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Exercise sheet 3

Exercise 3 - 1

Bob flips a (probably) manipulated coin as long as he gets tails. The moment the coin lands heads up, he stops tossing. Let n denote the number of tails he got and f the coins' intrinsic probability to land tails up. Bob's strategy is denoted by B.

a) Calculate $P(n \mid f, B)$ (1 point).

b) Calculate the expected number of tails, i.e. $\langle n \rangle_{(n|f,B)}$(2 points).

c) Bob performs one tossing experiment from a) and gets n tails in a row, which he tells Alice. So far Alice does not know how the experiment was conducted and likes to infer the unfairness of the coin.

- Until now Alice believes that Bob performed a coin toss experiment of predetermined length $n+1$. This strategy is called A. Calculate the most probable f using $P(f \mid n, A)$ (2 points).
- Now Bob tells Alice that he ended the tossing when he got the first head. She therefore infers the most probable f using $P(n \mid f, B)$ and Bayes Theorem. Calculate $P(f \mid n, B)$.
- Compare the results and discuss wether the finding is surprising (1 point).

d) As Alice knows that the maximum of a non-symmetric probability distribution is not equal to its expectation value she uses a computer algebra system of her choice to plot the probability distribution of f.
Calculate $\langle f \rangle_{(f|n,B)}$ and compare with your results from b) (2 points).

Exercise 3 - 2

Information is transmitted digitally as a binary sequence known as bits. However, noise on the channel corrupts the signal, in that a digit transmitted as *digit* is received as *its complement* with probability $(1-\alpha)$. It has been observed that, across a large number of signals to be transmitted, the 0s and 1s are **transmitted** in the ratio 3 : 4.
Given that one can only measure sequences of length three and the received data is 101, what is the probability distribution over transmitted signals? Derive an expression for the probability that the transmitted signal is equal to the received data. Assume that the signal selection and the noise are independent of each other (4 points).

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Exercise 3 - 3

You know that I = "Ravens are either black or white birds and there exist only an unknown, but finite numbers of ravens and birds". Let the statement A be "All ravens are black".

a) You observe a black raven, which is your data d = "one (randomly chosen) bird is a black raven". Does this data d increase, decrease, or leave your confidence in A unchanged (2 points)? (Prove your claim)

Hint: Use the notation n(bird type) = (the number of birds of a given type) to express probabilities and the statement A

b) Now, you observe a black swan, which is now our data d' = "one (randomly chosen) bird is not a raven". Does the data d' increase, decrease, or leave our confidence in A unchanged (1 point)? (Prove your claim)

c) Again, you observe a black raven, which is our data d = "one (randomly chosen) bird is a black a raven". But now, you believe in the additional information $J = \{J_1 \text{ and } (J_A \text{ or } J_{\overline{A}})\}$ with

$$J_1 = \text{There exist } 10^{11} \text{ birds} \tag{1}$$

$$J_A = 10^7 \text{ ravens exist and they are black} \tag{2}$$

$$J_{\overline{A}} = 10^8 \text{ ravens exist and } n(\text{white raven}) = 10^7 \text{ of them are white.} \tag{3}$$

Does the data d increase, decrease, or leave our confidence in A unchanged given the information I and J (3 points)? (Quantify your claim)

This exercise sheet will be discussed during the exercises.
Group 01, Wednesday 18:00 - 20:00, Theresienstr. 37, A 449,
Group 02, Thursday, 10:00 - 12:00, Theresienstr. 37, A 249,
Group 03, Thursday, 16:00 - 18:00, Theresienstr. 37, A 449

https://wwwmpa.mpa-garching.mpg.de/ ensslin/lectures/lectures.html