Accretion

introduction accretion physics:

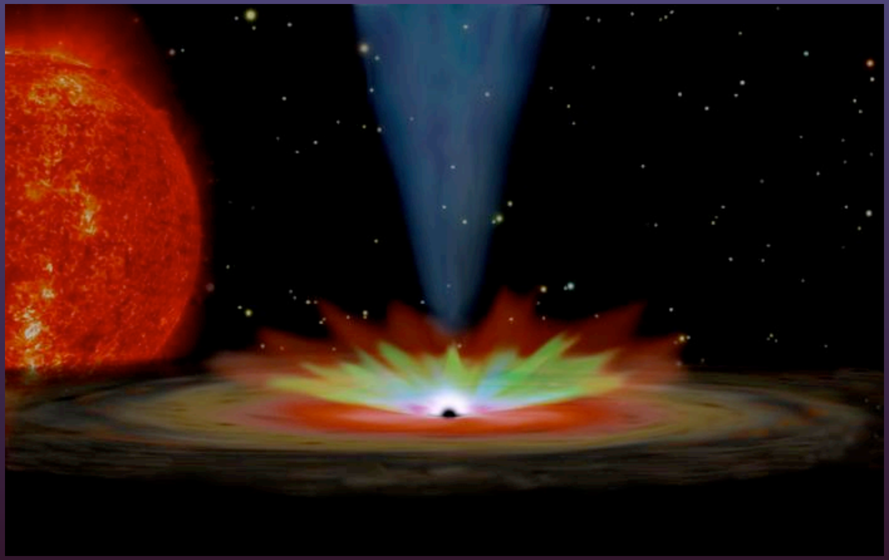
http://www.mpa-garching.mpg.de/~henk/disksn.pdf

(to be updated ...)

Textbook: Frank, King & Raine, 'accretion power in astrophysics' (CUP)

This presentation:

http://www.mpa-garching.mpg.de/~henk/imprsaccretion.pdf



Accretion:

Mass accumulating on a (compact) object by the action of its gravity

Examples:

- Formation of stars, planetary systems
- 'cataclysmic variables' (accreting white dwarfs)
- X-ray binaries (accreting black holes, neutron stars)
- Active galactic nuclei (massive accreting black holes)

Close connection with jets:

- protostellar jets (Herbig-Haro objects) $v \sim 100 - 300\ \mathrm{km/s}$
- microquasars, SS433
- radio galaxies
- gamma-ray bursts

} relativistic: $v \approx c$ $(\Gamma = 10 - 1000)$

Gravitational potential of spherically symmetric mass M of radius R

$$\Phi = -\frac{GM}{r} \qquad (r > R)$$

Acceleration of gravity

$$\mathbf{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{r}$$

Particles freely falling from $r \to \infty$ to r :

$$E_{\rm K} = \frac{1}{2}v^2$$ (kinetic energy per unit mass)

Energy conservation: $E_{\rm K} + \Phi = E = {\rm cst.}$

At r : $v^2 = \frac{2GM}{r}$ (free-fall or escape speed)

(def:) Compact star: M/R *large*

Example: neutron star, $M = 1.4M_\odot,\ R = 10\ {\rm km}$: $\frac{v_{\rm ff}}{c} \approx 0.6$

(Newtonian approximation!)

Accretion of gas

Equation of state of ideal gas: $P = \rho \mathcal{R} T/\mu$
(pressure P, density ρ, gas constant $\mathcal{R}$, 'molecular weight' μ)

Internal energy of gas at temperature T: $u = \frac{P}{\rho(\gamma-1)}$ $\quad (\gamma = \frac{c_p}{c_v} = \mathrm{cst.})$

Freely falling gas, dissipating its kinetic energy at accreting surface:

$$T = (\gamma - 1) T_{\mathrm{v}} \qquad T_{\mathrm{v}}: \text{ virial temperature, } \qquad T_{\mathrm{v}} \equiv \frac{GM}{\mathcal{R} r}$$

(assumption: *adiabatic* flow) (=?)

Neutron star, $v_{\mathrm{ff}}/c = 0.6$: $T_{\mathrm{vir}} \sim 10^{12}$ K $\approx 300\,\mathrm{MeV}$

$$v_{\rm ff}/c = 0.6 \quad \longrightarrow \quad T_{\rm vir} \sim 10^{12}\ {\rm K} \ \approx 300\,{\rm MeV}$$

$$kT \sim m_{\rm p}c^2 \gg m_{\rm e}c^2$$

Actual temperatures limited by:

- radiative energy loss during accretion (disks)
- energy density in radiation
- $e^{\pm}$ pair creation
- energy loss by neutrinos

(the last three: core collapse SN, GRB)

But: if $T_{\rm electron} \neq T_{\rm ion}$:

ion *temperatures* 10^{12} *possible in 'ion supported' accretion*

Radiative loss

BB: $F_{\rm rad} = \sigma T^4$

Accretion rate $\dot{M}$ (g/s) on star of radius R:

$$\dot{E}_{\rm rad} = -\Phi \dot{M} = \frac{GM\dot{M}}{R} = 4\pi R^2 \sigma T^4$$

$$\dot{M} \approx 10^{-8} M_\odot \ {\rm yr}^{-1} \rightarrow \ T \sim 1{\rm keV} \sim 10^7{\rm K}$$

Blackbody approx: never very good, but can be fair for high *optical depth* τ

$$\tau = n\sigma_{\rm c} R$$

particle density ↗ ↖ cross section

Escape time of photon from object of size R : $t_{\rm esc} \sim \frac{R}{c}\tau^2 \quad (\tau > 1)$

Accretion time : $t_{\rm acc} = R/v_r$

Radiation cooling important when $t_{\rm esc} < t_{\rm acc}$

Radiation processes

Photon production:

- *Atomic transitions*
- *Fully ionized: bremsstrahlung*
- *in* B *: synchrotron/cyclotron emission*
- $e^{\pm}$ *pair production* $(T > 100\ \mathrm{keV})$

Opacity :

- same processes, plus
- electron scattering

Thomson: elastic $e - \gamma$ scattering

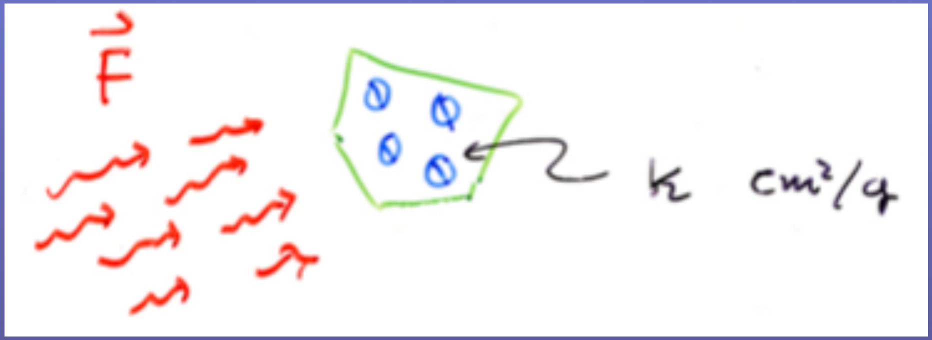
$$\sigma_{\mathrm{T}} = \frac{8\pi}{3} r_0^2 \qquad (<\sim 100\mathrm{keV})$$

$$\kappa_{\mathrm{T}} = \sigma_{\mathrm{T}}/m_{\mathrm{p}} \approx 0.3\ \mathrm{cm}^2/\mathrm{g}$$

Comptonization (inelastic, $>\sim 100\mathrm{keV}$):

- $e - \gamma$ scattering with change of photon energy

Eddington limit



Radiative force: $\frac{F_{\rm rad}}{c}\kappa$

gravity: $\frac{GM}{r^2}$

Equate: $F_{\rm E} \equiv \frac{c}{\kappa}\frac{GM}{r^2}$ (Eddington flux)

Eddington Luminosity (spherical, $\times 4\pi r^2$*) :* $L_{\rm E} = 4\pi GM\frac{c}{\kappa}$

$M = 1M_\odot,\ \kappa = 0.3 \qquad \rightarrow L_{\rm E} \approx 10^6 L_\odot$

If L from accretion: $\eta\frac{GM\dot{M}}{R} = L_{\rm E} \rightarrow \dot{M} = \dot{M}_{\rm E} \equiv \frac{1}{\eta}4\pi R\frac{c}{\kappa}$

$\eta = 1: \qquad \dot{M}_{\rm E} \sim 10^{-8} M_\odot\ {\rm yr}^{-1} \quad (10\ {\rm km},\ \kappa = 0.3)$

Eddington critical accretion rate.

Eddington limit, optically thick

static:

$$\nabla(P_{\rm gas} + P_{\rm rad}) = \rho \mathbf{g}$$

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}\tau} = F_{\rm rad}$$

$$\mathrm{d}\tau = \kappa\rho\mathrm{d}x$$

radiative flux balances gravity if $F_{\rm rad} = F_{\rm E}$

→ maximum luminosity is Eddington *if energy carried by radiation*

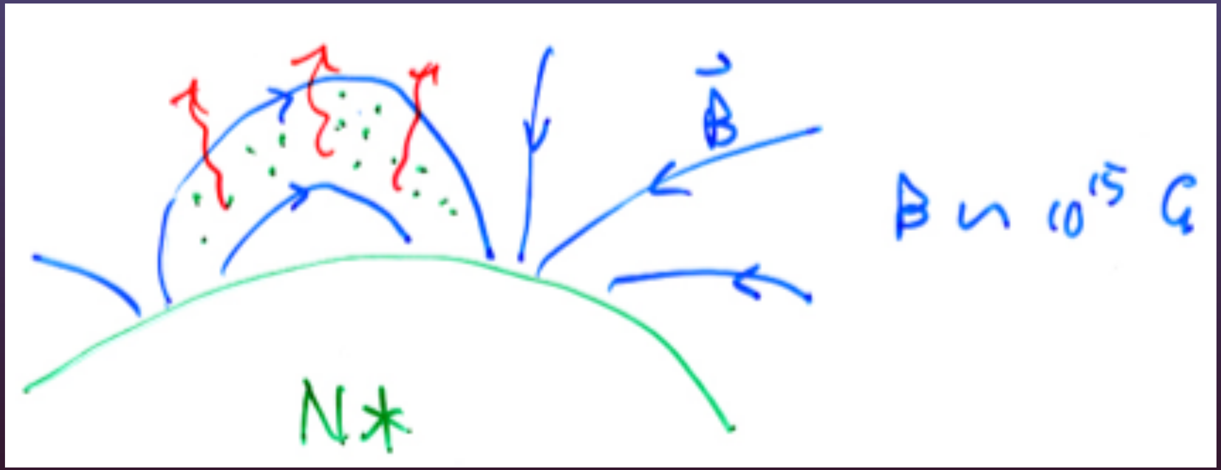
Applicability of the Eddington luminosity limit

assumptions made were:

1 - static radiating object
2 - only gravity, no other restraining forces
3 - energy not transported by something else (convection, B-fields)

Examples of exception to applicability

ad 1: nova, supernova explosions
ad 2: other forces: magnetic fields (e.g. *magnetars*)



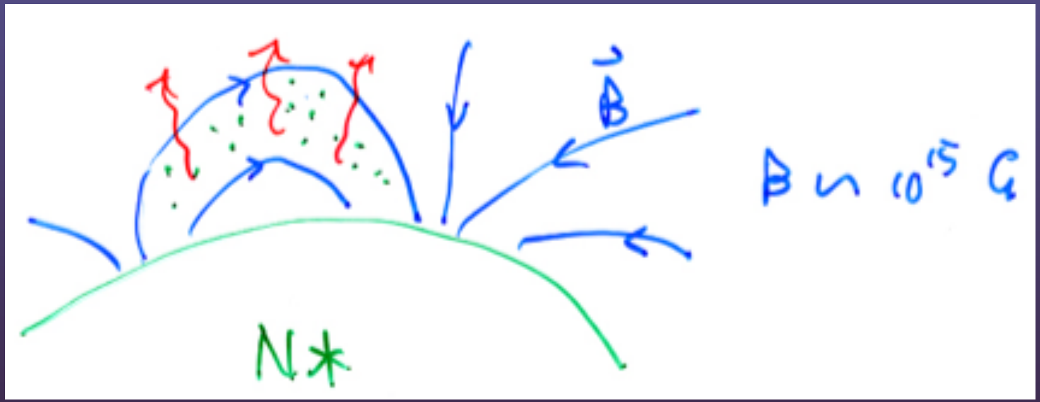
$$U_{\rm rad} = aT^4 \quad F_{\rm rad} = \frac{1}{4} acT^4$$

Magnetic confinement: $U_{\rm rad} < E_{\rm mag} = \frac{B^2}{8\pi}$

→ $F_{\rm rad} < \frac{B^2}{8\pi} c$

Can be $\gg F_{\rm Edd}$

Example: magnetars



$$L_{\rm max} = 4\pi R^2 \frac{B^2}{8\pi} c$$

$$3\,10^{16} L_\odot \; B_{14}^2 R_6^2$$

(decrease of B w. distance!)

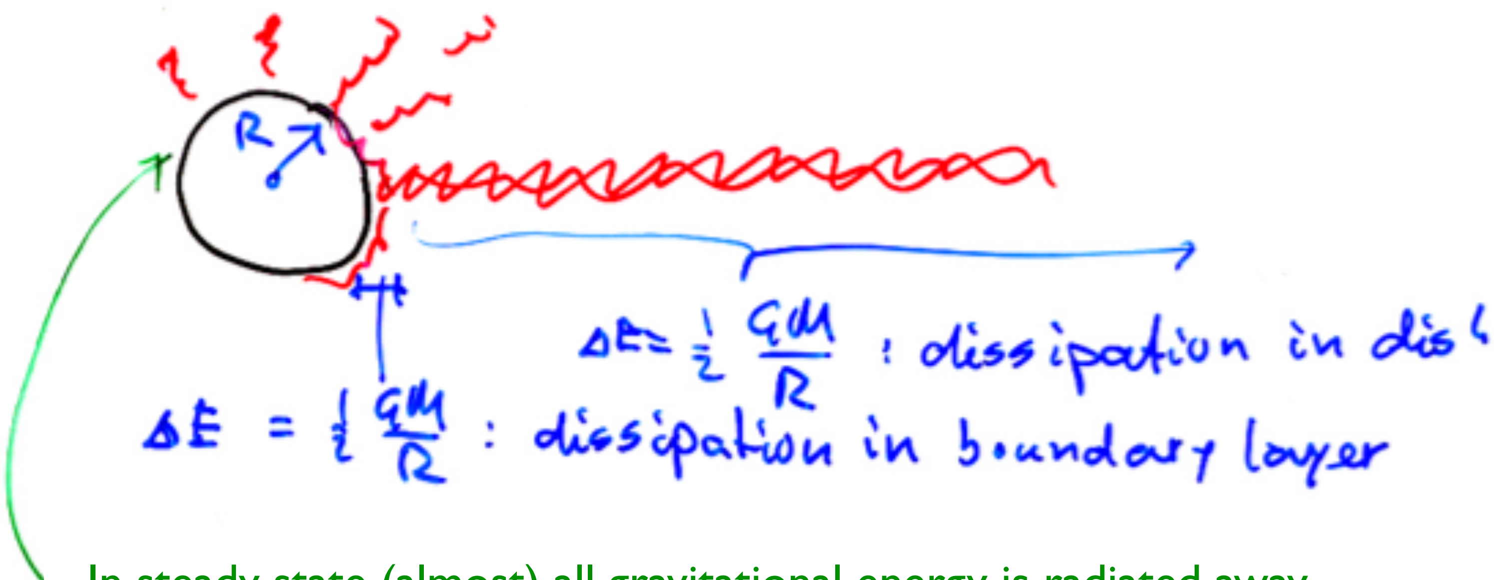
observed in magnetar outbursts:
up to $\sim 10^9 L_\odot \sim 10^4 L_{\rm Edd}$

Eddington accretion rate:

the difference between black hole and neutron star accreters

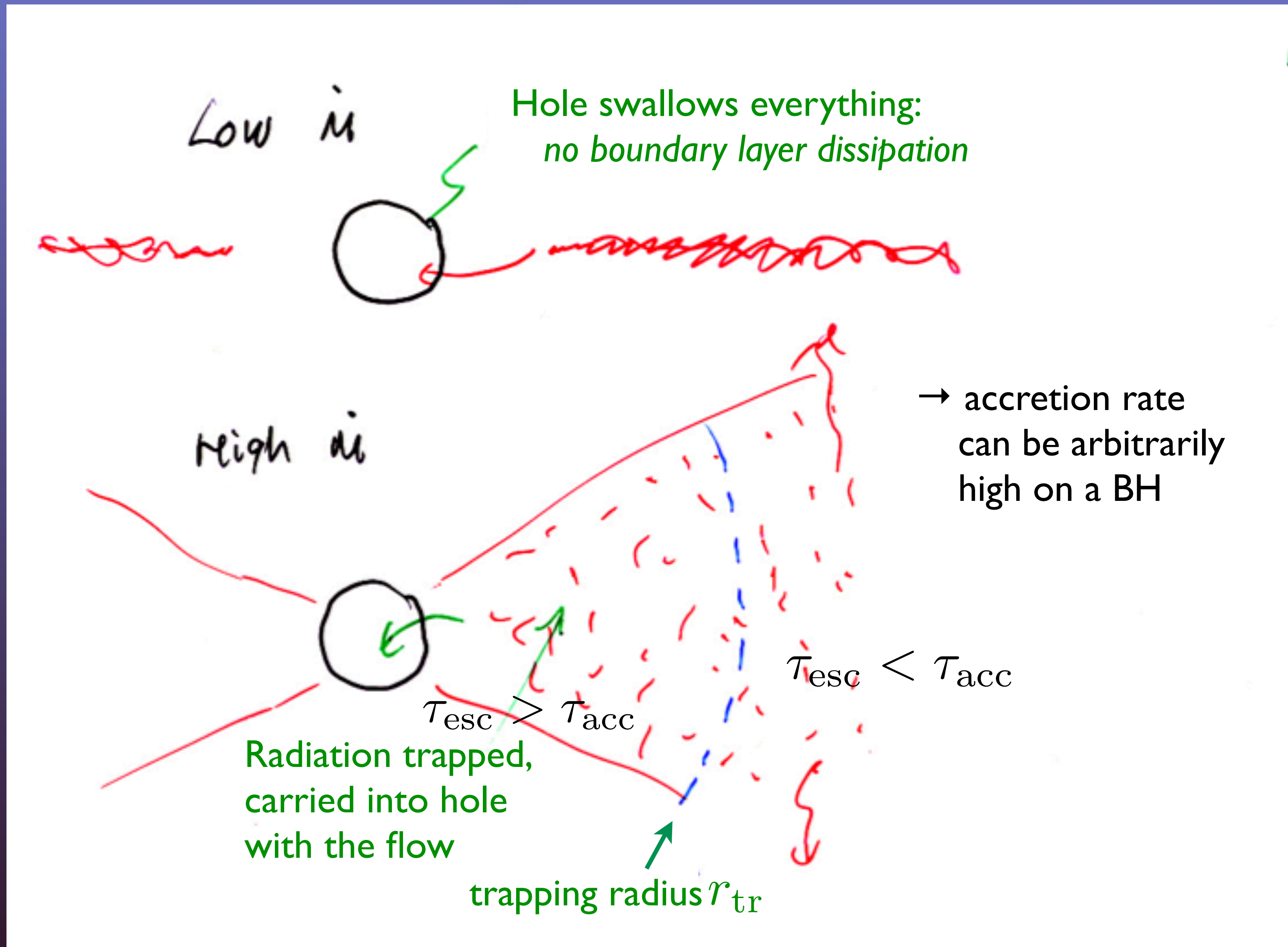
Accretion on *neutron stars*:

Orbital kinetic energy: $E_{\rm orb} = \frac{1}{2}\frac{GM}{r}$ *(Kepler orbits)*



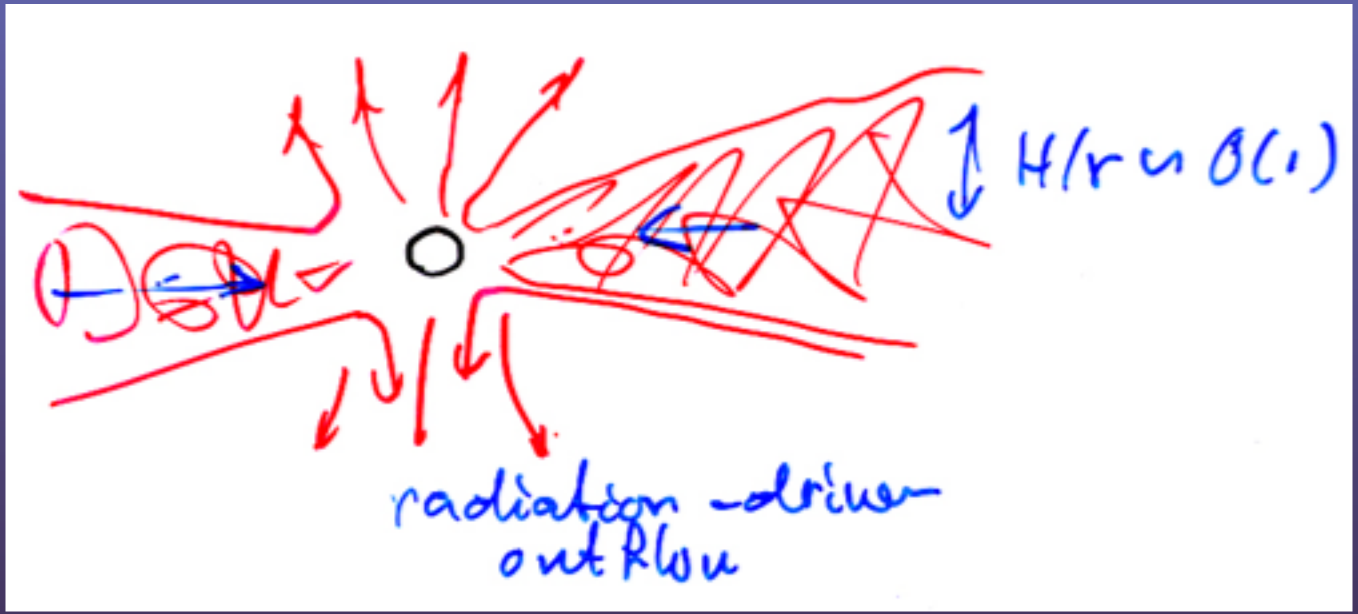
In steady state (almost) all gravitational energy is radiated away.
Half from disk, half from boundary layer on star.
(Some stays as thermal and rotation energy on the star)

Accretion on *black holes*:



non-spherical super-Eddington accretion:

n-star with $\dot{M} > \dot{M}_{\mathrm{E}}$:

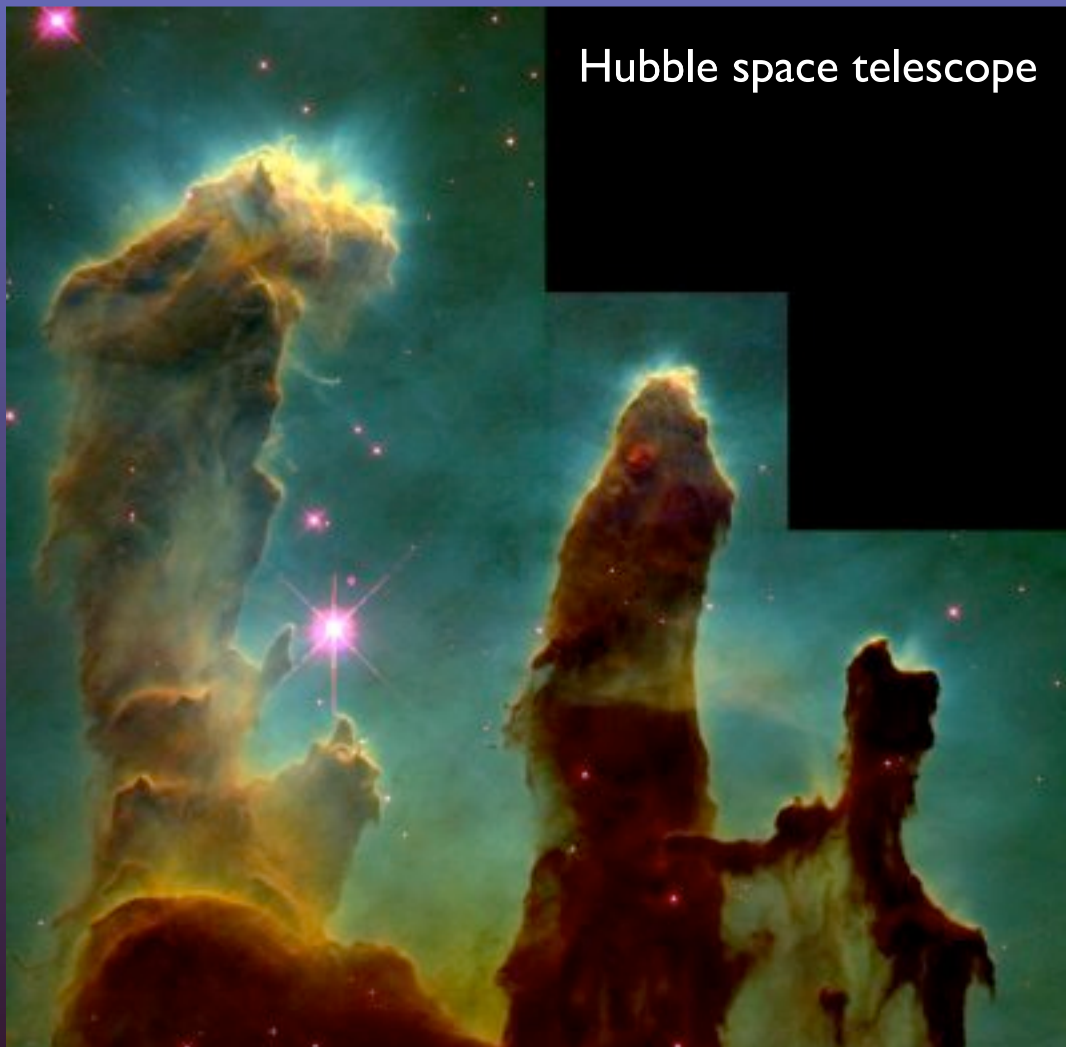


Example: SS433 ?

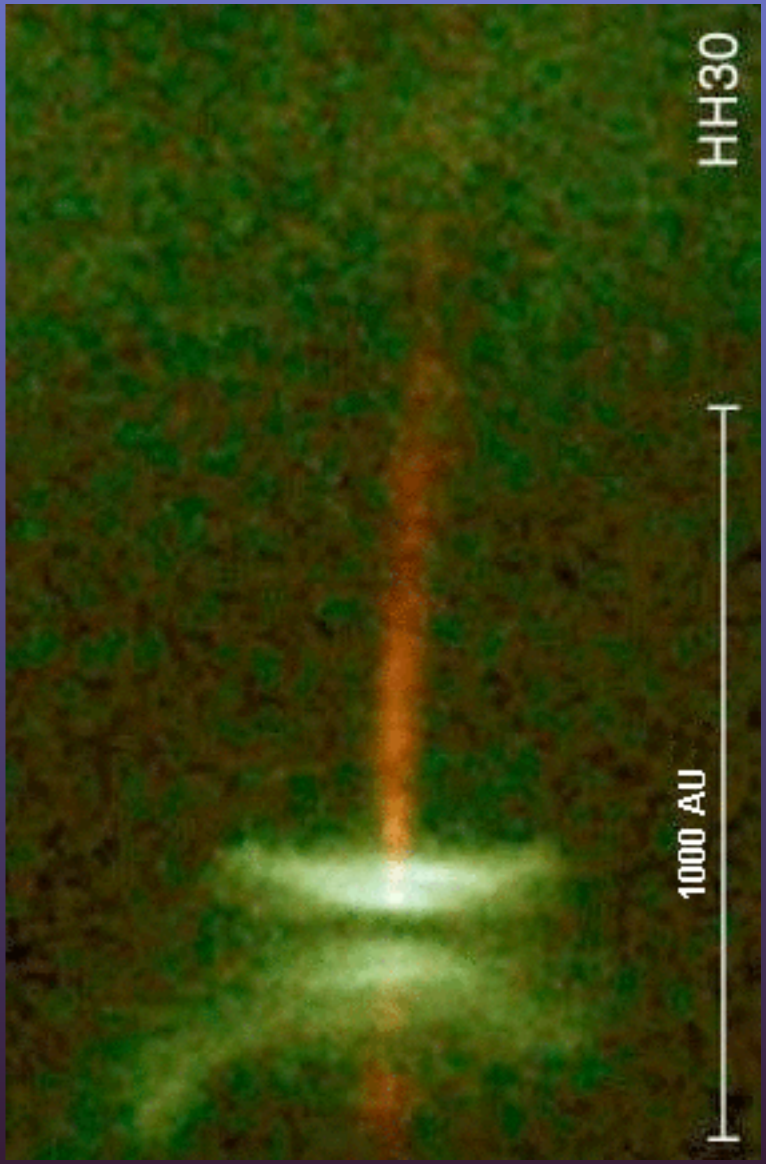
Where disks form

Star formation
Galactic nuclei
Mass transfer in close binary stars

Star forming region

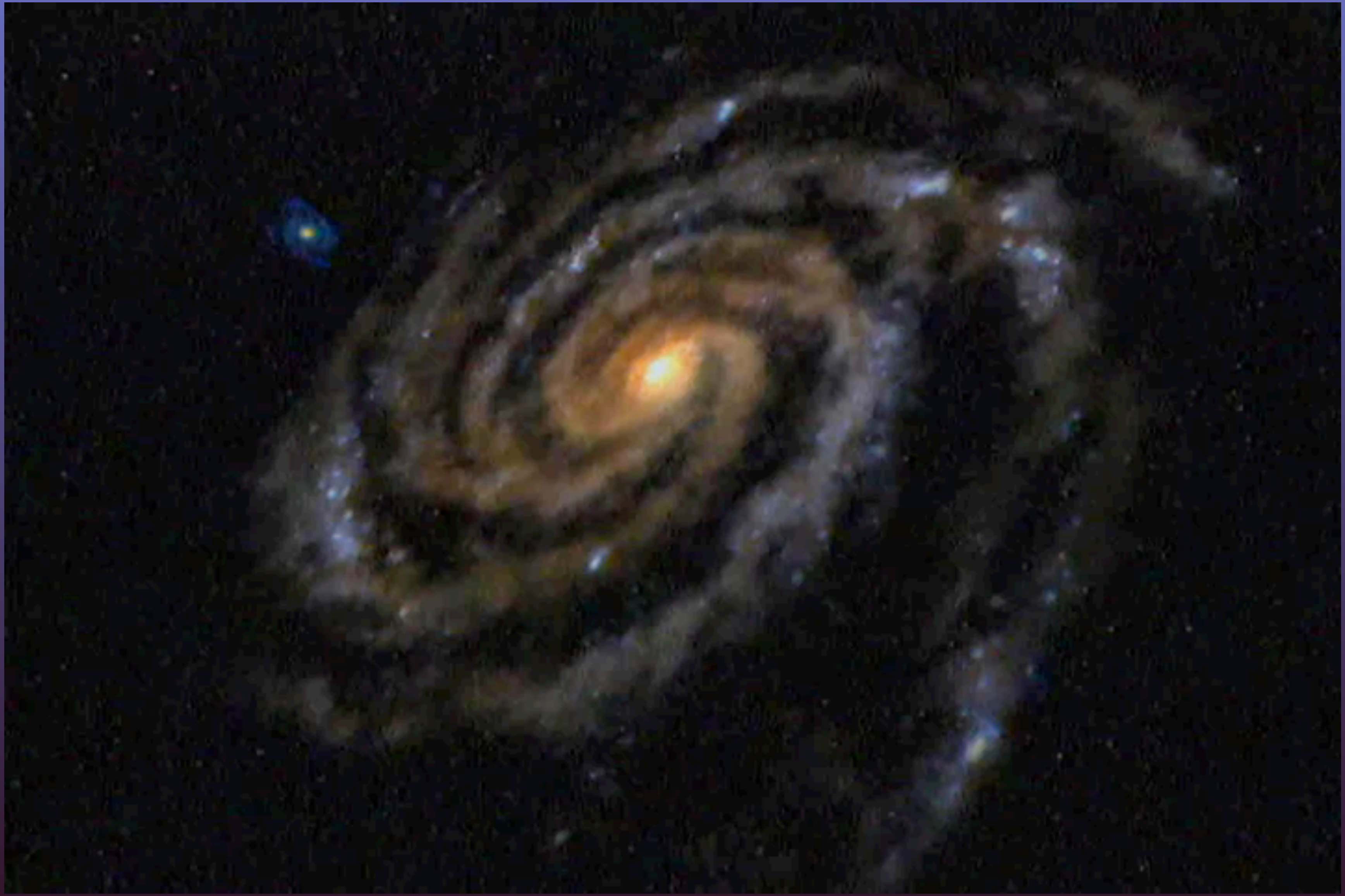


'protodisks'

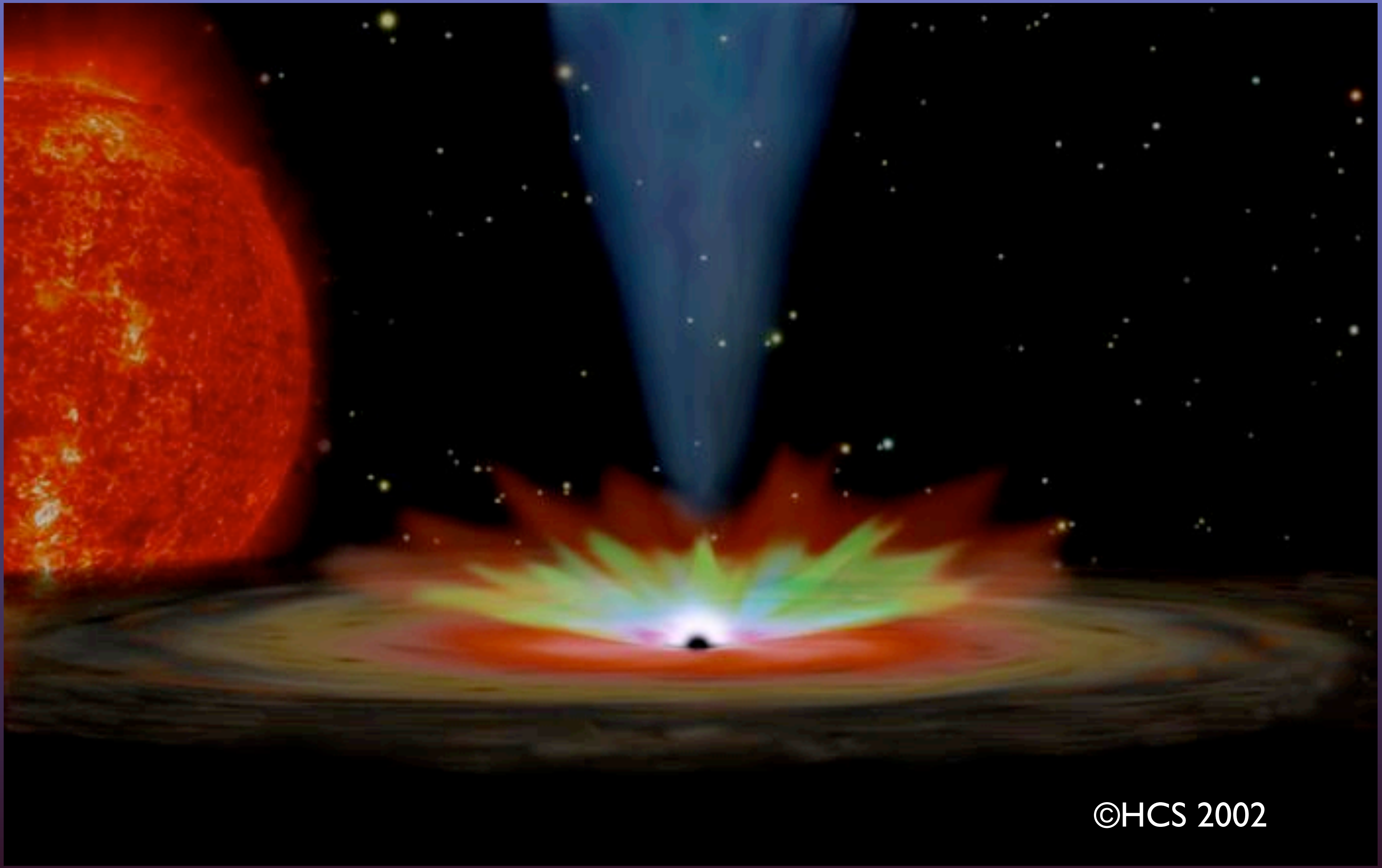

HH30
1000 AU

Disks around active galactic nuclei

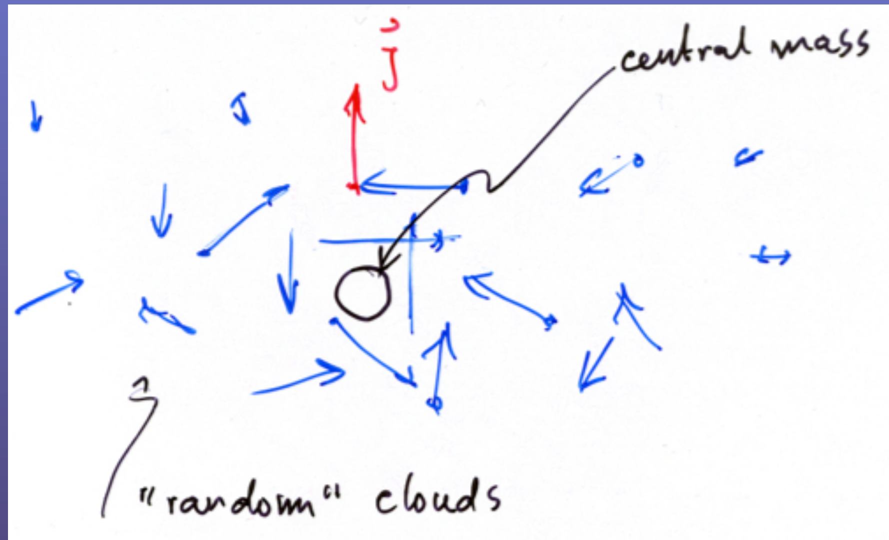
Disks around active galactic nuclei

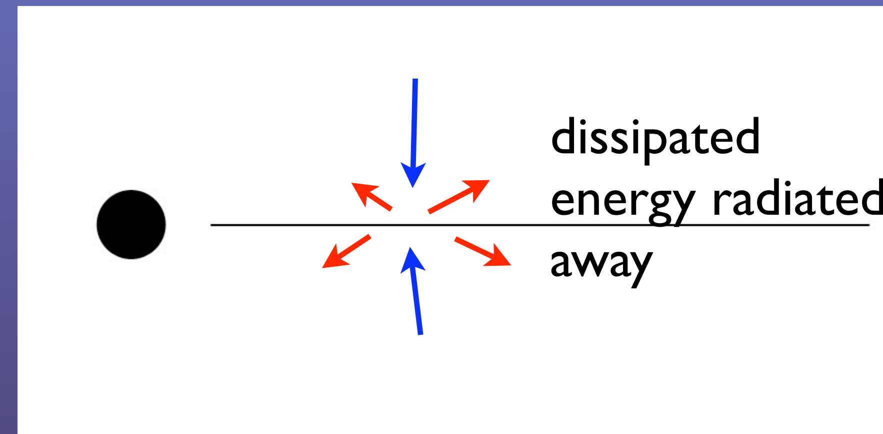


Disks formed by mass transfer in a binary

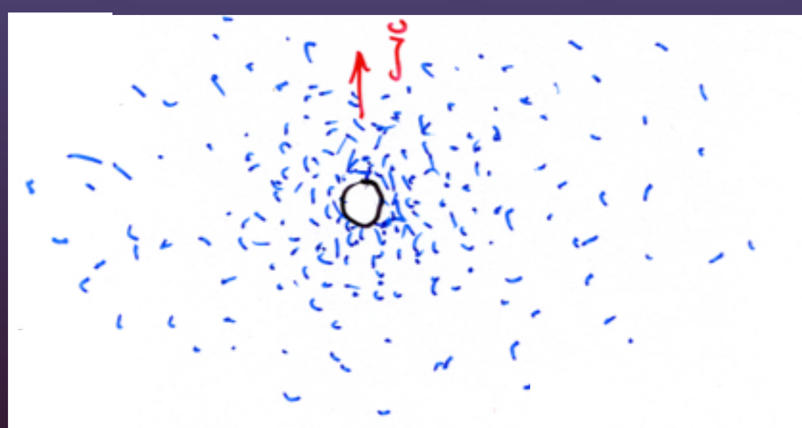


Why disks form: the role of angular momentum



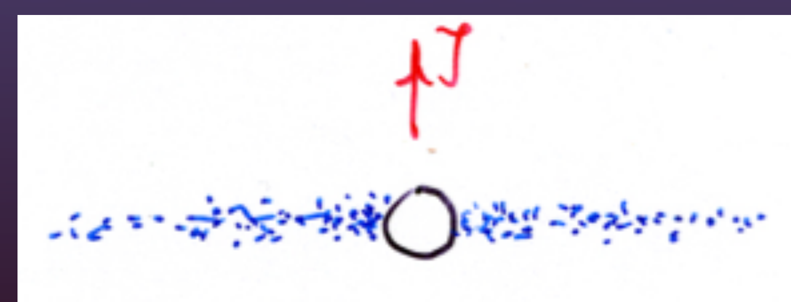


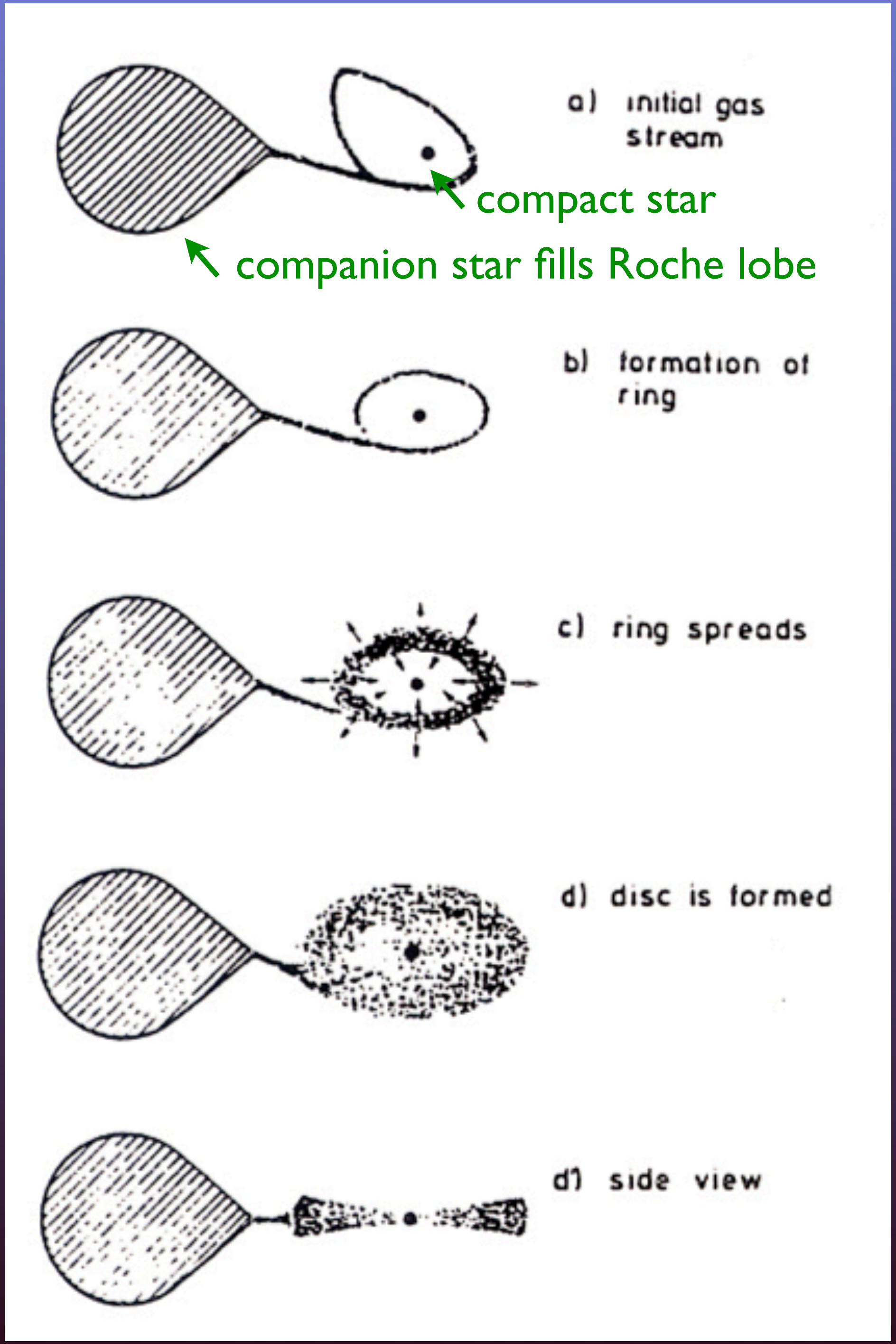
Virialized:



→

cooled:
(example: gas disks in galaxies)





Formation of a disk in a mass-transfering binary

Mass transfering binaries

Frank, King & Raine, "Accretion Power in Astrophysics", CUP, Ch.4

X-ray binary: n-star or BH + main sequence star
Cataclysmic variable: white dwarf + main sequence star (or WD)

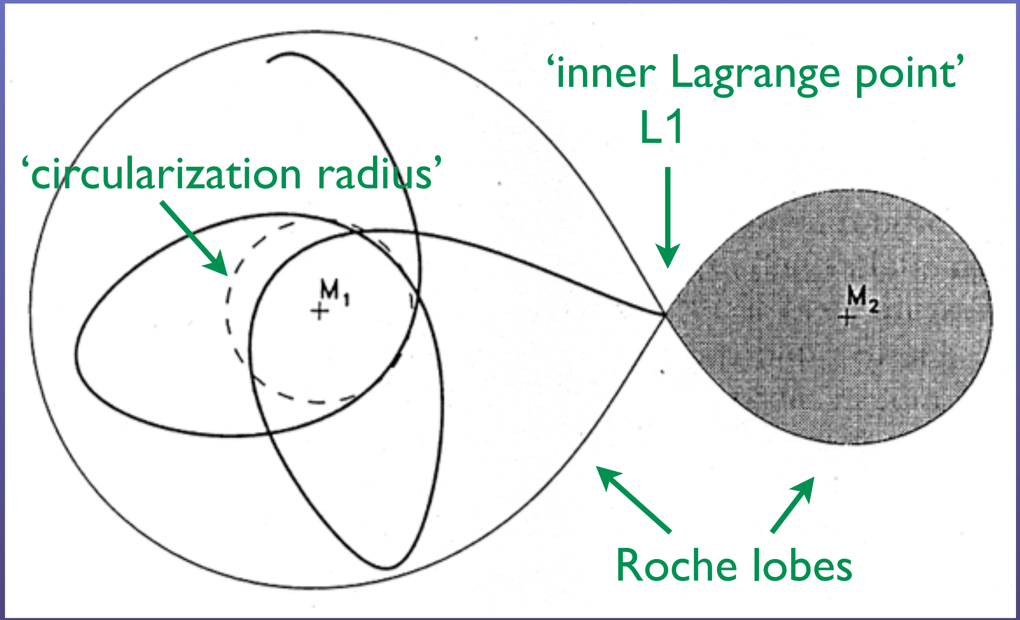
Binary: $M_1,\ M_2,\ \ q \equiv M_2/M1$ (mass ratio)
Circular orbit, separation a, orbital frequency $\Omega = 2\pi/P$

Kepler III: $\Omega^2 = \dfrac{G(M_1+M_2)}{a^3}$ ('period-mean density relation')

$$\rightarrow \quad a = 3.5\,10^{10}\ \mathrm{cm}\ (\frac{M_1}{M_\odot})^{1/3}(1+q)^{1/3}P_{\mathrm{hr}}^{2/3}$$

(c.f. $R_\odot = 7\,10^{10}\mathrm{cm}$)

Roche lobe overflow



Roche surface:

first equipotential surface in a rotating frame (with centrifugal force) connecting the two stars

Nomenclature:

'Roche lobe' ↔ 'Hill sphere'
M1: primary, accreter
M2: secondary, donor

Useful approximations for
- location of Lagrange points,
- shape, volume of Roche lobes:

B. Warner, 1995, "Cataclysmic Variable Stars", CUP, pp 30-40

Equation of motion of free test-particles in a binary

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_1 - \nabla\Phi_2 \qquad \Phi_{1,2} = -\frac{GM_{1,2}}{|\mathbf{r}-\mathbf{r}_1(t)|}$$

In corotating frame (rate Ω):

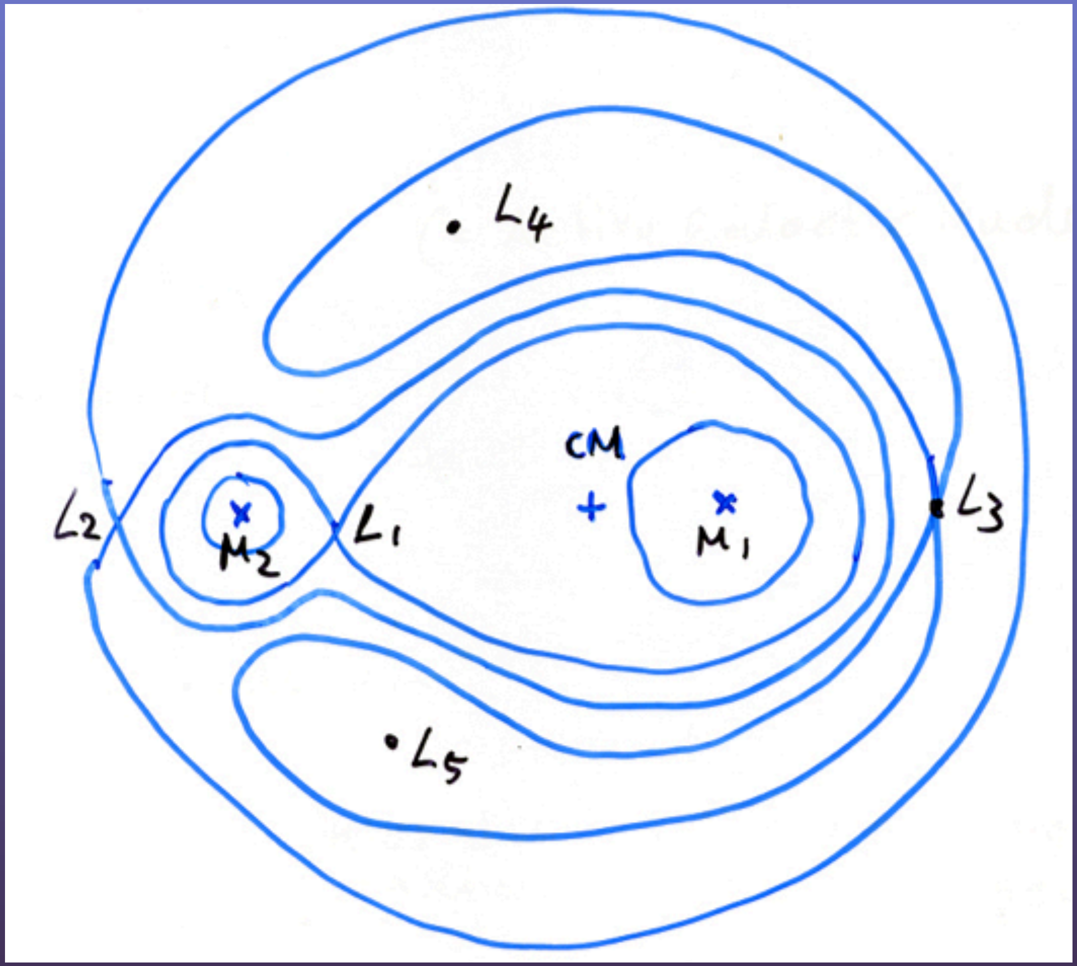
$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_1 - \nabla\Phi_2 + 2\mathbf{v}\times\boldsymbol{\Omega} + (\Omega\times\mathbf{r})\times\Omega$$

Coriolis — Centrifugal

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla\Phi_{\mathrm{R}} + 2\mathbf{v}\times\boldsymbol{\Omega} \qquad \Phi_{\mathrm{R}} = -\frac{GM_1}{|\mathbf{r}-\mathbf{r_1}|} - \frac{GM_2}{|\mathbf{r}-\mathbf{r_2}|} - \frac{1}{2}(\Omega\times\mathbf{r})^2$$

'Centrifugal potential'

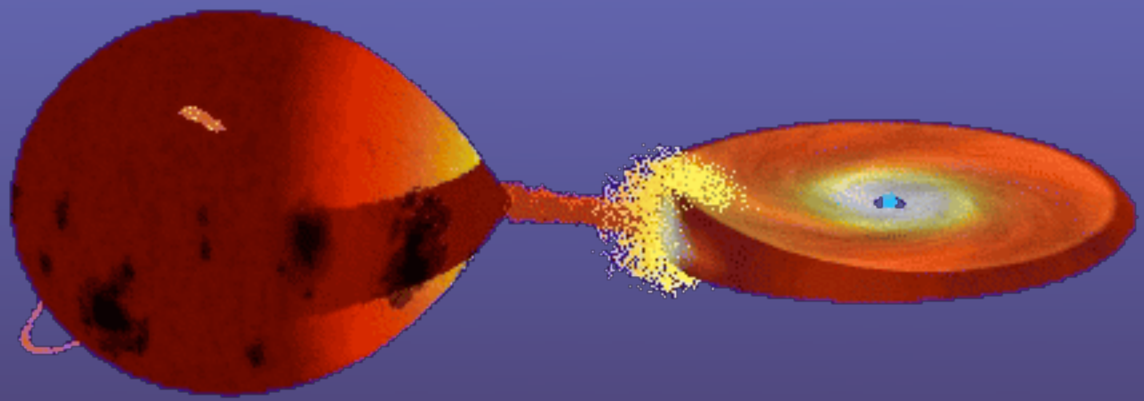
Equipotential surfaces



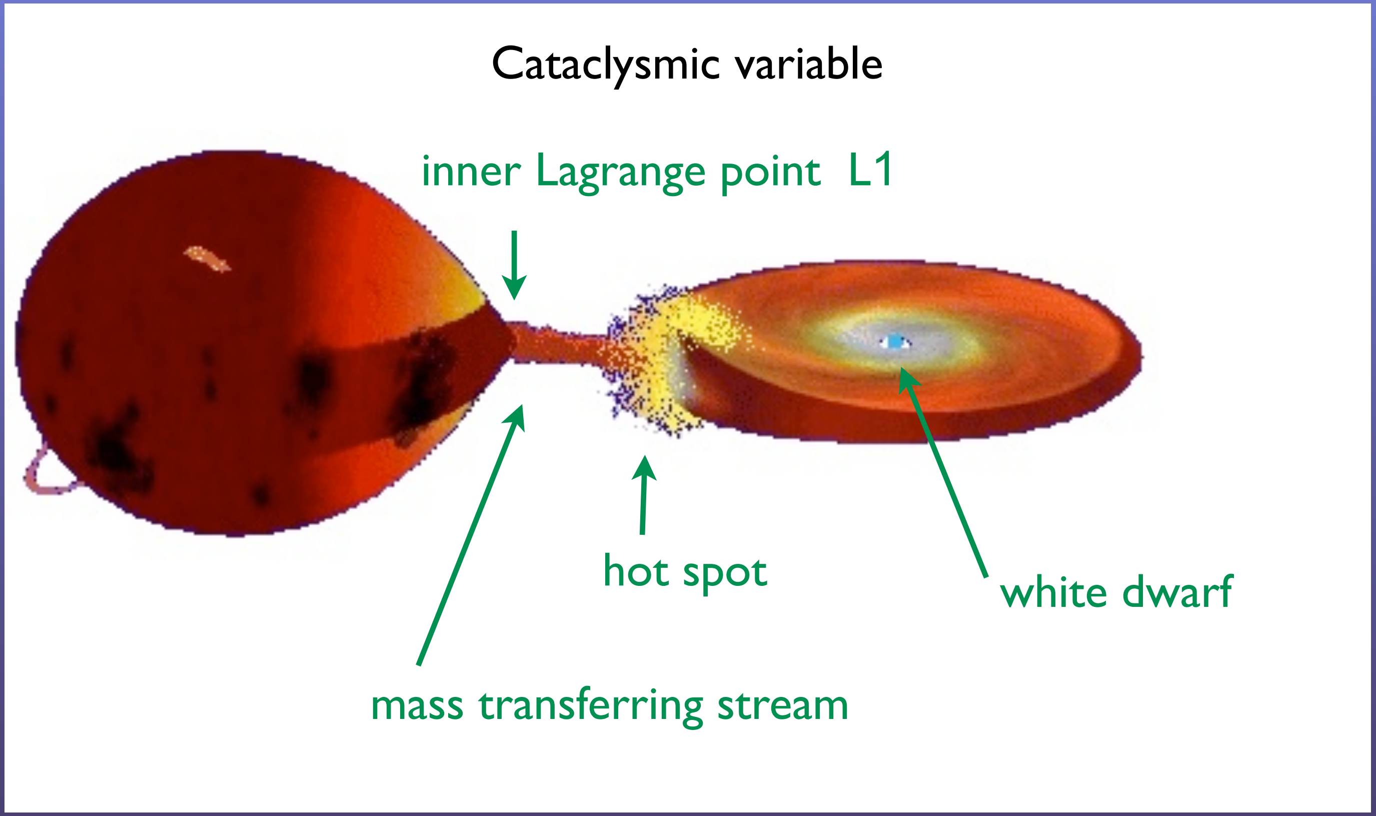
5 Equilibrium points:

Langrange points
unstable: L_1, L_2, L_3
stable: L_4, L_5
('Greeks & Trojans')

Validity of Φ_{R} :
- strictly corotating objects
(the stars, not for orbits between them!)



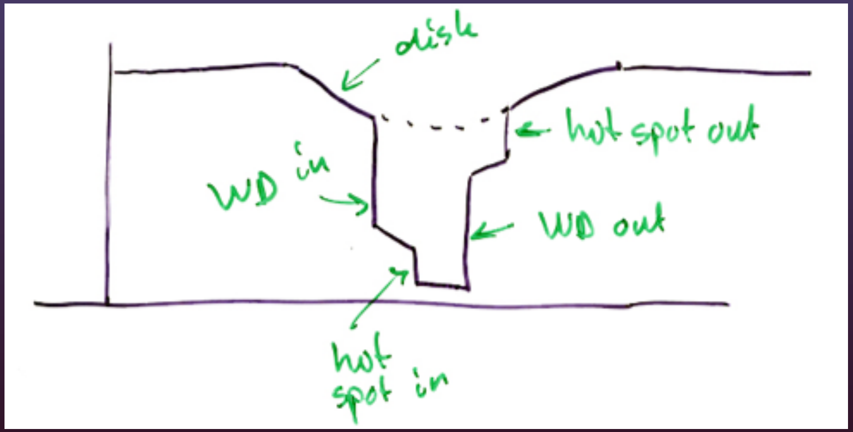
http://physics.technion.ac.il/~astrogr/research.html



Disk size in steady state:
- viscous spreading → r↑
- tidal torques → r↓

equilibrium → *tidal radius*

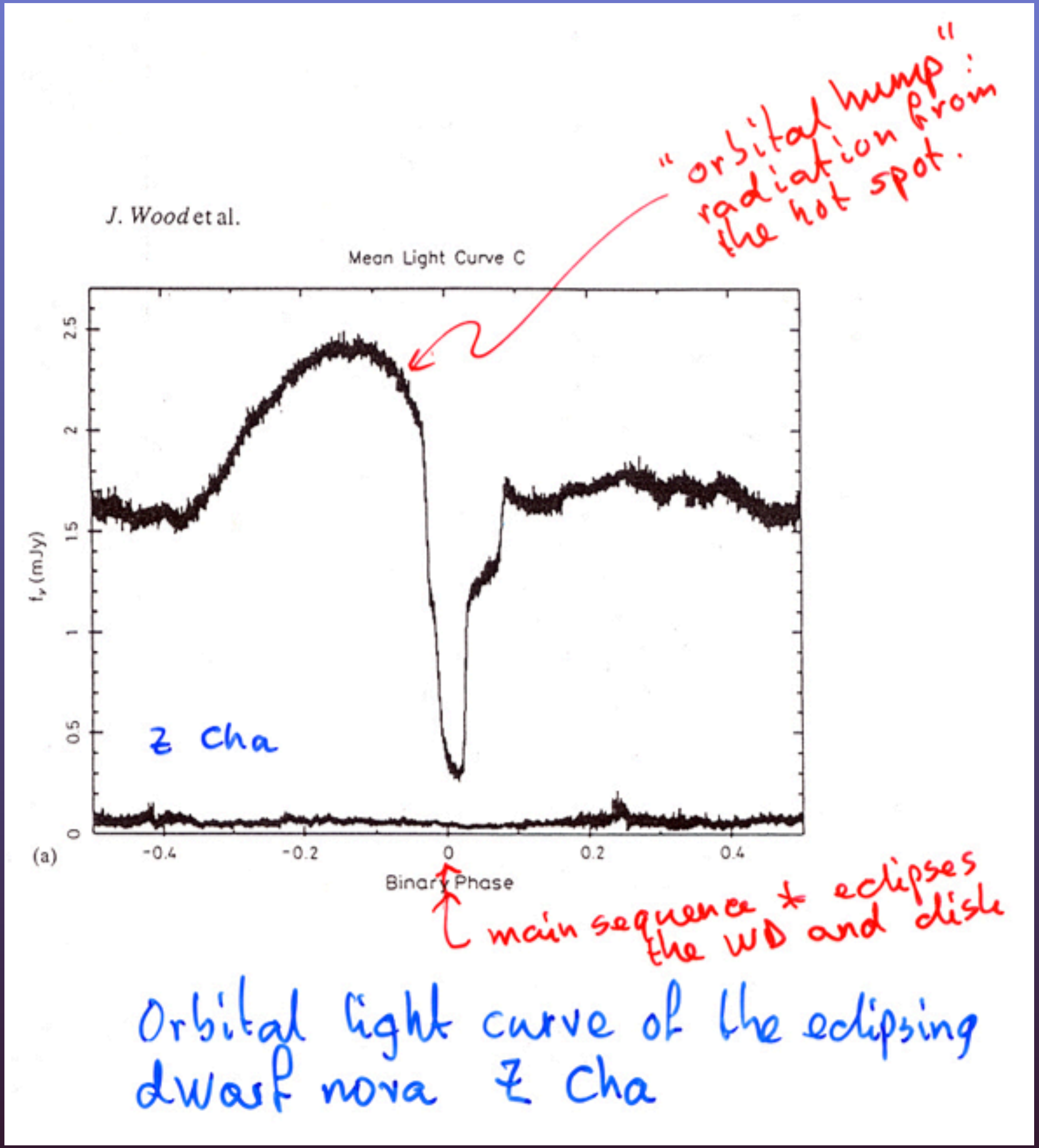
Predicted eclipse light curve:



Actual light curve

used to determine
- orbital parameters
- disk size
- stream impact region
- 'disk turbulence'

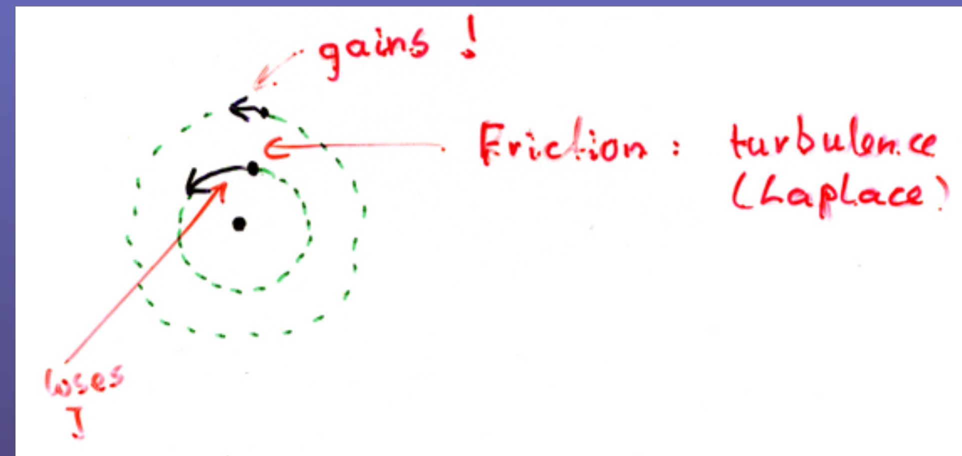
CV's are the best studied disk systems



Angular momentum transport and viscosity in disks

('central problem in accretion disks theory')

For accretion, angular momentum loss necessary

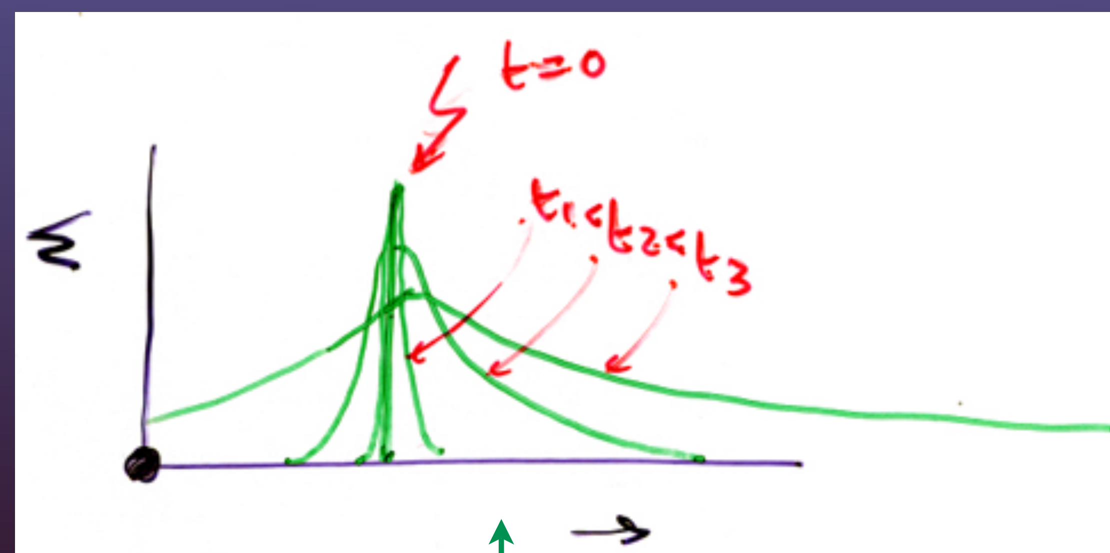


Model: viscous friction inner orbits faster than outer: shear flow
→ *viscous spreading*

Predicted spreading of a ring, $t \to \infty$:

- (almost) all mass accreted
- (almost) all J to $r \to \infty$

(mass and angular momentum conserved !)



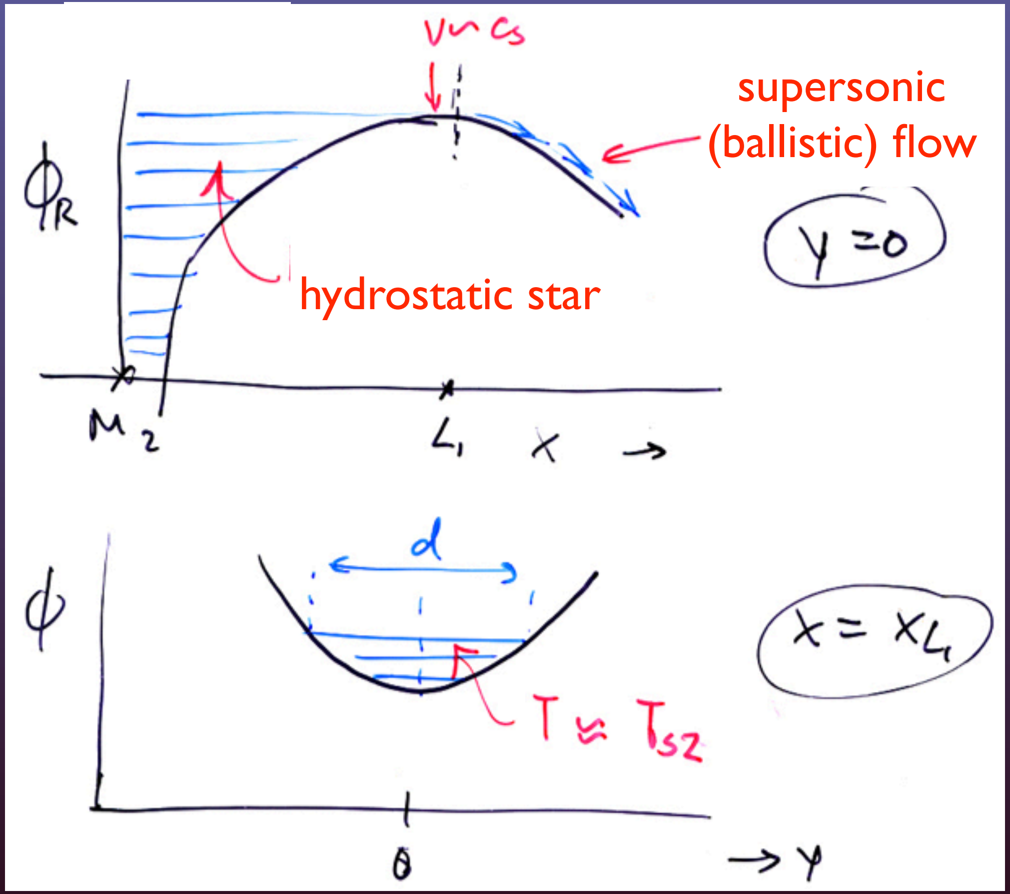
Find the error in this sketch!

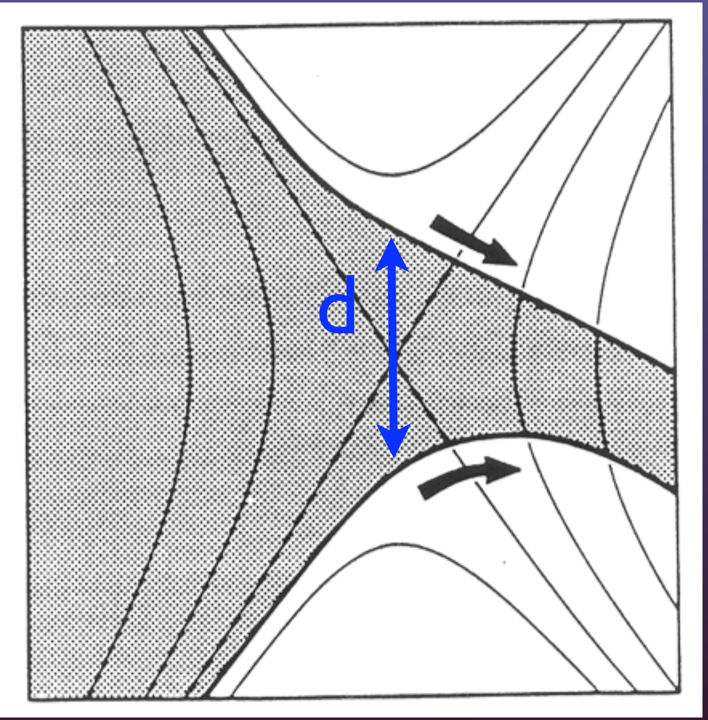
Flow through L1

How wide is the stream? Determined by temperature of gas.

$d \sim (H_2 R_2)^{1/2} \; (\ll R_2)$

pressure scale height at L1: $H_2 = \frac{\mathcal{R} T_{s2}}{\mu g_2} \qquad (g_2 = \frac{GM}{R_2^2})$





Viscosity: how large?

Observations of CV outbursts: decay time of outburst

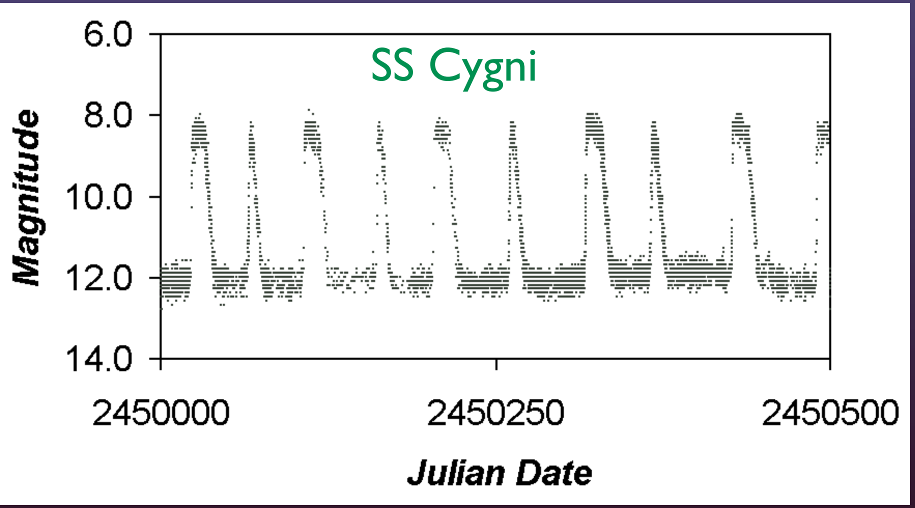
Theory: disk instability → decay on viscous time scale

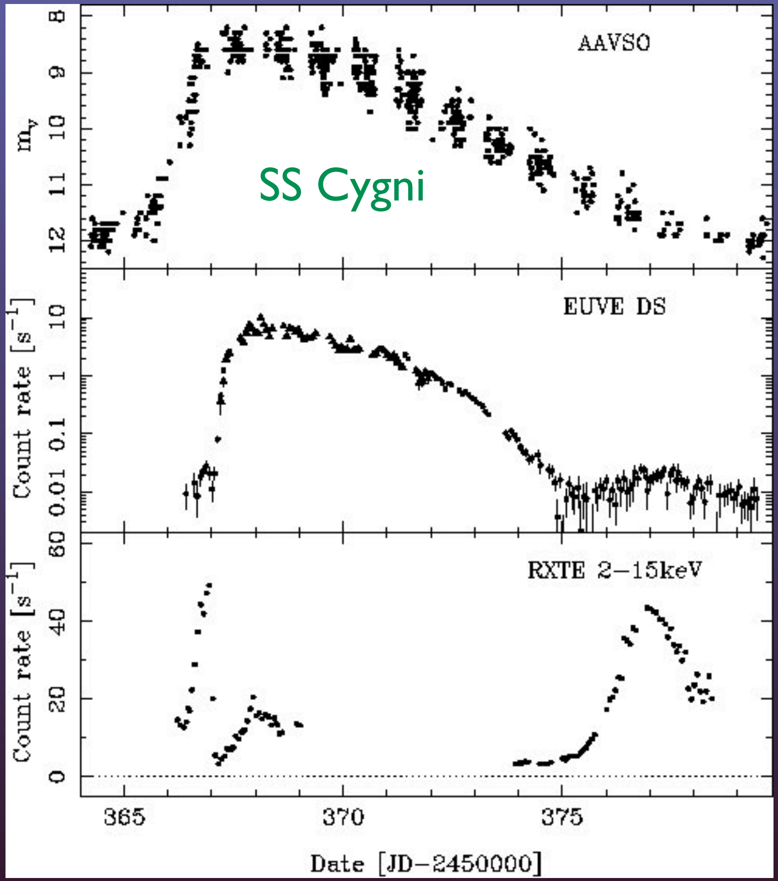
$$t_{\rm decay} = \frac{r_{\rm disk}^2}{\nu}$$

$t_{\rm decay} \sim 5{\rm d}$, $r_{\rm disk} \sim 2\,10^{10}$ cm

$$\rightarrow \quad \nu \sim \frac{4\,10^{20}}{4\,10^{5}} \sim 10^{15}\ {\rm cm^2 s^{-1}}$$

Ionized gas: $\nu \sim 1 - 10\ {\rm cm^2 s^{-1}}$





Cool disks

Gas pressure is unimportant in cool disks (cool: $T \ll T_{\rm vir}$)

Equation of motion in potential of a point mass

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho}\nabla P - \frac{GM}{r^2}\hat{\mathbf{r}}$$

fluid velocity — gas density — gas pressure — gravity

'typical' scales of quantities:

length: arbitrary distance r_0

time: Kepler time scale $t_0 = \Omega_0^{-1} = (\frac{r_0^3}{GM})^{1/2}$

velocity: Kepler velocity $v_0 = \Omega_0 r_0$

Isothermal gas (assume): $\nabla P = \mathcal{R}T\nabla\rho$

write $\mathbf{v} = \tilde{\mathbf{v}}\, v_0,\ \ \mathbf{r} = \tilde{\mathbf{r}}\, r_0,\ \ t = \tilde{t}\, t_0$

$$\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}} + \tilde{\mathbf{v}}\cdot\tilde{\nabla}\tilde{\mathbf{v}} = -\frac{T}{T_{\rm vir}}\tilde{\nabla}\ln\rho - \frac{\hat{\mathbf{r}}}{\tilde{\mathbf{r}}^2} \qquad T_{\rm v} \equiv \frac{GM}{\mathcal{R}r}$$

$\mathcal{O}(1)$ $\mathcal{O}(1)$ $\mathcal{O}(1)$ $\mathcal{O}(1)$

gas pressure comes in with factor $T/T_{\rm v}$

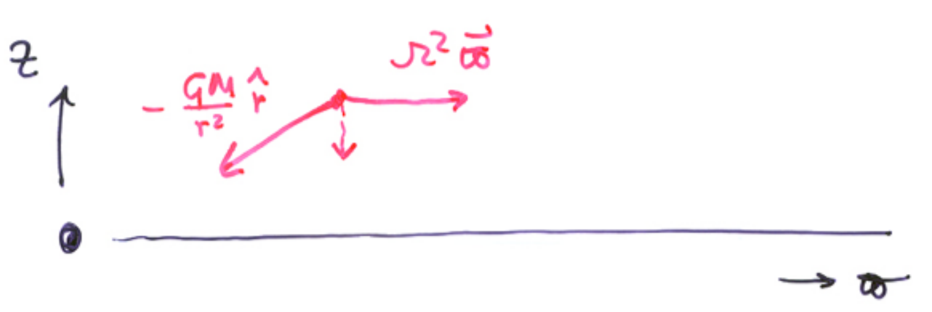
Thin disks

'disk aspect ratio'

Cool disks ($\frac{T}{T_{\rm vir}} \ll 1$) *are thin* ($\frac{H}{r} \ll 1$)

Calculate disk thickness for an 'isothermal' disk: $\partial_z T = 0$

coordinates:
cylindrical (ϖ, φ, z)



Forces on particle rotating with Kepler rate $\Omega_{\rm K}$ at ϖ :

$$\Omega_{\rm K}^2 \varpi = \frac{GM}{\varpi^2} \quad (z=0)$$

$$g_z \approx \Omega^2 \varpi - \frac{GM}{r^2}\hat{\mathbf{r}} \approx -\Omega_{\rm K}^2 z + \mathcal{O}(z^3) \qquad z/\varpi \ll 1$$

$$\frac{{\rm d}P}{{\rm d}z} = g_z \rho \;\rightarrow\; \frac{\mathcal{R}T}{\mu}\frac{{\rm d}\rho}{{\rm d}z} = -z\Omega_{\rm K}\rho \;\rightarrow\; \frac{{\rm d}\ln\rho}{{\rm d}z} = -z\frac{\mu\Omega_{\rm K}^2}{\mathcal{R}T} \;\rightarrow\; \rho = \rho_0 \exp[-\frac{z^2}{2H^2}]$$

where H nominal disk thickness

$$H = (\frac{\mathcal{R}T}{\mu\Omega_{\rm K}})^{1/2} = \frac{c_{\rm si}}{\Omega_{\rm K}}$$

(isothermal sound speed: $c_{\rm si} = (\mathcal{R}T/\mu)^{1/2}$)

Cool disks

Consequences of thin disk approximation:

$\partial_r P$ negligible

1 radial equation of motion: $\Omega^2 r = \frac{GM}{r^2}$ (circular Kepler orbits)

Internal energy $u = P/\rho$:

2 → advection of internal energy negligible

→ viscous dissipation is radiated away locally:

$$Q_{\rm visc} = \rho\nu(r\partial_r\Omega)^2 = (\frac{3}{2}\Omega)^2\nu\rho = {\rm div}\,\mathbf{F}_{\rm rad}$$

$$\int {\rm d}z\ {\rm div}F = \text{surface flux} \quad \rightarrow \quad \frac{9}{4}\Omega^2\int_{-\infty}^{\infty}\nu\rho{\rm d}z = 2\sigma T_{\rm s}^4$$

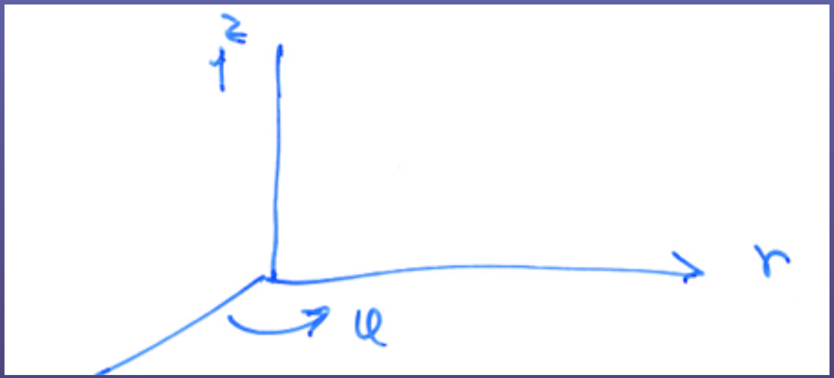
[note: this is ***not*** equal to the local gravitational energy release. Radial energy flux associated with viscous torques is to be accounted for]

3 All unknown physics enters through viscosity ν

Viscous disk theory

Thin axisymmetric viscous disks

Frank, King & Raine Accretion power Astrophsyics
Pringle, J.E., 1981, Ann rev. Astron. Astrophys.



midplane of disk at $z = 0$

surface mass density: $\Sigma = \int_{-\infty}^{\infty} \rho \, \mathrm{d}z$

radial (accretion-)velocity v_r

continuity (mass conservation): $\partial_t(r\Sigma) + \partial_r(r\Sigma v_r) = 0$

radial equation of motion: $v_\varphi^2 = \Omega^2 r^2 = GM/r$ (Kepler orbits)

azimuthal equation of motion:

Angular momentum

$$\partial_t(r\Sigma\Omega r^2) + \partial_r(r\Sigma v_r\Omega r^2) = \partial_r(r^2\Sigma\nu\partial_r\Omega)$$

local rate of change of angular momentum

'advection' of angular momentum

viscous torque = viscous angular momentum flux

Viscous disk theory

$$\partial_t(r\Sigma) + \partial_r(r\Sigma v_r) = 0 \quad (1)$$

$$\partial_t(r\Sigma\Omega r^2) + \partial_r(r\Sigma v_r\Omega r^2) = \partial_r(r^2\Sigma\nu\partial_r\Omega) \quad (2)$$

(2) $-\Omega r^2$ (1) $\rightarrow$ $r\Sigma v_r\partial_r(\Omega r^2) = \partial_r(r^2\nu\Sigma r\partial_r\Omega)$ (3)

Accretion rate: $\dot{M} = -2\pi r\Sigma v_r$

$\Omega \sim r^{-3/2}$ (Kepler) → $\dot{M} = 6\pi r^{1/2}\partial_r(r^{1/2}\nu\Sigma)$ (4)

(2) + (3) $\rightarrow$ $r\partial_t\Sigma = 3\partial_r[r^{1/2}\partial_r(\nu\Sigma r^{1/2})]$

(*thin disk evolution equation*)

- all relevant physics condensed in the viscosity ν
- diffusion equation: *viscous spreading*

Steady thin disks

$\partial_t = 0 \;\rightarrow\; \dot{M} = \mathrm{cst.} \qquad \dot{M} = 6\pi r^{1/2}\partial_r(r^{1/2}\nu\Sigma)$

(For accretion: $\dot{M} > 0$*)*

(4) $\rightarrow\; r^{1/2}\nu\Sigma = \frac{\dot{M}}{3\pi}r^{1/2} + c_2$ (c_2 : integration cst)

equivalent: $\nu\Sigma = \frac{\dot{M}}{3\pi}[1 - \beta(\frac{r_\mathrm{i}}{r})^{1/2}]$ (3) (r_i: inner edge of the disk)

Interpretation: Angular momentum flux (= 'torque') F_J

$F_J = \dot{M}\Omega r^2 + 2\pi r^2\nu\Sigma r\partial_r\Omega$

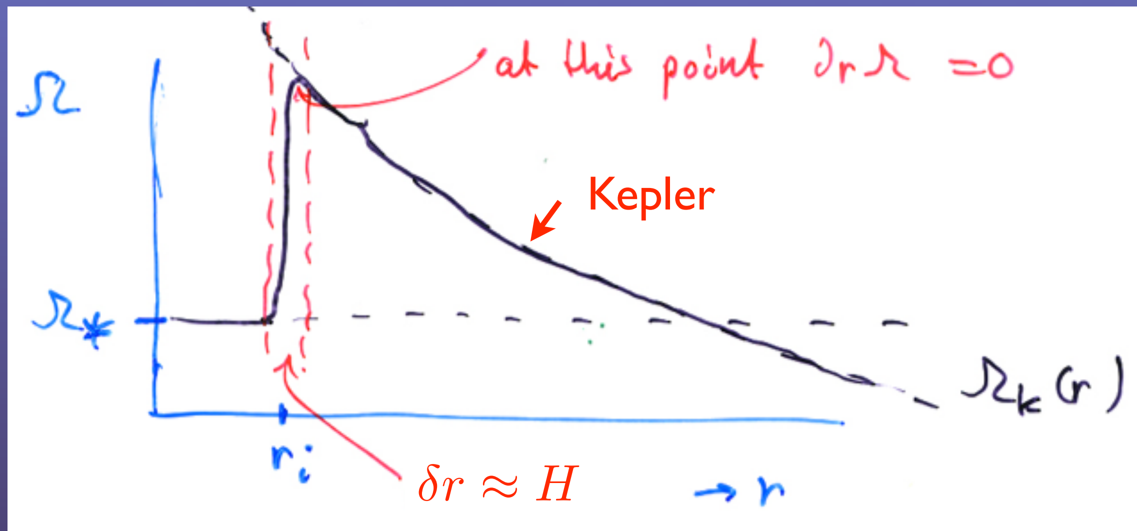
$r\partial_r\Omega = -\frac{3}{2}\Omega$ (Kepler) + (3) :

$F_J = \beta\dot{M}\Omega(r_\mathrm{i})r_\mathrm{i}^2 \;\; (= (GMr_\mathrm{i})^{1/2})$

→ β *is the angular momentum flux through the disk, in units of the angular momentum advected with* $\dot{M}$, *at* r_i

Steady thin disks

Thin disk solution for accretion onto a "slowly rotating" object



at r_i $\partial_r\Omega = 0$: no shear, no viscous stress

$$\rightarrow F_J = \dot{M}\Omega(r_\mathrm{i})r_\mathrm{i}^2 \rightarrow \beta = 1 \quad \rightarrow \nu\Sigma = \frac{\dot{M}}{3\pi}[1-(\frac{r_\mathrm{i}}{r})^{1/2}]$$

Standard steady thin disk result.
Holds (to lowest order in H/r) for all stars rotating with $\Omega_* < \Omega_\mathrm{K}(r_\mathrm{i})$ (i.e. all stars)

Exceptions : 1 stars rotating near maximum ('breakup')
2 stars with a magnetosphere

Surface density profile, relation with angular momentum flux

$\beta = 1$

'standard disk': accretion on slow rotator

$0 < \beta \leq 1$

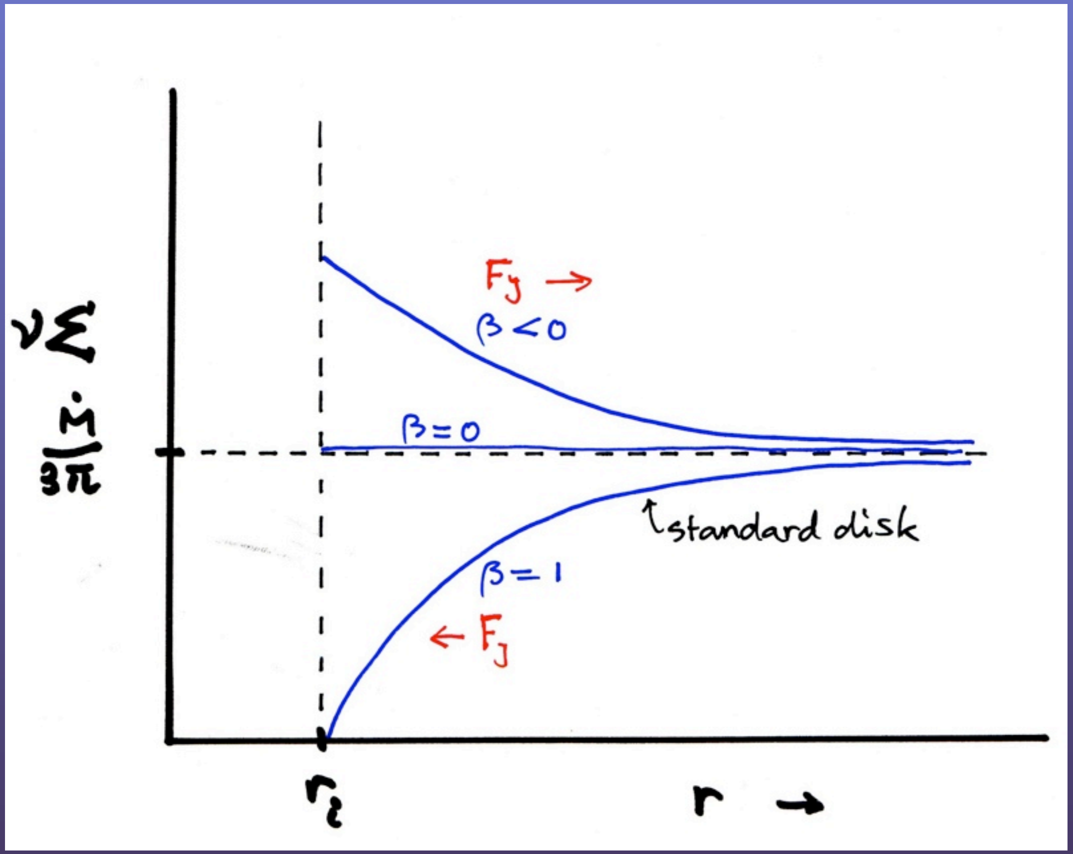
angular momentum flux inward ('spinup')

$\beta = 0$

zero net angular momentum flux

$\beta < 0$

outward angular momentum flux ('spindown'): *can happen in accretion on a star with a strong magnetic field*



$$\nu\Sigma = \frac{\dot{M}}{3\pi}[1 - \beta(\frac{r_i}{r})^{1/2}]$$

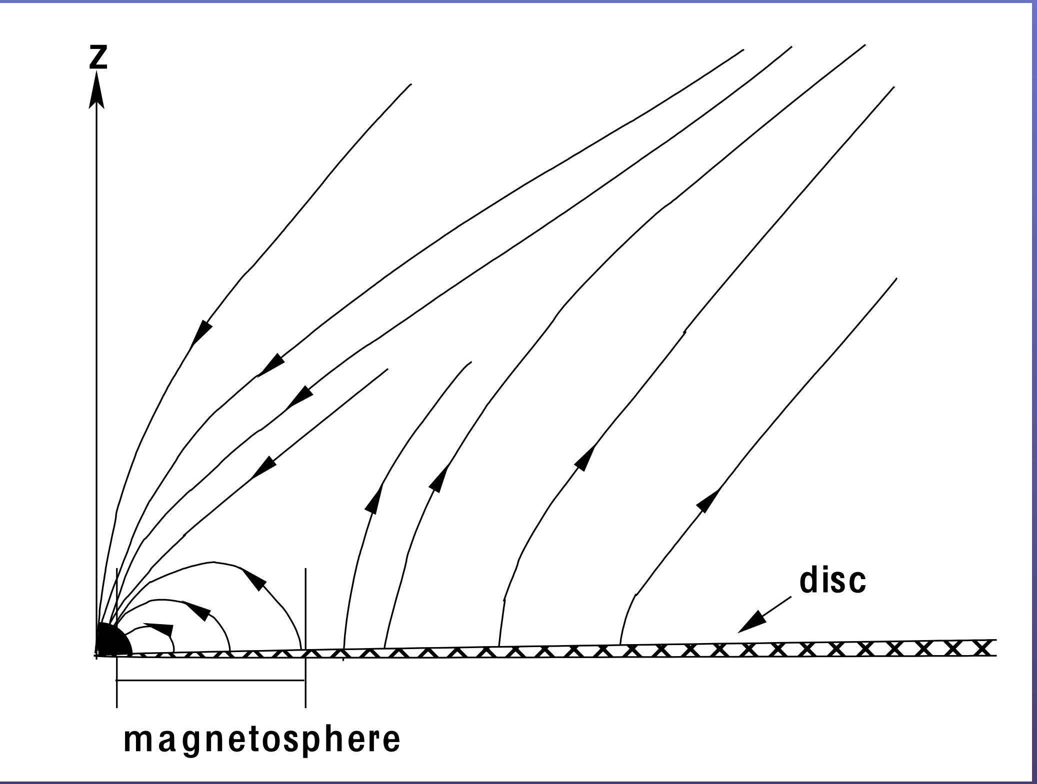
'Magnetospheric' accretion

(accretion disks around
magnetic neutron stars,
protostars)

magnetosphere radius $r_{\rm m}$
$r > r_{\rm m}$: Kepler rotation
$r < r_{\rm m}$: corotation with *

corotation radius $r_{\rm c}$:

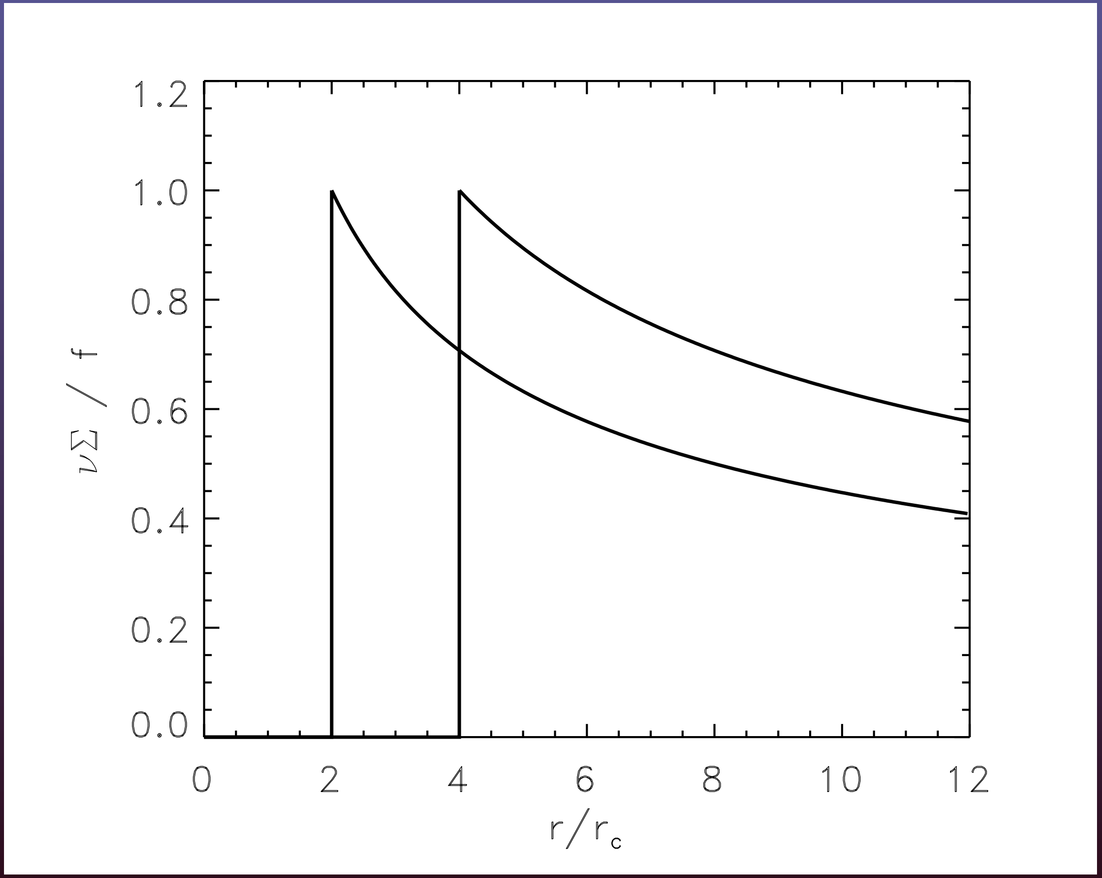
$$\Omega_* = \Omega_{\rm K}(r_c)$$



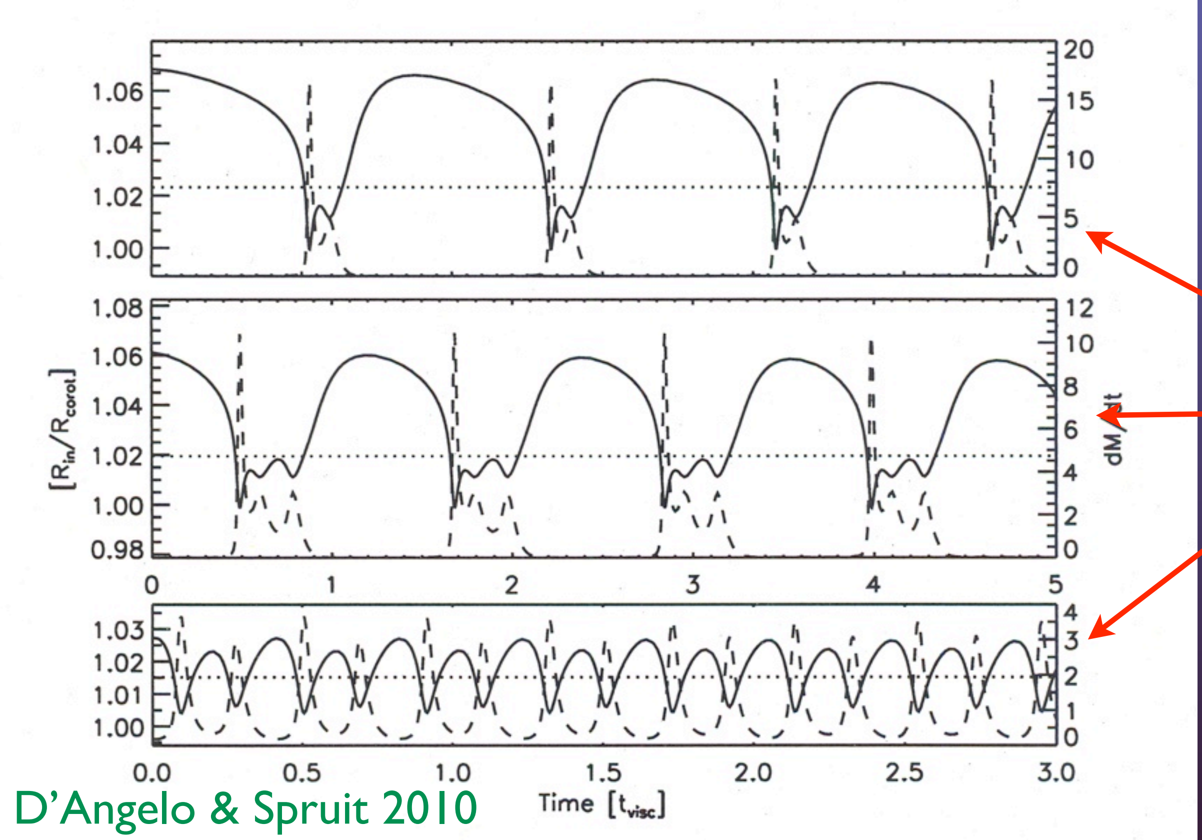
Magnetospheric accretion

$r_m > r_c$: 'Dead disks' (Sunyaev & Shakura 1977)

$r_m \gg r_c$: 'propellering' (e.g. AE Aqr)



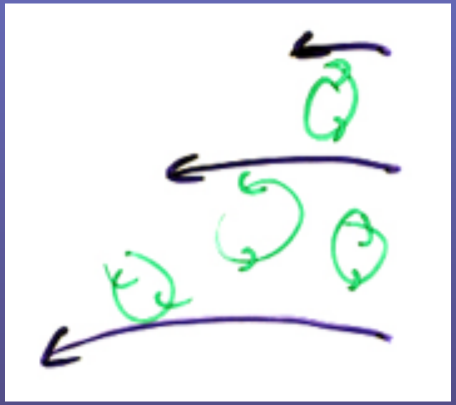
cyclic accretion possible between quiescent and accreting states *cyclic* accretion between quiescent and accreting states



parameter:
time-average
accretion rate

Source of viscosity: a limit

'Molecular' (microscopic) viscosity too low by many orders of magnitude



"Turbulence" (Laplace)

assume: hydrodynamic instability generates turbulence

"reason": Reynolds number:

$$\mathrm{Re} = \frac{LV}{\nu_{\mathrm{mic}}} = \frac{r_{\mathrm{disk}}^2 \Omega_{\mathrm{K}}}{\nu_{\mathrm{mic}}} \gg 1$$ (CV: $\mathrm{Re} \sim 10^{14}$)

Upper limit on turbulent viscosity: causality

speed of sound limits physical connection in radial direction

shear rate $\eta \equiv |\partial_r v| \sim \Omega_{\mathrm{K}}$

Eddy velocity $< c_{\mathrm{s}}$:

$$V = L\eta < c_{\mathrm{s}} \;\rightarrow\; L < c_{\mathrm{s}}/\Omega = H$$

➔ 1: $\nu \approx LV = L^2\eta < c_{\mathrm{s}}^2/\Omega$

➔ 2: 'vortices' in disks have radial length $L < H$

no 'large scale vortices'

digression: vortices in disks

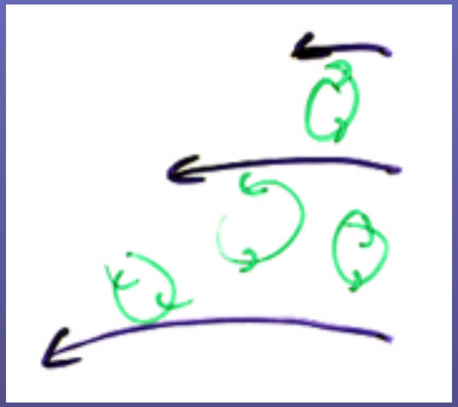
Idea:
- disk is thin sheet of rotating fluid
- earth's atmosphere is thin, rotating
  → approximations for hydrodynamics known,
    describing vortices (weather systems ...)
- *use these for computing large scale vortices in accretion disks*

Approximations used: incompressible (anelastic ...)

The error made here: accretion disk is a supersonic flow.
Causality (sound speed) limits horizontal scale of vortices to $L \sim H$.

Source of viscosity: a limit

'Molecular' (microscopic) viscosity too low by many orders of magnitude



"Turbulence" (Laplace)

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Upper limit on turbulent viscosity: causality

speed of sound limits physical connection in radial direction ('causality')

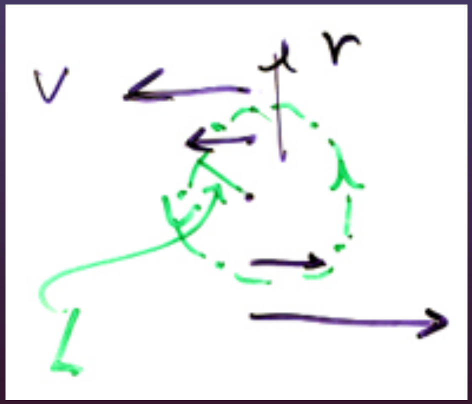
shear rate $\eta \equiv |\partial_r v| \sim \Omega_{\rm K}$

Eddy velocity $< c_{\rm s}$:

$$V = L\eta < c_{\rm s} \ \rightarrow \ L < c_{\rm s}/\Omega = H$$

1: $\nu \approx LV = L^2\eta < c_{\rm s}^2/\Omega$

2: 'vortices' in disks have radial length $L < H$



properties of alpha viscosity model

$\nu = \alpha c_\mathrm{s}^2/\Omega_\mathrm{K}$ (Causality: $\alpha < 1$) (Shakura & Sunyaev 1973)

Accretion velocity $v_\mathrm{acc} \approx \nu/L; \quad L = r: \quad v_\mathrm{acc} \approx \alpha \frac{c_\mathrm{s}^2}{\Omega r}$

Accretion time scale

$$t_\mathrm{a} = r/v_\mathrm{acc} \approx \frac{1}{\alpha\Omega}\left(\frac{\Omega r}{c_\mathrm{s}}\right)^2 = \frac{1}{\alpha\Omega}\frac{T_\mathrm{vir}}{T} \gg 1/\Omega \qquad T_\mathrm{vir}/T = (r/H)^2$$

Surface density $\rightarrow \quad \nu\Sigma = \frac{\dot{M}}{3\pi}[1 - (\frac{r_\mathrm{i}}{r})^{1/2}]$

$$r \gg r_\mathrm{i}: \quad \Sigma \approx \frac{\Omega_\mathrm{K}}{\alpha c_\mathrm{s}^2}\frac{\dot{M}}{3\pi}$$

Surface temperature T_s: viscous dissipation balanced by radiation:

$$(\frac{3}{2}\Omega)^2\nu\rho = \mathrm{div}\,\mathbf{F}_\mathrm{rad} \quad : \quad \frac{9}{4}\Omega^2\int_{-\infty}^{\infty}\nu\rho\,\mathrm{d}z = \frac{9}{4}\Omega^2\nu\Sigma = 2\sigma T_\mathrm{s}^4$$

$$\rightarrow \quad F_\mathrm{rad} = \sigma T_\mathrm{s}^4 = \frac{3}{8\pi}\Omega^2\dot{M}[1 - (\frac{r_\mathrm{i}}{r})^{1/2}] \rightarrow \quad T_\mathrm{s} \sim (M\dot{M})^{1/4}r^{-3/4}$$

independent of viscosity (for steady, thin disk) *warning: viscous stress transports energy through disk*

$$(\frac{3}{2}\Omega)^2\nu\rho = \mathrm{div}\,\mathbf{F}_{\rm rad} \quad \rightarrow \quad \frac{9}{4}\Omega^2\nu\Sigma = 2\sigma T_{\rm s}^4 \qquad (5)$$

viscous dissipation equals surface heat flux

Compare: release of *gravitational energy* . $\Phi = -\frac{GM}{r}$
Half stays in orbital motion. Released is:

$$W_{\rm grav} = \frac{1}{2}\dot{M}\partial_r\Phi = \frac{1}{2}\frac{GM\dot{M}}{r^2}$$

Thin disk solution: $\nu\Sigma = \frac{\dot{M}}{3\pi}f; \qquad f = 1 - (\frac{r_{\rm i}}{r})^{1/2}$

viscous dissipation, integrated in φ

$$W_{\rm visc} = (2\pi r)\frac{9}{4}\Omega^2\nu\Sigma = 2\pi\frac{9}{4}\frac{GM}{r^2}\nu\Sigma = \frac{3}{2}\frac{GM}{r^2}\dot{M}f = 3fW_{\rm grav}$$

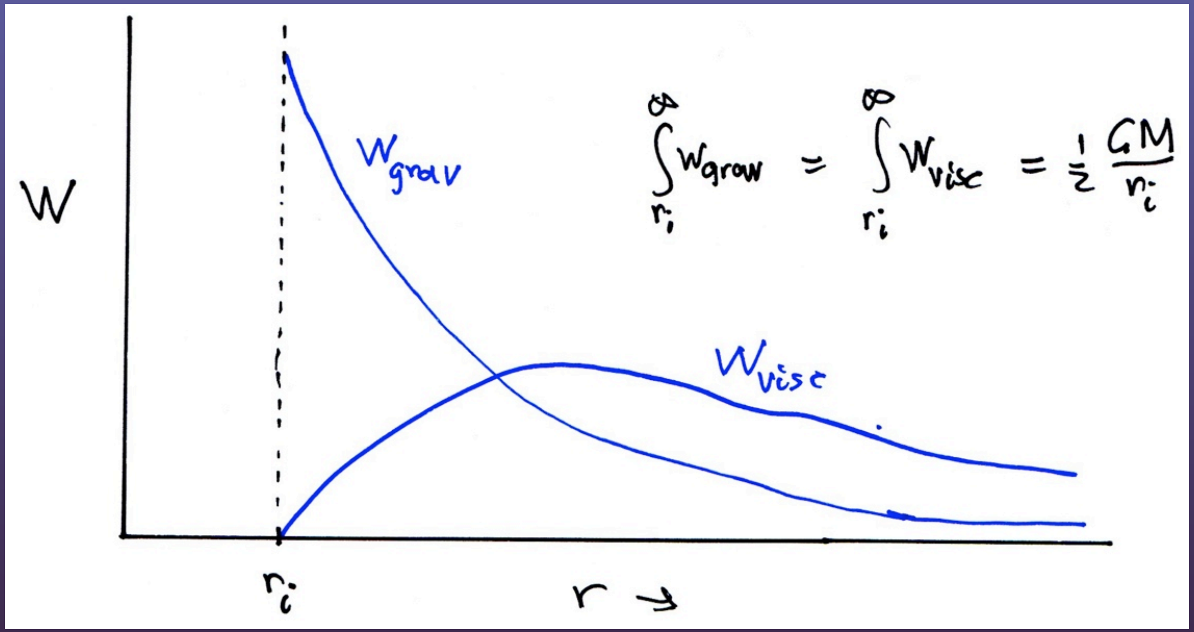
➜ $W_{\rm visc} \neq W_{\rm grav}$

warning (ct'd)

$$W_{\rm visc} = 3fW_{\rm grav}$$

$$r \to \infty \quad : (f \to 1) \quad W_{\rm visc} = 3W_{\rm grav}$$

Energy transport by viscous stress must be included in balance



'excess dissipation' useable for outflows? 'ADIOS': Begelman & Blandford 1999 *thin disk time scales*

orbital
$$t_{\rm dyn} = \frac{1}{\Omega_{\rm K}} = (\frac{r^3}{GM})^{1/2}$$

thermal

heating ...
$$t_{\rm th} = \frac{\int u\,{\rm d}z}{\int Q_{\rm visc}\,{\rm d}z} \approx \frac{u}{Q_{\rm visc}}$$

... = cooling
$$= \int u\,{\rm d}z/F_{\rm rad}$$

→
$$t_{\rm th} = \frac{P/(\gamma-1)}{\frac{9}{4}\rho\nu\Omega^2} \sim \frac{1}{\alpha\Omega_{\rm K}}$$

viscous

(=accretion time)
$$t_{\rm visc} = \frac{r^2}{\nu} = \frac{r^2}{\alpha c_{\rm s}^2/\Omega} = \frac{1}{\alpha\Omega}(\frac{r}{H})^2$$

sound travel time
$$t_{\rm s\,vertical} = \frac{H}{c_{\rm s}} = \Omega_{\rm K}^{-1}$$
$$t_{\rm s\,radial} = \frac{r}{c_{\rm s}} = \frac{r}{H}\Omega_{\rm K}^{-1}$$

summary of thin disk properties

- pressure, internal energy negligible
- aspect ratio $\frac{H}{r} = \frac{c_\mathrm{s}}{\Omega r} = \frac{1}{\mathrm{Ma}} = (\frac{T}{T_\mathrm{vir}})^{1/2} \ll 1$
- orbits Keplerian $\Omega = \Omega_\mathrm{K}$
- all (known, unknown) physics is local, enters thru $\nu(\Sigma, r)$ only
- viscous model: alpha-disk $\nu = \alpha c_\mathrm{s}^2/\Omega$
- viscously dissipated energy radiated locally (in r)
- *difference between local gravitational energy release and viscous dissipation*
- accretion onto $\Omega_* < \Omega_\mathrm{K}(r_\mathrm{i})$: $\nu\Sigma \sim 1 - (\frac{r_\mathrm{i}}{r})^{1/2}$

  → density at inner edge r_i is *low!*
- time scales $\frac{1}{\Omega} < \frac{1}{\alpha\Omega} << \frac{1}{\alpha\Omega}(\frac{r}{H})^2$

  dynamical   thermal   accretion

typical surface temperatures

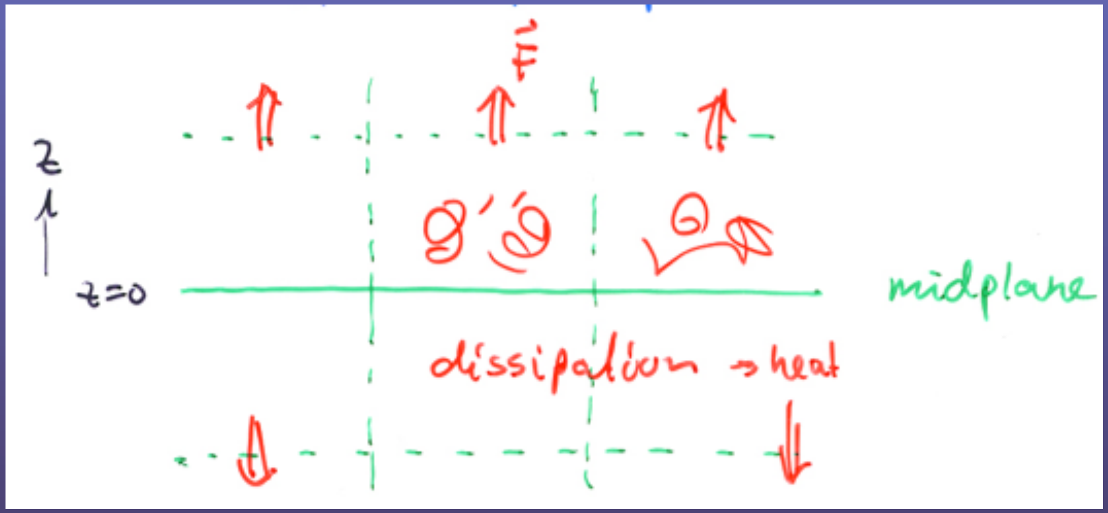
Most energy dissipated in inner regions → dominate overall radiation
(but not at all wavelengths)

	M	$\dot{M}$	$(GM/r)^{1/2}$	$T_{\rm s}$	$T_{\rm vir}$
protostar	$1M_\odot$	$10^{-7} M_\odot/\mathrm{yr}$	$500\ \mathrm{km/s}$	$5000\ \mathrm{K}$	$10^7\ \mathrm{K}$
neutron star	$1M_\odot$	10^{-8}	$c/3$	10^7	$10^{12}\ \mathrm{K}$
AGN	$10^8 - 10^9 M_\odot$	1	$c/3$	$10^4 - 10^5$	$10^{12}\ \mathrm{K}$

Surface temperature is not internal disk temperature.
Need energy transport model for $T(z)$

disk temperature

Internal disk temperature $T(z)$ depends on energy transport process → disk thickness H depends on same



Vertical structure of disk ↔ stellar structure

Simple case: transport by *radiation* only
(inner regions of CV and XRB disks)

optical depth $\tau(z) = \int_z^\infty \kappa\rho \mathrm{d}z$

optical depth to midplane: $\tau_0 = \int_0^\infty \kappa\rho \mathrm{d}z \approx \frac{1}{2}\kappa\Sigma$

disk temperature: the thickness of the disk

$$\tau = \int_z^\infty \kappa\rho \, \mathrm{d}z$$

Assume *radiative* energy transport

LTE, Eddington approximation: $$\frac{\mathrm{d}}{\mathrm{d}\tau}(\sigma T^4) = \frac{3}{4} F_{\rm rad}$$

F constant with z (all energy deposited at midplane): $F = \sigma T_{\rm eff}^4$

effective temperature

boundary condition (radiative atmosphere):

$$\sigma T^4(\tau = 2/3) \approx F \qquad T(2/3) \approx T_{\rm eff}$$

$$\sigma T^4(\tau) = \frac{3}{4}(\tau + \frac{2}{3})F$$ with $$F_{\rm rad} = \sigma T_{\rm s}^4 = \frac{3}{8\pi}\Omega^2 \dot{M}[1 - (\frac{r_{\rm i}}{r})^{1/2}]$$

→ $T = ...$

→ $H = (\mathcal{R}T)^{1/2}/\Omega$

example: $\kappa = \kappa_{\rm es} \approx 0.3,\ r \gg r_{\rm i},\ P_{\rm rad} \ll P_{\rm g}$

→ $$\delta = \frac{H}{r} \approx 5\,10^{-3} \alpha^{\frac{-1}{10}} \dot{M}_{16}^{\frac{1}{5}} (\frac{M}{M_\odot})^{\frac{-7}{20}} r_6^{\frac{1}{20}}$$ (black hole binary)

(see Frank, King & Raine for more)

Excercises:

http://www.mpa-garching.mpg.de/~henk/IMRSexc.pdf *radiation pressure*

- Temperature increases with accretion rate, depth of gravitational potential
- at high T pressure dominated by radiation

$$P = P_{\rm g} + \frac{1}{3} a T^4$$

$c_{\rm s}^2 = P/\rho$ (as before)

$\sigma T_{\rm s}^4 = \ldots$ (as before)

but (in limit $P_{\rm rad} \gg P_{\rm g}$) : $H = \frac{\kappa}{c} \frac{3}{8\pi} \dot{M} f \quad (f = 1 - (\frac{r_{\rm i}}{r})^{1/2})$

Eddington accretion rate on star of radius R_* : $\dot{M}_{\rm E} = 4\pi R_* \frac{c}{\kappa}$

$$\rightarrow \quad \frac{H}{R_*} = \frac{3}{2} \frac{\dot{M}}{\dot{M}_{\rm E}} f$$

for $\dot{M} \rightarrow \dot{M}_{\rm E}$
- disk not thin near R_* ,
- other radiation effects important ("rad. drag")

irradiated disks

- Most radiation from central regions of disk,
- illuminates outer regions

→ 'reprocessing': X ⇒ UV, O

- ... but effect on thickness of the disk small

Lyutyi & Sunyaev 1976, SvA 20, 290

beware: often wrong in litt.

Superoutbursts: The SU UMa Stars 189

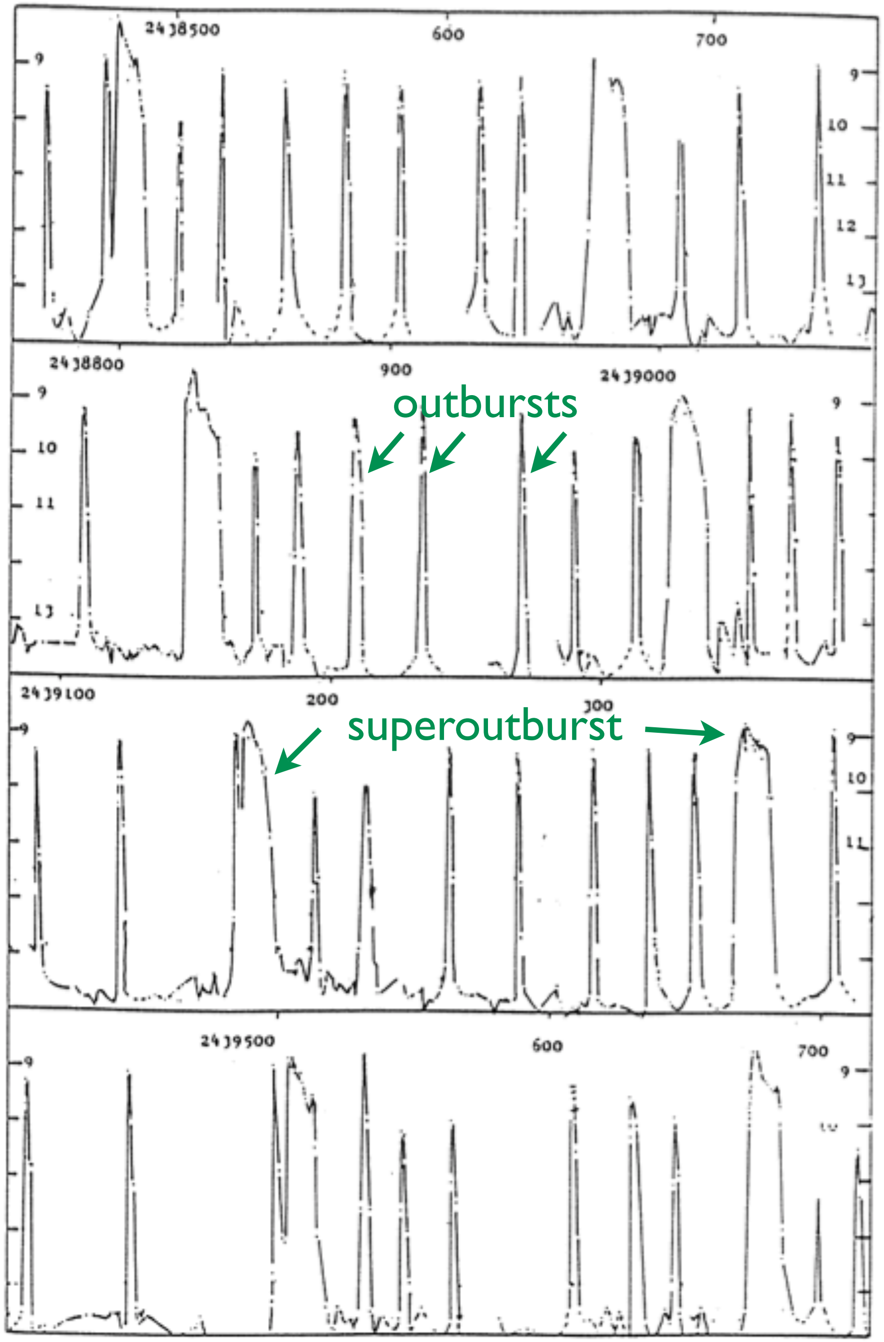


Figure 3.35 Light curve of VW Hyi, from observations made by the Variable Star Section of the Royal Astronomical Society of New Zealand. Adapted from Bateson (1977b).

Dwarf Nova outbursts:

most disks are not steady

Interpretation: disk instability model

Thin disk diffusion equation $\partial_t \Sigma = \frac{3}{r}\partial_r[r^{1/2}\partial_r(r^{1/2}\nu\Sigma)]$ (1)

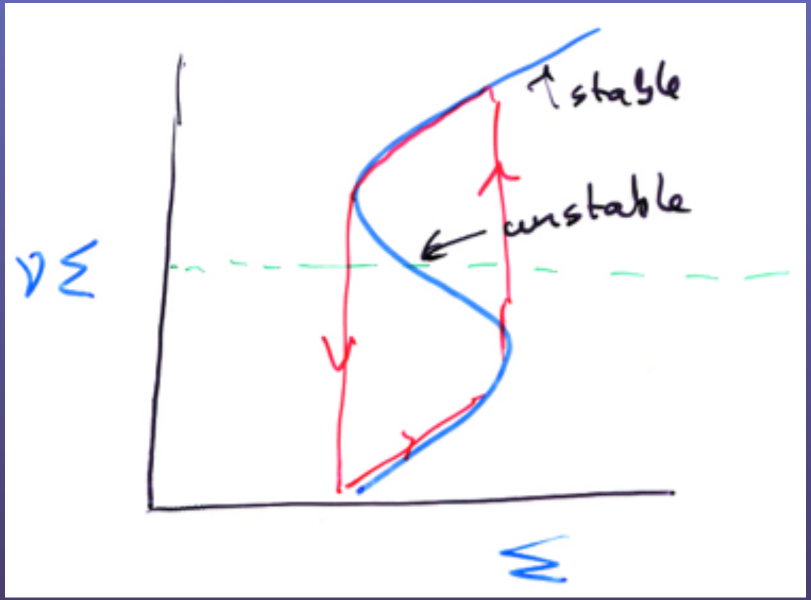
assume $\nu = \alpha c_s^2/\Omega_K$

Behavior of solution depends on $\nu(\Sigma)$

If ν independent of Σ :

all solutions stable.

Need 'interesting' dependence $\nu(\Sigma)$



Physics of energy transport,
equation of state → dependence $c_s(\Sigma)$

Experimental result (1-D simulations of 1):

stable if $\alpha = \mathrm{cst.}$

Excercise: use Sturm-Liouville theory to show that *necessary* for instability is that dependence of $\nu\Sigma$ on Σ is negative somewhere in the disk

'S-curve instability'

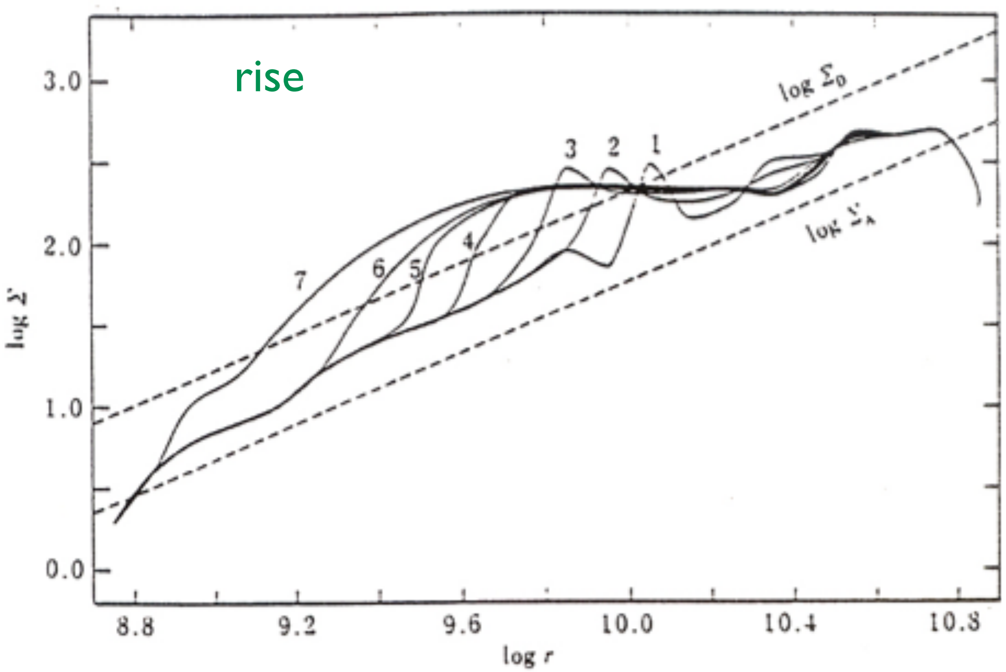
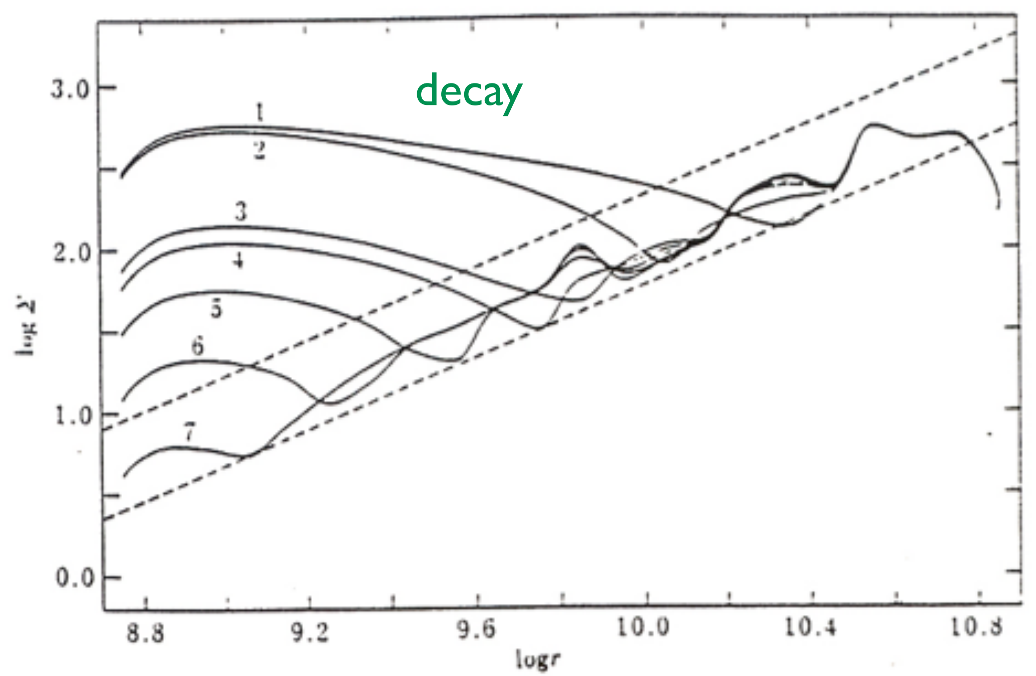
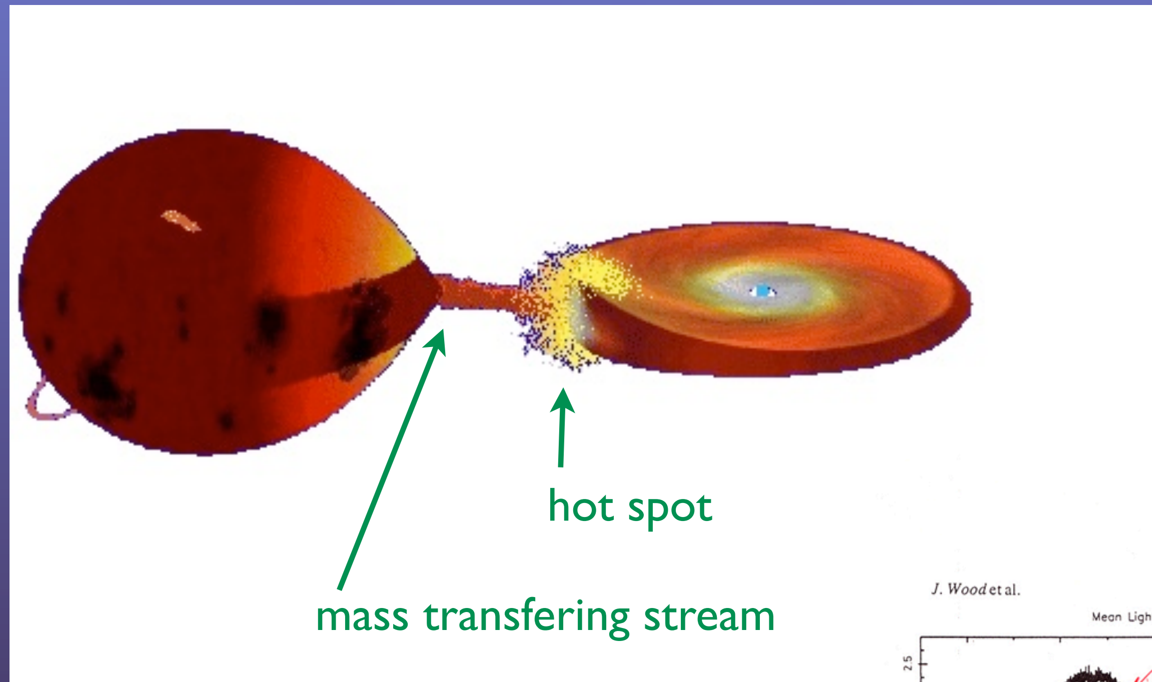


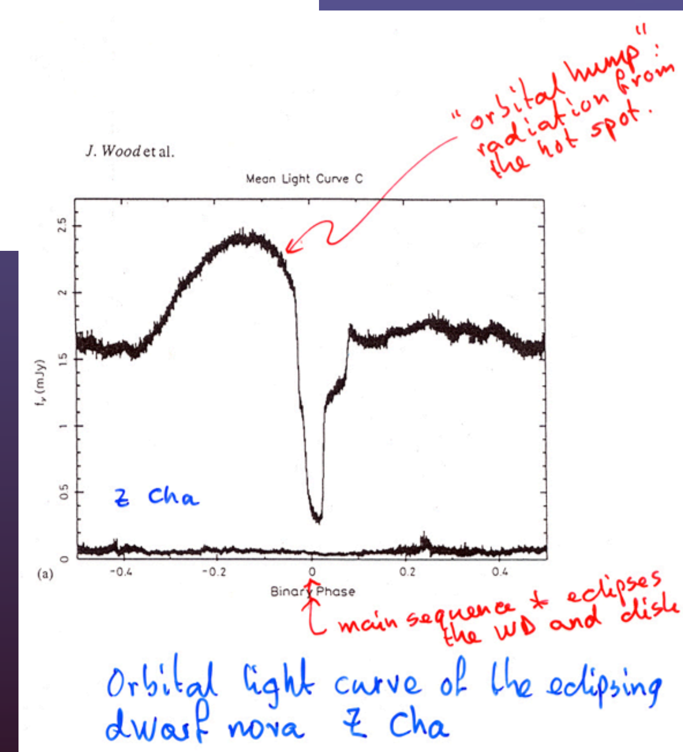
Figure 3.26 Surface density distribution $\Sigma(r)$ at different stages on the rise of a Type A outburst. $\Sigma_D \equiv \Sigma_{max}$ and $\Sigma_A \equiv \Sigma_{min}$. In numerical order, the elapsed times from the initial state are 63.48, 63.82, 64.04, 64.16, 64.25 and 64.41 in units of 10^5 s. From Mineshige & Osaki (1985).



Theoretical ‘outside-in’ dwarf nova outburst

stream impact on binary disk



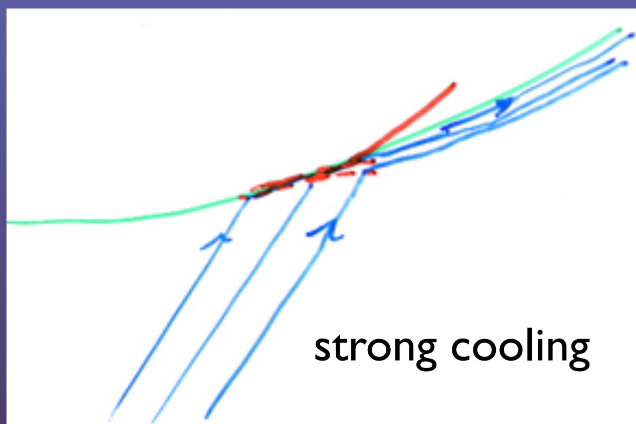


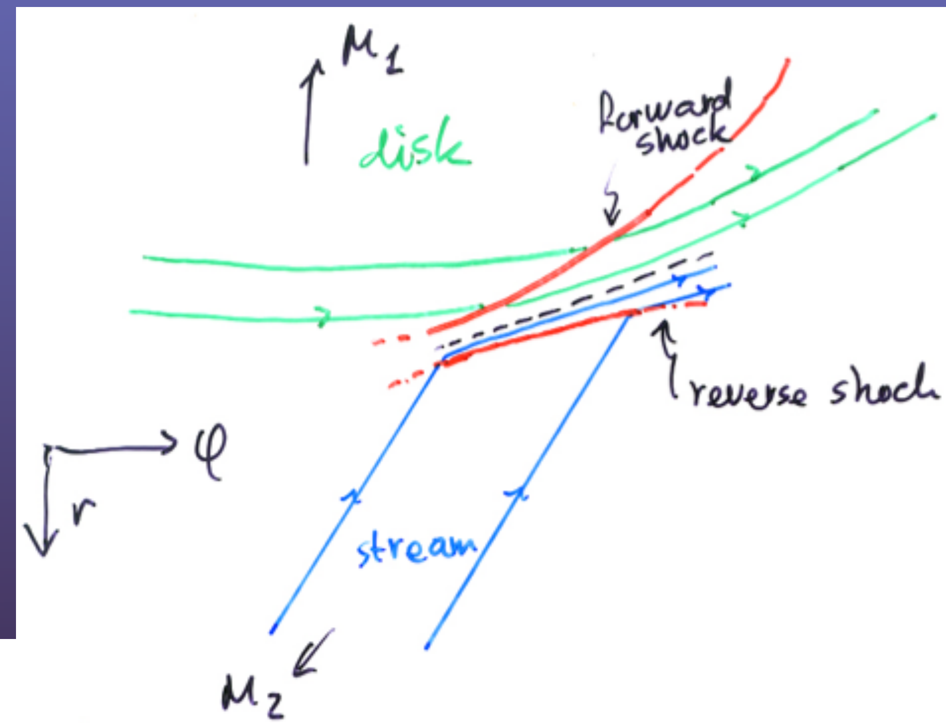
much observational detail in vis, UV, X

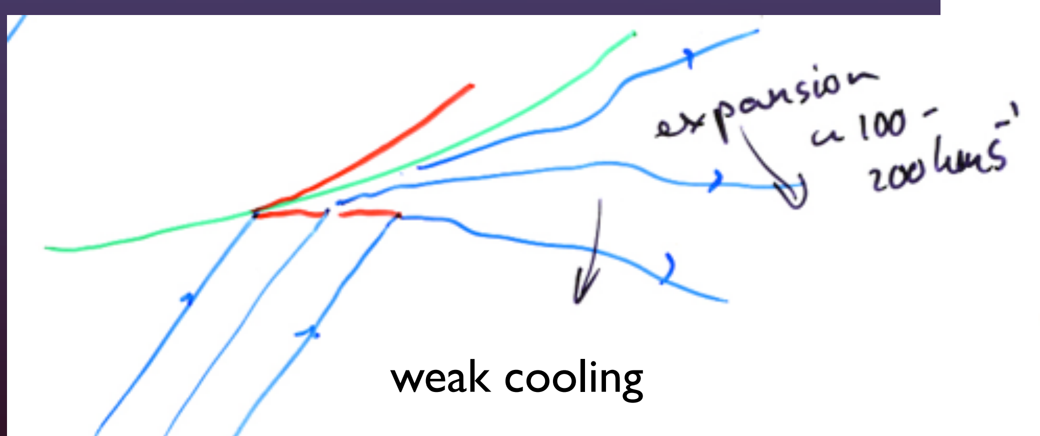
Effect of impact seen in
- X-ray emission
- visible radiation
- X-ray, UV-absorption by orbiting 'bounced' gas

Stream highly supersonic ($v \sim 100 - 1000\ \mathrm{km/s}$, $c_s \sim 10\ \mathrm{km/s}$)

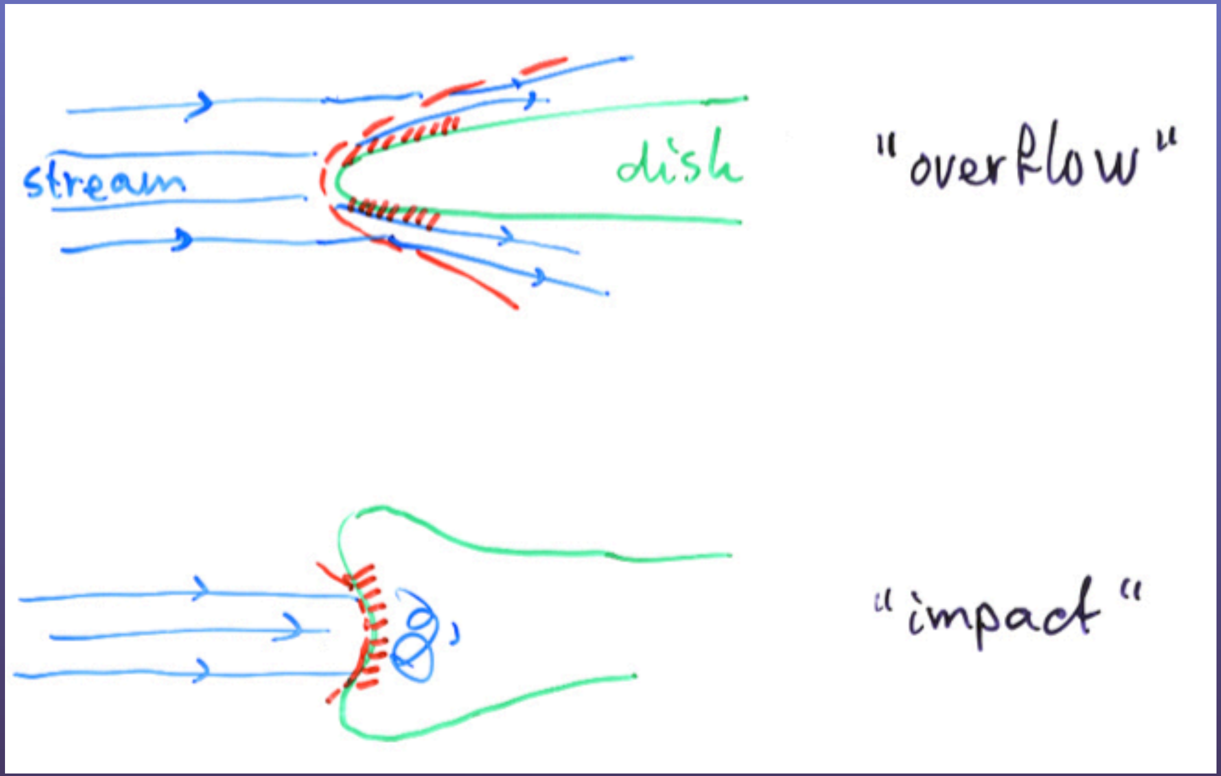
Impact is a double shock structure separated by a contact discontinuity: (Różycka & Czerny 1986)







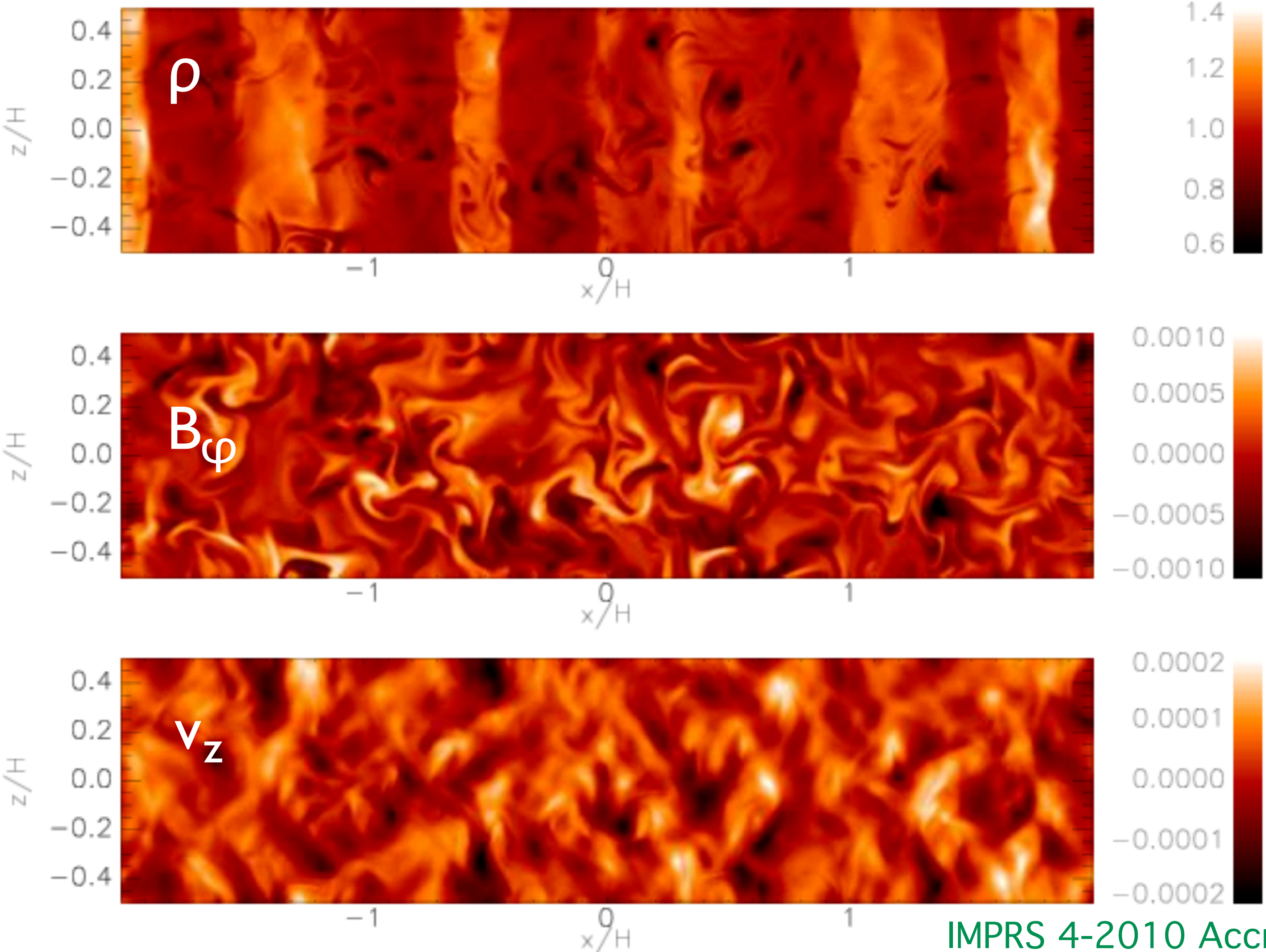
vertical structure of impact region: impact or overflow?



observations: both seem to occur

3-D Numerical simulations: see Armitage & Livio 1998

Disk viscosity: magnetic turbulence



Fromang & Stone 2009 (unstratified)

Disk viscosity: 'MRI turbulence'

Operates on (Keplerian) shear flow only ...
... without the aid of a turbulent velocity field

Hydrodynamic turbulence?
Probably does not happen in Ap disks even at Re → ∞
Experimental result (Princeton Couette expt):
no hydro turbulence seen at Re ~ 10^6.

- Shear flow instability is enabled by a weak B-field
- Produces a small scale magnetic field, Maxwell stress dominates over hydro stress.
- Field strength: B^/8π ~ α P < P

Success of MRI simulations 'bypasses' the hydrodynamic instability question

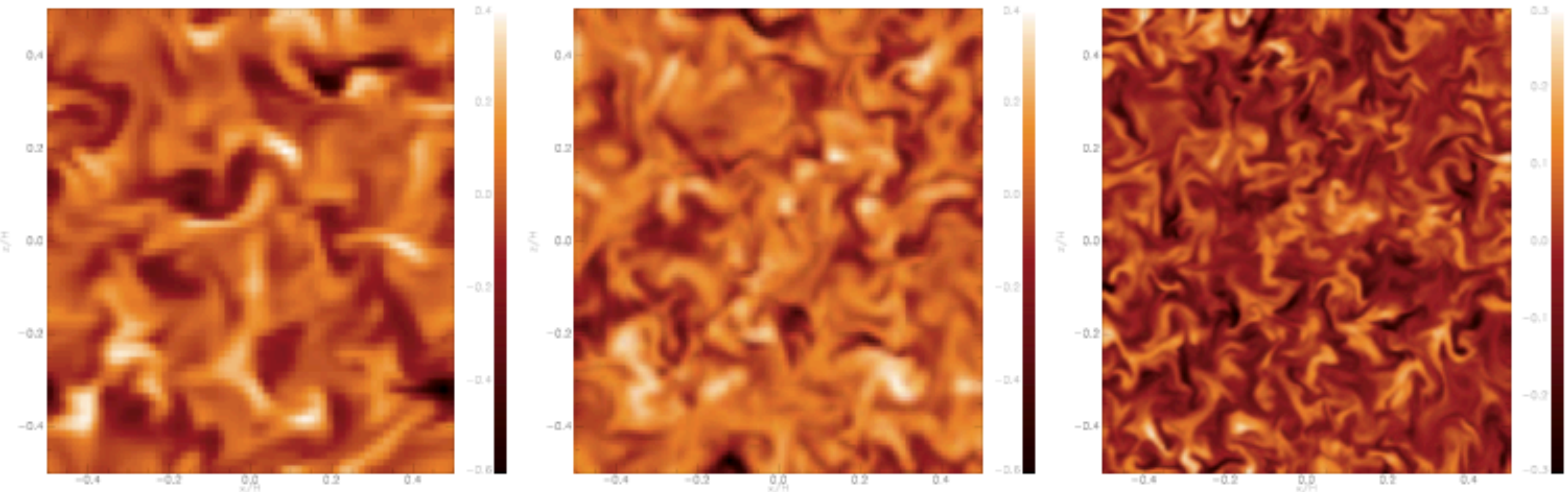
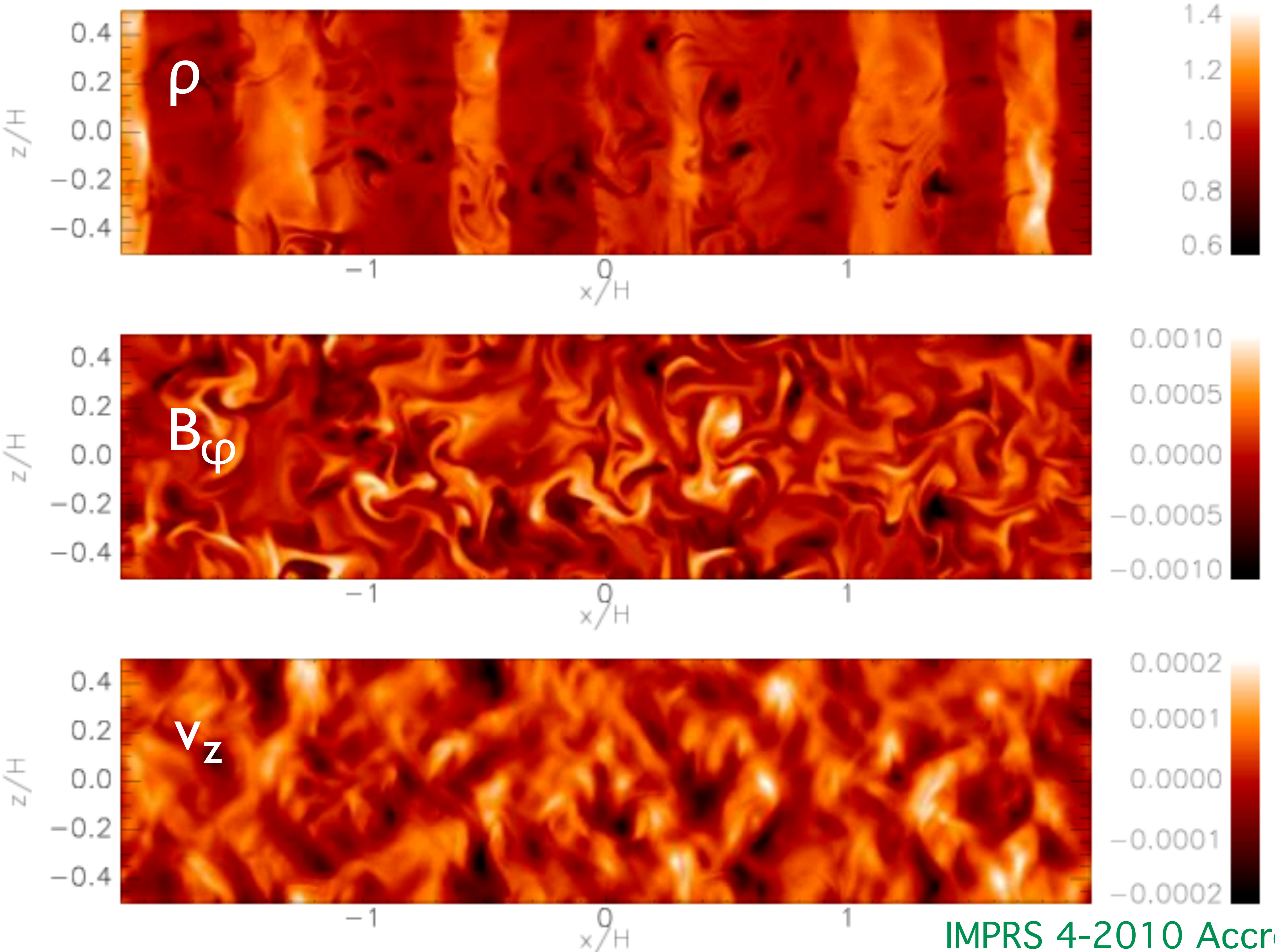


Fig. 4. Snapshots showing contours of the y–component of the magnetic field in the (x, z) plane $(y = 0)$ for the runs STD64 (*left panel*), STD128 (*middle panel*) and STD256 (*right panel*). Smaller and smaller scale features in the magnetic field are seen as the resolution of the simulation increases.

MRI-generated fields *decorrelate on scale L<H*



Fromang & Stone 2009 (unstratified)

Modifications: vertical stratification

- Include a low density atmosphere
- problem: $V_A \to \infty$: much more expensive

Expected difference:
Buoyant escape of magnetic field loops creates a field amplification cycle that is absent in unstratified simulations.

1. Stretching in Kepler-shear → B_φ dominates
2. Buoyant instability → 'Ω-loops'
3. Coriolis force during loop rise turns the loop
4. → produces a B_r component (→ back to 1.)

This cycle might not be dependent on diffusivities, and α might converge with resolution.
Claims to this effect: recent (2009) papers, e.g. Guan & Gammie, ApJ 697, 1901

Radiatively inefficient accretion flows

(+ *transition* from cool disk to radiatively ineff. flow) *radiatively inefficient flow*

Accretion time scale: $t_{\rm acc} = t_{\rm visc} = \frac{r^2}{\nu} = \frac{1}{\alpha\Omega}(\frac{r}{H})^2$

If $H/r \ll 1$, *then* $t_{\rm acc}$ long,
then cooling efficient, *then* $H/r \ll 1$: but this is a *circular argument*

What if $H/r \sim \mathcal{O}(1)$ and accretion time short?
⇒ *radiatively inefficient accretion flows* $\int Q_{\rm visc} {\rm d}z \neq 2\sigma T^4$

→ *disk is not thin*
advection of internal energy with the flow important

2 types: (Begelman, Blandford, Phinney & Rees 1982)
- optically thick: *radiation supported* accretion flow ('radiation torus')
- optically thin: *ion supported* accretion flow ('ion torus')

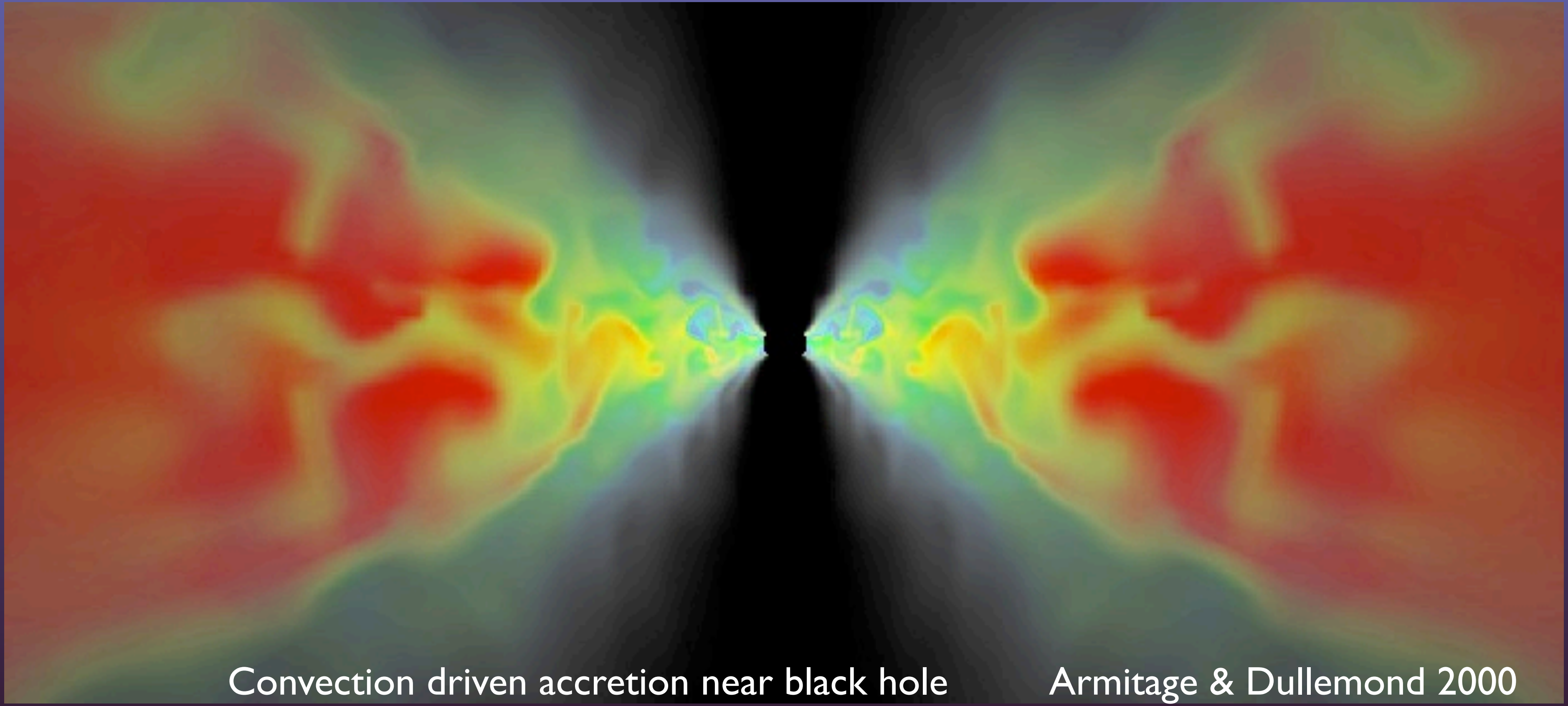
} 'ADAF'

- hydrodynamics the same

$$t_{acc} = t_{visc} = \frac{r^2}{\nu} = \frac{1}{\alpha\Omega}(\frac{r}{H})^2$$

Angular momentum transport easier in thick accretion

(also easier to simulate numerically)



Thin disks are more difficult numerically than thick ones

time scale problem:

$$\frac{1}{\Omega} \ll \frac{r}{c_{\rm s}}(= \frac{1}{\Omega}\frac{r}{H}) \ll \frac{1}{\alpha\Omega}(\frac{r}{H})^2$$

orbital — sound crossing — accretion time scale

time step problem: $\Delta t < \Delta x/v$ (Courant)

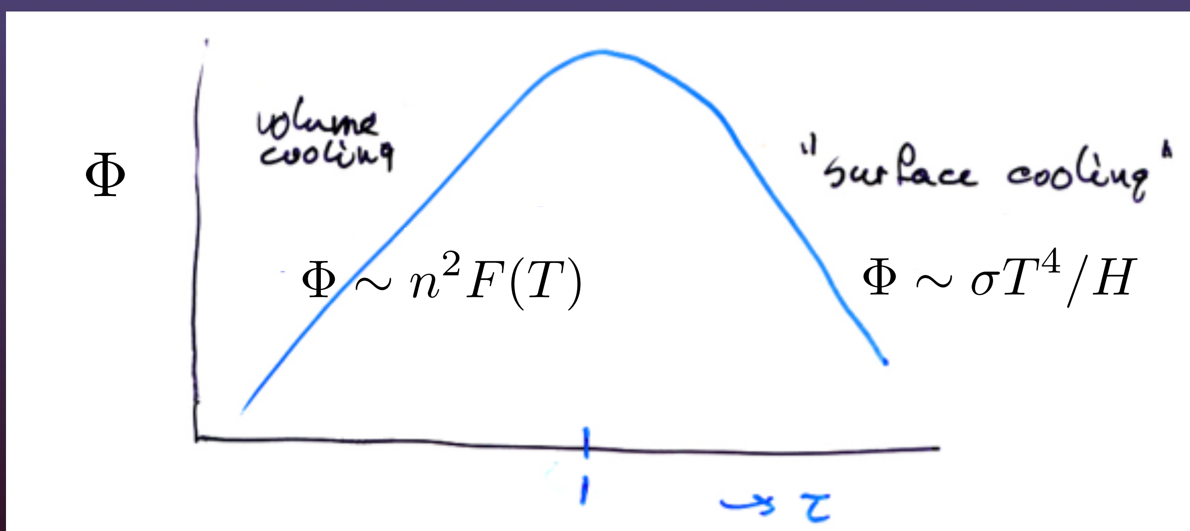
can be circumvented (FARGO: F. Masset 2000A&AS..141..165M)

(by separating purely orbital motion from the flow) *radiatively inefficient flow*

Cooling is ineffective:
- at very high optical depth: escape of heat too slow
- at low density: radiation processes inefficient

Cooling at low optical depth: $\Phi_c \sim n^2 F(T)$ erg cm^{-3}s^{-1}
Thermal energy density: $u \sim nkT$ erg cm^{-3}s

$$\rightarrow\ t_{\rm cool} \sim \frac{u}{\Phi_c} \sim \frac{1}{n} \qquad (\tau \ll 1)$$



$F(T)$: (optically thin) *cooling function* (atomic physics)

holds for ordinary thermal equilibrium plasma ($\sim 10^4 - 10^8$ K)

Thermal instability

Hot flows accrete faster:

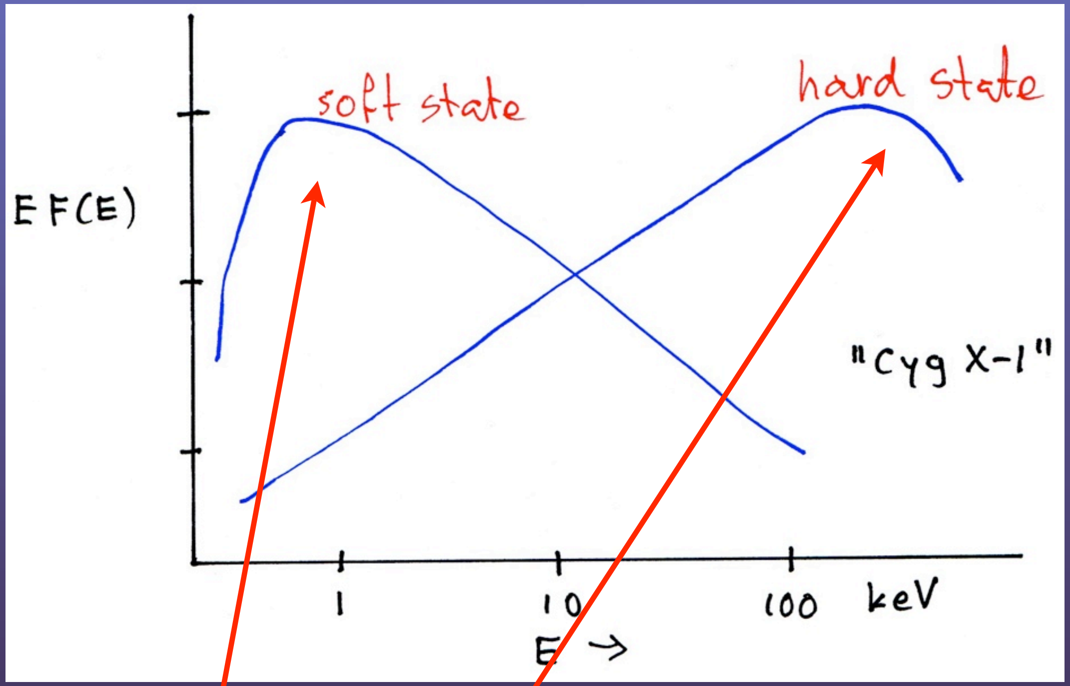
$$t_{\rm accr} = \frac{1}{\alpha\Omega}(\frac{r}{H})^2 = \frac{1}{\alpha\Omega}\frac{T_{\rm vir}}{T}$$

→ density lower (for given mass accretion rate)
→ *if cooling rate decreases* with decreasing density:
(thermal-viscous) *instability* (Lightman & Eardley)

→ if unstable, disk will cool down or heat up to a stable state.

Typical stable states:
- cool (Shakura-Sunyaev) disks
- radiatively inefficient flow: adiabatic ($\neq$ *isentropic*)

Observations indicate existence of separate ‘accretion states’:



surprising: (optically thin) 100keV plasma
should be *highly thermally unstable*

unsurprising: could be a thin (Shakura-Sunyaev disk)

Optically thin, radiatively ineffective flow near N-star or BH

Physics:
- Coulomb interaction in ionized plasma
- interaction of electrons with radiation

if loss inefficient: $T \sim T_{\rm vir}$
if energy shared between electrons and ions:

$$T_{\rm e} = T_{\rm ion} \sim T_{\rm vir} \sim 150\ MeV\ \frac{r}{3r_{\rm S}}$$

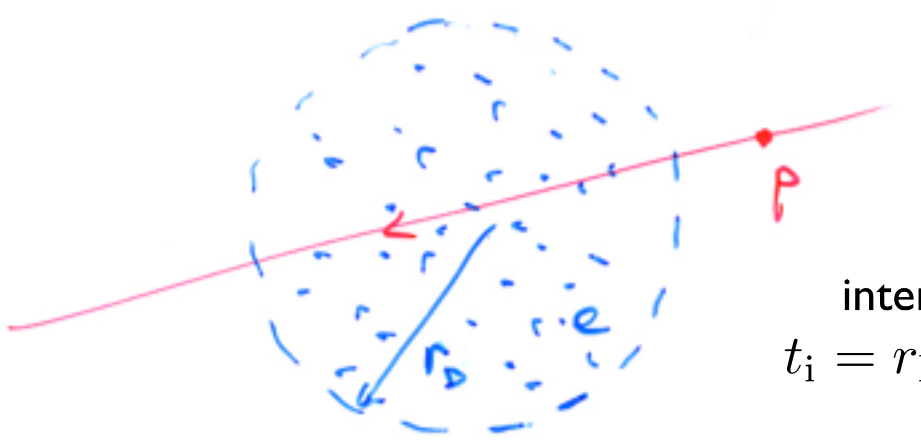
ions: subrelativistic
electrons: $\gamma \sim 500$

energy loss of electrons (inverse Compton) $\sim \gamma^2$
→ optically thin plasma cannot be in thermal equilibrium at this temperature

electrons radiate, → $T_{\rm e} \ll T_{\rm ion}$ 'two-temperature flow'

optically thin radiatively inefficient flow

Coulomb interaction in two-temperature plasma $T_{\rm ion} \gg T_{\rm e}$
interaction *decreases* with ion temperature:



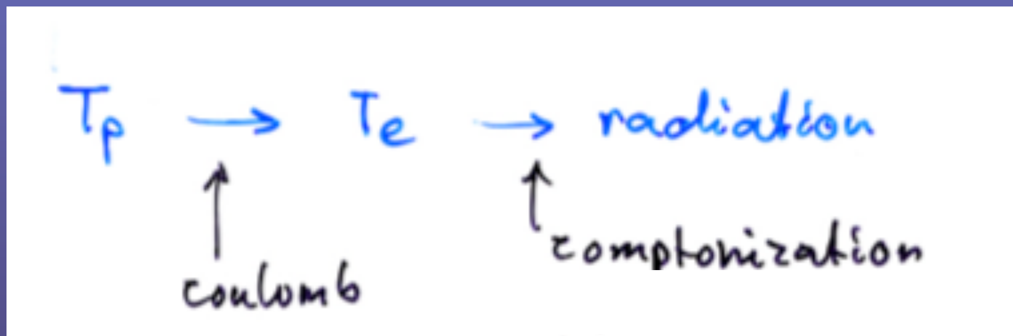
interaction time in $r_{\rm D}$:

$$t_{\rm i} = r_{\rm D}/v_{\rm ion} \sim (T_{\rm ion})^{-1/2}$$

$r_{\rm D}$ Debye sphere: "effective range" of electrostatic interaction in an ionized plasma

optically thin radiatively inefficient flow

Electrons loose energy by inverse Compton on radiation field of accretion flow



Loss of equilibrium p-e when radiation loss high
Ions heat up → transfer to electrons reduced → runaway disequilibrium

Shapiro, Lightman & Eardley 1976

Two-temperature accretion flow ('ion supported torus')
Ions loose little energy: $T_{\rm ion} \approx T_{\rm vir}$ (MeV's)
Electrons cool by inverse Compton. How cool?
Theory: depends on geometry of radiation environment.
Observed: $T_{\rm e} \sim 50 - 200\ {\rm keV}$

Energy is carried by ions. 'Thick' flow
- Thin disk approximation does not apply
- Internal energy advected with flow to be accounted for: 'ADAF'
- Inefficient radiator (bottleneck: Coulomb energy transfer ions → electrons

optically thin radiatively inefficient flow

near a BH or n-star 2 accretion states:
- optically thick cool disk $T_{\rm s} \sim 0.1 - 1$ keV $(10^6 - 10^7$ K$)$ $(\ll T_{\rm vir})$
- optically thin ion supported flow $T_{\rm ion} \approx T_{\rm vir}, T_{\rm e} \sim 30 - 100$ keV

X-ray observations
- 'soft state' spectrum $\sim 0.1 - 1$ keV
- 'hard state' spectrum ~ 100 keV

Problems:
- what determines which state chosen & when.
- geometry of the ion supported flow, source of the soft radiation comptonized by the electrons.

hydrodynamics of radiatively inefficient flow

Thin disk approximation not valid: $H/r \approx 1$ because
2-temperature flow: $T_{\rm ion} \approx T_{\rm vir}$
radiation supported flow: disk inflated by radiation pressure.

Problem 2-D, 3-D
Simplifications:
Ignore vertical dimension anyway, but keep pressure terms (deviations from Kepler rotation, advection of thermal energy with flow). Steady flow:

$$\Sigma 2\pi r v_r = \dot{M} = \text{ cst.}$$ mass

$$r\Sigma v_r \partial_r(\Omega r^2) = \partial_r(\nu \Sigma r^3 \partial_r \Omega)$$ angular momentum

$$v_r \partial_r v_r - (\Omega^2 - \Omega_{\rm K}^2) r = -\frac{1}{\rho}\partial_r P$$ radial

$$\Sigma v_r T \partial_r S = q^+ - q^-$$ energy

Entropy: $S = c_{\rm v} \ln(\frac{P}{\rho^\gamma})$ (ideal gas with constant ratio of specific heats γ)

$$q^+ = \int Q_{\rm visc} {\rm d}z; \quad q^- = \int {\rm div}\, F_{\rm rad}\, {\rm d}z$$

viscosity: keep $\nu = \alpha c_{\rm s}^2/\Omega$

hydrodynamics of radiatively inefficient flow

Self-similar solutions (Gilham 1981)

$$\Omega \sim r^{-3/2} \qquad (\Omega/\Omega_{\rm K} = {\rm cst.})$$

$$\rho \sim r^{-3/2}$$

$$H \sim r$$

$$T \sim 1/r \qquad (T/T_{\rm vir} = {\rm cst.})$$

solution for $\alpha \ll 1$:

$$v_r = -\alpha \Omega_{\rm K} r (9 \frac{\gamma - 1}{5 - \gamma})$$

$$\Omega = \Omega_{\rm K} (2 \frac{5 - 3\gamma}{5 - \gamma})^{1/2}$$

$$c_{\rm s}^2 = \Omega_{\rm K}^2 r^2 \frac{\gamma - 1}{5 - \gamma}$$

$$\frac{H}{r} = (\frac{\gamma - 1}{5 - \gamma})^{1/2}$$

Properties:

- $\Omega < \Omega_{\rm K}$ for $\gamma > 1$
- *no rotating solution for* $\gamma \geq 5/3$
- $H/r \rightarrow 0$ for $\gamma \downarrow 1$

self-similar radiatively inefficient flows

Special cases: $\gamma \downarrow 1, \ \gamma = 5/3$

$\gamma \downarrow 1$? $\quad \gamma = c_{\rm p}/c_{\rm v} = (\mathcal{R} + c_{\rm v})/c_{\rm v}$

large $c_{\rm v}$: energy goes into internal degrees of freedom, → temperature low

fully ionized gas (nonrelativistic) has $\gamma = 5/3$
→ (steady, self-similar...) ion-supported flow should not rotate ?!
Why is this case special? look at entropy.
Self-similar scaling:

$$\begin{matrix} \rho \sim r^{3/2} \\ T \sim r^{-1} \end{matrix} \rightarrow P \sim r^{-5/2} \quad \rightarrow S \sim \ln P/\rho^\gamma \sim (-\frac{5}{2} + \frac{3}{2}\gamma) \ln r$$

If $\gamma < 5/3$: Entropy increases inward. Consistent with dissipation taking place. $\gamma = 5/3$: entropy constant. Possible only in absence of viscous dissipation → *rotation must vanish.*

self-similar radiatively inefficient flows: the case $\gamma = 5/3$

Solution: (G. Ogilvie MNRAS 1999)
the flow is *time dependent*

Rotation slows down with time
(describes viscous spreading for thick accretion flows)

Time-dependent self-similar solution $f(\xi), \quad \xi = rt^{-2/3}$
$\Omega \downarrow 0 \quad (\xi \downarrow 0)$ boundary of slowly rotating inner region increases with time as $r \sim t^{2/3}$

→ *adiabatic accretion flow with* $\gamma = 5/3$ *does not rotate, or is time-dependent* *radiation supported radiatively inefficient flows (RSAF), 'slim disks'*

Inefficient cooling at high optical depth → high T

$u_{\rm rad} = aT^4$ high: radiation pressure dominates

Adiabatic approximation (photons trapped in flow:)

$$\rho\frac{1}{2}\frac{GM}{r} \approx u_{\rm gas} + u_{\rm rad} \qquad P_{\rm rad} = \frac{1}{3}u_{\rm rad} = \frac{1}{3}aT^4$$

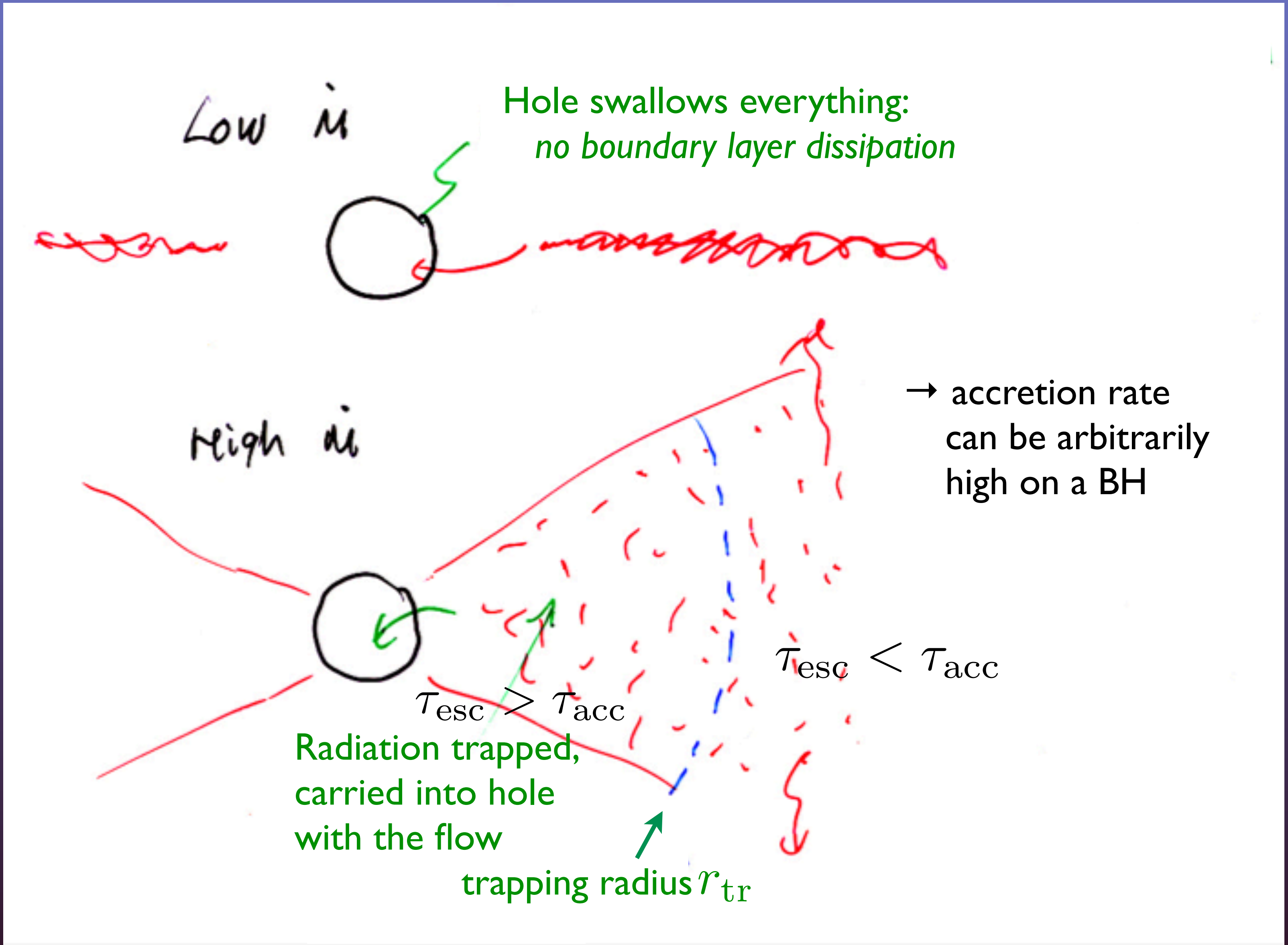
$$\rightarrow \quad \frac{1}{2}\frac{GM}{r} \approx \frac{P}{(\gamma-1)\rho} + \frac{1}{\rho}3P_{\rm rad}$$

$$T_{\rm vir} = \frac{GM}{r\mathcal{R}} \,,\quad P = \frac{\rho\mathcal{R}T}{\mu} : \rightarrow \quad \frac{T}{T_{\rm vir}} = \left(\frac{2}{\gamma-1} + 6\frac{P_{\rm r}}{P}\right)^{-1}$$

$P_{\rm r} \gg P \;\rightarrow\; T \ll T_{\rm vir}$ (thermal energy is in radiation, not particles)

$$H = c_{\rm si}/\Omega = \left(\frac{P_{\rm r}+P}{\rho}\right)^{1/2}\frac{1}{\Omega} \quad\rightarrow\quad H/r \sim \mathcal{O}(1)$$

radiation supported radiatively inefficient flows (RSAF)



radiation supported radiatively inefficient flows (RSAF)

RSAF has a luminosity of order Eddington

$$F_{\rm rad} = \frac{c}{3}\frac{\rm d}{{\rm d}\tau}u_{\rm rad} \qquad \approx \frac{c}{3}\frac{u_{\rm rad}}{\tau}$$

radiation energy density due to gravitational energy release:

$u_{\rm rad} \approx \rho\frac{GM}{2r}$ and vertical optical depth $\tau = \kappa\rho H$:

$$F_{\rm rad} = \frac{1}{3}\frac{GM}{2r}\frac{c}{\kappa H} = \frac{1}{6}F_{\rm E}\frac{r}{H}$$

($F_{\rm E} = \frac{GM}{r^2}\frac{c}{\kappa}$: Eddington flux)

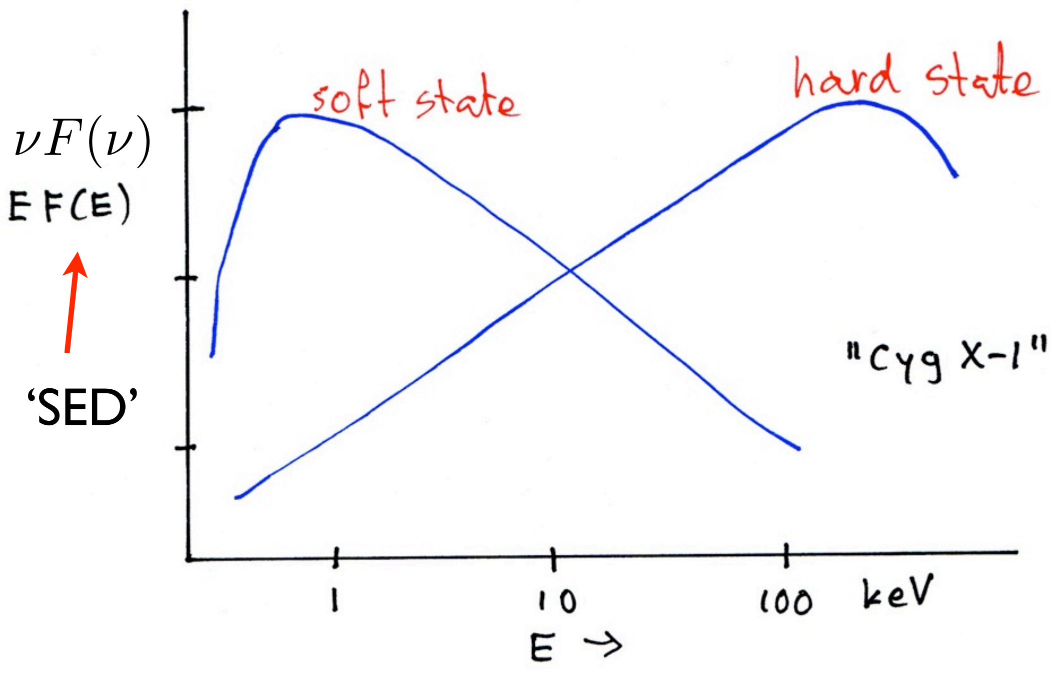
→ radiatively supported accretion flow has $L/L_{\rm E} \sim \mathcal{O}(1)$

→ luminosity of accreting black hole approaches $nL_{\rm E}$
(n:'a few')
independent of accretion rate (if above Eddington rate)

formation of ion supported accretion flow

At low accretion rate 2 states
- optically thick, cool disk (Shakura-Sunyaev), soft X-ray spectrum
- optically thin, ion supported (2-temperature) flow, hard X-ray spectrum

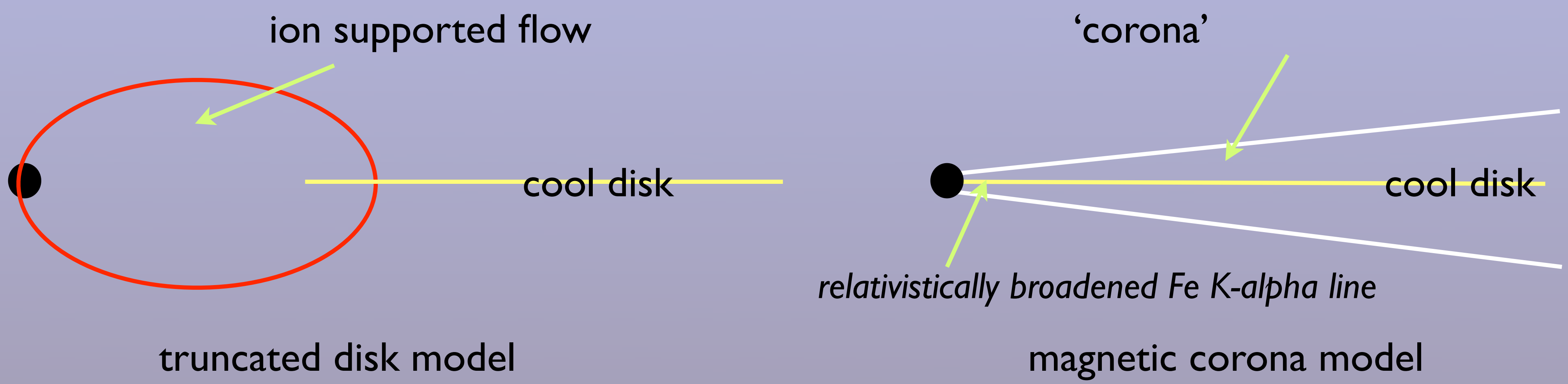
Observations: Both happen, transitions gradual



Both components sometimes observed to be present at same time:
cool disk and hard X-ray producing regions can coexist

How?

Geometry of the hard X-ray producing region



Variability of Cyg X-1

Gilfanov, Churazov, Revnivtsev, 1999 A&A 352, 182

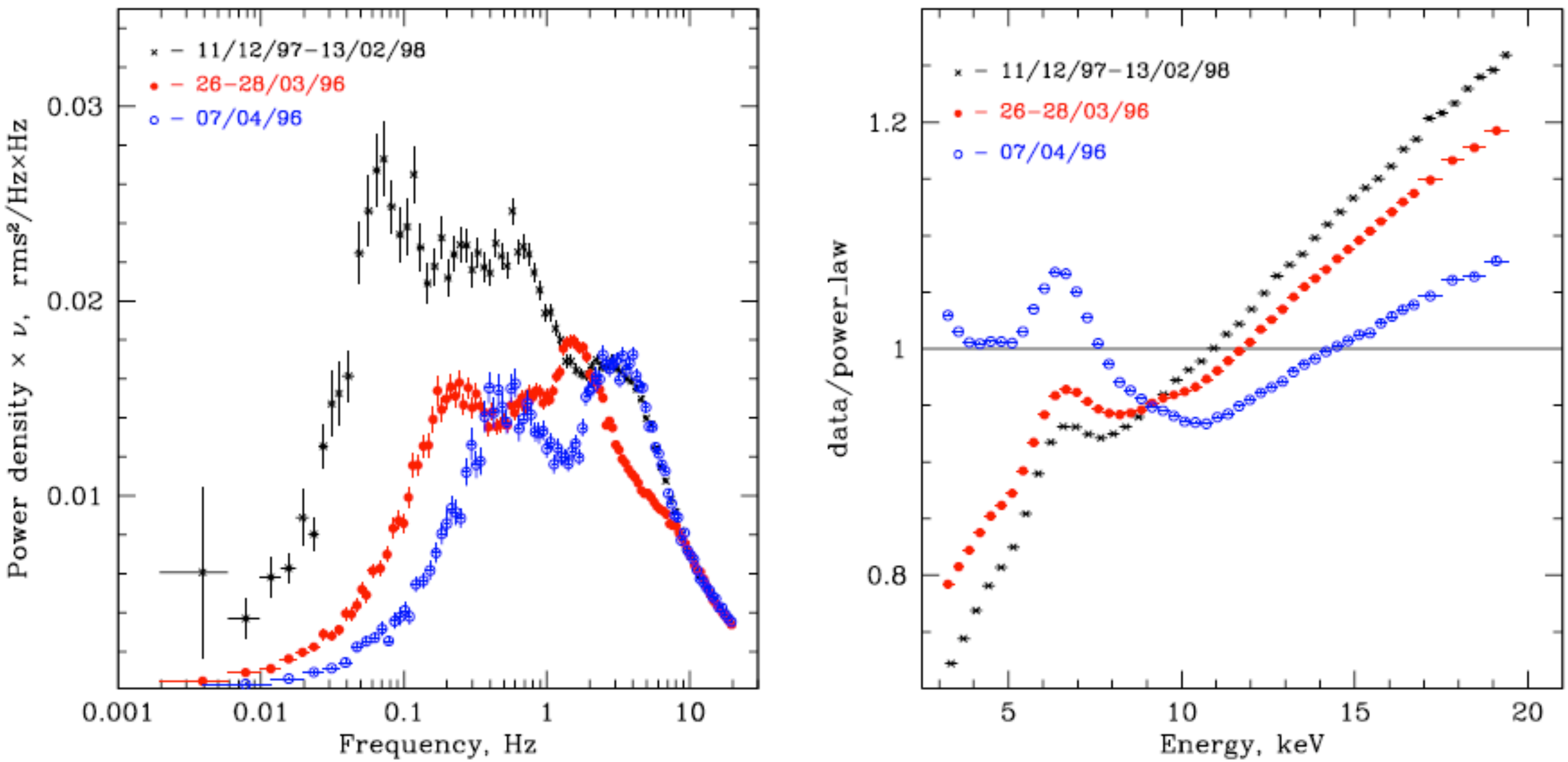


Fig. 1. The power density (*left*) and counts (*right*) spectra of Cyg X–1 at different epoch. The power density spectra are plotted as frequency × (power density), i.e. in units of Hz × rms^2/Hz. The counts spectra are shown as a ratio to a single slope power law model. The symbols are the same in the left and right panels. The higher characteristic noise frequencies correspond to steeper energy spectra with stronger reflection features.

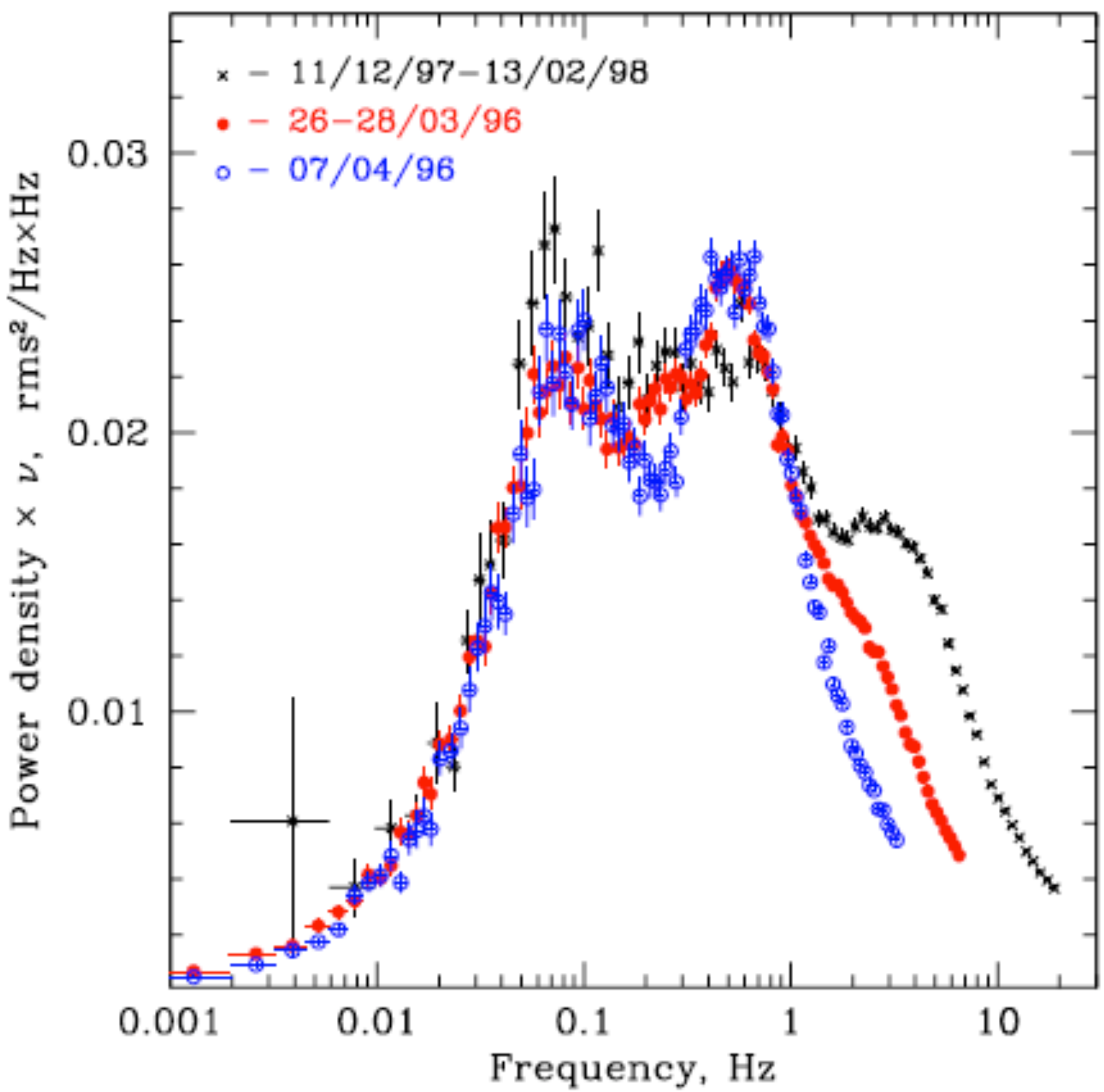


Fig. 2. The power density spectra of Cyg X–1 for the same datasets as in Fig. 1 but logarithmically shifted along the frequency axis and renormalized to match the spectrum averaged over 11/12/97–13/02/98 (thin black crosses) at low frequencies. The power density spectra are plotted as frequency × (power density), i.e. in units of Hz × rms^2/Hz. The symbols are the same as in Fig. 1.

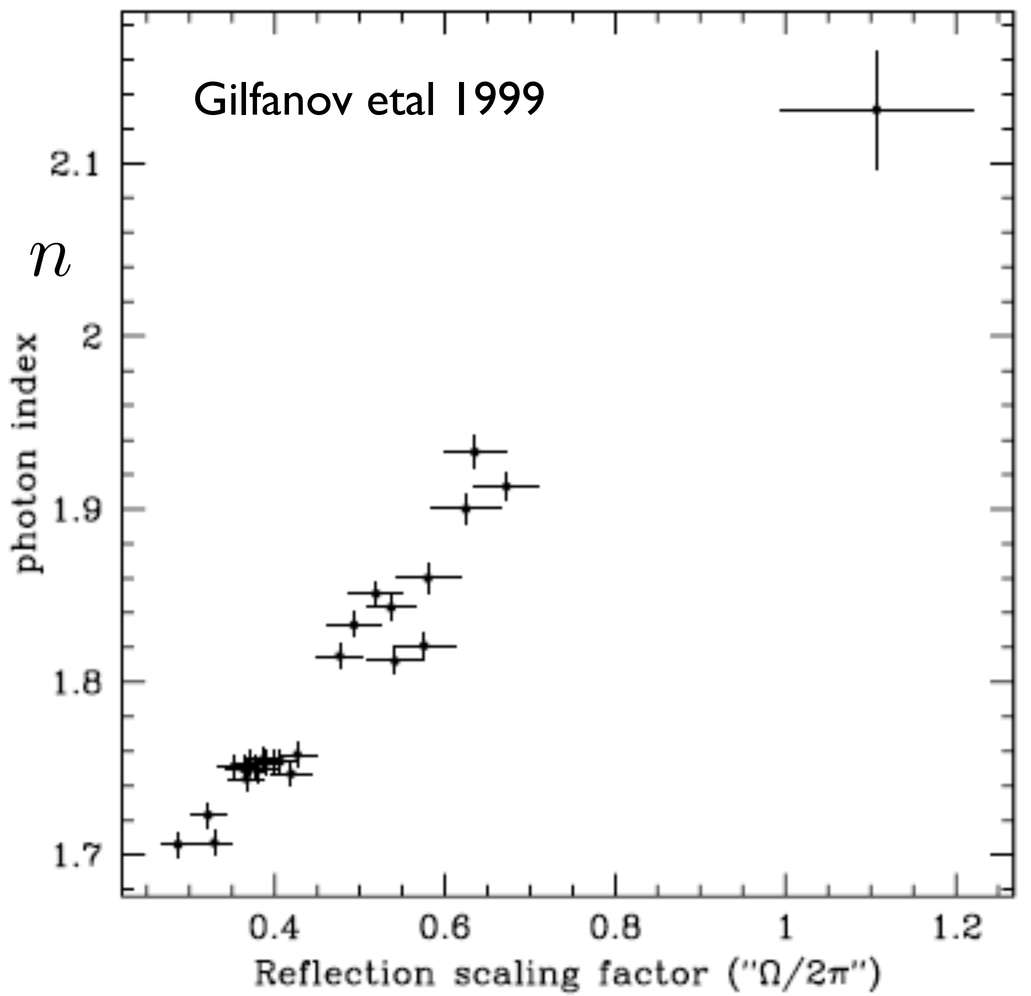


Fig. 3. The photon index of the underlying power law plotted vs. reflection scaling factor. See text for discussion of the spectral model.

Photon rate

$N(E) \sim E^{-n}$

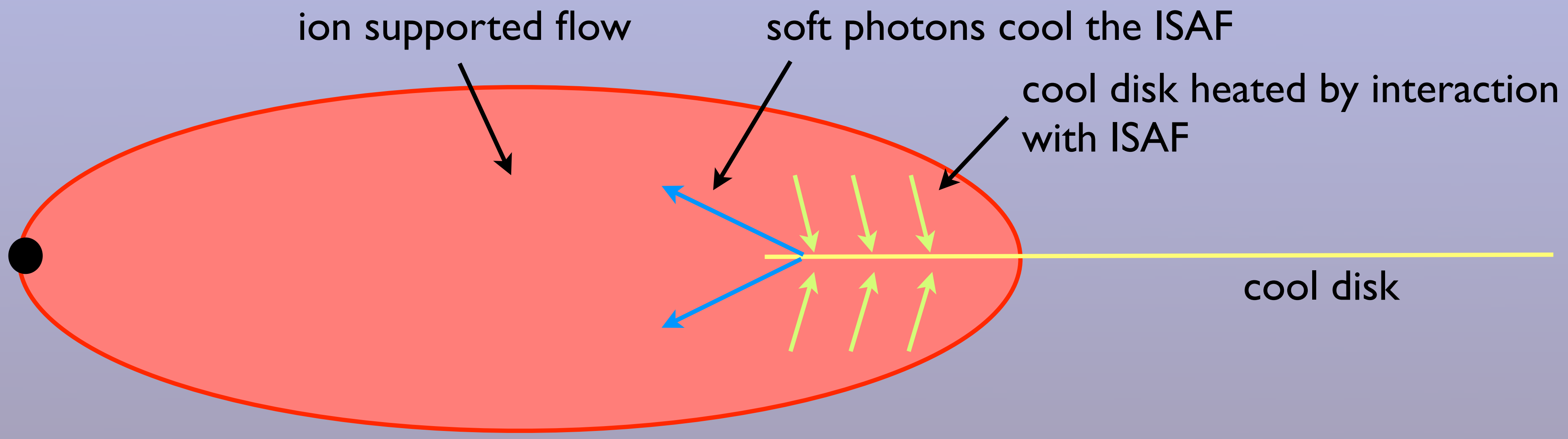
energy flux per keV

$F(E) = EN(E) \sim E^{-s} = E^{-(n-1)}$

'SED'

$EF(E) = E^2N(E) = E^{-(n-2)}$

X-ray sources in the truncated disk model



Transition from cool disk to ion-supported accretion flow

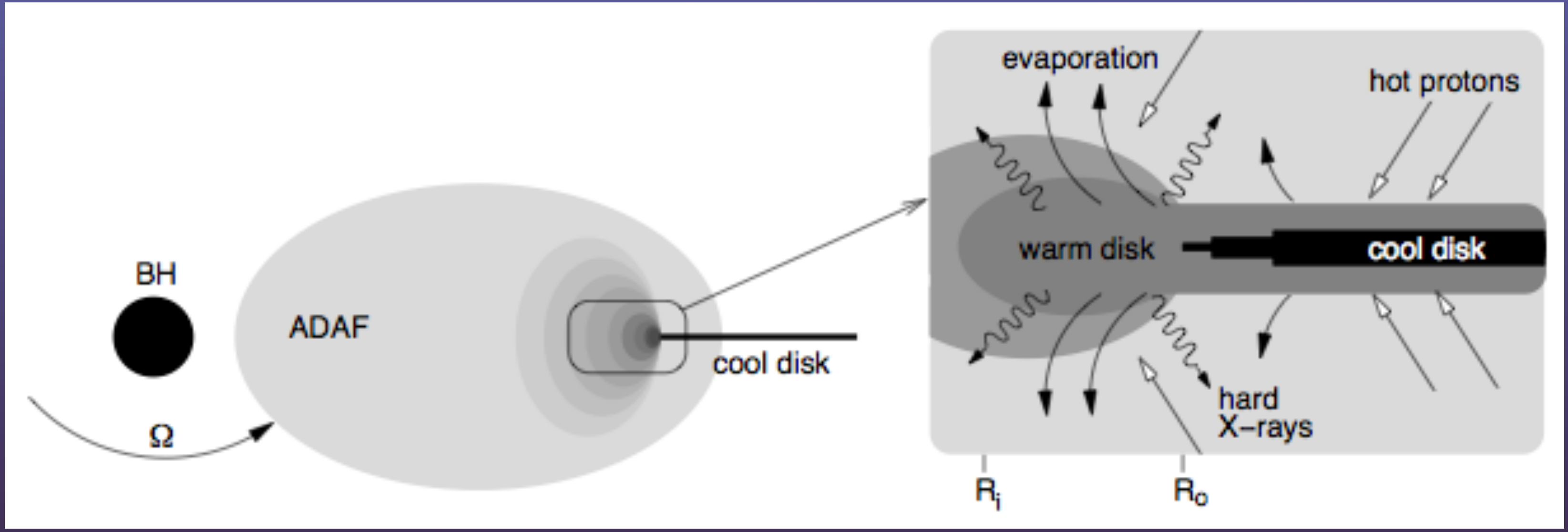
Bernhard Deufel
Kees Dullemond
Dimitris Giannios
Caroline D'Angelo
HS

Physics:
- alpha viscosity
- Coulomb interactions
- no magnetic fields!

two-step process:
1 'Ion illumination'
2 thermal instability

formation of ion supported accretion flow

Truncated disk: cool (SS) disk outside, hot ion supported flow inside
Interaction between the two:
- hot flow spreads outward over cool disk
- hot ions ($10\text{-}100\ \mathrm{MeV}$) hit cool disk



Ions heat surface of cool disk to $\sim 100\ \mathrm{keV}$
thermal instability at inner edge → transition to ion supported flow

Latest: see D'Angelo, Giannios, Dullemond & Spruit 2008

Ion illumination of cool disk

suppose there is a cool disk (~0.1 keV) extending into an ion supported flow (ISAF)
Q: can this be a self-sustained accretion state?

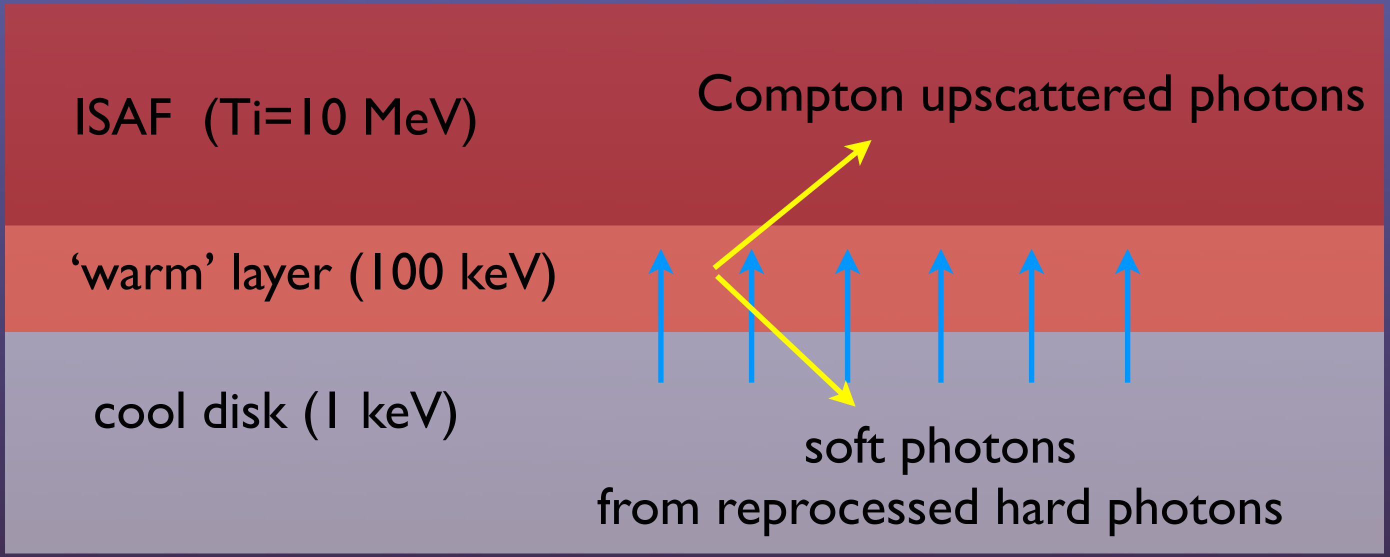
- the ISAF spreads viscously over the cool disk
- its ions have energies 10-100 MeV
- cool disk bombarded by energetic ions

Q: how does it respond?

A: same problem as in a cloud chamber particle detector:
ion penetrates into the disk and gradually gives up its energy to the disk electrons

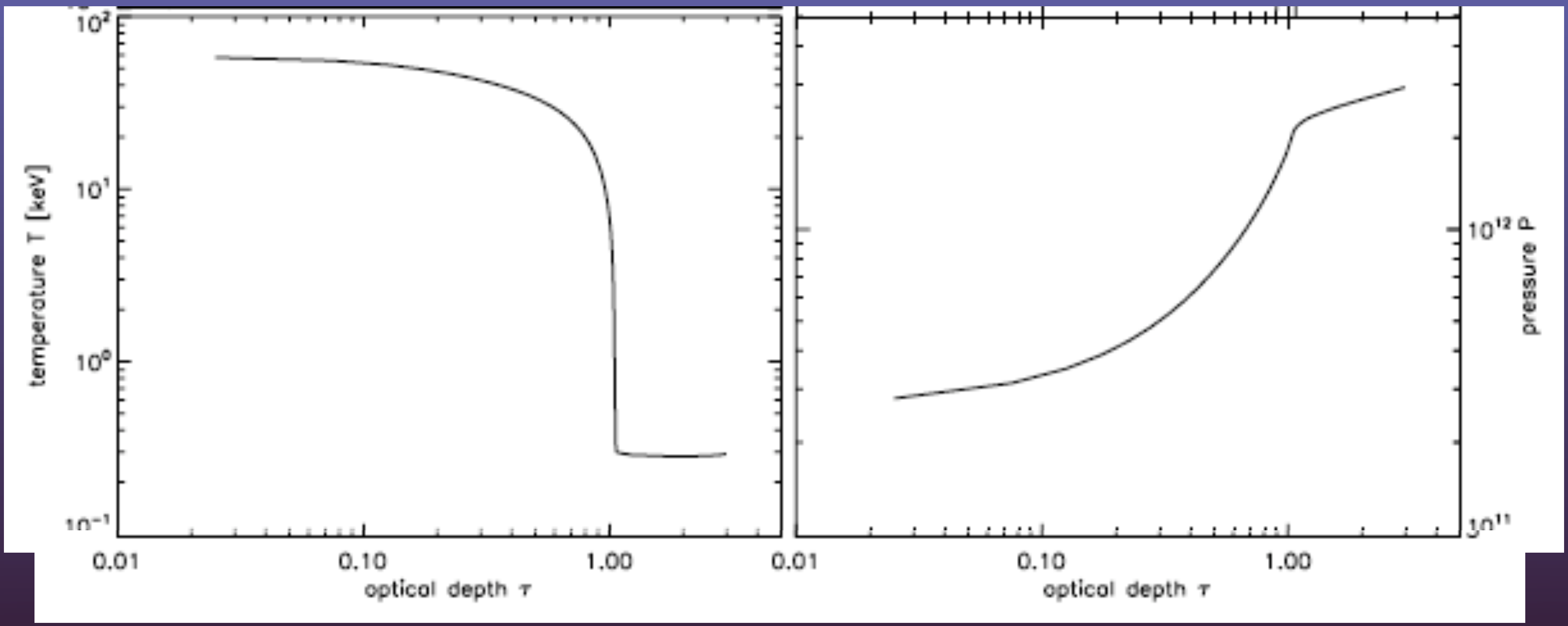
Model content:

- Coulomb interaction ('stopping')
- Compton scattering (Monte Carlo)

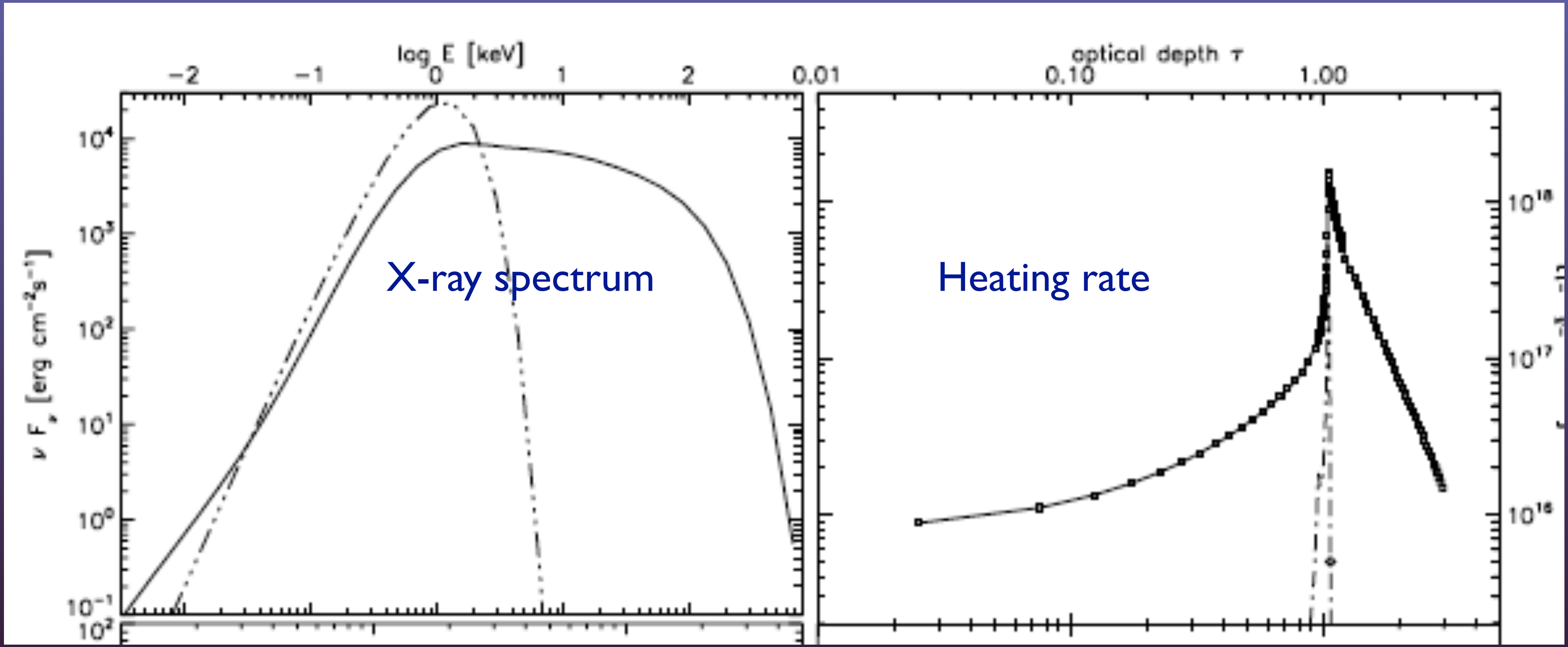


Ion-heated surface layer of a disk ('warm layer'):
balance between Coulomb heating and Compton cooling

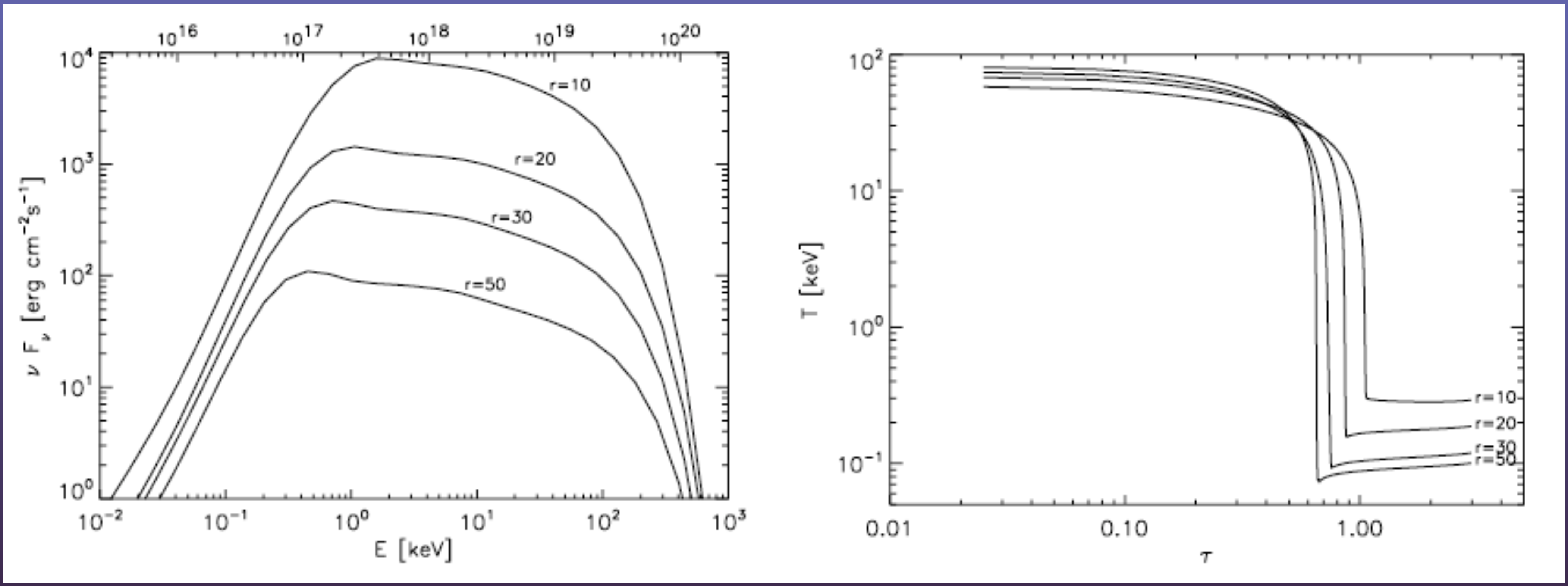
(Deufel et al 2002)

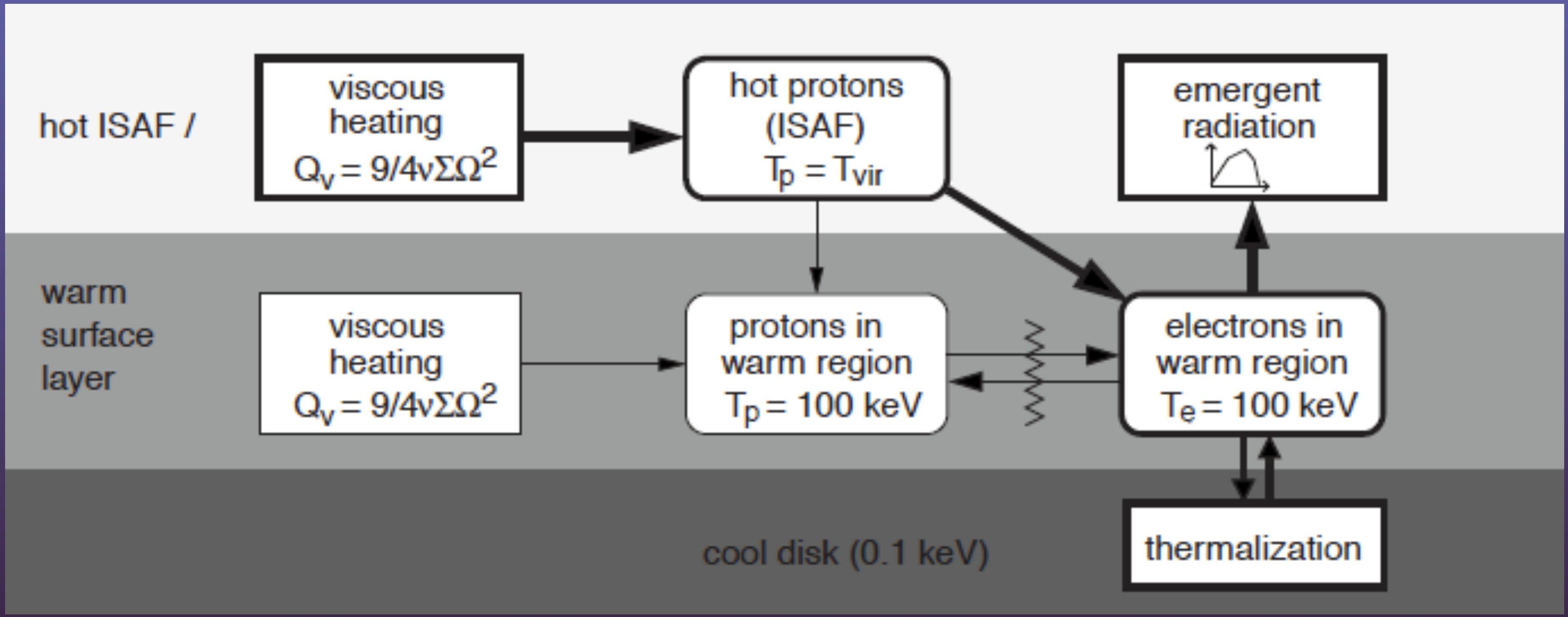


Ion-heated surface layer of a disk ('warm layer'):
balance between Coulomb heating and Compton cooling
(Deufel et al 2002)

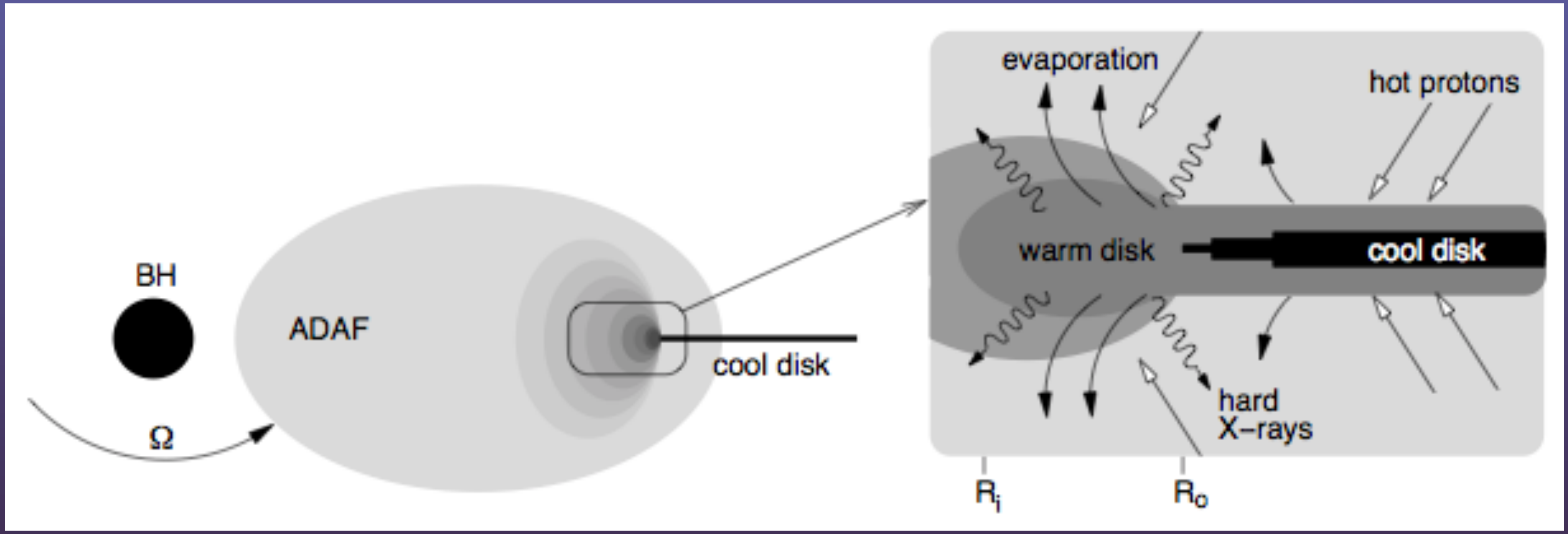


(Deufel et al 2002)

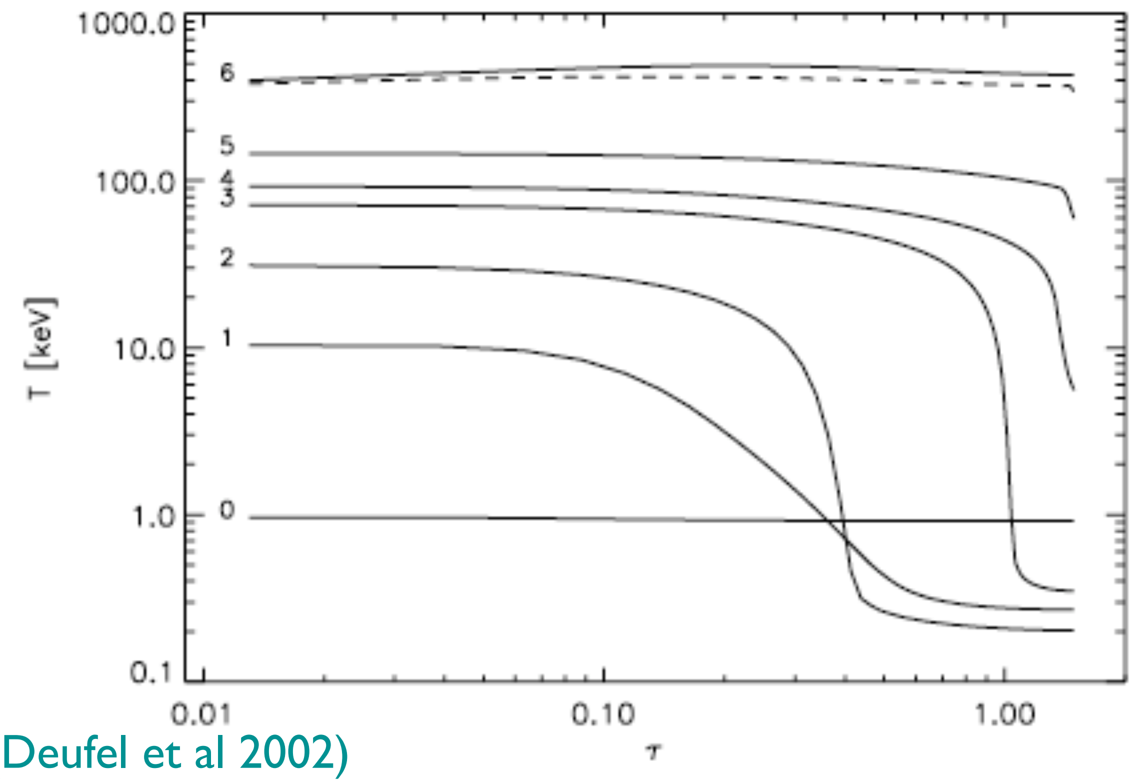



hot ISAF /
viscous heating
$Q_v = 9/4\nu\Sigma\Omega^2$
hot protons (ISAF)
$T_p = T_{vir}$
emergent radiation
warm surface layer
viscous heating
$Q_v = 9/4\nu\Sigma\Omega^2$
protons in warm region
$T_p = 100$ keV
electrons in warm region
$T_e = 100$ keV
cool disk (0.1 keV)
thermalization

ion heating at the inner edge of the cool disk



Heating of an optically *thin* cool disk



(Deufel et al 2002)

Fig. 5. Evolution from a cool disk to a warm disk by exposure to virialized protons at $r = 10$. Initial temperature $T_e = 1$ keV The numbers at the lines denote the sequential stages, referred to in the text. The final stage [6] shows an equilibrium at $T_e \approx 400$ keV. The dashed line shows the temperature profile after one dynamical time-scale.

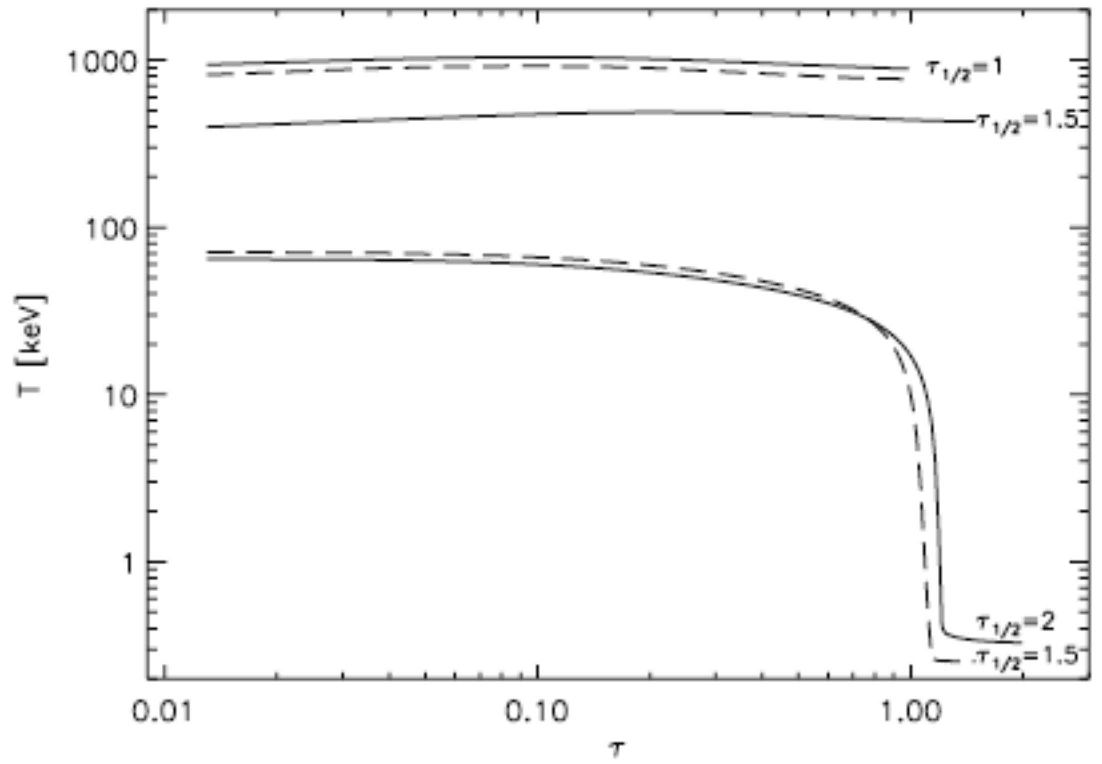
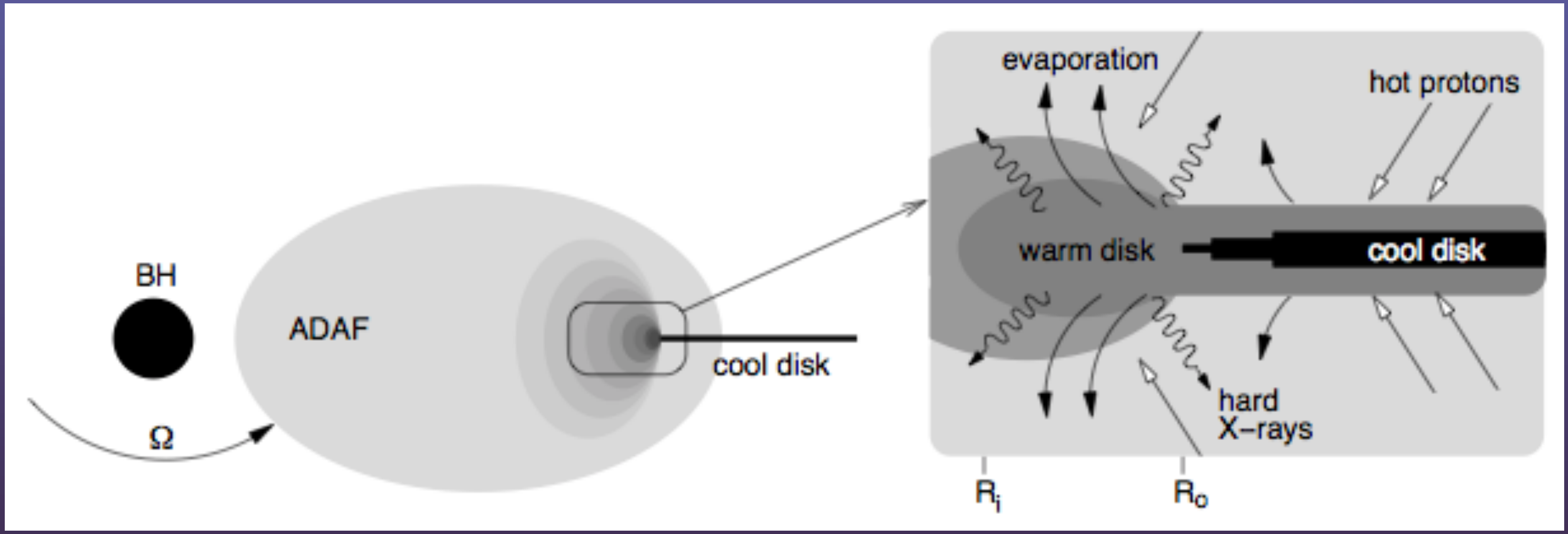
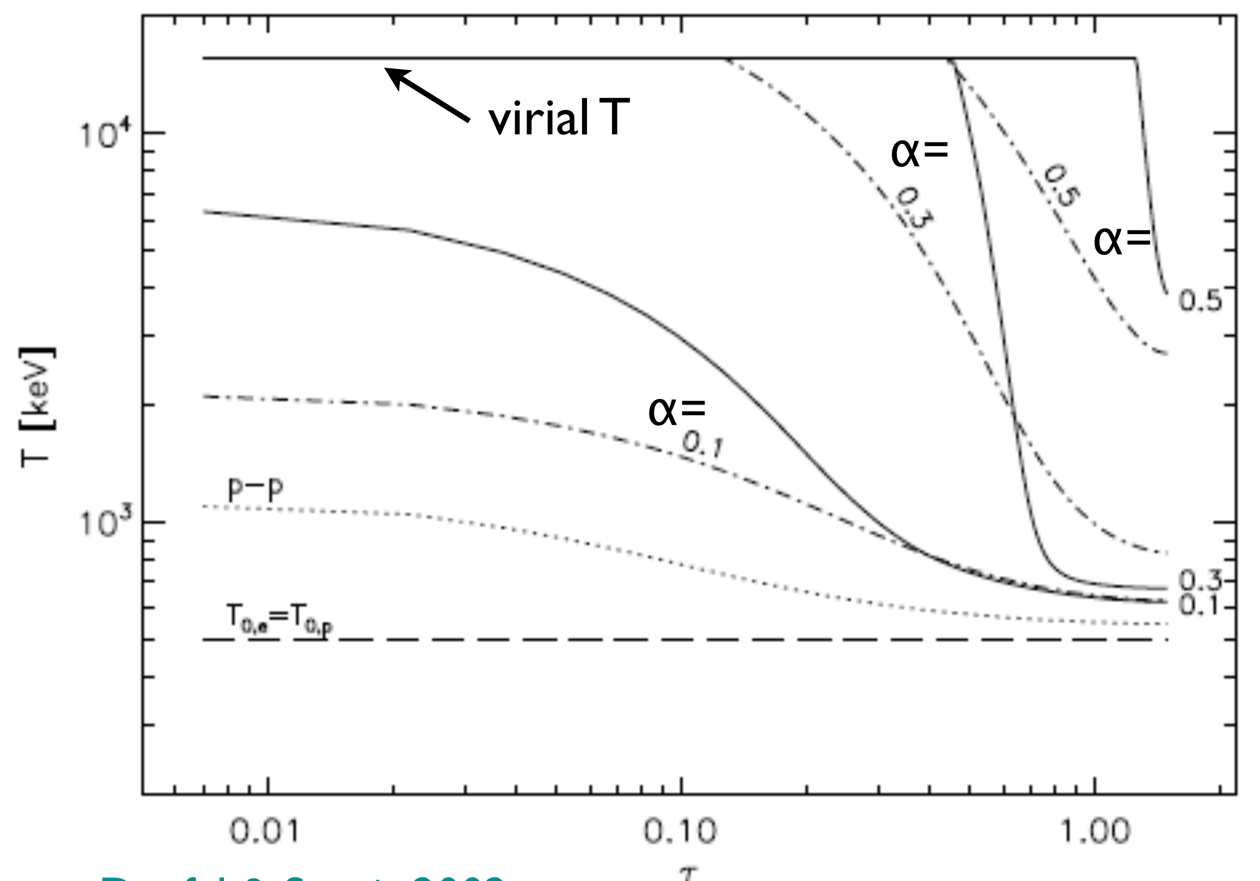


Fig. 6. Equilibrium temperature profiles of warm disks exposed to virialized protons at $r = 10$ (solid line) and $r = 15$ (dashed line) with different initial optical depths $\tau_{1/2} = 2, 1.5, 1.$ for $r = 10$ and $\tau_{1/2} = 1.5, 1$ for $r = 15$.

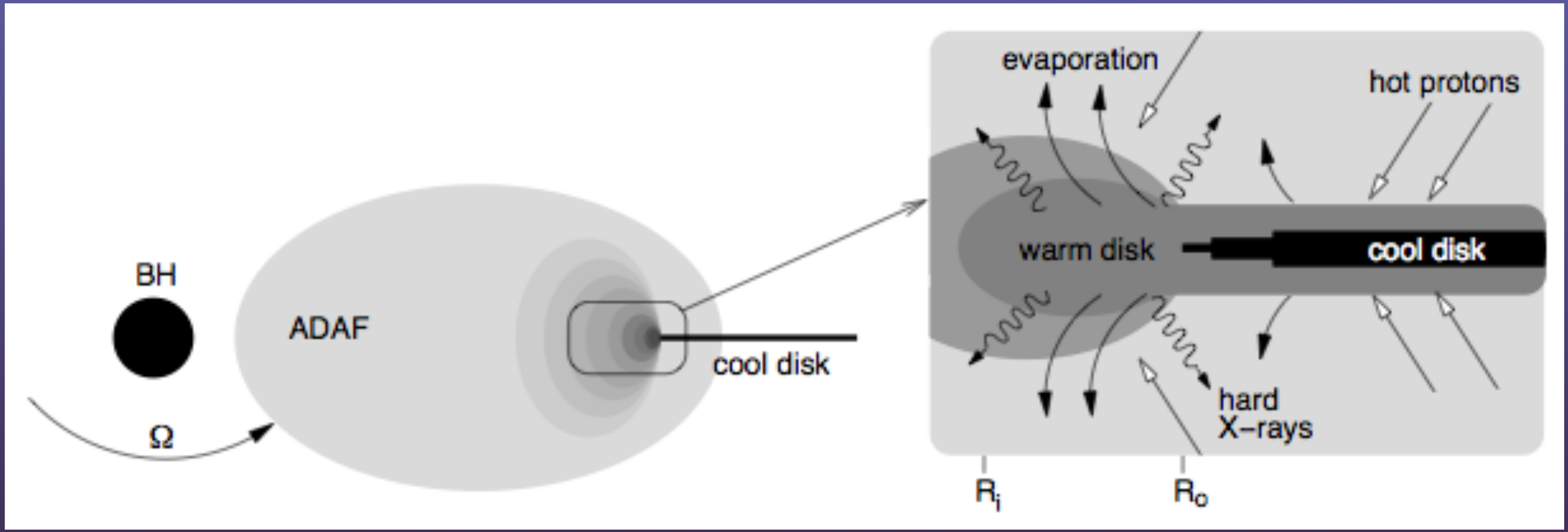
'evaporation' of the warm disk into the ISAF: thermal instability



Thermal instability of a 500 keV disk (without cooling photons...)



Deufel & Spruit 2002


BH
ADAF
Ω
cool disk
evaporation
hot protons
warm disk
cool disk
hard
X–rays
R_i
R_o

Summary ion supported accretion:

- simultaneous presence of cool disk and hot flow implies *interaction*
- heating by ions dominates interaction
- a surface layer of cool disk heated to ~ 100 keV
- at inner edge optical thickness of cool disk decreases
- thermal instability at inner edge → evaporation into ISAF
- ISAF spreads inward and outward over cool disk (loop closes here)

spectrum of warm ion heated layer must be an important component of the hard state spectrum