Cosmological Parity Violation

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1 Introduction

1.1 History and Observations

Prior to 1956, it seemed obvious to most physicists that the universe should respect the parity symmetry. Parity is a spacetime transformation under which spatial coordinates are negated,

$$\mathbf{x} \to -\mathbf{x} \,. \tag{1}$$

Under this transformation, vectors transform to minus themselves and pseudovectors (sometimes called axial vectors) are invariant. For example, angular momentum and spin are axial vectors, as they are invariant under parity. The belief that the laws of physics respect parity was not completely unjustified. A great deal of atomic and subatomic experimental physics in the early 20^{th} century was based on the spectroscopy of atoms and nuclei. Transitions between energy levels in these bound systems satisfy selection rules that strongly support parity invariance for electromagnetism and the strong nuclear force. The weak force has no such bound states, so it was not immediately clear that the weak nuclear force violated parity.

The discovery of parity violation arose from the work of T. D. Lee and C. N. Yang in 1956. They were working on a problem known as the τ-θ puzzle. Experimentalists had identified two mesons, known at the time as τ^+ and θ^+. Both had the same mass and the same lifetimes, but different decay products,

$$\theta^+ \to \pi^+ + \pi^0 \,, \tag{2}$$
$$\tau^+ \to \pi^+ + \pi^+ + \pi^- \,, \tag{3}$$

decaying into two and three pions respectively. At that time, it seemed the equal masses and lifetimes of these two particles could not just be a coincidence; τ^+ and θ^+ had to be related somehow. Since the pion has negative parity in its ground state, this implied that the final state of the θ^+ decay has positive parity while the final state of the τ^+ decay has negative parity. Yang and Lee proposed that τ^+ and θ^+ were the same particle [10], which is now known as K^+. This assertion implies that the weak nuclear force responsible for the K^+ decay is not symmetric under parity, and so it does not conserve parity. Left-handed and right-handed particles have different weak nuclear interactions.

Lee and Yang proposed experiments that could directly and unambiguously determine whether or not the weak force conserves parity [10]. One of these was carried out by C. S. Wu between 1956 and 1957 [19]. In this experiment, depicted in Fig. 1, ^{60}Co was cooled and then polarized using a magnetic field. The spins of the ^{60}Co nuclei align with the magnetic field, providing a known pseudovector $\mathbf{j}$. When ^{60}Co beta decays, it emits

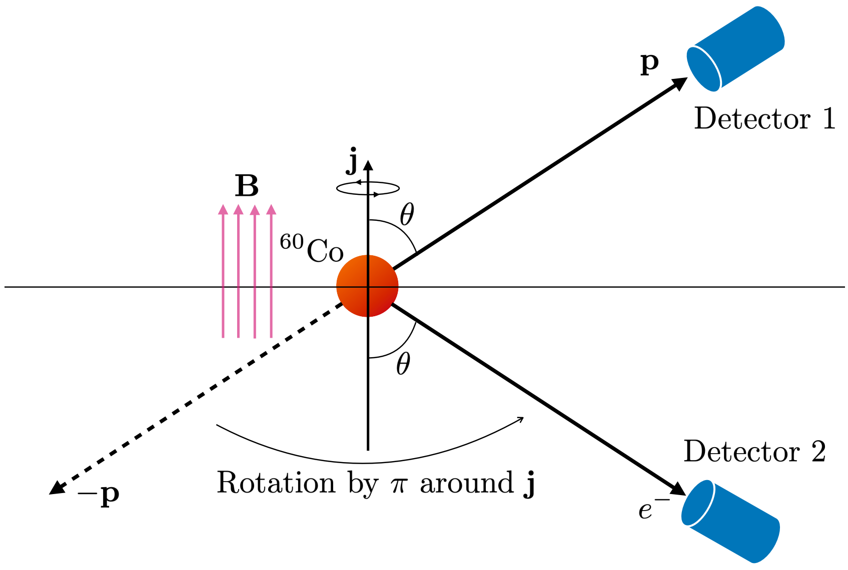

Figure 1: Diagram of C. S. Wu's experiment [19]. A magnetic field $\mathbf{B}$ polarizes the spin $\mathbf{j}$ of a ^{60}Co nucleus. Detector 1 is arranged to detect electrons that emerge from the beta decay event with angle $\mathbf{j} \cdot \mathbf{p} = jp\cos(\theta)$. Detector 2 is arranged to detect electrons that emerge from the beta decay event with angle $\mathbf{j} \cdot \mathbf{p} = -jp\cos(\theta)$. The events detected by detectors 1 and 2 are related through the combination of a parity transformation $\mathbf{p} \to -\mathbf{p}$ and a rotation by an angle π around $\mathbf{j}$.

an electron and an unobserved electron antineutrino. The decay product nucleus, ^{60}Ni, is populated in an excited state that emits two gamma rays before reaching its ground state. The total decay process is,

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e + \gamma + \gamma\,. \tag{4}$$

The gamma rays can be observed and the anisotropy of their angular distribution indicates the level of polarization in the original ^{60}Co. The momentum of the electron can be detected, and used to form the pseudoscalar quantity,

$$\mathbf{p} \cdot \mathbf{j} = pj\cos(\theta), . \tag{5}$$

Here θ is the angle between the orientation of the polarized ^{60}Co and the electron's momentum. Since momentum is a vector, which is odd under parity, and spin is a pseudovector, which is even, $\mathbf{p} \cdot \mathbf{j}$ is parity odd.

A parity transformation takes the azimuthal and polar angles,

$$\mathbb{P} : (\phi, \theta) \mapsto (\phi + \pi, \pi - \theta). \tag{6}$$

The azimuthal angle can be rotated back to its original value without changing the orientation of the polar axis, which is parallel with the fixed vector $\mathbf{j}$ in the experiment. Up to a rotation, parity transforms,

$$\mathbb{P} : \cos(\theta) \mapsto \cos(\pi - \theta)\,. \tag{7}$$

By counting the rate of electrons resulting from the polarized ^{60}Co decays at two angles, θ and $\pi - \theta$, the amount of parity asymmetry can be determined,

$$\alpha = \frac{N(\theta) - N(\pi - \theta)}{N(\theta) + N(\pi - \theta)}\,. \tag{8}$$

Where N is the total number of electrons detected in a fixed amount of time for a fixed amount of ^{60}Co. In Wu's original experiment, the electron detector's angle was fixed and the orientation of the magnetic field was inverted between runs to change the detector's angle.

Wu's experiment clearly indicated a large amount of parity violation. Due to uncertainty in the exact amount of polarization achieved, Wu could only put a lower bound on the magnitude of α, but that lower bound was a staggering 70% [19]. We now know that the value is consistent with 100% parity violation. That is, the weak nuclear force is maximally parity violating.

The sign of α from Wu's experiment is negative, meaning the electron's momentum is anti-aligned with the ^{60}Co spin. The ^{60}Co have spin $J = 5$, and in the magnetic field $J_z = 5$. The resulting ^{60}Ni has $J_z = 4$, so from the conservation of angular momentum, both the electron and the antineutrino must have $J_z = +1/2$. From the anti-alignment of the electron momentum with the nuclear spin, we can deduce that the electron is anti-aligned with its own spin, so it is a left-handed particle. This proves that the weak nuclear force couples to only left-handed particles and right-handed antiparticles, and not right-handed particles or left-handed antiparticles.

Another experiment was conducted at the same time by L. Lederman's group, who had been in contact with Lee, Yang, and Wu about their parity investigations. This experiment involved the decay of pions into muons, which then decay into electrons. Lederman's results were ready for publication before Wu's team had completed all of their tests, and Lederman agreed to coordinate the publication of both experiments since communications with Wu had been crucial to Lederman's experiment. Both articles appear one after another in the same issue of Physical Review [19, 8]. Lee and Yang were awarded the Nobel Prize within a year of these publications.

1.2 Spacetime Symmetries

Parity is a spacetime transformation. To understand its role in physics and the implication of its violation, we first need to understand its role as a candidate spacetime symmetry. In special relativity, the spacetime symmetries are Poincaré transformations, which include both Lorentz transformations and translations. Lorentz transformations are global, homogenous, linear transformations that leave to form of the metric tensor,

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

unchanged. Consider the linear coordinate transformation,

$$x^\mu = \Lambda^\mu{}_\nu \tilde{x}^\nu \,. \tag{10}$$

The metric tensor in the new coordinate system $\tilde{x}^\mu$ is

$$\tilde{\eta}_{\mu\nu} = \frac{\partial x^\rho}{\partial \tilde{x}^\mu} \eta_{\rho\lambda} \frac{\partial x^\lambda}{\partial \tilde{x}^\nu} \tag{11}$$

$$= \Lambda^\rho{}_\mu \eta_{\rho\lambda} \Lambda^\lambda{}_\nu \,. \tag{12}$$

If this transformation is an isometry (it leaves the form of the metric unchanged), then,

$$\eta_{\mu\nu} = \Lambda^{\rho}{}_{\mu}\eta_{\rho\lambda}\Lambda^{\lambda}{}_{\nu}\,. \tag{13}$$

This is the defining property of Lorentz transformations. As a matrix equation, without indices for components, we could simply write it as $\Lambda^T\eta\Lambda = \eta$. Any matrix that satisfies this property is a Lorentz transformation, and the set of these matrices form the Lorentz group.

Taking the determinant of Eq. (13) we find, $\mathrm{Det}(\Lambda)^2 = 1$, so there are two types of Lorentz transformation, ones with determinant +1 and ones with determinant -1. The former are known as *proper* Lorentz transformations. An important subset of the proper Lorentz transformations is the set of matrices that can be parameterized in such a way that the limit of the parameters going to zero recovers the identity matrix. For example, consider rotations around the z-axis,

$$R^{\mu}{}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) & 0 \\ 0 & \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\,. \tag{14}$$

If we take $\phi \to 0$, we recover the identity. Similarly, if we look at a boost by velocity $v^{\mu} = (1, v, 0, 0)$,

$$B^{\mu}{}_{\nu} = \begin{pmatrix} \cosh(w) & \sinh(w) & 0 & 0 \\ \sinh(w) & \cosh(w) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\,, \tag{15}$$

where $v = \tanh(w)$. Taking $v \to 0$ ($w \to 0$) also recovers the 4×4 identity matrix. The subset of matrices that satisfy this property can also be shown to preserve the direction of time. If $\Delta t > 0$ in one reference frame, then $\Delta\tilde{t} > 0$ in another that is related to the first through one of these transformations. Thus, this subgroup is called the *proper orthochronous* Lorentz group. It is the largest subgroup that is not equal to the whole group.

All other Lorentz transformations are obtained by applying a parity transformation,

$$\mathbb{P}^{\mu}{}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\,, \tag{16}$$

a time reversal transformation,

$$\mathbb{T}^{\mu}{}_{\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\,, \tag{17}$$

or both to a proper orthochronous transformation. Both of these discrete transformations have determinant -1.

Asking whether the laws of physics are invariant under parity, time reversal, both, or neither, is another way to ask the question: how Lorentz invariant is the universe? Is the universe invariant under the full Lorentz group, or only invariant under a subgroup of the full group?

At a minimum, the principle of relativity seems to require invariance under proper orthochronous Lorentz transformations. These are the ones that relate the coordinates of observers living in the same universe. If we find two inertial observers, we can always relate their measurements of the same events via a proper orthochronous Lorentz transformation.

The question of parity violation and time reversal is less clear. We cannot imagine a physical observer living in the same universe as us whose coordinates are related to our own through a parity or time reversal transformation. Instead, we must perform a more abstract thought experiment: what would a different universe that is related to our own through a parity transformation or time reversal look like? Since such a universe is not causally connected to our own, we cannot observe the same events and compare our measurements directly, so there would be no inconsistency if the laws of physics violated these discrete transformations.

From particle physics, we now know that the universe does not respect $\mathbb{P}$. There is another discrete spacetime symmetry in quantum field theory known as charge conjugation ($\mathbb{C}$), which swaps particles for their antiparticles, leaving their chirality unchanged. Under this symmetry, a left-handed fermion would be swapped for a left-handed antifermion. The former couples to the weak force, while the latter does not. Thus, Wu's and Lederman's experiments also demonstrate that charge conjugation is violated, as Lee and Yang had already predicted [10, 19, 8].

In 1964, observations of neutral kaon decays by J. Cronin and V. Fitch showed that the combination $\mathbb{CP}$ is also violated [4]. There is a famous theorem, which mathematically proves any quantum field theory that is invariant under the proper orthochronous Lorentz group must be invariant under the combination $\mathbb{CPT}$, known as the CPT theorem. Thus, observations of $\mathbb{CP}$ violation imply $\mathbb{T}$ violation.

Based on these results, the universe appears to be quite stingy with the spacetime symmetries it respects. The laws of physics are invariant under only the bare minimum subgroup of the full Lorentz group required for the principle of relativity to hold. Since this is true on small scales for particle physics, this might also be true on large scales for cosmology. There are several phenomena, such as dark matter, dark energy, and the inflationary physics of the early universe, that could involve parity-violating physics. Since there is no reason these phenomena need to respect the parity symmetry, we should not assume that they do.

1.3 Parity for Fields

1.4 Preliminaries

Fourier transformations of fields are defined,

$$f(\mathbf{k}) = \int \mathrm{d}^3 x e^{-i\mathbf{x}\cdot\mathbf{k}} f(\mathbf{x}) \,, \tag{18}$$

where the same symbol is used for the field and its modes. The two are distinguished by the symbol used for the spatial argument. Inverse Fourier transforms are,

$$f(\mathbf{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{x}\cdot\mathbf{k}} f(\mathbf{k}) \,. \tag{19}$$

Importantly, this means each Dirac delta function in Fourier space is always accompanied by a factor of $(2\pi)^3$. For example, the matter power spectrum is defined as,

$$\langle \delta_{\mathrm{m}}(\tau, \mathbf{k}) \delta_{\mathrm{m}}(\tau, \mathbf{k}') \rangle = (2\pi)^3 \delta_{\mathrm{D}}^{(3)}(\mathbf{k} + \mathbf{k}') P_{\mathrm{mm}}(\tau, k) \,. \tag{20}$$

We will make use of the short-hand notation for Fourier space integrals,

$$\int_{\mathbf{q}} f(\mathbf{q}) \equiv \int \frac{\mathrm{d}^3 q}{(2\pi)^3} f(\mathbf{q}) \,, \tag{21}$$

$$\int\limits_{\mathbf{q},\mathbf{q}'} f(\mathbf{q}) g(\mathbf{q}') \equiv \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{d}^3 q'}{(2\pi)^3} f(\mathbf{q}) g(\mathbf{q}') \,, \tag{22}$$

and so on.

1.5 Scalar Fields

Under parity, a scalar field $f(\mathbf{x})$ transforms as,

$$\mathbb{P} : f(\mathbf{x}) \mapsto f(-\mathbf{x}) \,. \tag{23}$$

Similarly, the modes of the field transform according to

$$\mathbb{P} : f(\mathbf{k}) \mapsto f(-\mathbf{k}) \,. \tag{24}$$

If $f(\mathbf{x})$ is a real-valued field, its modes satisfy $f(-\mathbf{k}) = f(\mathbf{k})^*$. Parity replaces the modes of a field with the complex conjugate of the modes. One immediate implication of this is that the N-point autocorrelation function of the field in Fourier space is mapped to its complex conjugate by the parity transformation,

$$\mathbb{P} : \langle f(\mathbf{k}_1) f(\mathbf{k}_2) ... f(\mathbf{k}_{N-1}) f(\mathbf{k}_N) \rangle \mapsto \langle f(\mathbf{k}_1)^* f(\mathbf{k}_2)^* ... f(\mathbf{k}_{N-1})^* f(\mathbf{k}_N)^* \rangle \,. \tag{25}$$

Therefore, the parity-violating part of any scalar field's N-point spectrum is the imaginary part of that spectrum.

We can immediately conclude that a field's auto power spectrum,

$$\langle f(\tau, \mathbf{k}) f(\tau, \mathbf{k}') \rangle = (2\pi)^3 \delta_{\mathrm{D}}^{(3)}(\mathbf{k} + \mathbf{k}') P(k) \,, \tag{26}$$

cannot be sensitive to parity. The power spectrum is an average over the squared amplitudes of the field's modes: $f(\tau, \mathbf{k}) f(\tau, -\mathbf{k}) = |f(\tau, \mathbf{k})|^2$, which is purely real.

The situation for the bispectrum,

$$\langle f(\mathbf{k}_1) f(\mathbf{k}_2) f(\mathbf{k}_3) \rangle = (2\pi)^3 \delta_{\mathrm{D}}^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3) \,, \tag{27}$$

is less immediately clear. One helpful observation is that we need to form a pseudoscalar quantity to have a parity-odd, scalar-like statistic. Since there are no clear candidates for a fundamental pseudoscalar, we must construct one ourselves, by taking the dot product of a pseudovector and a vector.

We can construct a pseudovector by taking the cross product of two vectors, for example $\mathbf{k}_2 \times \mathbf{k}_3$. Now we need a third vector that is linearly independent of $\mathbf{k}_2$ and $\mathbf{k}_3$ to form the pseudoscalar triple product $\mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)$. However, the Dirac delta function that imposes translational invariance on the bispectrum enforces $\mathbf{k}_1 = -(\mathbf{k}_2 + \mathbf{k}_3)$, so the

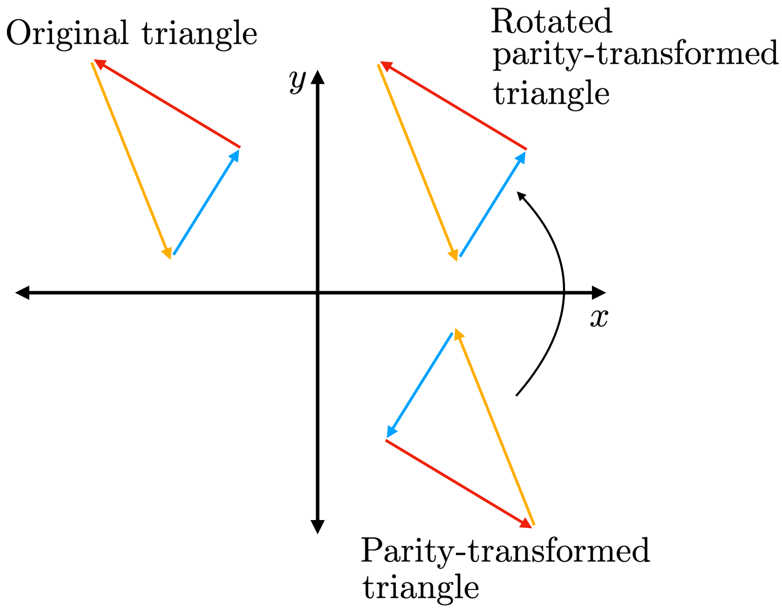

Figure 2: Diagram showing the equivalence of parity transformation and a rotation for a triangle.

triple product must vanish. Rotational invariance implies that there are no other vectors that the bispectrum can depend on, aside from $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$, so there is no way to form a parity-odd three-point function.

A similar line of reasoning is that the three wave vectors of the bispectrum must form a closed triangle. The parity transformation of a triangle in three dimensions is equivalent to a rotation. For example, if the triangle lies in the $k_z = 0$ plane, its parity transformation is the same as a rotation around the k_z-axis by π, as shown in Fig. 2. Thus the bispectrum cannot carry information about parity due to isotropy. This also means models that break isotropy can result in parity-violating bispectra.

The lowest-order correlation function that is sensitive to parity for a scalar-like field is the four-point function or trispectrum,

$$\begin{aligned}\langle f(\mathbf{k}_1)f(\mathbf{k}_2)f(\mathbf{k}_3)f(\mathbf{k}_4)\rangle = {} & (2\pi)^3\delta_{\mathrm{D}}^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4) \\ & \times T(k_1,k_2,k_3,k_4,|\mathbf{k}_1+\mathbf{k}_2|,|\mathbf{k}_1+\mathbf{k}_4|)\,.\end{aligned} \tag{28}$$

In this case, we have three potentially linearly independent vectors to form the pseudoscalar triple product $\mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)$. The trispectrum can be decomposed into its real, parity-even part, and its imaginary, parity-odd part,

$$T = T^{(+)} + iT^{(-)}\,. \tag{29}$$

The parity-odd part must be proportional to the triple product, so we can further parameterize its shape,

$$\begin{aligned}T^{(-)}(k_1,k_2,k_3,k_4,|\mathbf{k}_1+\mathbf{k}_2|,|\mathbf{k}_1+\mathbf{k}_4|) = {} & i\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3) \\ & \times\,\tau^{(-)}(k_1,k_2,k_3,k_4,|\mathbf{k}_1+\mathbf{k}_2|,|\mathbf{k}_1+\mathbf{k}_4|)\,.\end{aligned} \tag{30}$$

The six-dimensional function $\tau^{(-)}$ describes the full shape of the parity-odd trispectrum. Due to translational invariance, the sum of the four wave vectors must vanish, so they

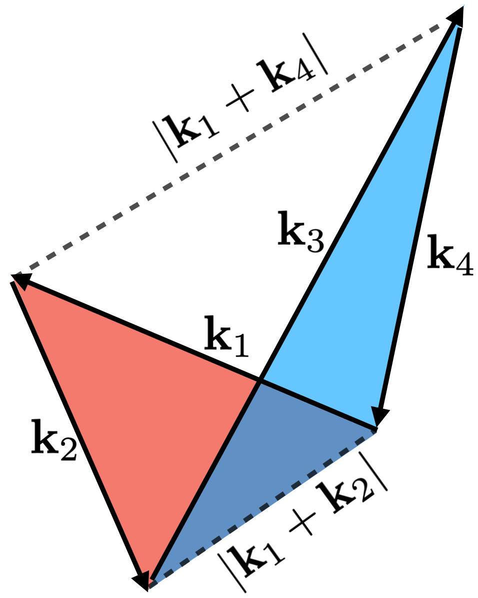

Figure 3: Diagram of a Fourier space tetrahedron representing a possible trispectrum configuration.

form a tetrahedron in Fourier space, as in Fig. 3. The wave vector magnitudes k_1, k_2, k_3, and k_4 form four sides of the tetrahedron and the other two sides are diagonals: $|\mathbf{k}_1 + \mathbf{k}_2| = |\mathbf{k}_2 + \mathbf{k}_3|$ and $|\mathbf{k}_1 + \mathbf{k}_4| = |\mathbf{k}_2 + \mathbf{k}_3|$. The third diagonal, $|\mathbf{k}_1 + \mathbf{k}_3| = |\mathbf{k}_2 + \mathbf{k}_4|$ is not independent,

$$|\mathbf{k}_1 + \mathbf{k}_3|^2 = k_1^2 + k_2^2 + k_3^2 + k_4^2 - |\mathbf{k}_1 + \mathbf{k}_2|^2 - |\mathbf{k}_1 + \mathbf{k}_4|^2 \,. \tag{31}$$

Since the left-hand side of Eq. (28) is symmetric under the interchange of any two wave vectors, the function $\tau^{(-)}$ must be antisymmetric under the interchange of any two sides k_i and k_j.

As an example, consider a random Gaussian field $\phi(\mathbf{x})$ with a power spectrum $P(k)$. We can construct a non-Gaussian field that has a term of order ϕ^3 and that involves a triple product of derivatives. For that triple product not to vanish, we need to transform the field so that the derivatives all act on distinct fields. For example, consider transforming the modes of the field,

$$\phi^{(\alpha)}(\mathbf{k}) = k^\alpha \phi(\mathbf{k}) \,. \tag{32}$$

Here α is a constant exponent. We can construct the non-Gaussian field,

$$\Phi(\mathbf{x}) = \phi(\mathbf{k}) + g_- \nabla\phi^{(\alpha)}(\mathbf{x}) \cdot \left[\nabla\phi^{(\beta)}(\mathbf{x}) \times \nabla\phi^{(\gamma)}(\mathbf{x})\right] , \tag{33}$$

with g_- being a constant that determines the level of non-Gaussianity in the field. We will assume that there is only a small amount of non-Gaussianity and keep only terms that are linear in g_- in what follows. The modes of the non-Gaussian field are

$$\begin{aligned}\Phi(\mathbf{k}) = \phi(\mathbf{k}) - i g_- \int_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3} & (2\pi)^3 \delta_{\mathrm{D}}^{(3)}(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2 - \mathbf{q}_3)\, \mathbf{q}_1 \cdot (\mathbf{q}_2 \times \mathbf{q}_3)\, q_1^\alpha q_2^\beta q_3^\gamma \\ & \times \phi(\mathbf{q}_1)\phi(\mathbf{q}_2)\phi(\mathbf{q}_3) \,. \end{aligned} \tag{34}$$

We will compute the leading trispectrum for this field. The full trispectrum is computed from the four-point correlator $\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle$. If we replace the first three Φ with ϕ, we obtain one contribution that is linear in g_-,

$$\langle\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle = -ig_- \int_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3} (2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)\,\mathbf{q}_1\cdot(\mathbf{q}_2\times\mathbf{q}_3)\,q_1^\alpha q_2^\beta q_3^\gamma \\ \times\langle\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\phi(\mathbf{q}_1)\phi(\mathbf{q}_2)\phi(\mathbf{q}_3)\rangle\,. \tag{35}$$

The trispectrum results from the six-point function of the Gaussian field being integrated down to a four-point function.

We could compute the full six-point function of ϕ using Wick's theorem. However, this would result in many *disconnected* terms that vanish. For example, one such term would be,

$$\langle\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\rangle\langle\phi(\mathbf{k}_3)\phi(\mathbf{q}_1)\rangle\langle\phi(\mathbf{q}_2)\phi(\mathbf{q}_3)\rangle = (2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{k}_2)P(k_1) \\ \times(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_3+\mathbf{q}_1)P(k_3)(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{q}_2+\mathbf{q}_3)P(q_1)\,. \tag{36}$$

In this term, we have $\mathbf{q}_2=-\mathbf{q}_3$. But these two wave vectors are in a triple product in Eq. (35), so if they are parallel this term will vanish. In general, all terms where two $\mathbf{q}$'s are in the same Dirac delta function will vanish.

The only terms that we need to compute are the ones where each $\phi(\mathbf{k}_i)$ is contracted with a $\phi(\mathbf{q}_i)$. One such term is

$$\langle\phi(\mathbf{k}_1)\phi(\mathbf{q}_1)\rangle\langle\phi(\mathbf{k}_2)\phi(\mathbf{q}_2)\rangle\langle\phi(\mathbf{k}_3)\phi(\mathbf{q}_3)\rangle = (2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{q}_1)P(k_1) \\ \times(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{q}_2+\mathbf{k}_2)P(k_2)(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_3+\mathbf{q}_3)P(q_1)\,. \tag{37}$$

This sets $\mathbf{q}_i=-\mathbf{k}_i$ for all wave vectors after integration and we find one term in Eq. (35),

$$(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)i\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)\,g_-k_1^\alpha P(k_1)k_2^\beta P(k_2)k_3^\gamma P(k_3)\,. \tag{38}$$

This has all of the features of a parity-odd trispectrum:

- Dirac delta function for translational invariance
- Purely imaginary
- Pseudoscalar triple product
- Shape is not symmetric under interchange of any two $\mathbf{k}$'s

We can obtain another term by interchanging k_1 with k_2. This is the term in the Wick expansion where $\phi(\mathbf{k}_1)$ is contracted with $\phi(\mathbf{q}_2)$ and $\phi(\mathbf{k}_2)$ is contracted with $\phi(\mathbf{q}_1)$. In this case we get $k_1^\beta k_2^\alpha$ instead of $k_1^\alpha k_2^\beta$ and we get the triple product $\mathbf{k}_2\cdot(\mathbf{k}_1\times\mathbf{k}_3) = -\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)$, so overall this term is,

$$-(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)i\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)\,g_-k_1^\beta P(k_1)k_2^\alpha P(k_2)k_3^\gamma P(k_3)\,. \tag{39}$$

Repeating this reasoning, there are $3!=6$ terms that are obtained by permuting $\{k_1,k_2,k_3\}$. Terms with an even permutation have a positive sign and odd permutations have a negative sign, so the whole contribution from Eq. (35) is

$$(2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)i\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)\,g_-P(k_1)P(k_2)P(k_3) \\ \left(\left(k_1^\alpha k_2^\beta-k_1^\beta k_2^\alpha\right)k_3^\gamma+\left(k_2^\alpha k_3^\beta-k_2^\beta k_3^\alpha\right)k_1^\gamma+\left(k_3^\alpha k_1^\beta-k_3^\beta k_1^\alpha\right)k_2^\gamma\right) \tag{40}$$

The full leading trispectrum has three more contributions. We could have taken the non-Gaussian term from $\Phi(\mathbf{k}_1)$, $\Phi(\mathbf{k}_2)$, or $\Phi(\mathbf{k}_3)$ instead of from $\Phi(\mathbf{k}_4)$. These contributions have the same form as the above expression, interchanging on $\mathbf{k}_i$ for $\mathbf{k}_4$. Consider the term that swaps $\mathbf{k}_1$ with $\mathbf{k}_4$. It has a triple product $\mathbf{k}_4 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)$. From the Dirac delta function, $\mathbf{k}_4 = -\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3$, so $\mathbf{k}_4 \cdot (\mathbf{k}_2 \times \mathbf{k}_3) = -\mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)$. That is, we can always put the triple product in the form $\mathbf{k}_1 \cdot (\mathbf{k}_2 \times \mathbf{k}_3)$ up to a sign. Using Eq. (30), we can isolate the trispectrum shape function τ_- by stripping off the delta function and triple product,

$$\tau_-(k_1, k_2, k_3, k_4) = g_- \left(k_1^\alpha k_2^\beta k_3^\gamma P(k_1)P(k_2)P(k_3) \mp 23 \text{ permutations} \right) , \tag{41}$$

where the permutations are of $\{k_1, k_2, k_3, k_4\}$, even permutations receiving a positive sign and odd permutations receiving a negative sign. Due to the antisymmetry of the shape, the trispectrum vanishes unless all three exponents, α, β, and γ are distinct. In this example, the trispectrum has no dependence on the diagonals, so it is only a four-dimensional function.

This kind of template can be used to set up initial conditions for N-body simulations to investigate how the presence of a primordial parity violating trispectrum affects large-scale structure (LSS), as was done in Ref. [6].

1.6 Vector Fields

Under parity, a vector field $\mathbf{V}(\mathbf{x})$ transforms,

$$\mathbb{P} : \mathbf{V}(\mathbf{x}) \mapsto -\mathbf{V}(-\mathbf{x}) . \tag{42}$$

For the modes of the field, we can choose an orthonormal basis $\mathbf{e}_i(\mathbf{k})$ such that,

$$\mathbf{V}(\mathbf{k}) = \mathbf{e}_i(\mathbf{k}) V^i(\mathbf{k}) , \tag{43}$$

and

$$\mathbf{e}_i(\mathbf{k}) \cdot \mathbf{e}_j(\mathbf{k})^* = \delta_{ij} . \tag{44}$$

One natural candidate for a basis vector is the unit vector associated with the gradient operator, which corresponds to longitudinal modes,

$$\mathbf{e}_\parallel(\mathbf{k}) = i\frac{\mathbf{k}}{k} , \tag{45}$$

Next, we need to choose a basis for the transverse space perpendicular to $\mathbf{e}_\parallel(\mathbf{k})$. To do this, pick any unit vector $i\mathbf{n}(\mathbf{k})$ that is not parallel with $\mathbf{k}$. The factor of i is included so that $\mathbf{n}(\mathbf{x})$ is a real-valued field if $\mathbf{n}(\mathbf{k})$ is real and $\mathbf{n}(-\mathbf{k}) = -\mathbf{n}(\mathbf{k})$. Then

$$\mathbf{e}_+(\mathbf{k}) = -\frac{\mathbf{k} \times \mathbf{n}(\mathbf{k})}{\sqrt{k^2 - (\mathbf{k} \cdot \mathbf{n})^2}} , \tag{46}$$

is a unit vector perpendicular to $\mathbf{e}_\parallel(\mathbf{k})$, and so is,

$$\mathbf{e}_-(\mathbf{k}) = -\frac{i\mathbf{k} \times (\mathbf{k} \times \mathbf{n}(\mathbf{k}))}{k\sqrt{k^2 - (\mathbf{k} \cdot \mathbf{n})^2}} . \tag{47}$$

The basis formed by $\{\mathbf{e}_{\parallel}(\mathbf{k}), \mathbf{e}_{+}(\mathbf{k}), \mathbf{e}_{-}(\mathbf{k})\}$ is the parity basis, since $\mathbf{e}_{\pm}(\mathbf{k})$ are explicitly even $(+)$ and odd $(-)$ under parity if $i\mathbf{n}(\mathbf{k})$ is taken to be a vector. Then we can decompose the vector field's modes,

$$\mathbf{V}(\mathbf{k}) = \mathbf{e}_{\parallel}(\mathbf{k})V^{\parallel}(\mathbf{k}) + \mathbf{e}_{+}(\mathbf{k})V^{-}(\mathbf{k}) + \mathbf{e}_{-}(\mathbf{k})V^{+}(\mathbf{k})\,. \tag{48}$$

Here we label the components $V^{\mp}(\mathbf{k})$ with the opposite sign from their associated basis vectors $\mathbf{e}_{\pm}(\mathbf{k})$ since the components have opposite transformation properties to the basis vectors under parity. That is,

$$\mathbb{P}: \{\mathbf{e}_{\parallel}(\mathbf{k}), \mathbf{e}_{+}(\mathbf{k}), \mathbf{e}_{-}(\mathbf{k})\} \mapsto \{-\mathbf{e}_{\parallel}(-\mathbf{k}), \mathbf{e}_{+}(-\mathbf{k}), -\mathbf{e}_{-}(-\mathbf{k})\}\,. \tag{49}$$

$$\mathbb{P}: \{V^{\parallel}(\mathbf{k}), V^{-}(\mathbf{k}), V^{+}(\mathbf{k})\} \mapsto \{V^{\parallel}(-\mathbf{k}), -V^{-}(-\mathbf{k}), V^{+}(-\mathbf{k})\}\,. \tag{50}$$

We can see that $\langle V^{+}(\mathbf{k})V^{-}(\mathbf{k}')\rangle$ is parity odd, so for a vector field we can access parity information at the level of the power spectrum. To gain more insight, consider the way the curl operator, or $\mathbf{e}_{\parallel}$ acts on the transverse basis vectors,

$$i\frac{\mathbf{k}}{k} \times \mathbf{e}_{\pm}(\mathbf{k}) = \pm\mathbf{e}_{\mp}(\mathbf{k})\,. \tag{51}$$

This follows directly from the definitions of the basis vectors and the fact that they form an orthonormal basis. We can use this to form new basis vectors that diagonalize the curl operator,

$$\mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) = \frac{1}{\sqrt{2}}\left(\mathbf{e}_{+}(\mathbf{k}) \mp i\mathbf{e}_{-}(\mathbf{k})\right)\,. \tag{52}$$

Then we find,

$$\frac{\mathbf{k}}{k} \times \mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) = \pm\mathbf{e}_{\mathrm{R/L}}(\mathbf{k})\,. \tag{53}$$

The modes that have a positive curl are right-handed and the modes with negative curl are left-handed, hence the labels of these basis vectors. Under parity, these transform as,

$$\mathbb{P}: \mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) \mapsto \mathbf{e}_{\mathrm{L/R}}(-\mathbf{k})\,, \tag{54}$$

so parity swaps the right-handed and left-handed modes.

The parity odd power spectrum can be rewritten,

$$\langle V^{+}(\mathbf{k})V^{-}(\mathbf{k}')\rangle = \frac{1}{2}\langle V^{\mathrm{R}}(\mathbf{k})V^{\mathrm{R}}(\mathbf{k}')\rangle - \frac{1}{2}\langle V^{\mathrm{L}}(\mathbf{k})V^{\mathrm{L}}(\mathbf{k}')\rangle \tag{55}$$

so the parity-odd cross-power spectrum from the transverse components of the vector field measures the difference between power in right-handed modes and left-handed modes. If the universe violates parity, then the universe distinguishes between right and left.

2 Generating Parity Violation in the Early Universe

In this section, we will study one possible mechanism for generating primordial parity violation during inflation. As an example, we will analyze the axion-U(1) model. This section summarizes some results that were first calculated in Refs. [1, 2].

2.1 Inflation

Rather than give an overview of all of the details of slow-roll inflation, here we will consider only the most essential feature of this model: the requirement for an early epoch of near-exponential expansion. If the universe is in a state where it expands exponentially,

$$a(t) \simeq e^{Ht} . \tag{56}$$

Then the Hubble rate H is nearly constant. In the absence of spatial curvature, the Friedmann equation,

$$H^2 = \frac{1}{3M_\mathrm{p}^2}\bar{\rho} , \tag{57}$$

requires that the mean energy density of the universe, $\bar{\rho}$ is nearly constant. Here M_p is the reduced Planck mass. The continuity equation,

$$\dot{\bar{\rho}} + 3H\bar{\rho}\,(1+w) = 0 \tag{58}$$

where the equation of state $w \equiv \bar{P}/\bar{\rho}$ is the ratio of pressure to energy density, implies that $w \simeq -1$. The dot denotes the derivative with respect to time.

Suppose the energy density is dominated by some scalar(-like) field $\varphi(t,\mathbf{x}) = \bar{\varphi}(t) + \delta\varphi(t,\mathbf{x})$. If the fluctuations of φ are small, then its total energy density is

$$\bar{\rho}_\varphi = \frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi}) , \tag{59}$$

and its pressure is

$$\bar{P}_\varphi = \frac{1}{2}\dot{\bar{\varphi}}^2 - V(\bar{\varphi}) , \tag{60}$$

with $V(\varphi)$ the field's potential energy density. The equation of state is,

$$w_\varphi = \frac{\frac{1}{2}\dot{\bar{\varphi}}^2 - V(\bar{\varphi})}{\frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi})} . \tag{61}$$

If the scalar field's potential energy dominates, $V(\bar{\varphi}) \gg \dot{\bar{\varphi}}^2$, then $w_\varphi \simeq -1$. In summary, we can achieve near-exponential expansion if

- the energy density of the universe is dominated by a scalar field
- that scalar field's energy density is potential dominated.

Generically, if these conditions are satisfied as some initial condition, the scalar field would quickly evolve to minimize its potential energy and the exponential expansion would cease. To have sustained exponential expansion, we also require that the scalar field evolves slowly. Usually this is achieved by concocting a potential that is flat enough, so that the scalar field approaches some terminal velocity $\dot{\bar{\varphi}} \sim$ constant. However, other scenarios such as noncanonical kinetic terms and higher derivative interactions can also achieve the same effect.

The equation of motion for the scalar field's background is,

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} = -V_{,\varphi}(\bar{\varphi}) , \tag{62}$$

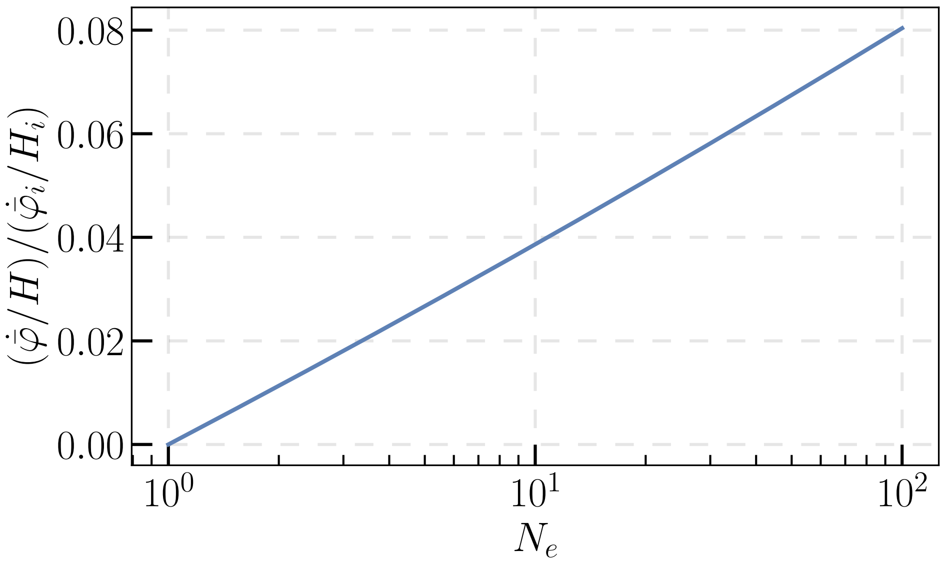

Figure 4: Evolution of the inflaton derivative divided by the Hubble rate for an α-attractor inflaton potential that agrees well with the CMB measurements of the amplitude and the spectral tilt of the primordial power spectrum.

where $V_{,\varphi}(\bar{\varphi})$ is the slope of the potential. In the standard slow-roll scenario, assuming the potential is sufficiently flat, we neglect the acceleration so the background equation of motion reduces to,

$$\frac{\dot{\bar{\varphi}}}{H} = -\frac{V_{,\varphi}(\bar{\varphi})}{3H^2} \,. \tag{63}$$

Using the Friedmann equation,

$$\frac{\dot{\bar{\varphi}}}{H} = -M_{\mathrm{p}}^2 \frac{V_{,\varphi}(\bar{\varphi})}{V(\bar{\varphi})} \,. \tag{64}$$

To leading order, this quantity is nearly constant, but it can evolve by 10 percent throughout inflation, as shown in Fig. 4. This will be quantitatively important below, as the generation of parity violation will be exponentially sensitive to $\dot{\bar{\varphi}}/H$. However, the evolution of this quantity does not change the qualitative features of dynamical parity violation in the axion–U(1) model, so we will treat it as a constant.

We will use conformal time,

$$\tau = \int_{-\infty}^{t} \mathrm{d}t \, a(t) \,, \tag{65}$$

when analyzing the evolution of fluctuations during inflation. If the universe expands exponentially (de Sitter space), then

$$\tau = -\frac{1}{Ha(\tau)} \,, \tag{66}$$

$$= -\frac{1}{\mathcal{H}(\tau)} \,. \tag{67}$$

Here $\mathcal{H} = aH$ is the conformal Hubble rate. The conformal time spans $\tau \in (-\infty, 0)$, where $\tau \to 0$ is the asymptotic future. In these coordinates the metric has the form $g_{\mu\nu} = a(\tau)^2 \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric.

2.1.1 The Axion-U(1) Model

In this subsection, we will describe the Axion-U(1) model in expanding spacetime. We will refer to the U(1) field as the *electromagnetic* field. However, it does not need to correspond to the electromagnetic field of the stand model.

The fundamental quantity of electromagnetism is the Maxwell tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{68}$$

where the electromagnetic potential 4-vector A_μ has components $(-\phi, \mathbf{A})$. The electric field is related to $F_{\mu\nu}$ though

$$\begin{aligned} F_{0i} &= A_i' + \nabla_i \phi & (69)\\ &= -E_i \,. & (70)\end{aligned}$$

The magnetic field is related,

$$\begin{aligned} F_{ij} &= \nabla_i A_j - \nabla_j A_i & (71)\\ &= \epsilon_{ijk} B_k \,. & (72)\end{aligned}$$

The standard Lagrangian of the electromagnetic field is,

$$\begin{aligned} \mathcal{L}_{U(1)} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,, & (73)\\ &= \frac{a^{-4}}{2}\left(|\mathbf{E}|^2 - |\mathbf{B}|^2\right) . & (74)\end{aligned}$$

Using this, we can find the energy density and isotropic pressure,

$$\begin{aligned} \rho_{U(1)} &= \frac{a^{-4}}{2}\left(|\mathbf{E}|^2 + |\mathbf{B}|^2\right) , & (75)\\ P_{U(1)} &= \frac{a^{-4}}{6}\left(|\mathbf{E}|^2 + |\mathbf{B}|^2\right) . & (76)\end{aligned}$$

Although the electric field is a parity-odd vector and the magnetic field is a parity-even pseudovector, the Lagrangian is explicitly parity even since it involves only the magnitudes of these fields.

A possible generalization of this construction is to include the dual electromagnetic tensor,

$$\tilde{F}_{\mu\nu} = \frac{\sqrt{-g}}{2} \epsilon_{\mu\nu\rho\lambda} F^{\rho\lambda} \,. \tag{77}$$

where $\epsilon_{\mu\nu\rho\lambda}$ is totally antisymmetric and $\epsilon_{0123} = 1$. We find that the dual Maxwell tensor swaps the electric and magnetic fields in the following sense,

$$\begin{aligned} \tilde{F}_{0i} &= B_i \,, & (78)\\ \tilde{F}_{ij} &= \epsilon_{ijk} E_k \,. & (79)\end{aligned}$$

This means that,

$$-\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = a^{-4} \mathbf{E} \cdot \mathbf{B} \,, \tag{80}$$

which is parity odd. Including this in the Lagrangian would seem to explicitly violate parity, but in fact, this can be shown to be a total (covariant) derivative, so it does not affect the classical equations of motion.

We can include a term like this by coupling it to a pseudoscalar field φ,

$$\mathcal{L}_{\rm CS} = -\frac{g}{4}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu} \,. \tag{81}$$

Here g is the coupling constant between φ and the U(1) field, it has dimensions of $(\text{mass})^{-1}$. This is known as a Chern-Simons coupling between the U(1) electromagnetic gauge field and the pseudoscalar φ, which in this context is called an *axion-like* field, or sometimes just an *axion*. As long as φ is a pseudoscalar, the Chern-Simons term is explicitly parity invariant, since it is the product of two parity-odd quantities. However, as we will see below, this coupling can dynamically break parity.

A straightforward method for figuring out how the Chern-Simons term alters the Maxwell equations is to factor the two terms involving $F_{\mu\nu}$ in the Lagrangian,

$$-\frac{1}{4}F_{\mu\nu}\big(F^{\mu\nu} + g\varphi\tilde{F}^{\mu\nu}\big) \,. \tag{82}$$

Then the terms in the parentheses have contributions of the form,

$$F^{0i} + g\varphi\tilde{F}^{0i} = a^{-4}\left(E^i - g\varphi B^i\right) \,, \tag{83}$$

$$F^{ij} + g\varphi\tilde{F}^{ij} = a^{-4}\epsilon^{ijk}\left(B_k + g\varphi E_k\right) \,. \tag{84}$$

The equations of motion, the sourced Maxwell equations (Coulomb's law and the Ampère-Maxwell law), are obtained by varying Eq. (82) by the gauge potential A_μ. We can obtain those equations by varying only the gauge potential in the $F_{\mu\nu}$ outside of the parentheses. So we can replace $\mathbf{E} \to \mathbf{E} - g\varphi\mathbf{B}$ and $\mathbf{B} \to \mathbf{B} + g\varphi\mathbf{E}$ in the standard sourced Maxwell equations to obtain the Axion-U(1) equations.

$$\nabla \cdot (\mathbf{E} - g\varphi\mathbf{B}) = 0 \,, \tag{85}$$

$$(\mathbf{E} - g\varphi\mathbf{B})' - \nabla \times (\mathbf{B} + g\varphi\mathbf{E}) = 0 \,. \tag{86}$$

The unsourced Maxwell equations are unchanged,

$$\nabla \cdot \mathbf{B} = 0 \,, \tag{87}$$

$$\mathbf{B}' - \nabla \times \mathbf{E} = 0 \,. \tag{88}$$

Using these, we can simplify the sourced equations,

$$\nabla \cdot \mathbf{E} = g\nabla\varphi \cdot \mathbf{B} \,, \tag{89}$$

$$\mathbf{E}' + \nabla \times \mathbf{B} = g\left(\varphi'\mathbf{B} + \nabla\varphi \times \mathbf{E}\right) \,. \tag{90}$$

Substituting the left-hand side of the above equation for the definition of the electric and magnetic fields in terms of the gauge potential,

$$\mathbf{E}' + \nabla \times \mathbf{B} = -\mathbf{A}'' + \nabla^2\mathbf{A} + \nabla\left(\phi' + \nabla \cdot \mathbf{A}\right) \,. \tag{91}$$

Choose a gauge where $\varphi' + \nabla \cdot \mathbf{A} = 0$,

$$\mathbf{E}' + \nabla \times \mathbf{B} = -\mathbf{A}'' + \nabla^2\mathbf{A} \,. \tag{92}$$

We also find

$$\nabla \cdot \mathbf{E} = -\nabla \cdot \mathbf{A}' - \nabla^2 \phi\,, \tag{93}$$
$$= \phi'' - \nabla^2 \phi\,. \tag{94}$$

Defining the D'Alembert wave operator $\Box f \equiv -f'' + \nabla^2 f$, the equations of motion become

$$-\Box \phi = g \nabla \varphi \cdot \mathbf{B}\,, \tag{95}$$
$$-\Box \mathbf{A} = -g\left(\varphi' \mathbf{B} + \nabla \varphi \times \mathbf{E}\right)\,. \tag{96}$$

2.1.2 Linearized Axion-U(1) Inflation

We want to solve the linearized version of Eqs. (95–96) assuming φ is the inflaton. First, note that $\nabla \varphi$ and $\mathbf{B} = \nabla \times \mathbf{A}$ are both linear in fluctuations, so Eq. (95) indicates that the sourced fluctuations of ϕ are at least second order. We can thus neglect the scalar electromagnetic potential. The equation for the linearized vector potential becomes,

$$-\Box \mathbf{A} = -g \bar{\varphi}' \nabla \times \mathbf{A}\,. \tag{97}$$

In Fourier space, we can decompose the vector potential into its left and right modes,

$$\mathbf{A}(\tau, \mathbf{k}) = \mathbf{e}_{\mathrm{R}}(\mathbf{k}) A_{\mathrm{R}}(\tau, \mathbf{k}) + \mathbf{e}_{\mathrm{L}}(\mathbf{k}) A_{\mathrm{L}}(\tau, \mathbf{k})\,, \tag{98}$$

where the right and left polarization vectors satisfy Eq. (53). Using,

$$\bar{\varphi}' = a \dot{\bar{\varphi}}\,, \tag{99}$$
$$= -\frac{\dot{\bar{\varphi}}}{H\tau}\,. \tag{100}$$

and defining,

$$\xi \equiv \frac{g \dot{\bar{\varphi}}}{2H}\,, \tag{101}$$

We find the equation of motion for the right and left modes of the gauge field,

$$A_{\mathrm{R/L}}(\tau, \mathbf{k})'' + k^2 \left(1 \pm \frac{2\xi}{k\tau}\right) A_{\mathrm{R/L}}(\tau, \mathbf{k}) = 0\,. \tag{102}$$

This equation can be put in a familiar form by letting $x = -k\tau$ and $A_{\mathrm{R/L}}(\tau, \mathbf{k}) = f_{\mathrm{R/L}}(x)$,

$$\frac{\mathrm{d}^2 f_{\mathrm{R/L}}}{\mathrm{d}x^2} + \left(1 \mp \frac{2\xi}{x}\right) f_{\mathrm{R/L}} = 0\,. \tag{103}$$

To remember where we have most likely encountered this differential equation before, remember that in quantum mechanics the wavefunction for a charged particle of charge q and mass m in the presence of a Coulomb potential from a charge Q can be decomposed through partial wave expansion and separation of variables. Then the Schrödinger equation for the radial wave function is,

$$\frac{\mathrm{d}^2 R_\ell}{\mathrm{d}r^2} + \left(1 - \frac{2\eta}{r} - \frac{\ell(\ell+1)}{r^2}\right) R_\ell = 0\,. \tag{104}$$

where $\eta = mqQ\alpha/k$ and α is the fine structure constant. By comparing with Eq. (103), we see that the right and left modes of the gauge field in axion-U(1) inflation satisfy the radial Coulomb wavefunction equation with $\ell = 0$, $\eta = \pm\xi$, and $r = -k\tau$.

Notice that this means asymptotically far from the center of the potential, $r \to \infty$, corresponds to asymptotically far in the past $\tau \to -\infty$ in our case. This makes sense, since for the Coulomb wave function, the charged particle is essentially a freely propagating particle with plane wave solutions for its wave function in this regime. Similarly, in our case, the right and left modes of the photon field have plane wave solutions in the asymptotic past, which correspond to vacuum fluctuations deep inside the horizon. This can be seen directly from Eq. (102) in the limit $\tau \to -\infty$,

$$A_{\mathrm{R/L}}(\tau, \mathbf{k})'' + k^2 A_{\mathrm{R/L}}(\tau, \mathbf{k}) = 0\,, \tag{105}$$

which has solutions,

$$A_{\mathrm{R/L}}(\tau, \mathbf{k})'' \propto e^{\pm ik\tau}\,. \tag{106}$$

Importantly, in this regime, the right and left modes behave the same. We can set up initial conditions that do not distinguish between right and left photons and therefore are even under parity. That is, we can assume that inflaton begins with the fields initialized as vacuum fluctuations, and therefore they have no parity-violating components to begin with. After evolution under Eq. (102), this party invariant state will evolve to a state that clearly distinguishes between right and left, as the right and left modes undergo very different evolution in the regime where $\tau \to 0$.

The solutions are expressed in terms of Coulomb Wave functions F_0 and G_0,

$$A_{\mathrm{R/L}}(\tau, \mathbf{k}) \propto \Big(G_0(\pm\xi, -k\tau) + iF_0(\pm\xi, -k\tau)\Big)\,. \tag{107}$$

Unfortunately, to go further and access the degree of parity violation dynamically generated in this model we must venture into the territory of special functions and their asymptotic expansions. It is difficult to determine how the amplitudes of the right and left modes differ at late times if they begin the same. To gain some insight here, we will make use of a rather crude but instructive approximation. We will analyze the solution in the asymptotic regimes where $x \to \infty$ and $x \to 0$, and then extrapolate both of these to match their amplitudes when $x \simeq 2\xi$. This will give a rough estimate of how different the amplitudes of the left and right modes have become at late times.

We already know that in early times the modes of both helicities have the same amplitude. To be concrete we will say,

$$A_{\mathrm{R/L}} = A_i e^{-ik\tau}\,, \tag{108}$$

in the limit $\tau \to -\infty$. Next, we assume that $g > 0$. Making the opposite assumption simply swaps the behaviour of the right and left modes. For the left-handed modes, in the $\tau \to 0$ limit we have,

$$\frac{\mathrm{d}^2 A_{\mathrm{L}}}{\mathrm{d}x^2} + \frac{2\xi}{x} A_{\mathrm{L}} = 0\,, \tag{109}$$

which is solved by

$$A_{\mathrm{L}} = \sqrt{2\xi x}\left(A_{\mathrm{L}J} J_1\left(2\sqrt{2\xi x}\right) + A_{\mathrm{L}Y} Y_1\left(2\sqrt{2\xi x}\right)\right)\,, \tag{110}$$

where J_1 and Y_1 are Bessel functions of the first and second kind. At early times, $-k\tau = x$ is large. When the argument of the Bessel functions is large,

$$J_1(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{3}{4}\right) , \tag{111}$$

$$Y_1(z) = \sqrt{\frac{2}{\pi z}} \sin\left(z - \frac{3}{4}\right) , \tag{112}$$

Since the initial vacuum fluctuations are random, we are equally likely to source either of the J_1 or Y_1 modes, so to match the expected amplitude from Eq. (108) at $x = 2\xi$, we take

$$|A_{\mathrm{L}Y}|^2 = |A_{\mathrm{L}J}|^2 = |A_i|^2 \frac{\pi}{4\xi} \tag{113}$$

At late times, $-k\tau = x$ is small and approaching zero from below. In the small x limit,

$$J_1(z) \simeq \frac{z}{2} , \tag{114}$$

$$Y_1(z) \simeq -\frac{2}{\pi z} , \tag{115}$$

so the J_1 term is decaying while the Y_1 is growing. We approximate

$$|A_{\mathrm{L}}(x \to 0)| \simeq \frac{|A_i|}{\sqrt{4\xi}} . \tag{116}$$

The left modes saturate, and their amplitude approaches a constant, which is suppressed by a factor of $\sim \xi^{-1/2}$ relative to the initial amplitude.

The right handed-modes satisfy,

$$\frac{\mathrm{d}^2 A_{\mathrm{R}}}{\mathrm{d}x^2} - \frac{2\xi}{x} A_{\mathrm{R}} = 0 , \tag{117}$$

which has the solution

$$A_{\mathrm{R}} = \sqrt{2\xi x} \left(A_{\mathrm{R}I} I_1 \left(2\sqrt{2\xi x} \right) + A_{\mathrm{R}K} K_1 \left(2\sqrt{2\xi x} \right) \right) , \tag{118}$$

where I_1 and K_1 are the modified Bessel functions of the first and second kind. When the arguments of these Bessel functions are larger than 1,

$$I_1(z) \simeq \sqrt{\frac{1}{2\pi z}} e^{z} \tag{119}$$

$$K_1(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z} . \tag{120}$$

We set the expected amplitude for each component to be equal, and match it to the plane wave solution at the transition when $x = 2\xi$. Then

$$|A_{\mathrm{R}I}| = \sqrt{\frac{2\pi}{\xi}} e^{-4\xi} |A_i| , \tag{121}$$

$$|A_{\mathrm{R}K}| = \sqrt{\frac{2}{\pi\xi}} e^{4\xi} |A_i| . \tag{122}$$

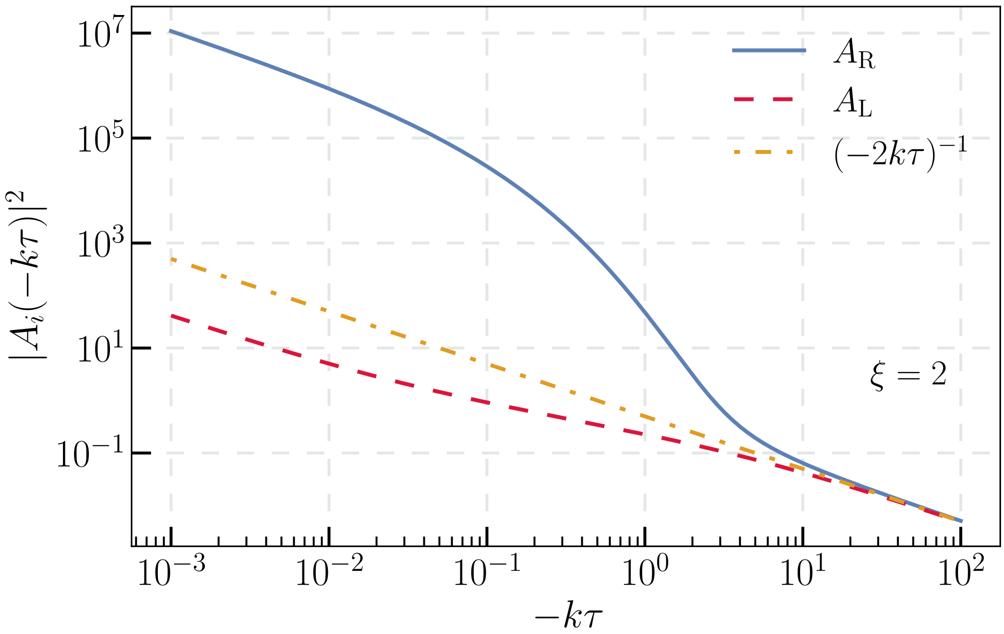

Figure 5: Amplitude of the right and left modes of the gauge field during Axion-U(1) inflation. The blue curve, for the right-handed modes are exponentially enhanced $-k\tau \simeq 2\xi = 4$ and saturate around $-k\tau \simeq 1/(8\xi) = 6.25 \times 10^{-2}$. The left-handed modes, shown as the dashed red curve, are mildly suppressed across this same regime. The yellow dot-dashed line shows the expected vacuum fluctuations in the absence of the axion-like field for reference.

In the future, when $x \to 0$, the behaviour of the modified Bessel functions becomes,

$$I_1(z) \simeq \frac{z}{2} \tag{123}$$

$$K_1(z) \simeq \frac{1}{z}\,. \tag{124}$$

so again we find a decaying solution, I_1 and a growing solution, K_1. We find the amplitude for the right modes asymptotes to,

$$|A_{\rm R}(x \to 0)| \simeq \frac{1}{\sqrt{2\pi\xi}} e^{4\xi} |A_i|\,. \tag{125}$$

Based on this reasoning, we find an expected enhancement of right modes of left modes,

$$\frac{|A_{\rm R}(x \to 0)|}{|A_{\rm L}(x \to 0)|} \propto e^{4\xi}\,. \tag{126}$$

There is an exponential increase in the amount of right modes. The dynamically generated parity violation is exponentially sensitive to the value of ξ which is proportional to the axion-gauge coupling. The amplitudes of the linearized mode functions are shown in Fig. 5. A more careful analysis reveals that the factor should be $e^{\pi\xi}$. However, due to the exponential sensitivity to the ratio $\dot{\varphi}/H$ and the assumption that this is constant, one shouldn't take either of these analyses too quantitatively seriously.

While this exponential, and thus nearly maximal parity violation result is promising, we have not yet identified how it affects cosmological observables. The first issue is that photons quickly redshift away, so even though we may produce an abundance of right modes compared to left modes, by the end of inflation these make a negligible contribution to the total energy density. We will not directly observe the photons, and therefore it may seem that this parity-violating phenomenon is unobservable.

The observable quantity at the end of inflation is the primordial curvature perturbation ζ, which is determined by the fluctuations in the inflaton field,

$$\zeta(\tau, \mathbf{k}) = -\frac{H}{\dot{\bar{\varphi}}}\delta\varphi(\tau, \mathbf{k})\,. \tag{127}$$

While the gauge field is not directly observable, it affects the axion-like inflation through the inverse decay process,

$$A_\mu + A_\mu \to \delta\varphi\,, \tag{128}$$

To see this, we need to look at the axion/inflaton equation of motion with its gauge field coupling,

$$-\Box\varphi + 2\mathcal{H}\varphi' = -a^2 V_{,\varphi}(\varphi) - \frac{g}{a^2}\mathbf{E}\cdot\mathbf{B} \tag{129}$$

The first term on the left is the standard wave operator accounting for the propagation of the field's modes. The second term on the left is Hubble drag. The first term on the right is the slope of the scalar field's potential. The last term describes the inverse decay process, where the excited gauge field modes back react onto the inflaton's background and source inflaton fluctuations. Since this is a parity-odd term, it will source parity-odd fluctuations. By the end of inflation, the inflaton will have two sources of fluctuations: the usual vacuum fluctuations that are parity even and the gauge-sourced parity-odd fluctuations.

Interestingly, changing the sign of g cannot affect the sourced inflaton fluctuations. If we change from $g > 0$ to $g < 0$, then we excite the left modes instead of the right modes. Then we would change the sign of $\mathbf{E}\cdot\mathbf{B}$, so the product $g\mathbf{E}\cdot\mathbf{B}$ would stay the same. We would find,

$$\frac{|A_{\rm L}(x \to 0)|}{|A_{\rm R}(x \to 0)|} \propto e^{-4\xi}\,. \tag{130}$$

but ξ now has the opposite sign, thus we are only sensitive to $e^{4|\xi|}$.

Since the gauge-sourced fluctuations are of order $\mathcal{O}(A^2)$, the trispectrum of the gauge modes contributes a correction to the primordial power spectrum.

$$P(k) = \frac{2\pi^2}{k^3}A_{\rm s}\left(\frac{k}{k_{\rm p}}\right)^{n_s-1}\left[1 + A_{\rm s} f_2(\xi) e^{4\pi\xi}\right]. \tag{131}$$

Here $f_2(\xi)$ is a slowly varying, nearly power-law function of ξ. The gauge field six-point spectrum gives rise to an inflaton bispectrum with a nearly equilateral shape, proportional to $e^{6\pi\xi}$. There will also be a parity-violating inflaton trispectrum. However, the computation of the full shape of the trispectrum is an active area of research, as it is difficult to evaluate such a high-dimensional integral (see [7, 18, 11]). In addition to these scalar fluctuations, the gauge field in this model also sources chiral gravitational waves that could be probes using the EB correlations of the CMB fluctuations.

3 Probes of Cosmic Parity Violation

3.1 Trispectrum

3.1.1 CMB

CMB analysis involves decomposing the temperature and polarization anisotropies into spherical harmonics on the sky. The CMB polarization is decomposed into E-modes and B-modes using Stoke's parameters. The result of these decompositions is a set of amplitudes $a^X_{\ell m}$, where X can either be T, E, or B. The T and E modes are sourced by scalar fluctuations during inflation, while the B modes are sourced by tensor fluctuations. Under parity,

$$\mathbb{P} : a^T_{\ell m} \to (-1)^\ell a^T_{\ell -m} \,, \tag{132}$$

$$\mathbb{P} : a^E_{\ell m} \to (-1)^\ell a^E_{\ell -m} \,, \tag{133}$$

$$\mathbb{P} : a^B_{\ell m} \to -(-1)^\ell a^B_{\ell -m} \,. \tag{134}$$

There are many CMB trispectra that we can compute,

$$\langle a^{X_1}_{\ell_1 m_1} a^{X_2}_{\ell_2 m_2} a^{X_3}_{\ell_3 m_3} a^{X_4}_{\ell_4 m_4} \rangle = T^{X_1 X_2 X_3 X_4}_{\ell_1 \ell_2 \ell_3 \ell_4} \,. \tag{135}$$

The average on the left also averages over all of the m_i values. If there is an even number of B's among the X_i's, then the trispectrum is parity-odd when $\ell_1 + \ell_2 + \ell_3 + \ell_4$ is odd. If there are an odd number of B's then the trispectrum is parity-odd if $\ell_1 + \ell_2 + \ell_3 + \ell_4$ is even. The trispectrum on the right of the above equation is related to the primordial trispectrum from inflation (see Ref. [13] for details).

In general, the CMB is particularly sensitive to one specific shape of primordial trispectra. This is because, in the flat-sky limit, corresponding to high ℓ for all ℓ_i, the tetrahedra reduces to 2D shapes in a plane, for which parity is equivalent to a rotation. Then the trispectrum from scalar fluctuations cannot yield a signal in this limit. The B-modes can still yield a signal in this regime, although there has been no detection of even the B-mode power spectrum to date, so these fluctuations may be prohibitively small.

Another regime is the full-sky limit, where ℓ can be small. Here we are severely cosmic variance limited for many configurations. Since the tetrahedra that we correlate must have all of their vertices on the last-scattering surface, we cannot probe a significant number of fully equilateral tetrahedra. If there is a primordial trispectrum that peaks for these configurations, the CMB will not be very sensitive to it.

However, trispectra peaks on configurations where the diagonal of the tetrahedron is small, sometimes called collapsed configurations, essentially correlate the power spectrum in one region of the sky with a power spectrum in another region. One model that produces a trispectrum of this shape has a U(1) field Lagrangian [**?**, 17],

$$\mathcal{L} \propto -\frac{f(\varphi)}{4} F_{\mu\nu} \left(F^{\mu\nu} + g\tilde{F}^{\mu\nu} \right) \,. \tag{136}$$

In this model, isotropy is broken during inflation by generating a small electric field expectation value. This provides a vector which can be used to form the triple-product ubiquitous in parity-violating statistics, so the structure of the trispectrum is significantly different than in the Axion-U(1) case. This model is well constrained by the CMB. So far no evidence of parity violation has been found in CMB trispectrum analysis [5, 13, 16].

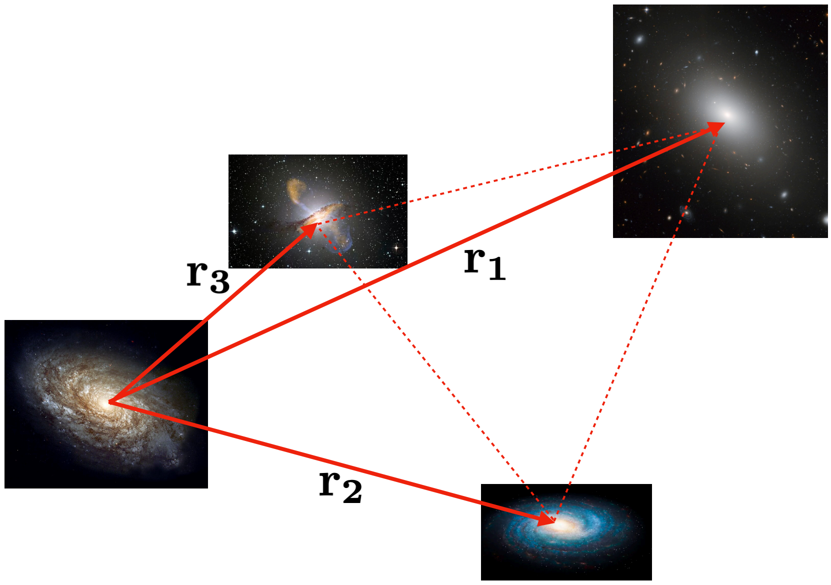

Figure 6: The four-point galaxy correlation function probes tetrahedral shapes formed by four galaxies, which is sensitive to parity.

3.1.2 LSS

A collection of four galaxies forms a tetrahedron, with one galaxy at each corner. By selecting one of the galaxies as a reference, there are three vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ pointing to the other three galaxies. The lengths of these vectors give the separation between each galaxy and the reference galaxy. By imposing an ordering $r_1 < r_2 < r_3$, a sign, or handedness can be assigned to each collection of four galaxies through the sign of the triple-product $\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_2)$. Right-handed collections of four galaxies have a positive triple product and left-handed collections have a negative triple product. By counting the abundances of right-handed and left-handed groups of four galaxies, we can determine if the galaxy statistics is symmetric with respect to right and left, or if they violate parity. If we bin the configurations of tetrahedra formed by collections of four galaxies, we are measuring the galaxy four-point function.

Let $\delta_{\rm g}(\mathbf{x})$ be the galaxy density contrast. The four-point function is,

$$\zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle \delta_{\rm g}(\mathbf{x})\delta_{\rm g}(\mathbf{x}_1)\delta_{\rm g}(\mathbf{x}_2)\delta_{\rm g}(\mathbf{x}_3)\rangle \, . \tag{137}$$

with $\mathbf{r}_i = \mathbf{x}_i - \mathbf{x}$. If we count only right-handed configurations we have,

$$\zeta_{\rm R}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle \delta_{\rm g}(\mathbf{x})\delta_{\rm g}(\mathbf{x}_1)\delta_{\rm g}(\mathbf{x}_2)\delta_{\rm g}(\mathbf{x}_3)\rangle_{\mathbf{r}_1\cdot(\mathbf{r}_2\times\mathbf{r}_3)>0} \, . \tag{138}$$

and for left-handed configurations,

$$\zeta_{\rm L}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle \delta_{\rm g}(\mathbf{x})\delta_{\rm g}(\mathbf{x}_1)\delta_{\rm g}(\mathbf{x}_2)\delta_{\rm g}(\mathbf{x}_3)\rangle_{\mathbf{r}_1\cdot(\mathbf{r}_2\times\mathbf{r}_3)<0} \, . \tag{139}$$

Under parity $\zeta_{\rm R/L}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \mapsto \zeta_{\rm L/R}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$, so the difference between these two four-point functions,

$$\zeta_{(-)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \zeta_{\rm R}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) - \zeta_{\rm L}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \, , \tag{140}$$

is odd under parity. This is equivalent to weighting the galaxy tetrahedra by the sign of their triple-product,

$$\zeta_{(-)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle {\rm sgn}(\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3))\delta_{\rm g}(\mathbf{x})\delta_{\rm g}(\mathbf{x}_1)\delta_{\rm g}(\mathbf{x}_2)\delta_{\rm g}(\mathbf{x}_3)\rangle \, . \tag{141}$$

The four-point correlation function in real space is the Fourier transform of the trispectrum,

$$\zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \int\limits_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} e^{i\mathbf{r}_1\cdot\mathbf{k}_1} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\,. \tag{142}$$

The effect of weight by the sign of the triple product in real space is to weight by the triple product over wave vectors in Fourier space, which isolates the parity-odd trispectrum,

$$\zeta_-(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \int\limits_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} e^{i\mathbf{r}_1\cdot\mathbf{k}_1} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} \frac{-i\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)}{|\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)|} T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\,, \tag{143}$$

$$= \int\limits_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} e^{i\mathbf{r}_1\cdot\mathbf{k}_1} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} e^{i\mathbf{r}_3\cdot\mathbf{k}_3} |\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)| \tau_-(k_1, k_2, k_3, k_4, K_{12}, K_{14})\,. \tag{144}$$

with $\mathbf{k}_4 = -(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$ and $\mathbf{K}_{ij} = \mathbf{k}_i + \mathbf{k}_j$. The relative abundances of right-handed and left-handed tetrahedra are directly related to the parity-odd trispectrum.

Binning the full four-point function directly is computationally slow, so measurements of the four-point function rely on a trick using spherical harmonic transformations of the density fields to reduce the angular structure of the tetrahedra to a discrete list of angular indices ℓ_1, ℓ_2, and ℓ_3 [3, 15]. When the sum of these is odd, the configuration is parity-odd.

Two analyses of the BOSS data detected parity violation in the galaxy distribution [9, 12]. However, a reanalysis of the data with more sophisticated covariance modelling significantly decreased the detection, so the results are unclear [14]. Future surveys will help decide if the galaxy distribution preserves parity or not.

3.2 Nonlinear LSS

References

[1] Neil Barnaby, Ryo Namba, and Marco Peloso. Phenomenology of a Pseudo-Scalar Inflaton: Naturally Large Nongaussianity. *Journal of Cosmology and Astroparticle Physics*, 2011(04):009–009, April 2011. arXiv:1102.4333 [astro-ph, physics:hep-ph, physics:hep-th].

[2] Neil Barnaby and Marco Peloso. Large Nongaussianity in Axion Inflation. *Physical Review Letters*, 106(18):181301, May 2011. arXiv:1011.1500 [astro-ph, physics:hep-ph, physics:hep-th].

[3] Robert N. Cahn, Zachary Slepian, and Jiamin Hou. A Test for Cosmological Parity Violation Using the 3D Distribution of Galaxies, October 2021. arXiv:2110.12004 [astro-ph].

[4] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. Evidence for the 2 pi Decay of the K 2 0 Meson. *Physical Review Letters*, 13(4):138–140, July 1964.

[5] Planck Collaboration. Planck intermediate results. XLIX. Parity-violation constraints from polarization data. *Astronomy & Astrophysics*, 596:A110, December 2016. arXiv:1605.08633 [astro-ph].

[6] William R. Coulton, Oliver H. E. Philcox, and Francisco Villaescusa-Navarro. Signatures of a Parity-Violating Universe, June 2023. arXiv:2306.11782 [astro-ph, physics:gr-qc, physics:hep-ph, physics:hep-th].

[7] Tomohiro Fujita, Tomoaki Murata, Ippei Obata, and Maresuke Shiraishi. Parity-violating scalar trispectrum from a rolling axion during inflation, October 2023. arXiv:2310.03551 [astro-ph].

[8] Richard L. Garwin, Leon M. Lederman, and Marcel Weinrich. Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon. *Physical Review*, 105(4):1415–1417, February 1957.

[9] Jiamin Hou, Zachary Slepian, and Robert N. Cahn. Measurement of Parity-Odd Modes in the Large-Scale 4-Point Correlation Function of SDSS BOSS DR12 CMASS and LOWZ Galaxies. *Monthly Notices of the Royal Astronomical Society*, 522(4):5701–5739, May 2023. arXiv:2206.03625 [astro-ph, physics:gr-qc, physics:hep-th, physics:physics].

[10] T. D. Lee and C. N. Yang. Question of Parity Conservation in Weak Interactions. *Physical Review*, 104(1):254–258, October 1956.

[11] Xuce Niu, Moinul Hossain Rahat, Karthik Srinivasan, and Wei Xue. Parity-Odd and Even Trispectrum from Axion Inflation. *Journal of Cosmology and Astroparticle Physics*, 2023(05):018, May 2023. arXiv:2211.14324 [astro-ph, physics:hep-ph, physics:hep-th].

[12] Oliver H. E. Philcox. Probing Parity-Violation with the Four-Point Correlation Function of BOSS Galaxies. *Physical Review D*, 106(6):063501, September 2022. arXiv:2206.04227 [astro-ph, physics:gr-qc, physics:hep-th].

[13] Oliver H. E. Philcox. Do the CMB Temperature Fluctuations Conserve Parity?, September 2023. arXiv:2303.12106 [astro-ph, physics:gr-qc, physics:hep-ex, physics:hep-ph, physics:hep-th].

[14] Oliver H. E. Philcox and Julia Ereza. Could Sample Variance be Responsible for the Parity-Violating Signal Seen in the BOSS Galaxy Survey?, January 2024. arXiv:2401.09523 [astro-ph, physics:gr-qc, physics:hep-ph, physics:hep-th].

[15] Oliver H. E. Philcox, Jiamin Hou, and Zachary Slepian. A First Detection of the Connected 4-Point Correlation Function of Galaxies Using the BOSS CMASS Sample, August 2021. arXiv:2108.01670 [astro-ph, physics:gr-qc, physics:hep-ex, physics:hep-ph].

[16] Oliver H. E. Philcox and Maresuke Shiraishi. Testing Parity Symmetry with the Polarized Cosmic Microwave Background, August 2023. arXiv:2308.03831 [astro-ph, physics:gr-qc, physics:hep-ex, physics:hep-ph, physics:hep-th].

[17] Maresuke Shiraishi. Parity violation in the CMB trispectrum from the scalar sector. *Physical Review D*, 94(8):083503, October 2016. arXiv:1608.00368 [astro-ph, physics:gr-qc, physics:hep-ph].

[18] David Stefanyszyn, Xi Tong, and Yuhang Zhu. Cosmological Correlators Through the Looking Glass: Reality, Parity, and Factorisation, September 2023. arXiv:2309.07769 [astro-ph, physics:gr-qc, physics:hep-ph, physics:hep-th].

[19] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson. Experimental Test of Parity Conservation in Beta Decay. *Physical Review*, 105(4):1413–1415, February 1957.