PD Dr. Torsten Enßlin
Andreas Popp, Philipp Haim,
Vishal Johnson, Fabian Sigler, Hanieh Zandinejad

Exercise sheet 1

Exercise 1 - 1: Boolean Algebra

a) The NAND and NOR operators are defined as $A \uparrow B = \overline{AB}$ and $A \downarrow B = \overline{A+B}$, respectively. Construct the following expressions using only the statements A and B and the $\uparrow$- and $\downarrow$-operators (3 points):

$$\overline{A}, \qquad A+B, \qquad AB$$

b) What is the minimum number of operations that need to be defined on logical statements so that the negation, conjunction, and disjunction can be expressed? (1 point)

c) If 1 (True) and 0 (False) are a representation of logical values, how do we need to modify the rules of arithmetics to represent the Boolean algebra? (1 point)

Exercise 1 - 2

A sports game in which two players play for a point in each round, the game is won by the first player who leads by two points. Player A always has the probability θ to win a round. What are his chances?

a) State the three probabilities (as a function of θ) that after two rounds player has A won, has lost, or has to continue the game. Call them in the following w, l, and c, respectively (1 points).

b) State now the three probabilities (as a function of w, l, and c) that after (at most) four rounds player A has won, has lost, or that the game still continues (1 points).

c) State now the three probabilities that after (at most) an infinite numbers of rounds player A has won, lost, or that the game still continues (2 points).

d) Now you hear that player A has won the game. What is the probability $P(n|W)$ (as a function of θ) that the game ended with round n? If you were not able to solve c) use the following:

$$P(W, n \leq \infty) = \frac{\theta^2}{1-2\theta(1-\theta)} \qquad P(L, n \leq \infty) = \frac{(1-\theta)^2}{1-2\theta(1-\theta)} \qquad P(C, n \leq \infty) = 0$$

(3 points)

Exercise 1 - 3: Weather in Markovia

You are traveling to the beautiful country of Markovia. Your travel guide tells you that the weather w_i in Markovia on a particular day i is sunny, $w_i = \mathrm{s}$, for 80% of all days or it is cloudy, $w_i = \mathrm{c}$, for 20% of all days. There are no other weather conditions in Markovia and the weather changes only during nights. The probability for a weather change is 10% if it is sunny,

$$P(w_{i+1} = \mathrm{c}|w_i = \mathrm{s}) = 0.1, \tag{1}$$

and 40% if it is cloudy,

$$P(w_{i+1} = \mathrm{s}|w_i = \mathrm{c}) = 0.4, \tag{2}$$

irrespective of what it has been on earlier days, $P(w_{i+1}|w_i,\, w_{i-1},\, w_{i-2},\, \ldots) = P(w_{i+1}|w_i)$.

a) You arrive on a sunny day, $w_i = \mathrm{s}$, in Markovia. Calculate the probability that it was cloudy there the day before, $P(w_{i-1} = \mathrm{c}|w_i = \mathrm{s})$ Hint: Use Bayes-Theorem. (2 points).

b) What is the total probability for a weather change $P(w_{i+1} \neq w_i)$ in Markovia during an arbitrary night (1 point)?

c) The Markovian weather forecast for some day i predicts a sunshine probability of

$$p_i = P(w_i = \mathrm{s}\,|\text{forecast}). \tag{3}$$

What is the sunshine probability there for the following day,

$$p_{i+1} = P(w_{i+1} = \mathrm{s}\,|\text{forecast})? \tag{4}$$

(1 point)

d) Verify or correct the travel guide's statement on the frequency of 80% sunny and 20% cloudy days in Markovia (1 point).

Hint: Your result of question **c)** might be useful for this.

e) Implement an algorithm to simulate the weather in Markovia and verify your results numerically (optional).

This exercise sheet will be discussed during the exercises.
Group 01, Wednesday 18:00 - 20:00, Theresienstr. 37, A 449,
Group 02, Thursday, 10:00 - 12:00, Theresienstr. 37, A 249,
Group 03, Thursday, 16:00 - 18:00, Theresienstr. 37, A 449

https://wwwmpa.mpa-garching.mpg.de/ ensslin/lectures/lectures.html