Critical Tests of Inflation as a Mechanism for Generating Observed Cosmological Fluctuations in the Universe

Eiichiro Komatsu[1] [(a),(b),(c)]

[(a)] *Texas Cosmology Center and the Department of Astronomy, The University of Texas at Austin, 1 University Station, C1400, Austin, TX 78712, USA*
[(b)] *Kavli Institute for the Physics and Mathematics of the Universe, Todai Institutes for Advanced Study, the University of Tokyo, Kashiwa, Japan 277-8583 (Kavli IPMU, WPI)*
[(c)] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany*

Abstract

Can we rule out inflation as a mechanism for generating the observed cosmological fluctuations in the universe? There now exists a well-defined approach to rule out single-field inflation models (using non-adiabaticity and non-Gaussianity); however, we are yet to find a way to convincingly rule out multi-field inflation models. In this contribution we will discuss these two methods (particularly non-Gaussianity) as a way to critically test inflation, as well as the current observational constraints. We first begin by showing how to rule out single-field inflation models, and then proceed onto discussing how to rule out multi-field inflation models.

1 Introduction

The idea of *cosmic inflation*, a period of accelerated expansion in the very early universe, was proposed more than three decades ago as a solution to the horizon and flatness problems [1–3], and has now become an indispensable ingredient of the standard model of cosmology. During inflation, the universe expands quasi-exponentially as

$$a(t) = a(t_0)\exp\left(\int_{t_0}^{t} H(t')dt'\right), \tag{1}$$

with a slowly-varying $H(t)$. Here, $a(t)$ is the Robertson-Walker scale factor that appears in the unperturbed metric as $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$. In order for inflation to occur, it is necessary for $H(t)$ to vary only slowly; thus, one requires the following "slow-roll parameter" to be small:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1. \tag{2}$$

The dots denote derivatives with respect to time.

Not only does inflation make the observable part of the universe homogeneous, isotropic, and flat, it also provides a natural built-in mechanism for creating quantum fluctuations which seed the observed large-scale structure of the universe today [4–8]. It is also the latter that makes inflation a testable (and potentially falsifiable) model of the early universe, as inflation makes specific predictions for statistical properties of the observed cosmological fluctuations in the universe. We write the perturbed metric as

$$ds^2 = -N^2 dt^2 + a^2(t)e^{2\mathcal{R}}[e^h]_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{3}$$

where N and N^i are the lapse function and the shift vector, respectively. We shall call the trace of the space-space part of the metric perturbation, $\mathcal{R}$, the "curvature perturbation," and demand $\det[e^h]_{ij} = 1$ (hence $\mathrm{Tr}[h] = 0$). In Fourier space, $\mathcal{R}$ is related to the perturbation to the 3-dimensional Ricci scalar as $\delta^{(3)}R = \frac{4k^2}{a^2(t)}\mathcal{R}$.

It is likely that we need something like inflation (i.e., accelerated expansion of the universe in the very early time) to dynamically explain homogeneity, isotropy, and flatness of the observable universe.

[1]Email address: komatsu@mpa-garching.mpg.de

However, one cannot quite rule out inflation using these properties, as one may simply postulate that these properties arose from the initial condition of our universe, rather than explaining them dynamically from generic initial conditions.

On the other hand, statistical properties of the observed cosmological fluctuations can be used to test various models of inflation. In this article, we pose a more general question: *can we falsify inflation as a mechanism for generating the observed cosmological fluctuations in the universe?*

2 Ruling out single-field inflation

2.1 Invariant curvature perturbation, "ζ"

Before we discuss more general models of inflation, let us discuss single-field models of inflation. Here, we define single-field models of inflation as "models of inflation in which one scalar field is responsible for driving the accelerated expansion of the universe *and* creating the observed cosmological fluctuations." The action for a minimally-coupled canonical scalar field is given by

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi)\right]. \tag{4}$$

Throughout this article, we shall take $8\pi G \equiv 1$. Single-field inflation models may have a non-canonical kinetic term or a non-minimal coupling to R. While in this article we shall use this action for carrying out concrete calculations, our conclusions do not depend on the exact form of the action, as long as it involves only one scalar field.

During inflation, quantum fluctuations generate scalar perturbations such as $\delta\phi$ and $\mathcal{R}$ [4–8], as well as tensor perturbations, h_{ij} [9, 10]. For the linear perturbation, it is convenient to define the following quantity,

$$\zeta \equiv \mathcal{R} - \frac{H}{\dot\phi}\delta\phi. \tag{5}$$

This quantity is convenient because it is invariant under infinitesimal transformation of coordinates [11], and becomes a constant on super-horizon scales (for which the wavenumber satisfies $k \ll aH$) [8, 12]. Also, the action given in Eq. (4) can be expanded to quadratic order in ζ: $S = S^{(0)} + S^{(2)} + \dots$, where [13, 14]

$$S^{(2)} = \int d^4x\ \epsilon\left[a^3\dot\zeta^2 - a(\partial\zeta)^2\right]. \tag{6}$$

With this variable, the temperature anisotropy of the cosmic microwave background in a direction $\hat n$ in the Sachs–Wolfe limit is given by $\delta T(\hat n)/T = -\zeta(\hat n r_*)/5$, where $r_* = 14$ Gpc is the comoving distance to the surface of last scattering, $z_* = 1090$. The Poisson equation (in the sub-horizon limit, $k \gg aH$) relates ζ to the matter-density fluctuation, $\delta_M \equiv \delta\rho_M/\rho_M$, in Fourier space as $\delta_{M,\mathbf{k}} = \frac{2k^2}{5H_0^2\Omega_M}\zeta_{\mathbf{k}}T(k)D(z)$, where $T(k)$ is the linear matter transfer function and $D(z)$ is the linear growth factor normalized such that $(1+z)D(z) = 1$ during the matter dominated era. Therefore, a positive ζ gives a negative temperature anisotropy in the Sachs–Wolfe limit and a positive matter density fluctuation.

2.2 Non-adiabaticity

The curvature perturbation generates fluctuations in matter as well as in radiation. As there is no other source of fluctuations in single-field models, fluctuations in different energy components must be equal up to the respective equation of state [15]

$$\frac{\delta\rho_i}{\rho_i(1+w_i)} = 3\zeta\frac{H}{a}\int\frac{da}{H}, \tag{7}$$

where $w_i \equiv P_i/\rho_i$ is the equation of state parameter of an energy component i, $\delta\rho_i$ is the fluctuation in the energy density, and P_i and ρ_i are the mean pressure and energy density, respectively. This implies that the matter density fluctuation, $\delta\rho_M$, and the radiation density fluctuation, $\delta\rho_R$, must satisfy the following adiabatic condition:

$$\frac{\delta\rho_M}{\rho_M} = \frac{3}{4}\frac{\delta\rho_R}{\rho_R}. \tag{8}$$

This is one of the fundamental predictions of single-field models of inflation. Therefore, detection of a violation of the adiabatic condition rules out all single-field models of inflation.

However, the adiabatic condition is a sufficient condition for single-field models of inflation: while detection of a violation of the adiabatic condition rules out all single-field models of inflation, the *lack of* detection of a violation of the adiabatic condition does *not* prove single-field models of inflation. While multi-field models of inflation usually generate non-adiabatic fluctuations which violate the adiabatic condition, the adiabatic condition will be restored if the matter and radiation become local thermal equilibrium with no non-zero conserved quantities [16].

Therefore, detection of a violation of the adiabatic condition has a profound implication for cosmology: not only does it rule out all single-field models of inflation regardless of details of models, but also it rules out any scenarios in which matter enters local thermal equilibrium with radiation before any non-zero conserved quantities are generated. For example, if axions constitute the majority of cold dark matter particles, then one would expect a violation of the adiabatic condition between the dark matter fluctuations and radiation density fluctuations, as the currently viable production mechanism for axions (called the misalignment mechanism) predicts that axions are not in local thermal equilibrium with radiation after inflation (see Section 3.6.3 of [17]; Section 4.4 of [18] and references therein). Another possibility is that the baryon number (more precisely $B-L$, which is a conserved charge in the Standard Model of particle physics) is produced at the end of inflation by some non-thermal process such as the Affleck-Dine mechanism [19], well before baryons enter local thermal equilibrium with radiation. Therefore, detection of a violation of the adiabatic condition between matter and radiation has a profound implication for the nature of dark matter or the physics of baryogenesis. As these two examples (axion and Affleck-Dine baryogenesis) require an additional light field (whose mass is much smaller than H during inflation) which acquires quantum fluctuations on super-horizon scales during inflation, they are not considered as single-field inflation.

We have used the 7-year *WMAP* data of temperature and polarization anisotropies to constrain a violation of the adiabatic condition, and obtained [18]

$$\frac{\left|\frac{\delta\rho_{\rm cdm}}{\rho_{\rm cdm}} - \frac{3\delta\rho_\gamma}{4\rho_\gamma}\right|}{\frac{1}{2}\left(\frac{\delta\rho_{\rm cdm}}{\rho_{\rm cdm}} + \frac{3\delta\rho_\gamma}{4\rho_\gamma}\right)} < 0.092, \tag{9}$$

at the 95% C.L. In order to obtain this constraint, we have assumed that there is no correlation between $\mathcal{S} \equiv \frac{\delta\rho_{\rm cdm}}{\rho_{\rm cdm}} - \frac{3\delta\rho_\gamma}{4\rho_\gamma}$ and ζ, which is a valid assumption for axions.

Where does this constraint come from? On large angular scales where the Sachs–Wolfe approximation is valid, the temperature anisotropy is given by [20]

$$\frac{\delta T}{T} = -\frac{1}{5}\zeta - \frac{2}{5}\mathcal{S}. \tag{10}$$

Therefore, when ζ and $\mathcal{S}$ are uncorrelated, their contributions to the observed angular power spectrum of CMB add in quadrature, increasing the amplitude of the power spectrum on large angular scales, $l \ll 100$. On smaller angular scales, the contribution from $\mathcal{S}$ becomes much smaller than that from ζ. While this change in the CMB power spectrum can be partially canceled by increasing the spectral tilt, n_s, and decreasing the matter density [17], the *WMAP* data combined with the other astrophysical data-sets such as the large-scale structure and the local expansion rate constrain the difference between $\delta\rho_{\rm cdm}/\rho_{\rm cdm}$ and $3\delta\rho_\gamma/4\rho_\gamma$ to be less than 9%.

This limit provides an interesting constraint on the axion decay constant, f_a. Assuming that axions constitute the majority of cold dark matter particles, we obtain [18]

$$f_a > 3.2 \times 10^{32}\ {\rm GeV}\ \gamma^2 \left(\frac{r}{10^{-2}}\right)^2, \tag{11}$$

at the 95% C.L. Here, $r \equiv \langle h_{ij}h^{ij}\rangle/\langle\zeta^2\rangle$ is the tensor-to-scalar ratio, and $\gamma \le 1$ is a "dilution factor" by which the axion density would have been diluted due to a potential late-time entropy production by, e.g., decay of some (unspecified) heavy particles, between 200 MeV and the epoch of nucleosynthesis, 1 MeV. Therefore, detection of the tensor-to-scalar ratio at the level of $r = 10^{-2}$ in the future would rule out axions constituting the majority of dark matter particles, unless one invokes a super-Planckian decay constant, $f_a \gg M_{\rm pl} = 2.4\times 10^{18}$ GeV, or a significant dilution of axion density due to entropy production, $\gamma \ll 1$, or both. The current limit on the tensor-to-scalar ratio is $r < 0.24$ (95% C.L.) [18]. A future satellite mission such as *LiteBIRD* is expected to detect r down to $r = 10^{-3}$ [21].

2.3 Non-Gaussianity

2.3.1 Single-field theorem for the squeezed-limit bispectrum

Inflation usually predicts that $\delta\phi$ (hence ζ according to Eq. (5)) is a Gaussian random variable. How can we use this property to test inflation? It turns out that a deviation from Gaussianity, i.e., *non-Gaussianity*, offers a powerful test of inflation [22–25].

Before we discuss general models of inflation, let us first focus on single-field models of inflation. While the derivation of the main result (Eq. (24)) we present below uses a minimally-coupled slowly-rolling scalar field with canonical kinetic term and the Bunch-Davies initial vacuum state, the final result does not depend on these assumptions [14, 26–31].

As H also depends on the scalar-field value and the scalar field contains a perturbation, one may obtain the following expression (see Section 2.4 of [32]):

$$\zeta = -\frac{H}{\dot\phi}\delta\phi - \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\delta\phi^2 + \mathcal{O}(\delta\phi^3). \tag{12}$$

Here, we have chosen the "flat hypersurface," on which $\mathcal{R} = 0$. As ζ is invariant under the change of hypersurfaces, we are free to choose any hypersurfaces which are convenient for our purposes.

This relation makes ζ *non-Gaussian* even when $\delta\phi$ is Gaussian.[2] Now, going to Fourier space and computing the *bispectrum* (Fourier transform of the 3-point function), we obtain (to the leading order)

$$\begin{aligned}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle &= -\left(\frac{H}{\dot\phi}\right)^3\langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\delta\phi_{\mathbf{k}_3}\rangle \\ &\quad -(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(\frac{H}{\dot\phi}\right)^2\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\left[P_\phi(k_1)P_\phi(k_2)+(2\text{ perm.})\right],\end{aligned} \tag{14}$$

where P_ϕ is the power spectrum of $\delta\phi$: $\langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)P_\phi(k_1)$.

In the first line, let us define $\zeta_c \equiv -(H/\dot\phi)\delta\phi$. In the second line, one can use this relation to replace P_ϕ with the power spectrum of ζ_c, $P_\zeta(k) = (\dot\phi/H)^{-2}P_\phi(k)$. We obtain

$$\begin{aligned}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle &= \langle\zeta_{c,\mathbf{k}_1}\zeta_{c,\mathbf{k}_2}\zeta_{c,\mathbf{k}_3}\rangle - (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(\frac{\dot\phi}{H}\right)^2\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\left[P_\zeta(k_1)P_\zeta(k_2)+(2\text{ perm.})\right] \\ &= \langle\zeta_{c,\mathbf{k}_1}\zeta_{c,\mathbf{k}_2}\zeta_{c,\mathbf{k}_3}\rangle + (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\dot\phi}{2H}\frac{\partial\ln\epsilon}{\partial\phi}\left[P_\zeta(k_1)P_\zeta(k_2)+(2\text{ perm.})\right] \\ &= \langle\zeta_{c,\mathbf{k}_1}\zeta_{c,\mathbf{k}_2}\zeta_{c,\mathbf{k}_3}\rangle + (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(2\epsilon-\eta)\left[P_\zeta(k_1)P_\zeta(k_2)+(2\text{ perm.})\right],\end{aligned} \tag{15}$$

where we have used $\epsilon \equiv -\frac{\dot H}{H^2} = \frac{\dot\phi^2}{2H^2}$ and $\eta \equiv -\frac{\ddot\phi}{H\dot\phi}+\epsilon$. Therefore, if $\delta\phi$ (hence ζ_c) is a Gaussian random variable, only the second term contributes to the bispectrum. While the second term has been known since 1990 [33], a complete computation of the first term had not been done until 2003 [14].

[2] As shown in [32], this result agrees with a modern calculation of ζ on large scales using the so-called "δN formalism" [33], according to which ζ is related to $\delta\phi$ on the flat hypersurface via (for single-field models)

$$\zeta = \int_{\bar\phi+\delta\phi}^{\bar\phi}\frac{H}{\dot\phi}d\phi = -\frac{H}{\dot\phi}\delta\phi - \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\delta\phi^2 + \mathcal{O}(\delta\phi^3). \tag{13}$$

Also see [34, 35].

Before we discuss the first term, let us introduce the so-called "squeezed configuration," where one of the wavenumbers is small. As the power spectrum roughly scales as $P_\zeta(k) \propto k^{-3}$ (or more precisely $P_\zeta(k) \propto k^{n_s-4}$ with $n_s = 0.96\pm0.01$ [18]), the second term is maximized in the squeezed configuration. Let us take k_3 to be the smallest wavenumber. The squeezed configuration then corresponds to $k_3 \ll k_1 \approx k_2$. In this limit, the second term goes to $(4\epsilon - 2\eta)P_\zeta(k_1)P_\zeta(k_3)$.

In order to compute the first term, one needs to expand the action (Eq. (4)) to the third order in terms of ζ_c (see Eq. (3.13) of [14]):

$$S^{(3)} = \int d^4x\ 4\epsilon^2 H a^5 \dot{\zeta}_c^2 \partial^{-2}\dot{\zeta}_c. \tag{16}$$

Then the first term is given by (see Eq. (4.2) of [14]):

$$\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle = -i\int_{t_0}^{t} dt' \langle [\zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t), H_{\rm int}(t')]\rangle, \tag{17}$$

where t_0 is the time at which initial conditions are specified. The interaction Hamiltonian, $H_{\rm int}$, is given by $\int dt'\ H_{\rm int}(t') = -S^{(3)}$. Now, let us expand ζ_c into the creation and annihilation operators,

$$\zeta_c(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^3}\left[a_{\mathbf{k}} u_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^\dagger u_k^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]. \tag{18}$$

One then obtains

$$\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle = i\frac{8\epsilon^2}{H^2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}+\frac{1}{k_3^2}\right) u_{k_1}u_{k_2}u_{k_3}\int_{\eta_0}^{\eta}\frac{d\eta'}{(\eta')^3} u'^*_{k_1}u'^*_{k_2}u'^*_{k_3} + \text{c.c.} \tag{19}$$

where $\eta = \int dt/a$ is the conformal time.

How should we choose the mode function, u_k? This must be a solution to the equation of motion obtained from the quadratic action, $S^{(2)}$, given in Eq. (6): $(a^2\epsilon u_k')' + k^2a^2\epsilon u_k = 0$, where the primes denote derivatives with respect to the conformal time. We choose the following normalization of the mode function:

$$u_k(t) = \frac{H^2}{\dot\phi}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}, \tag{20}$$

which gives its conformal-time derivative as $u_k' = \frac{H^2}{\dot\phi}\sqrt{\frac{k}{2}}\eta e^{-ik\eta}$. The power spectrum, $P_\zeta(k)$, is given by $P_\zeta(k) = |u_k(\eta\to 0)|^2 = \frac{H^4}{\dot\phi^2}\frac{1}{2k^3}$.

This choice of the mode function is a reasonable one (although it is not the only one - more later) in the following sense: when the wavenumber is large, modes should see a flat, Minkowski space. We know how to quantize a massless scalar field in Minkowski space, and the mode function of a massless scalar field in the vacuum state (ground state/minimum-energy state) in Minkowski space is given by $\frac{1}{\sqrt{2k}}e^{-ikt}$. Let us take $k\to\infty$ limit of Eq. (20):

$$u_k(t) \to -\frac{H}{\dot\phi}\frac{ie^{-ik\eta}}{\sqrt{2k}a(t)}. \tag{21}$$

Therefore, this gives a mode function for a scalar field fluctuation in the high-k limit as $\delta\phi_k \to \frac{1}{\sqrt{2k}a}e^{-ik\eta}$ (up to an unimportant phase factor i). Taking the Minkowski limit ($a\to 1$), one reproduces the mode function of a massless scalar field in the vacuum state in Minkowski space. This choice is called the Bunch-Davies initial vacuum state.

Now, all we need to do is to insert the mode function given by Eq. (20) into Eq. (19) and perform the integral. We shall take the initial conformal time to be infinite past: $\eta_0 \to -\infty$. In order to suppress the exponential terms such as $e^{i(k_1+k_2+k_3)\eta_0}$, we shall go slightly to the complex plane: $\eta_0 \to -\infty(1-i\epsilon)$ (this ϵ is not the slow-roll parameter, but just denotes a small number). This corresponds to suppressing excited states and choosing the vacuum [36]. While the result of this computation is a bit complicated, it simplifies to

$$\langle \zeta_{c,\mathbf{k}_1}\zeta_{c,\mathbf{k}_2}\zeta_{c,\mathbf{k}_3}\rangle \to (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)2\epsilon P_\zeta(k_1)P_\zeta(k_3), \tag{22}$$

in the squeezed configuration [14]. Therefore, in the squeezed configuration, single-field models of inflation predict

$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \to (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(6\epsilon-2\eta)P_\zeta(k_1)P_\zeta(k_3). \tag{23}$$

Remarkably, the combination of the slow-roll parameters that appears on the right hand side is equal to $1-n_s$ at the first order in the slow-roll parameters [37]. Thus, we write

$$\boxed{\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \to (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(1-n_s)P_\zeta(k_1)P_\zeta(k_3)} \tag{24}$$

This is called the consistency condition of single-field inflation models.

While we have used the (i) action for a minimally-coupled scalar field with canonical kinetic term, (ii) slow-roll approximation, and (iii) Bunch-Davies initial vacuum state to derive the above consistency condition, a remarkable fact is that this result holds for *all* single-field inflation models regardless of the above three assumptions [14, 26–30]. Therefore, detection of a violation of this condition would rule out *all* single-field inflation models regardless of the details of models. This is the single-field theorem for the squeezed-limit bispectrum.

2.3.2 Local-form bispectrum and observational limits

How can we measure the squeezed-limit bispectrum and test Eq. (24)? Inspired by the consistency condition, let us define [38]

$$\frac{6}{5}f_{\rm NL} \equiv \frac{B_\zeta(k_1,k_2,k_3)}{P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)}, \tag{25}$$

where B_ζ is defined by

$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3). \tag{26}$$

In the squeezed limit, we find $f_{\rm NL}\to\frac{5}{12}(1-n_s)=\mathcal{O}(10^{-2})$. While in general $f_{\rm NL}$ should depend on wavenumbers in a complex way, a momentum-independent $f_{\rm NL}$ is called the "local-form bispectrum." This provides a good template for testing the single-field consistency condition.

The local-form $f_{\rm NL}$ (which is independent of wavenumbers) can be measured experimentally [39–41], and thus detection of the primordial bispectrum signal of order $f_{\rm NL}\gg\mathcal{O}(10^{-2})$ would rule out all single-field inflation models. The limit on $f_{\rm NL}$ from the *WMAP* 7-year CMB data is $f_{\rm NL}=32\pm 21$ (68% C.L.) [18]. Therefore, the current data are consistent with single-field inflation models. The *Planck* data are expected to reduce the error bar by a factor of four [38].

2.3.3 Non-Bunch-Davies initial state

However, there is an important subtlety: all single-field models predict the amplitude of the squeezed-limit bispectrum given by $1-n_s$ (as explained above) *only in the exact squeezed limit*, $k_3/k_1\to 0$. On the other hand, we can measure k_3/k_1 down only to $1/1500$ using anisotropies in the cosmic microwave background (whose map can be used to extract information about primordial fluctuations from $l=2$ to 3000 where $l=kr_*$); and down to $1/1000$ using the large-scale structure of the universe where we have access to $k\approx 10^{-3}$ to 1 Mpc^{-1}. It may be possible to reach $k_3/k_1\approx 10^{-8}$ using the so-called μ-type distortion of the thermal spectrum of the cosmic microwave background [42, 43]. The question is then, "*what happens if k_3/k_1 is small but finite?*" If there are models of inflation which produce a negligible bispectrum signal in $k_3/k_1\to 0$ (so that it does not violate the consistency condition) but produce sizable signals in small but finite k_3/k_1, then it may produce a signal in the *measured* local-form $f_{\rm NL}$.

Recently, Creminelli, D'Amico, Musso and Norena [31] have shown that all single-field inflation models with the Bunch-Davies initial state give

$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left[(1-n_s)+\mathcal{O}(k_3^2/k_1^2)\right]P_\zeta(k_1)P_\zeta(k_3), \tag{27}$$

when k_3/k_1 is small but finite. The correction to the consistency condition is thus small, as long as the initial state is the Bunch-Davies state. Therefore, this leaves a deviation from the Bunch-Davies initial state, a *non-Bunch-Davies initial state*, as the only source of a significant local-form $f_{\rm NL}$.

How do we choose a mode function to describe a non-Bunch-Davies initial state? One reasonable starting point would be a Bogoliubov transform of the Bunch-Davies mode function:

$$\tilde{u}_k(t) = \alpha_k u_k(t) + \beta_k u_k^*(t), \tag{28}$$

where u_k is the Bunch-Davies mode function given by Eq. (20). The commutation relation of creation and annihilation operators demands that the Bogoliubov coefficients, α_k and β_k satisfy $|\alpha_k|^2 - |\beta_k|^2 = 1$. As we still have a situation in which the spacetime must approach the Minkowski space in the high-k limit, the mode function must approach $u_k(t)$ in the high-k limit. In other words, we must satisfy

$$\alpha_k \to 1, \qquad \beta_k \to 0, \qquad \text{for } k \to \infty. \tag{29}$$

In other words, we do not modify the ultraviolet behavior of the mode function, but only modify the infrared behavior of the mode function. The new state corresponds to a state with "particles," with the occupation number given by $N_k = |\beta_k|^2$ [44]. Of course, the occupation number must vanish in the high-k limit: $N_k \to 0$ for $k \to \infty$. The power spectrum of ζ is now given by

$$P_\zeta(k) = |\tilde{u}_k(\eta \to 0)|^2 = \frac{H^4}{\dot{\phi}^2}\frac{1}{2k^3}|\alpha_k + \beta_k|^2. \tag{30}$$

As the current observation suggests $P_\zeta(k) \propto k^{n_s-4}$ with $n_s = 0.96 \pm 0.01$ [18], the factor $|\alpha_k + \beta_k|^2$ must not vary strongly with k. This motivates our writing the occupation number as $N_k \approx N_0 e^{-k^2/k_{\rm cut}^2}$, where N_0 is a constant and $k_{\rm cut}$ is some ultraviolet cutoff scale of the theory.

The bispectrum for this new mode function can be obtained by simply replacing u_k with $\tilde{u}_k$ in Eq. (19):

$$\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle = i\frac{8\epsilon^2}{H^2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}+\frac{1}{k_3^2}\right)\tilde{u}_{k_1}\tilde{u}_{k_2}\tilde{u}_{k_3}\int_{\eta_0}^{\eta}\frac{d\eta'}{(\eta')^3}\tilde{u}_{k_1}'^*\tilde{u}_{k_2}'^*\tilde{u}_{k_3}'^* + \text{c.c.} \tag{31}$$

Taking $k_3 \ll k_1 \approx k_2$ (but not taking $k_3 \to 0$), we find [43]

$$\begin{aligned}\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle \;\to\; & (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)2\epsilon P_\zeta(k_1)P_\zeta(k_3)\frac{1}{|\alpha_{k_1}+\beta_{k_1}|^2}\frac{1}{|\alpha_{k_3}+\beta_{k_3}|^2}\\ & \times\Re\left\{F_{\alpha\alpha\alpha}\left(1-e^{i(k_1+k_2+k_3)\eta_0}\right)+F_{\alpha\alpha\beta}\left(1-e^{i(k_1+k_2-k_3)\eta_0}\right)\right.\\ & \left.+2\frac{k_1}{k_3}\left[F_{\alpha\beta\alpha}\left(1-e^{i(k_1-k_2+k_3)\eta_0}\right)+F_{\beta\alpha\alpha}\left(1-e^{i(-k_1+k_2+k_3)\eta_0}\right)\right]\right\},\end{aligned} \tag{32}$$

where

$$\begin{aligned}F_{\alpha\alpha\alpha} \equiv\; & (\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3})\alpha_{k_1}^*\alpha_{k_2}^*\alpha_{k_3}^*\\ & -\left(\alpha_{k_1}^*+\beta_{k_1}^*\right)\left(\alpha_{k_2}^*+\beta_{k_2}^*\right)\left(\alpha_{k_3}^*+\beta_{k_3}^*\right)\beta_{k_1}\beta_{k_2}\beta_{k_3}\end{aligned} \tag{33}$$

$$\begin{aligned}F_{\alpha\alpha\beta} \equiv\; & (\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3})\alpha_{k_1}^*\alpha_{k_2}^*\beta_{k_3}^*\\ & -\left(\alpha_{k_1}^*+\beta_{k_1}^*\right)\left(\alpha_{k_2}^*+\beta_{k_2}^*\right)\left(\alpha_{k_3}^*+\beta_{k_3}^*\right)\beta_{k_1}\beta_{k_2}\alpha_{k_3}\end{aligned} \tag{34}$$

$$\begin{aligned}F_{\alpha\beta\alpha} \equiv\; & (\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3})\alpha_{k_1}^*\beta_{k_2}^*\alpha_{k_3}^*\\ & -\left(\alpha_{k_1}^*+\beta_{k_1}^*\right)\left(\alpha_{k_2}^*+\beta_{k_2}^*\right)\left(\alpha_{k_3}^*+\beta_{k_3}^*\right)\beta_{k_1}\alpha_{k_2}\beta_{k_3}\end{aligned} \tag{35}$$

$$\begin{aligned}F_{\beta\alpha\alpha} \equiv\; & (\alpha_{k_1}+\beta_{k_1})(\alpha_{k_2}+\beta_{k_2})(\alpha_{k_3}+\beta_{k_3})\beta_{k_1}^*\alpha_{k_2}^*\alpha_{k_3}^*\\ & -\left(\alpha_{k_1}^*+\beta_{k_1}^*\right)\left(\alpha_{k_2}^*+\beta_{k_2}^*\right)\left(\alpha_{k_3}^*+\beta_{k_3}^*\right)\alpha_{k_1}\beta_{k_2}\beta_{k_3}.\end{aligned} \tag{36}$$

The exponential factors in the first two terms of Eq. (32) vanish by taking the limit $\eta_0 \to -\infty(1-i\epsilon)$. On the other hand, one has to be careful about the exponential factors in the last two terms, as $k_1 - k_2 + k_3$ and $-k_1 + k_2 + k_3$ are also very small in the squeezed limit. One may ignore the exponential factors when $k_1 - k_2 + k_3$ etc. are finite; otherwise one has to retain the exponential factors.

Using $F_{\alpha\alpha\alpha} + F_{\alpha\alpha\beta} = (|\alpha_{k_1} + \beta_{k_1}|^2 + 2i\Im(\alpha_{k_1}^*\beta_{k_1}))|\alpha_{k_3} + \beta_{k_3}|^2$ and $F_{\alpha\beta\alpha} = F_{\beta\alpha\alpha}$ for $k_1 = k_2$, we obtain

$$\begin{aligned}\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle \;\to\; & (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)2\epsilon P_\zeta(k_1)P_\zeta(k_3)\\ & \times\left\{1+4\frac{k_1}{k_3}\frac{\Re\left[F_{\alpha\beta\alpha}\left(1-e^{ik_3\eta_0}\right)\right]}{|\alpha_{k_1}+\beta_{k_1}|^2|\alpha_{k_3}+\beta_{k_3}|^2}\right\}.\end{aligned} \tag{37}$$

Do we recover the single-field consistency condition in the exact squeezed limit, $k_3 \to 0$? To recover this, one needs to show $\langle \zeta_{c,\mathbf{k}_1}(t)\zeta_{c,\mathbf{k}_2}(t)\zeta_{c,\mathbf{k}_3}(t)\rangle \to (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)2\epsilon P_\zeta(k_1)P_\zeta(k_3)$ for $k_3 \to 0$, provided that a scale-dependence of $|\alpha_k+\beta_k|^2$ can be ignored.[3] The second term in Eq. (37) goes to $-4k_1\eta_0\Im[F_{\alpha\beta\alpha}]/(|\alpha_{k_1}+\beta_{k_1}|^2|\alpha_{k_3}+\beta_{k_3}|^2)$ as $k_3 \to 0$. One can show that $\Im[F_{\alpha\beta\alpha}] \to \Im[\alpha^*_{k_1}\beta_{k_1}]$ for $k_3 \to 0$, $\alpha_{k_3} \to 1$, and $\beta_{k_3} \to 0$; thus, this term does not quite vanish in the exact squeezed limit. This is puzzling, and how can this result be reconciled with the single-field theorem requires a further investigation.

A remarkable result from Eq. (32) is that, for a non-Bunch-Davies initial state, the bispectrum in the squeezed configuration can pick up another factor of $k_1/k_3 \gg 1$, changing the behavior of the bispectrum in this limit from $2\epsilon P_\zeta(k_1)P_\zeta(k_3)$ to $8\epsilon\frac{k_1}{k_3}P_\zeta(k_1)P_\zeta(k_3)\{\Re[F_{\alpha\beta\alpha}(1-e^{ik_3\eta_0})]/(|\alpha_{k_1}+\beta_{k_1}|^2|\alpha_{k_3}+\beta_{k_3}|^2)\}$, provided that $k_3\eta_0 = \mathcal{O}(1)$ (i.e., a mode corresponding to k_3 is still inside the horizon at the initial time η_0). This behavior has been found first by Agullo and Parker [45]. This suggests that, unless β_k (hence the occupation number of "particles" in the initial state) is very small, a non-Bunch-Davies initial state can change the behavior of the bispectrum in the squeezed configuration dramatically. This leads to profound observational consequences for the bispectrum of the cosmic microwave background temperature anisotropy [46], the scale-dependent bias of galaxies [43, 47], and the μ-type distortion of the thermal spectrum of the cosmic microwave background [43]. Therefore, the bispectrum in the squeezed configuration can be used to learn about the initial state of quantum fluctuations during inflation.

How does this affect our ability to rule out single-field inflation models using the bispectrum? While a non-Bunch-Davies initial state produces a significant bispectrum in the squeezed configuration, more generally it produces large signals in the so-called "co-linear" configurations, for which the sum of the magnitudes of two wavenumbers is equal to the magnitude of one wavenumber, e.g., $k_1 = k_2 + k_3$, etc. [48, 49]. Obviously this includes the squeezed configuration, $k_3 \ll k_1 \approx k_2$. Therefore, once we find a large signal in the squeezed configuration (say, by measuring the primordial local-form bispectrum of order $f_{\rm NL} \gg 1$), we should check whether we find similarly large signals in the other co-linear configurations for which k_3 is not small. If we do find signals in the co-linear configurations, we would conclude that the initial state is not in the Bunch-Davies initial state (and inflation can still be single-field). If we do *not* find signals in the co-linear configurations, we would rule out all single-field inflation models.

3 Ruling out multi-field inflation

Let us suppose that we find the primordial local-form bispectrum of order $f_{\rm NL} \gg 1$ without corroborating signals in the co-linear configurations where k_3 is not small. Single-field inflation models are gone - what should we do?

As single-field models are gone, we must be led to multi-field models of inflation. In order to calculate ζ from multi fields, one must extend Eq. (12). This can be achieved by using the δN formalism [34, 35]:

$$\zeta = \sum_I N_I\delta\phi^I + \frac{1}{2}\sum_{IJ}N_{IJ}\delta\phi^I\delta\phi^J + \mathcal{O}(\delta\phi^3), \tag{39}$$

where N is the number of e-folds (not to be confused with the lapse function) calculated as a function of multiple scalar fields at the epoch of the horizon crossing: $N \equiv \int Hdt = N(\phi^I)$, and $N_I \equiv \partial N/\partial\phi^I$ and $N_{IJ} \equiv \partial^2 N/\partial\phi^I\partial\phi^J$. Here, we evaluate $\delta\phi^I$ on the flat hypersurface. One can check that this formula yields the known single-field result (Eq. (12); also see footnote 2) by noting that, for single-field models, $N = \int\frac{H}{\dot\phi}d\phi$, $N_1 = -\frac{H}{\dot\phi}$, etc. ($N_1$ has a negative sign because the derivative is taken with respect to the initial field value at the epoch of the horizon crossing, rather than the final field value.)

In this section, we shall ignore contributions from $\langle\delta\phi^I\delta\phi^J\delta\phi^K\rangle$, as they are usually sub-dominant (slow-roll suppressed) for the local-form bispectrum. Using the definition of the local-form $f_{\rm NL}$ given in Eq. (25), one finds, to the lowest order [35]

$$\frac{6}{5}f_{\rm NL} = \frac{\sum_{IJ}N_{IJ}N_IN_J}{(\sum_I N_I^2)^2}. \tag{40}$$

[3]If a scale-dependence of $|\alpha_k+\beta_k|^2$ cannot be ignored, then one needs to check that the squeezed limit bispectrum reproduces $1-n_s$ given by

$$1-n_s = 6\epsilon - 2\eta - \frac{d\ln|\alpha_k+\beta_k|^2}{d\ln k}. \tag{38}$$

For a given model of multi-field inflation, it is possible to compute $f_{\rm NL}$ from this formula; thus, one can constrain multi-field inflation models by comparing this formula to the measured value of the primordial local-form $f_{\rm NL}$.

However, can we ever rule-out multi-field inflation using non-Gaussianity? Clearly the bispectrum alone cannot rule out multi-field models. Now is the time to go beyond the bispectrum and consider the *trispectrum* (four-point function). The δN formula (Eq. (39)) yields, to the lowest order, the following trispectrum:

$$\begin{aligned}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle &= (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\\ &\times\left\{\tau_{\rm NL}\left[P_\zeta(k_1)P_\zeta(k_2)\left\{P_\zeta(k_{13})+P_\zeta(k_{14})\right\}+(11\ \text{perm.})\right]\right.\\ &\left.+\frac{54}{25}g_{\rm NL}\left[P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+(3\ \text{perm.})\right]\right\},\end{aligned} \tag{41}$$

where $k_{ij}\equiv|\mathbf{k}_i+\mathbf{k}_j|$, with $\tau_{\rm NL}$ and $g_{\rm NL}$ given by [50, 51]

$$\tau_{\rm NL} = \frac{\sum_I(\sum_J N_{IJ}N_J)^2}{(\sum_I N_I^2)^3}, \tag{42}$$

$$\frac{54}{25}g_{\rm NL} = \frac{\sum_{IJK}N_{IJK}N^IN^JN^K}{(\sum_I N_I^2)^3}. \tag{43}$$

Now, let us look at $f_{\rm NL}$ and $\tau_{\rm NL}$. As both of these depend only on the first derivative, N_I, and the second derivative, N_{IJ}, there may be a universal relation between them. To see this clearly, let us change variables:

$$a_I = \frac{\sum_J N_{IJ}N_J}{(\sum_J N_J^2)^{3/2}}, \tag{44}$$

$$b_I = \frac{N_I}{(\sum_J N_J^2)^{1/2}}. \tag{45}$$

Then, Cauchy-Schwarz inequality,

$$\left(\sum_I a_I^2\right)\left(\sum_J b_J^2\right)\geq\left(\sum_I a_Ib_I\right)^2, \tag{46}$$

yields the following *Suyama-Yamaguchi inequality* [52]:

$$\boxed{\tau_{\rm NL}\geq\left(\frac{6f_{\rm NL}}{5}\right)^2} \tag{47}$$

While this inequality has been derived only for the leading-order contributions from Eq. (39) (i.e., $(1\text{st})^2\times(2\text{nd})$ for $f_{\rm NL}$; $(1\text{st})^2\times(2\text{nd})^2$ for $\tau_{\rm NL}$), we have shown that they hold even when including higher-order contributions (e.g., $(2\text{nd})^3$ for $f_{\rm NL}$; $(2\text{nd})^4$ for $\tau_{\rm NL}$; etc) [53, 54]. This is a powerful relation - one may potentially rule out all multi-field inflation models if we find a violation of this relation observationally. For example, if *Planck* finds $f_{\rm NL}=30$ (which is the central value obtained from the *WMAP* 7-year data), then it should also find $\tau_{\rm NL}\geq 1300$, which is about 5-σ level compared to the 1-σ uncertainty of $\tau_{\rm NL}$ expected from the *Planck* data [55]. In other words, if $f_{\rm NL}=30$ is confirmed by *Planck*, we'd better find $\tau_{\rm NL}$ also! For follow-up studies of the Suyama-Yamaguchi inequality from rather different perspectives, see [56, 57].

Finally, let us comment on an alternative view on the Suyama-Yamaguchi inequality reported by [58]. They show that the definitions of $f_{\rm NL}$ and $\tau_{\rm NL}$ alone yield an inequality between these two quantities, which does not rely on physics but relies only on statistics:

$$\tau_{\rm NL}\geq\left(\frac{6f_{\rm NL}}{5}\right)^2-\Delta\tau_{\rm NL}, \tag{48}$$

where $\Delta\tau_{\rm NL}$ depends on experiments. For *Planck*, for example, $\Delta\tau_{\rm NL}$ is 10 times as large as the 1-σ uncertainty of $\tau_{\rm NL}$ expected from *Planck* [58]. Therefore, detection of a violation of $\tau_{\rm NL}\geq\left(\frac{6f_{\rm NL}}{5}\right)^2$ would challenge inflation as a mechanism for generating the observed fluctuations.

4 Summary

We have discussed how to rule out single-field inflation models using non-adiabaticity and non-Gaussianity of primordial fluctuations. In particular, non-Gaussianity can be a powerful tool to rule out not only single-field models but also multi-field models. Specifically, all single-field inflation models predict that $f_{\rm NL} \to \frac{5}{12}(1-n_s)$ in the exact squeezed limit, $k_3/k_1 \to 0$. For a small but finite k_3/k_1, a non-Bunch-Davies initial state can yield a significant $f_{\rm NL}$; however, this signal must be accompanied by the other co-linear configurations (e.g., $k_1 = k_2 + k_3$) in which k_3 is not so small. Finally, once single-field models are excluded, one can use the Suyama-Yamaguchi inequality between the local-form bispectrum and trispectrum amplitudes, $\tau_{\rm NL} \geq (6f_{\rm NL}/5)^2$, to test multi-field models. Detection of a violation of this inequality potentially rules out all multi-field inflation models (although this statement is not as definitive as that for single-field models).

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