Jets

IMPRS April 2010

- Examples
    - knots, precession, superluminal motion
- magnetic jet model
- problem areas

introduction:

http://www.mpa-garching.mpg.de/~henk/pub/jetrevl.pdf (somewhat old)

current issues:

http://www.mpa-garching.mpg.de/~henk/pub/Jetissues.pdf
(=arXiv:0804.3096)

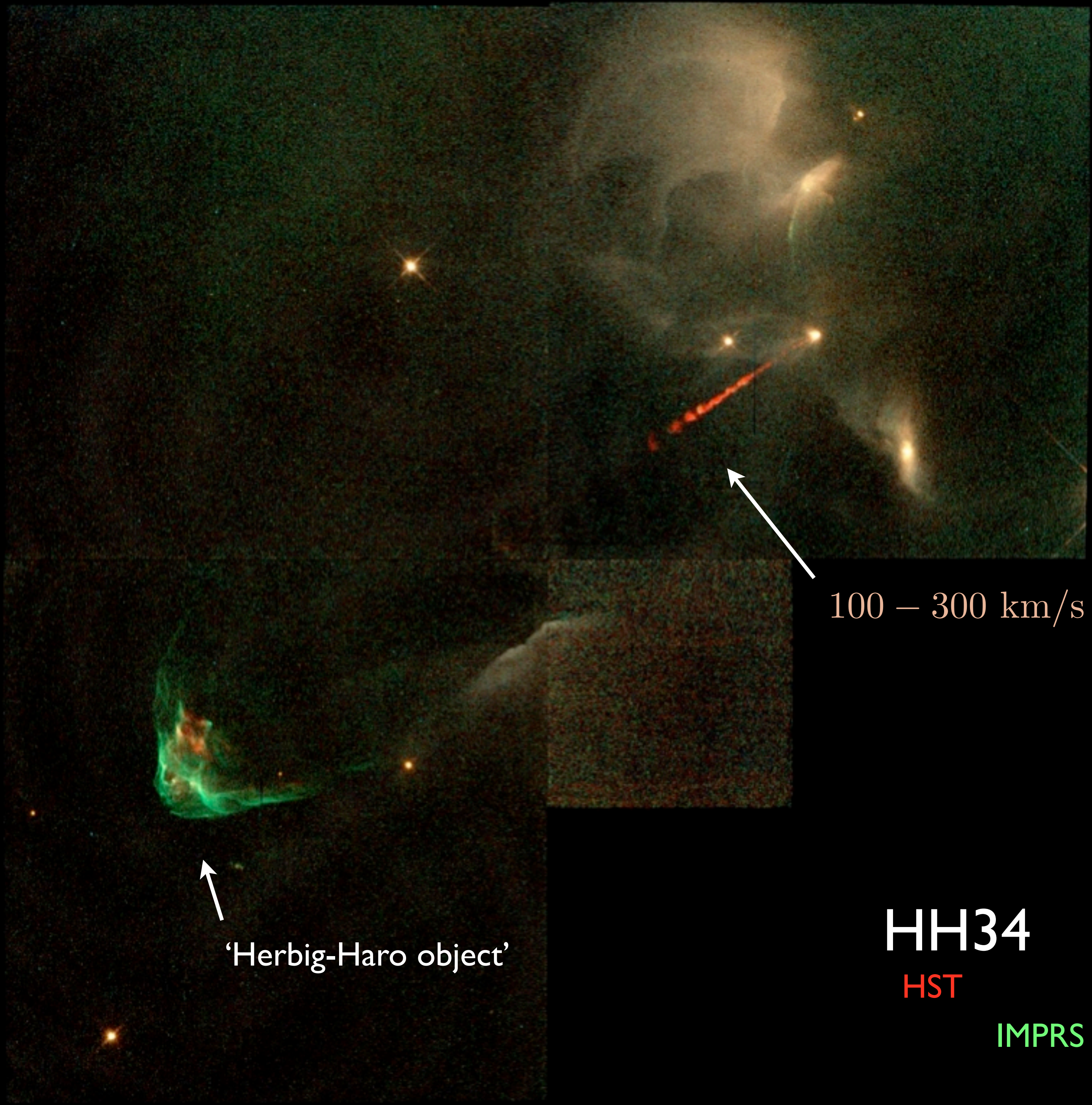
This presentation:

http://www.mpa-garching.mpg.de/~henk/imprsjets.pdf

Jets observed in:
- protostars
- ‘symbiotic’ binaries
- ‘supersoft’ X-ray sources
- SS433
- n-star binaries (Cir X-1)
- black hole binaries (‘microquasars’)
- active galaxies

Common: all involve accretion and disks

exceptional case (?) : planetary nebulae

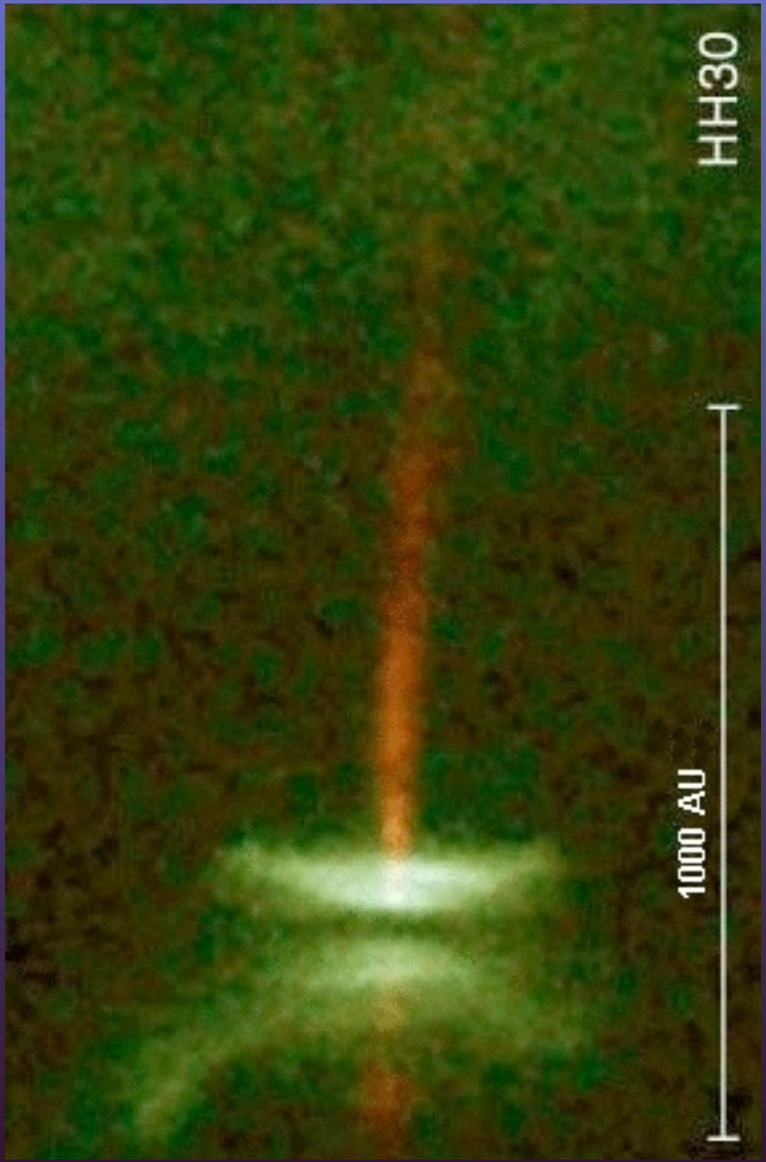


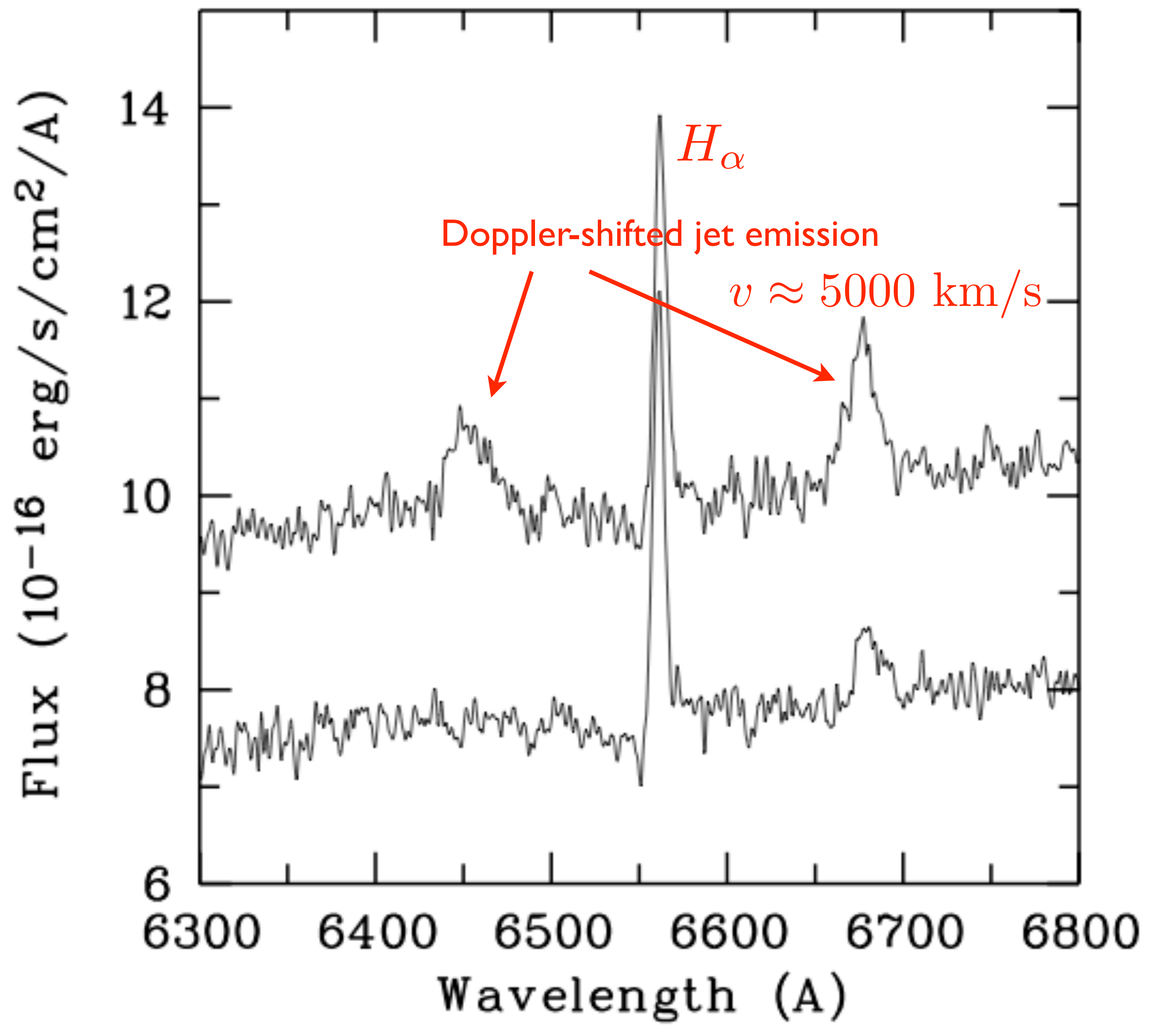
$100 - 300\ \mathrm{km/s}$

‘Herbig-Haro object’

HH34

HST


HH30
1000 AU

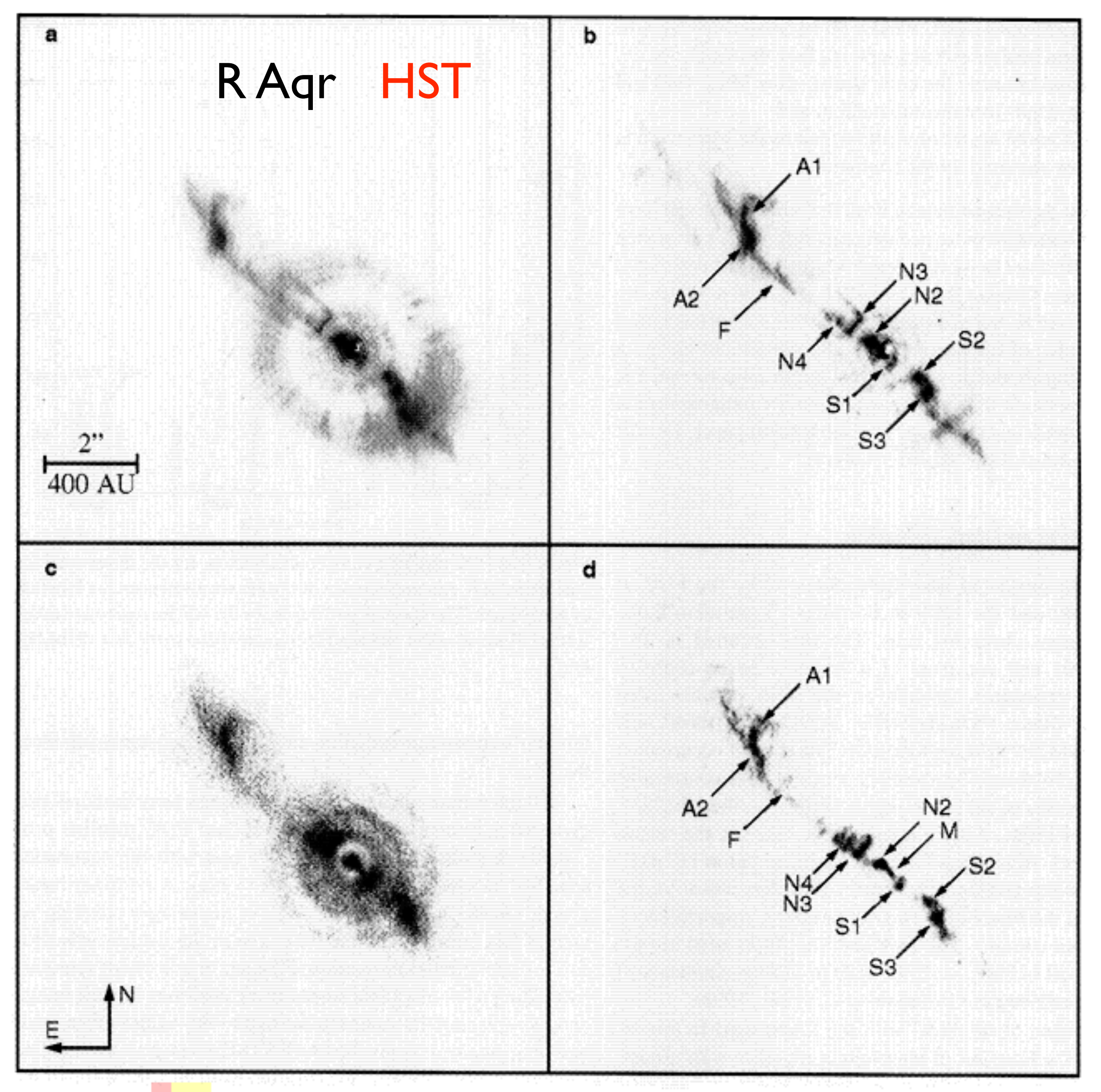


'Supersoft source' accreting WD burning H on its surface

↔ symbiotics

& CVs

C. Motch: The transient jet of the galactic supersoft X-ray source RX J0925.7-4758


a
R Aqr
HST
2"
400 AU
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A1
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N2
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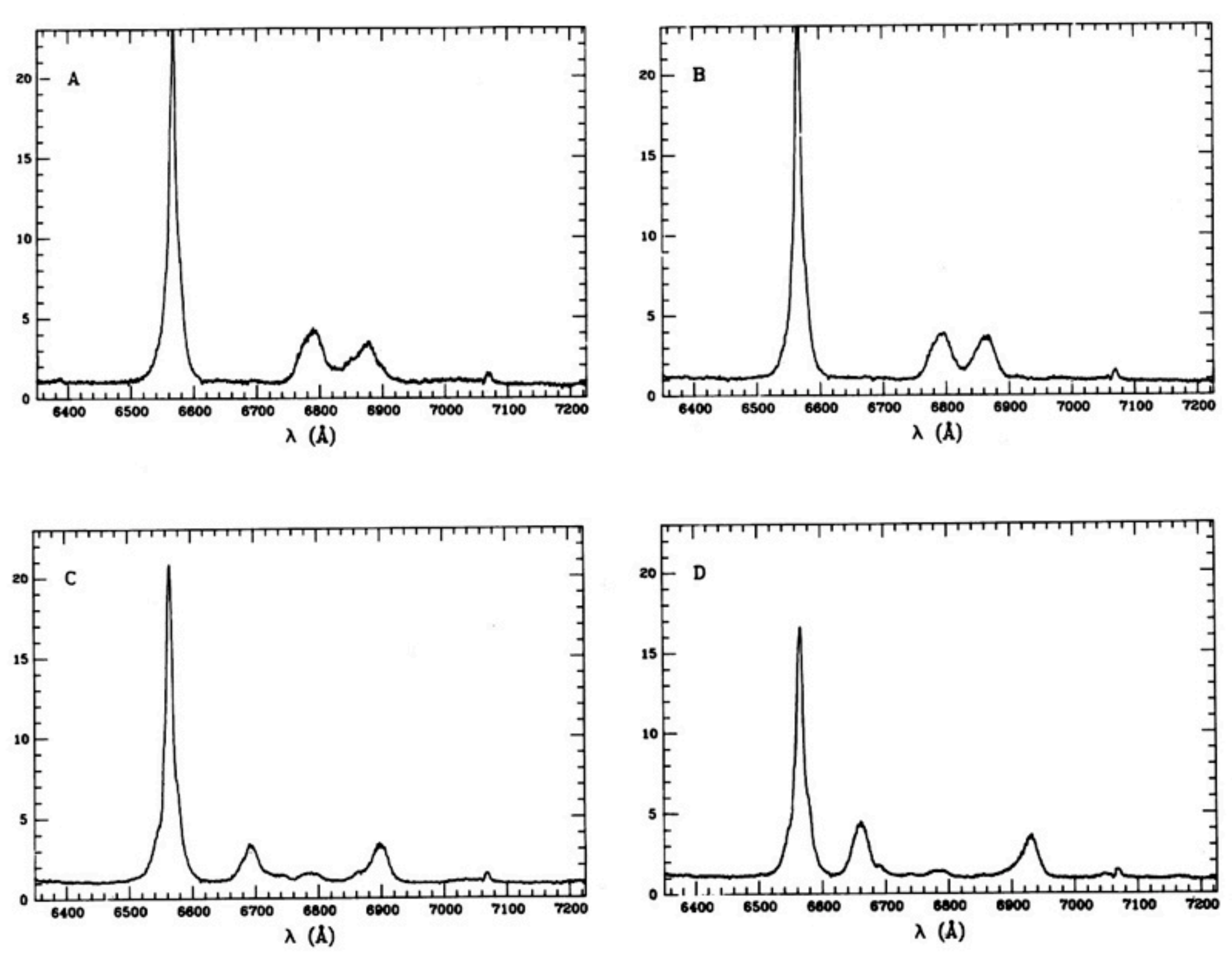
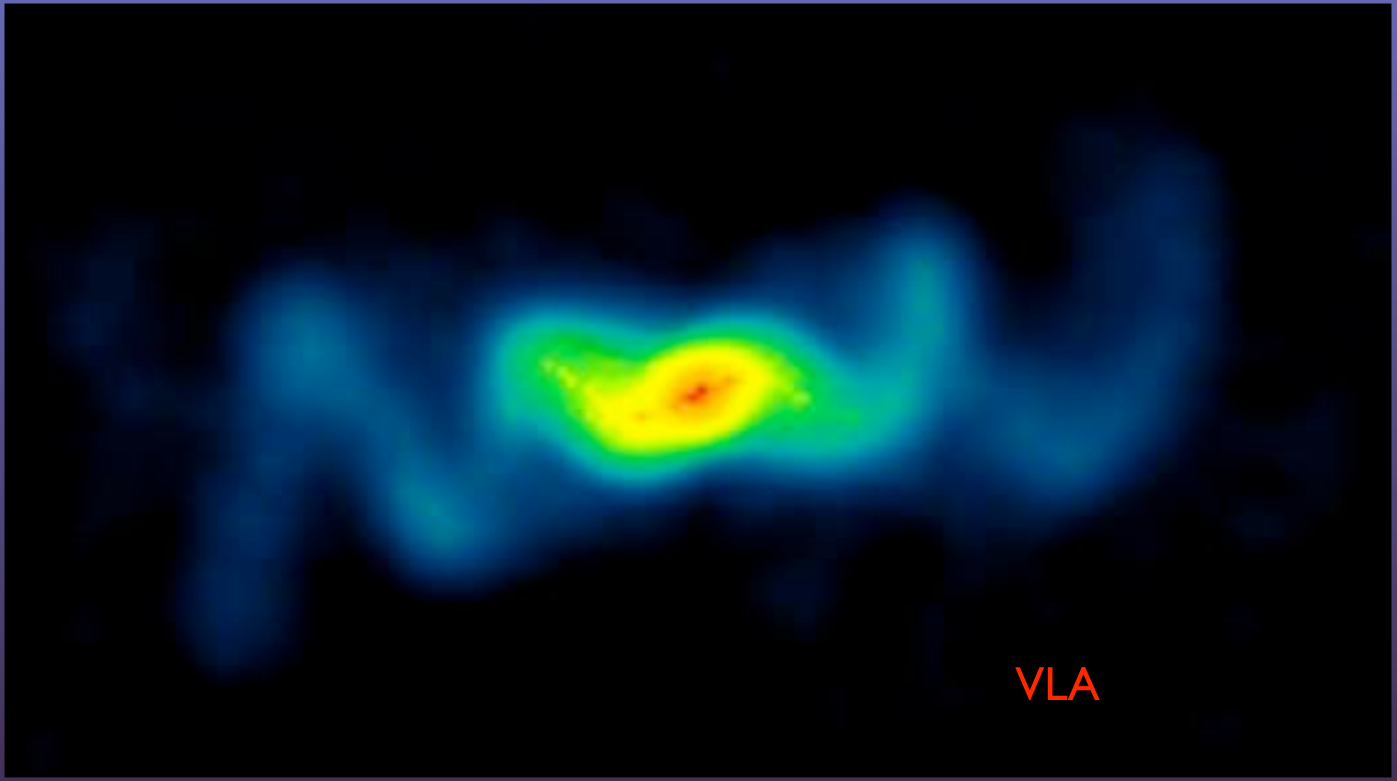
Jet precession



SS433 INT

$v = 0.26c$

The precessing jet of SS433 (Precession period =164d)



NASA

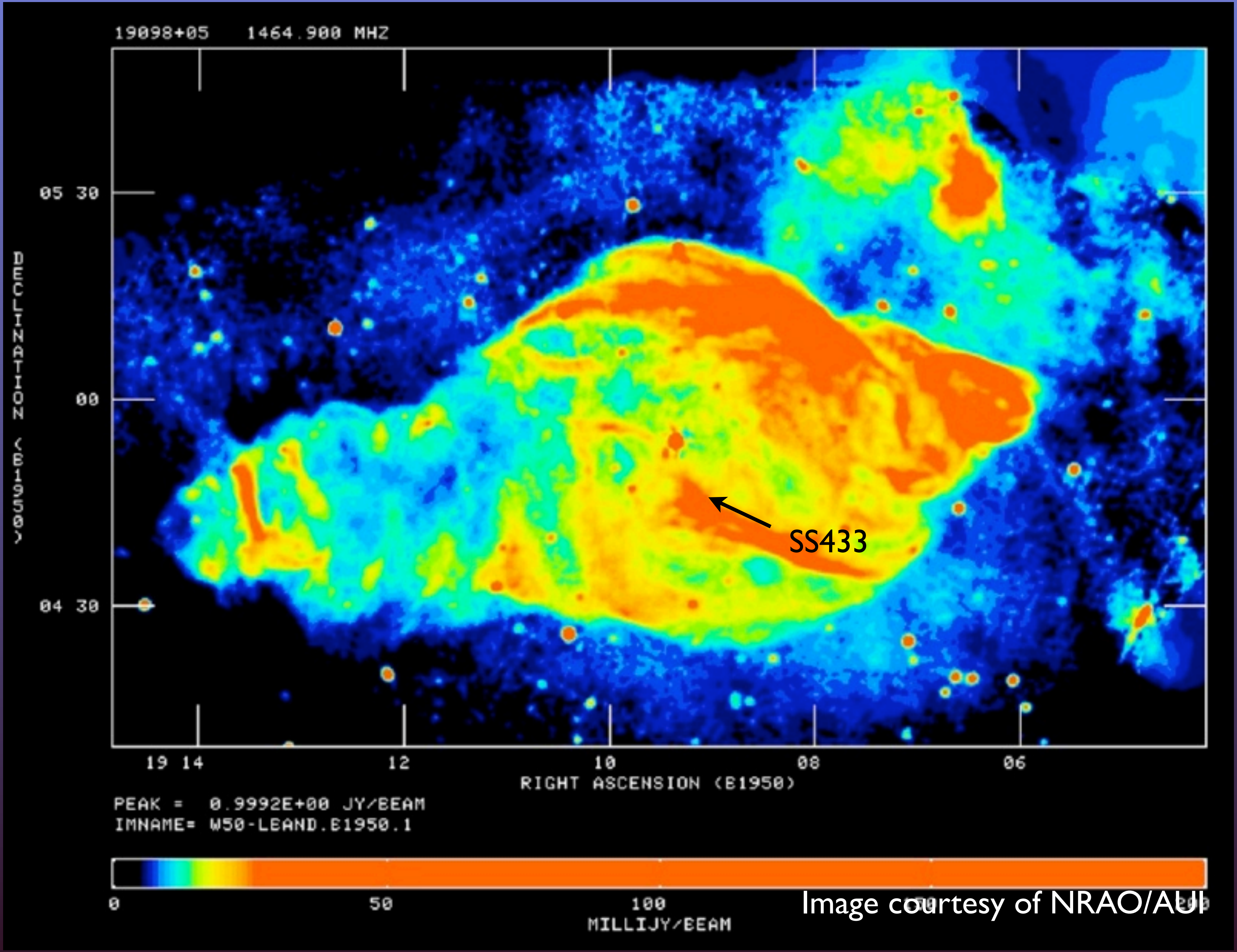


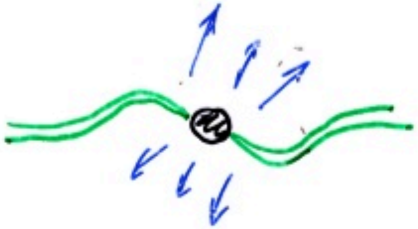
Image courtesy of NRAO/AUI

Jet precession

SS 433

AGN : - hot spot morphology
- lack of correlation
jet axis ↔ galactic plane

interpretation : precession of warped disk



Warps by instability due to irradiation

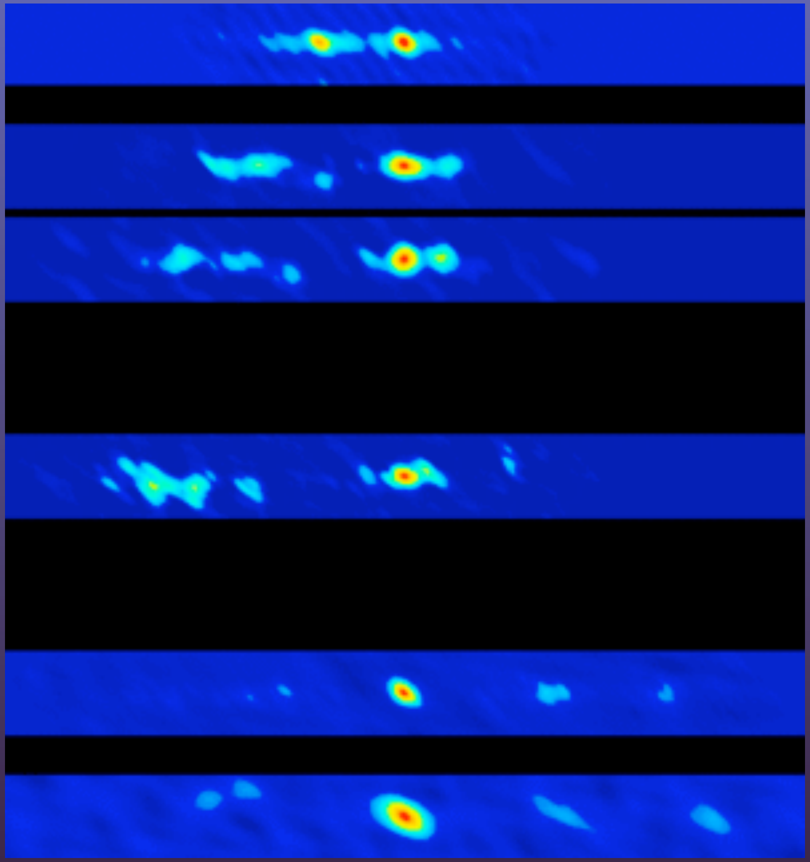
→ direct : radiation pressure
(Petterson 75(?) ; Pringle '95)

→ indirect : radiation-driven wind reaction
(Schandl & Meyer '94)

Definitive formalism : Ogilvie MRAS 1999

Slow precession : apparently "bent" jet :

Microquasars: black hole binaries with radio jets

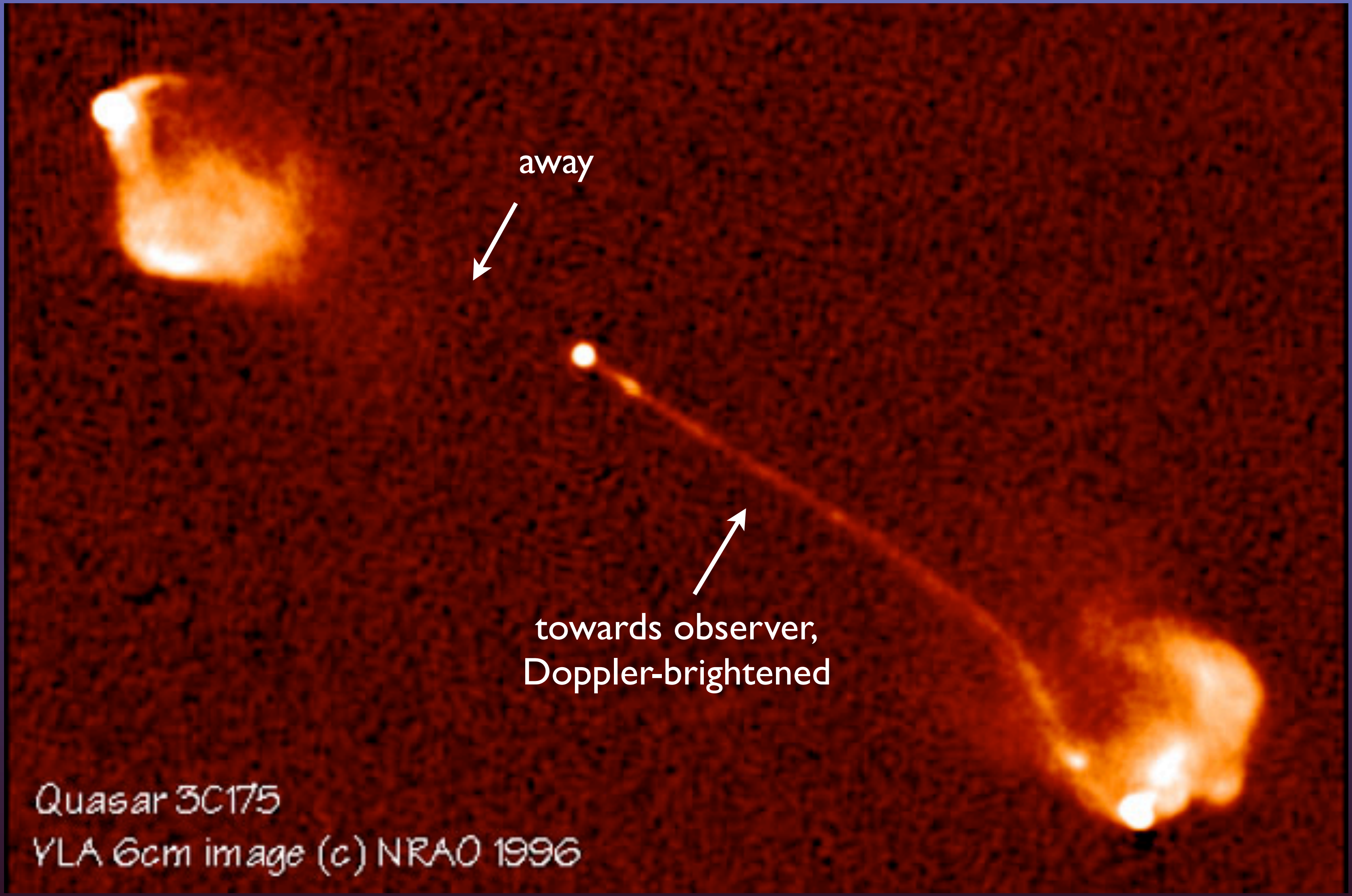


GRS 1655-40 VLBA (NRAO/AUI)

$\sim 10 M_\odot$ instead of $10^7 - 10^9$

'blobs' moving at 'superluminal' apparent speed $\gamma \sim 2 - 10$

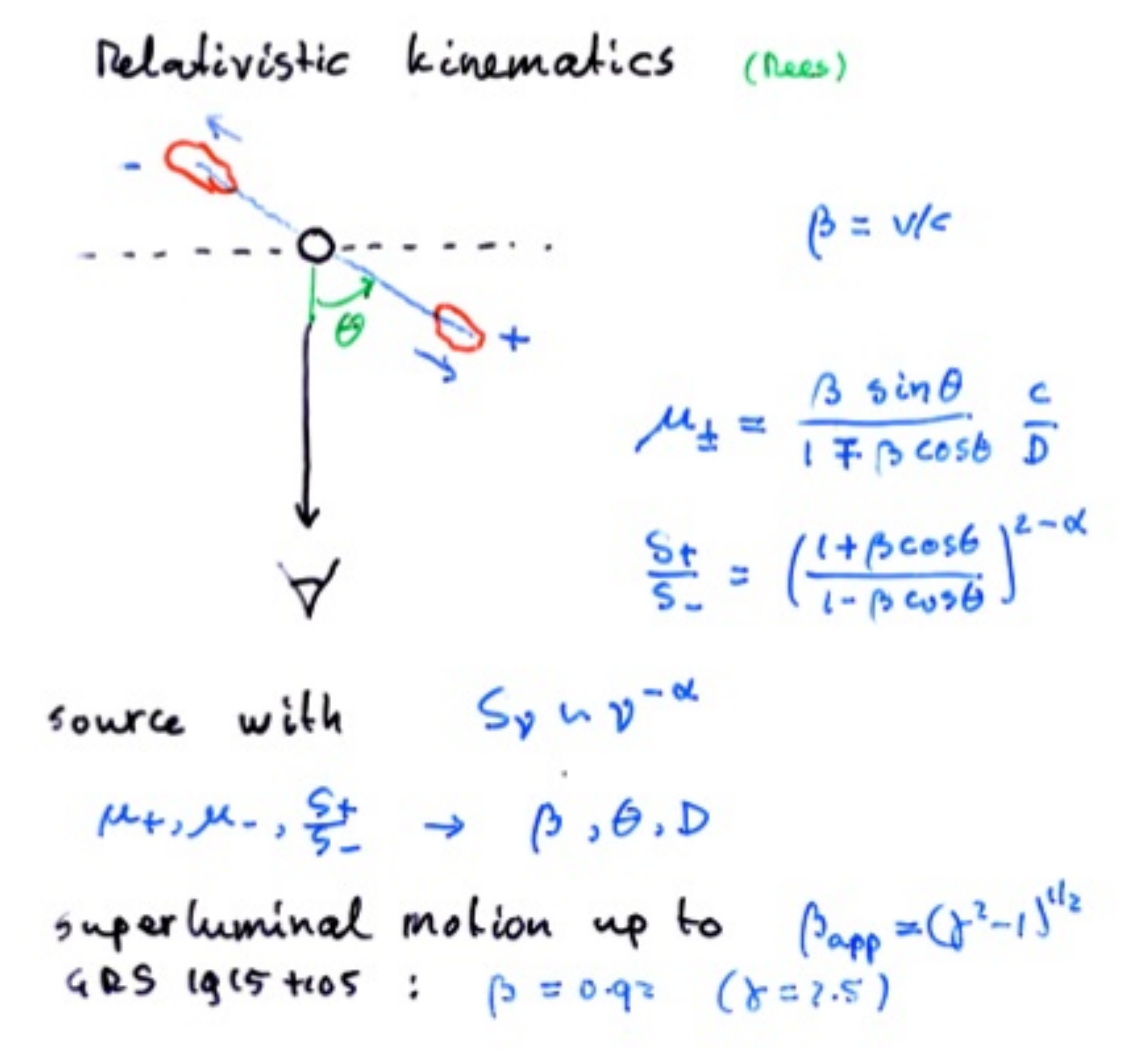
One-sided jets (but *not* one-sided radiolobes): evidence of relativistic flow speeds



away

towards observer, Doppler-brightened

Apparent ‘superluminal’ motion



Relativistic kinematics (Rees)

$\beta = v/c$

$$\mu_\pm = \frac{\beta \sin\theta}{1 \mp \beta\cos\theta}\,\frac{c}{D}$$

$$\frac{S_+}{S_-} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{2-\alpha}$$

source with $S_\nu \sim \nu^{-\alpha}$

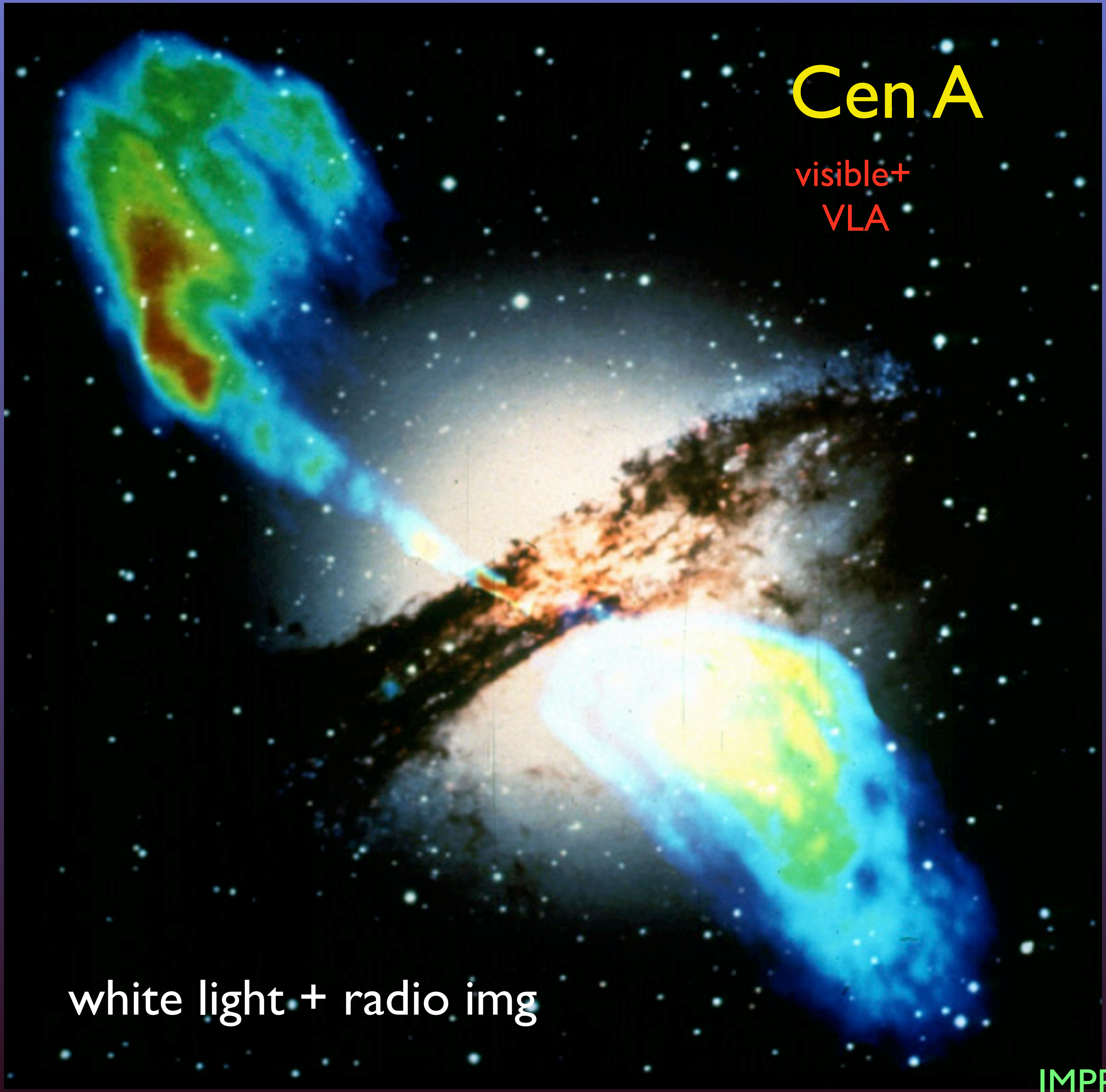
$\mu_+, \mu_-, \frac{S_+}{S_-} \rightarrow \beta, \theta, D$

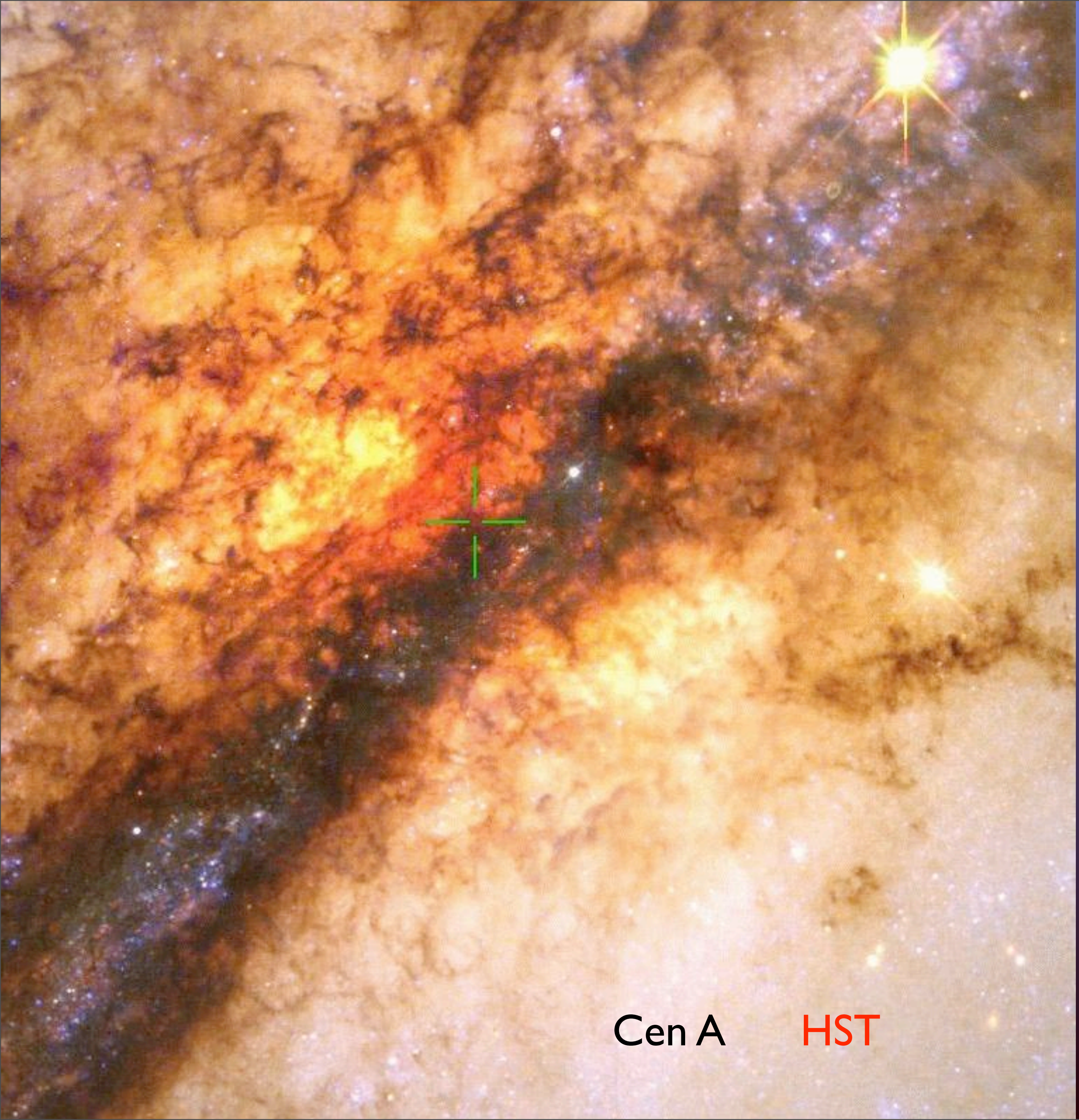
superluminal motion up to $\beta_{app} = (\gamma^2-1)^{1/2}$

GRS 1915+105 : $\beta = 0.92$ $(\gamma = 2.5)$

Doppler effect increases apparent proper motion of proximal jet (and slows down distal jet)

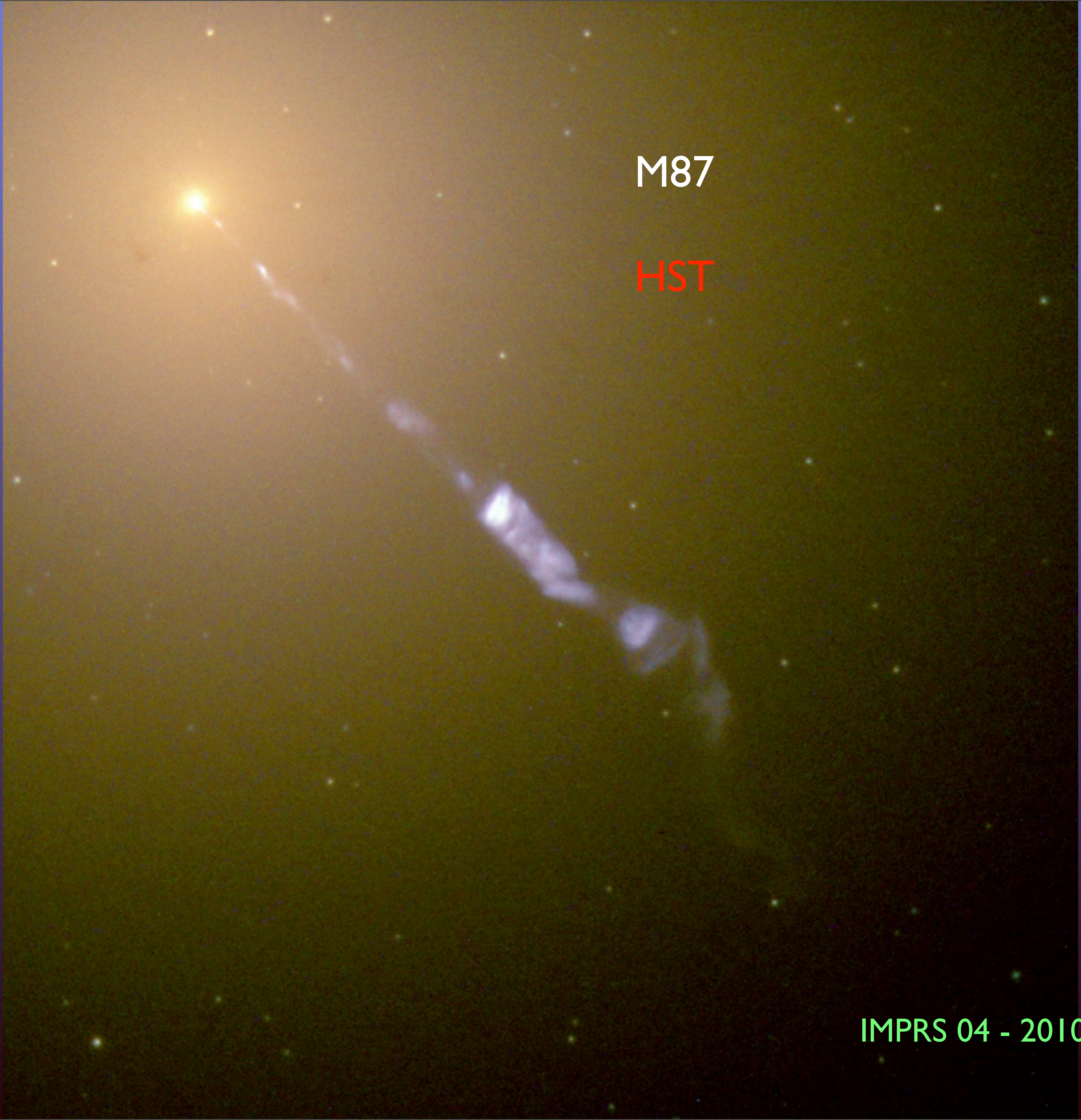
Lorentz factor and angle to line of sight derived from asymmetric proper motions and brightness


Cen A
visible+
VLA
white light + radio img

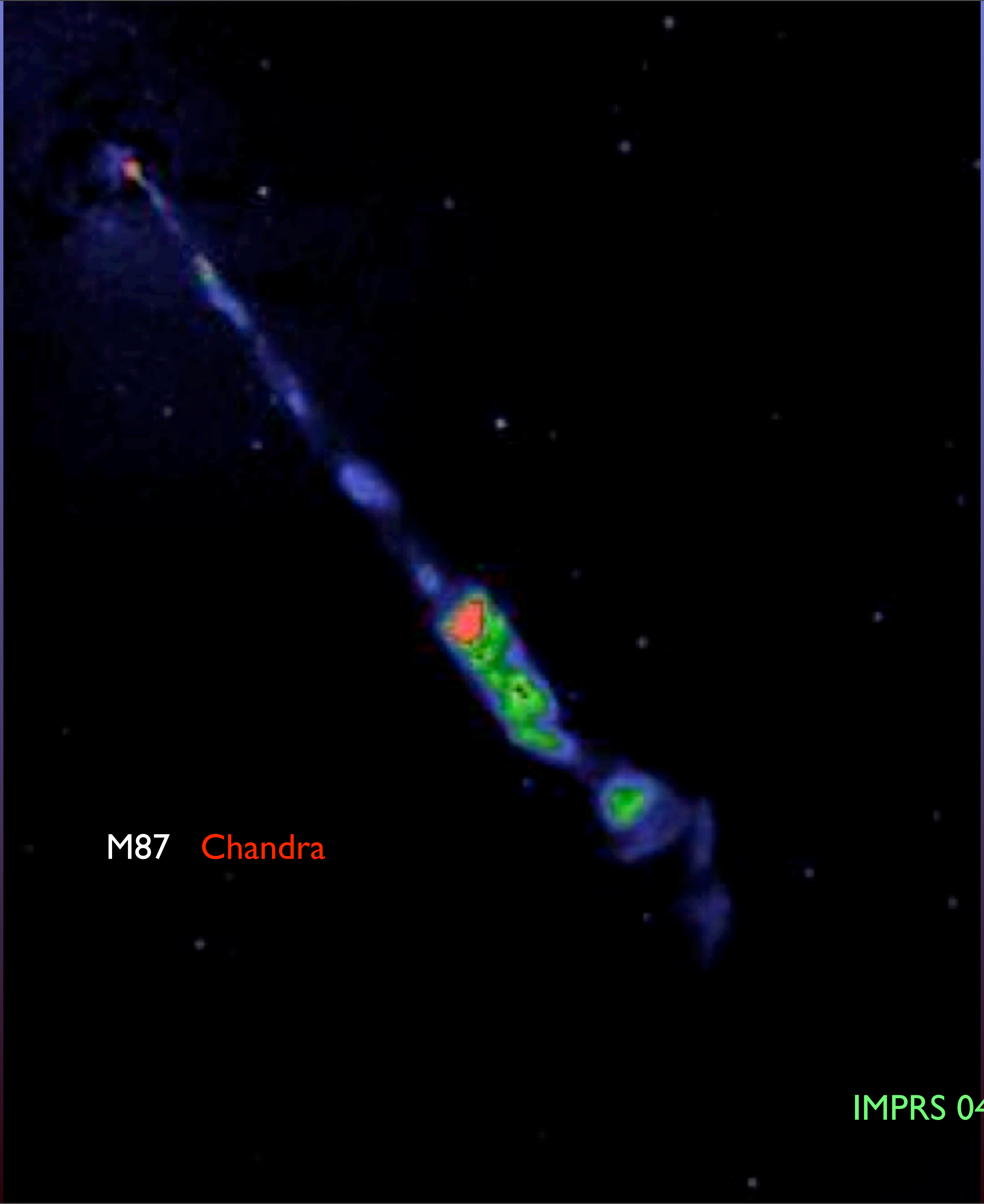


Cen A HST


Cen A
Chandra



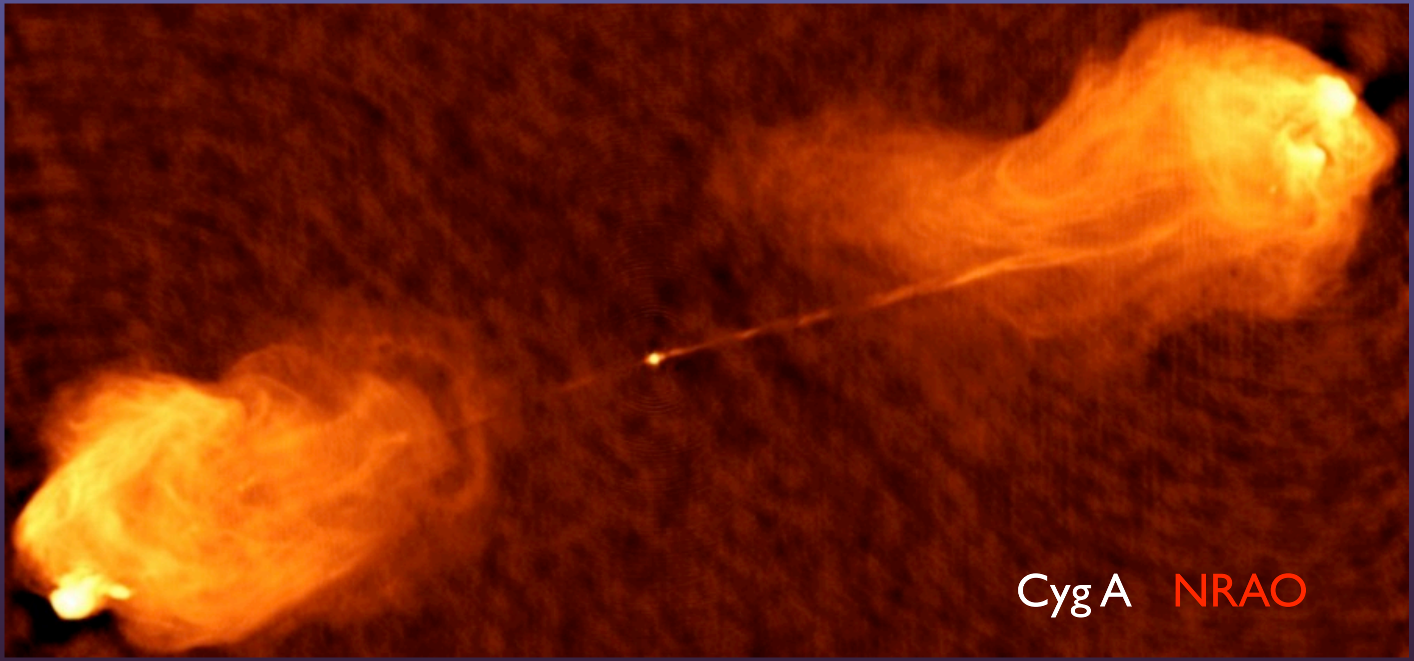
M87
HST

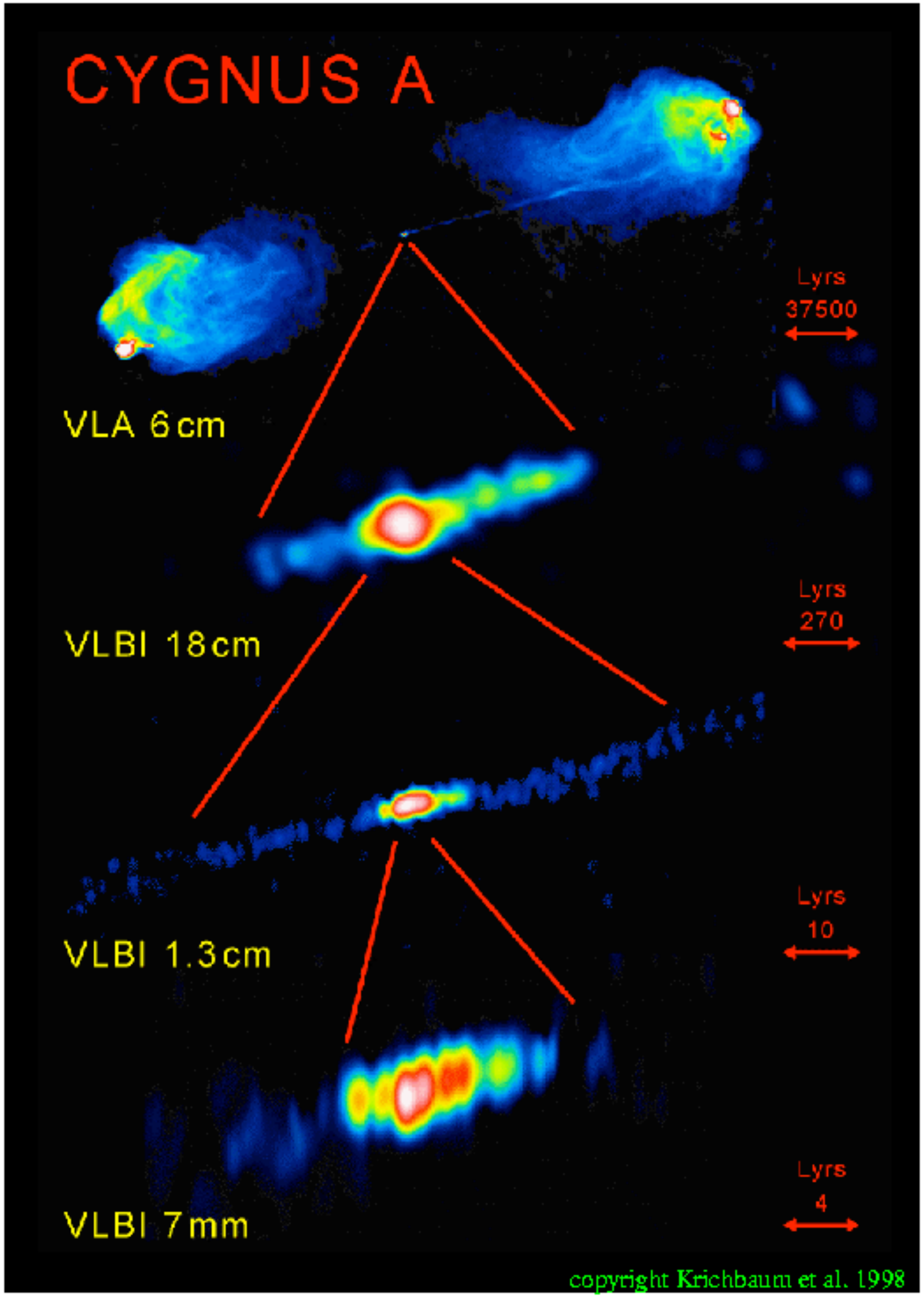


M87 Chandra

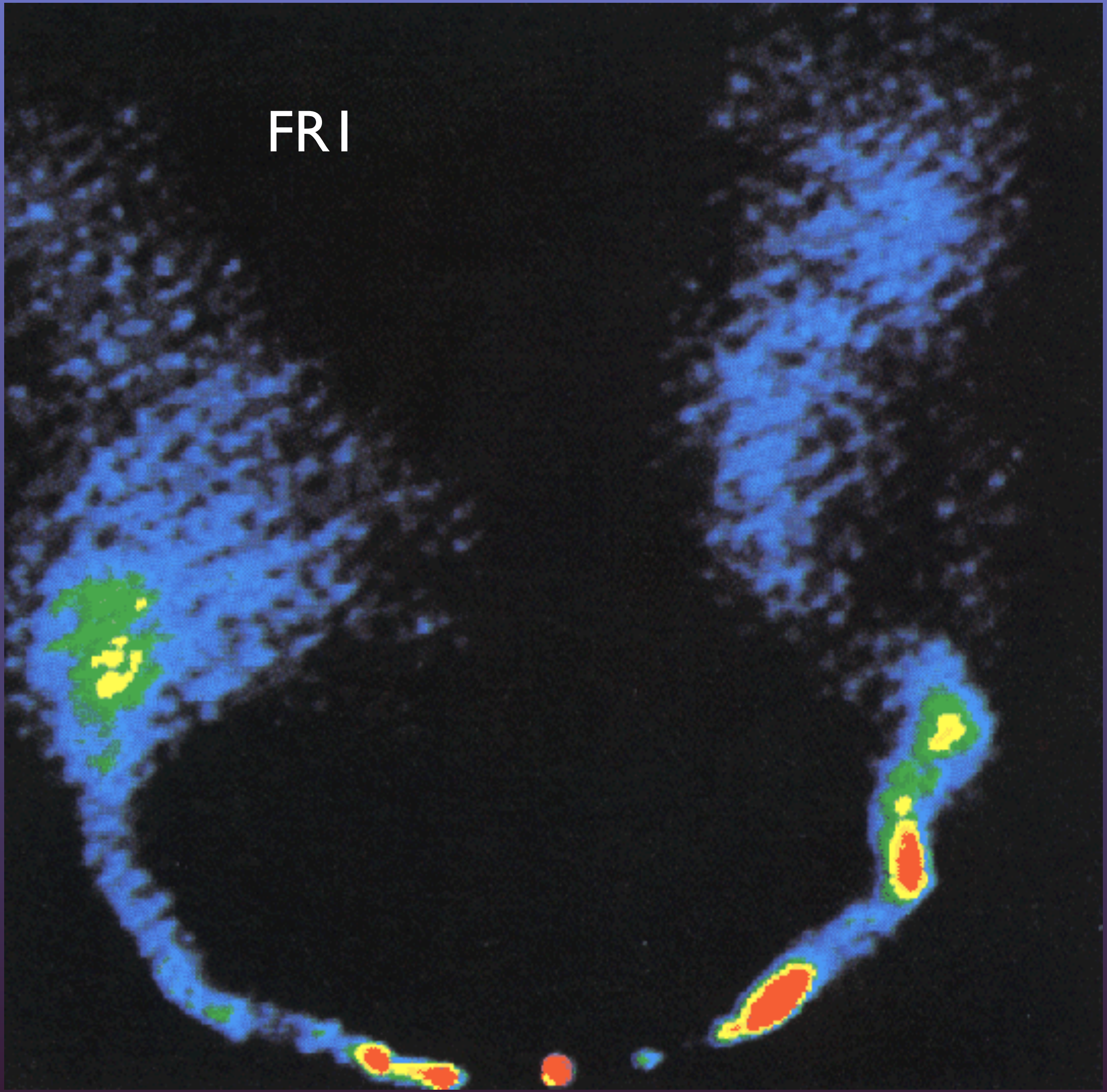
classical double-lobed radio source with jets visible

$\gamma \sim 10 - 30$



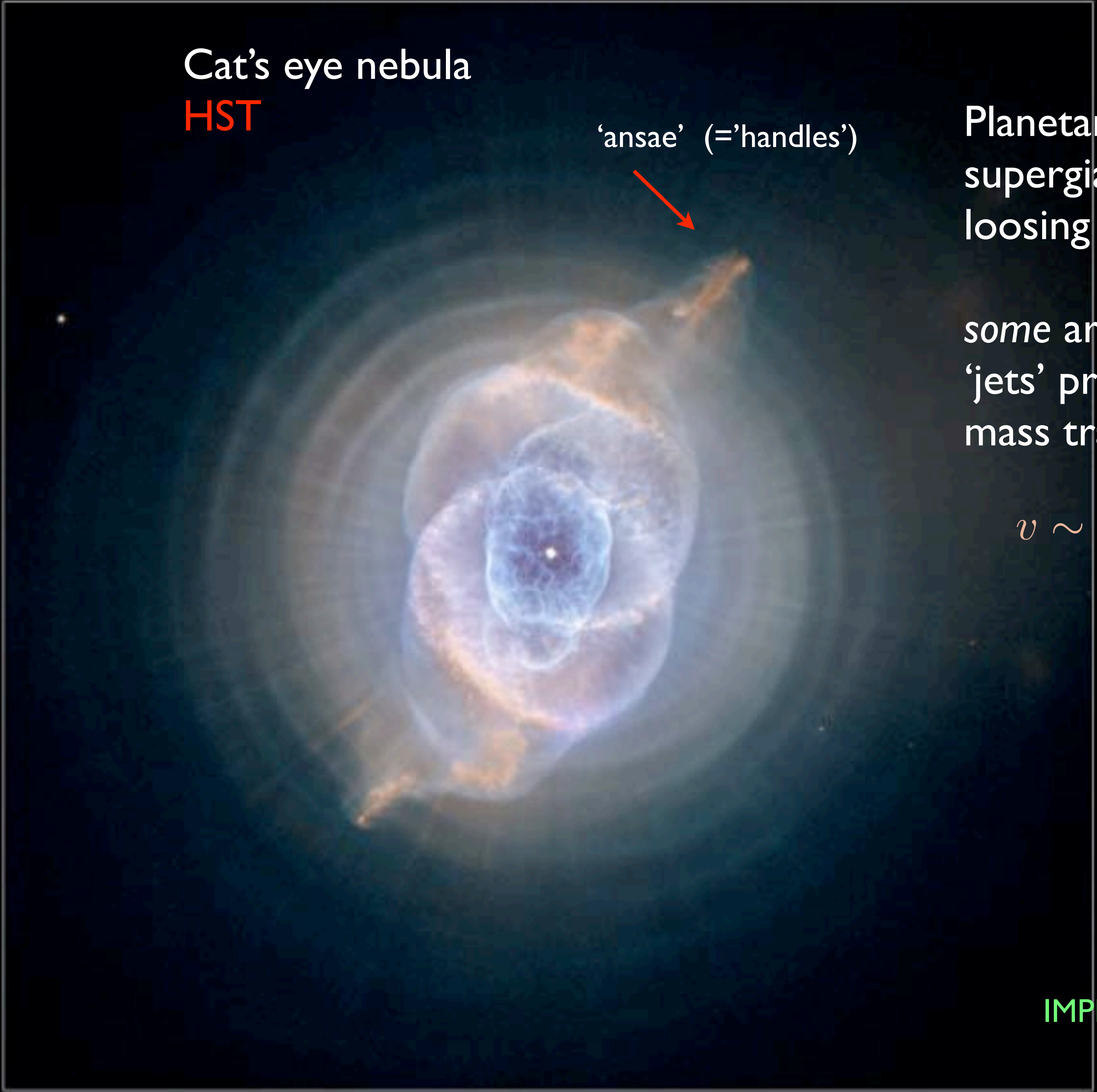


copyright Krichbaum et al. 1998



FR1 vs FR11 classification
FRII: lobes fed by narrow relativistic jet

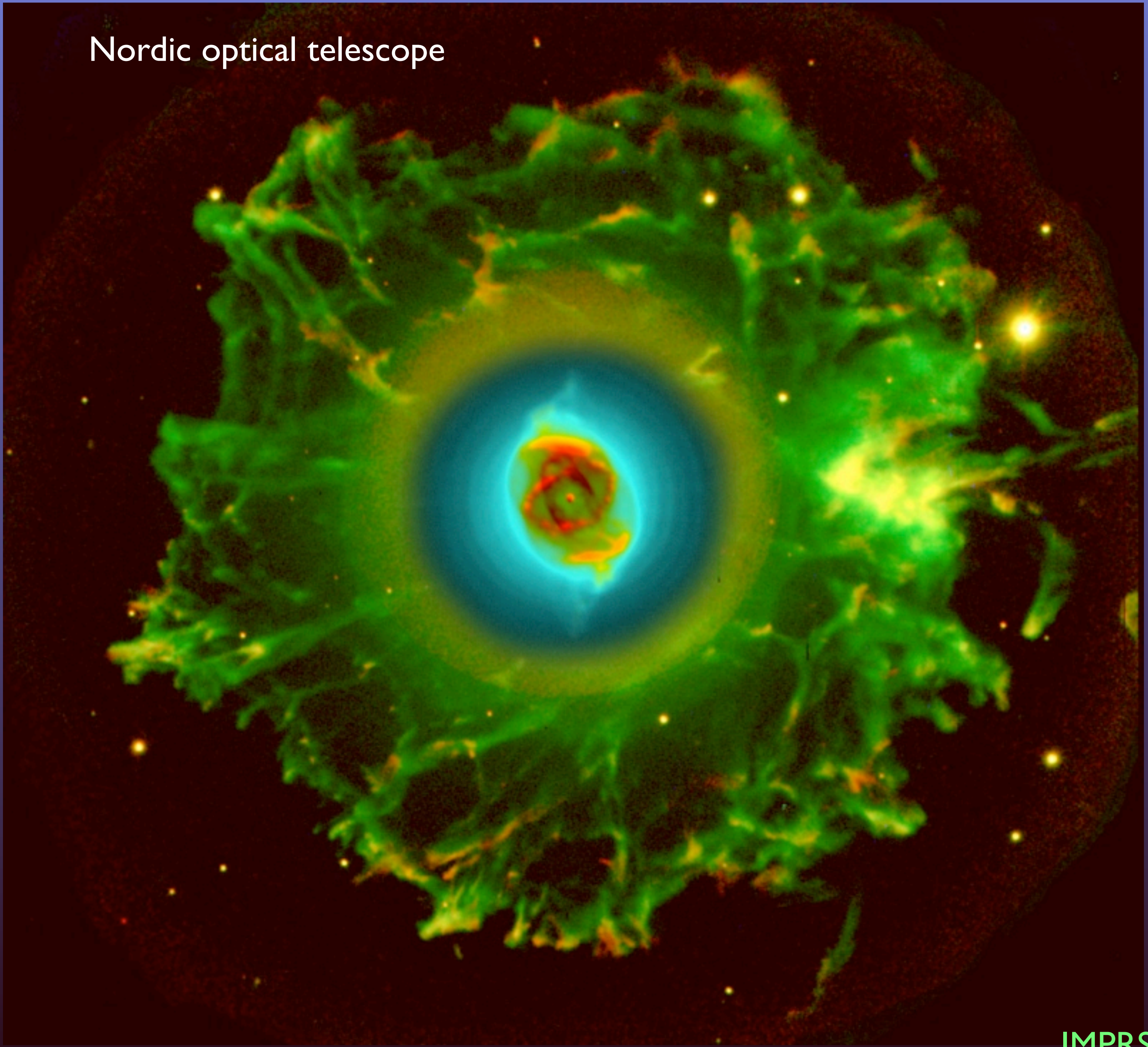
FRI: jet slowed by interaction with intergalactic medium

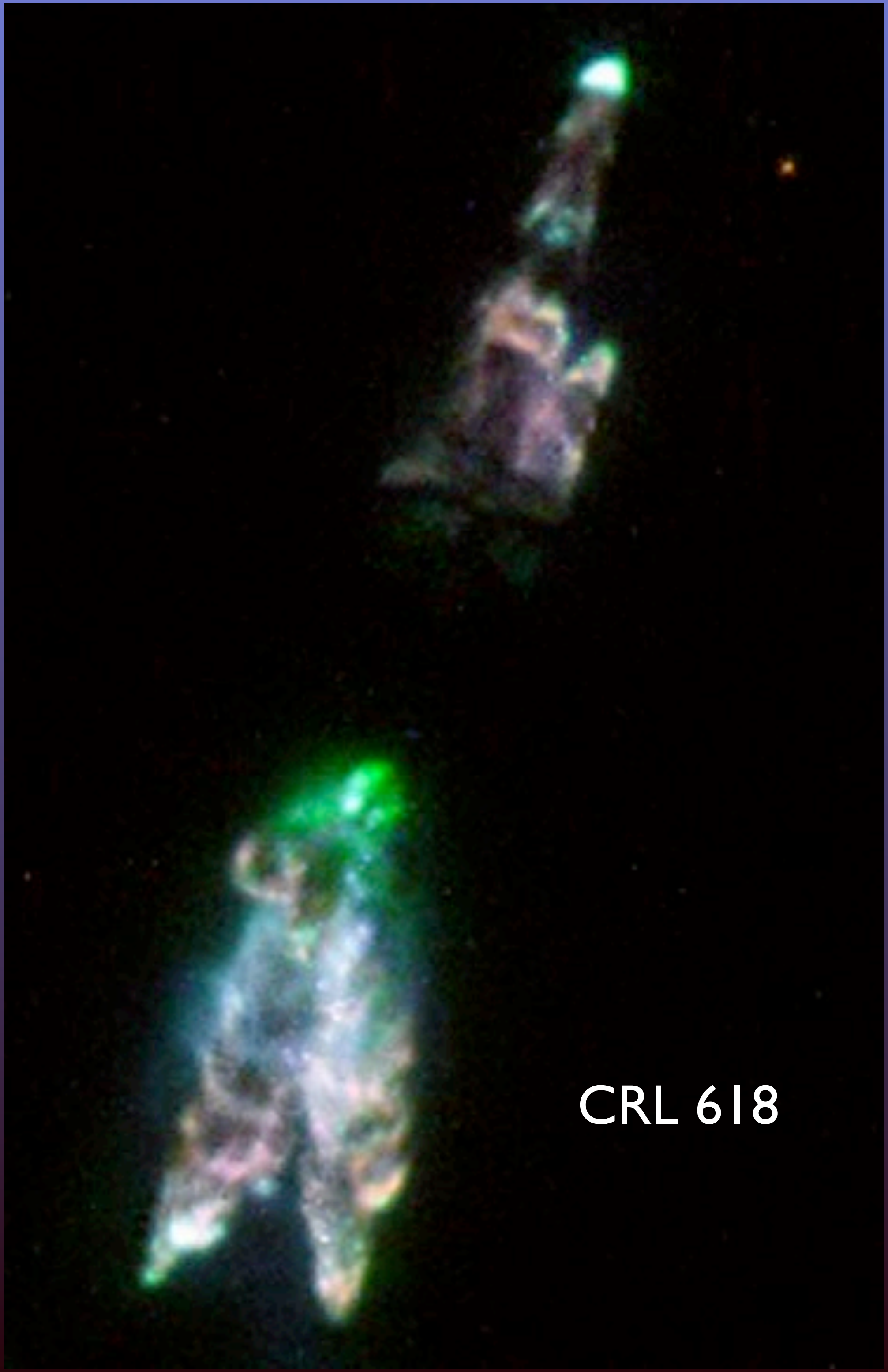


Planetary nebula: red supergiant star (AGB) loosing its envelope

some are in binaries, but 'jets' probably not due to mass transfer or accretion

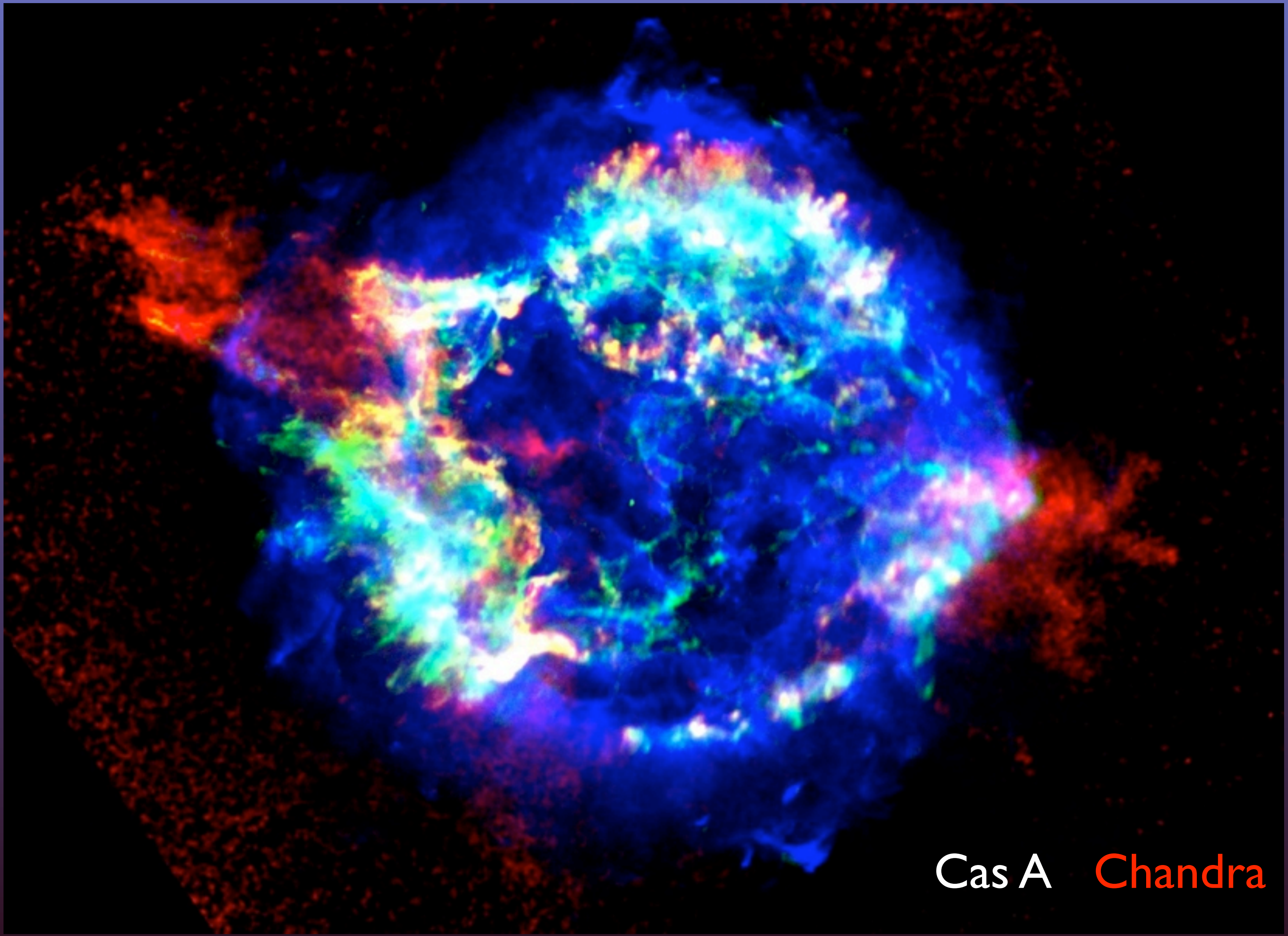
$$v \sim 100 \text{ km/s}$$


Nordic optical telescope



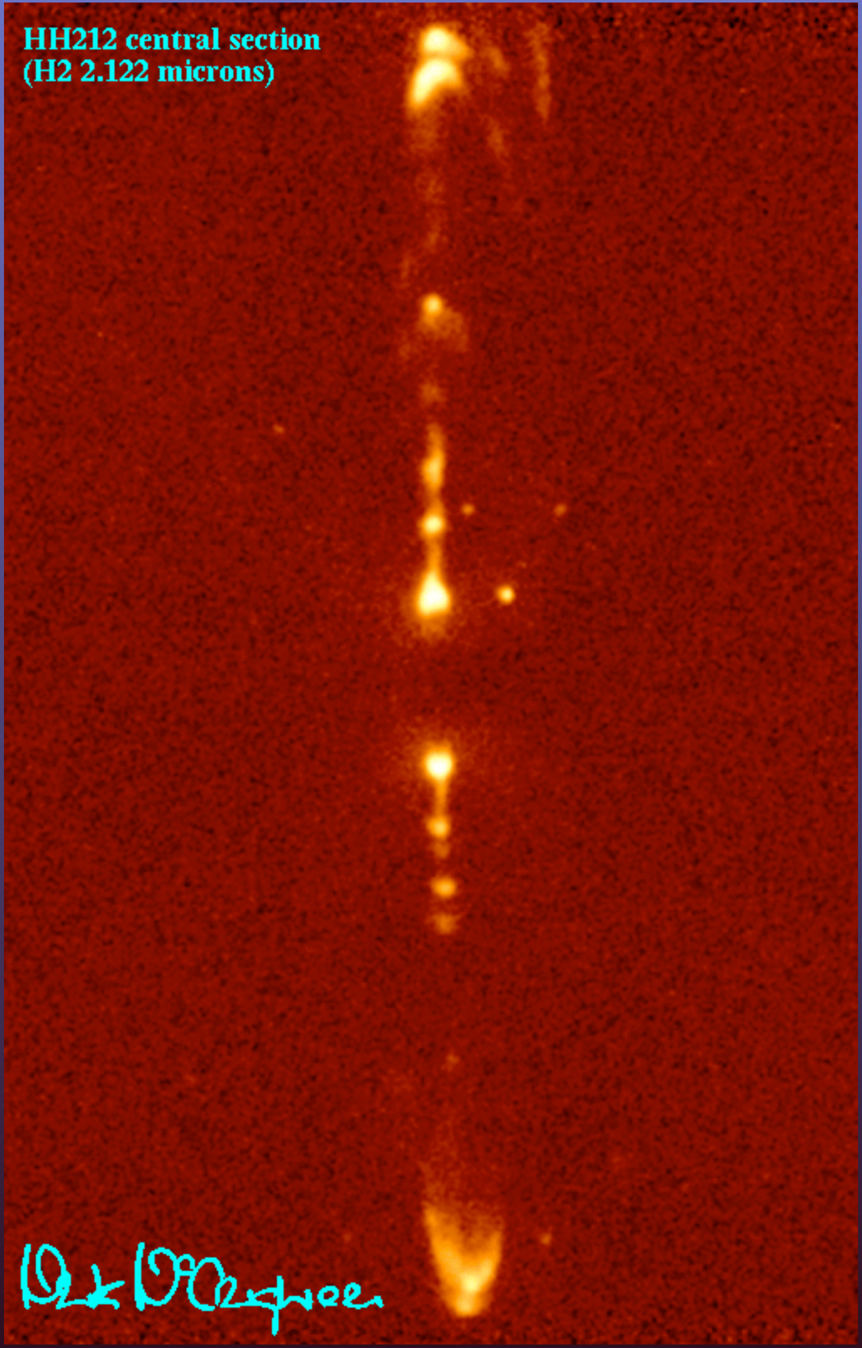
First phases of the formation of a planetary nebula

‘ansae’ in a supernova remnant



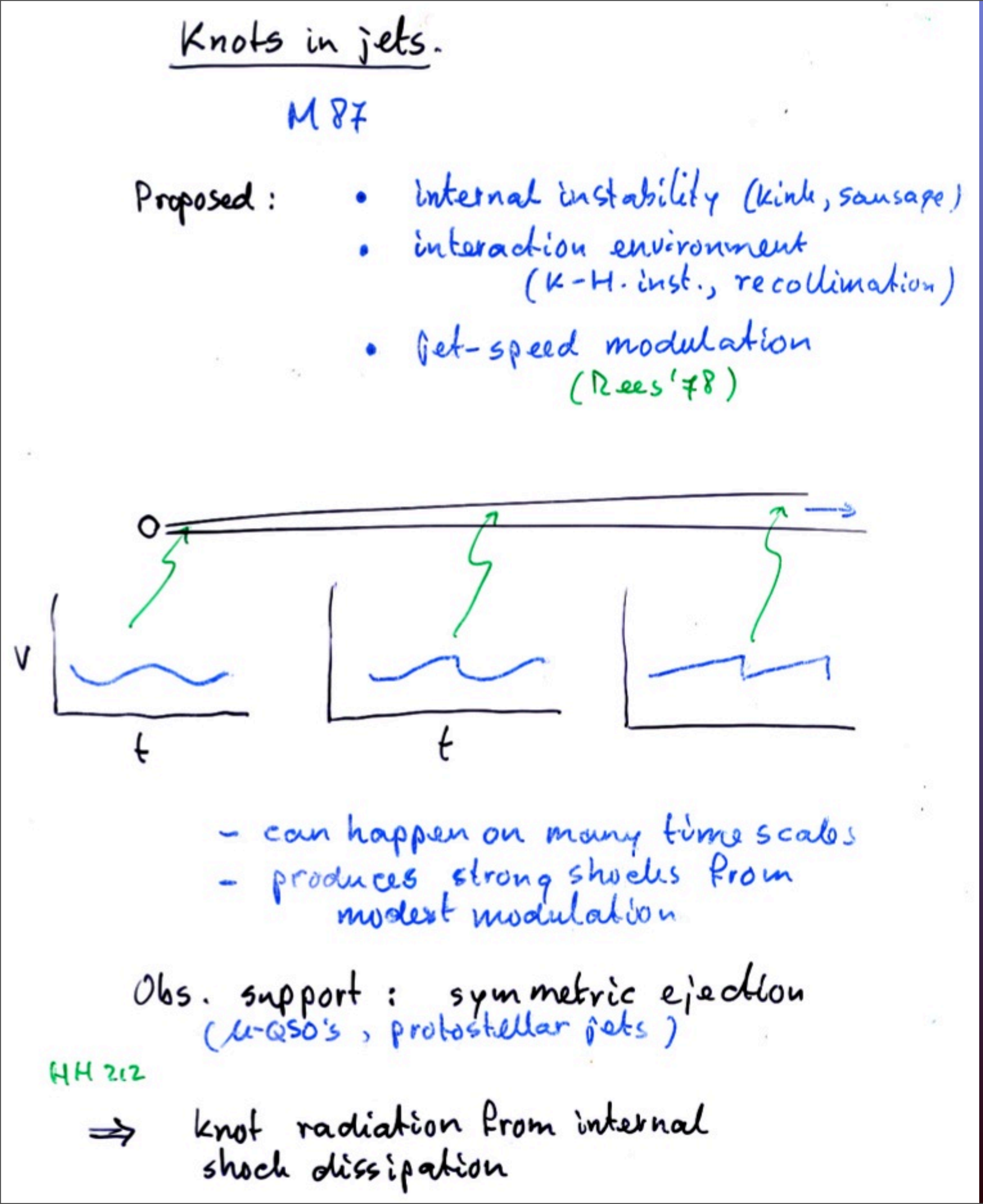
‘Observability’ of the source of the jet

	inner radius of disk	distance	angular scale (”)
	r_0	D	$100\ r_0/D$
nearby protostar	$3R_\odot$	500 pc	$0''.003$ ←
nearby AGN	10 AU	10 Mpc	$0''.0001$
galactic BHC	100 km	2 kpc	$3\,10^{-8}$ ”



Knots in protostellar jets

- often symmetric
- source produces variable mass outflow
- flow speed from proper motion of knots



Knots in jets.

M 87

Proposed:
- internal instability (kink, sausage)
- interaction environment
  (K-H. inst., recollimation)
- jet-speed modulation
  (Rees '78)

- can happen on many time scales
- produces strong shocks from modest modulation

Obs. support: symmetric ejection
(μ-QSO's, protostellar jets)

HH 212

⇒ knot radiation from internal shock dissipation

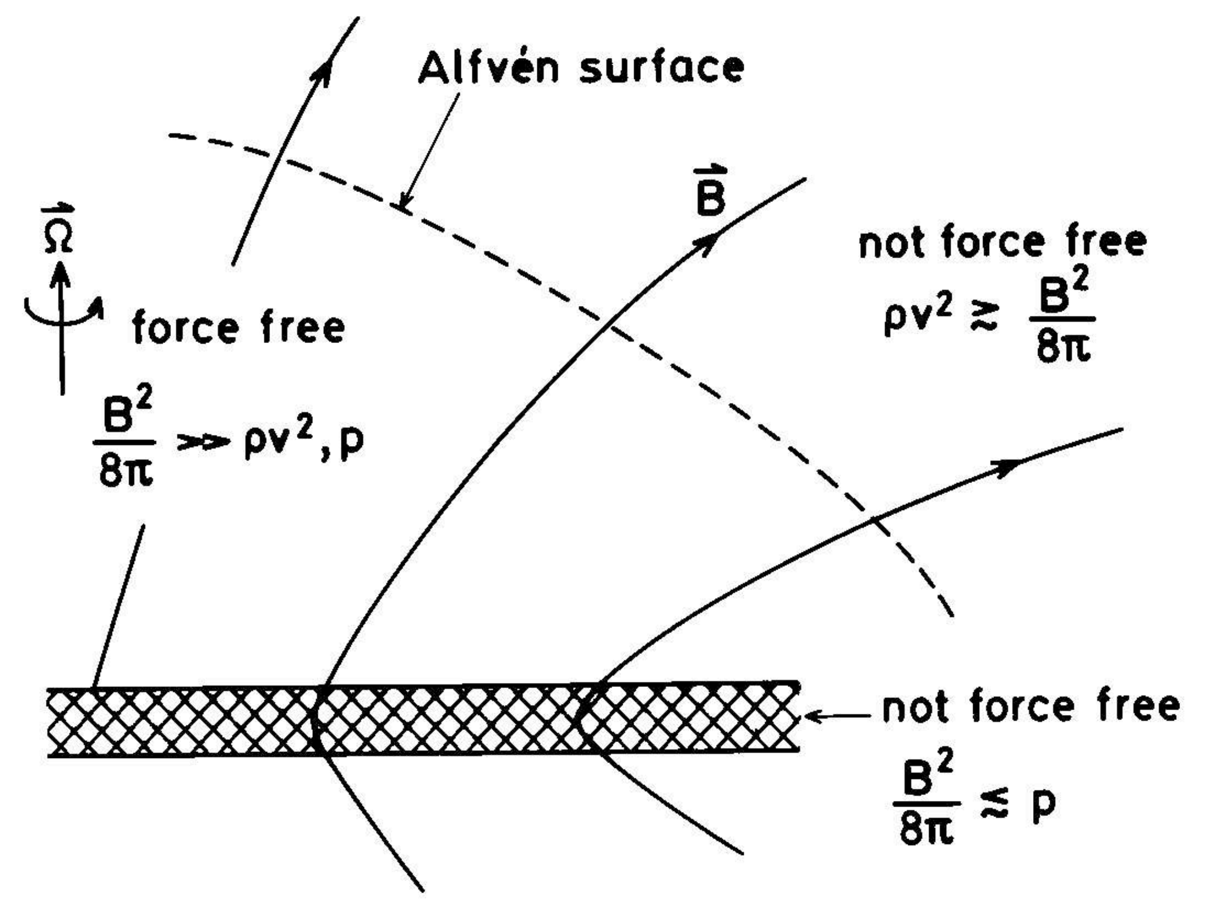
Knot formation by modulation of flow speed:
internal shocks
- model for time variability
in blazars and GRB

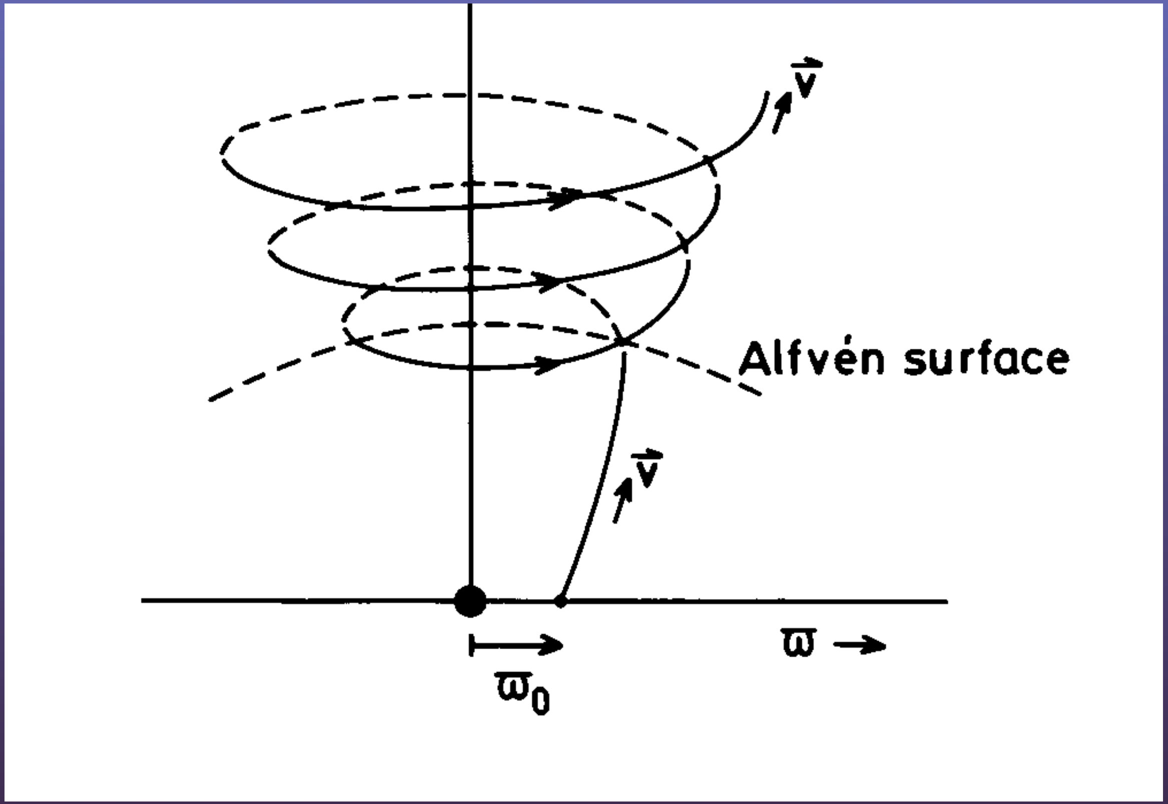
Magnetic jets: history

- Schatzman 1962 proposes spindown of the Sun by magnetic field in the solar wind
- Weber & Davis '67, Mestel '61-'67 formal MHD theory developed
- F.C. Michel '69, '73: relativistic wind from pulsars
- 1976: application to jets (Blandford, Bisnovatyi-Kogan & Ruzmaikin
- Blandford & Payne 1982: selfsimilar model
- '80s, '90s 2-D (axisymmetric numerical simulations)
- '00s: 3-D simulations

The magnetic model

Gravitation → rotation → magnetic → kinetic




$\vec{v}$
Alfvén surface
$\vec{v}$
ϖ_0
$\varpi \rightarrow$

Magnetic acceleration

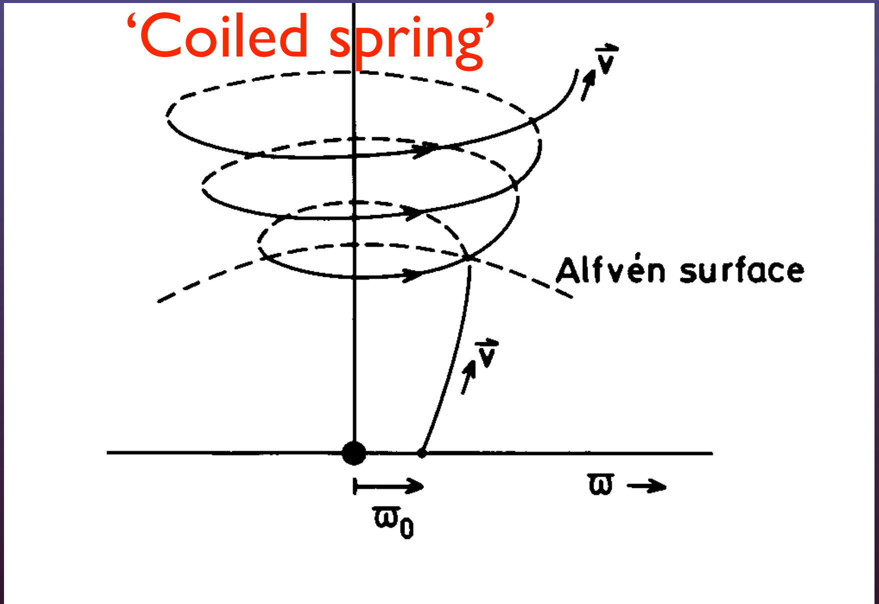
rotation → *magnetic* → *kinetic*

region $r \sim r_{\mathrm{Alfven}}$

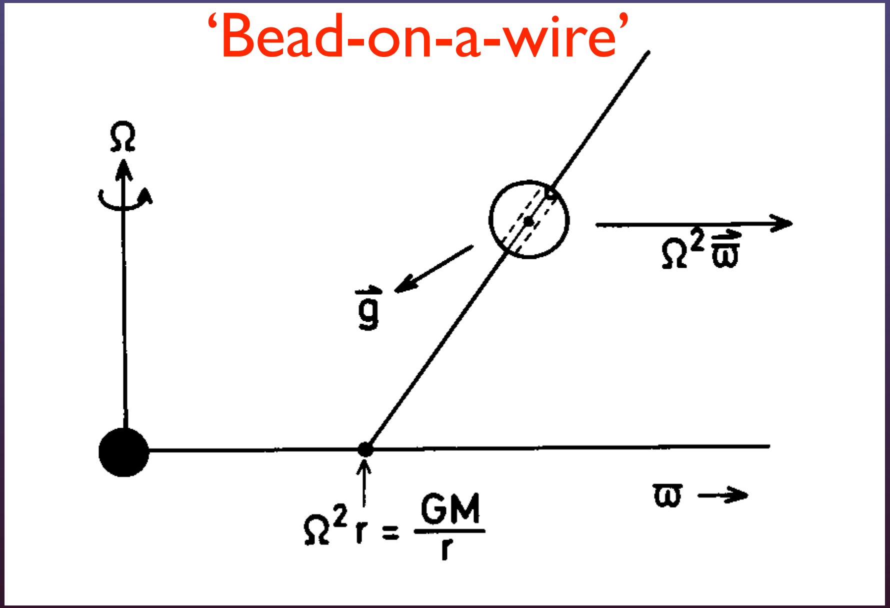
- Magnetic pressure
- Centrifugal acceleration
- Poynting flux conversion
- 'Magnetic towers'

} *Equivalent*

'Coiled spring'



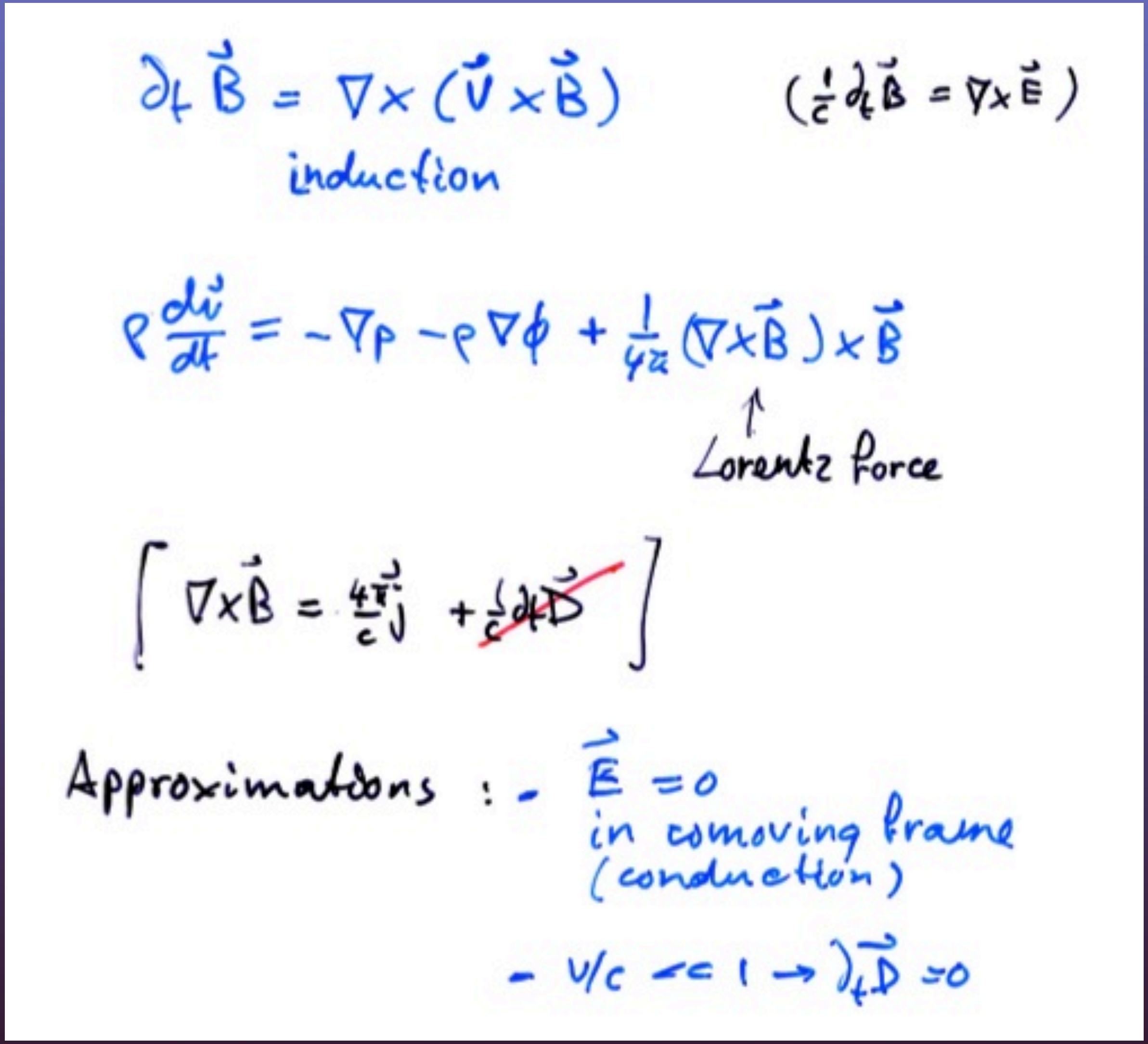
'Bead-on-a-wire'



centrifugal acceleration ↔ collimation

centrifugal acceleration requires field bent outward
→ need collimation after acceleration
demanding: AGN jets often < 3 degrees

Magnetohydrodynamics



$$\partial_t \vec{B} = \nabla\times(\vec{v}\times\vec{B}) \qquad \left(\frac{1}{c}\partial_t\vec{B} = \nabla\times\vec{E}\right)$$

induction

$$\rho\frac{d\vec{v}}{dt} = -\nabla p - \rho\nabla\phi + \frac{1}{4\pi}(\nabla\times\vec{B})\times\vec{B}$$

↑ Lorentz force

$$\left[\nabla\times\vec{B} = \frac{4\pi}{c}\vec{j} + \cancel{\frac{1}{c}\partial_t\vec{D}}\right]$$

Approximations :
- $\vec{E} = 0$ in comoving frame (conduction)
- $v/c \ll 1 \rightarrow \partial_t\vec{D} = 0$

2 vectors: $\mathbf{v}$, $\mathbf{B}$

(current, charge density and electric field irrelevant)

Magnetic fluid theory
not *electromagnetism*

$\mathbf{B}$ not *'generated by currents'*

$\mathbf{B}$ *evolves in interaction with fluid*

Analogy: elastic media

Magnetohydrodynamics

- component of magnetic force along B vanishes

- a flow perpendicular to B carries field lines with it

- 2 regimes depending on strength of B :

$$\beta \equiv \frac{8\pi P}{B^2}$$ ('plasma beta')

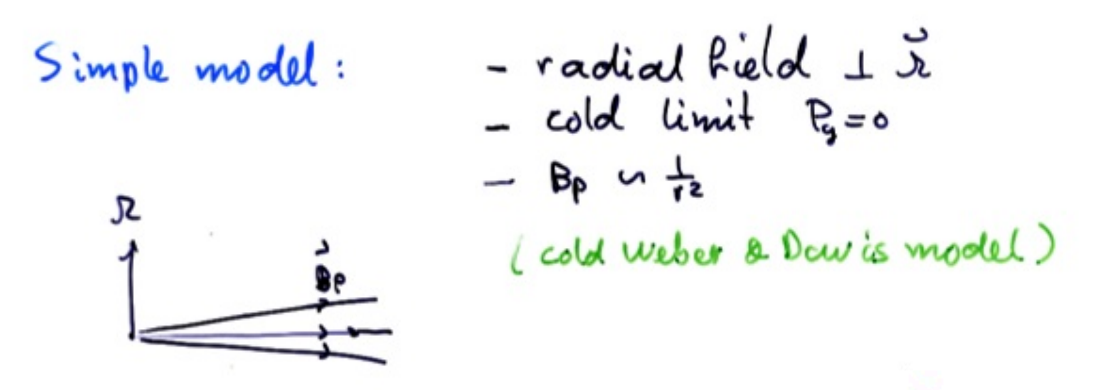
$\beta \ll 1$: magnetic field dominates. Fluid forced to flow along field lines

$\beta \gg 1$: fluid dominates, carries field lines (and wraps them around)

Steady, rotating, axisymmetric magnetic flow
- flow accelerated along field lines
- compute asymptotic speed

Model: 'Weber-Davis' (1967)
derivation: Mestel, L. Stellar magnetism, Oxford U Press, 1999
Sakurai, T. 1985, A&A 152, 121
http://www.mpa-garching.mpg.de/~henk/pub/jetrevl.pdf

Cold Weber-Davis model



Simple model :

- radial field $\perp \vec{\Omega}$
- cold limit $P_g = 0$
- $B_p \sim \frac{1}{r^2}$

(cold Weber & Davis model)

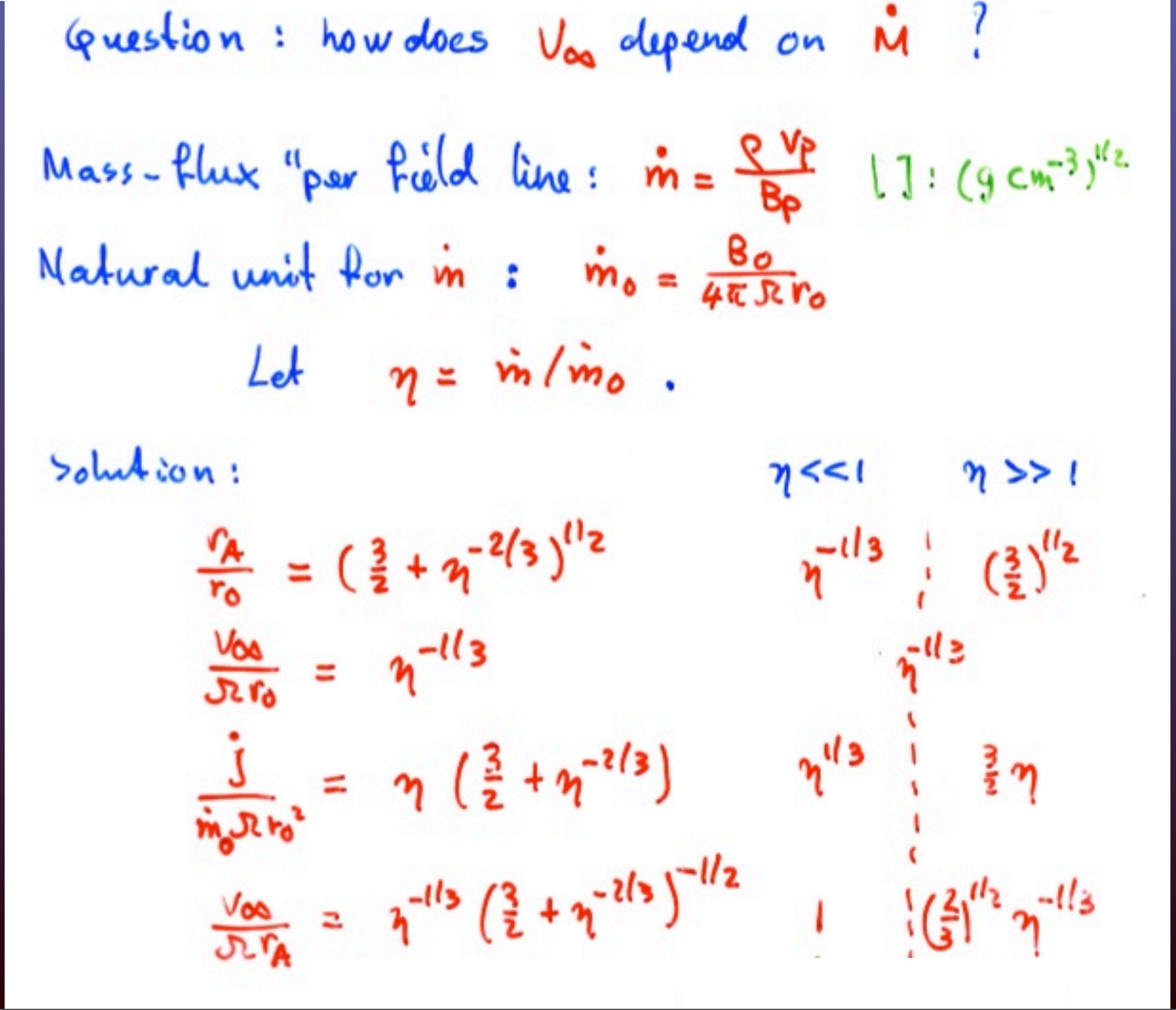
Visualize: equatorial plane. (Applies at all latitudes.)

Assumed:

- poloidal field fixed
- gas pressure neglected

compute:

- azimuthal field B_ϕ
- flow speed



Question : how does V_∞ depend on $\dot{M}$?

Mass-flux "per field line : $\dot{m} = \frac{\rho v_p}{B_p}$ [ ] : $(g\,cm^{-3})^{1/2}$

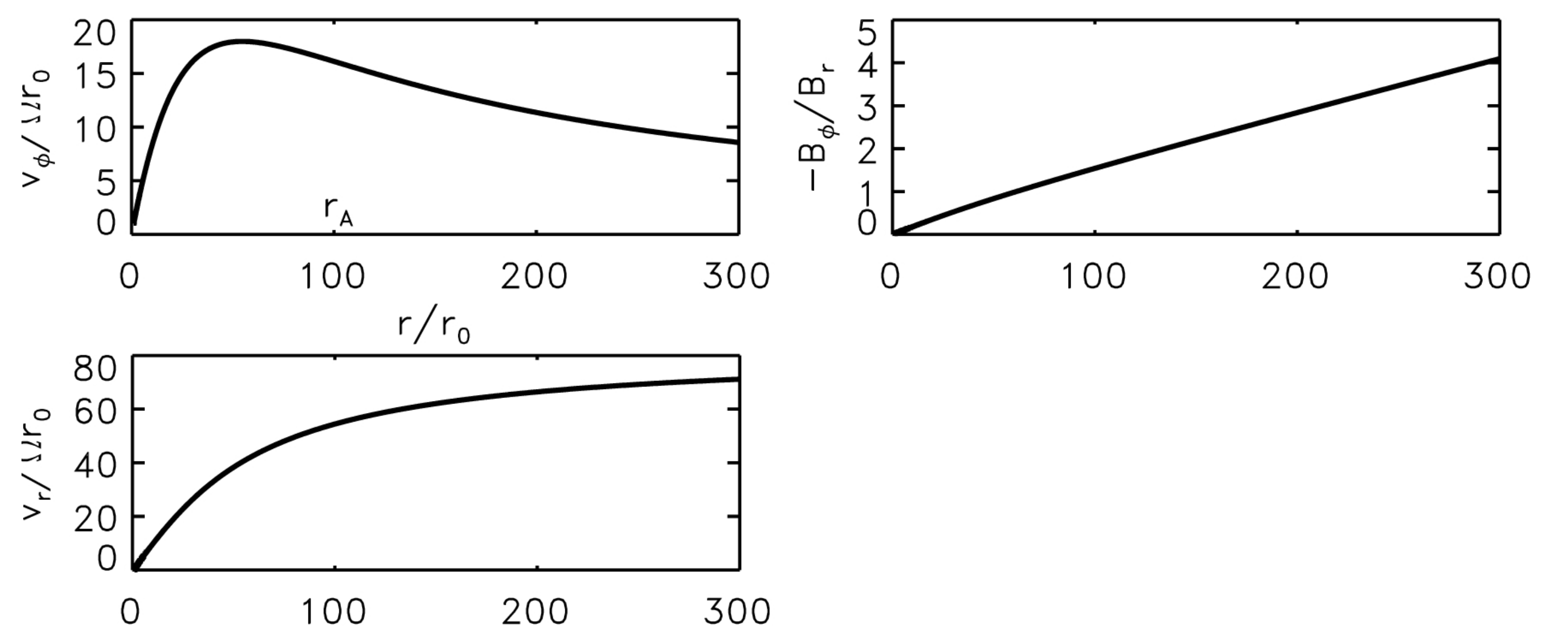
Natural unit for $\dot{m}$: $\dot{m}_0 = \frac{B_0}{4\pi \Omega r_0}$

Let $\eta = \dot{m}/\dot{m}_0$.

Solution :

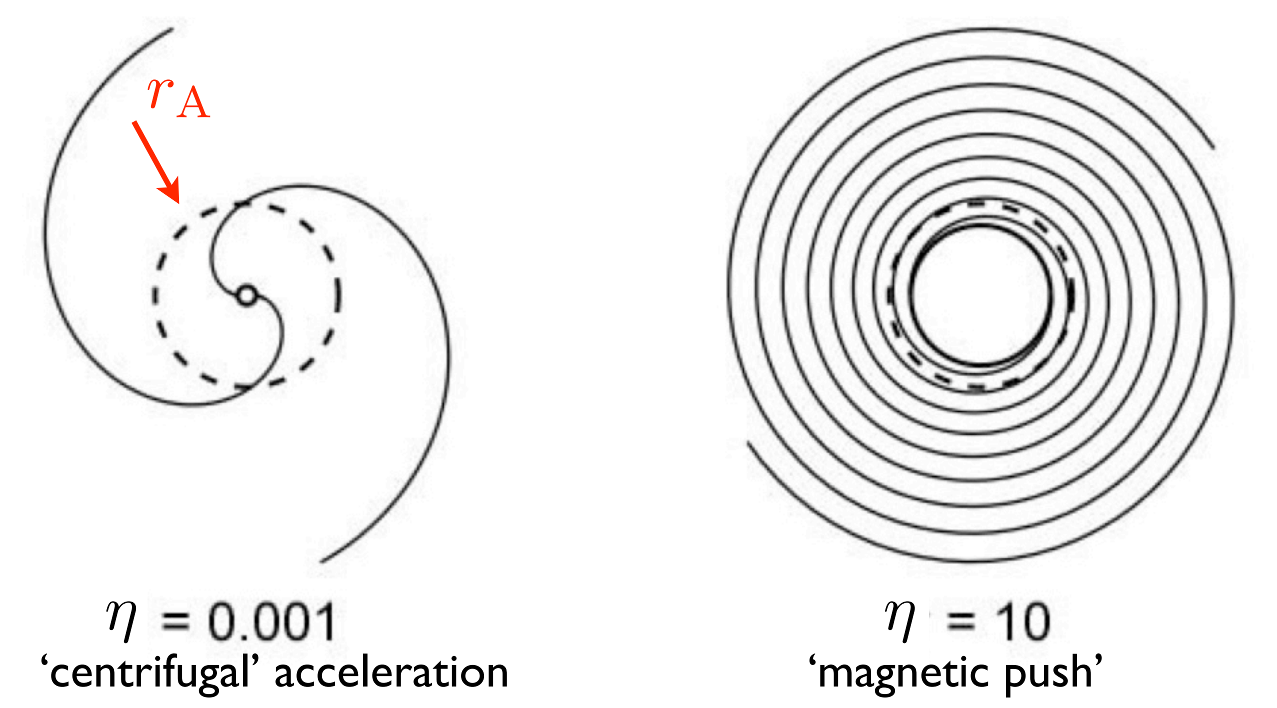
	$\eta << 1$	$\eta >> 1$
$\frac{r_A}{r_0} = \left(\frac{3}{2} + \eta^{-2/3}\right)^{1/2}$	$\eta^{-1/3}$	$\left(\frac{3}{2}\right)^{1/2}$
$\frac{V_\infty}{\Omega r_0} = \eta^{-1/3}$		$\eta^{-1/3}$
$\frac{\dot{J}}{\dot{m}_0 \Omega r_0^2} = \eta\left(\frac{3}{2} + \eta^{-2/3}\right)$	$\eta^{1/3}$	$\frac{3}{2}\eta$
$\frac{V_\infty}{\Omega r_A} = \eta^{-1/3}\left(\frac{3}{2} + \eta^{-2/3}\right)^{-1/2}$	1	$\left(\frac{2}{3}\right)^{1/2} \eta^{-1/3}$

Cold Weber-Davis model: example



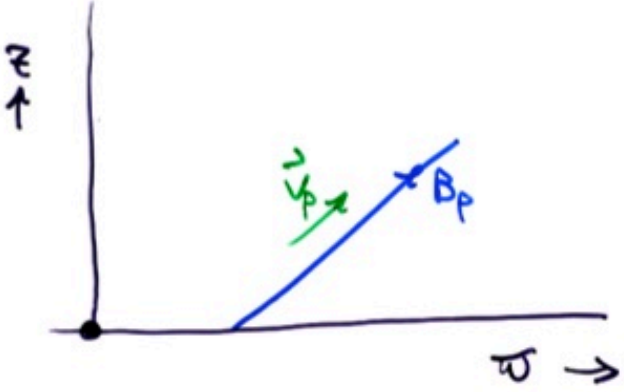
Cold Weber-Davis model

Shape of the field lines



η = 0.001
'centrifugal' acceleration

η = 10
'magnetic push'



Acceleration (in inertial frame):

a) $(j \times B)_p$

$$- \hat{\imath} \cdot \left(\nabla \frac{B_\varphi^2}{8\pi} + \frac{B_\varphi^2}{4\pi\varpi} \vec{e}_\varpi \right)$$ (>0 for acceleration)

equiv.

b) In corotating frame: Bernoulli integral:

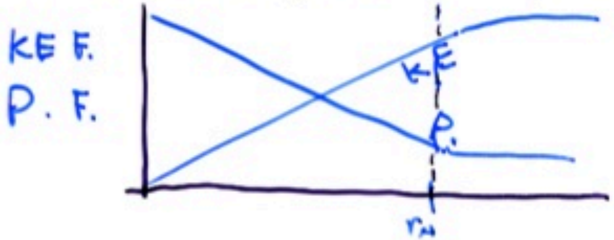
$$\frac{1}{2} v_p^2 - \frac{1}{2} \Omega^2 \varpi^2 + \cancel{\frac{1}{2} (v_\varphi - \Omega\varpi)^2} - \frac{GM}{r} + \text{enthalpy} = E = \text{cst}$$

centrifugal acceleration

small if corotating

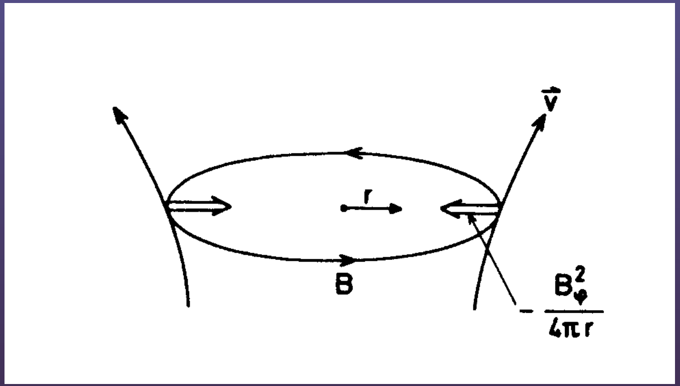
c) "Energy fluxes":

kinetic energy flux ⇔ poynting flux



Equivalent descriptions of magnetic acceleration

- ‘centrifugal’
- magnetic pressure
- ‘Poynting flux conversion’



Poynting flux in MHD

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}$$ (Gaussian units)

in MHD: $\mathbf{E} = \mathbf{v} \times \mathbf{B}/c$ (perfect conductivity)

$$\rightarrow \ \mathbf{S} = \frac{1}{4\pi}(\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \qquad = \mathbf{v}_\perp \frac{B^2}{4\pi}$$

$$u_\mathrm{m} = \frac{B^2}{8\pi}$$ magnetic energy density

$$P_\mathrm{m} = \frac{B^2}{8\pi}$$ magnetic pressure

$\mathbf{S} = \mathbf{v}_\perp(u_\mathrm{m} + P_\mathrm{m})$ ‘magnetic enthalpy flux’

Steps in jet formation

1 “launching”.

Transition from disk to flow

- how much mass flows into the jet?

2 Acceleration

- magneto-centrifugal picture
- ‘push’ from magnetic pressure B_ϕ^2

3 collimation

- how/where does external medium determine opening angle of flow?

Problem areas and current topics

http://www.mpa-garching.mpg.de/~henk/pub/spruitv3.pdf

- ‘length scales’
- net magnetic flux of a disk
- ‘hoop stress’ collimation
- acceleration ‘by dissipation’
- 3-D stability of jets
- disk-jet transition

launching

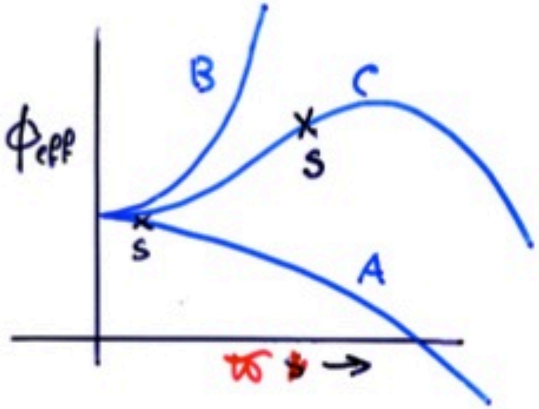
How much mass is launched?
(In num. simulations: $\dot{m}$ set by hand)

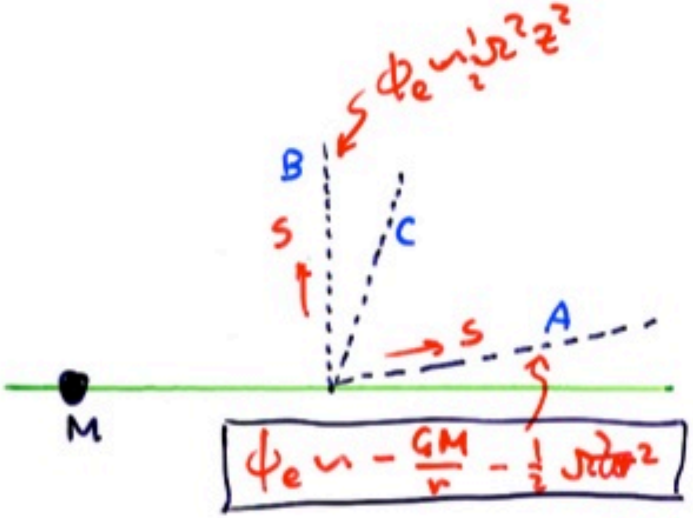
Depends on

- details of temperature structure of disk atmosphere
  → need to know energy dissipation in atmosphere
- strength and inclination of field lines at disk surface

Better defined in hot (near virial) accretion:
flow already ‘loosely bound’ in gravitational potential
→ perhaps only radiatively inefficient flows make jets ?

Sonic point, potential barrier.





$$\boxed{\Phi_e \sim -\frac{GM}{r} - \frac{1}{2}\Omega^2\varpi^2}$$

Transition between disk and jet, the 'launching region'

s-point : $V_p = V_s = \left(\frac{V_{Ap}^2 c_s^2}{V_{Ap}^2 + c_s^2}\right)^{1/2}$; $V_{Ap}^2 = \frac{B_p^2}{4\pi\rho}$

(near Φ-max)

cusp speed

$V_s \approx c_s$ $(V_A >> c_s)$

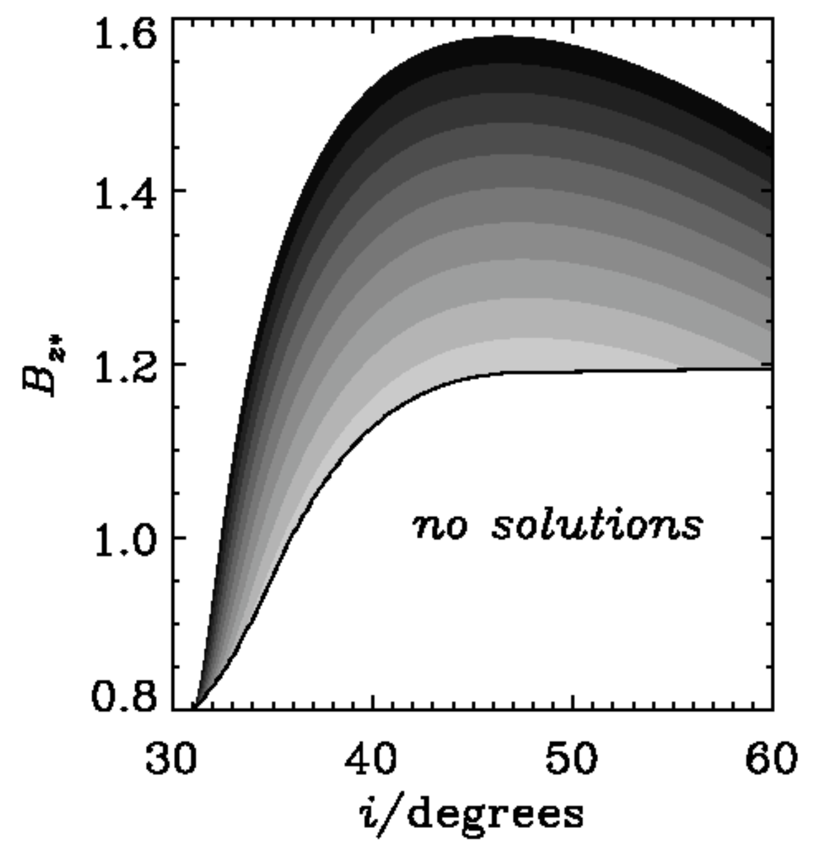
$\theta_{crit} = 60^0$

Mass flux : $\approx \rho(r_s)\, c_s$

$r < r_s$: hydrostatic $(v \simeq 0)$

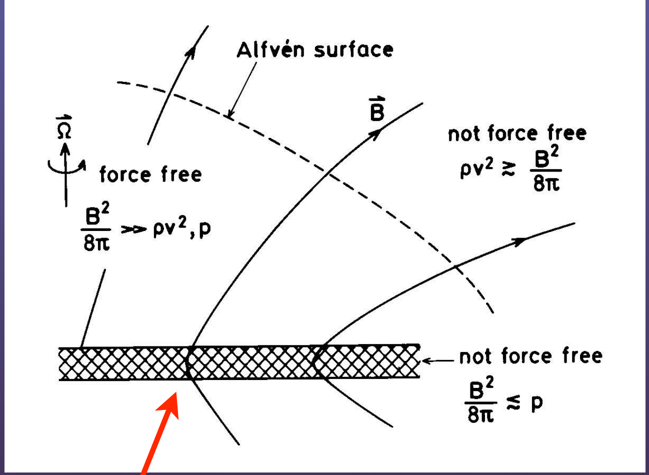
$>$: supersonic $(P \simeq 0)$

Low T : $\rho(r_s)$ low $\rightarrow$ $\dot{M}_w$ low *launching*



Dependence of mass flux on strength and inclination of $\mathbf{B}$

Ogilvie and Livio 2001



tension force (outward) reduces rotation rate
→ centrifugal force less
→ potential barrier increased

Below a minimum field strength no steady flow solutions

launching

‘Poloidal’ (p): in a plane containing
the rotation axis
‘toroidal’ = azimuthal (ϕ)

Shape of field above the disk

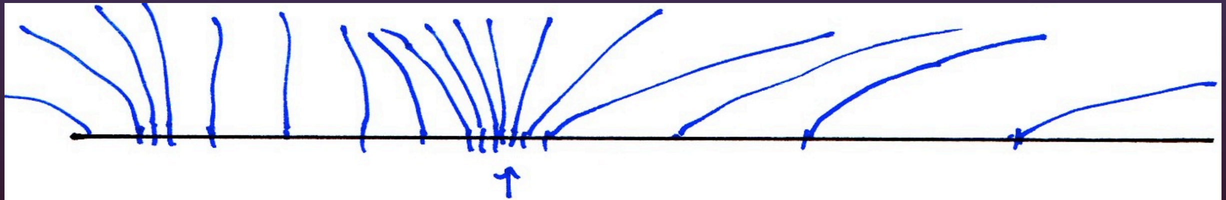
- (well) inside $r_{\rm A}$:

  Magnetic field dominates over other forces
  → field *force free*, $(\nabla\times\mathbf{B})\times\mathbf{B}=0$
  (well) inside $r_{\rm A}$: $B_\phi \ll B_{\rm p}$, neglect.

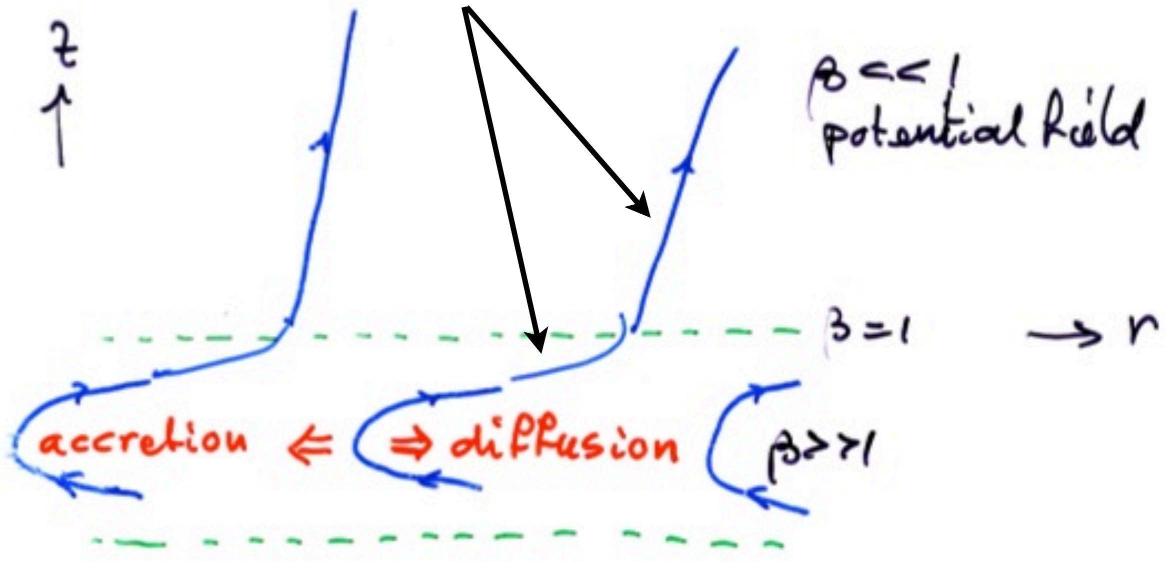
- → field approx. *potential*, $\nabla\times\mathbf{B}=0\,,\;\mathbf{B}=-\nabla\Phi_{\rm m}$

- potential field: field lines fan out away from concentrations (like bar magnets)

→ field line shape,
inclination at surface
are *global* problem



inclination governed by different physics!



- Inclination above disk
  global problem (Grad-Shafranov.)
- In disk: local balance
  diffusion ↔ advection

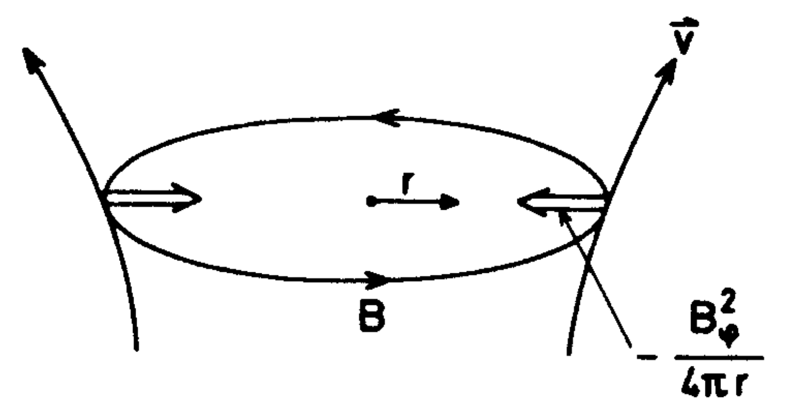
beware: literature confusing

‘Hoop stress

$(\nabla \times \mathrm{B}) \times \mathrm{B}$: tension *along* field lines

: pressure $\perp$ field lines

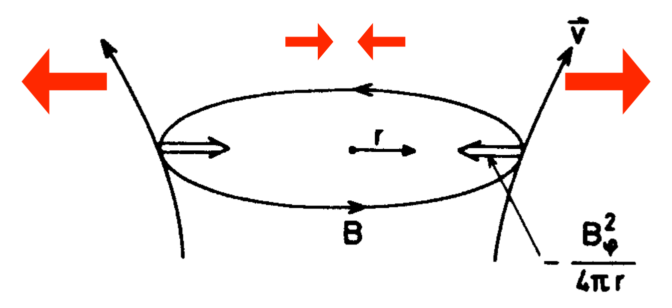
loop of field lines wants to contract



Field beyond r_{A} mostly
azimuthal
contraction towards:
jet ‘collimated by hoop stress’?
‘self-collimation’?

collimation

Def. *Collimation*: angle between flow lines
not *width* of jet



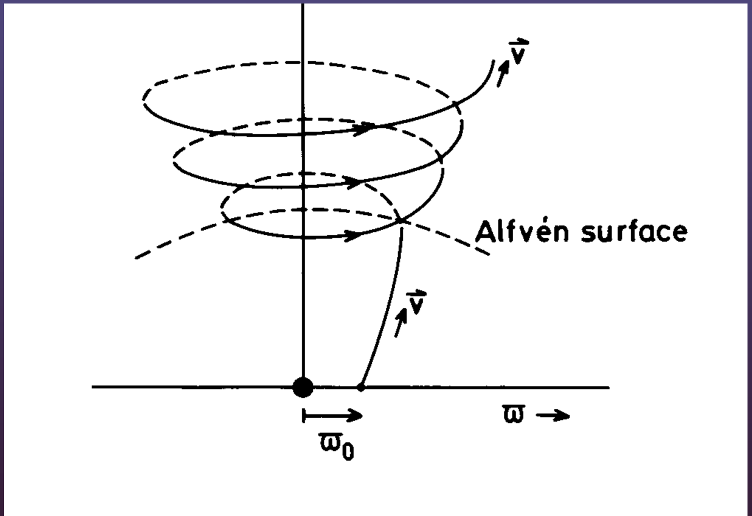
Magnetic fields are expansive
(↔ ‘tensor virial theorem’)

Azimuthal field adds energy density

azimuthal field *decollimates*

B_ϕ can collimate a jet *core*, but only at expense of overall expansion (*cf.* E.N. Parker 1979)

collimation ultimately
due to something external



Expansive nature of magnetic fields

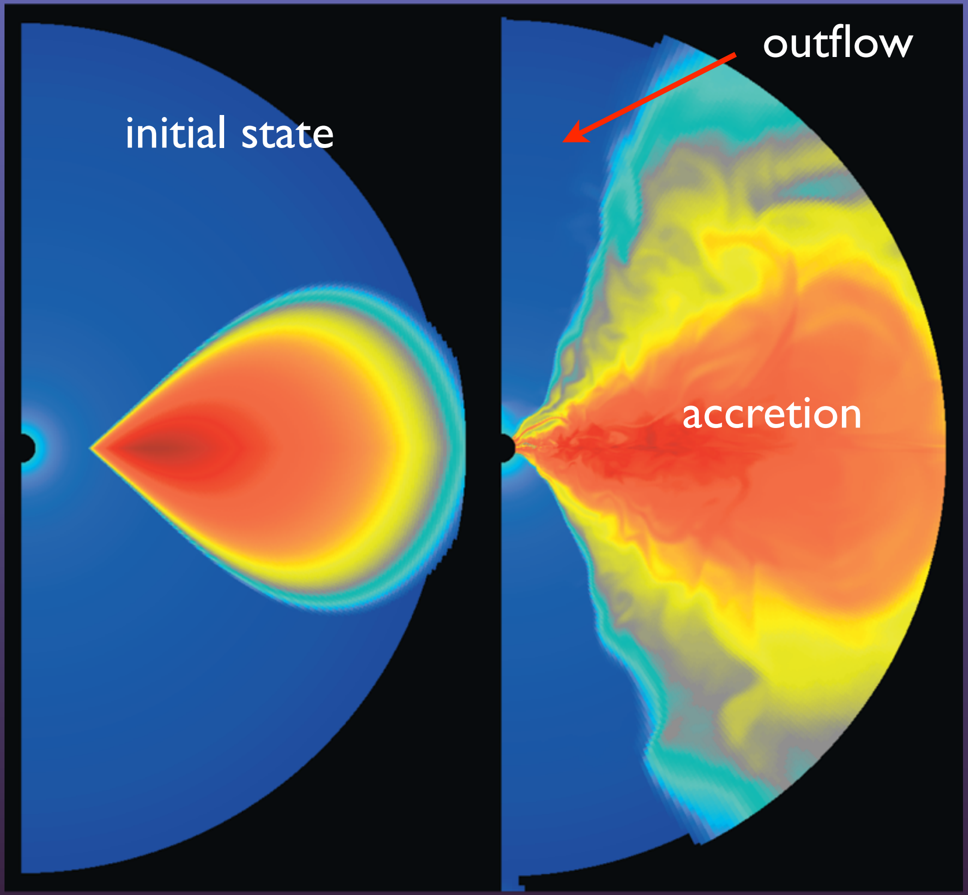
Useful theorem (‘the vanishing force-free field’):

A field which is force free $(\nabla \times \mathbf{B}) \times \mathbf{B} = 0$ everywhere (and finite) *vanishes identically*

Physics: there has to be a boundary that takes up the stress in the field and keeps it together.

(beware of the literature)

Collimation in numerical simulations



Equilibrium at boundary between flow and surroundings (assume field dominated by B_ϕ :

$$P_{\rm in} + B_\phi^2/8\pi = P_{\rm ext}$$

→ toroidal field increases pressure on boundary of the flow, *widens* the flow.

core of flow can be collimated by tension force in B_ϕ but stress must be taken up by an external medium collimating agents?

- disk surface → *toroidal field has to extend all the way from axis to disk surface*
- gas in the star-forming cloud
- material in the broad line outflow (AGN)
- a poloidal magnetic field in (the outer parts of) the disk

- Nothing. Ballistic flow, sideways expansion unconfined.
  (relativity helps: sideways expansion reduced by time dilatation)

observed opening angle, nonrelativistic: $\theta = v_{\rm expansion}/v_{\rm jet}$

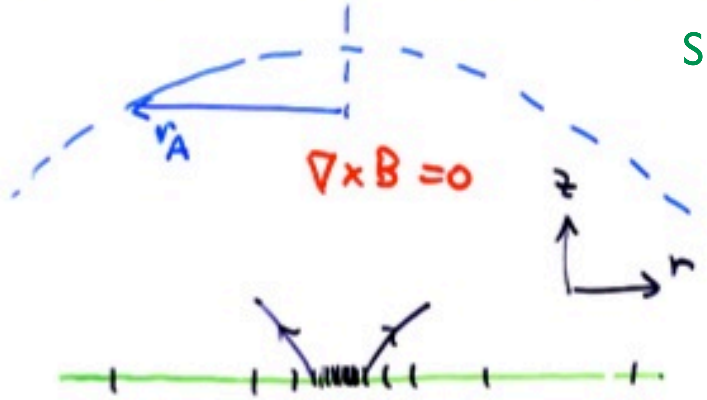
" " flow at Lorentz factor Γ : $\theta = \frac{1}{\Gamma} v_{\rm exp,comoving}/c$

flow of relativistic plasma: ($v_{\rm expansion} \approx c_{\rm s} = c/\sqrt{3}$):

$$\theta \approx \frac{1}{\Gamma\sqrt{3}}$$

* "Poloidal" collimation, by disk field.

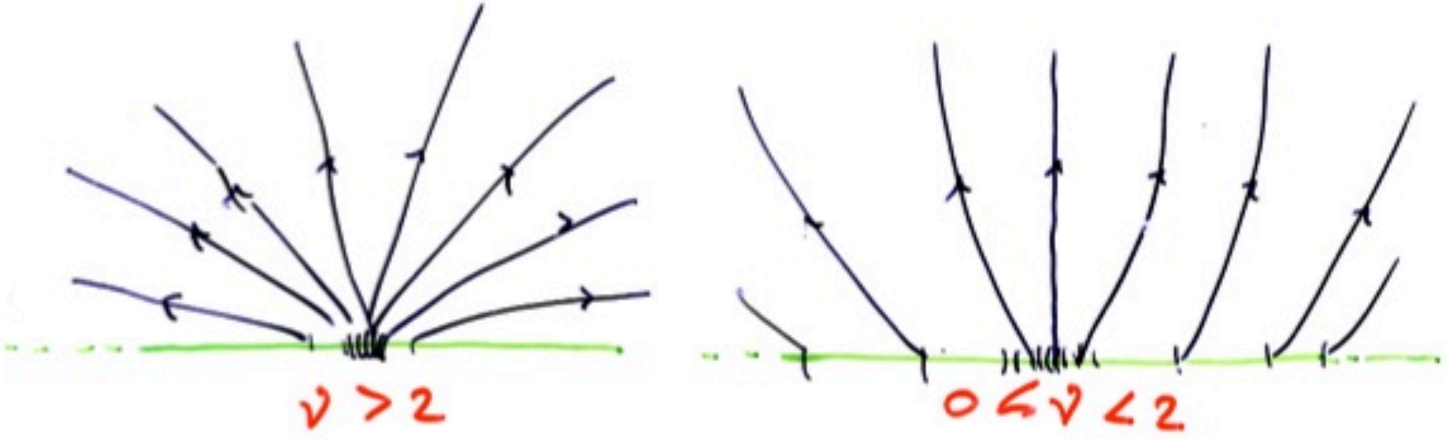
Spruit, Foglizzo & Stehle 1997



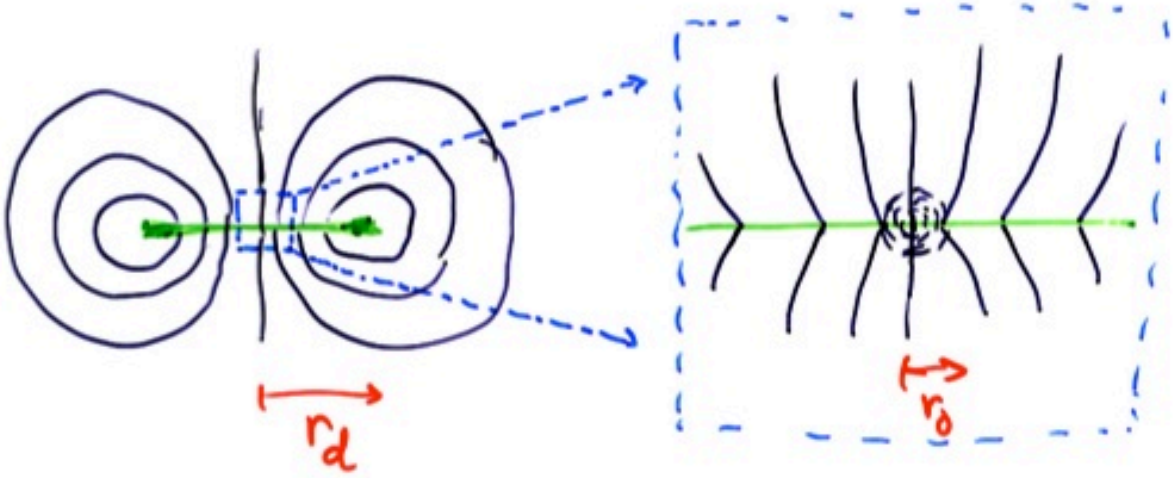
Magn. flux: $\phi(r) = r^2 B_z(z=0, r)$

Assume $B_z \downarrow$ as $r \uparrow$
$\phi \uparrow$ as $r \uparrow$

exa: $B_z = (r^2 + r_c^2)^{-\nu/2}$ $\quad 0 < \nu < 2$



Effect of finite disk size.



Good collimation only if $z_A < r_d$

→ $\theta_{min} \approx \left(\frac{r_0}{r_d}\right)^{1/2}$

	r_0	r_d	θ_{min}
proto*	0.01 AU	100 AU	0.01
LMXB	10 km	10^5 km	0.01
AGN	1 AU	$>10^4$ AU	<0.01
CV	$5\,10^3$ km	$2\cdot10^5$ km	0.2 ←
R Aqr	$5\,10^3$ km	$5\,10^8$ km	0.003

predicts:
no collimated jets
from cataclysmic
variables

‘Ordered’ magnetic fields

ordered: - net flux crossing the disk,
- sufficiently strong

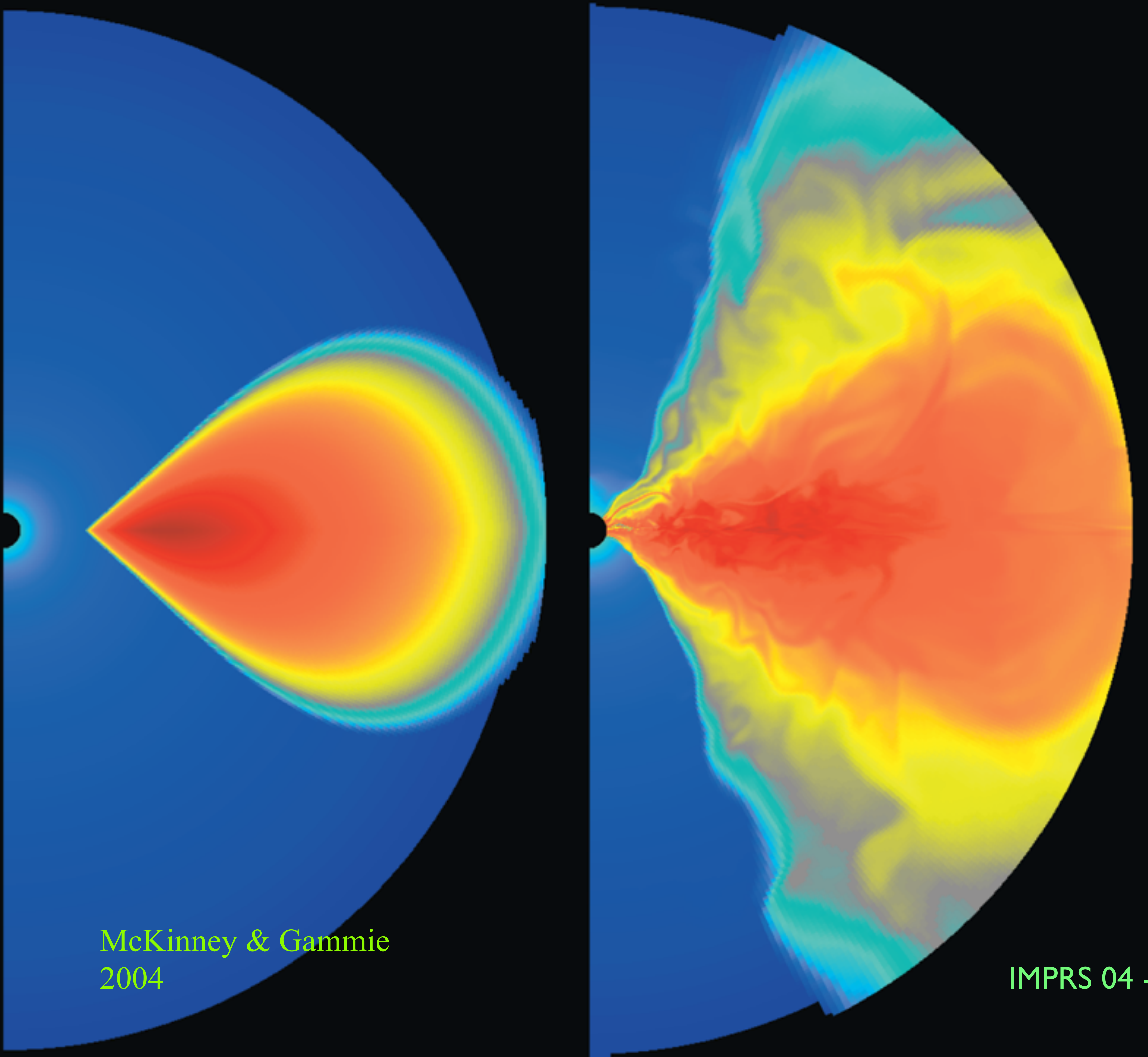
How strong can such a field be?
$\mathbf{B}$ must be less than orbital KE:

$$\frac{B^2}{8\pi} < \frac{1}{2}\rho\Omega^2 r^2 \qquad = \frac{1}{2}\frac{P}{c_\mathrm{s}^2}\Omega^2 r^2 = \frac{1}{2}P(\frac{r}{H})^2$$

MRI turbulence: $\frac{B^2_\mathrm{turb}}{8\pi} < P$

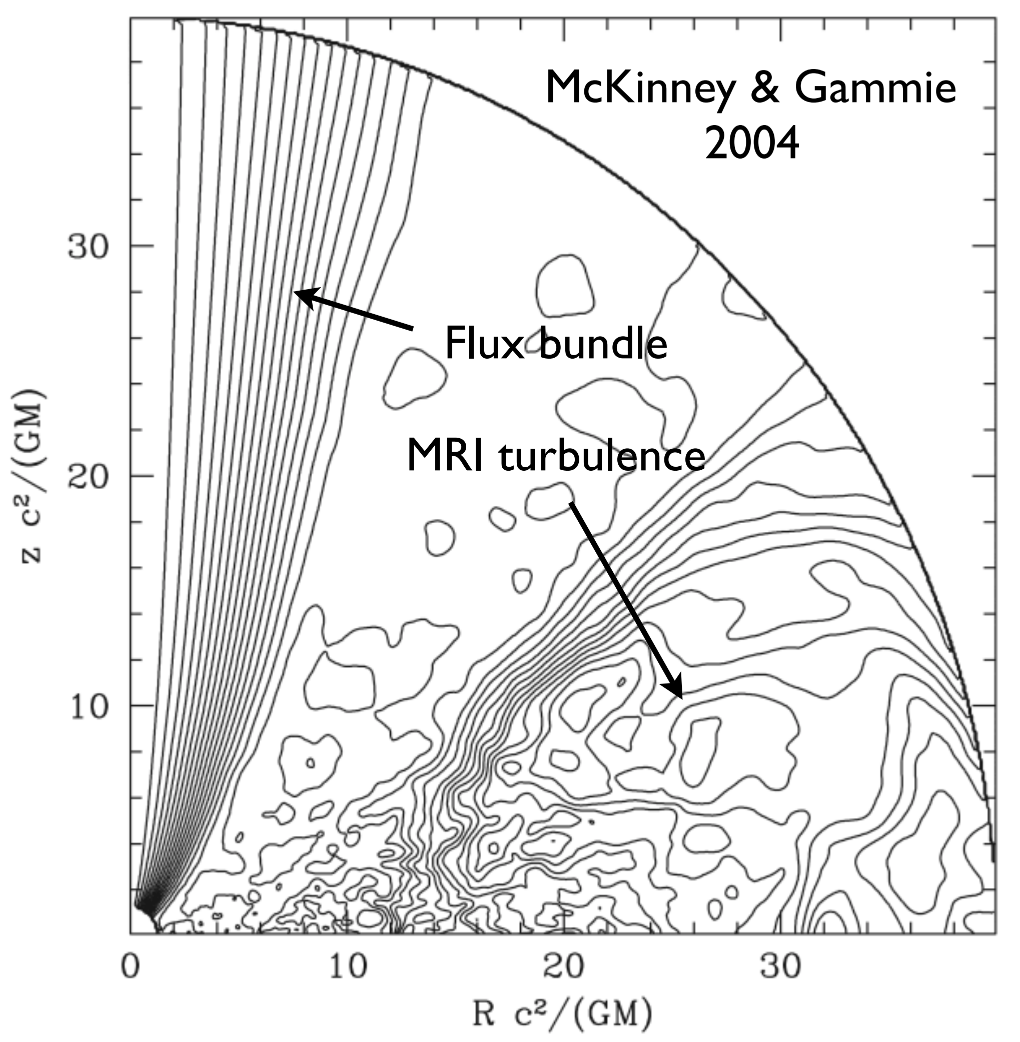
is *suppressed* in an ordered external field B_ordered when

$$\frac{B^2_\mathrm{ordered}}{8\pi} > P$$



McKinney & Gammie
2004

How do ‘good’ field configurations come about?



$\mathrm{div}\mathbf{B} = 0$: Net magnetic flux Φ through the disk surface cannot change by internal processes.

Φ can only enter or leave through outer disk boundary.

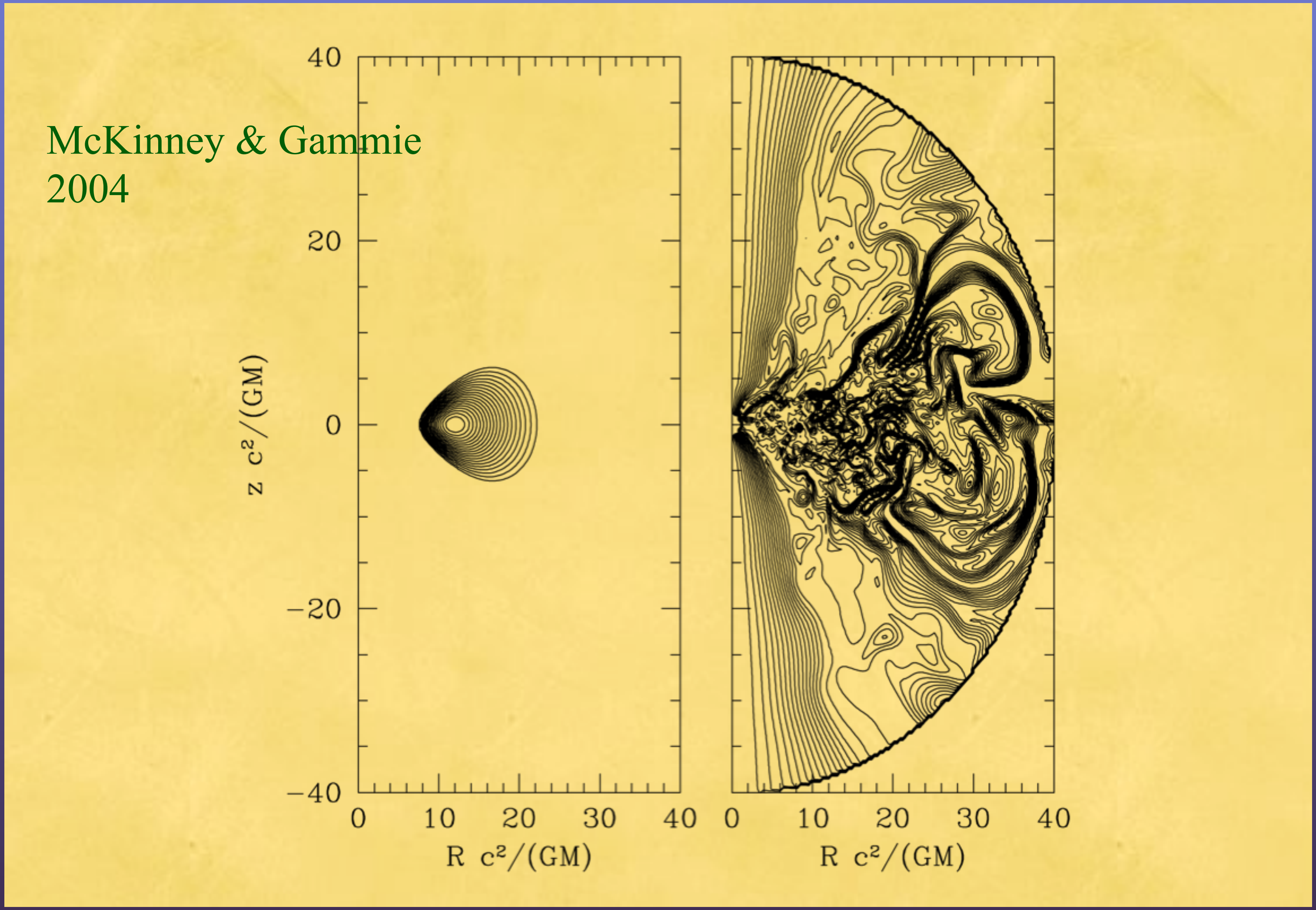
→ net flux is *inherited,*

or advected in at outer boundary:

$$\partial_t \Phi = \int \mathrm{d}r\mathrm{d}\phi \; r[\nabla \times (\mathbf{v} \times \mathbf{B})]_z \qquad \Phi = \int B_z r \mathrm{d}\phi \mathrm{d}r$$

$$v_r(0,\phi,z) = B_r(0,\phi,z) = 0$$

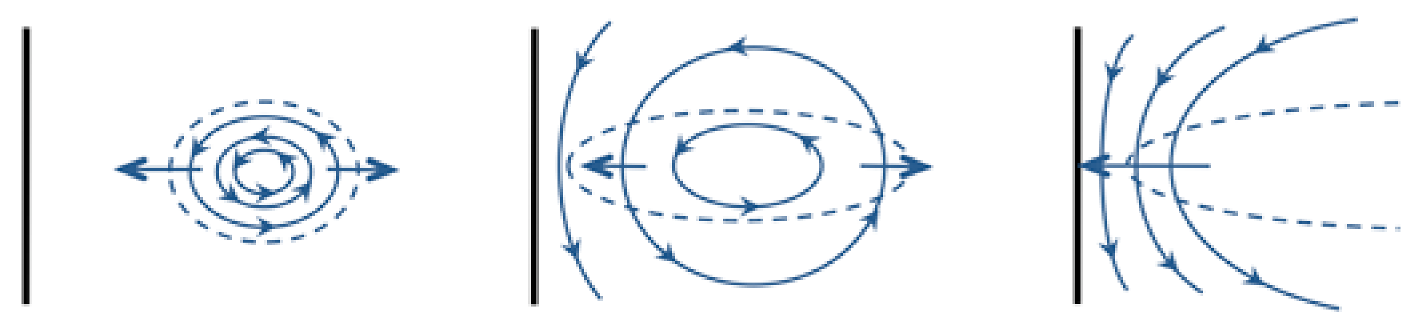
$$\rightarrow \quad \partial_t \Phi = -\int \mathrm{d}\phi \; R[\underbrace{v_z B_r - v_r B_z}_{= \; \mathbf{v}_\perp B_\mathrm{p}}]$$



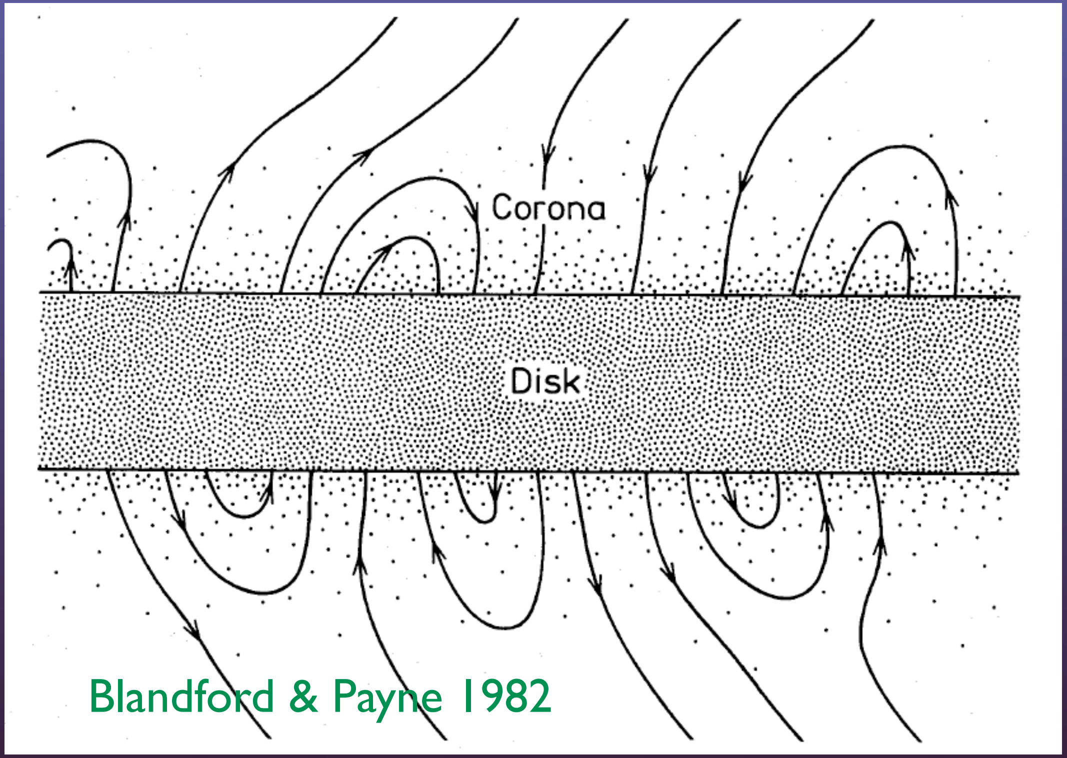
Ordered poloidal flux reflects initial conditions (deVilliers et al 2004)

→ *origin of poloidal flux (if needed) still t.b.d.*

Formation of a magnetic flux bundle through the hole



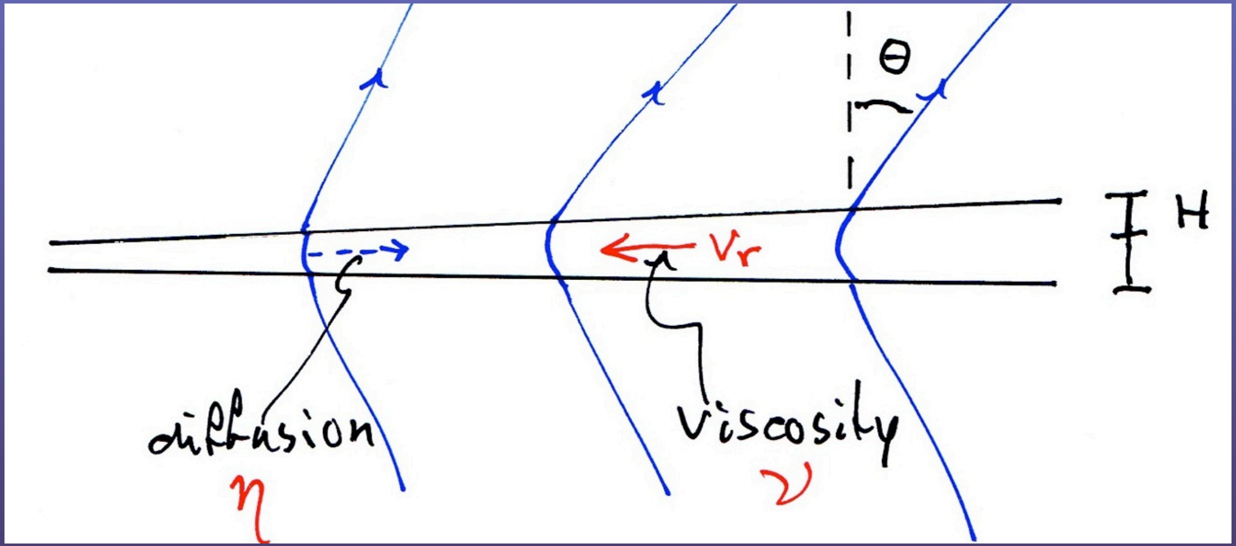
Magnetic jets from chaotic field?
Not seen in simulations, so far



Blandford & Payne 1982

Accretion of external flux

Accretion of ordered (net, poloidal) magnetic flux from environment



If accretion due to (magnetic) turbulence,

$$\eta \approx \nu$$

Balancing outward diffusion vs accretion of field, find

$$\Theta_{\rm max} \approx H/r$$

Reason: diffusion acts on curvature of field where it crosses the disk:

$$\rightarrow v_{\rm diff} \sim \frac{\eta}{H}\frac{B_r}{B_z} \qquad\qquad v_{\rm acc} \sim \nu/r$$

→ accretion of external field difficult in a diffusive disk model

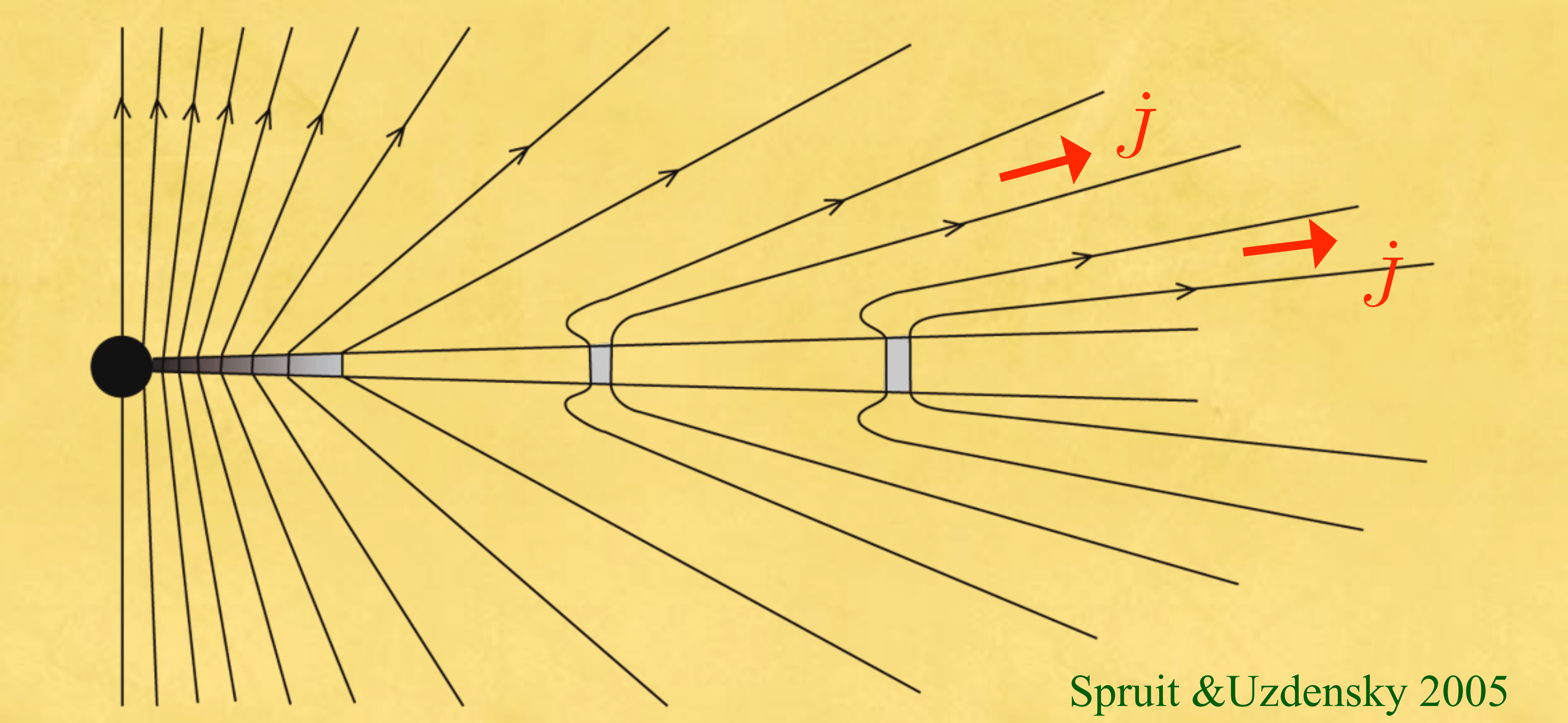
Accretion of external flux

Diffusive disk model. Viscosity ν, magnetic difusion η:

$\eta \approx \nu$ → no flux accreted

Alternative: patchy magnetic field
seen in MRI simulations

Fromang, Papaloizou. Lesur, Heinemann 2008

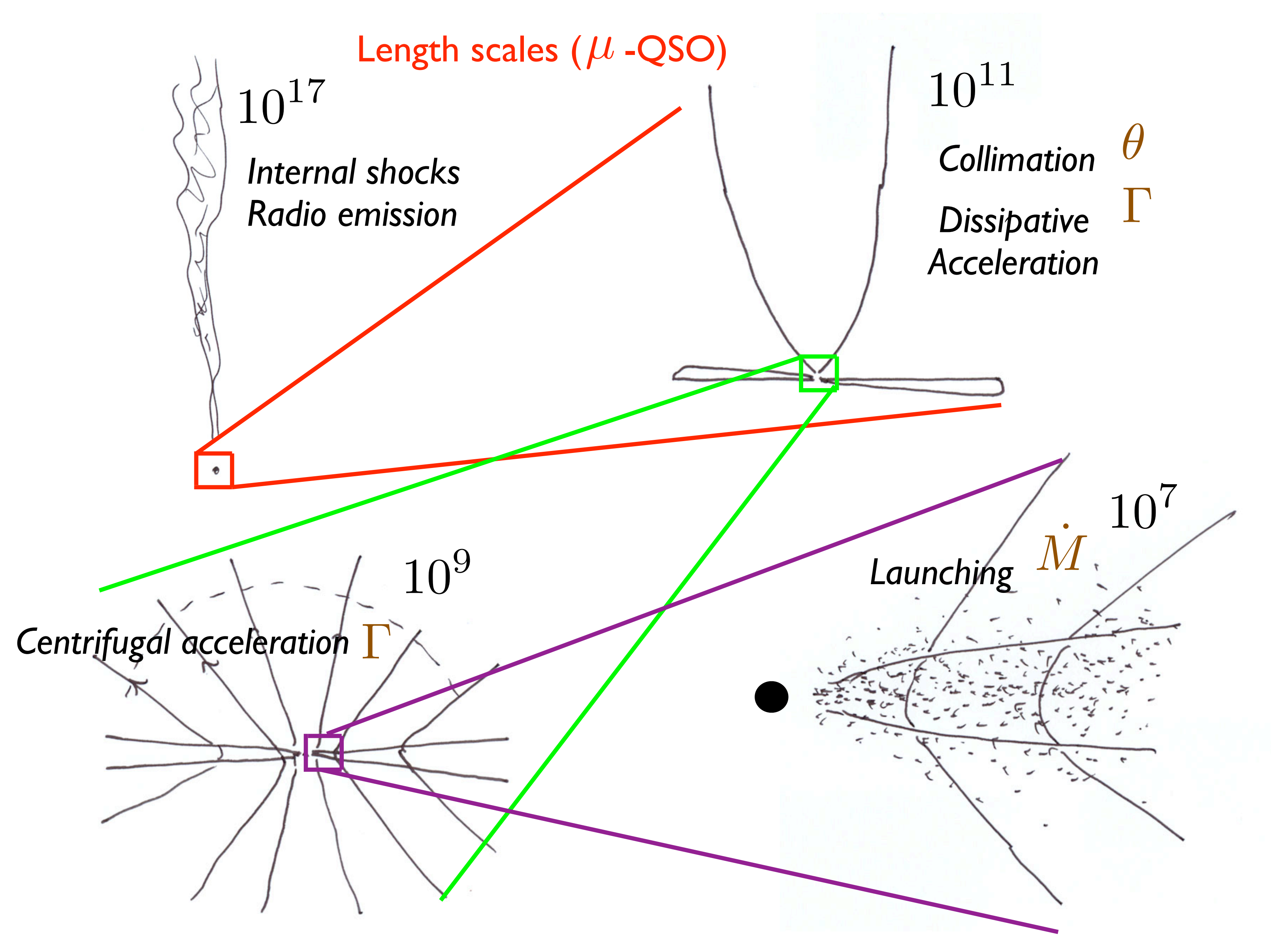


Spruit &Uzdensky 2005

Why need disks with net magnetic flux?

- geometry good for jets
- could be stronger than internally generated fields
- could be involved as ‘second parameter’ in the X-ray states of X-ray binaries

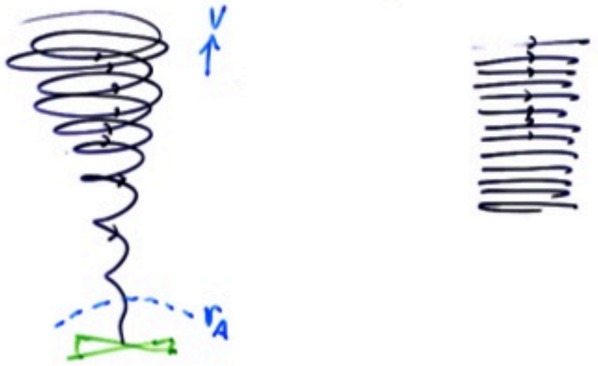
Numerical problem: length scales



Instability of toroidal field in jets.

(Choudhuri & Königl '86 ; Eichler '94)

*



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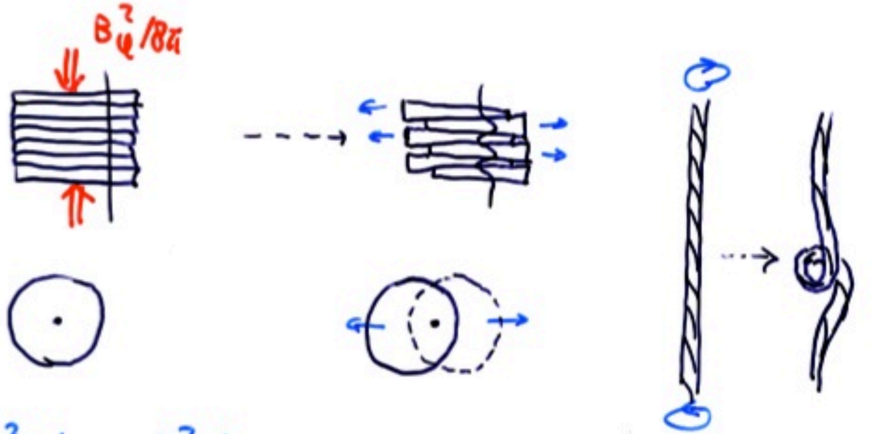
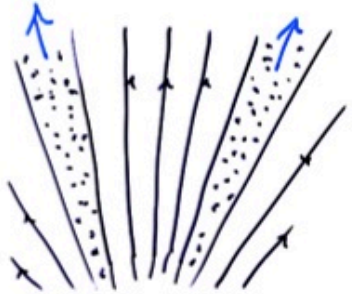
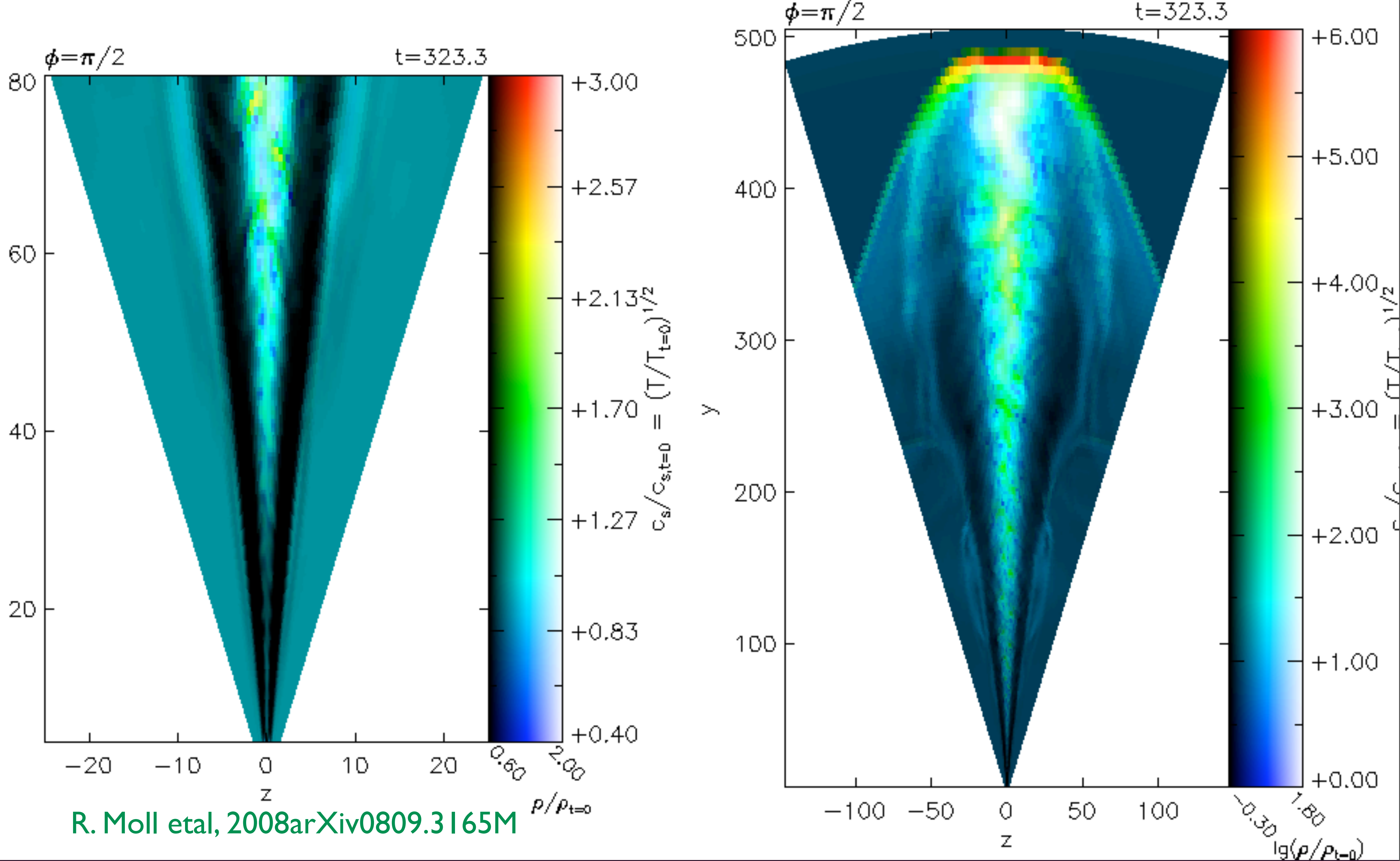


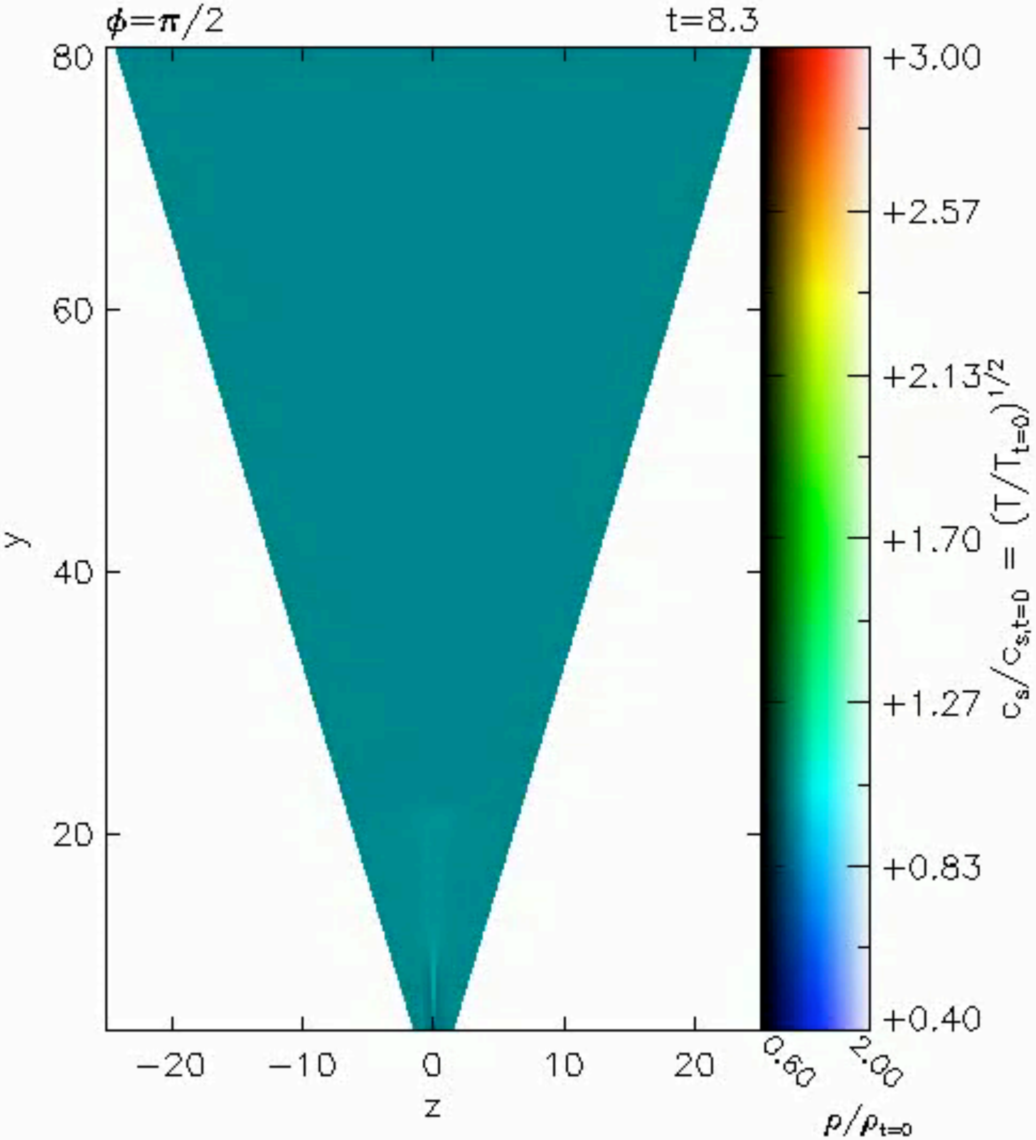
$B_\varphi^2 \downarrow$, $B_z^2 \uparrow$, net energy release if $B_\varphi \gtrsim B_z$

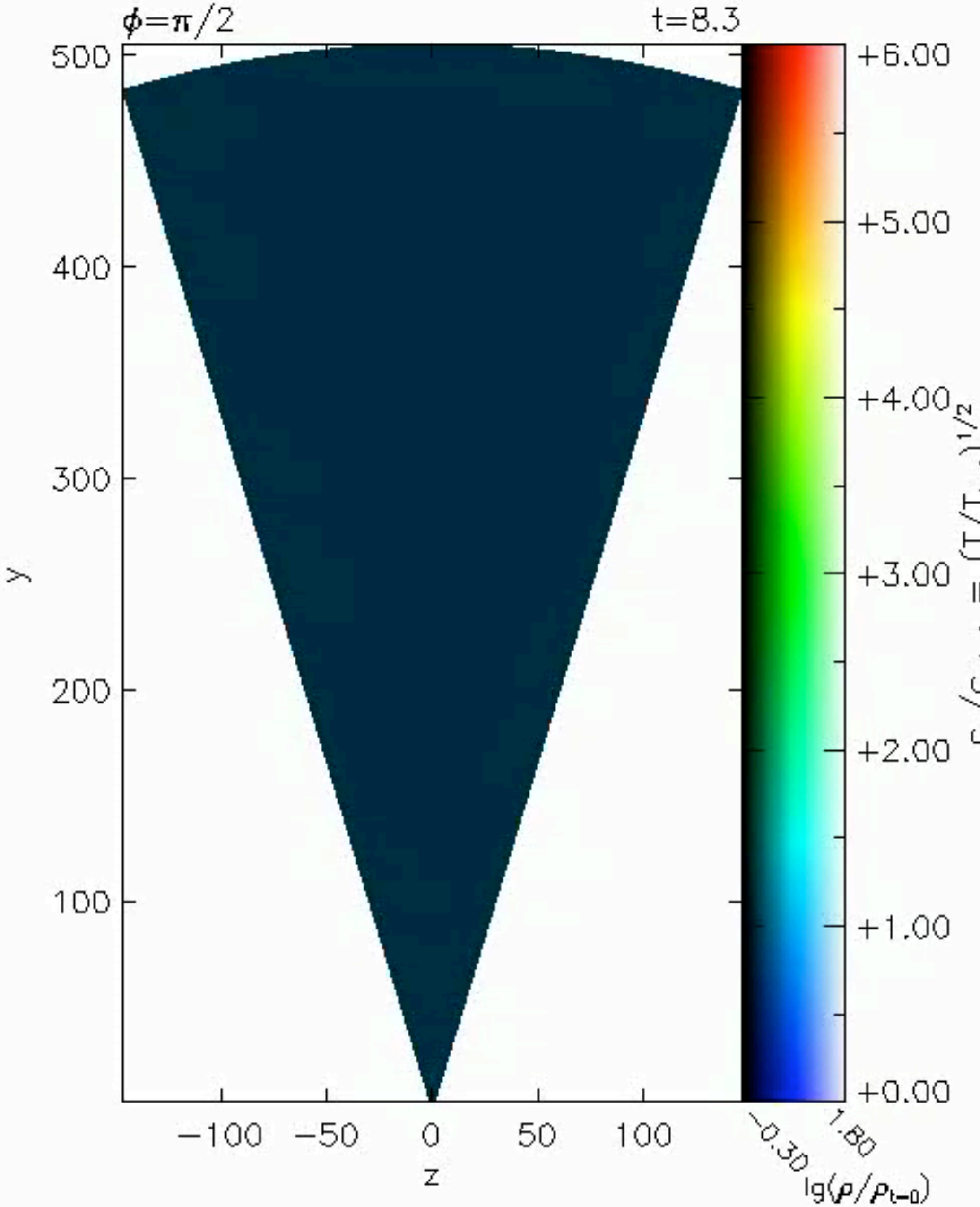
* Stabilizing effect of neighboring untwisted fields:

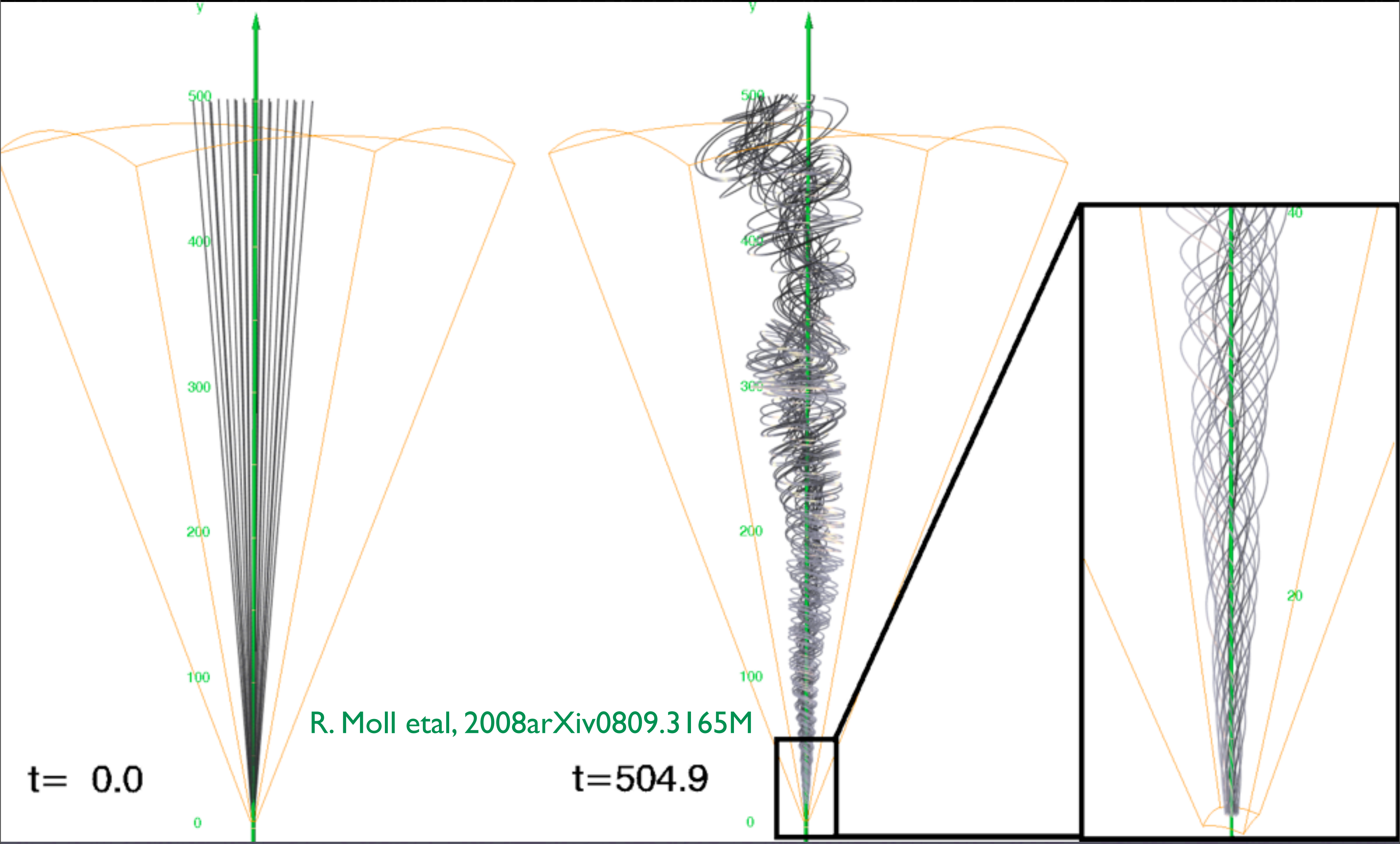




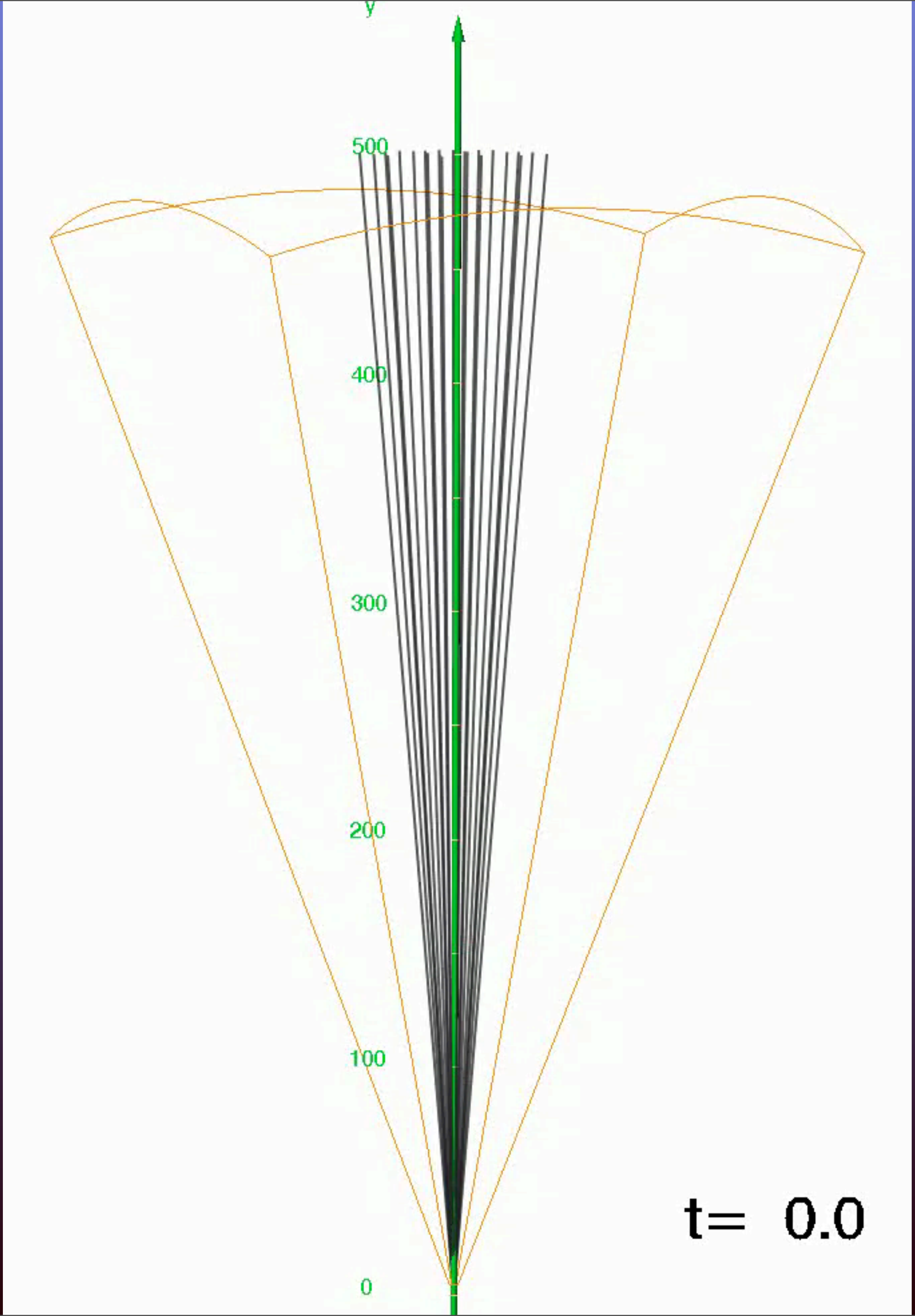
R. Moll etal, 2008arXiv0809.3165M


φ=π/2
t=8.3
80
60
40
20
y
−20
−10
0
10
20
z
+3.00
+2.57
+2.13
+1.70
+1.27
+0.83
+0.40
$c_s/c_{s,t=0} = (T/T_{t=0})^{1/2}$
0.60
2.00
$\rho/\rho_{t=0}$


φ=π/2
t=8.3
500
400
300
200
100
y
−100
−50
0
50
100
z
+6.00
+5.00
+4.00
+3.00
+2.00
+1.00
+0.00
$c_s/c_{s,t=0} = (T/T_{t=0})^{1/2}$
−0.30
1.80
$\lg(\rho/\rho_{t=0})$



R. Moll etal, 2008arXiv0809.3165M



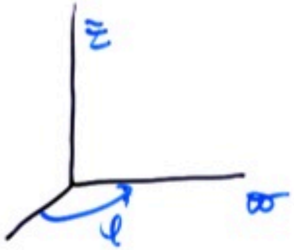
R. Moll, 2009, A&A 507, 1203

Consequences of kink instability
- Flow highly time dependent
- collimation influenced
- dissipation of magnetic energy source for radiation
- *increases the flow speed*

End jets

Stationary, axisymmetric MHD

Stationary :



$$\nabla\times(v\times B)=0$$

$$\rho v\cdot\nabla v = -\nabla p - \rho\nabla\phi + \frac{1}{4\pi}(\nabla\times B)\times B$$

$$\nabla\cdot(\rho \vec v)=0$$

$$\nabla\cdot B = 0$$

Axisymmetry : decompose into poloidal and toroidal components:

$$\vec B = \vec B_p + B_\varphi \vec e_\varphi \ ; \ \vec v = \vec v_p + v_\varphi \vec e_\varphi$$

$$\begin{array}{l} B_p \\ v_p \end{array} = \begin{array}{l} (B_\varpi, B_z) \\ (v_\varpi, v_z) \end{array}$$

B_p can be written as :

$$B_p = \frac{1}{\varpi}\nabla\psi\times\vec e_\varphi$$

$\partial_\varphi = 0$
$\nabla\cdot B = 0$

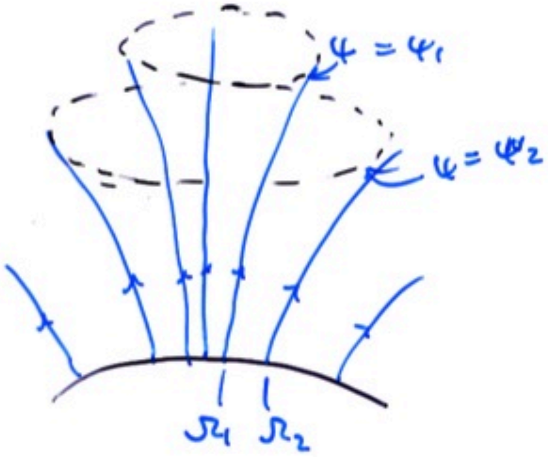
or

$$B_z = \frac{1}{\varpi}\partial_\varpi\psi$$

$$B_\varpi = -\frac{1}{\varpi}\partial_z\psi$$

$B\cdot\nabla\psi = 0$ → ψ "numbers field lines"

ψ : stream function



$$\nabla\times(v\times B) = 0 \ : \qquad v\times B = \nabla f \quad (\text{some } f)$$

$$v_p \times B_p + v_\varphi\, e_\varphi \times B_p + B_\varphi\, v_p \times e_\varphi = \nabla f \qquad (A)$$

toroidal component: $v_p \times B_p = 0$

$$\downarrow$$

$$v_p = k(\varpi, z)\, B_p$$

$$// \quad !$$

$B_p \cdot (A)$: $$\left.\begin{array}{l} B_p\cdot\nabla f = 0 \\ (\, B_p\cdot\nabla\psi = 0 \,) \end{array}\right\} \rightarrow f = f(\psi)$$

$(A) \rightarrow$

$$\rightarrow \quad v_\varphi - k B_\varphi = \varpi f'(\psi)$$

$$\nabla\cdot(\rho\vec{v}) = \nabla\cdot(\rho v_p) = \nabla\cdot(\rho k \vec{B}_p) = 0$$

$$\rightarrow \quad B_p\cdot\nabla(\rho k) = 0 \rightarrow$$

$$\rightarrow \rho k = \eta(\psi) \ \rightarrow \ \rho\frac{|v_p|}{|B_p|} = \eta(\psi)$$

"Rotation of a field line"

$$\kappa = \frac{\eta(\psi)}{\rho} \rightarrow v_\varphi - \frac{\eta}{\rho} B_\varphi = \varpi f'(\psi)$$

Deep inside rotating object : $\rho \rightarrow \infty$: $v_\varphi = \varpi f'(\psi)$

$\rightarrow f(\psi) = \Omega(\psi)$

elsewhere :

$\frac{v_\varphi}{\varpi} \neq \Omega$!

In frame rotating w. Ω :

$$v' = v - \varpi \Omega(\psi) \hat{e}_\varphi$$

$$\rightarrow \quad \underline{v' = \kappa B}$$

$\Rightarrow$ In a frame "corotating with the field line", $v \,/\!/\, B$.

Equation of motion : toroidal component using $(\nabla \times B) \times B = - \nabla B^2/2 + (B \cdot \nabla) B$:

$$\rho (v \cdot \nabla v)_\varphi = \frac{1}{4\pi} (B \cdot \nabla B)_\varphi$$

using $(a \cdot \nabla b)_\varphi = a \cdot \nabla (\varpi b_\varphi)/\varpi$:

$$B_p \cdot \nabla (\rho \kappa \varpi v_\varphi) = \frac{1}{4\pi} B_p \cdot \nabla (\varpi B_\varphi)$$

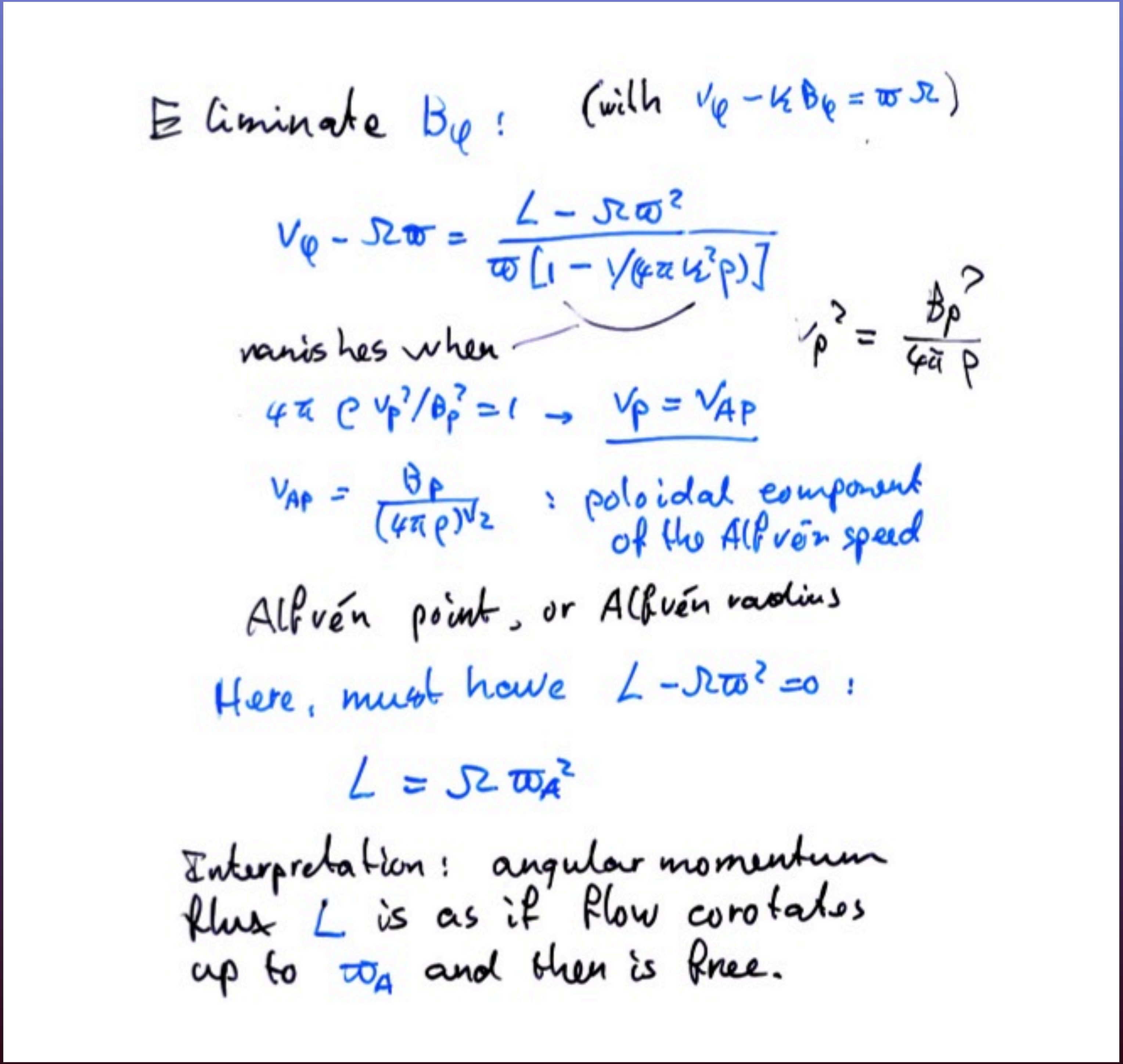
$$\rightarrow \quad \frac{1}{B_p} (\rho \psi \varpi v_\varphi - \frac{\varpi}{4\pi} B_\varphi B_p) = \eta L(\psi)$$

$$\rightarrow \quad \varpi (v_\varphi - \frac{1}{4\pi \eta} B_\varphi) = L(\psi)$$

ang. mom. flux

magnetic torque

$$\eta(\psi) = \rho \frac{|v_p)}{|B_p|}$$



Eliminate B_φ : (with $v_\varphi - \kappa B_\varphi = \varpi \Omega$)

$$v_\varphi - \Omega\varpi = \frac{L - \Omega\varpi^2}{\varpi\left[\underbrace{1 - 1/(4\pi\kappa^2\rho)}\right]}$$

vanishes when

$$v_p^2 = \frac{B_p^2}{4\pi\rho}$$

$$4\pi\rho v_p^2/B_p^2 = 1 \rightarrow \underline{v_p = v_{Ap}}$$

$v_{Ap} = \frac{B_p}{(4\pi\rho)^{1/2}}$: poloidal component of the Alfvén speed

Alfvén point, or Alfvén radius

Here, must have $L - \Omega\varpi^2 = 0$:

$$L = \Omega\varpi_A^2$$

Interpretation: angular momentum flux L is as if flow corotates up to ϖ_A and then is free.