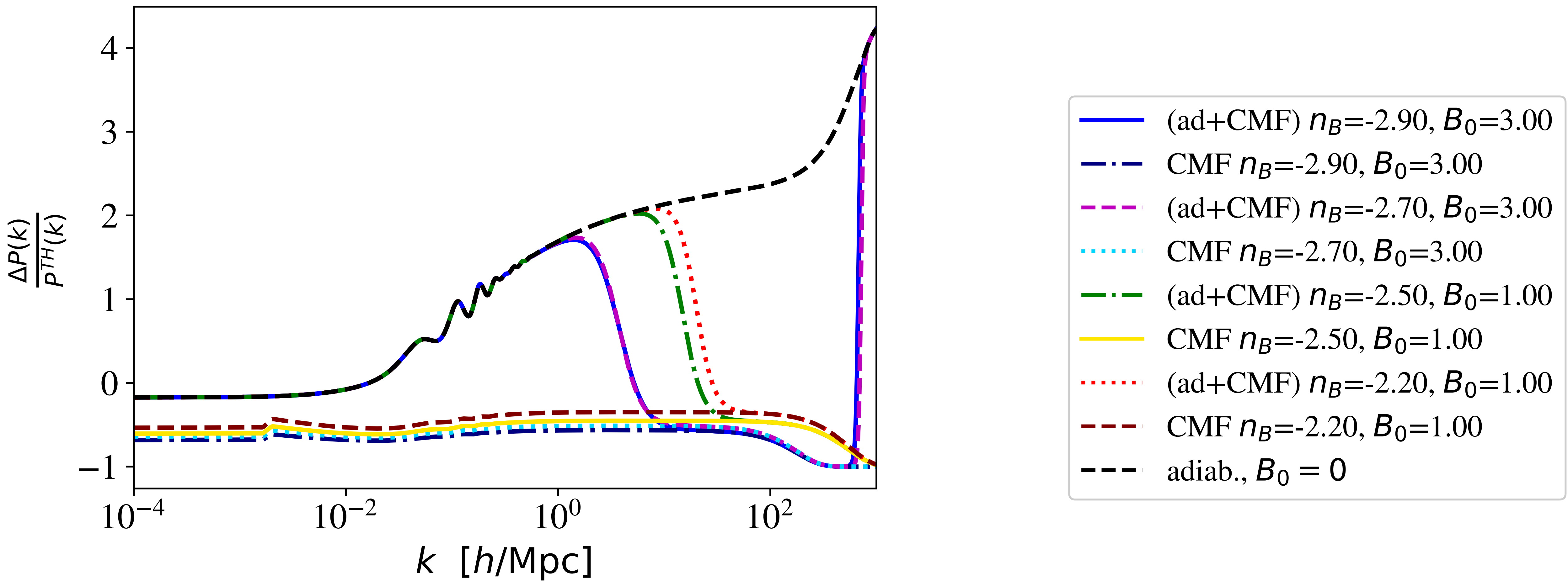
Atacama Large Millimeter/submillimeter Array (ALMA): ALMA -the largest astronomical project in existence- is a single telescope of revolutionary design, composed of 66 high precision antennas located on the Chajnantor plateau, 5000 meters altitude in northern Chile.

KK ‘14

Effect of magnetic mode on the linear matter power spectrum and 21cm line signal

- Total linear matter power spectrum: primordial curvature mode + magnetic mode





21 cm line signal: change in brightness temperature of CMB due to hyperfine transition in neutral hydrogen atoms along line of sight

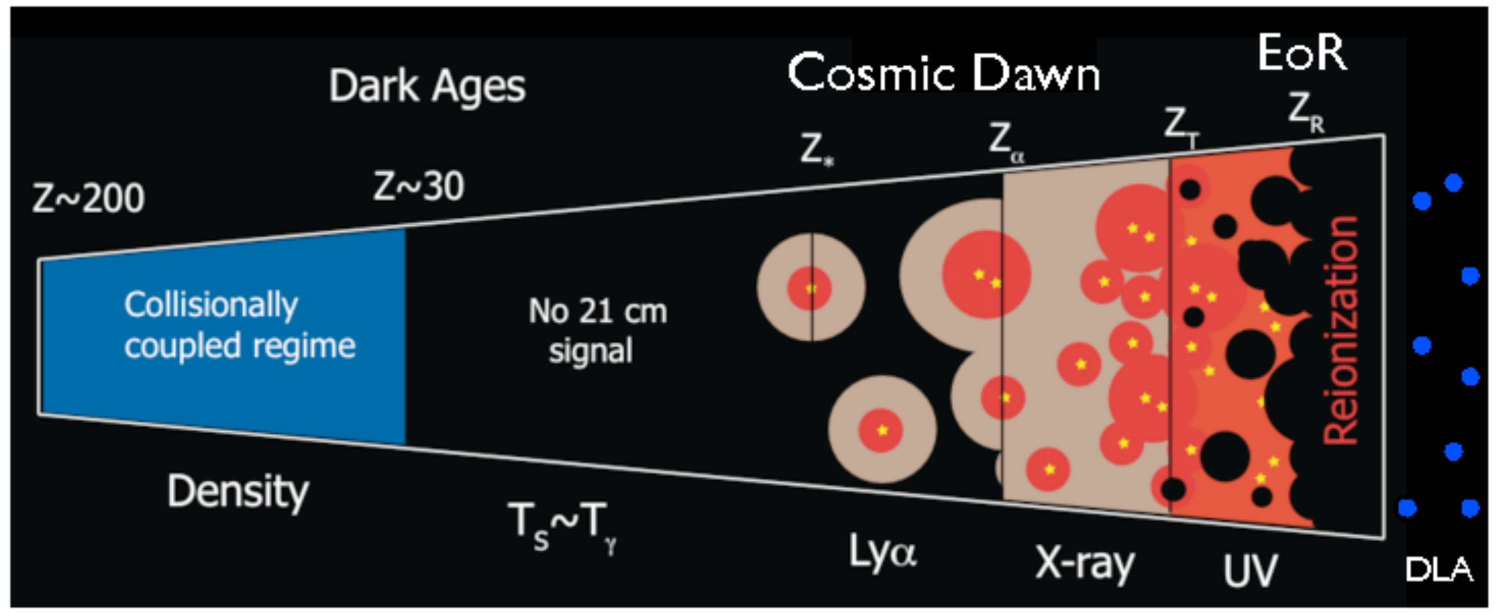


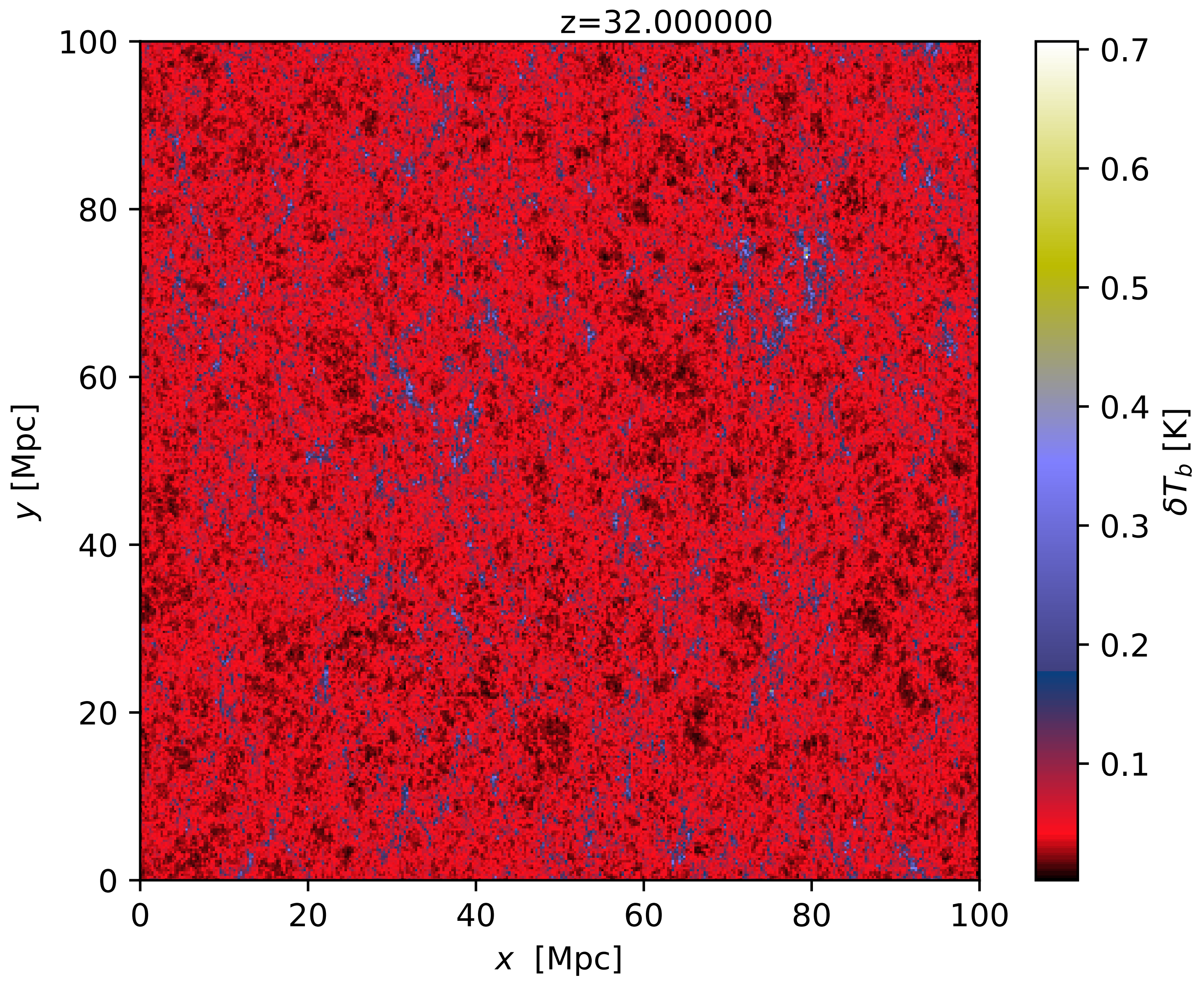
Figure 7. Linear matter power spectrum for three thermal neutrinos (TH) and three non-thermal neutrinos (NT, $z_{\rm eq}^{\rm TH}$) with distribution function (2.1) for different choices of the magnetic field parameters (B_0[nG], n_B). *Upper panel:* the total linear matter power spectrum of the adiabatic mode (ad) and the compensated magnetic mode (CMF) is shown together with data points from BOSS DR9 Ly-α forrest [42] and SDSS [43]. The light dotted and dashed-dottted lines indicate the three neutrino thermal magnetic mode and the three neutrino non-thermal pure magnetic mode solutions, respectively. $z_{\rm eq}^{\rm TH}$ denotes that the cosmological parameters have been adjusted so that the redshift of radiation-matter equality in the non-thermal model is the same as that in the non-thermal one (see details in the text). *Lower panel:* relative change of the linear matter power spectrum w.r.t. to the three thermal neutrino model (TH).

KK 2021

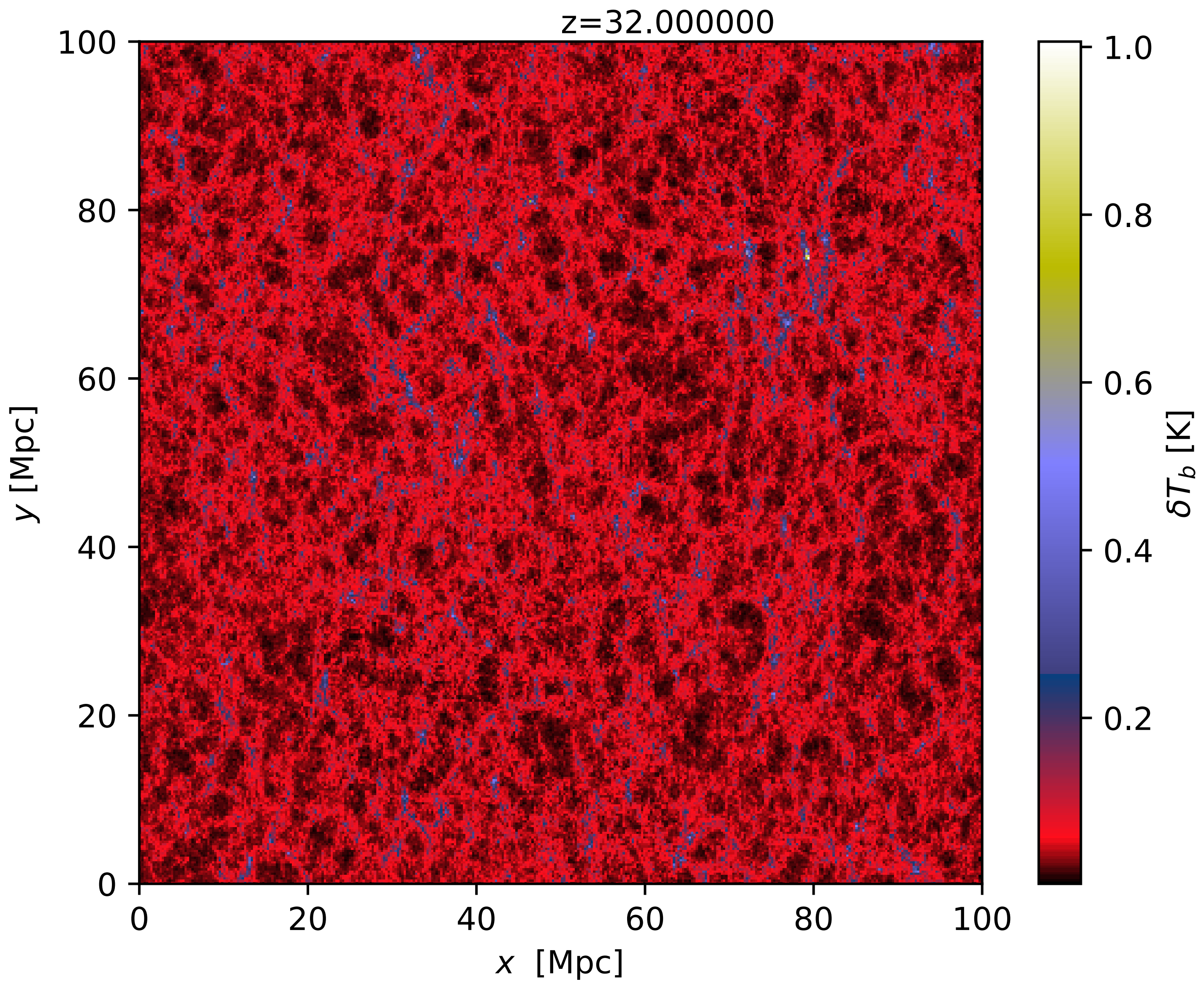
Simulations: 21cm line signal

z=32

B=5 nG, nB=-1.5



B=5 nG, nB=-2.2

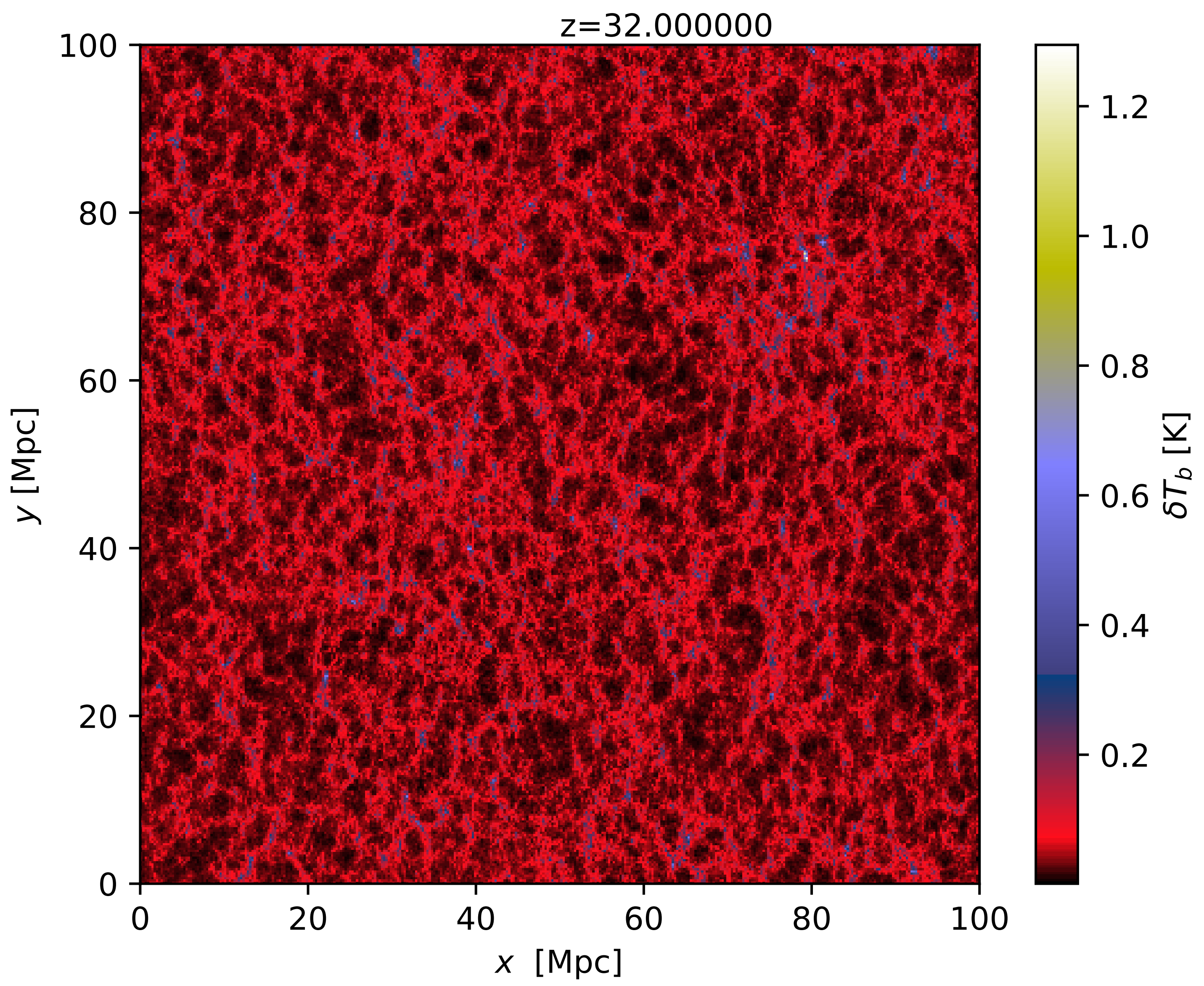


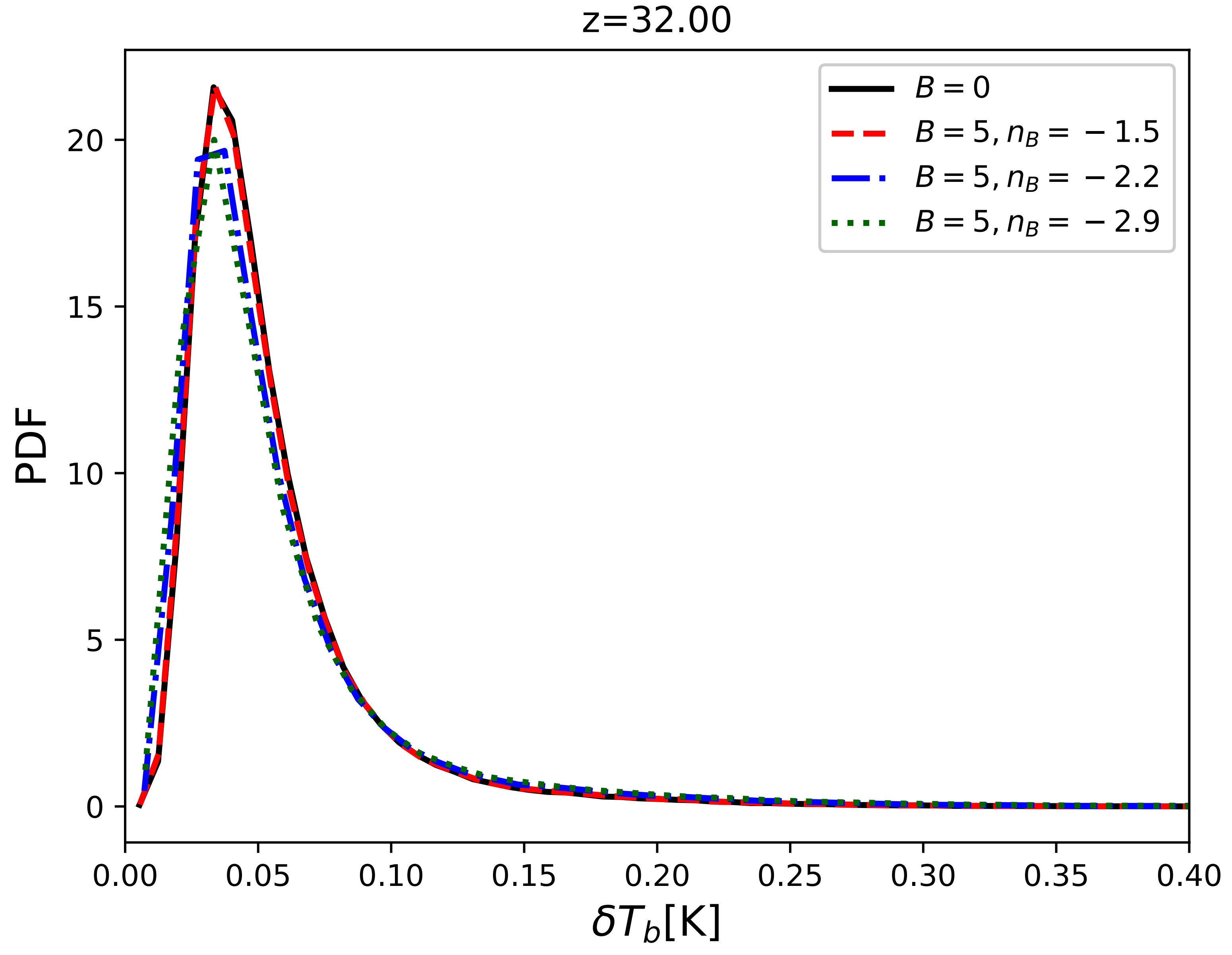
KK 2019

Simulations: 21cm line signal

z=32

B=5 nG, nB=-2.9

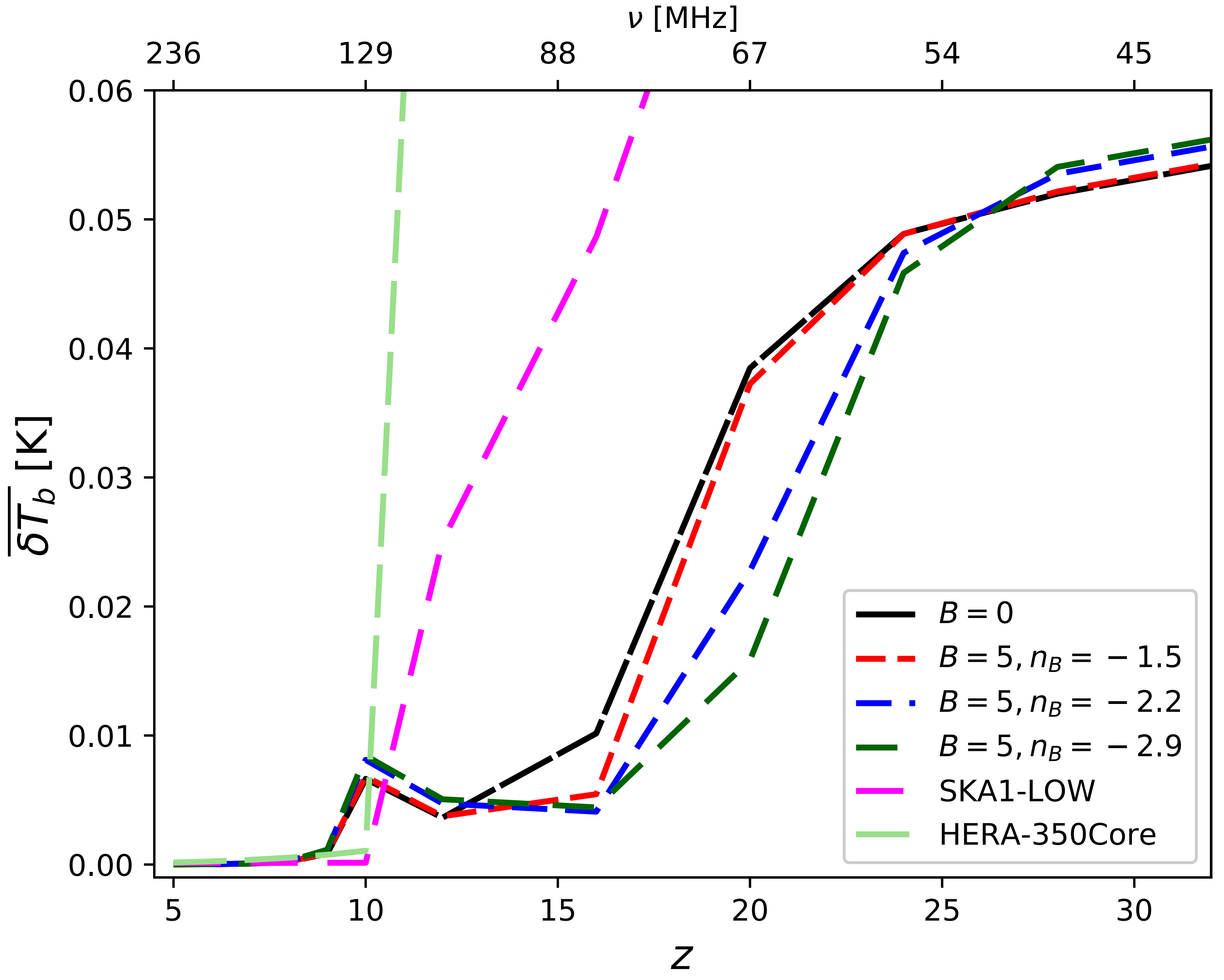


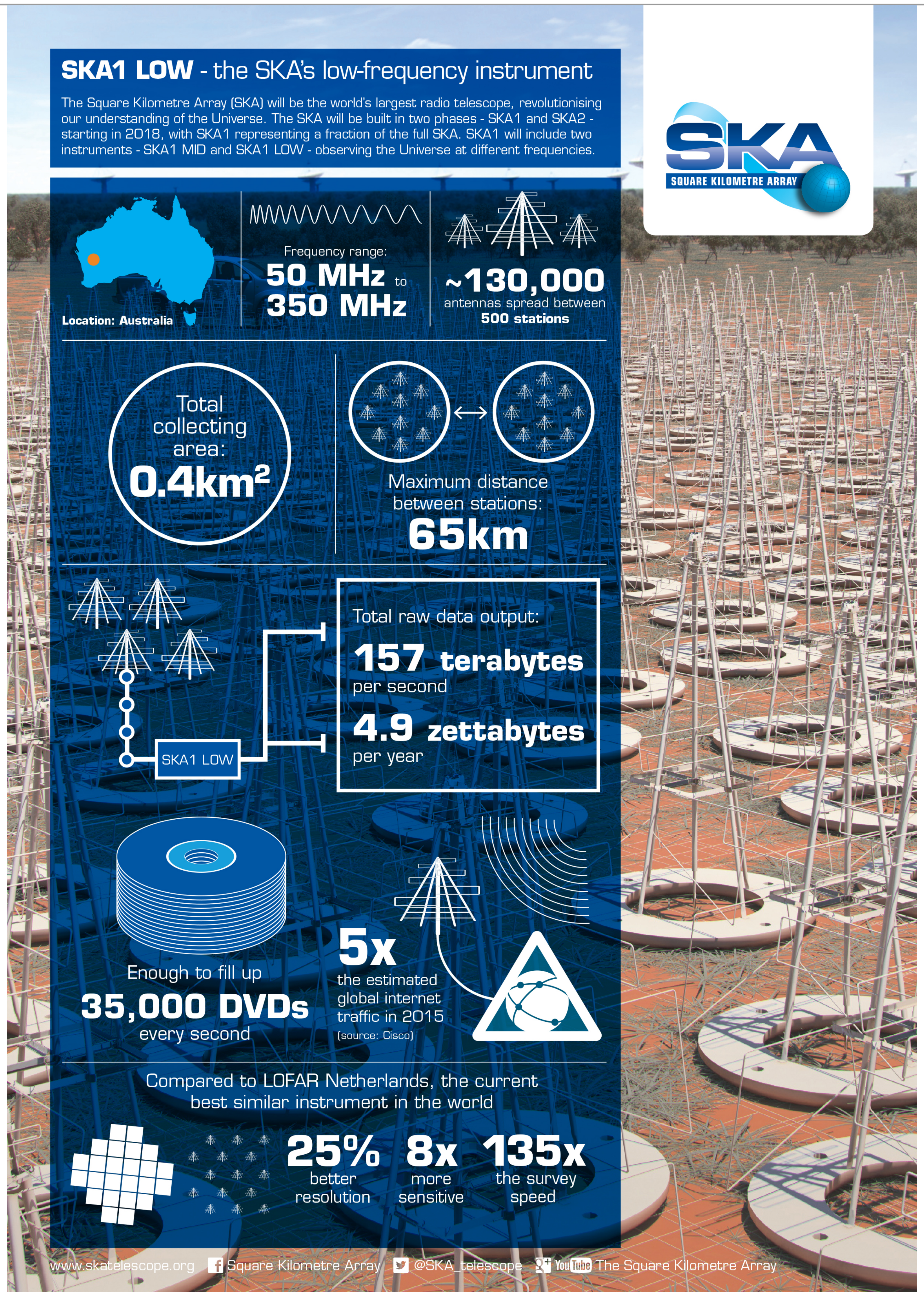


KK 2019

Observations

- Average 21 cm line signal





KK 2019

Magnetic Fields in Cosmology

3

Evolution
and
dissipation

Observational
signature?

Cosmological magnetic fields and the CMB

Damping of magnetic fields

- **Before decoupling of photons**
  - viscous damping
- **After decoupling of photons**
  - decaying MHD turbulence
  - ambipolar diffusion

There is also damping around neutrino decoupling at around z~ 10^{10} when a black body spectrum is always restored. (no spectral distortions)

energy injection

change in thermal and ionisation history

Damping in the pre-decoupling era

Subramanian, Barrow 1998
(nonlinear treatment)

In a magnetized plasma: 3 additional modes

Jedamzik, Katalinic, Olinto
1998

- Fast magnetosonic modes: damp similarly to sonic waves (Silk damping)

damping wave number $k_d = k_\gamma$

inverse of usual photon diffusion scale

- Slow magnetosonic and Alfvén modes: overdamped limit

damping wave number $k_d = \dfrac{k_\gamma}{v_A \cos\theta}$

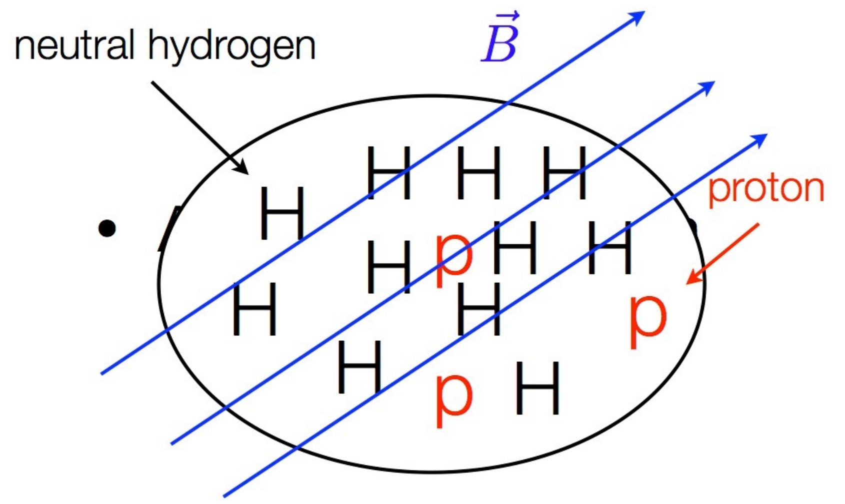
angle between background field direction and wave vector

Alfvén velocity $v_A \sim B_0/\sqrt{\rho + p}$

$$v_A = 3.8 \times 10^{-4} \left(\frac{B_0}{1\mathrm{nG}}\right)$$

Damping in the post-decoupling era

Ambipolar diffusion



Lorentz force only acts on ionized component

↓

$\vec{v}_{ion} \neq \vec{v}_{neutral}$

↓

Resisting the relative drift is a frictional (drag) force due to mutual collisions between ions and neutrals.

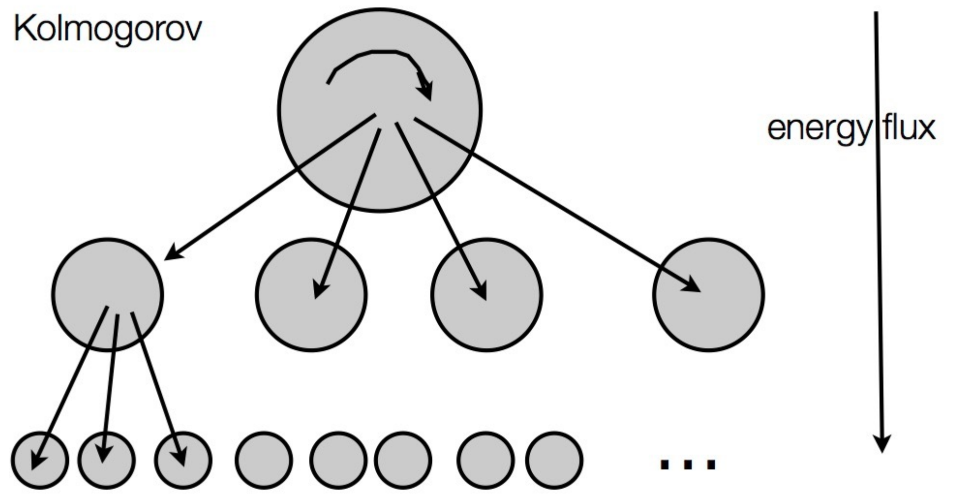
↓

Heating of IGM and dissipation of magnetic energy

After decoupling radiative viscosity dramatically drops. However magnetic fields can still be damped.

Decaying MHD turbulence

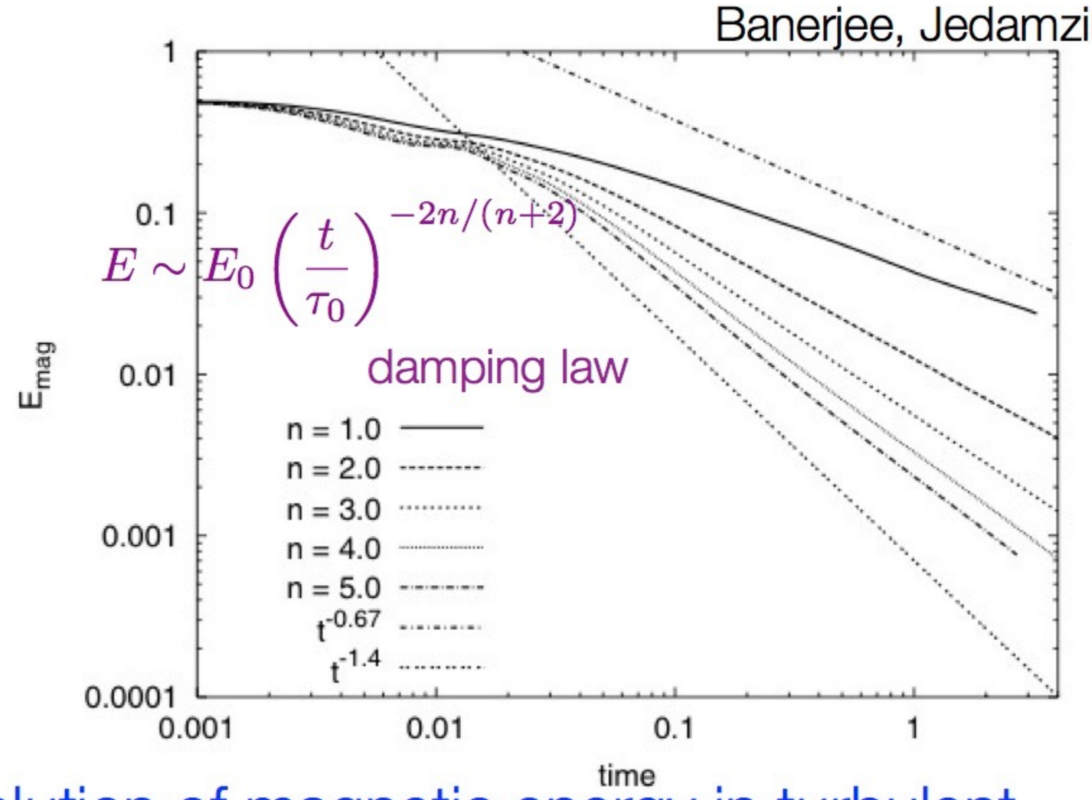
After decoupling turbulence no longer suppressed, nonlinear interactions transfer energy to smaller scales, dissipating magnetic field on large scale, inducing MHD turbulence to decay.



flow eddies break up into smaller eddies

non helical

Banerjee, Jedamzik 2004



Evolution of magnetic energy in turbulent regime for different initial energy spectra n. $(E_k \sim k^n)$

(open debate on inverse cascade for non helical fields: Kahniashvili et al. '13, '14, '15)

Ambipolar diffusion

Decaying MHD turbulence

Evolution of electron temperature (Sethi, Subramanian '05)

$$\dot{T}_e = -2\frac{\dot{a}}{a}T_e + \frac{x_e}{1+x_e}\frac{8\rho_\gamma\sigma_T}{3m_e c}(T_\gamma - T_e) + \frac{x_e\Gamma}{1.5k_B n_e}$$

$$\Gamma = \Gamma_{\rm in} + \Gamma_{\rm decay}$$

$$\Gamma_{\rm in} \propto (1+z)^{3.625}(1-x_e)/x_e$$

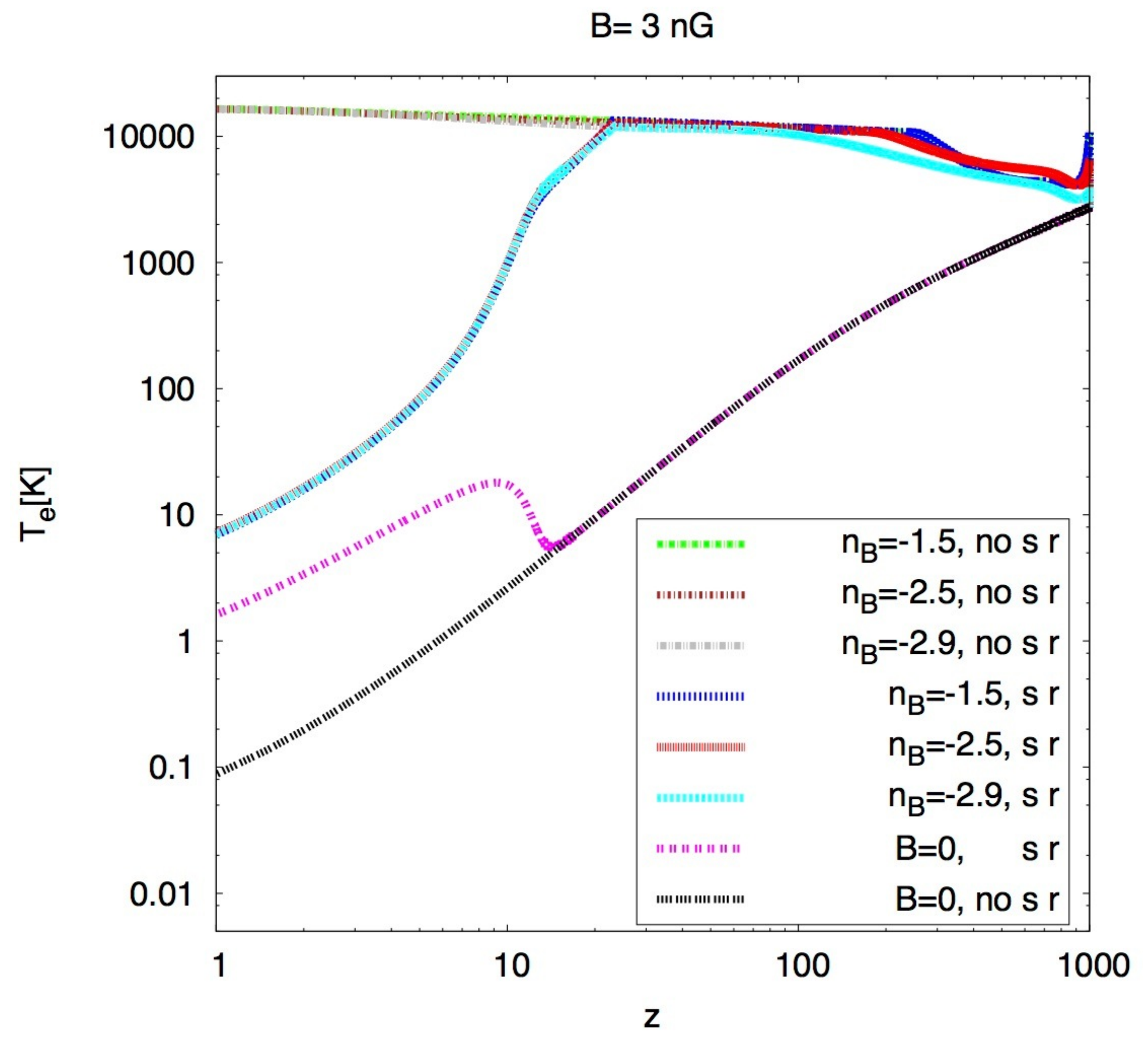
important at late times

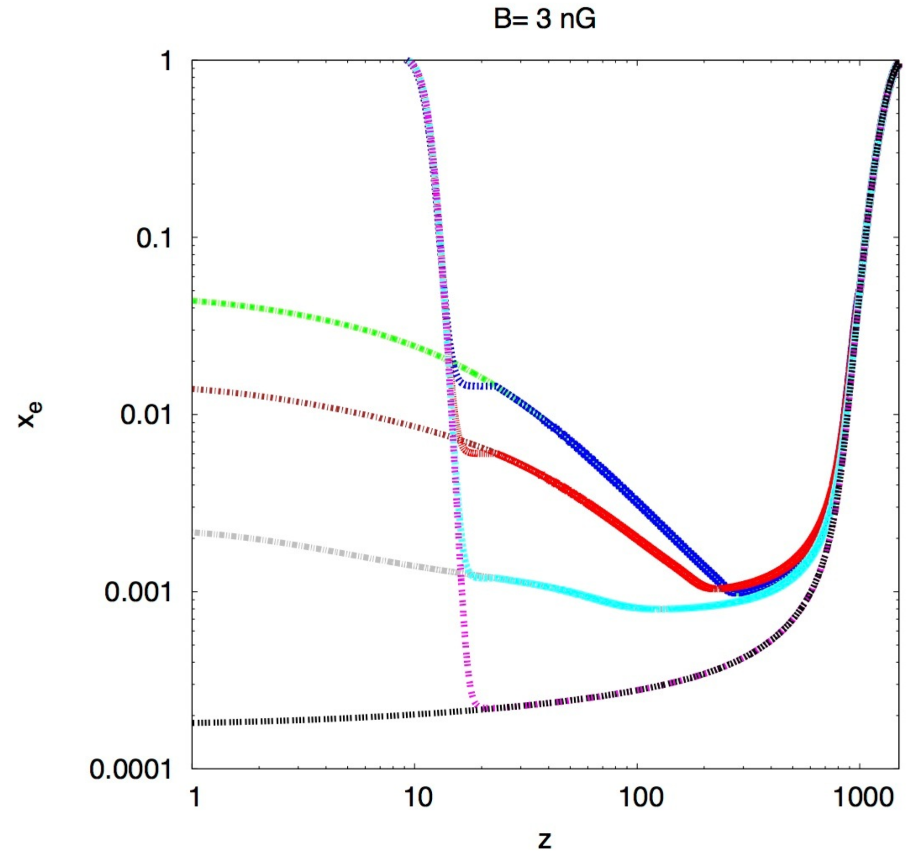
$$z < 100$$

$$\Gamma_{\rm decay} \propto (1+z)^{5.5}$$

important at early times

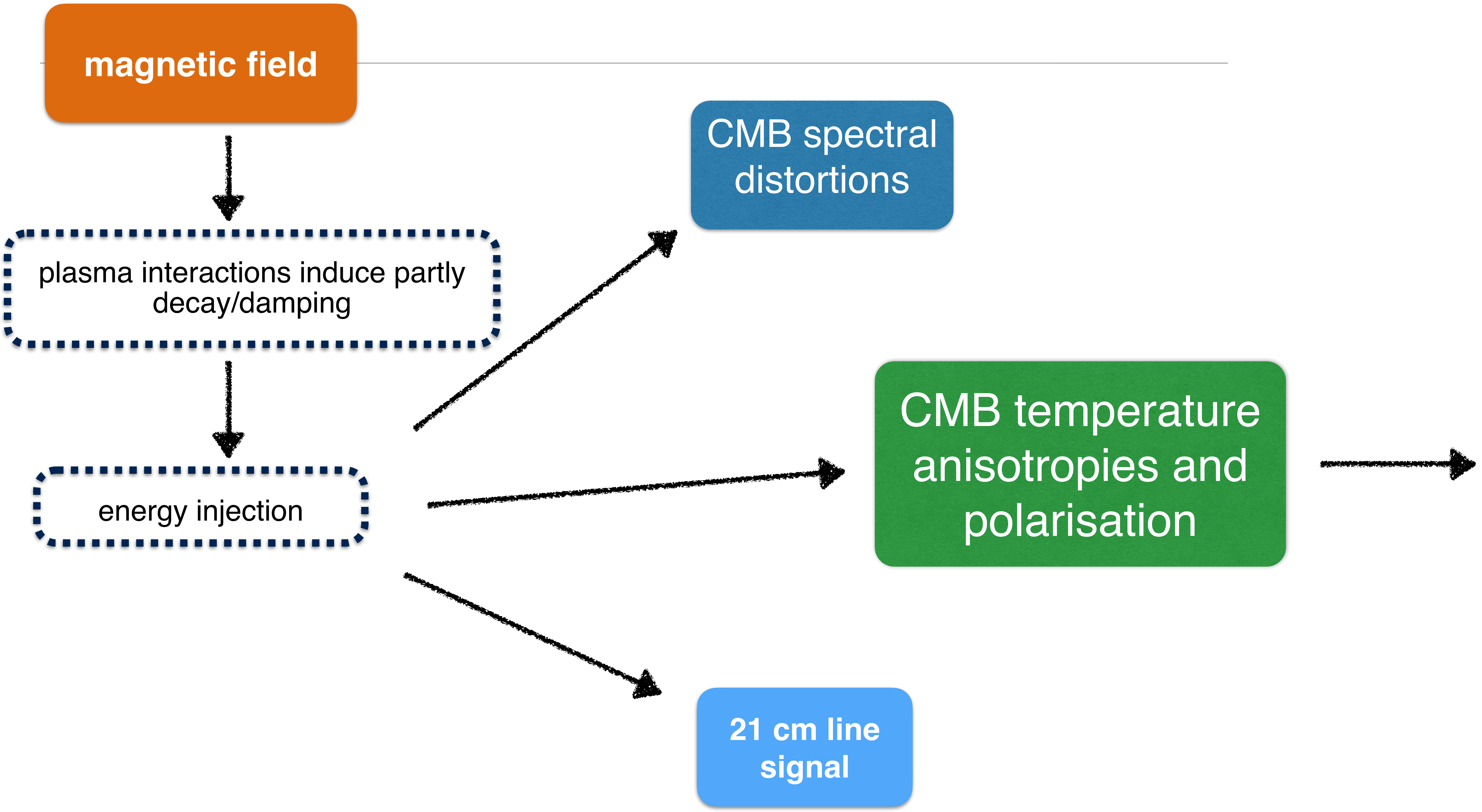
$$z > 100$$





KK, Komatsu '15

Evolution/dissipation and observational effects



Effect of post-decoupling magnetic field damping on CMB anisotropies

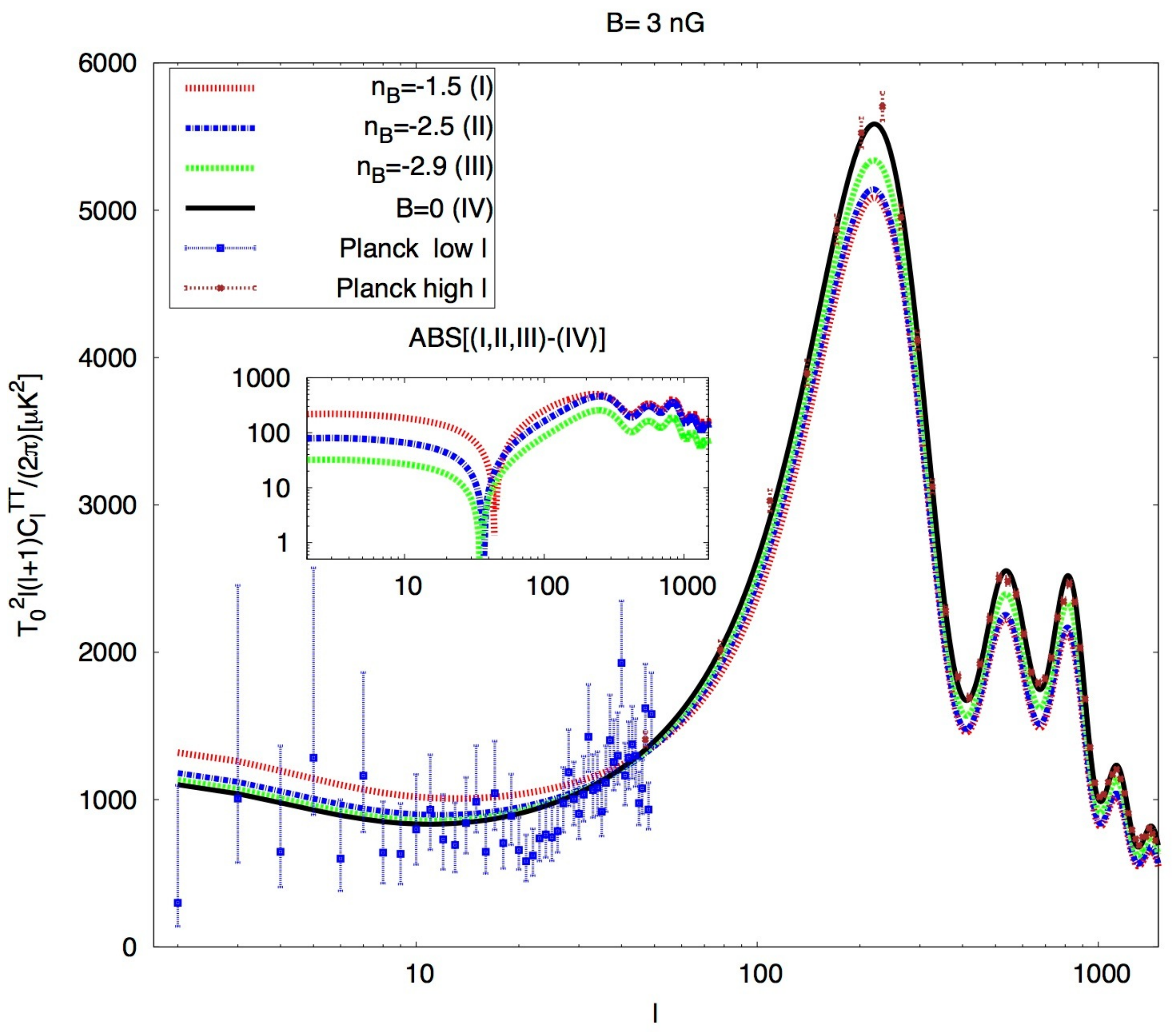
$$\tau_{tot} = \tau_{reio} + \tau_{high-z}$$

$$C_\ell^{TT} \rightarrow C_\ell^{TT} e^{-2\tau_{tot}}$$

C_ℓ^{EE} at low ℓ not affected

magnetic field dissipation contributes

determined by $\Delta\tau(B_0, n_B)$



KK, Komatsu '15

The 95% CL upper bounds are B0 < 0.63, 0.39, and 0.18 nG for nB = –2.9, –2.5, and –1.5, respectively.

modified version of CLASS + montepython

Conclusions

- Many open questions with respect to cosmological magnetic fields:
  - Generation mechanism?
  - Observational tracers? Effects are small and often difficult to disentangle from other effects ("degeneracy" with other cosmological parameters)
  - Prospects:
    - more data, also different frequencies…cross correlate different data sets…
    - advance model building: theoretical understanding of evolution of large scale magnetic fields: numerical simulations