PD Dr. Torsten Enßlin
Margret Westerkamp
Johannes Harth-Kitzerow

Exercise sheet 2

Exercise 2 - 1: Happy Birthday

A number of persons, k, meet. Assume that the probability of a person to have his/her birthday is the same for every day of the year. Assume further that the number of days per year is always 365.

a) How high is the probability that the birthday of at least q of these people is on the first of January (2 Points)?

b) How high is the probability of at least two persons in the room having their birthday on the same day (1 Point)?

c) For which k is this probability larger than 50% (1 Point)?

Exercise 2 - 2

You go to the doctor and he performs a test for a certain disease. You know from the literature that two percent of the population are afflicted by this particular disease and your doctor tells you that his test is very accurate since it tests 95 percent of the people afflicted by the disease positive and only 10 percent of the people not afflicted by it.
How high is the probability that you are afflicted by the disease if your test result is positive and how high if it is negative (2 points)?

Exercise 2 - 3

You take part in a gameshow. At one point in the show the host presents you with three doors, each hiding one prize. You get to choose one of the doors and get to keep whatever is behind it. Two of the doors are hiding a goat and one is hiding a sportscar. After you have made your choice the host, who knows which door is hiding the car, opens one of the doors you have not chosen, making sure he is revealing a goat. Now he asks you if you want to stick to your original choice or if you would like to get what is behind the third door.
Should you change to the third door (assuming you prefer cars over goats)? Give a formal proof of your answer using Bayes' Theorem (2 points).

Exercise 2 - 4: Generalized Sum Rule

a) Prove the *generalized* sum rule

$$P(A+B|C) = P(A|C) + P(B|C) - P(AB|C) \tag{1}$$

using the sum rule and the product rule (2 points).

b) Prove or disprove the following statement (1 point):

$$P(C|A+B) = P(C|A) + P(C|B) - P(C|AB)\ \forall\ A, B, C \text{ with } \mathrm{A} \neq \overline{\mathrm{B}}. \tag{2}$$

c) Generalize the generalized sum rule to three statements, i.e., find an expression for $P(A+B+C|D)$ analogous to Equation (1) (2 points).

d) Given a set of propositions $\{A_1, \dots, A_n\}$ which are mutually exclusive, given C, i.e., $P(A_i A_j|C) = \delta_{ij} P(A_i|C)$, show that

$$\sum_{i=1}^{n} P(A_i|C) = P(A_1 + \dots + A_n|C). \tag{3}$$

(2 points)

e) Given a set of propositions $\{A_1, \dots, A_n\}$ which are mutually exclusive, given C, i.e., $P(A_i A_j|C) = \delta_{ij} P(A_i|C)$, show that

$$P(B|(A_1 + \dots + A_n)C) = \frac{\sum_{i=1}^{n} P(A_i|C) P(B|A_i C)}{\sum_{i=1}^{n} P(A_i|C)}, \tag{4}$$

i.e., the left hand side is a weighted average of the separate plausibilities $P(B|A_i C)$ (1 points).

f) Given a set of propositions $\Omega^{(n)} = \{A_1, \dots, A_n\}$, define $\Lambda_i^{(n)}$ as the set of all statements that arise from the combination of i $(1 \leq i \leq n)$ different elements of $\Omega^{(n)}$ with the AND operator; e.g., for $\Omega^{(n=3)}$: $\Lambda_1^{(3)} = \{A_1, A_2, A_3\}, \Lambda_2^{(3)} = \{A_1 A_2, A_1 A_3, A_2 A_3\}, \Lambda_3^{(3)} = \{A_1 A_2 A_3\}$.
Show the sum rule

$$P(A_1 + \dots + A_n|C) = \sum_{i=1}^{n} \sum_{\pi \in \Lambda_i^{(n)}} (-1)^{i+1} P(\pi|C). \tag{5}$$

(2 points)

This exercise sheet will be discussed during the exercises.
group A, Wednesday 16:00 - 18:00 Theresienstr. 39 - room B 101,
group B, Thursday, 10:00 - 12:00, Geschw.-Scholl-Pl. 1 (B) - room B 015
group C, Thursday, 16:00 - 18:00, Theresienstr. 37 - room A 449
backup slot, Wednesday, 8:00 - 10:00, Theresienstr. 37 - A 450
(to be used whenever Thursday is a vacation day)
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