Lecture 13

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Hydrodynamical Simulations of Galaxy Formation

Methods, recent progress, and results from the EAGLE project

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Rendered with TYPHOON Crain / Geach

Motivation for simulations

Observations:

- Only single time snapshot
- Limited physical information

Analytic models:

- Severe approximations
- Real life is complicated!

Simulating DM / gravity only

Brief recap:

- Gravity is the dominant force over long distances
- "N-body simulations"
- Example: MXXL

Advantages:

- Relatively simple
- Physics "understood"
- Large volumes possible

Disadvantages:

- No galaxies (!!)
- Ignore effect of baryon physics on dark matter



Matter density in MXXL simulation (R. Angulo et al., 2012, MNRAS 426, 2046)

Semi-Analytic models

Deals with problem 1: Galaxies included into DM simulations "by hand" (see Lecture 11)

- Analytic prescriptions for galaxy evolution processes (Star formation, feedback, mergers, AGN, ...)
- Quick to run
- Easy to explore model variations
- But: severely simplified
- Baryons cannot influence DM
- No predictions about the inter-galactic medium





Hydrodynamical simulations

More fundamental approach: Include baryons directly into simulation

- Two fundamental additions to DM-only simulations:
- I.) Hydrodynamics: Gas responds to pressure as well as gravity
- 2.) Baryonic physics: Gas can do more than simply move around (star formation, chemical evolution, black holes, ...)

Need to treat gas as *fluid*: Not collisionless, mean free path is short. This requires a fundamentally different simulation technique from DM.

Relevant scales often unresolved: Need to implement as *sub-grid physics* modules

How to deal with fluids in simulations

Fluid simulation: Grid method

Basic idea: Parcel up the simulation *volume* into a finite number of *cells*

- Each cell has a finite volume and contains a finite gas mass: get density
- Compute fluxes between neighbouring cells and update masses
- Often combined with adaptive mesh refinement (AMR) to make cells smaller in densest areas



Illustration of AMR: Grid becomes finer in dense regions (yellow)

Fluid simulation: Particle method

Basic idea: Parcel up the simulation *mass* into a finite number of *particles*.

- Conceptually similar to DM-only simulation: Particles conserve mass and move through space.
- But particles occupy only infinitesimally small volume
 --> to find the local density, one must
 interpolate over
 neighbouring particles...



+Spatial resolution adapted automatically +Galilean-invariant (no preferred frame or direction)

+Lagrangian: can trace individual particle's history

Smoothed Particle Hydrodynamics (SPH)

$$F_s(\mathbf{r}) = \int F(\mathbf{r}) W(\mathbf{r} - \mathbf{r}', b) d\mathbf{r}'.$$

Some field F(**r**)

h is the characteristic width of the Kernel (normalized to unity)

$$W(\mathbf{r}, b) = w(\frac{|\mathbf{r}|}{2b}) \text{ with } w_{3D}(q) = \frac{8}{\pi} \begin{cases} 1 - 6q^2 + 6q^3, & 0 \le q \le \frac{1}{2}, \\ 2(1-q)^3, & \frac{1}{2} < q \le 1, \\ 0, & q > 1, \end{cases}$$

Integrate over finite volume

Kernel W(**r**, h)

Assume we know the field values only at a finite set of points: $F_i = F(\mathbf{r}_i)$. Each point has mass m_i and density $\rho_i \Rightarrow$ volume $\Delta \mathbf{r}_i \sim m_i / \rho_i$

$$F_s(\mathbf{r}) \simeq \sum_j \frac{m_j}{\rho_j} F_j W(\mathbf{r} - \mathbf{r}_j, b)$$

$$\rho_s(\mathbf{r}) \simeq \sum_j m_j W(\mathbf{r} - \mathbf{r}_j, b)$$

F_s defined everywhere and differentiable!

Want to have $h \ge d$ (mean interparticle separation): Minimum of ~33 neighbours

(V. Springel, 2010, ARA&A 48, 391)

SPH in some more detail - gas dynamics



(V. Springel, 2010, ARA&A 48, 391)

SPH in some more detail - gravity

Also need to account for gravity!

This leads to an extra term in the equations of motions:

$$m_{i}\mathbf{a}_{i}^{\text{grav}} = -\frac{\partial E_{\text{pot}}}{\partial \mathbf{r}_{i}}$$

$$= -\sum_{j} Gm_{i}m_{j}\frac{\mathbf{r}_{ij}}{r_{ij}}\frac{[\phi'(r_{ij},\varepsilon_{i}) + \phi'(r_{ij},\varepsilon_{j})]}{2}$$

$$-\frac{1}{2}\sum_{jk} Gm_{j}m_{k}\frac{\partial\phi(r_{jk},\varepsilon_{j})}{\partial\varepsilon}\frac{\partial\varepsilon_{j}}{\partial\mathbf{r}_{i}},$$

where E_{pot} is the potential energy:

$$E_{\text{pot}} = \frac{1}{2} \sum_{i} m_i \Phi(\mathbf{r}_i) = \frac{G}{2} \sum_{ij} m_i m_j \phi(r_{ij}, \varepsilon_j)$$

This term is only non-zero if the gravitational softening length is variable in space (usually not the case)

Final note:

SPH is by construction *inviscid* and cannot capture shocks!

 \Rightarrow "artificial viscosity"

(convert kinetic into thermal energy when there is local convergence of the fluid flow)

Fluid simulation: Method comparison

Two different schemes for same job: compare results!



Fluid simulation: Method comparison

Two different schemes for same job: compare results!



Disruption of a gas blob in a supersonic flow (O. Agertz et al., 2007, MNRAS 380, 963)

Result of calculation *can* depend strongly on numerical method (more later) !!

Note:

In recent years, much work has gone into solving these problems with the "traditional" SPH formalism. Details are beyond the scope of this course, but modern SPH codes are able to make accurate predictions about galaxy formation (see later). How to deal with physics on unresolved scales: sub-grid modules











Concentrate on this scale here: most useful for understanding the observed population of galaxies as a whole*

(Galaxies well-resolved at $M_{star} \gtrsim 10^9 M_{\odot}$: 1000s of objects for robust comparisons to observations)

*: Except for those living in rare environments such as massive galaxy clusters (→ zoom simulations)



Which of these is a `typical' galaxy...?

Sub-grid modules: overview

Baryons are subject to physical effects beyond gravity and hydrodynamics. Broadly divide into three categories [obviously heavily dependent on research interest!]

Directly interesting / observable	Impact on interesting quantities	Irrelevant
 Star formation Nucleosynthesis SMBHs / Active Galactic Nuclei 	- Supernova feedback - AGN feedback - Cooling/ heating	 Planet formation Chemistry Biology / Life /

Need to be implemented into simulation

Important note:

This is an active area of research, and there are many different ways in which these effects can be implemented into a simulation. Impossible to list them all!!

This lecture mostly follows the implementation in the EAGLE simulations.

I: Radiative cooling

Gas can radiate away its internal energy and cool: Essential for formation of dense structures like galaxies and stars.



Cooling function depends strongly on metallicity: H, He, C, N, O, Ne, Mg, Si, S, Ca and Fe

Look up tabulated cooling rates on element-byelement basis.

I: Radiative cooling



(A. Rahmati et al., 2013, MNRAS 431, 2261)

II: Reionization

Include a time-dependent, but spatially uniform UVbackground at $z \le 11.5$ (consistent with Planck measurements).

Also inject 2 eV energy per proton mass to account for boost in photoheating rates during reionization

 \Rightarrow heats photoionized gas quickly to ~ 10⁴ K

III: Star formation

Starting point: Observed Kennicutt-Schmidt law

$$\dot{\Sigma}_* = A \left(\Sigma_g \,\mathrm{M}_\odot^{-1} \,\mathrm{pc}^2 \right)^n$$

with A = 1.5 x 10⁻⁴ M_{\odot} yr⁻¹ kpc⁻² and n = 1.4

 \Rightarrow SFR from surface density Implement stochastically with probability of gas \rightarrow star given by particle's star formation rate

But this is *not* a "direct" variable of the simulation - would need to identify "galaxies", find disk orientation, ...



(R. Kennicutt, 1998, ApJ 498, 541)

III: Star formation as a pressure law

Assume that gas is self-gravitating: disc scale-height \sim Jeans scale L_J

$$\begin{split} \Sigma_{\rm g} &\sim \Sigma_{\rm g,J} \equiv \rho_{\rm g} L_{\rm J}, \\ &= \left(\frac{\gamma k}{\mu G X}\right)^{1/2} (f n_{\rm H} T)^{1/2}, \\ &= \left(\frac{\gamma}{G}\right)^{1/2} (f_{\rm g} P_{\rm tot})^{1/2}, \end{split}$$

So the KS law becomes: $\dot{m}_* = m_g A \left(1 \,\mathrm{M_{\odot} \, pc^{-2}}\right)^{-n} \left(\frac{\gamma}{G} f_g P\right)^{(n-1)/2}$



Test simulation reproduces original Kennicutt-Schmidt law

(J. Schaye & C. Dalla Vecchia, 2008, MNRAS 383, 1210)

III: Star formation threshold

Observationally, star formation (SF) occurs only in cold molecular gas $(T \le 10^4 \text{ K})$. Not modelled well in current simulations for resolution and physics reasons, so set the SF threshold at threshold density for cold phase formation:

$$n_{\rm H}^*(Z) = 10^{-1} \,{\rm cm}^{-3} \left(\frac{Z}{0.002}\right)^{-0.64}$$

(Schaye, 2004)

[Higher $Z \rightarrow$ more dust \rightarrow form cold molecular phase at lower density]

Impose effective equation of state above this threshold:

$$P_{\rm tot} = P_{\rm tot,c} \left(rac{
ho_{\rm g}}{
ho_{\rm g,c}}
ight)^{\gamma_{\rm eff}} \quad {
m with} \; \gamma_{\rm eff} = 4/3$$

(J. Schaye & C. Dalla Vecchia, 2008, MNRAS 383, 1210)



 $\gamma_{eff} = 4/3$: Realistic disk galaxy

γ_{eff} = I: Artificial fragmentation of the disk

IV: Stellar recycling

Stars do not lock up matter indefinitely: Significant fraction (~40%) *returned* to gas phase during star's life.

Important for two reasons: (i) Significant contribution to gas mass (ii) All elements except H and He produced in stars

 \Rightarrow Impact on large-scale structure

- Metallicity = observable test
- Cooling rates
- (Future) star formation

Requires both modelling of stellar evolution and implementation of recycling...



IV: Stellar recycling

- Current simulations have (mass) resolution of $\gtrsim 10^5 M_{\odot}$
- Star particle = "Simple Stellar Population" (SSP)
- At each time step, compute which stellar masses leave the main sequence: these return mass
- AGB, SN II, SN Ia
- Masses of individual elements can be calculated to give detailed chemical composition of ISM
- Neighbouring gas particles increase their mass and metallicity

N.B.: "Mixing" of metals poorly modelled in SPH simulations!





Stars inject energy into surrounding ISM through stellar winds, radiation, and supernovae

Naive implementation: Star particle heats up neighbouring gas particles

But energy is *radiated away far too quickly* and feedback does not have much effect...



Several routes to solve this:

- (i) Inject energy kinetically (by giving nearby star particles velocity kicks). Need to specify details by hand.
- (ii) Artificially disable cooling for some time, to allow conversion to kinetic energy.
- (iii) Artificially decouple different thermal phases: Prolongues cooling time for SN ejecta.
- (iv) Implement feedback stochastically...

Reality: Energy distributed over ~ 1 M_{\odot} ejecta, so ratio M_{star} / M_{ej} is large (1 SN per 10² M_{\odot} of new stars)

Simulation: Energy from 1 star particle distributed over ~ 48 gas particles: ratio of M_{star} / M_{ej} is small

Reducing the heated gas mass would make feedback (much) more effective!



(C. Dalla Vecchia & J. Schaye, 2012, MNRAS 426, 140)

Stochastic approach:

Specify desired heating temperature ΔT (specific energy $\Delta \epsilon$) and calculate probability pthat given gas particle is heated:

$$p = f_{\rm th} \frac{\epsilon_{\rm SN\,II}}{\Delta \epsilon} \frac{m_*}{\sum_{i=1}^{N_{\rm ngb}} m_i}$$

 f_{th} = fraction of SN energy going into feedback



(C. Dalla Vecchia & J. Schaye, 2012, MNRAS 426, 140)



log t_{cool} / t_{sound-crossing} (solid: primordial, dotted: solar)

Impact of heating temperature on outflows in $10^{12}\ M_{\odot}$ galaxy



$$\Delta T = 10^{6.5} \text{ K}$$

 $\Delta T = 10^{7.5} \text{ K}$

(C. Dalla Vecchia & J. Schaye, 2012, MNRAS 426, 140)

For the animations, see *http://www.strw.leidenuniv.nl/DS12/*

Two parts: (i) BH formation and growth (ii) Feedback effect on surrounding gas

"Seed" ~10⁵ M $_{\odot}$ black holes in FOF haloes above threshold mass (10¹⁰ M $_{\odot}$)



(V. Springel et al., 2005, arXiv:1312.0598; C. Booth & J. Schaye, 2009) BH "sphere of influence":

$$\begin{split} |E_{pot}| &> E_{internal} \\ r_A \sim GM_{BH} / c_s^2 \\ \dot{m}_{BH} &= 4\pi r_A^2 \rho_A c_s(r_A) \end{split}$$

$$\dot{m}_{
m Bondi} = rac{4\pi G^2 m_{
m BH}^2
ho}{(c_{
m s}^2 + v^2)^{3/2}} \quad imes 100 \; (!!)$$

Also impose Eddington limit:

$$\dot{M}_{
m Edd} = rac{4\pi G m_{
m p} M_{
m BH}}{\sigma_{
m T} \, c \, \epsilon_{
m r}}$$

Stochastically swallow gas particles near black hole whenever subgrid BH mass has increased sufficiently

+ allow nearby BHs to merge

Two parts: (i) BH formation and growth (ii) Feedback effect on surrounding gas



So that
$$\dot{m}_{\rm BH} = (1 - \epsilon_{\rm r}) \dot{m}_{\rm accr}$$

and
$$E_{\rm BH} = \epsilon_f \, \epsilon_r \, m_{\rm accr} \, c^2$$

(Y. Rosas-Guevara et al., 2013, arXiv:1312.0598)

Feedback implementation similar to SN feedback: Stochastically heat nearby gas particles by temperature ΔT_{AGN} (energy $\Delta \epsilon_{AGN}$):

$$P = rac{E_{
m BH}}{\Delta \epsilon_{
m AGN} N_{
m ngb} \langle m_{
m g}
angle}$$

N.B.: Typically higher gas density than around newly formed star particles \Rightarrow higher ΔT

But: Too efficient in Milky-Way mass galaxies. Need to artificially limit impact in these cases ("radio-/quasar-mode")...



(Y. Rosas-Guevara et al., 2013, arXiv:1312.0598)

Additional limit on BH accretion rate:



disk structure (unresolved)

$$egin{aligned} rac{t_{
m Bondi}}{t_{
m visc}} &= rac{r_{
m Bondi}c_{
m s}^{-1}}{C_{
m visc}[r_{
m Bondi}V_{\phi}]^3[GM_{
m BH}]^{-2}} \ &= rac{1}{C_{
m visc}}rac{c_{
m s}^3}{V_{\phi}^3}. \end{aligned}$$

Find V_{$$\phi$$} as $V_{\phi} = \Big| \sum_{i=0}^{N_{\rm SPH}} \mathbf{r_i} \times \mathbf{v_i} \, m_i W(\mathbf{r_i}, h) \frac{1}{\rho h} \Big|$



(Y. Rosas-Guevara et al., 2013, arXiv:1312.0598)

momentum limits AGN effect in massive haloes to observed level

Aside: sub-grid physics vs. hydro solver



(C. Scannapieco, 2012, MNRAS 423, 1726)

much larger effect

The EAGLE Simulations

Evolution and Assembly of GaLaxies and their Environments



The Illustris Simulation

Towards a predictive theory of galaxy formation.

www.illustris-project.org

The EAGLE Simulations

Evolution and Assembly of GaLaxies and their Environments



N.B.: "Illustris" is another recent cosmological hydrodynamical simulation with similar results (and very different simulation code)

EAGLE: overview

Large suite of cosmological hydrodynamical simulations including all the subgrid recipes discussed above

Simulation aims:

- "Realistic" galaxy population: Especially match to observed galaxy stellar mass function
- Sub-grid physics only dependent on physically motivated, local quantities
- Resolve Jeans mass in warm ISM (i.e. $m_{gas} \approx 10^6 M_{\odot}$)



Not always the case: For example, SNdriven outflows often assumed to scale with velocity dispersion of DM halo

EAGLE: calibration

Main free parameters of above subgrid models (i.e. not well-constrained by either observations or theory):

- Black hole heating temperature ΔT , efficiency ϵ_f , and viscosity parameter C_{Visc}
- Supernova feedback f_{th}

Strong influence only on most massive haloes (galaxy groups): $\Delta T = 10^{8.5} \text{ K \& } \epsilon_{f} = 0.15 \text{ used as}$ default

C_{Visc} calibrated to give break in GSMF at M* ~ $10^{10.5}$ M_{\odot}



(J. Schaye et al., 2015, MNRAS 446, 521; R. Crain et al., 2015, arXiv:1501.01311)

EAGLE: calibration

Main free parameters of above subgrid models (i.e. not well-constrained by either observations or theory):

- Black hole heating temperature ΔT , efficiency ϵ_f , and viscosity parameter C_{Visc}
- Supernova feedback f_{th}
 Most influential parameter:

calibrated to GSMF and stellar sizes

$$f_{\mathrm{th}} = f_{\mathrm{th,min}} + rac{f_{\mathrm{th,max}} - f_{\mathrm{th,min}}}{1 + \left(rac{Z}{0.1 \, \mathrm{Z}_{\odot}}
ight)^{n_Z} \left(rac{n_{\mathrm{H,birth}}}{n_{\mathrm{H,0}}}
ight)^{-n_n}}$$

$$\label{eq:fth,min} \begin{split} f_{th,\,min} &= 0.3 \text{ and } f_{th,\,max} = 3.0 \\ n_Z &= n_n \approx \ I \\ n_{H,\,0} &= 0.67 \ \text{cm}^{-3} \end{split}$$

(J. Schaye et al., 2015, MNRAS 446, 521; R. Crain et al., 2015, arXiv:1501.01311) Strong influence only on most massive haloes (galaxy groups): $\Delta T = 10^{8.5}$ K & $\epsilon_f = 0.15$ used as default

C_{Visc} calibrated to give break in GSMF at M* ~ $10^{10.5}$ M_{\odot}



EAGLE results: Stellar masses



Galaxy Stellar Mass Function (GSMF) in EAGLE compared to two Semi-Analytic Models: equally good fit to data

EAGLE GSMF compared to other hydro simulations: Significantly better fit Mock images of EAGLE galaxies along the Hubble Sequence from SPS models + ray-tracing accounting for dust





(J. Schaye et al., 2015, MNRAS 446, 521)



See http://eagle.strw.leidenuniv.nl/index.php/eagle-visualisation/

The EAGLE simulations

EVOLUTION AND ASSEMBLY OF GALAXIES AND THEIR ENVIRONMENTS

A project of the Virgo consortium

z = 19.9 L = 25.0 cMpc

See http://eagle.strw.leidenuniv.nl/index.php/eagle-visualisation/

Summary

- Hydrodynamic simulations have made tremendous progress in last few years and now match observations as good as semi-analytic models, with far fewer free parameters.
- Fundamental difficulties are accurate modelling of hydrodynamics, and (especially) unresolved physical processes that influence structure formation on larger (resolved) scales
- Current simulations achieve resolution of ~10⁶ M_{\odot} in boxes of ~100 Mpc sidelength: 1000s of galaxies like Milky Way

Together with simulations on larger and smaller scales, they are a very powerful tool to learn more about how the galaxies we observe formed and evolve

> Rendered with TYPHOON Crain / Geach

Note:

The following slides were not shown in the lecture, but are nevertheless interesting.

They show a range of other predictions from the EAGLE simulations and the (generally very good) level of agreement with observations.

EAGLE results: Galaxy sizes



Simulated galaxies have the **right sizes**

(non-trivial success recall angular momentum catastrophe)

N.B.: No fully "blind" prediction - stellar feedback model was chosen to reproduce this

(J. Schaye et al., 2015, MNRAS 446, 521)

Calibration caveat: GSMF is not all!



EAGLE results: Mstar-Mhalo



Relation between galaxy stellar mass and halo mass:

The right galaxies live in the right haloes

⁽J. Schaye et al., 2015, MNRAS 446, 521)

EAGLE results: Black hole masses



Large scatter in observations, but simulations generally produce **black holes of reasonable mass**

(Simulation volume too small to fully sample $M_{star} \gtrsim 10^{11} M_{\odot}$)

N.B.: Mostly influenced by ε_f

(J. Schaye et al., 2015, MNRAS 446, 521)

Some more EAGLE results...



(J. Schaye et al., 2015, MNRAS 446, 521)

EAGLE results: Galaxy groups



EAGLE results: build-up of stellar mass

