Magnetic field amplification in GRB shocks

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1. Why magnetic field amplification?
   - Phenomenology of electron acceleration
   - Phenomenology of proton acceleration

2. How to amplify fields
   - Weibel instability
   - Non-resonant streaming instability

3. Implications for particle acceleration
The story of $\epsilon_B$

“We put all the physics into $p$, $\epsilon_e$ and $\epsilon_B$”

$$\epsilon_B = \frac{B^2}{8\pi nk_B T} \approx 1\% \text{ (RSN)}$$

$$\approx 1\% \text{ (GRB)}$$
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- Hydrodynamics: $B_\perp \rightarrow rB_\perp$
  
  \[ r = (\gamma + 1)/(\gamma - 1). \]

- MHD: slow mode: $B$ decreases,
  fast mode: less compressive than hydrodynamics

- Relativistic shocks: $B_\perp \rightarrow rB_\perp$
  
  \[ r \rightarrow 3\Gamma \text{ (proper frames)} \quad r \rightarrow 3 \quad \text{(lab. frame/shock frame)}. \]
Why magnetic field amplification? How to amplify fields Implications for particle acceleration

Phenomenology of electron acceleration

The story of $\epsilon_B$

“We put all the physics into $p$, $\epsilon_e$ and $\epsilon_B$”

$$\epsilon_B = \frac{B^2 / 8\pi}{nk_B T} \approx 1\% \text{ (RSN)}$$

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$$\epsilon_B \approx r \frac{V_{A}^2}{V_{s}^2} \approx 4 \times 10^{-4} \text{ (RSN)}$$
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$$\epsilon_B \approx r \frac{v_A^2}{c^2} \approx 3 \times 10^{-7} \text{ (GRB)}$$
X-ray filaments

- Chandra image of Cas A
X-ray filaments

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- Non-thermal X-rays near rim
X-ray filaments

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- Filament width given by synchrotron burn-off: $B = 250 - 300 \, \mu G$
X-ray filaments

- Chandra image of Cas A
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- Filament width given by synchrotron burn-off: 
  \[ B = 250 - 300 \mu G \]
- Electrons accelerated to 
  \[ \gamma = 10^8 \] (Vink 2006)
The CR acceleration problem

Acceleration rate

\[
\frac{\dot{E}}{E} \sim \frac{v_s^2}{\kappa}
\]

Diffusion coefficient \(\kappa = \text{mean-free path} \times \text{velocity}/3\)
The CR acceleration problem

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Bohm diffusion: mean-free path = gyro-radius

\[ E_{\text{max}} \approx 2 \times 10^{13} \left( \frac{ZB}{3 \mu \text{G}} \right) \text{ eV} \]

(Lagage & Cesarsky A&A 125, 249, (1983))
The CR acceleration problem

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But, knee in CR spectrum is at \( 10^{15} - 10^{16} \text{eV} \)
Weibel Instability:

- Growth of current filaments fuelled by streaming
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- Seen in rel. P.I.C. simulations of $e^+ - e^-$ shocks
Weibel instability

Something from nothing?

Weibel Instability:

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- Short wavelength across stream, long wavelength along it
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- Saturation at low amplitude in $e$-$\rho$ plasmas?

Lyubarsky & Eichler (2006)
Weibel Instability:

- Growth of current filaments fuelled by streaming
- Short wavelength across stream, long wavelength along it
- Seen in rel. P.I.C. simulations of $e^+ - e^-$ shocks
- Saturation at low amplitude in e-p plasmas? *Lyubarsky & Eichler (2006)*
- May help thermalize electron component
**SNR shocks**

- Diffusive shock acceleration: CR density constant downstream, falls off exponentially upstream.
- In plasma frame, CR streaming speed $\approx$ shock speed.
Why magnetic field amplification?

How to amplify fields

Implications for particle acceleration

Non-resonant streaming instability

SNR shocks

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CR density

Upstream

Downstream

Modification of low freq. wave modes unimportant $\rightarrow$

Alfven waves grow at the CR cyclotron resonance
Bell’s (2004) instability

- But, shorter wavelength modes with
  \[ r_{\text{thermal}}^{-1} > k > r_{CR}^{-1} \]
  strongly modified.
- Plasma uncompensated: helicon/whistler-type modes
- Strong, nonresonant growth driven by “uncompensated” current.
Nonlinear development

Saturation expected when

$$\left| \vec{k} \wedge \vec{B} \right| \approx \frac{4\pi}{c} j_{\text{CR}}$$

$$\Rightarrow \frac{B^2}{8\pi} \approx \frac{1}{2} \frac{v_{\text{CR}}}{c} U_{\text{CR}}$$

SNR shock: $v_s/c = 1/50$, $M_A = 200$, $\beta \approx 1$:

$$U_{\text{CR}} \approx M_A^2 B_{\text{ISM}}^2 / 8\pi$$

$$\Rightarrow B_{\text{shock}} \approx 30 B_{\text{ISM}}$$

Acceleration to $> 10^{15} \text{ eV}$?
Relativistic case

- Relativistic proton beam $\Gamma_b \gg 1$
- Warm electron/proton plasma $kT/m = \Theta < \Gamma$
- Charge neutrality, zero net current

\[ \Rightarrow \omega^2 \chi \approx -\frac{\omega'_p^2 \omega'}{\epsilon \omega_c} + \frac{\omega'_p^2 \omega'}{\epsilon \omega_c - \omega'} + \frac{\omega^2}{v_A^2} + \frac{\omega'_p^2 \omega}{\epsilon \omega_c^3} \left( k^2 - \omega^2 \right) \langle \gamma^2 v_{\perp}^2 \rangle \]

- Plasma current
- Beam response
- Thermal effects
Relativistic case

Cold plasma, $\epsilon = -1$: purely growing modes, max. growth rate

$$\text{Im}(\omega) \approx \frac{n_b}{n_p} \omega_p$$

at

$$k_\parallel \approx \frac{n_b \omega_p}{2 v_A n_p}$$

Thermal effects reduce current drive when

$$k_\parallel > \frac{\omega_p v_A}{c} \left( \frac{n_b}{4 \Gamma n_p \langle \gamma^2 v^2_\perp \rangle} \right)^{1/2}$$

i.e.,

$$\Theta > \frac{v^2_A}{c^2} \sqrt{\frac{n_p}{\Gamma n_b}}$$
Non-resonant streaming instability

Relativistic case

\[ v_A = 2 \times 10^{-5}, \Gamma = 10, n_b/n_p = 1/3, \epsilon = -1, \epsilon = +1 \]
Similar to cosmic ray scattering in SNR shocks, where a nonresonant, current driven instability is important. Bell (2004), Pelletier, Marcowith et al (2006)
Non-resonant streaming instability

Relativistic case - summary

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- Short wavelength turbulence generated, \( k_\parallel \approx \frac{n_b \omega_p}{n_p v_A} \). Strong amplification of seed magnetic field
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- Saturates when ambient plasma heats to “shock” temperature
- Nonlinear evolution uncertain...
Particle transport

Average field orientation: \( B_{\parallel} = B'_{\parallel}, \quad B_{\perp} = \Gamma_{\text{shock}} B'_{\perp} \).
Large \( \Gamma \Rightarrow \) perpendicular shocks.  
Begelman & Kirk 1990
Particle transport

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\( N_{\text{cross}} \leq 3 \) for \( \Gamma \to \infty \)

\cite{Lemoineetal2006}
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Short-wavelength turbulence needed
First-order Fermi at relativistic shocks

- Average field strong $\Rightarrow$ no stochastic acceleration:
  - Random B-field, Niemiec & Ostrowski (2006)
First-order Fermi at relativistic shocks

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  P.I.C simulations, *Spitkovsky (2006)*
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- For GRBs incoming $B$ irrelevant ($\epsilon_B \approx 0.01$)
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- Standard synchtron theory applies provided

$$\frac{k_{\text{max}}}{k_{\text{synch}}} = \frac{m_e}{4\Gamma m_p} \left( \frac{\Gamma n_b m_p}{B^2/8\pi} \right) = \frac{m_e}{4\Gamma m_p \epsilon_B} \ll 1$$
Conclusions

- Amplification required in GRB, SNR, RSN…
- Suitable mechanism identified. Nonlinear evolution not yet clear
- Standard acceleration theory and radiation mechanisms apply