# Exercise sheet 3

### Exercise 3 - 1

Bob flips a (probably) manipulated coin as long as he gets tails. The moment the coin lands with head up, he stops the tossing. $n$ denotes the number of tails he got. The coins' probability to land with tail up may be $f$. Bob's strategy is denoted by $B$.

**a)** Calculate $P(n \mid f, B)$ (1 point).

**b)** Calculate the expected number of tails, i.e. $\langle n \rangle_{(n|f,B)}$( 2 points).

**c)** Bob performs one tossing experiment from $a$) and gets n tails in a row, which he tells Alice. So far Alice does not know how the experiment was conducted and likes to infer the unfairness of the coin.

- Until now Alice believes that Bob performed a coin toss experiment of predetermined length $n+1$. This strategy is called $A$. Calculate the most probable $f$ using $P(f \mid n, A)$ (2 points).
- Now Bob tells Alice that he ended the tossing when he got the first head. She therefore infers the most probable $f$ using $P(n \mid f, B)$ and Bayes Theorem. Calculate $P(f \mid n, B)$.
- Compare the results and discuss wether the finding is surprising (1 point).

**d)** As Alice knows that the maximum of a non-symmetric probability distribution is not equal to its expectation value she uses a computer algebra system of her choice to plot the probability distribution of $f$.
Calculate $\langle f \rangle_{(f|n,B)}$ and compare with your results from $b$) (2 points).

### Exercise 3 - 2

Information is transmitted digitally as a binary sequence known as bits. However, noise on the channel corrupts the signal, in that a digit transmitted as *digit* is received as *as its complement* with probability $(1-\alpha)$. It has been observed that, across a large number of signals to be transmitted, the 0s and 1s are transmitted in the ratio 3 : 4
Given that one can only measure sequences of length three and the received data is 101, what is the probability distribution over transmitted signals? Derive an expression for the probability that the transmitted signal is equal to the received data. Assume that the signal selection and the noise are independent of each other (4 points).

**Exercise 3 - 3**

Assume that loosing a fraction $x$ of your budget hits you as $l(x) = x/(1-x)$, being your personal loss function.

**a)** Up to which fraction $y < x < 1$ of your budget should you invest to insure against the risk of loosing the budget fraction $x = x_E$ by an event $E$ occurring with probability $p = P(E)$ (2 points) ?

**b)** 1 An insurance company asks you to pay a budget fraction $z = \alpha\, p\, x$ to insure your loss under $E$ with $\alpha > 1$ (to ensure that the company makes profit on average). Under which conditions should you take their offer (1 point)?

**c)** Show that a tiny monetary loss $x \ll 1$ should never be insured (1 point).

**d)** The insurance company knows the true $p_E$ perfectly well, whereas the beliefs of its customers on it are uniformly distributed in $p \in [0, 1]$ What is the expected profit per potential costumer as a function of the insurance price z (2 points) ?

*This exercise sheet will be discussed during the exercises.*
*Group 01, Wednesday 16:00 - 18:00, online,*
*Group 02, Thursday, 10:00 - 12:00, online,*
*Group 03, Thursday, 16:00 - 18:00, online,*
*Please hand in your solution by Monday, May 3rd 2021, noon online:*
*https://moodle.lmu.de/course/view.php?id=14930*